

On the Strategies for Disaggregating Between-Person Time Slope Effects from Within-Person Effects in Longitudinal Data

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Slides available at: <https://www.lesahoffman.com/Workshops/index.html>

Associations in Longitudinal Data

- Sampling multiple persons over multiple occasions creates at least **two distinct levels of analysis**:
- **Between-person** variation IN means over time
 - Are people higher on predictor x *than other people* also higher on outcome y *than other people*?
 - “**Level-2**” or “macro-level” relation among person means
- **Within-person** variation AROUND means over time
 - *When* a person is higher on predictor x *than usual*, are they also higher on outcome y *than usual*?
 - “**Level-1**” or “micro-level” relation among mean deviations
 - But what about within-person change over time?

Associations in Longitudinal Data

- **Presence of within-person (WP) change over time** requires new vocabulary and new modeling strategies
- e.g., **Long-term relations** of health (x) with cognition (y) in which there are WP effects of time in each variable
 - People who are healthier (*than other people at time 0*) may have better cognition → BP relation of intercepts (not “means”)
 - People whose health declines less over time (*than other people*) may decline less in cognition → BP relation of WP time slopes
 - When a person feels relatively better (*than predicted by their time trend*), they may then also have relatively better cognition
 - WP relation of time-specific residuals (whose extent can differ BP as well)
 - Feel better *next time* instead? WP “lagged” relation (that can differ BP)

Associations in Longitudinal Data

- “Change over time” includes **ALL kinds of time trends**, each of which can also show between-person (BP) variation
- e.g., **Short-term relations** of health (x) with bad mood (y)
 - People who tend to be less healthy (*than other people*) may tend to be grumpier than other people → BP relation of means
 - When people feel worse (*than usual*), they may also be grumpier (*than usual*) → WP relation of mean deviations
- How about a **Monday effect***? It needs a WP slope, too!
 - If some people are more adversely affected by Mondays (*than other people*), then that WP Monday slope can have BP variation!
 - People who feel relatively worse on Mondays (*than other people*) may also be grumpier on Mondays* → BP relation of time slopes

* Office Space movie “Case of the Mondays” <https://www.youtube.com/watch?v=2AB9zPfXqQQ>

Associations in Longitudinal Data

- No matter the time scale, any variable measured over time has the potential for **three distinct sources of variation**:
 - **BP** in some measure of overall level (mean or intercept)
 - **BP** differences in WP slopes for time and time-varying predictors
 - **WP** time-specific deviations from BP-predicted trajectory
- In theory each source can relate to those of other variables, but **common practice has two common problems**:
 - Time-varying “outcomes” are treated differently than “predictors”
 - “Time” may not be considered as sufficiently in short-term studies
- Result? **Missing BP time slope relations will create bias!**
 - Today's demo: In WP slope main effects and lagged effects

Presentation Overview

- Introduce **simulation**: data generation and manipulations
- Show **recovery results** across different types of longitudinal models for distinguishing BP and WP sources of variance
 - Try to link ideas, buzz words, diagrams, and equations to show what each type of model can or cannot do (well), including:
 - **Univariate models with observed predictors**—using person-mean-centered, baseline-centered, or time-detrended predictors
 - **Multivariate models with latent predictors**—requiring single-level or multilevel structural equation models with “latent” change factors
 - Auto-regressive cross-lag panel models for lagged effects
- Consider **best practice** in light of real-data complications
 - e.g., Unbalanced occasions, small samples, model complexity

Simulation Data Generation

- **2 variables** (x and y) with no missing data for **100 persons** (Level 2; i) over **5 occasions** (Level 1; t), indexed as $time = (0,1,2,3,4)^*$

	Unconditional Model for Change		Variances
Level 1 Occasions:	$x_{tix} = \beta_{0ix} + \beta_{1ix}(Time_{tix}) + e_{tix}$ $y_{tiy} = \beta_{0iy} + \beta_{1iy}(Time_{tiy}) + e_{tiy}$		$\sigma_{e_x}^2 = .40$ $\sigma_{e_y}^2 = .40$
Level 2 Intercepts:	$\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + U_{0iy}$	$\gamma_{00x} = 0$ $\gamma_{00y} = 0$	$\tau_{U_{0x}}^2 = .60$ $\tau_{U_{0y}}^2 = .60$
Level 2 Time Slopes:	$\beta_{1ix} = \gamma_{10x} + U_{1ix}$ $\beta_{1iy} = \gamma_{10y} + U_{1iy}$	$\gamma_{10x} = ?$ $\gamma_{10y} = ?$	$\tau_{U_{1x}}^2 = .06$ $\tau_{U_{1y}}^2 = .06$

- Total variance set = 1 at $time = 0$, so that:
 - Conditional ICC = .60 → Intercept variance for U_{0ix} and U_{0iy}
 - Slope Reliability = .60 → Time slope variance for U_{1ix} and U_{1iy}

Simulation Manipulations

- Fixed time effects (γ_{10} absent or present) collapsed here
 - Didn't matter because $time_{ti}$ was always a predictor of y_{ti}
- Key manipulation: **match across 3 types of relationships**
- Level-2 random effects (U_{0ix} , U_{0iy} , U_{1ix} , U_{1iy}) drawn from a multivariate normal distribution with 4 conditions:
 - **Intercept correlations:** $r(U_{0ix}, U_{0iy}) = 0 \text{ or } .3$
 - **Time slope correlations:** $r(U_{1ix}, U_{1iy}) = 0 \text{ or } .3$
 - All other Intercept–Time slope pairs of correlations = 0
- Level-1 **residuals** drawn from a separate multivariate normal distribution with 2 conditions: $r(e_{tix}, e_{tiy}) = 0 \text{ or } .3$

2 Longitudinal Modeling Families

- Univariate models: predict y_{ti} from **observed** x_{ti} **predictors**
 - *aka*, Multilevel models (MLMs) using person-mean-centered, baseline-centered, or detrended-residual predictors
 - Estimated in any software with mixed effects (e.g., MIXED in SAS, SPSS, or STATA; LME4 or NLME in R environment)
- Multivariate models: predict both y_{ti} and x_{ti} as **outcomes**
 - But x_{ti} can't predict y_{ti} in univariate mixed-effects software, so...
 - Can be specified as a single-level structural equation model (SEM)
 - e.g., "Multivariate latent growth curve models" (with or without "structured residuals"); "auto-regressive cross-lag panel models"
 - Can also be specified as a "multilevel SEM" (= multivariate MLM)
 - I will use ML estimation; *Mplus* "latent predictor centering" and lagged effects within "dynamic multilevel SEM" require Bayes MCMC instead

Unconditional Time Model for y_{ti} : 3 Ways

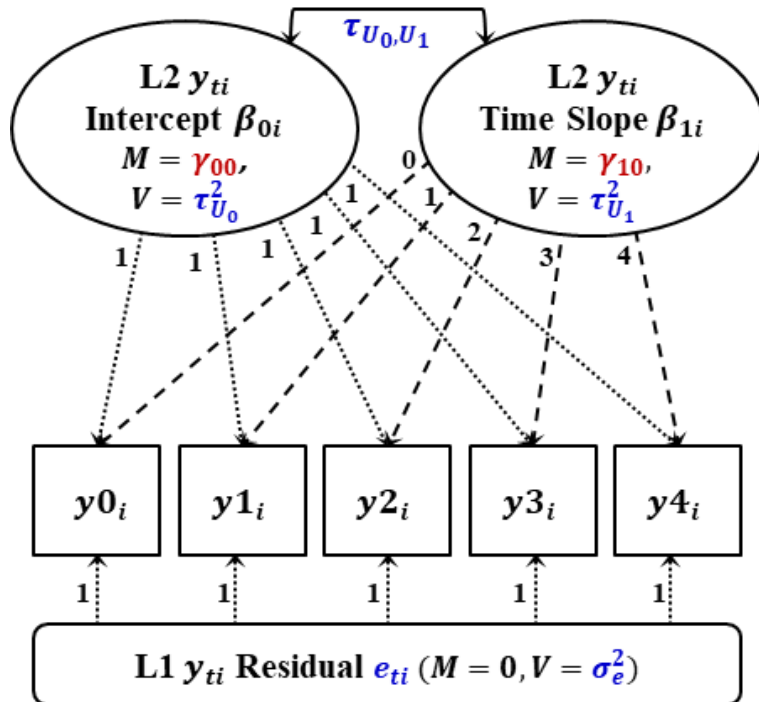
Unconditional Time Univariate Multilevel Model (long data)

L1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + e_{ti}$

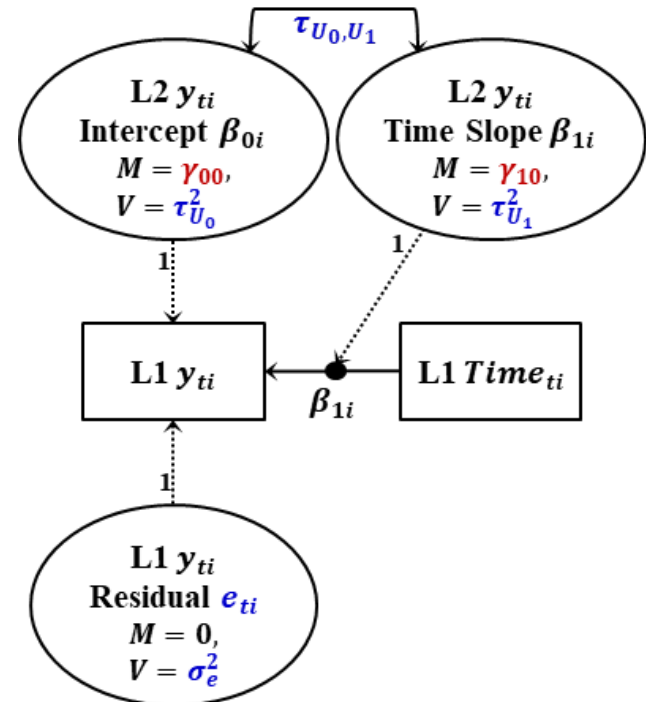
L2 Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

L2 Time Slope: $\beta_{1i} = \gamma_{10} + U_{1i}$

As Single-Level SEM* (wide data)



As Multilevel SEM* (long data)



* MLM = SEM because random effects = latent variables!

Naïve Addition of Time-Varying x_{ti}

Univariate MLM: TV x_{ti} has a Smushed Effect
(aka conflated, convergence, composite effect)

$$\text{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + e_{ti} \\ + \beta_{2i}(x_{ti})$$

$$\text{L2 Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

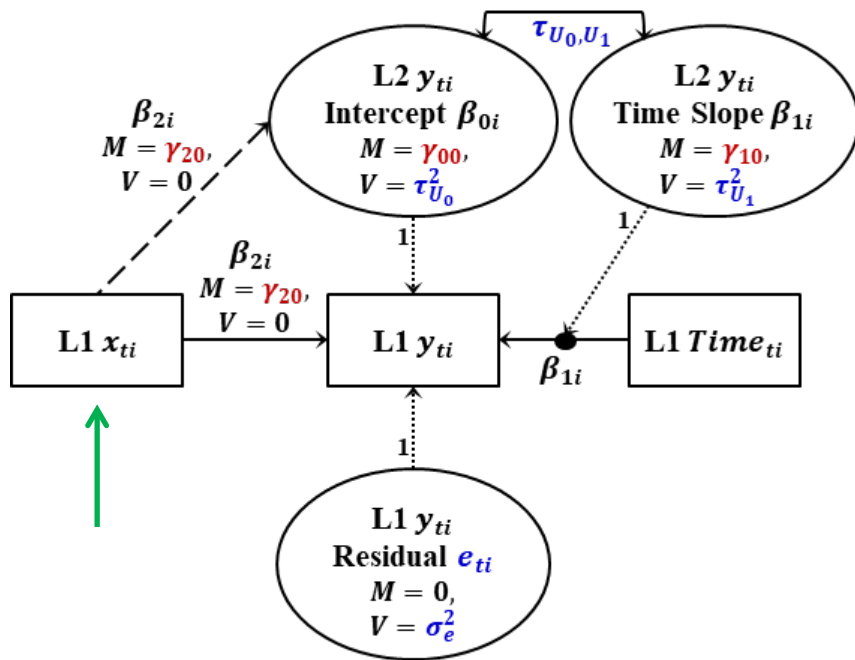
$$\text{L2 Time Slope: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{L2 } x_{ti} \text{ Slope: } \beta_{2i} = \gamma_{20}$$

