On the Strategies for Disaggregating Between-Person Relations across Individual Time Slopes from Within-Person Relations in Longitudinal Data

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Associations in Longitudinal Data

- Sampling multiple persons over multiple occasions creates at least **two distinct levels of analysis**:

- **Between-person** variation IN means over time
  - Are people higher on predictor x *than other people* also higher on outcome y *than other people*?
  - "**Level-2**" or "macro-level" relation among person means

- **Within-person** variation AROUND means over time
  - *When* a person is higher on predictor x *than usual*, are they also higher on outcome y *than usual*?
  - "**Level-1**" or "micro-level" relation among mean deviations

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  - *When* a person is higher on predictor x *than usual*, are they also higher on outcome y *than usual*?
  - "**Level-1**" or "micro-level" relation among mean deviations
  - **But what about within-person change over time?**

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- **Presence of within-person (WP) change over time** requires new vocabulary and new modeling strategies

- e.g., **Long-term relations** of health (x) with cognition (y) in which there is WP change over time in each variable
  - People who are healthier (*than others at time 0*) may have better cognition → **L2-BP relation of intercepts** (not "means")
  - People whose health declines less over time (*than others*) may decline less in cognition → **L2-BP relation of L1-WP time slopes**

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  - People whose health declines less over time (*than others*) may decline less in cognition → **L2-BP relation of L1-WP time slopes**
  - When a person feels relatively better (*than predicted by their time trend*), they may then also have relatively better cognition
    - **L1-WP relation of time-specific residuals** (can differ **L2-BP**)
    - Feel better *next time*? **L1-WP "lagged" relation** (can differ **L2-BP**)

Associations in Longitudinal Data

- "Change over time" includes **ALL kinds of time trends**, each of which can also show between-person (BP) variation

- e.g., **Short-term relations** of health (x) with bad mood (y)
  - People who tend to be less healthy (*than others*) may tend to be grumpier (*than others*)→ **L2-BP relation of person means**
  - When people feel worse (*than usual*), they may also be grumpier (*than usual*) → **L1-WP relation of mean deviations**

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- How about a **Monday effect***? It may need **L1-WP** slope, too!
  - If some people are more adversely affected by Mondays (*than others*), then that **L1-WP Monday slope has L2-BP variation**!
  - People who feel even worse on Mondays (*than others*) may be even grumpier on Mondays → **L2-BP relation** of **L1-WP time slopes**

* See *Office Space* movie: "Case of the Mondays" https://www.youtube.com/watch?v=2AB9zPfXqQQ

Associations in Longitudinal Data

- No matter the time scale, **any variable measured over time** has the potential for **three distinct sources of (co)variation**:
  - **L2-BP** in a measure of overall level (usually mean or intercept)
  - **L2-BP** differences in **L1-WP** slopes for time and time-varying predictors (including slopes for auto-regressive or "inertia" effects)
  - **L1-WP** time-specific deviations from BP-predicted trajectory

- But **common practice has two common problems**:
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- But **common practice has two common problems**:
  - Time-varying "outcomes" are treated differently than "predictors"
  - "Time" may not be considered as sufficiently in short-term studies
- **Missing L2-BP relations of L1-WP time slopes create bias!**
  - **Cue demo via simulation…**

Presentation Overview

- Introduce **simulation**: data generation and manipulations
- Show **recovery results** across different longitudinal models for distinguishing **L2-BP** and **L1-WP** sources of (co)variance
  - Try to link ideas, buzz words, diagrams, and equations to show what each type of model can or cannot do (well), including:
    - **Univariate models with observed predictors**—using person-mean-centered, baseline-centered, or time-detrended predictors
    - **Multivariate models with latent predictors**—requiring single-level or multilevel structural equation models with "latent" change factors
    - Auto-regressive cross-lagged panel models for lagged effects
- Consider **best practice** in light of real-data complications
  - e.g., Unbalanced occasions, small samples, model complexity

Simulation Data Generation

- **2 variables** (x and y) with no missing data for **100 persons** (Level 2; i) over **5 occasions** (Level 1; t), indexed as $Time = (0,1,2,3,4)$*****

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	Unconditional Model for Change	
Level 1 Occasions:	$x_{tix} = \beta_{0ix} + \beta_{1ix}(Time_{tix}) + e_{tix}$ $y_{tiy} = \beta_{0iy} + \beta_{1iy}(Time_{tiy}) + e_{tiy}$	
Level 2 Intercepts:	$\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + U_{0iy}$	$\gamma_{00x} = 0$ $\gamma_{00y} = 0$
Level 2 Time Slopes:	$\beta_{1ix} = \gamma_{10x} + U_{1ix}$ $\beta_{1iy} = \gamma_{10y} + U_{1iy}$	$\gamma_{10x} = ?$ $\gamma_{10y} = ?$

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Level 1 Occasions:	$x_{tix} = \beta_{0ix} + \beta_{1ix}(Time_{tix}) + e_{tix}$ $y_{tiy} = \beta_{0iy} + \beta_{1iy}(Time_{tiy}) + e_{tiy}$		$\sigma^2_{e_x} = .40$ $\sigma^2_{e_y} = .40$
Level 2 Intercepts:	$\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + U_{0iy}$	$\gamma_{00x} = 0$ $\gamma_{00y} = 0$	$\tau^2_{U_{0x}} = .60$ $\tau^2_{U_{0y}} = .60$
Level 2 Time Slopes:	$\beta_{1ix} = \gamma_{10x} + U_{1ix}$ $\beta_{1iy} = \gamma_{10y} + U_{1iy}$	$\gamma_{10x} = ?$ $\gamma_{10y} = ?$	$\tau^2_{U_{1x}} = .06$ $\tau^2_{U_{1y}} = .06$

- Total variance set to 1 at $time = 0$, so that:
  - Conditional ICC = .60 → Intercept variance for U_{0ix} and U_{0iy}
  - Slope Reliability = .60 → Time slope variance for U_{1ix} and U_{1iy}

Simulation Manipulations

- Fixed time effects (γ_{10} absent or present) collapsed here
  - Didn't matter because $Time_{ti}$ was always a predictor of y_{ti}

- Key manipulation: **match across 3 types of relationships**

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- Key manipulation: **match across 3 types of relationships**

- **L2-BP random effects** ($\boldsymbol{U_{0ix}}$, $\boldsymbol{U_{0iy}}$, $\boldsymbol{U_{1ix}}$, $\boldsymbol{U_{1iy}}$) drawn from a multivariate normal distribution with 4 conditions:
  - **Intercept correlations:** $\boldsymbol{r(U_{0ix}, U_{0iy}) = 0 \text{ or } .3}$
  - **Time slope correlations:** $\boldsymbol{r(U_{1ix}, U_{1iy}) = 0 \text{ or } .3}$
  - All other Intercept–Time slope pairs of correlations $= 0$

- **L1-WP residuals** drawn from a separate multivariate normal distribution with 2 conditions: $\boldsymbol{r(e_{tix}, e_{tiy}) = 0 \text{ or } .3}$

2 Longitudinal Modeling Families

- Univariate models: predict y_{ti} from **observed $\boldsymbol{x_{ti}}$ predictors**
  - *aka*, Multilevel models (MLMs) using person-mean-centered, baseline-centered, or detrended-residual predictors
  - Estimated in any software with mixed effects (e.g., MIXED in SAS, SPSS, or STATA; LME4 or NLME in R environment)

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- <u>Multivariate models:</u> predict both y_{ti} and x_{ti} as **outcomes**
  - But x_{ti} can't predict y_{ti} in univariate mixed-effects software, so…
  - Can be specified as a single-level structural equation model (SEM)
    - e.g., "Multivariate latent growth curve models" (with or without "structured residuals"); "auto-regressive cross-lagged panel models"
  - Can also be specified as a "multilevel SEM" (= multivariate MLM)
    - I will use ML estimation; M*plus* "latent predictor centering" and lagged effects within "*dynamic* multilevel SEM" require Bayes MCMC instead

Unconditional Time Model for y_{ti}: 3 Ways

Unconditional Time Univariate Multilevel Model (long data)	
L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$	**L2 Intercept:** $\beta_{0i} = \gamma_{00} + U_{0i}$ **L2 Time Slope:** $\beta_{1i} = \gamma_{10} + U_{1i}$

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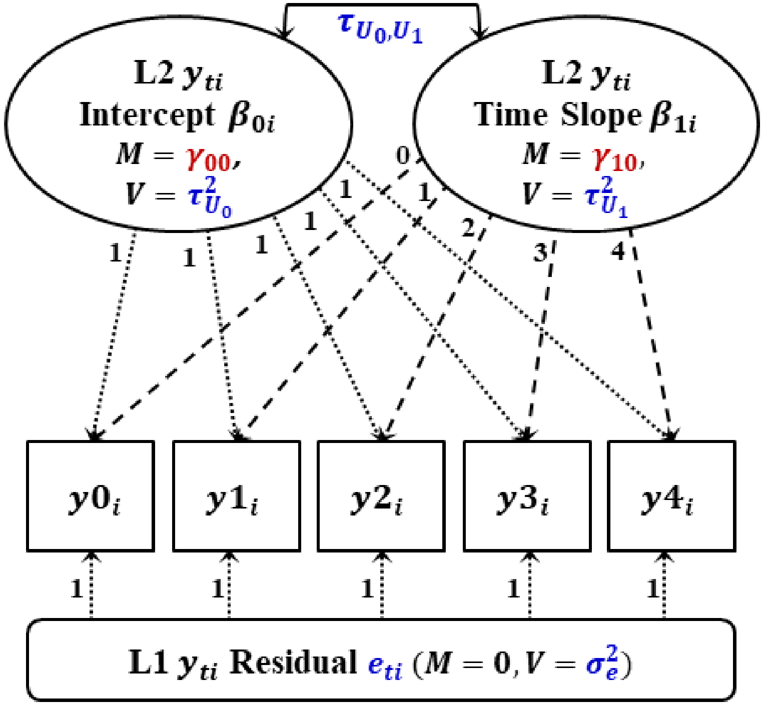
As Single-Level SEM (wide data)



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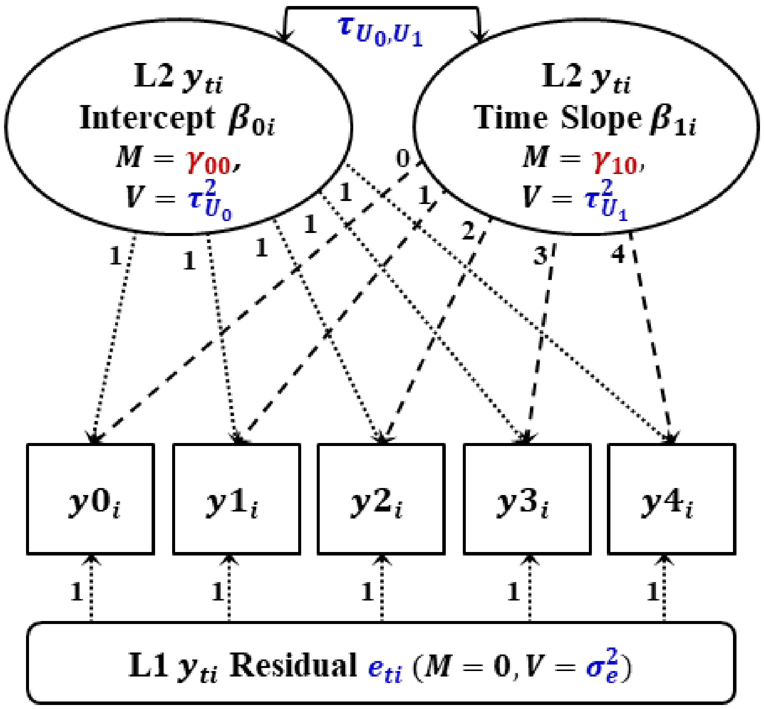

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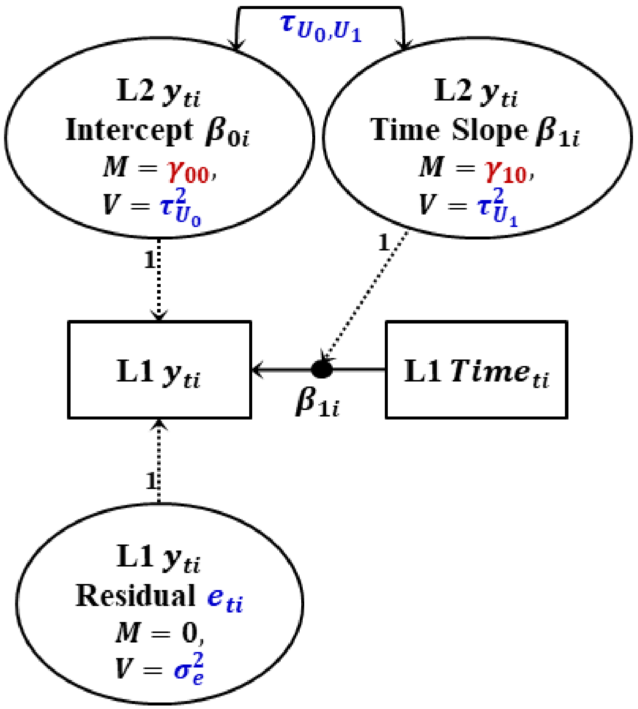

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Naïve Addition of Time-Varying x_{ti}

Univariate MLM: TV x_{ti} has a Smushed* Effect
(*aka conflated, convergence, composite effect)

L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$
$+ \beta_{2i}(x_{ti})$

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- Model is **bad news** if the L1 predictor has L2 variance (i.e., people differ in their mean of x_{ti} over time)
  - Could also be true for the L1 $time_{ti}$ predictor! (but not here)

- **Forces** level-1 (**WP**) and level-2 (**BP**) x_{ti} effects **to be equal**, which is unlikely to be true, *especially* in longitudinal data!

