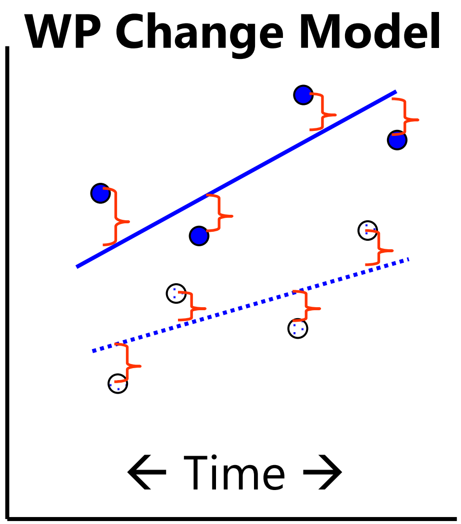
Time-Varying Predictors and Their Levels of Relations in Longitudinal Models

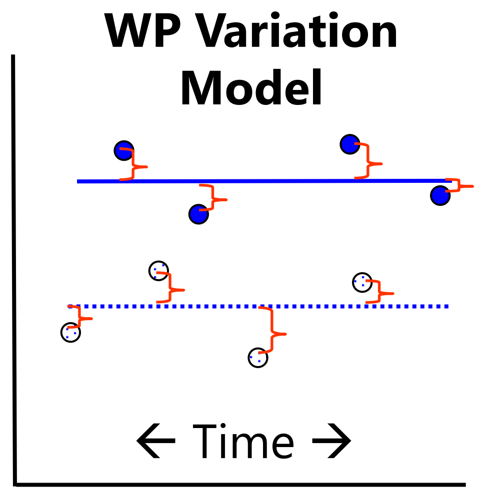
- Topics:
  - Time-varying predictors in models of fluctuation
    - **Concepts and what NOT to do in MLMs**
    - Univariate MLM: Person-(group)-mean-centering (+Example 5)
    - Univariate MLM: Grand-mean-(constant)-centering
  - Multivariate MLM (via SEM and M-SEM): Latent centering of time-varying predictors and models of change
  - Also what not to do: path models for longitudinal data with smushed effects (and how to fix it)

The Joy of Time-Varying Predictors

- TV predictors predict leftover **WP (residual) variation**:



If model for time works, then residuals should look like this →



- Modeling time-varying predictors is complicated because they represent an **aggregated effect**:
  - Effect of the *between-person* variation in the predictor x_{ti} on y_{ti}
  - Effect of the *within-person* variation in the predictor x_{ti} on y_{ti}
  - For now, we are assuming the predictor x_{ti} only **fluctuates** over time…
    - *We will need a **different model** when x_{ti} changes individually over time…*

The Joy of Time-Varying Predictors

- Time-varying (TV) predictors usually carry 2 kinds of effects because **they are really 2 predictor variables, not 1**

- Example: Stress measured daily
  - Some days are worse than others:
    - **WP variation in stress** *(represented as deviation from own mean)*
  - Some people just have more stress than others all the time:
    - **BP variation in stress** *(represented as person mean predictor over time)*

- Can quantify each source of variation with an **ICC**
  - ICC = (BP variance) / (BP variance + WP variance)
  - **ICC > 0?** TV predictor has **BP** variation (so it *could* have a BP effect)
  - **ICC < 1?** TV predictor has **WP** variation (so it *could* have a WP effect)

Between-Person vs. Within-Person Slopes

- Between-person and within-person slopes in <u>SAME</u> direction
  - Stress → Health?
    - **BP: People with more chronic stress than other people may have <u>worse</u> general health than people with less chronic stress**
    - **WP: People may feel <u>worse</u> than usual when they are currently under more stress than usual (regardless of what "usual" is)**

- Between-person and within-person slopes in <u>OPPOSITE</u> directions
  - Exercise → Blood pressure?
    - **BP: People who exercise more often generally have <u>lower</u> blood pressure than people who are more sedentary**
    - **WP: During exercise, blood pressure is <u>higher</u> than during rest**

- Variables have different **meanings** at different levels!
- Variables have different **scales** at different levels!