- Model is **bad news** if the L1 predictor has L2 variance (i.e., people differ in their mean of x_{ti} over time)
 - Could also be true for the L1 time_{ti} predictor! (but not here)
- **Forces** level-1 (**WP**) and level-2 (**BP**) x_{ti} effects **to be equal**, which is unlikely to be true, especially in longitudinal data!
- A predictor for x_{ti} is needed at any level it has variability

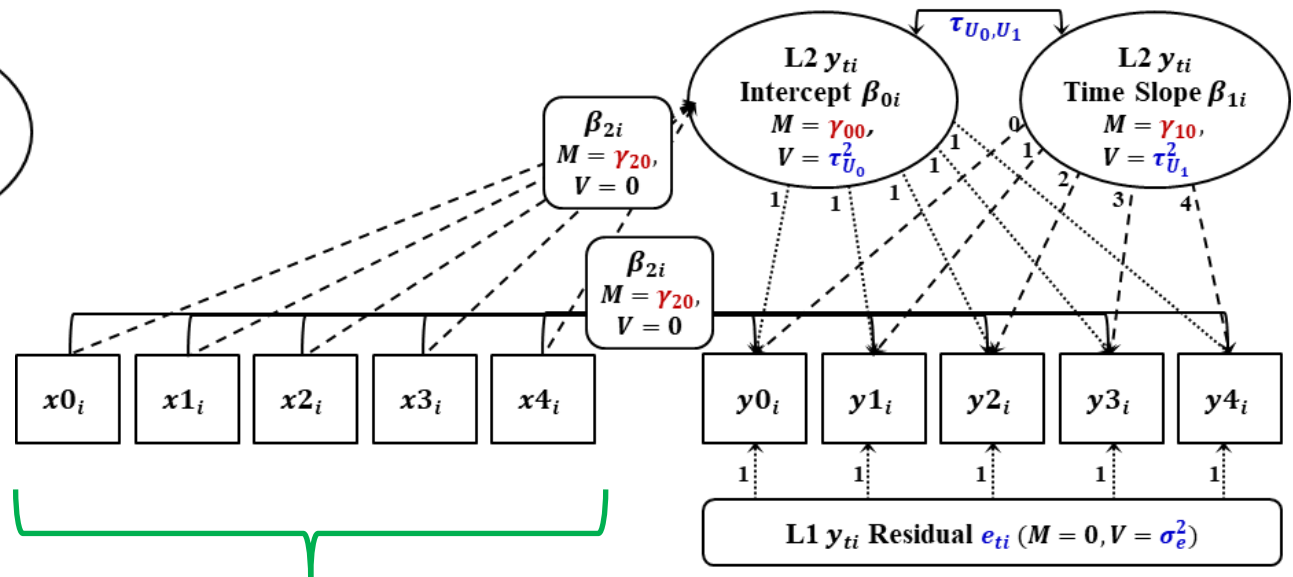
Naïve Addition of Time-Varying x_{ti}

As Multilevel SEM (long data)



Smushed Effect:
L1 (WP) and L2 (BP) effects of x_{ti} are forced to be equal (both γ_{20})

As Single-Level SEM (wide data)



Unsmushing the Effects of L1 x_{ti}

Univariate MLMs to Distinguish L2 BP and L1 WP Effects of x_{ti}

L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$

L2 Time Slope: $\beta_{1i} = \gamma_{10} + U_{1i}$

L2 x_{ti} Slope: $\beta_{2i} = \gamma_{20}$

**Person-Mean
(PM) Centering:**

$+ \beta_{2i}(x_{ti} - \bar{x}_i)$

L2 Int: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\bar{x}_i) + U_{0i}$

**Baseline
(BL) Centering:**

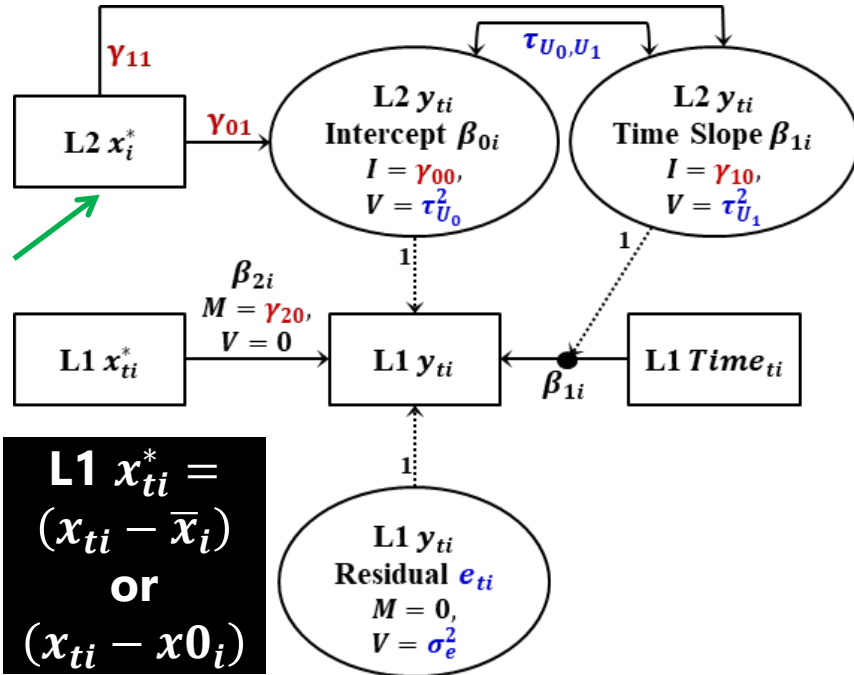
$+ \beta_{2i}(x_{ti} - x_{0i})$

L2 Int: $\beta_{0i} = \gamma_{00} + \gamma_{01}(x_{0i}) + U_{0i}$

- Either way, should be: L1 $\gamma_{20} \rightarrow$ **WP** effect; L2 $\gamma_{01} \rightarrow$ **BP** effect
- L2 PM (\bar{x}_i) uses all occasions so L1 errors should cancel...
 - ...But timing is off: x_{ti} L2 *average* predicts y_{ti} L2 intercept for *time 0*
- L2 BL (x_{0i}) matches timing to create L2 relation at *time 0*...
 - ...But still contains L1 error (is *actual* x_{0i} , not *predicted* x_{ti} at time 0)

Unsmushing the Effects of L1 x_{ti}

As Multilevel SEM (long data)



L1 $x_{ti}^* = (x_{ti} - \bar{x}_i)$ or $(x_{ti} - x_{0i})$

L2 x_i^* Model Variants:

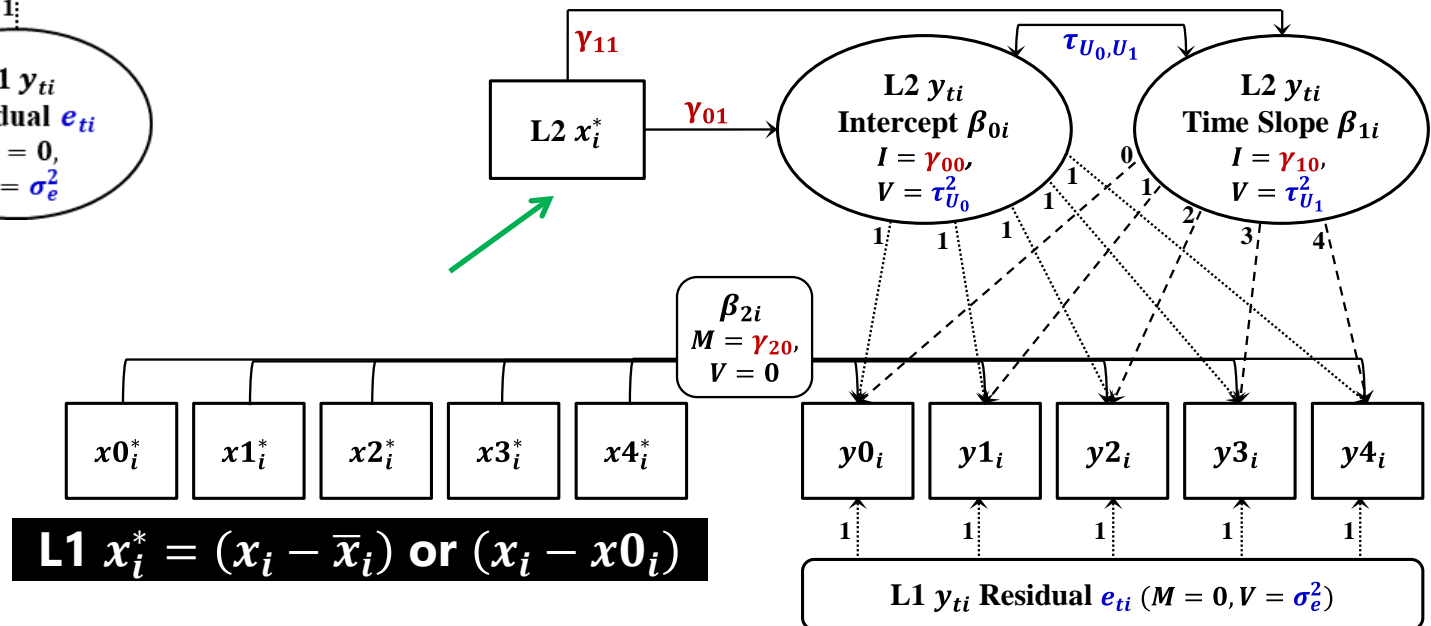
Person-Mean Centering uses \bar{x}_i

Baseline Centering uses x_{0i}

L2 x_i^* by $Time_{ti}$ slope γ_{11} added for comparability with next models:

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(x_i^*) + U_{1i}$$

As Single-Level SEM (wide data)



L1 $x_i^* = (x_i - \bar{x}_i)$ or $(x_i - x_{0i})$

Simulation Results: Univ MLMs

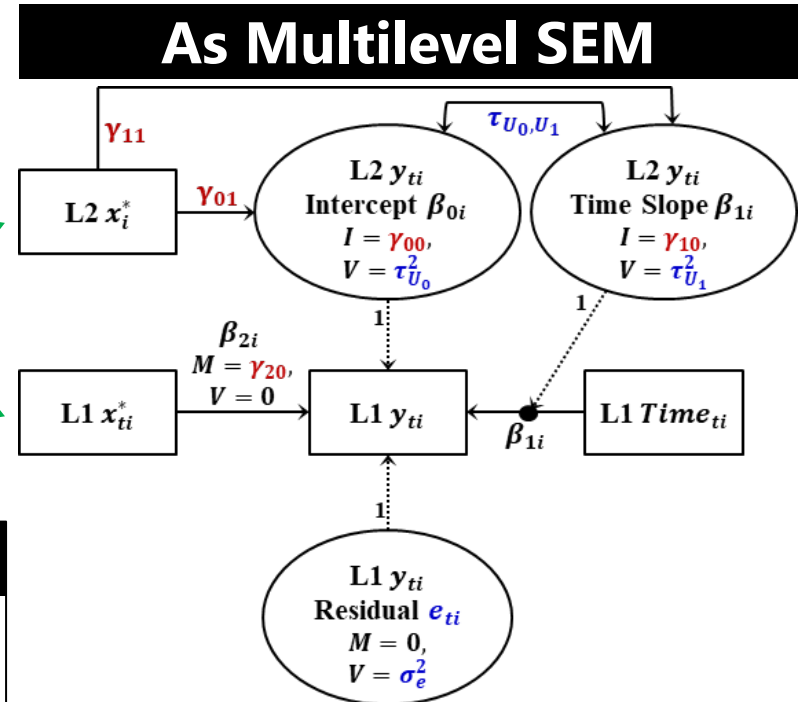
- How well did centering with the person mean (\bar{x}_i) or baseline ($x0_i$) **recover the 3 relations of x_{ti} with y_{ti}** ?