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- **Forces** level-1 (**WP**) and level-2 (**BP**) x_{ti} effects **to be equal**, which is unlikely to be true, *especially* in longitudinal data!
- A predictor for x_{ti} **is needed at any level it has variability**

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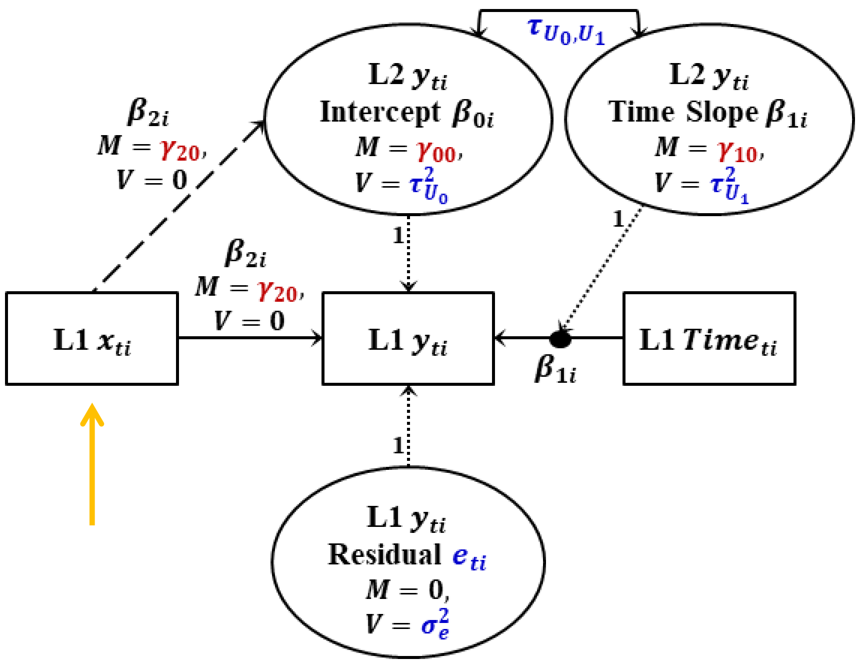
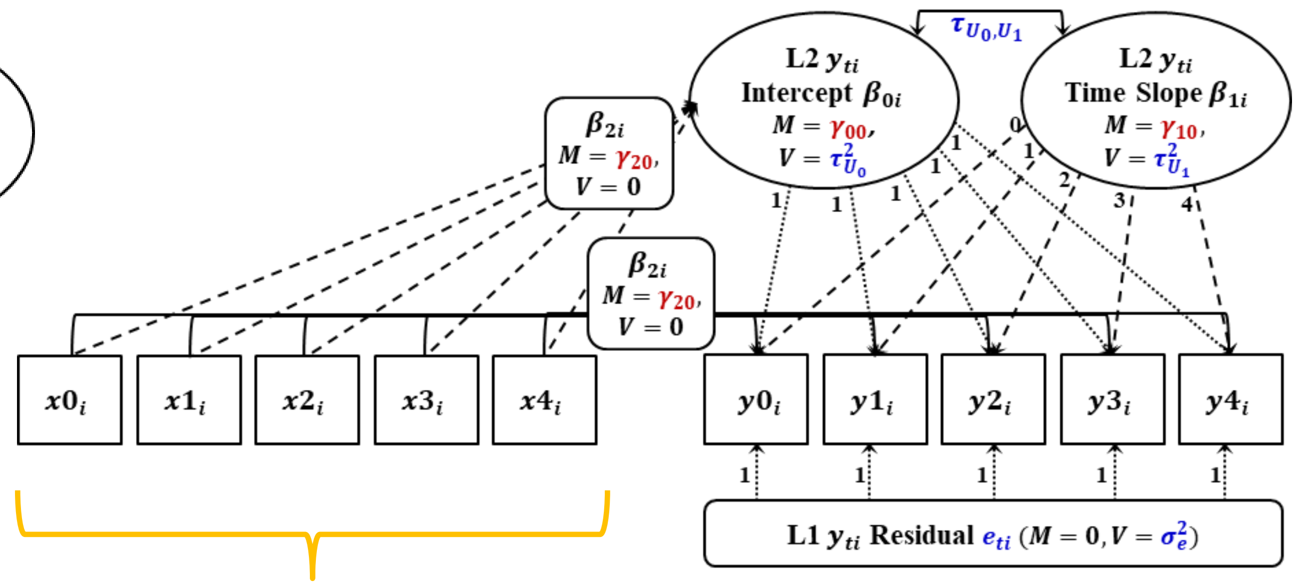


Smushed Effect:
L1-WP and L2-BP effects of x_{ti} are forced to be equal (both γ_{20})

As Single-Level SEM (wide data)



Unsmushing the Effects of L1 x_{ti}

MLMs: L2 BP and L1 WP Effects of x_{ti} as observed predictors		
L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$		**L2 Time Slope:** $\beta_{1i} = \gamma_{10} + U_{1i}$ **L2 x_{ti} Slope:** $\beta_{2i} = \gamma_{20}$
Person-Mean (PM) Centering:	$+ \beta_{2i}(x_{ti} - \bar{x}_i)$	**L2 Int:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(\bar{x}_i) + U_{0i}$
Baseline (BL) Centering:	$+ \beta_{2i}(x_{ti} - x0_i)$	**L2 Int:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(x0_i) + U_{0i}$

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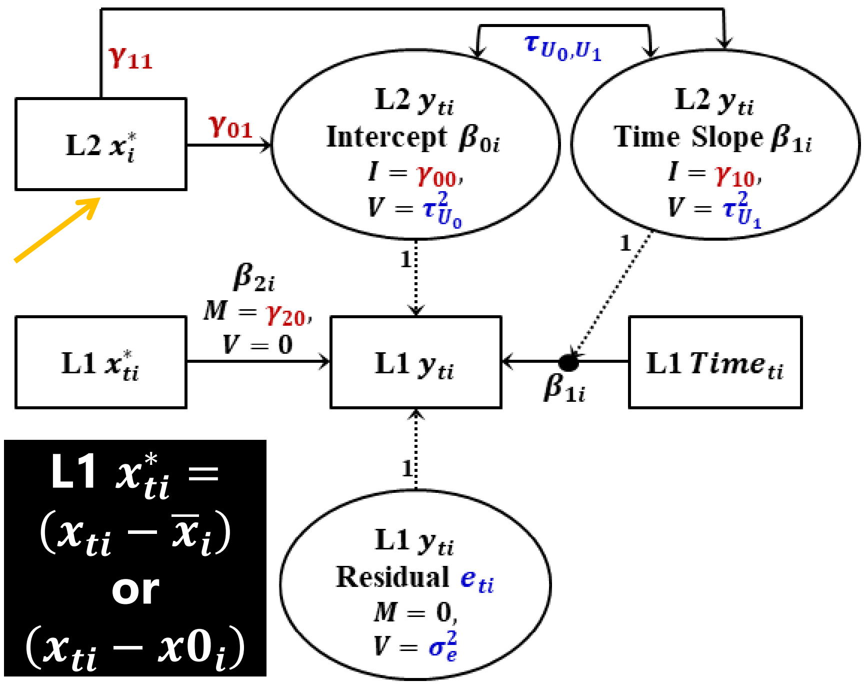
- Either should yield: γ_{20} → **L1-WP effect**; γ_{01} → **L2-BP effect**
- **L2 PM $\bar{x}_i$** uses all occasions so L1 residuals should cancel…
  - …**But timing is off:** L2 *average* x_{ti} predicts L2 y_{ti} *time 0* intercept
- **L2 BL $x0_i$** matches timing to create L2 relation at *time 0*…
  - …**But still has L1 residual:** Is *actual* $x0_i$, not *predicted* x_{ti} at time 0

Unsmushing the Effects of L1 x_{ti}

As Multilevel SEM (long data)



L1 $x_{ti}^* =$ $(x_{ti} - \bar{x}_i)$ or $(x_{ti} - x0_i)$

L2 x_i^* Model Variants:

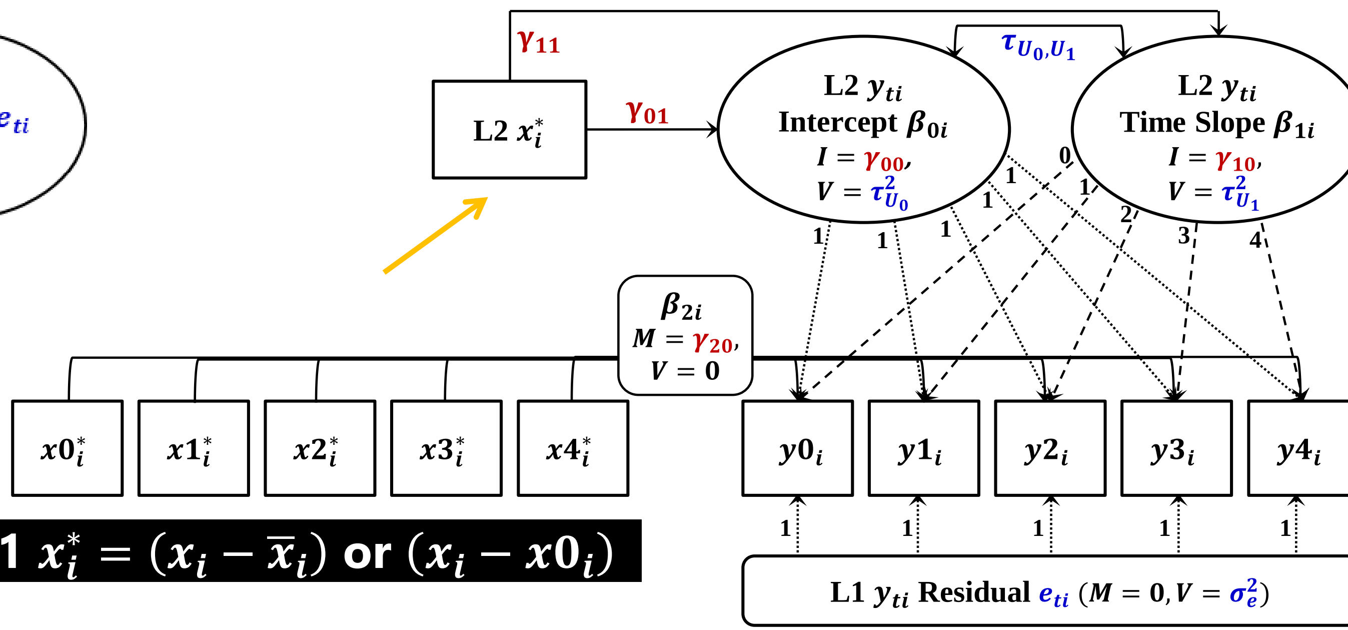
Person-Mean Centering uses $\bar{x}_i$

Baseline Centering uses $x0_i$

L2 x_i^* by $Time_{ti}$ slope γ_{11} added for comparability with next models:

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(x_i^*) + U_{1i}$$

As Single-Level SEM (wide data)



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Simulation Results: Univ MLMs

- How well did centering with the person mean ($\bar{x}_i$) or baseline ($x0_i$) **recover the 3 relations of x_{ti} with y_{ti}**?