3 Kinds of Fixed Slopes for TV Predictors

- **Is the Level-2 Between-Person (BP) slope significant?**
  - Are people with higher predictor values <u>than other people</u> (*on average over time*) also higher on Y <u>than other people</u> (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?

- **Is the Level-1 Within-Person (WP) slope significant?**
  - If you have higher predictor values <u>than usual</u> (*at this occasion*), do you also have higher outcomes values <u>than usual</u> (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ^2_e)?

- **Are BP and WP slopes different : Is there a level-2 contextual effect?**
  - After controlling for the absolute value of TV predictor at each occasion, is there still <u>an incremental contribution from having a higher person mean</u> of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond)?
  - If there is no contextual effect, then the BP and WP effects of the TV predictor show <u>*convergence*</u>, such that their effects are of equivalent magnitude

WRONG WAY: Within-Person Fluctuation Model with x_{ti} **represented at Level 1 Only:** → Its WP and BP Slopes are **Smushed Together**

x_{ti} is grand-mean-centered into TVx_{ti}, WITHOUT PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

$TVx_{ti} = x_{ti} - C$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}}$

Because TVx_{ti} still contains its original 2 different kinds of variation (BP and WP), its 1 fixed slope has to do the work of 2 predictors!

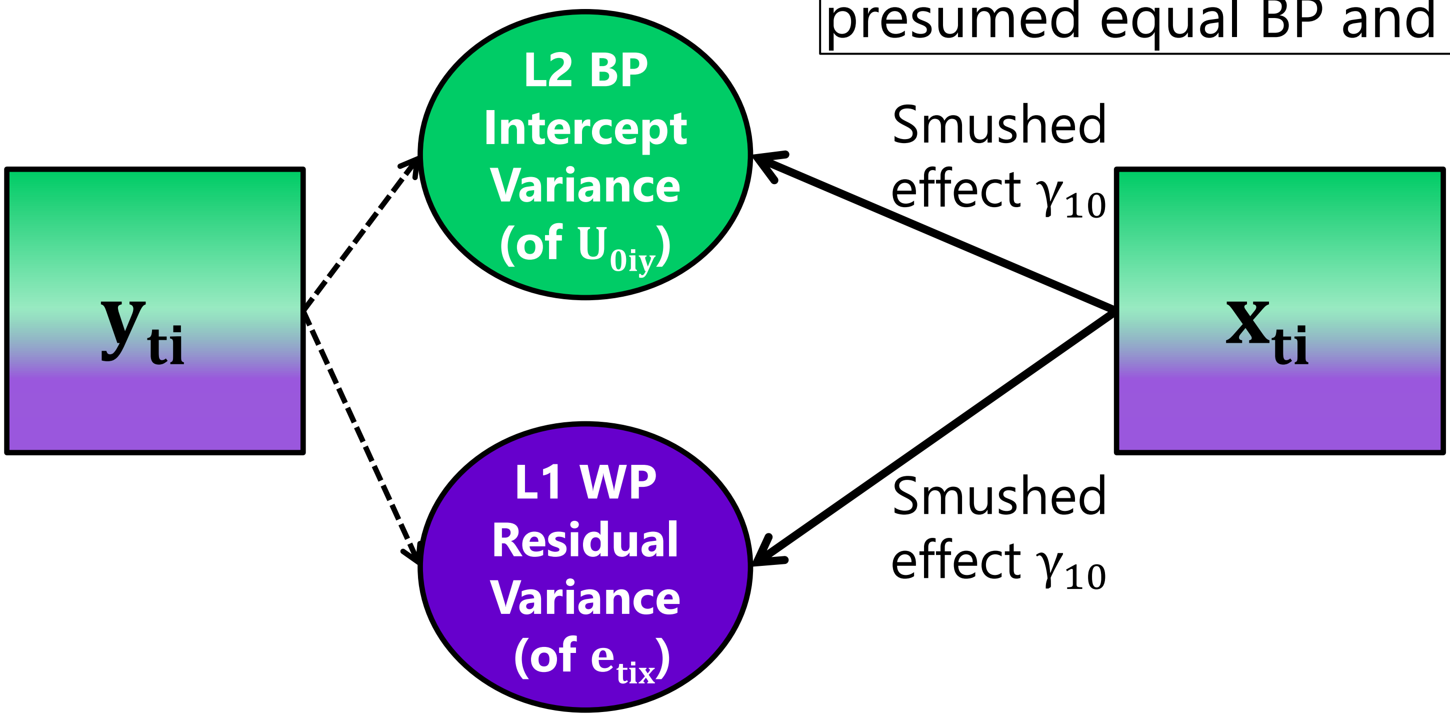
γ_{10} = *smushed* WP and BP effects

A *smushed* effect is also referred to as the *convergence*, *conflated*, or *composite* effect

Univariate MLM: Level-1 Predictor Without Level-2 Predictor = Smushing

Model-based partitioning of level-1 y_{ti} outcome variance into estimated **variance components**:

Observed level-1 x_{ti} has not been partitioned – AND – it has only **one fixed slope** in the model. Thus, that smushed effect reflects presumed equal BP and WP slopes.



Time-Varying Predictors and Their Levels of Relations in Longitudinal Models

- Topics:
  - Time-varying predictors in models of fluctuation
    - Concepts and what NOT to do in MLMs
    - **Univariate MLM: Person-(group)-mean-centering (Example 5)**
    - Univariate MLM: Grand-mean-(constant)-centering