➤ L2 x_i^* by $Time_{ti}$ slope $\gamma_{11} = 0$

1. L2 time slope missing (so ≈ 0)

2. L2 intercept \rightarrow L2 x_i^* slope γ_{01}

3. L1 residual \rightarrow L1 x_{ti}^* slope γ_{20}



As Univariate MLM

L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + \beta_{2i}(x_{ti}^*) + e_{ti}$

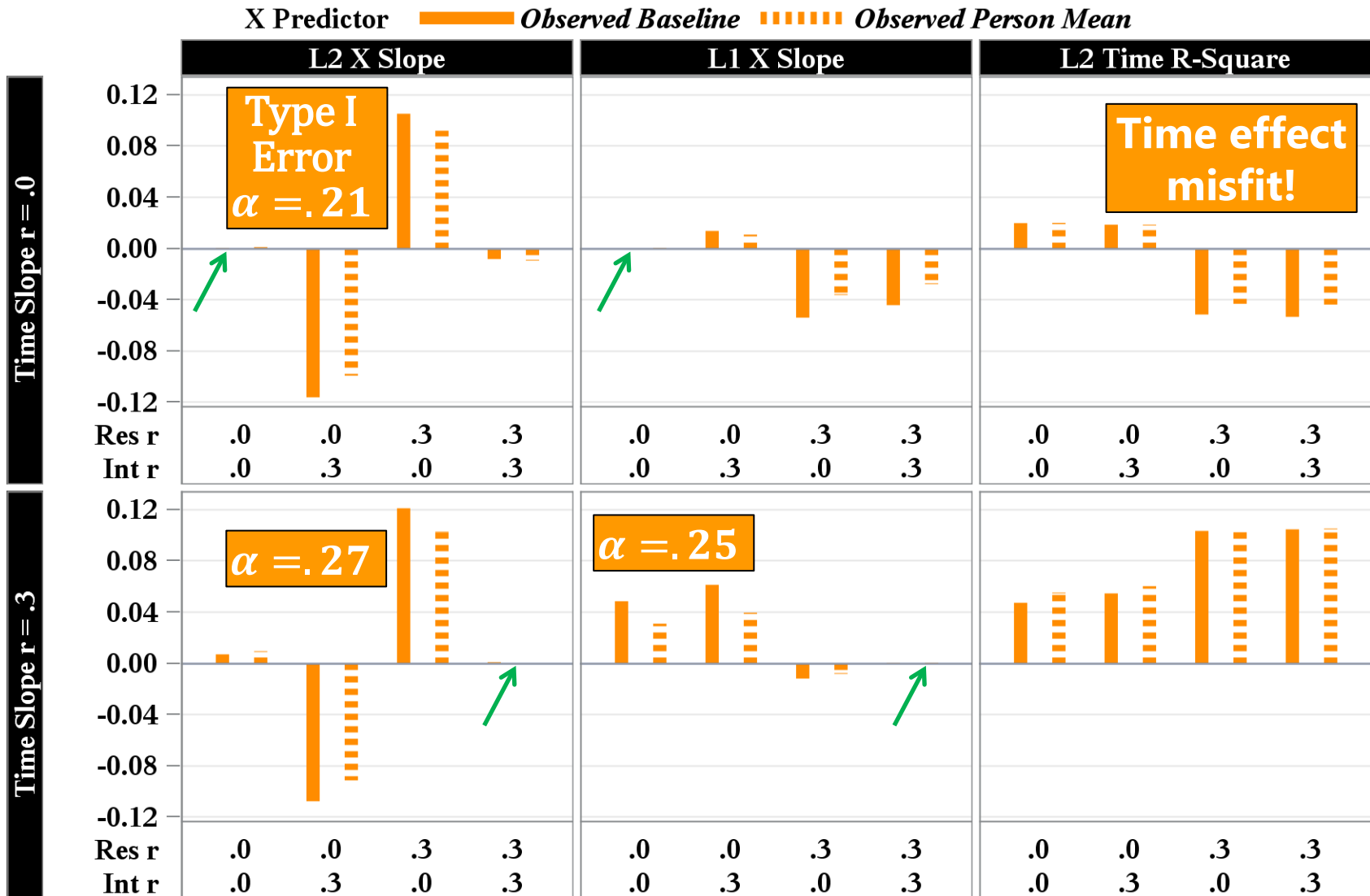
L2 Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(x_i^*) + U_{0i}$

L2 Time: $\beta_{1i} = \gamma_{10} + \gamma_{11}(x_i^*) + U_{1i}$

L2 x_{ti} Slope: $\beta_{2i} = \gamma_{20}$

L2 $x_i^* = \bar{x}_i$ or $x0_i$
L1 $x_{ti}^* = (x_{ti} - \bar{x}_i)$
or $(x_{ti} - x0_i)$

Univ Results: Time-Smushing Bias!



Fixing Level-1 Bias... Univariately

- “**Detrended residuals**” is a strategy designed to remove time-related variance from the level-1 x_{ti} predictor
- Is a two-stage approach also known as “slopes-as-outcomes”:
 - Fit **separate regression model** to each person’s data
 - Save time-specific x_{ti} **residuals** to use as **level-1** x_{ti}^*
 - Save **fixed intercept at time = 0** to use as **level-2** x_i^*

As Univariate MLM

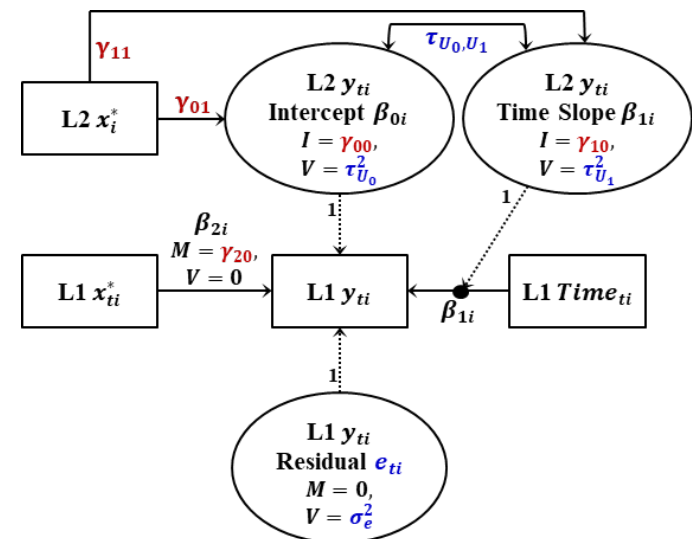
$$\text{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(x_{ti}^*) + e_{ti}$$

$$\text{L2 Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}(x_i^*) + U_{0i}$$

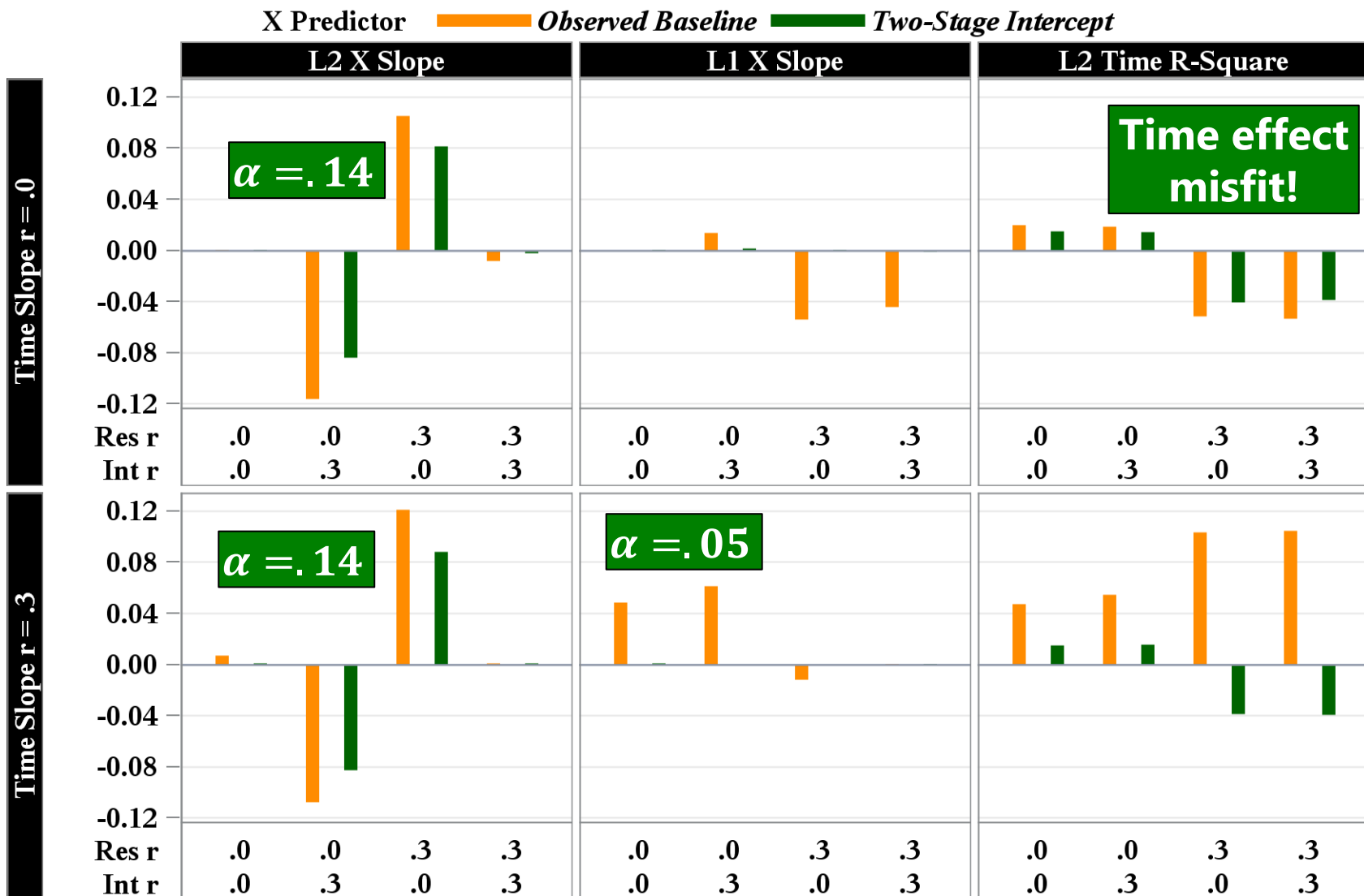
$$\text{L2 Time: } \beta_{1i} = \gamma_{10} + \gamma_{11}(x_i^*) + U_{1i}$$