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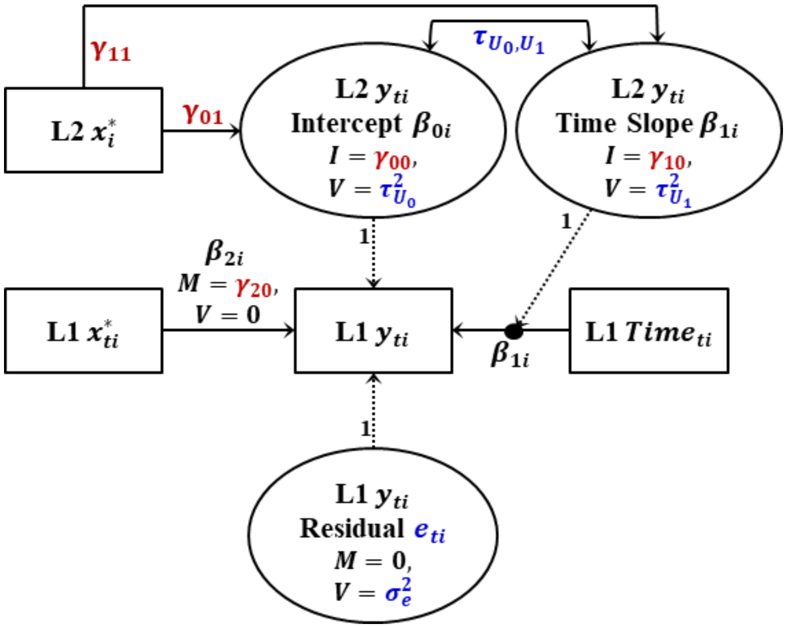

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L2 x_{ti} Slope: $\beta_{2i} = \gamma_{20}$

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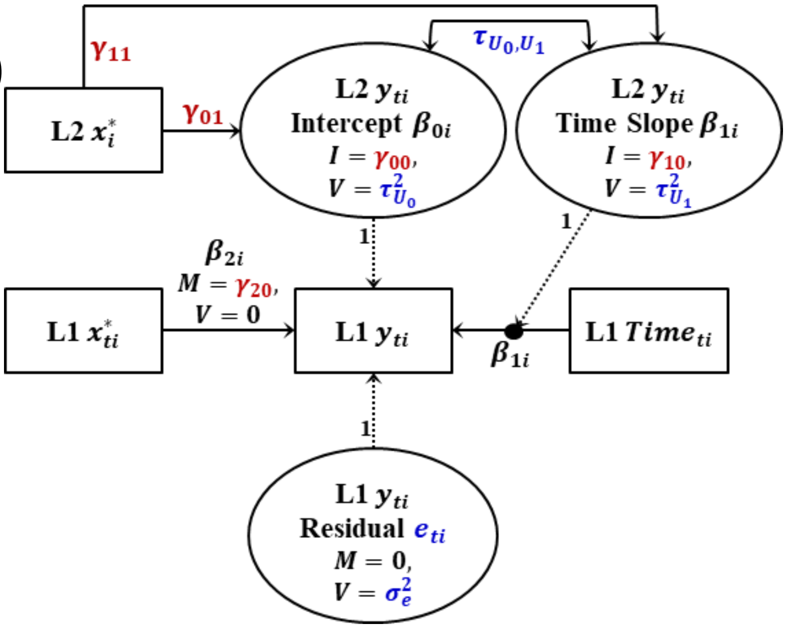
0. L2 x_i^* by $Time_{ti}$ slope $\boldsymbol{\gamma_{11}} = \mathbf{0}$
1. No L2 x_{ti} random time (slope r → 0)
2. L2 intercept r → L2-BP $\boldsymbol{x_i^*}$ slope $\boldsymbol{\gamma_{01}}$
3. L1 residual r → L1-WP $\boldsymbol{x_{ti}^*}$ slope $\boldsymbol{\gamma_{20}}$

As Multilevel SEM



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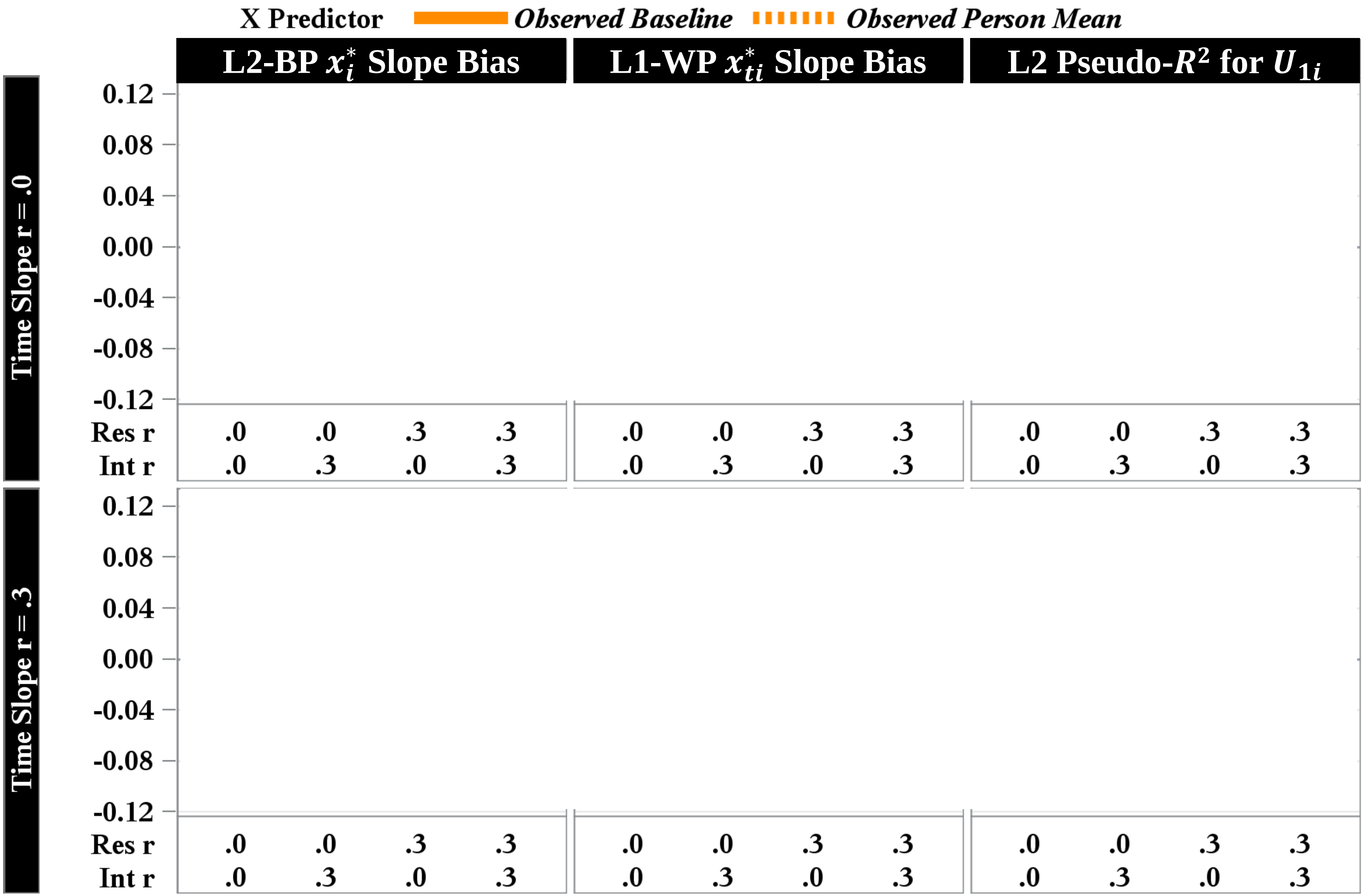
Univ Results: Time-Smushing Bias!



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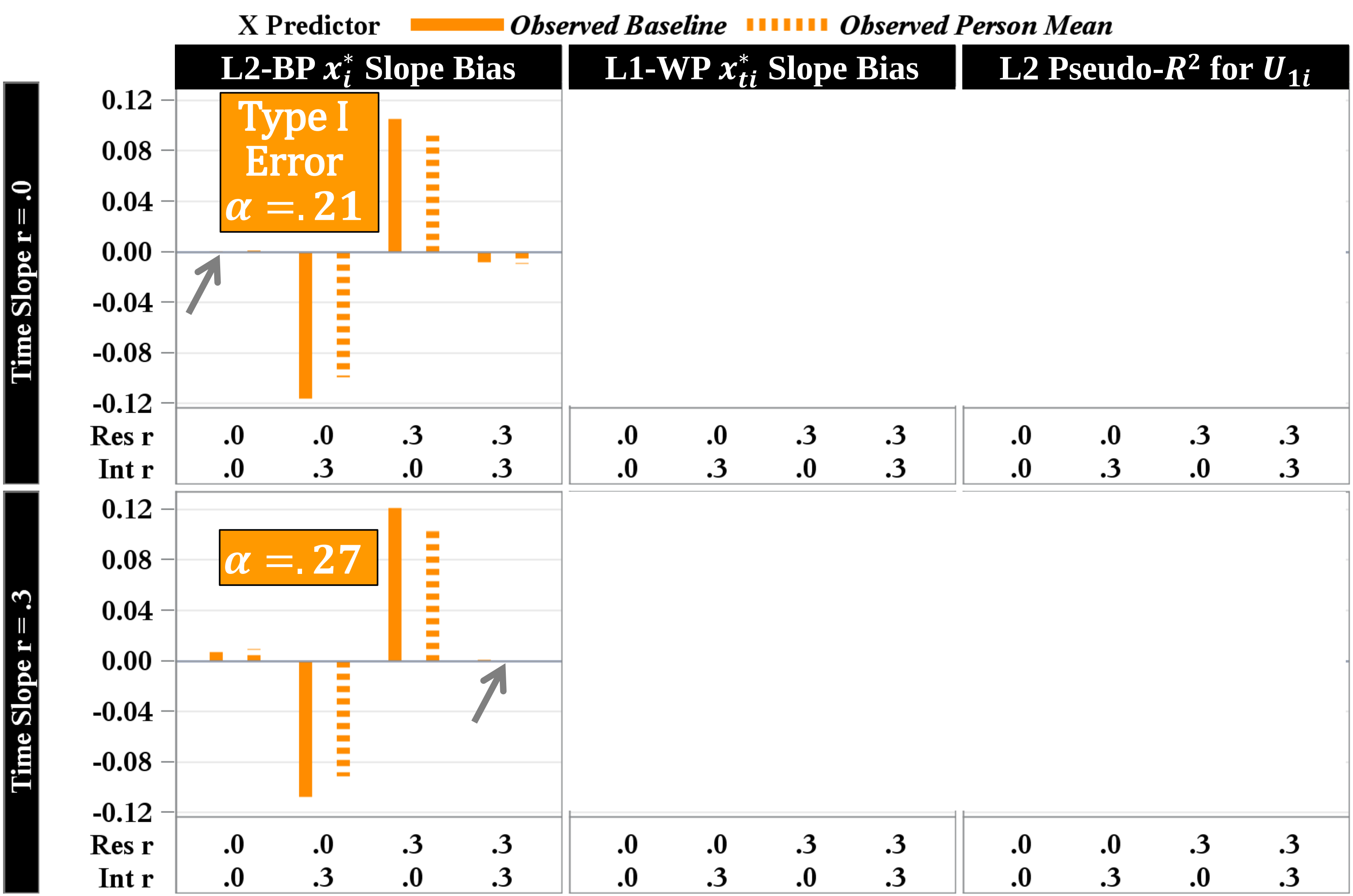