$$\text{L2 } x_{ti} \text{ Slope: } \beta_{2i} = \gamma_{20}$$

As Multilevel SEM



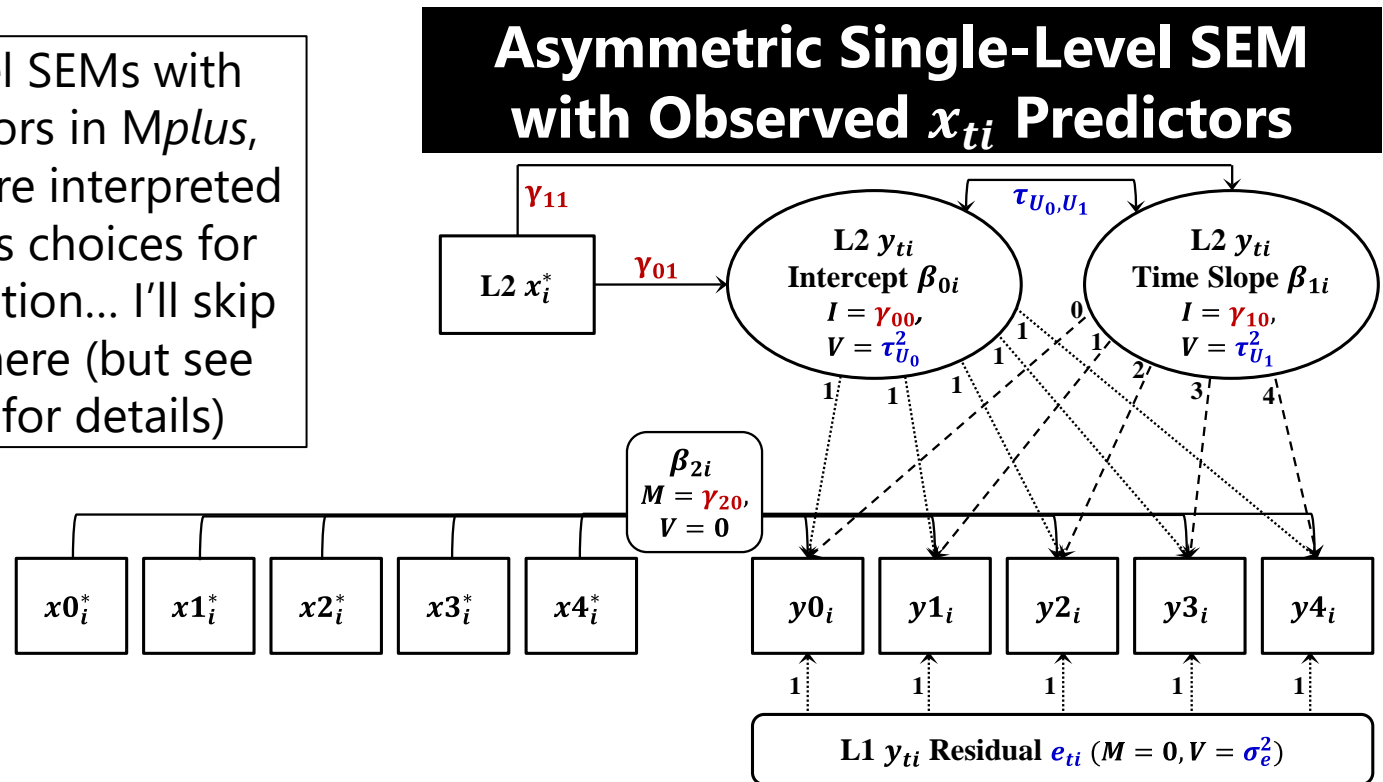
Univ Results: A Partial Fix



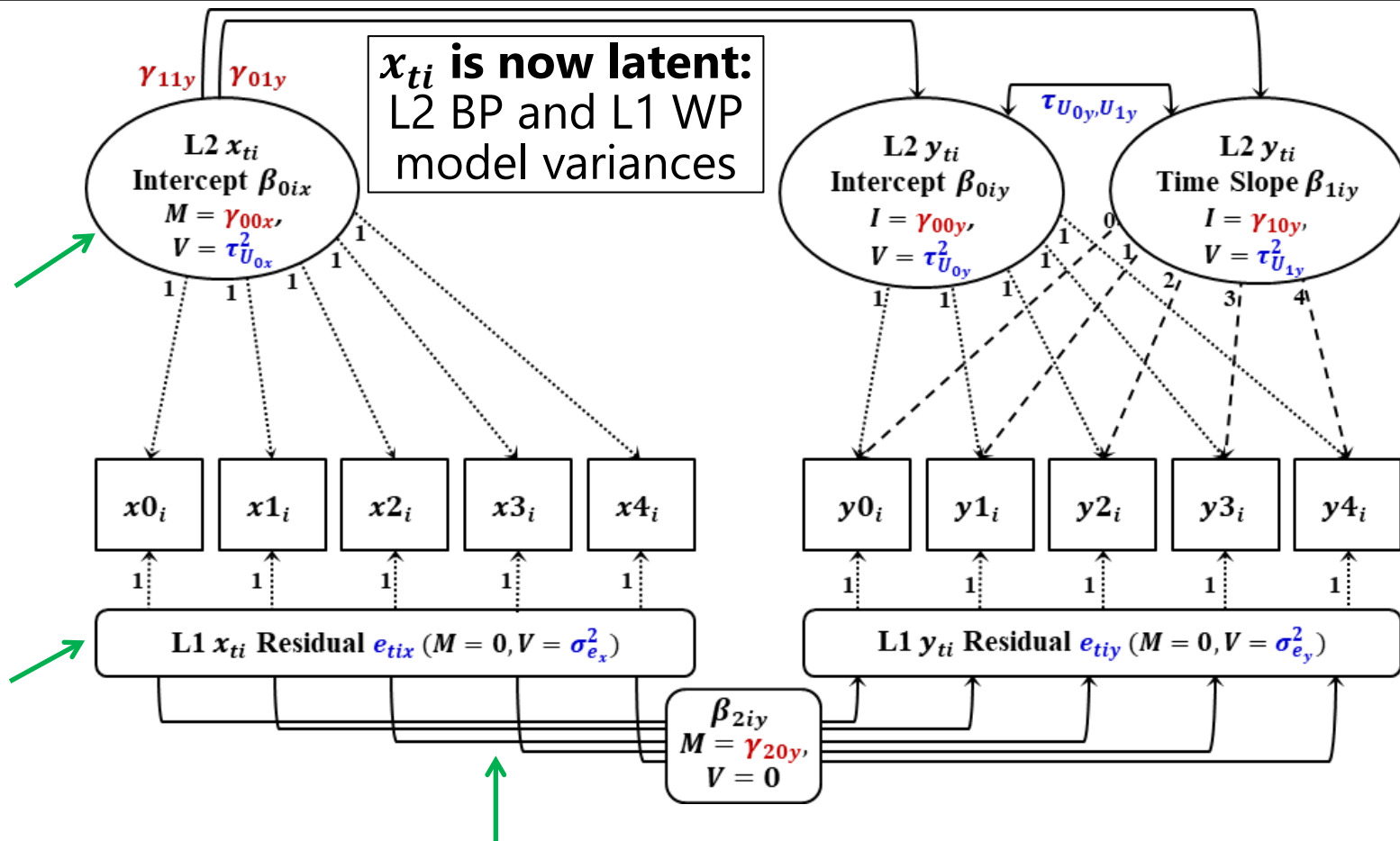
Why the asymmetry of x_{ti} and y_{ti} ?

- Why is y_{ti} **treated as latent** (i.e., three sources of variance partitioned by the model; in circles) **while x_{ti} is observed** (variance partitioned by brute-force predictors; in squares)?
- Primary benefit of multivariate models is to treat x_{ti} like y_{ti} *but still be able to include **fixed effects of x_{ti} that predict y_{ti}***

Btw: in multilevel SEMs with latent x_{ti} predictors in Mplus, how parameters are interpreted depends on one's choices for syntax and estimation... I'll skip this complexity here (but see Hoffman (2019 for details)



Symmetric Single-Level SEM



- This SEM uses "**Structured Residuals**": Level-1 x_{ti} effect between the x_{ti} and y_{ti} *residuals* (instead of between the observed variables)
 - Why? To get **level-2 BP effects** instead of level-2 *contextual* effects

Same Symmetric Single-Level SEM as a “Truly” Multivariate MLM

Fixed Effects of Intercept and Residual of Latent x_{ti}

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$
 $y_{tix} = \beta_{0iy} + yw_{tiy}$

w indicates a **L1** *within* variable

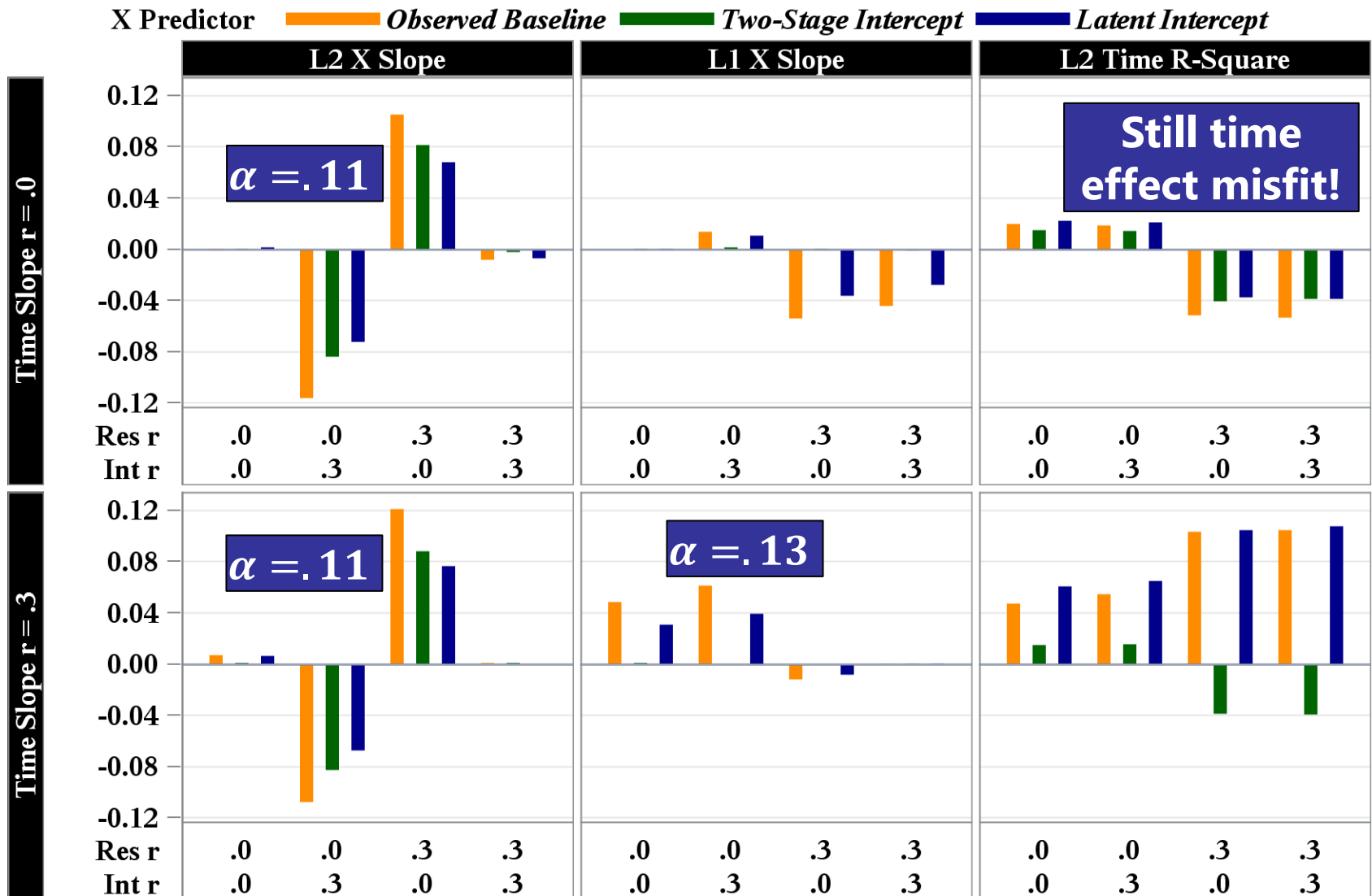
L1: $xw_{tix} = e_{tix}$
 $yw_{tiy} = \beta_{1iy}(\text{Time}_{tiy}) + \beta_{2iy}(xw_{tix}) + e_{tiy}$ $\beta_{2iy} = \gamma_{20y}$

L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$
 $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + U_{0iy}$