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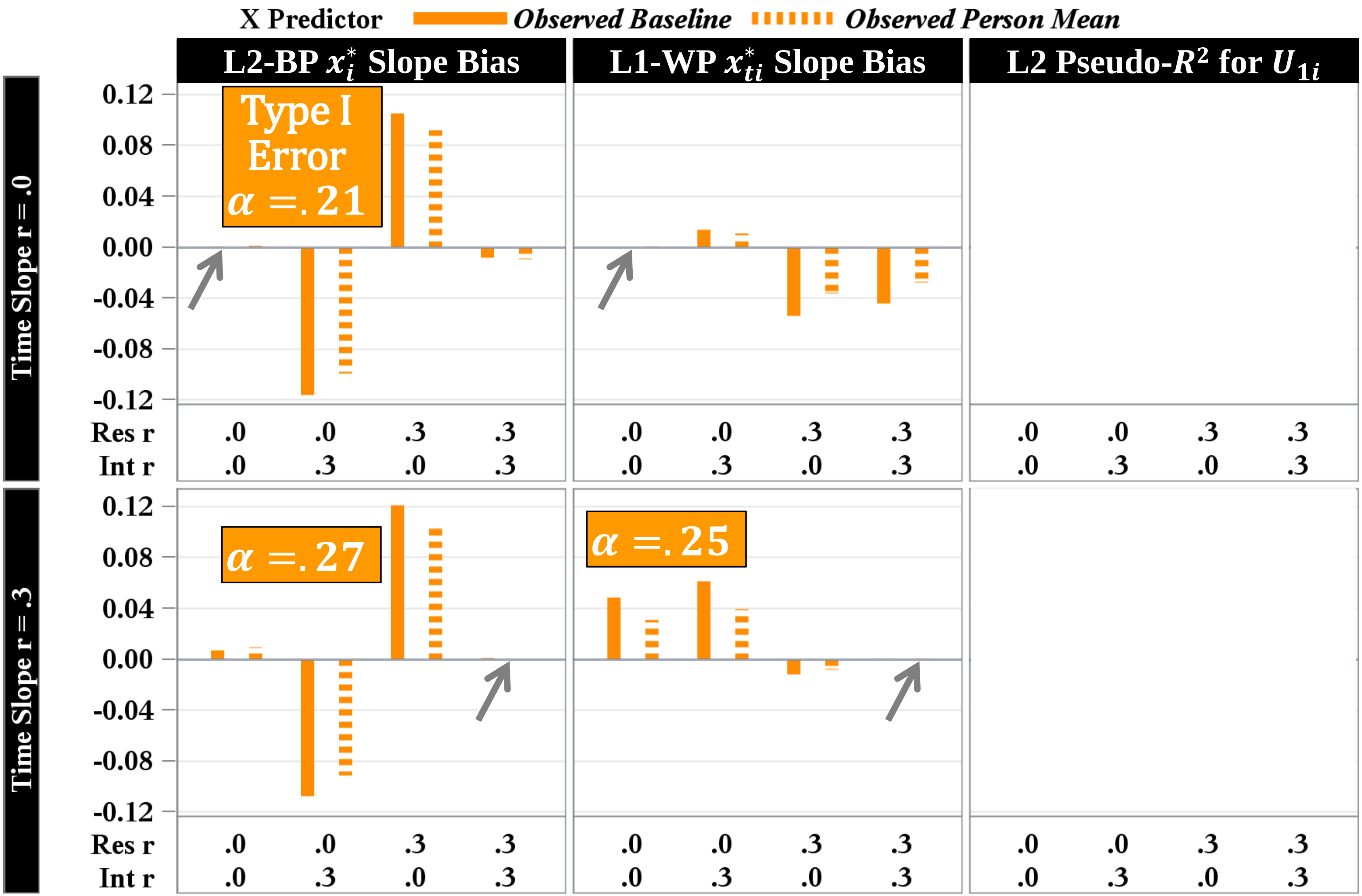

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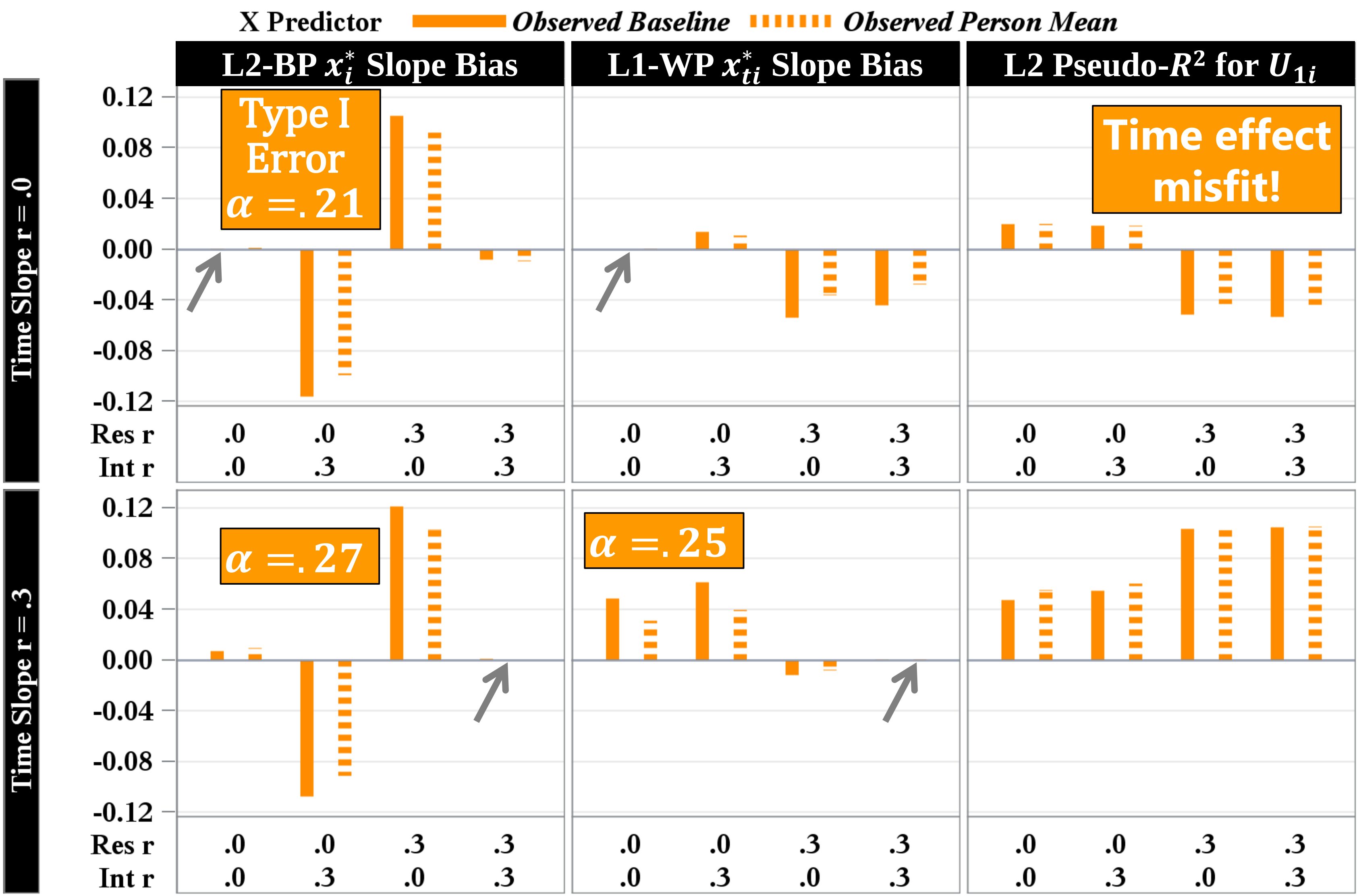
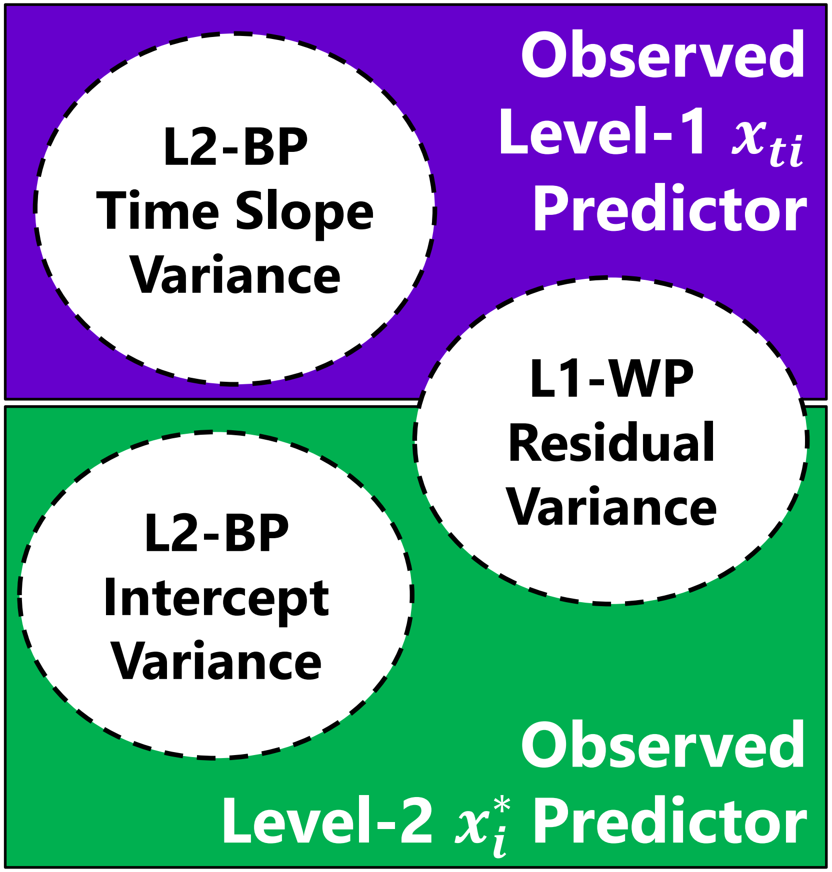


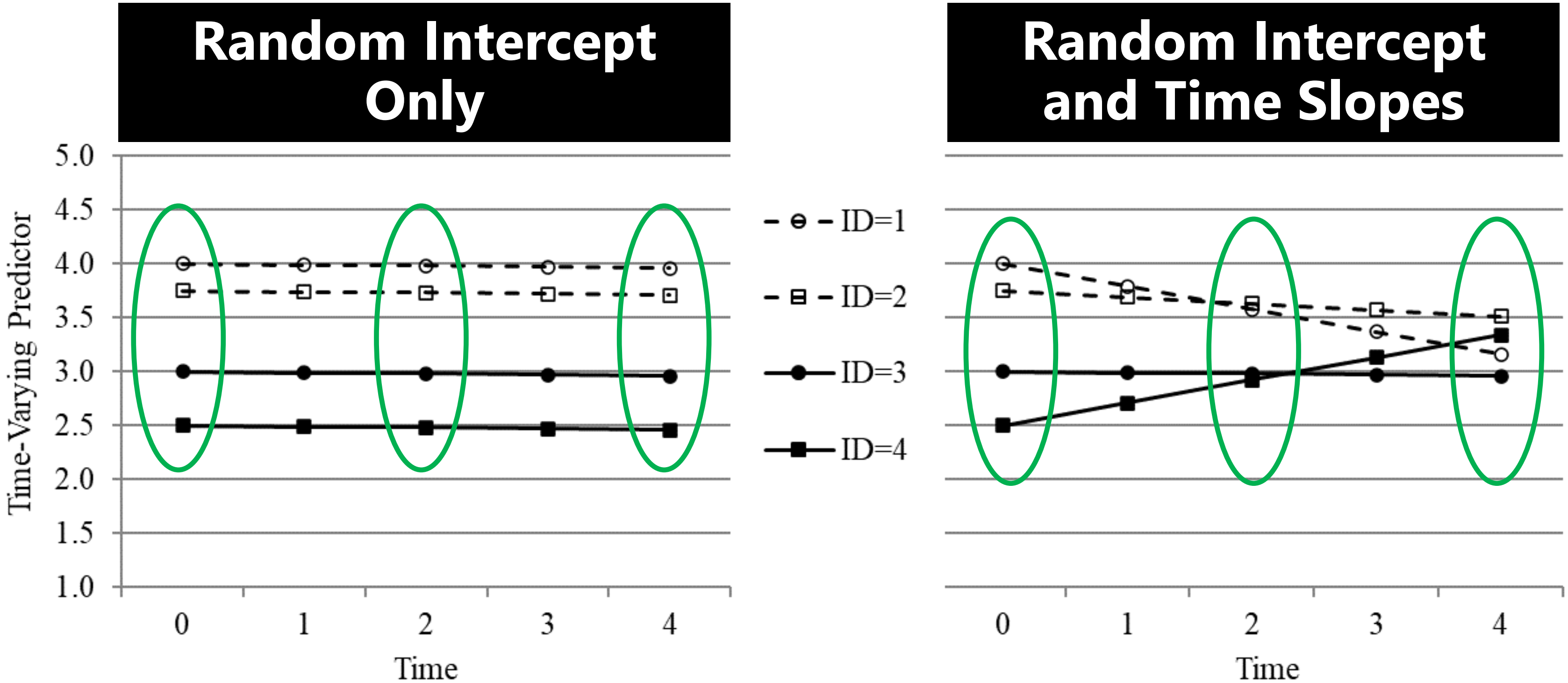
Why Time-Smushing Bias Happens

- **Ignoring L2-BP relationships between the time slopes** of longitudinal variables can contaminate their other relations:
  - Top: if the **L1-WP x_{ti}** still contains unmodeled L2-BP variance in time slopes, **the L1-WP effect will be smushed with the missing L2-BP time slope effect!**
  - Different than well-known problems of **intercept-smushed L1 WP effects OR bias from using observed mean** (bottom)



Why Level-2 BP Slopes are Affected

- **Ignoring L2-BP relationships between the time slopes** of longitudinal variables can contaminate their other relations:
  - Also in the **L2-BP Intercept**—because it must change over time!



Fixing Level-1 Bias… Univariately

- "**Detrended residuals**" is a univariate strategy designed to remove time-related variance from the level-1 x_{ti} predictor
- Is a **two-stage approach** analogous to "**slopes-as-outcomes**":
  - Fit **separate regression model** to each person's data
  - Save time-specific x_{ti} **residuals** to use as **level-1** x^*_{ti}
  - Save **fixed intercept at** $time = 0$ to use as **level-2** x^*_i

As Univariate MLM

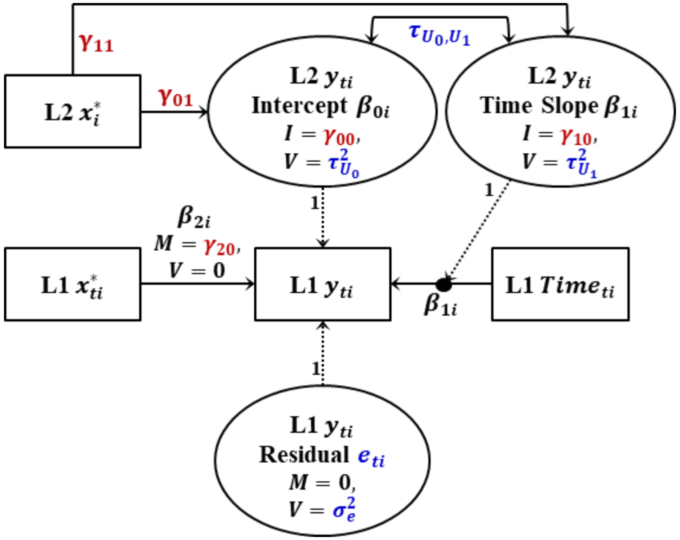
L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + \beta_{2i}(x^*_{ti}) + e_{ti}$

L2 Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(x^*_i) + U_{0i}$

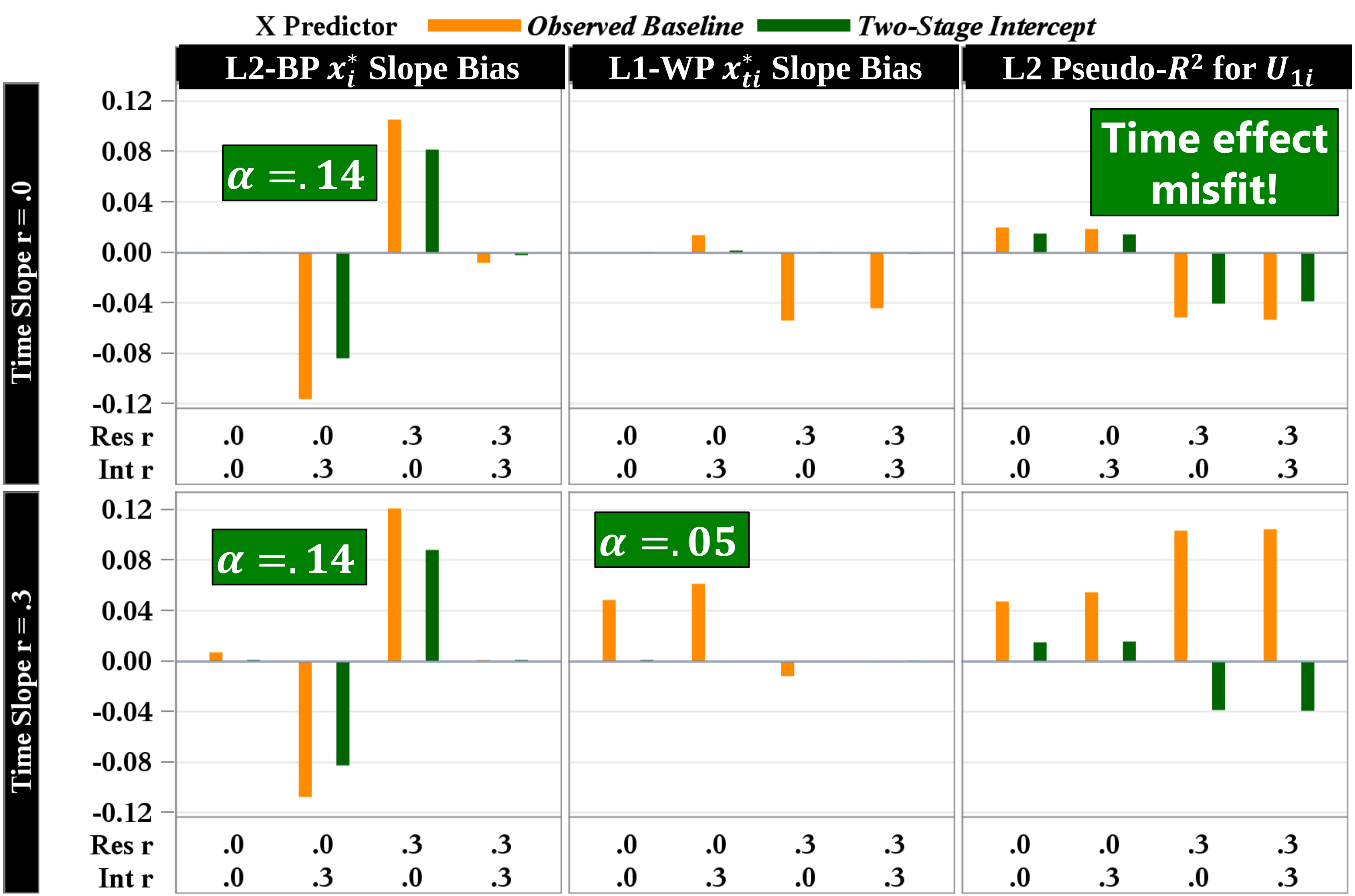
L2 Time: $\beta_{1i} = \gamma_{10} + \gamma_{11}(x^*_i) + U_{1i}$

L2 x_{ti} Slope: $\beta_{2i} = \gamma_{20}$

As Multilevel SEM



Univ Results: A Partial Fix

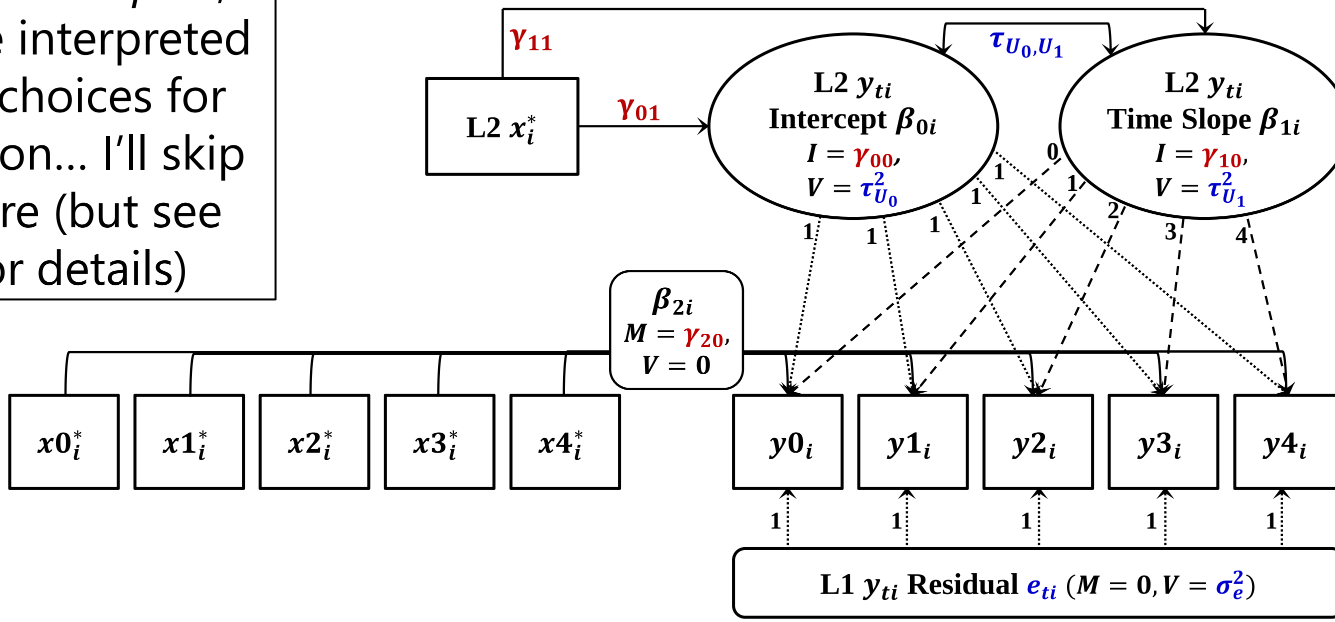


Why the asymmetry of x_{ti} and y_{ti}?

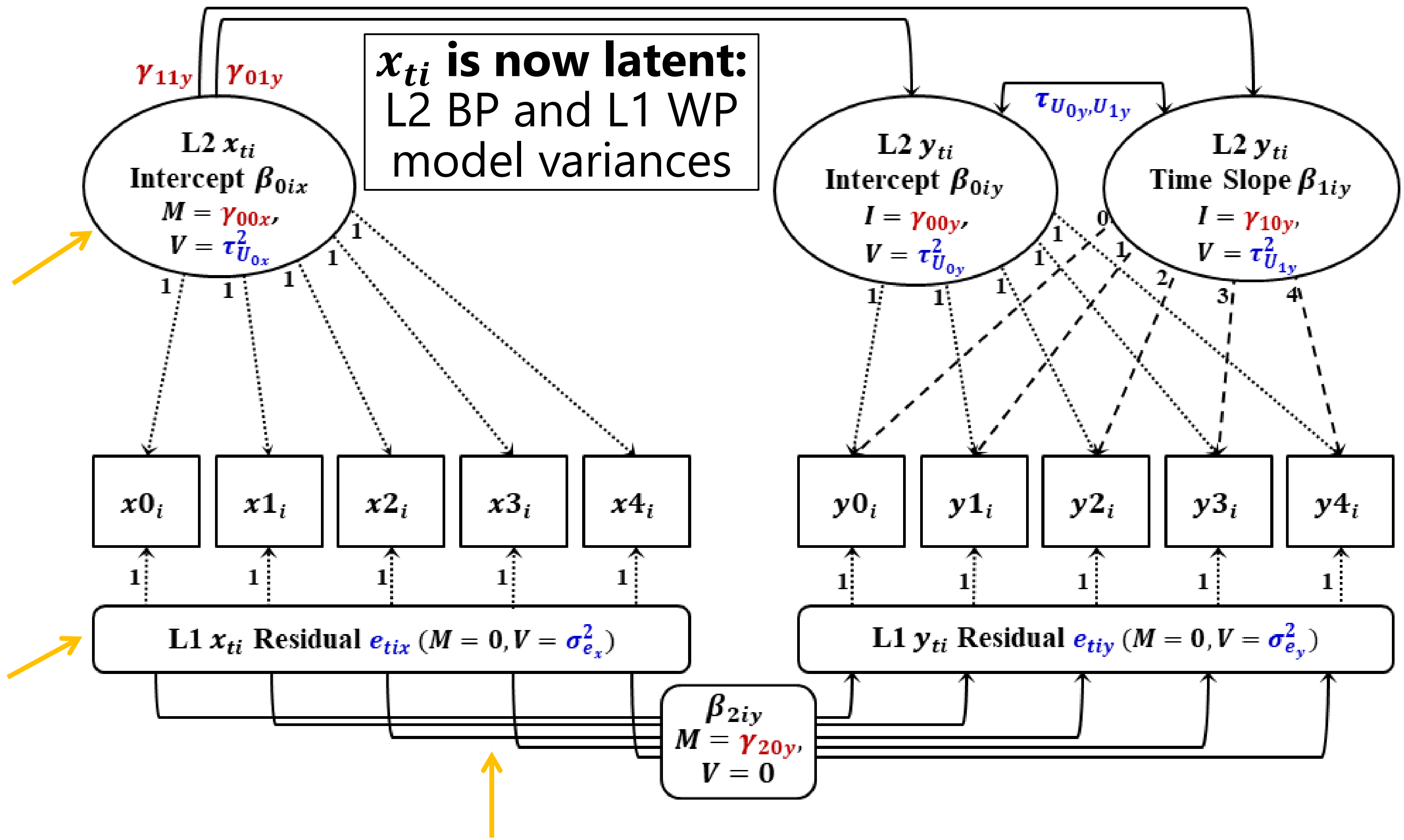
- Why is y_{ti} **treated as latent** (i.e., three sources of variance partitioned by the model; in circles) **while x_{ti} is observed** (variance partitioned by brute-force predictors; in squares)?
- Primary benefit of **multivariate models** is to treat x_{ti} like y_{ti} *but still be able to include **fixed effects of x_{ti} that predict y_{ti}***