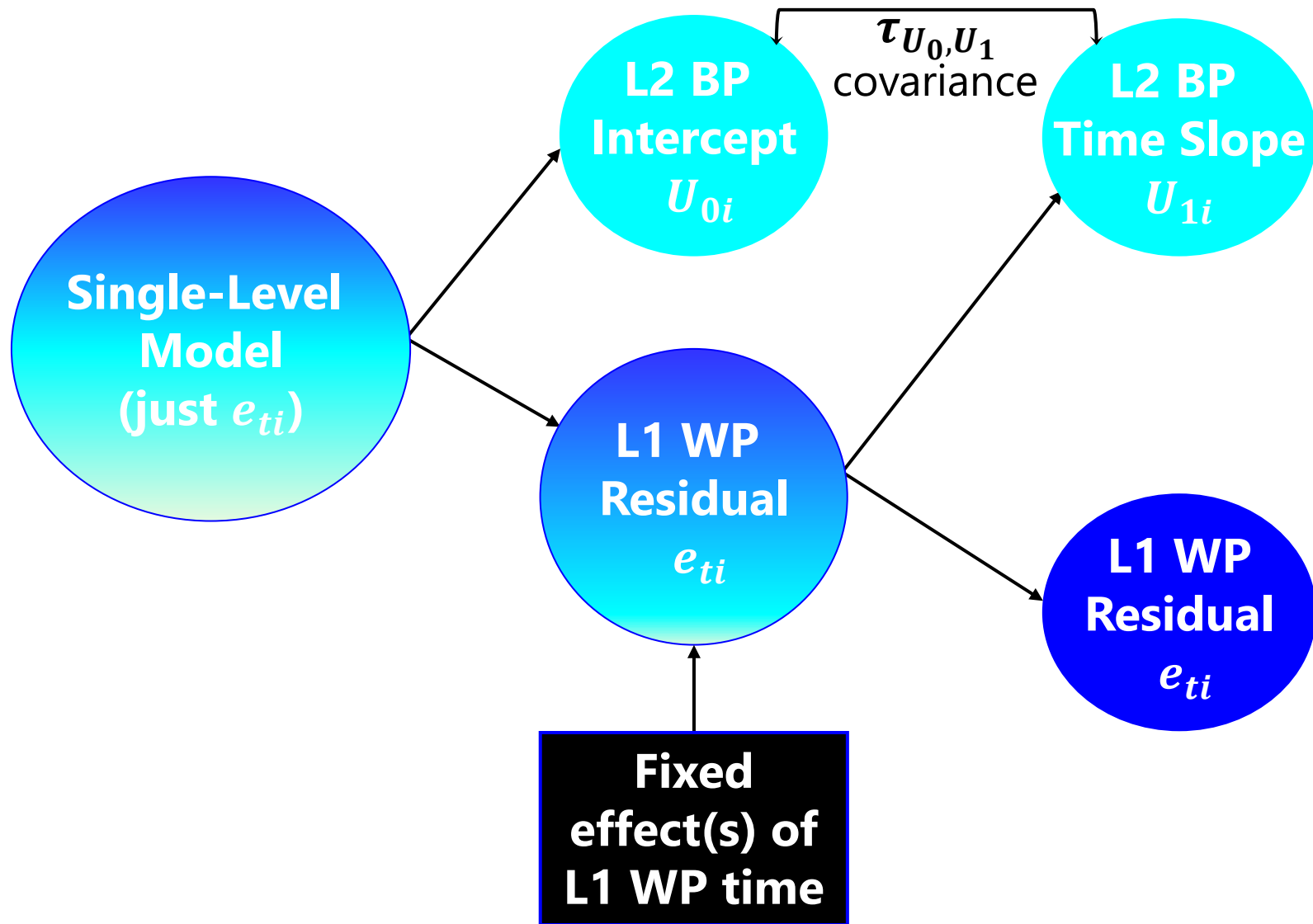
L2 Time Slopes: $\beta_{1iy} = \gamma_{10y} + \gamma_{11y}(\beta_{0ix}) + U_{1iy}$

- So how does using **latent x_{ti} predictors** compare with **observed x_{ti} predictors** (baseline or two-stage intercept)?

Latent x_{ti} → Less Bias? Not yet...



The Source of the Problem



A “Truly” Multivariate MLM with Random Time Slopes Predicting x_{ti}

Fixed Effects of Intercept, Time Slope, and Residual of Latent x_{ti}

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$
 $y_{tix} = \beta_{0iy} + yw_{tiy}$

w indicates a **L1** within variable

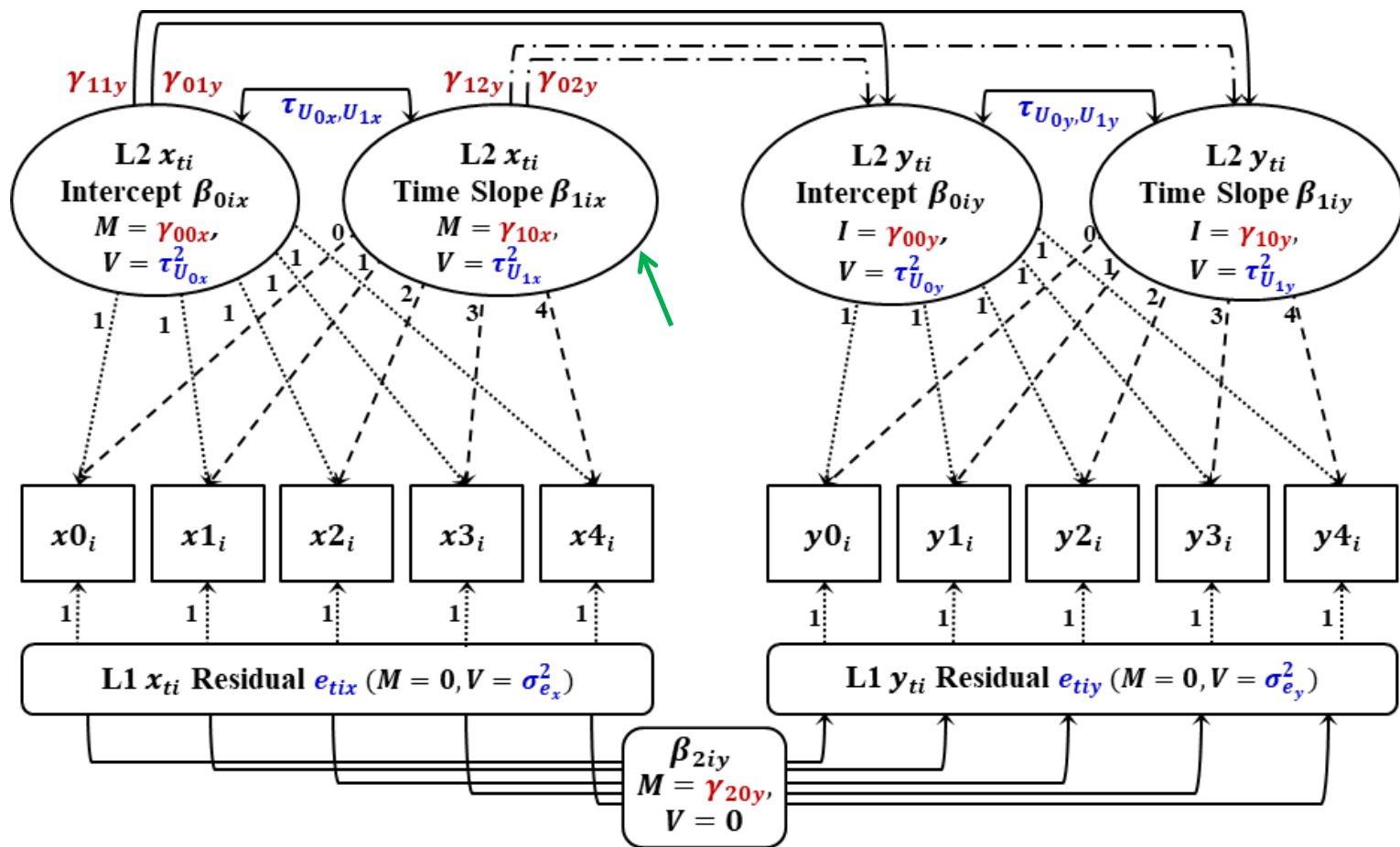
L1: $xw_{tix} = \beta_{1ix}(\text{Time}_{tix}) + e_{tix}$
 $yw_{tiy} = \beta_{1iy}(\text{Time}_{tiy}) + \beta_{2iy}(xw_{tix}) + e_{tiy}$

$\beta_{2iy} = \gamma_{20y}$

L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$
 $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + \gamma_{02y}(\beta_{1ix}) + U_{0iy}$

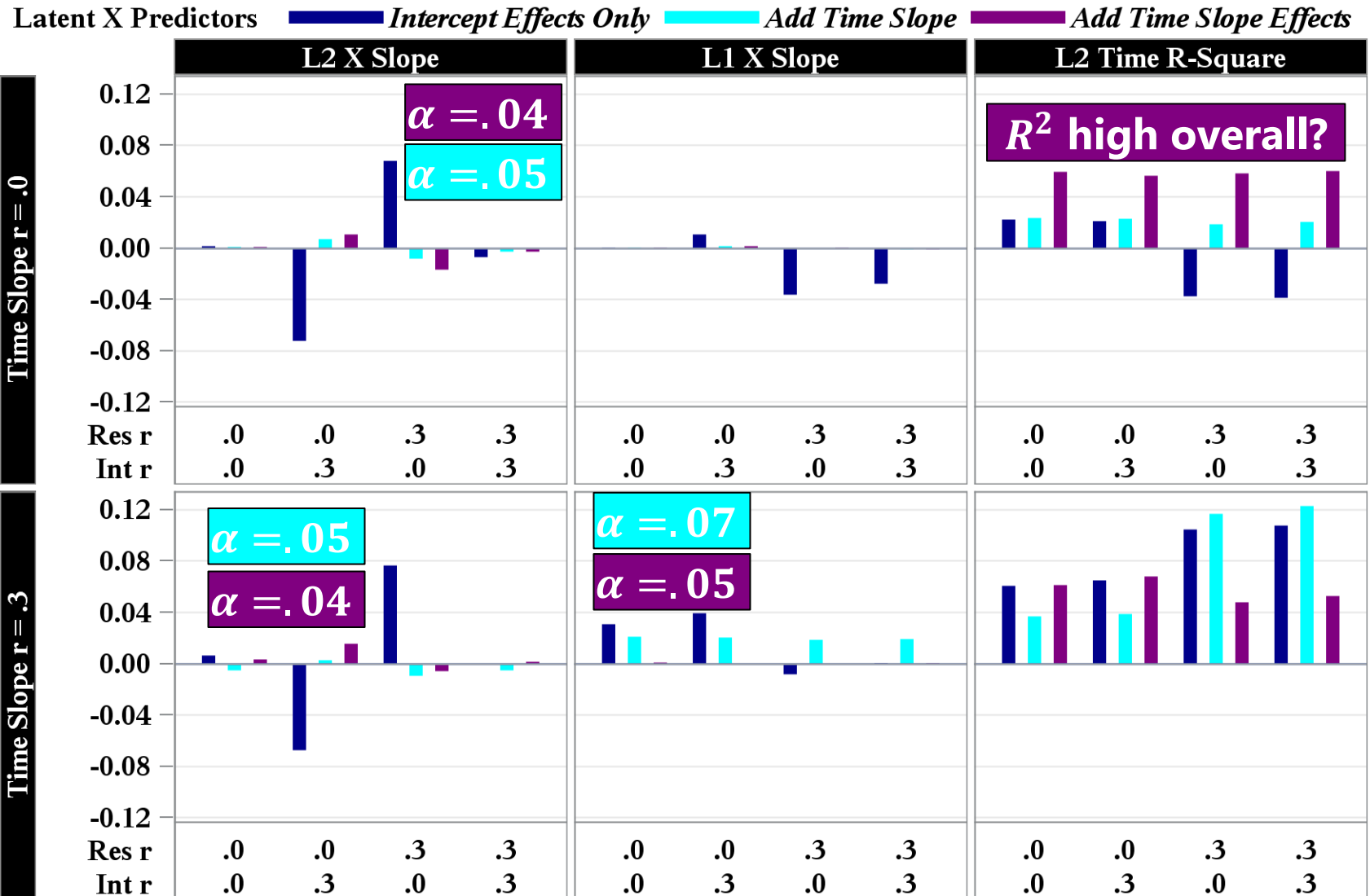
L2 Time Slopes: $\beta_{1ix} = \gamma_{10y} + U_{1iy}$
 $\beta_{1iy} = \gamma_{10y} + \gamma_{11y}(\beta_{0ix}) + \gamma_{12y}(\beta_{1ix}) + U_{1iy}$