Btw: in multilevel SEMs with latent x_{ti} predictors in M*plus*, how parameters are interpreted depends on one's choices for syntax and estimation… I'll skip this complexity here (but see Hoffman 2019 for details)

Asymmetric Single-Level SEM with Observed x_{ti} Predictors



Symmetric Single-Level SEM



- This SEM uses "**Structured Residuals**": Level-1 x_{ti} effect between the x_{ti} and y_{ti} *residuals* (instead of between the observed variables)
  - Why? To get **level-2 BP effects** instead of level-2 *contextual* effects

Multivariate MLM: From Single-Level SEM to Multilevel SEM

Fixed Effects of Intercept and Residual of Latent x_{ti}

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$

$y_{tix} = \beta_{0iy} + yw_{tiy}$

w indicates a **L1** *within* variable

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Total: $x_{tix} = \beta_{0ix} + xw_{tix}$ $y_{tix} = \beta_{0iy} + yw_{tiy}$ w indicates a **L1** *within* variable
L1: $xw_{tix} = e_{tix}$ $yw_{tiy} = \beta_{1iy}(Time_{tiy}) + \beta_{2iy}(xw_{tix}) + e_{tiy}$ $\beta_{2iy} = \gamma_{20y}$
L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + U_{0iy}$
L2 Time Slopes: (β_{1ix} doesn't exist yet) $\beta_{1iy} = \gamma_{10y} + \gamma_{11y}(\beta_{0ix}) + U_{1iy}$

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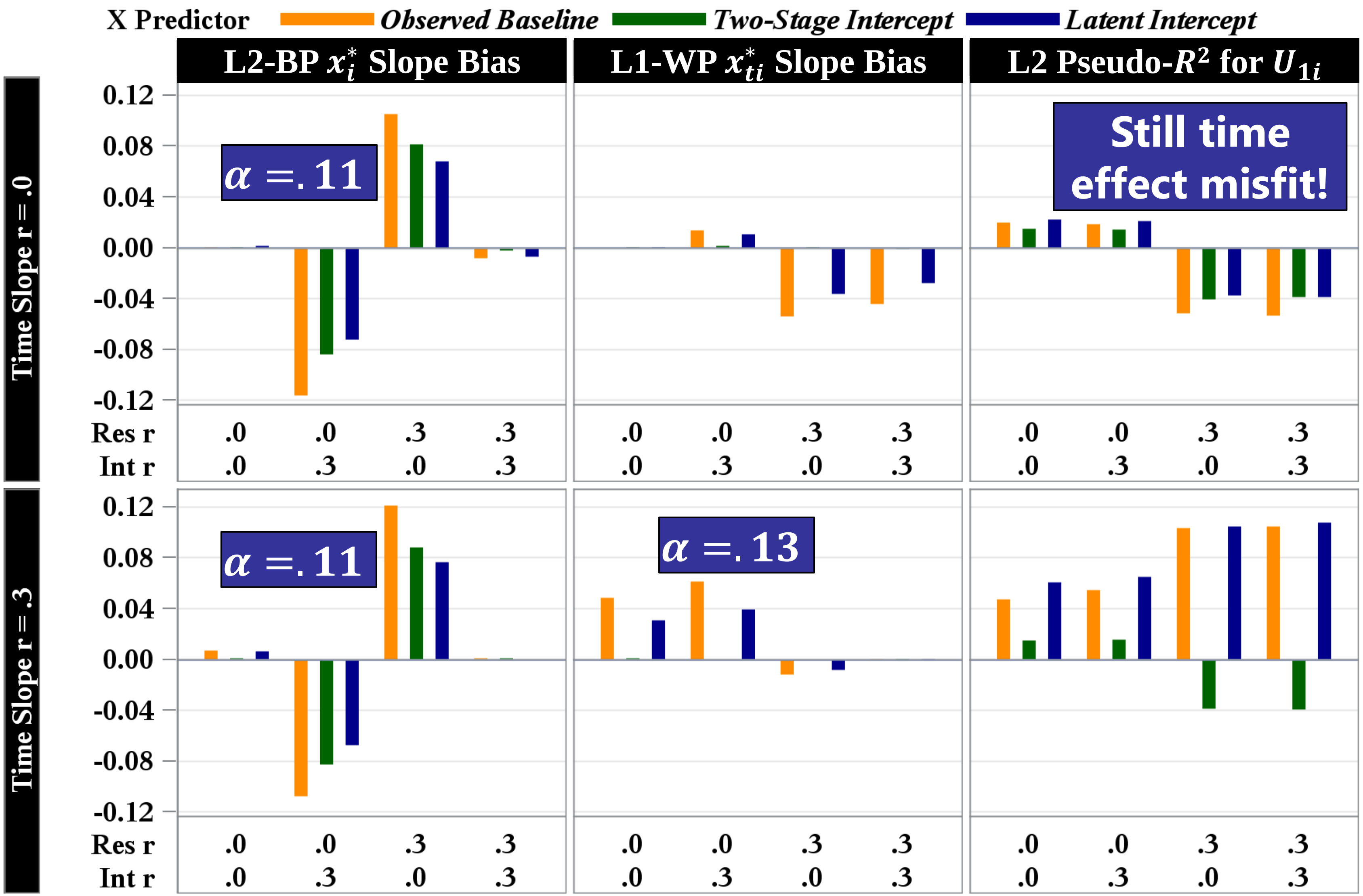
So how does using **latent x_{ti} predictors** compare with **observed x_{ti} predictors** (baseline or two-stage intercept)?

Latent x_{ti} → Less Bias? Not yet…

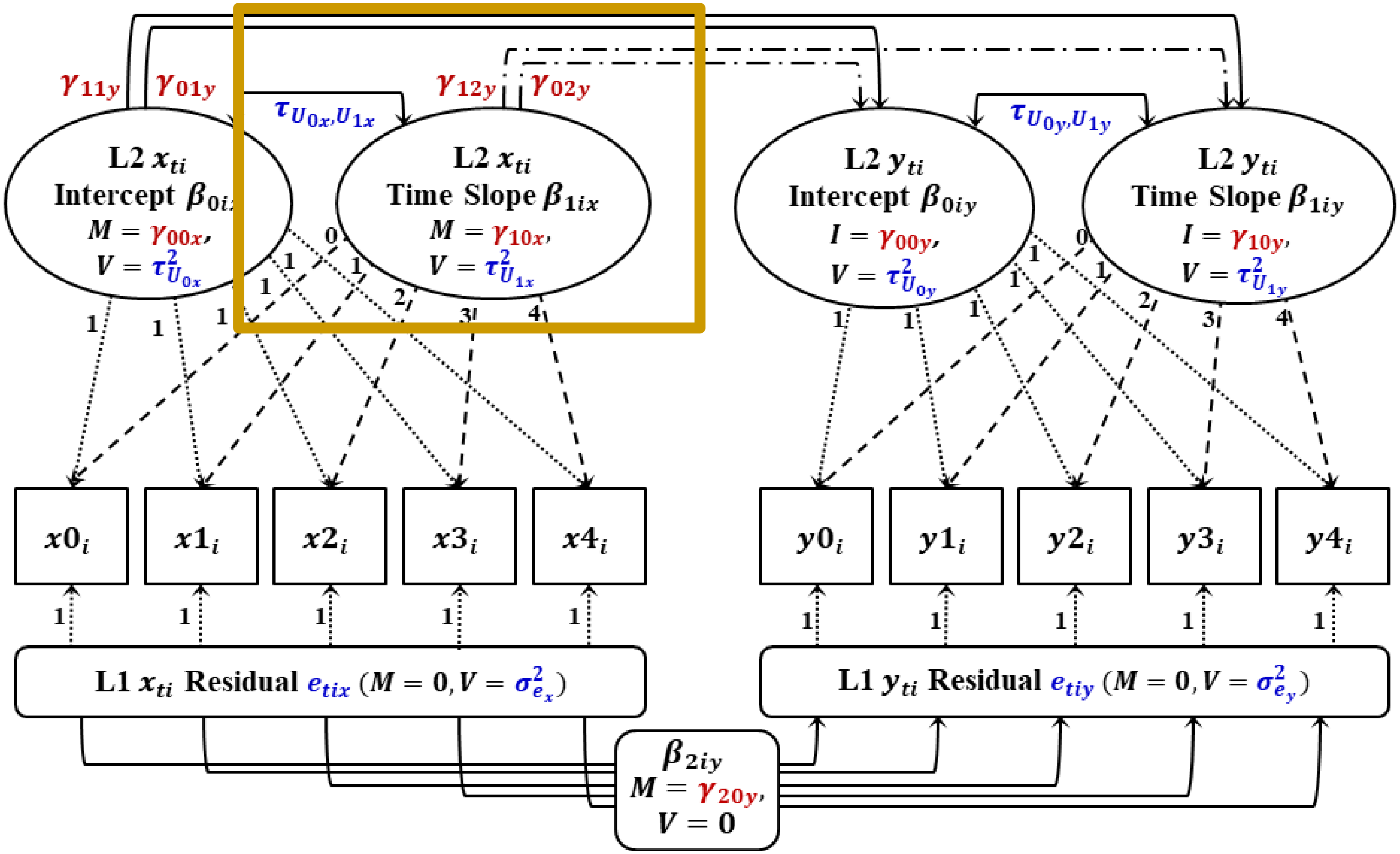


Multivariate MLM via Multilevel SEM: Add Random Time Slope for x_{ti}

Fixed Effects of Intercept, Time Slope, and Residual of Latent x_{ti}	
Total: $x_{tix} = \beta_{0ix} + xw_{tix}$ $y_{tix} = \beta_{0iy} + yw_{tiy}$	w indicates a **L1** *within* variable
L1: $xw_{tix} = \beta_{1ix}(Time_{tix}) + e_{tix}$ $yw_{tiy} = \beta_{1iy}(Time_{tiy}) + \beta_{2iy}(xw_{tix}) + e_{tiy}$	$\beta_{2iy} = \gamma_{20y}$
L2 Intercepts:	$\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + \gamma_{02y}(\beta_{1ix}) + U_{0iy}$
L2 Time Slopes:	$\beta_{1ix} = \gamma_{10y} + U_{1iy}$ $\beta_{1iy} = \gamma_{10y} + \gamma_{11y}(\beta_{0ix}) + \gamma_{12y}(\beta_{1ix}) + U_{1iy}$

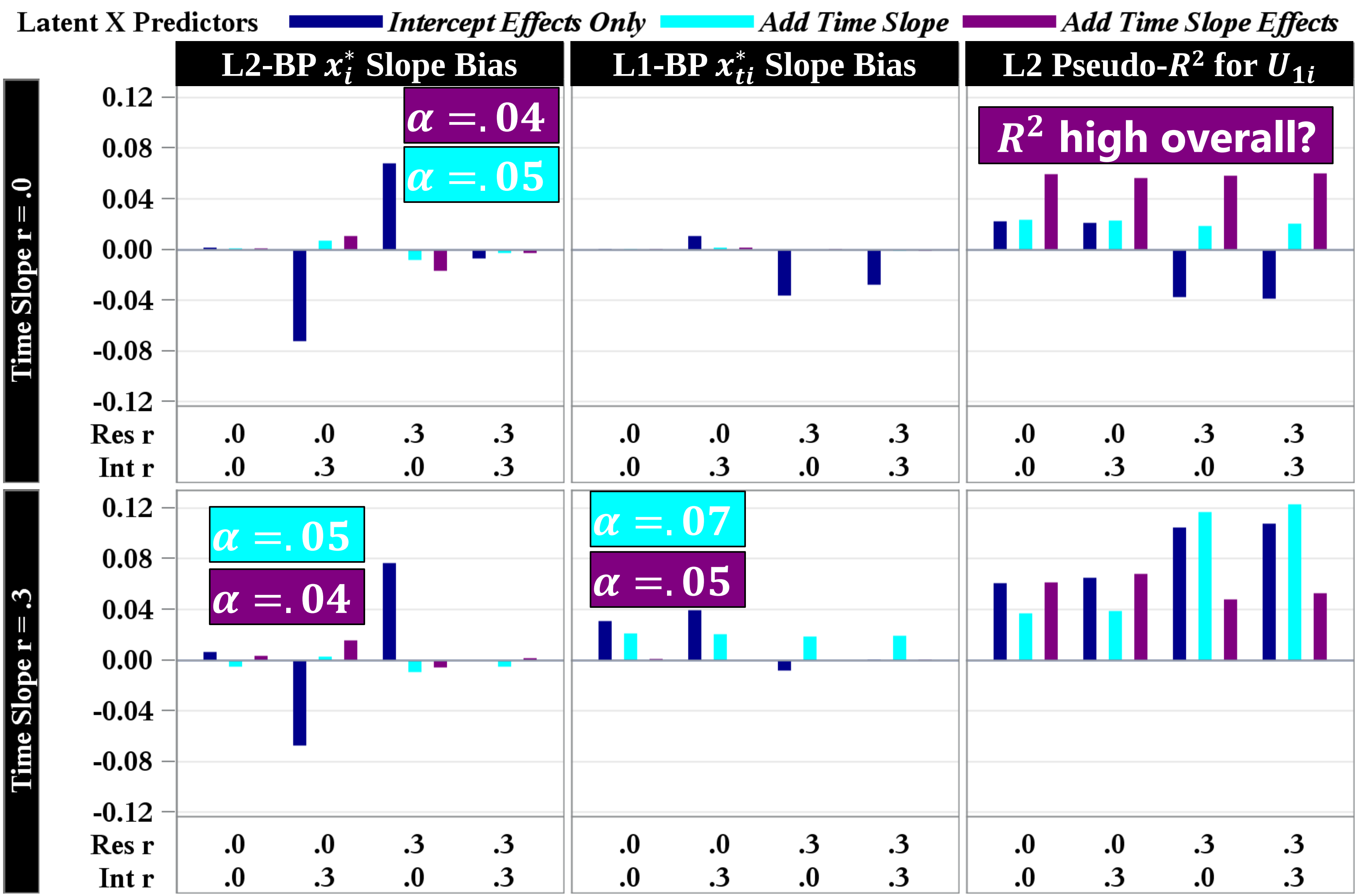
How well does this "multivariate latent growth curve model with structured residuals" recover the **3 types of relations of x_{ti} with y_{ti}**?

So Just Let $Time_{tix}$ Also Predict x_{ti}



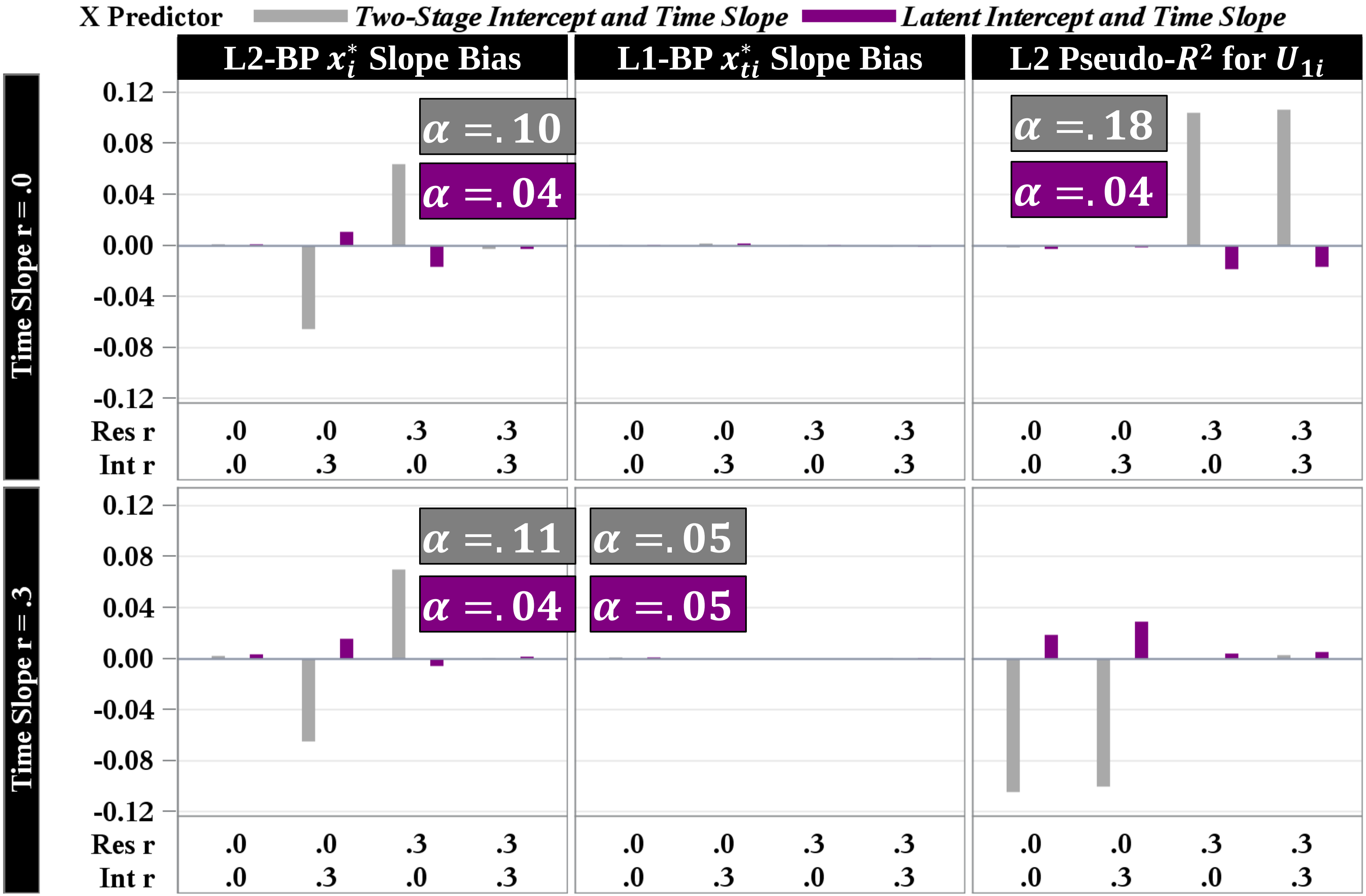
- L2 Intercept β_{0ix} is now specific to $time = 0$ (just like β_{0iy} has been)
- How well does this "multivariate latent growth curve model with structured residuals" recover the **3 types of relations of x_{ti} with y_{ti}**?

Results: Better! (But Not Perfect)



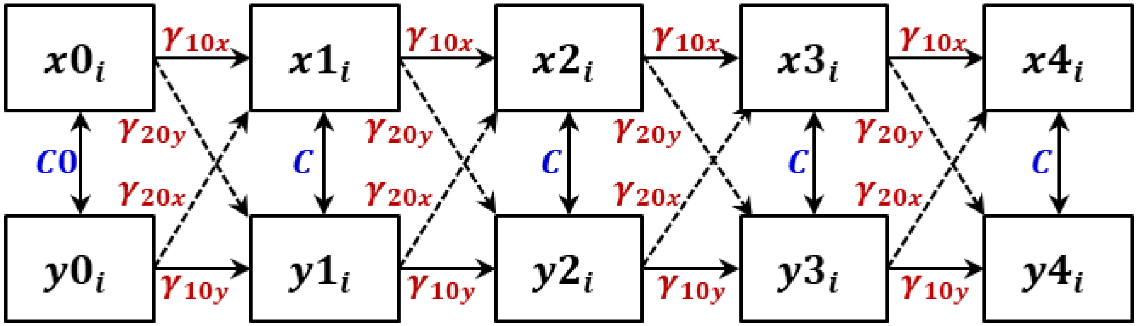
Slopes-as-Outcomes?

Slopes-as-Outcomes? Still Nope.



Smushed Effects in Related Models*

Auto-Regressive Cross-Lagged Panel Model (the “ARCL” or “CLPM”)



Path model with separate intercepts (and residual variances) per occasion, and lag-1 fixed effects:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + e_{tix}$$

$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + e_{tiy}$$

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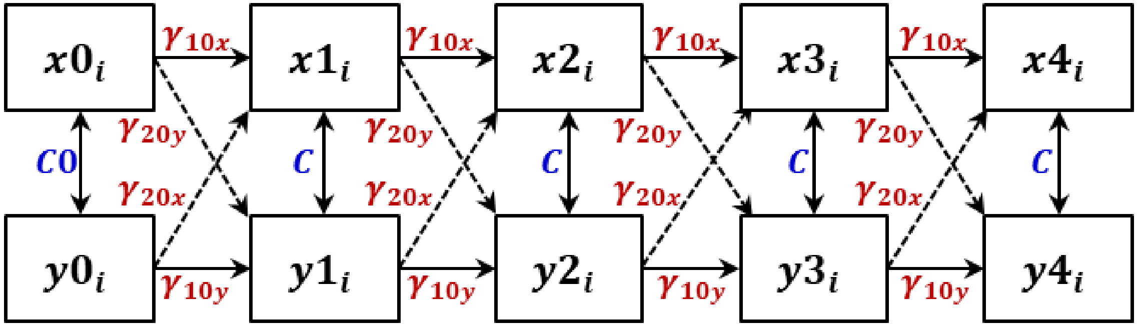

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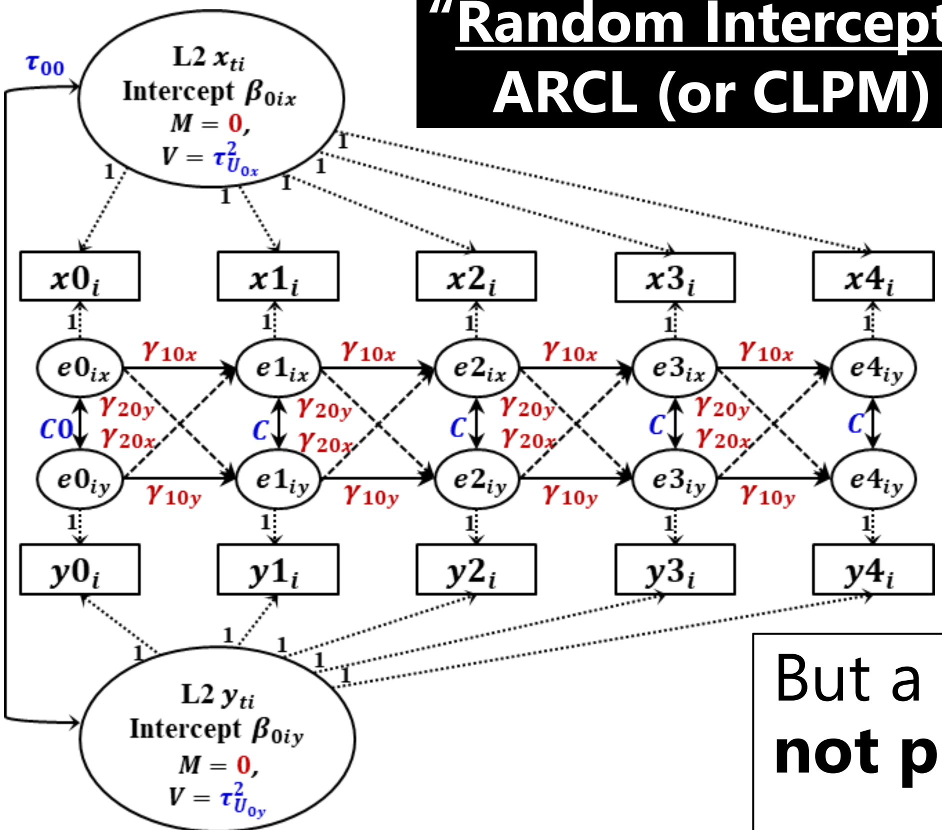
- **CLPM interpretation is problematic:**
  - Do the γ_{10} **auto-regressive (AR) effects** "control for stability"?
  - Which type of relation is given by γ_{20} **cross-lagged (CL) effects**?
  - Which type of relation is the **same-occasion** C **covariance**?