So Let $Time_{ti}$ Also Predict x_{ti}

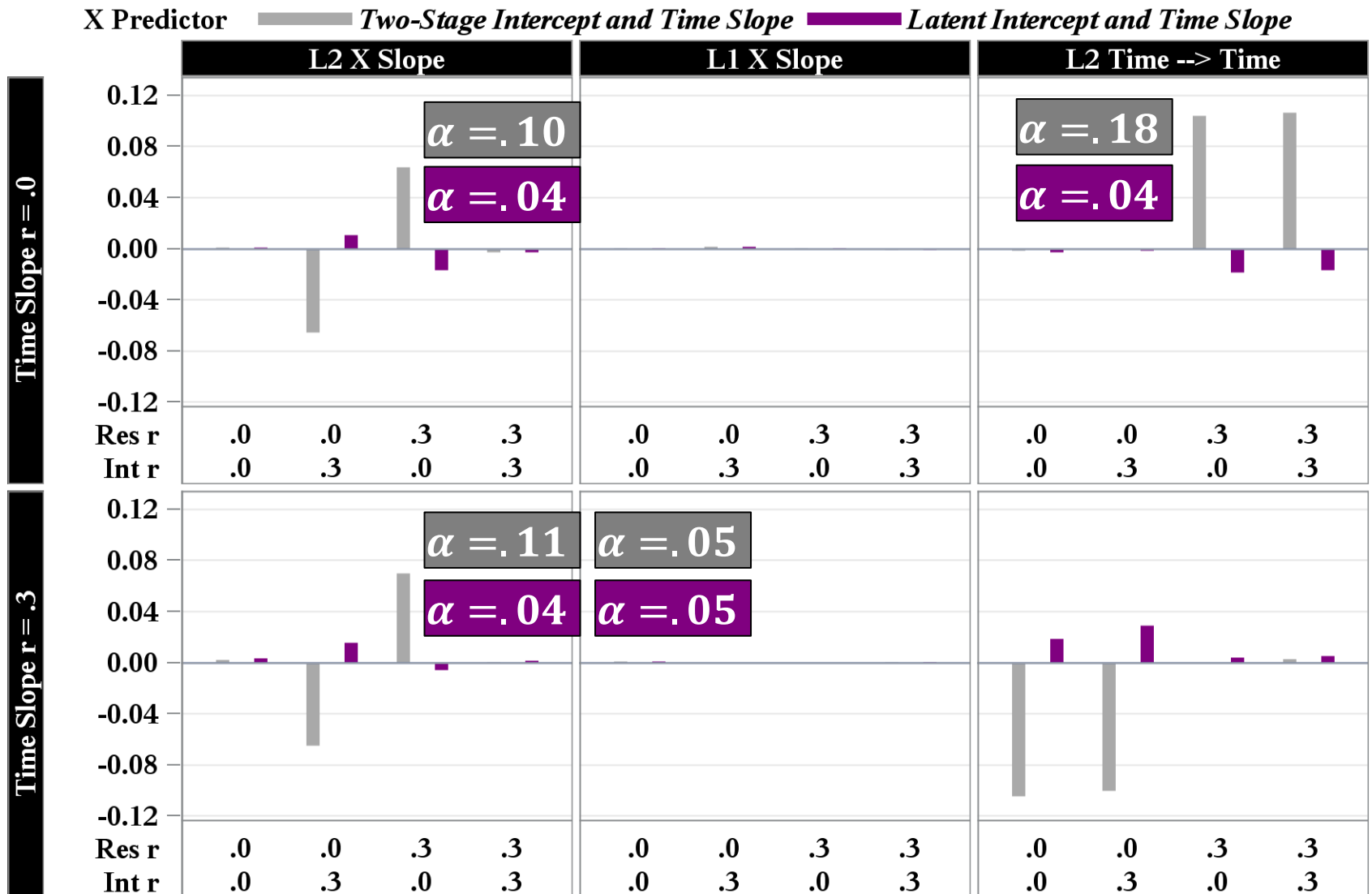


- L2 Intercept β_{0ix} is now specific to $time = 0$ (just like β_{0iy} has been)
- How well does this "multivariate latent growth curve model with structured residuals" recover the **3 types of relations of x_{ti} with y_{ti}** ?

Results: Better! (But Not Perfect)

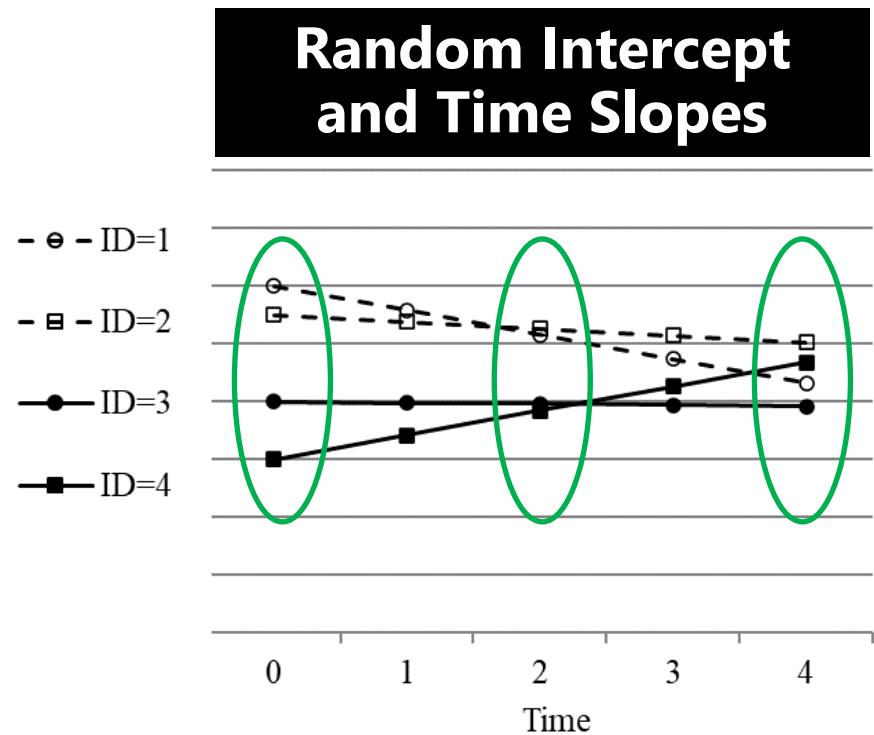
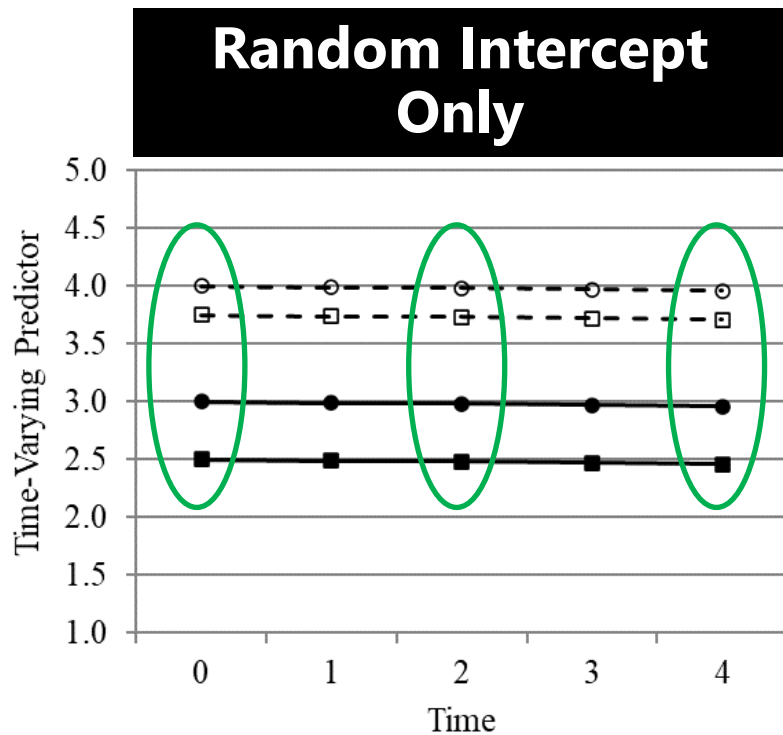


Slopes-as-Outcomes? Still Nope.



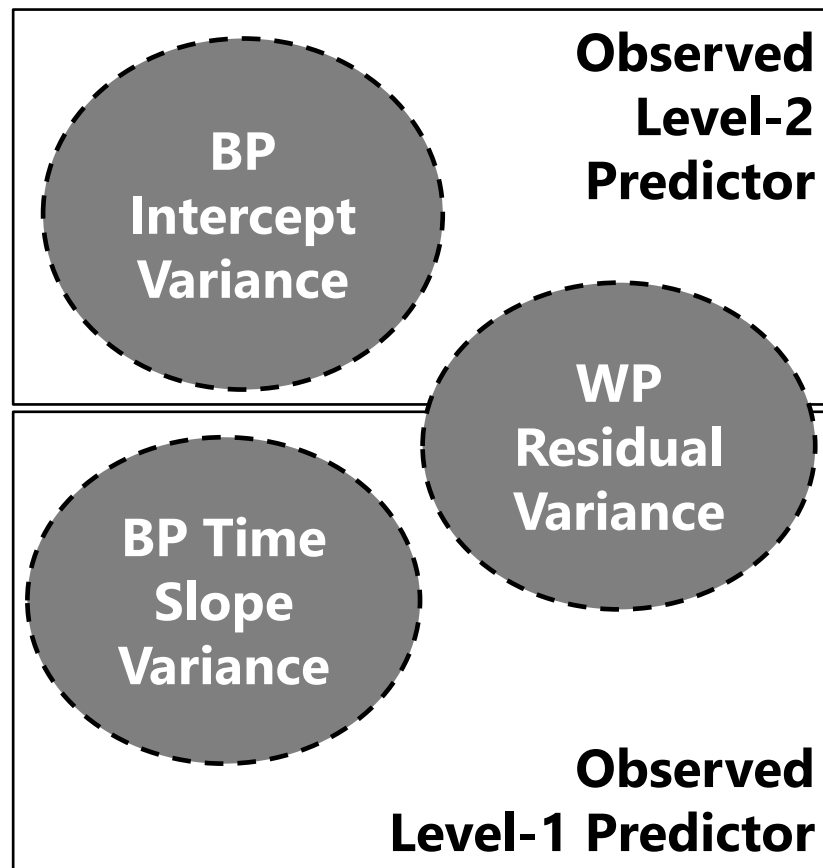
Summary: Part 1

- **Ignoring relationships between the BP Time Slopes** of longitudinal variables can contaminate their other relations:
 - Such as in the **BP Intercept**—because it must change over time!



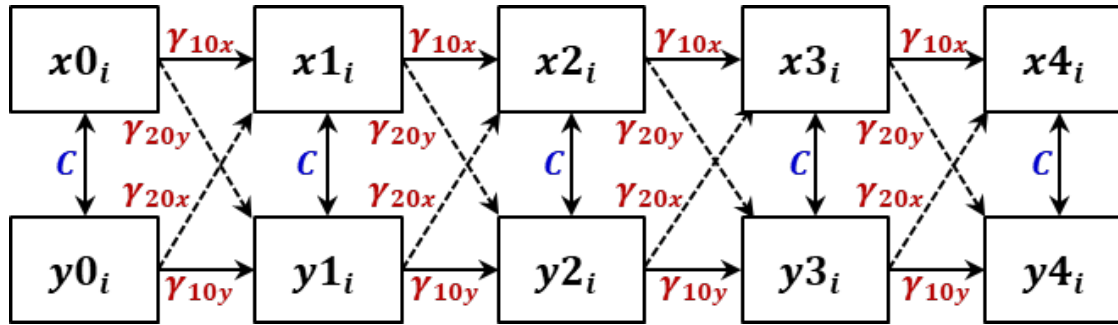
Summary: Part 1

- **Ignoring relationships between the BP Time Slopes** of longitudinal variables can contaminate their other relations:
 - If the **WP Residual** still contains the unmodeled BP time slope variance, **the level-1 effect will be smushed with the missing L2 time slope effect!** (bottom panel)
 - Different problem than more well-known result of **intercept-smushed L1 effects** (top panel)



Smushed Effects in Other Models*

Auto-Regressive Cross-Lag Panel Model (the "ARCL" or "CLPM")



Path model with separate intercepts (and residual variances) per occasion, and lag-1 fixed effects:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + e_{tix}$$

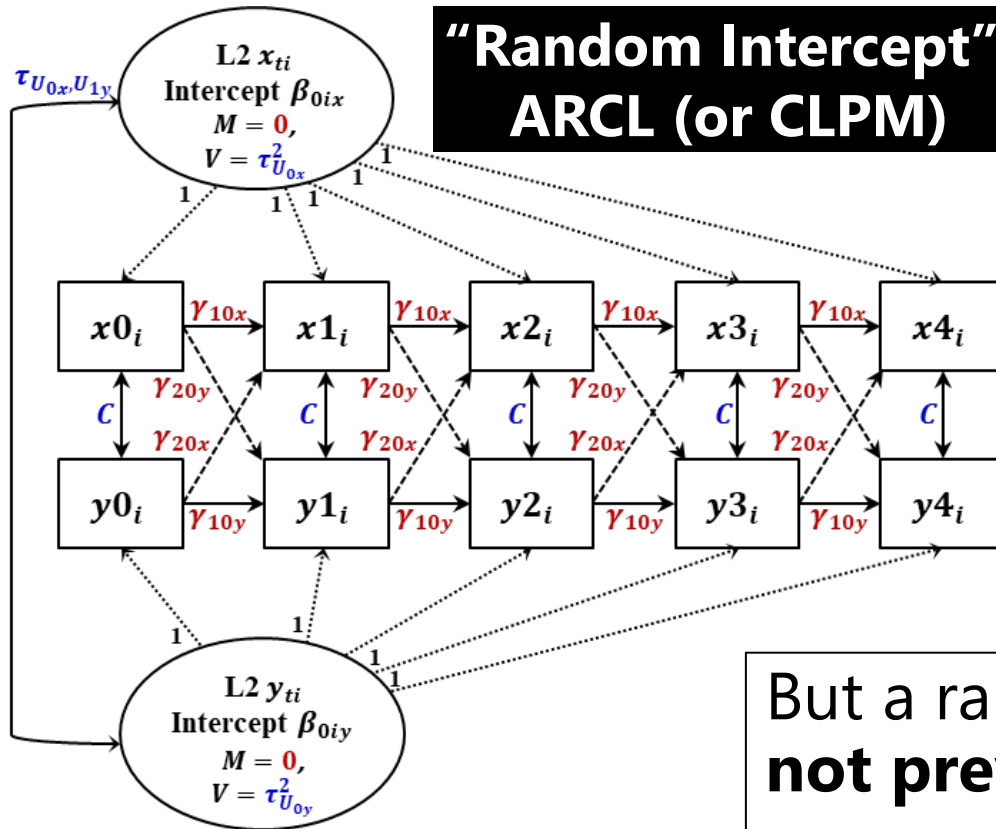
$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + e_{tiy}$$

- **ARCL model interpretation is problematic** (at best):
 - Do the γ_{10} within-variable **AR paths** really "control for time"?
 - Which type of relationship is given by the γ_{20} **cross-lag paths**?
 - Which type of relationship is the **same-occasion C covariance**?