* *Same problems apply to mediation variants (X → M → Y)*

Remedies for Intercept Smushing



"Random Intercept" ARCL (or CLPM)

Distinguish BP mean effects from WP residual effects:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + U_{0ix} + e_{tix}$$

$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + U_{0iy} + e_{tiy}$$

But a random intercept alone will **not prevent time-smushing**…

Do the **within-variable AR paths** protect against time smushing?

Let's find out!

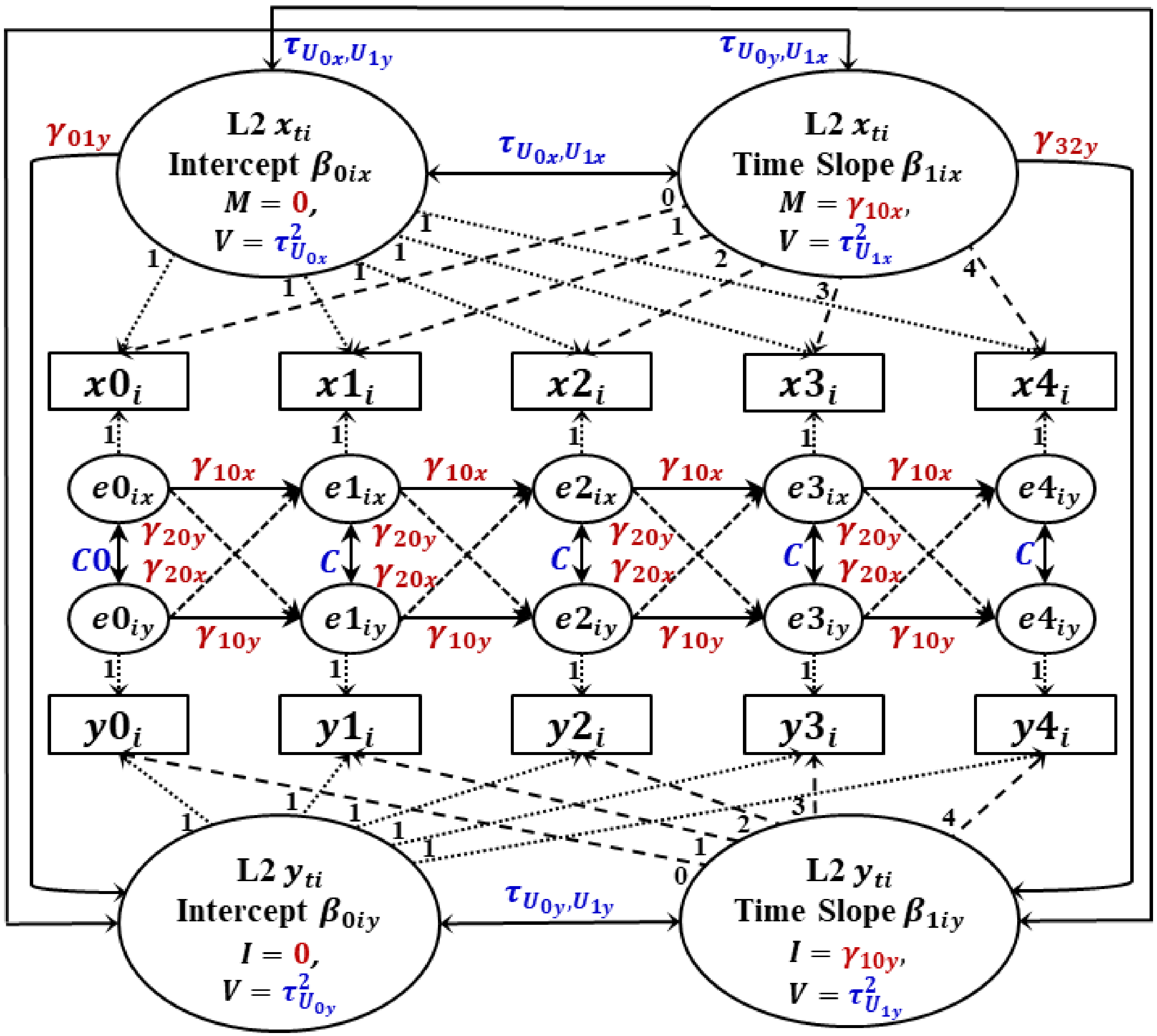
Simulation: Add CLPM Fixed Effects

Full X → Y Model: L2-BP Intercept Effects, L2-BP Time Slope Effects, L1-WP AR Effects, and L1-WP CL Effects	
Total: $x_{tix} = \beta_{0ix} + xw_{tix}$ $y_{tix} = \beta_{0iy} + yw_{tiy}$	w indicates a **L1** *within* variable
L1: $xw_{tix} = \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + \beta_{3ix}(Time_{tix}) + e_{tix}$ $yw_{tiy} = \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + \beta_{3iy}(Time_{tiy}) + e_{tiy}$	
L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + U_{0iy}$	**Intercept → Intercept**
L2 Time Slopes: $\beta_{3ix} = \gamma_{30y} + U_{3iy}$ $\beta_{3iy} = \gamma_{30y} + \gamma_{32y}(\beta_{1ix}) + U_{3iy}$	**Time slope → Time slope**

All L1-WP AR and CL Slopes had population values = 0

**Btw, this is also a "latent curve model with structured residuals"*

Simulation: Compare Model Variants



Full X → Y Model: L2-BP Intercept Effects, L2-BP Time Slope Effects, L1-WP AR Effects, and L1-WP CL Effects*
Drop Time Slope effect
Drop Time Slope, too
Drop Time Slope effect; drop L1 AR Slope
Drop Time Slope, too; drop L1 AR Slope
* *Always included*

Simulation Results: CLPM Fixed Effects

- If a random time slope for x_{ti} was omitted:
  - L1 AR slopes for x_{ti} were very positively biased ($\alpha = .98$)
- If the BP-L2 time slope relation for x_{ti} → y_{ti} was omitted:
  - L1 CL slopes for x_{ti} → y_{ti} were biased in that direction, even more so when including L1 AR slopes for x_{ti}!
  - L1 CL slopes for y_{ti} → x_{ti} had complex patterns of bias
- It seems like WP questions of "which came first" cannot be answered reliably until the BP model is complete
  - Same idea as "detrending" individual time series for time trends *before* looking at time-specific relations across variables
  - **So first check for *random* change in time-varying "predictors"!**

Recommendations for Practice

- **BP time-slope smushing** is a potential problem in longitudinal studies over **ANY TIME SCALE**!
  - "Time" is a more obvious predictor of long-term development
  - "Time" is a less obvious predictor of **short-term WP fluctuation**

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- e.g., L1 days within L2 persons
  - L1 Time = **day of study** for reactivity to measurement?
  - L1 Time = **day of week** for work or family routines?

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  - L1 Time = **day of week** for work or family routines?
- e.g., L1 occasions during the day (in L2 days in L3 persons)
  - L1 Time = **time since waking** for circadian rhythms?
  - L1 Time = **time at work** for functional rhythms?
  - Still need to consider L2 time (day of study, day of week…)

Recommendations for Practice

- **Treat time-varying "predictors" and "outcomes" the same** by starting with univariate models for each to explore *time*:
  - Consider design-informed **fixed effects** of time at ALL relevant levels
  - Consider corresponding **random effects** of time at ALL upper levels
  - Consider remaining **residual relations** (e.g., of adjacent occasions)
- Any **predictor with a random time slope** needs to be **treated as another outcome** in a multivariate model
  - i.e., as latent predictor → model-based partitioning of variances

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- Any **predictor with a random time slope** needs to be **treated as another outcome** in a multivariate model
  - i.e., as latent predictor → model-based partitioning of variances

- **Predictors with fixed effects of time only (no random time)**?
  - Time is controlled for—if you include those effects in outcome model
  - Do have choice of using **observed or latent predictor variables…**

Recommendations for Practice

- **Using <u>latent</u> instead of <u>observed</u> predictors means:**
  - Smaller level-2 samples and smaller ICCs → noisier results
  - SEM: No REML estimation and no denominator DF options
→ too small L2 variances and associated fixed effect SEs
  - Interactions of latent variables → greater estimation complexity
  - Non-normal level-1 variables→ greater estimation complexity

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- **Can Bayes fix it?** *The jury is still out…*
  - If your priors know the right answer, sure!
  - If your variance priors are "too diffuse", bad news!
  - Point estimates for variances: apples and oranges?
  - Useful as alternative to ML given ↑ estimation complexity

Recommendations for Practice

- **But using <u>observed</u> instead of <u>latent</u> predictors means:**
  - **Ignoring BP differences in unreliability** (i.e., caused by differing numbers of occasions or differential WP variance)
  - Result is "Lüdke's bias" → **too-small level-2 effects** (for intercept)
- Can **two-stage** approaches get around this? **Not likely***
  - "Slopes-as-outcomes" cannot be recommended for anything other than time-detrending residuals (but why do just that?)
    - Saved intercepts and time slopes did not provide accurate results here
    - * Corrections for unreliability may have more promise…
- Choosing a software option for **latent predictors** in multivariate MLMs: **Single-level or multilevel SEM…**

Single-Level vs. Multilevel SEM for Fitting Multivariate MLMs

- **Single-level SEM is designed for balanced occasions:**
  - All persons share **common measurement schedule** (or close enough)
  - Absolute fit tests are possible given saturated model covariance matrix
  - Availability of random WP non-time slopes varies by software
  - Structured residuals can create level-2 BP effects only in some cases

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  - Structured residuals can create level-2 BP effects only in some cases
- **Multilevel SEM is more flexible for unbalanced occasions:**
  - Much more realistic, especially for studying short-term fluctuations
  - But no absolute fit tests are provided without a saturated model!
  - Btw, "dynamic" multilevel SEM (in M*plus* terms) just adds options for fitting lagged effects of latent predictors (across rows) with missing data
  - Pay attention to centering methods, especially given random slopes!
    - See Hoffman (2019): EXACT SAME SYNTAX gives different versions of the level-2 parameters when estimated using ML vs Bayes in Mplus 8.0+!
    - This can lead to inadvertent smushing of all kinds using ML… be careful!

Thank you! Suggested Readings:

- Berry, D., & Willoughby, M. (2017). On the practical interpretability of cross-lagged panel models: Rethinking a developmental workhorse. *Child Development, 88*(4), 1186-1206.
- Curran, P. J., & Bauer, D.J. (2011). The disaggregation of within-person and between-person effects in longitudinal models of change. *Annual Review of Psychology 62*(1), 583-619.
- Curran, P. J., Howard, A. L., Bainter, S. A., Lane, S. T., & McGinley, J. S. (2014). The separation of between-person and within-person components of individual change over time: A latent curve model with structured residuals. *Journal of Consulting and Clinical Psychology, 82*(5), 879-894.
- De Haan-Rietdijk, S., Kuppens, P., & Hamaker, E. L. (2016). What's in a day? A guide to decomposing the variance in intensive longitudinal data. *Frontiers in Psychology: Quantitative Psychology and Measurement*, 7, Article 891: https://doi.org/10.3389/fpsyg.2016.00891
- Hoffman, L. (2015). *Longitudinal analysis: Modeling within-person fluctuation and change.* New York, NY: Routledge Academic.
- Hoffman, L. (2019). On the interpretation of parameters in multivariate multilevel models across different combinations of model specification and estimation. *Advances in Methods and Practices in Psychological Science, 2*(3), 288-311.
- Lüdtke, O., Marsh, H. W., Robitzsch, A., Trautwein, U., Asparouhov, T., & Muthén, B. (2008). The multilevel latent covariate model: A new, more reliable approach to group-level effects in contextual studies. *Psychological Methods, 13*(3), 203-229.
- McNeish, D., & Hamaker, E. L. (2020). A primer on two-level dynamic structural equation models for Intensive Longitudinal Data in Mplus. *Psychological Methods*, 25(5), 610-635.