* Same problems apply to mediation variants ($X \rightarrow M \rightarrow Y$)

Remedies for Intercept Smushing

**"Random Intercept"
ARCL (or CLPM)**



Several authors have pointed out the need to distinguish constant BP effects from WP effects via:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + \boxed{U_{0ix}} + e_{tix}$$

$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + \boxed{U_{0iy}} + e_{tiy}$$

But a random intercept alone will **not prevent time-smushing...**

Do the **within-variable AR paths** protect against time smushing?

Let's find out!

Simulation: ARCL Model for x_{ti}

Full Model: L2 Latent Intercept and Time Slope Effects, L1 Within-Variable AR Paths, and L1 Cross-Lag Paths

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$
 $y_{tix} = \beta_{0iy} + yw_{tiy}$

w indicates a **L1** *within* variable

L1: $xw_{tix} = \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + \beta_{3ix}(\text{Time}_{tix}) + e_{tix}$
 $yw_{tiy} = \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + \beta_{3iy}(\text{Time}_{tiy}) + e_{tiy}$

L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$
 $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + U_{0iy}$

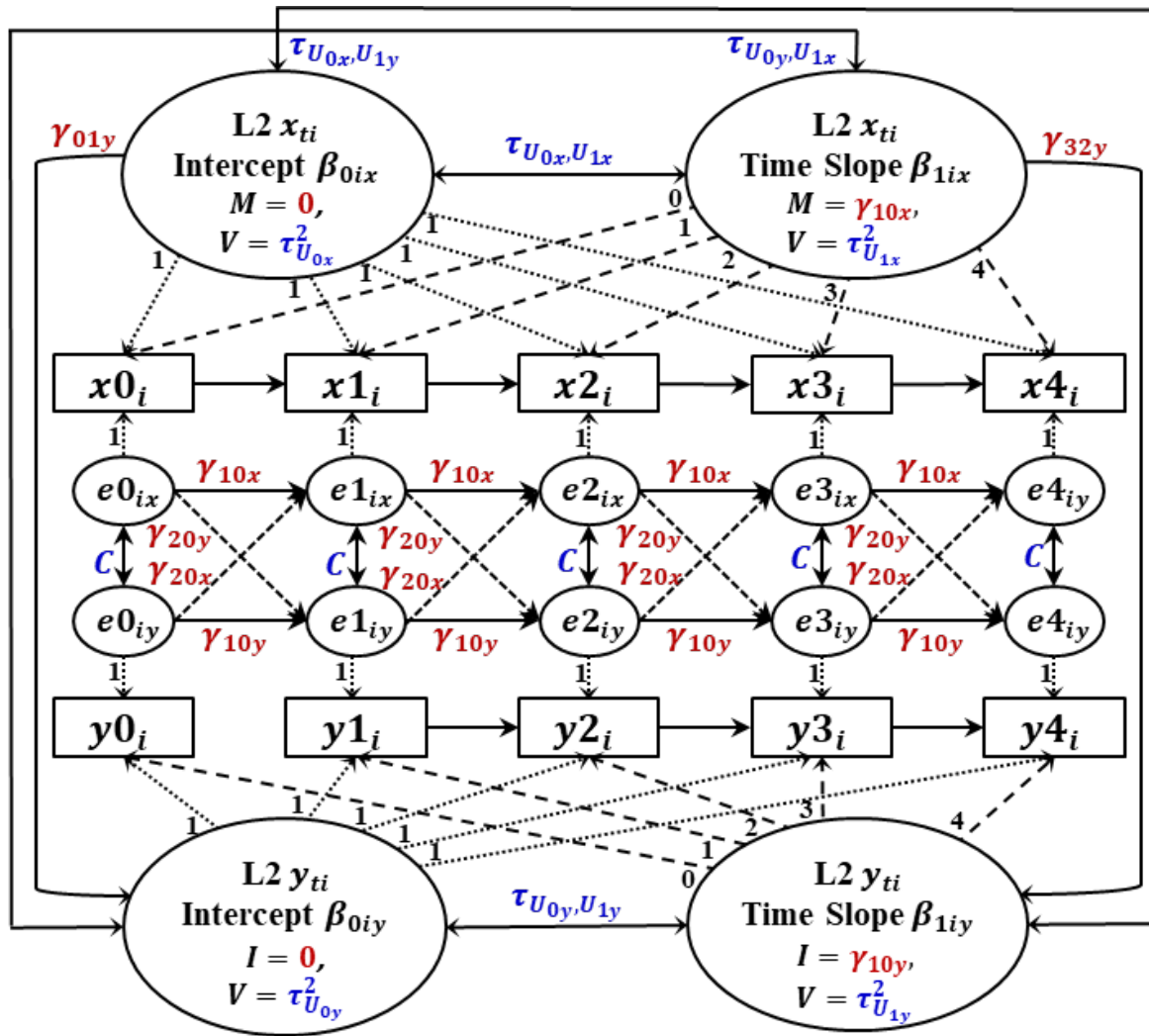
Intercept → Intercept

L2 Time Slopes: $\beta_{3ix} = \gamma_{30y} + U_{3iy}$
 $\beta_{3iy} = \gamma_{30y} + \gamma_{32y}(\beta_{1ix}) + U_{3iy}$

Time slope → Time slope

- All L1 AR paths and cross-lag paths had pop values = 0
- Also estimated (but with Pop=0): all L2 Intercept–Time slope covariances; all L1 lag-0 residual covariances (equal over time)

Comparison ARCL Models for x_{ti}



← Full Model:
L2 Latent Intercept*
and Time Slope Effects,
L1 AR Paths, and
L1 Cross-Lag Paths*

Drop Time Slope effect

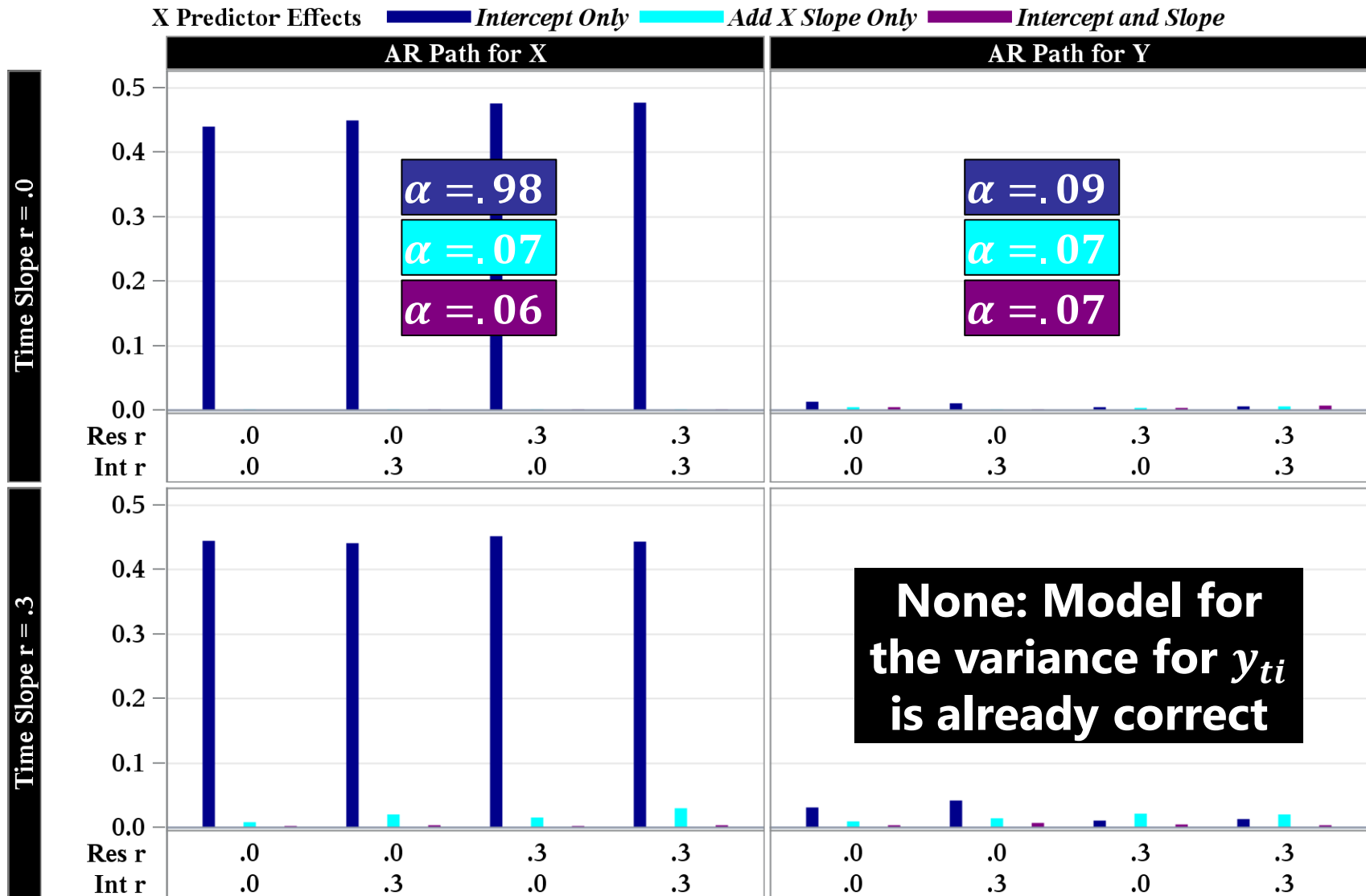
Drop Time Slope, too

Drop Time Slope effect;
drop L1 AR paths

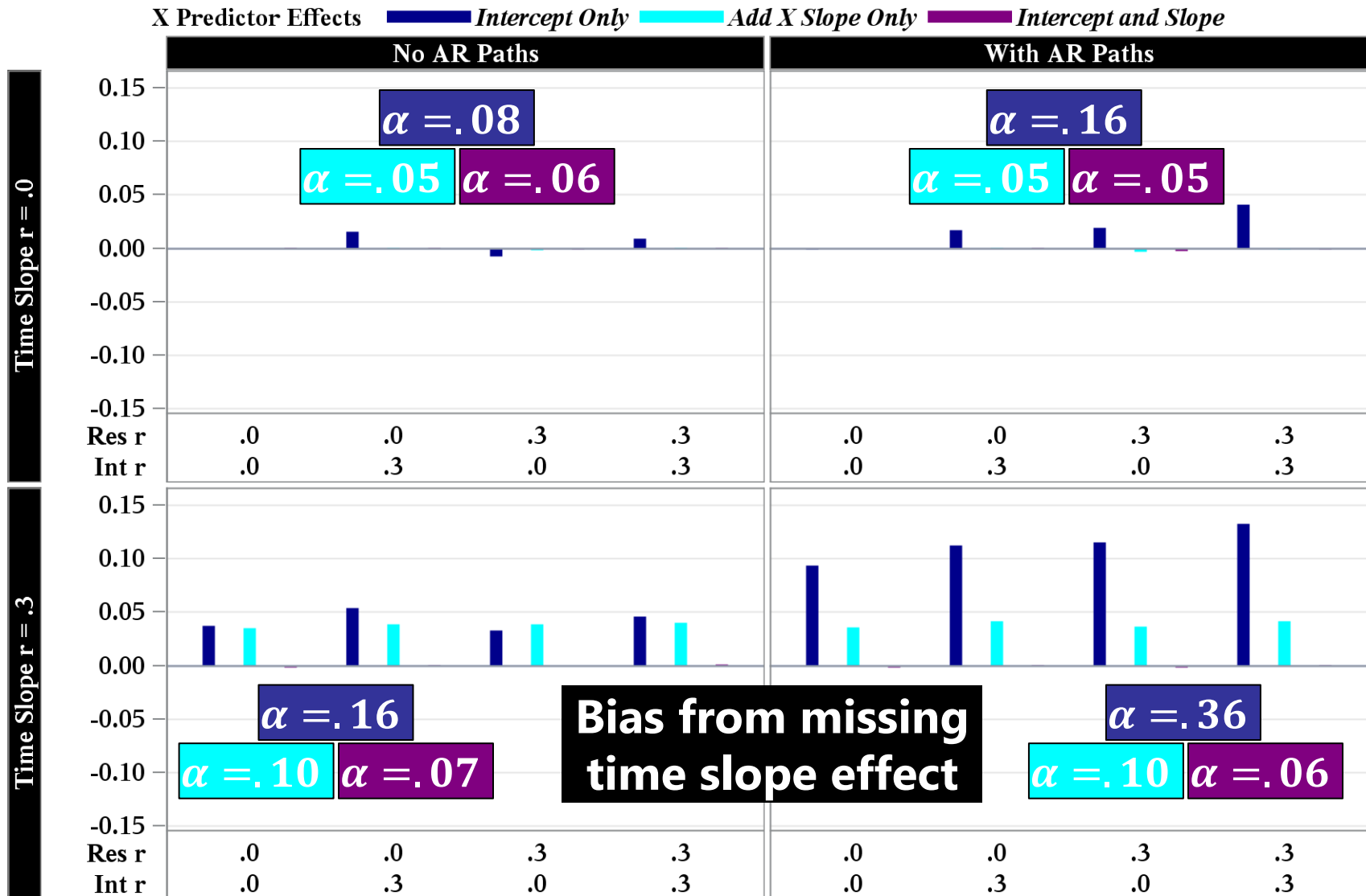
Drop Time Slope, too;
drop L1 AR paths

* Always modeled

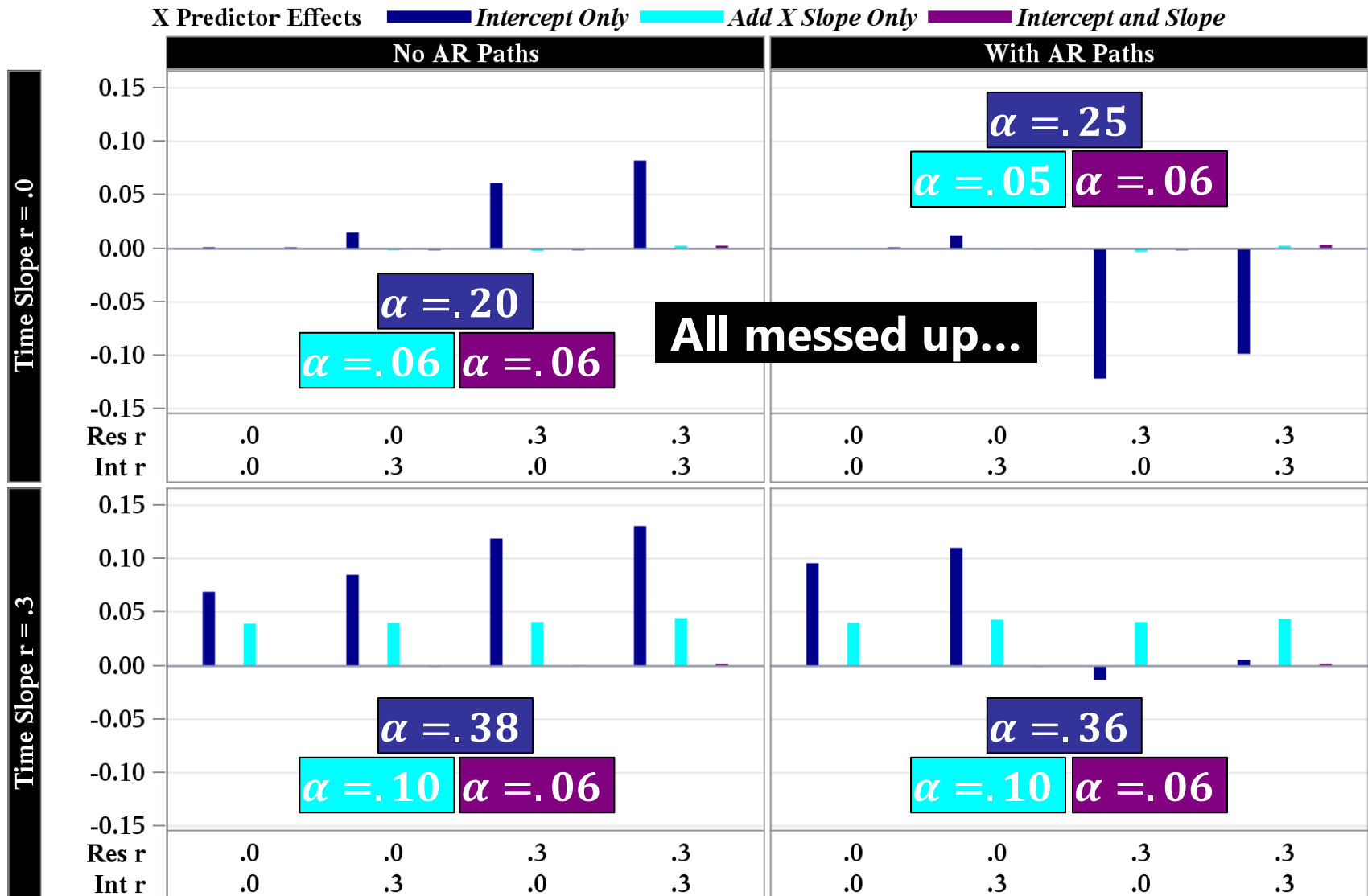
ARCL Results: Bias in L1 AR Paths



ARCL Results: Bias in L1 $X \rightarrow Y$ Paths



ARCL Results: Bias in L1 $Y \rightarrow X$ Paths



Summary: Part 2

- **Ignoring relationships between the BP Time Slopes** of longitudinal variables can contaminate ARCL relations:
 - For the **L1 AR path for the predictor with the unmodeled time slope**—because it's trying (unsuccessfully) to account for the nonconstant L1 residual correlation over time!
 - For the **L1 cross-lag paths**—which try to compensate for the missing L1 residual path (that was a covariance instead), the missing L2 time slope effect, and any biased L1 AR paths
 - Modification indices will never get to the real problem...
 - **The Point: WP questions of “which came first” cannot be answered reliably until BP model is properly specified**

Recommendations for Practice

- **BP time-slope smushing** is a potential problem in longitudinal studies over **ANY TIME SCALE!**
 - “Time” is more obvious predictor of long-term development
 - “Time” is less obvious predictor of **short-term WP fluctuation**
- e.g., L1 days within L2 persons
 - L1 Time = **day of study** for reactivity to measurement?
 - L1 Time = **day of week** for work or family routines?
- e.g., L1 occasions during the day (in L2 days in L3 persons)
 - L1 Time = **time since waking** for circadian rhythms?
 - L1 Time = **time at work** for functional rhythms?
 - Still need to consider L2 time (day of study, day of week...)

Recommendations for Practice

- **Treat time-varying “predictors” and “outcomes” the same** by starting with univariate models for each to explore *time*:
 - Consider design-informed **fixed effects** of time at ALL relevant levels
 - Consider corresponding **random effects** of time at ALL upper levels
 - Consider remaining **residual relations** (e.g., of adjacent occasions)
- Any **predictor with a random time slope** needs to be **treated as another outcome** in a multivariate model
 - i.e., as latent predictor → model-based partitioning of variances
- **Predictors with fixed effects of time only?**
 - Time is controlled for—if you include those effects in outcome model
 - Do have choice of using **observed or latent predictor variables...**

Recommendations for Practice

- **Using latent instead of observed predictors means:**
 - Smaller level-2 samples and smaller ICCs → noisier results
 - SEM: No REML estimation and no denominator DF options
→ too small L2 variances and associated fixed effect SEs
 - Interactions of latent variables → greater estimation complexity
 - Non-normal level-1 variables → greater estimation complexity
- **Can Bayes fix it?** *The jury is still out...*
 - If your priors know the right answer, sure!
 - If your variance priors are “too diffuse”, bad news!
 - Point estimates for variances: apples and oranges?
 - Useful as alternative to ML given ↑ estimation complexity

Recommendations for Practice

- But using observed instead of latent predictors means:
 - Ignoring BP differences in unreliability (i.e., caused by differing numbers of occasions or differential WP variance)
 - Result is “Lüdke’s bias” → **too-small level-2 effects** (for intercept)
- Can **two-stage** approaches get around this? **Not likely***
 - “Slopes-as-outcomes” cannot be recommended for anything other than time-detrending residuals (but why do just that?)
 - Saved intercepts and time slopes did not provide accurate results here
 - * Corrections for unreliability may have more promise...
- Choosing a software option for **latent predictors** in multivariate MLMs: **Single-level or multilevel SEM...**

Single-Level vs. Multilevel SEM for Fitting Multivariate MLMs

- **Single-level SEM is designed for balanced occasions:**
 - All persons share **common measurement schedule** (or close enough)
 - Absolute fit tests are possible given saturated model covariance matrix
 - Availability of random WP non-time slopes varies by software
 - Structured residuals can create level-2 BP effects only in some cases
- **Multilevel SEM is more flexible for unbalanced occasions:**
 - Much more realistic, especially for studying short-term fluctuations
 - But no absolute fit tests are possible without saturated model!
 - Btw, “dynamic” multilevel SEM (in *Mplus* terms) just adds options for fitting lagged effects of latent predictors (across rows) with missing data
 - Pay attention to centering methods, especially given random slopes!
 - See Hoffman (2019): EXACT SAME SYNTAX gives different level-2 parameters when estimated using ML vs Bayes in Mplus 8.0+!
 - This can lead to inadvertent smushing of all kinds using ML... be careful!

Thank you! Suggested Readings:

- Berry, D., & Willoughby, M. (2017). On the practical interpretability of cross-lagged panel models: Rethinking a developmental workhorse. *Child Development*, 88(4), 1186-1206.
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- De Haan-Rietdijk, S., Kuppens, P., & Hamaker, E. L. (2016). What's in a day? A guide to decomposing the variance in intensive longitudinal data. *Frontiers in Psychology: Quantitative Psychology and Measurement*, 7, Article 891: <https://doi.org/10.3389/fpsyg.2016.00891>
- Hoffman, L. (2015). *Longitudinal analysis: Modeling within-person fluctuation and change*. New York, NY: Routledge Academic.
- Hoffman, L. (2019). On the interpretation of parameters in multivariate multilevel models across different combinations of model specification and estimation. *Advances in Methods and Practices in Psychological Science*, 2(3), 288-311.
- Lüdtke, O., Marsh, H. W., Robitzsch, A., Trautwein, U., Asparouhov, T., & Muthén, B. (2008). The multilevel latent covariate model: A new, more reliable approach to group-level effects in contextual studies. *Psychological Methods*, 13(3), 203-229.
- McNeish, D., & Hamaker, E. L. (in press). A primer on two-level dynamic structural equation models for Intensive Longitudinal Data in Mplus. *Psychological Methods*.