  - Multivariate MLM (via SEM and M-SEM): Latent centering of time-varying predictors and models of change
  - Also what not to do: path models for longitudinal data with smushed effects (and how to fix it)

Modeling TV Predictors (x_{ti}) in Univariate MLM

- **Level-2 effect of x_{ti}:**
  - The level-2 effect of x_{ti} is usually represented by the person's mean of time-varying x_{ti} across time (to be labeled as **PMx$_i$** or $\bar{\mathbf{X}}_\mathbf{i}$)
  - **PMx$_i$** should be centered at a CONSTANT (grand mean or other) so that 0 is meaningful, just like any other time-invariant predictor

- **Level-1 effect of x_{ti} can be included in two different ways:**
  - "**Group-mean-centering**" → "**person-mean-centering**" in longitudinal data, in which level-1 predictors are centered using a level-2 VARIABLE
    - I call this "**variable-centering**" because the key idea is the subtraction of a variable
  - "**Grand-mean-centering**" → level-1 predictors are centered using a CONSTANT (not necessarily the grand mean; that's just most common)
    - I call this "**constant-centering**" because which constant you pick does NOT matter
  - Note that these 2 choices do NOT apply to the level-2 effect of x_{ti}
    - But the interpretation of the level-2 effect of x_{ti} WILL DIFFER based on which centering method you choose for the level-1 effect of x_{ti}!

Person-Mean-Centering (P-MC)

- In P-MC, we partition the TV predictor x_{ti} into 2 variables that directly represent its BP (level-2) and WP (level-1) sources of variation and **include these variables as the predictors instead**:

- **Level-2, PM predictor = person mean of x_{ti}**
  - $\mathbf{PMx_i} = \mathbf{\bar{X}_i} - \boldsymbol{C_2}$
  - $\mathrm{PMx_i}$ is centered at constant C_2, chosen for meaningful 0 (e.g., sample mean)
  - $\mathrm{PMx_i}$ is positive? Above sample mean → "more than other people"
  - $\mathrm{PMx_i}$ is negative? Below sample mean → "less than other people"

- **Level-1, WP predictor = deviation from person mean of x_{ti}**
  - $\mathbf{WPx_{ti}} = \mathbf{x_{ti}} - \mathbf{\bar{X}_i}$ *(note: uncentered person mean $\boldsymbol{\bar{X}_i}$ is used to center x_{ti})*
  - $\mathrm{WPx_{ti}}$ is NOT centered at a constant – it is centered at a VARIABLE
  - $\mathrm{WPx_{ti}}$ is positive? Above your own mean → "more than usual"
  - $\mathrm{WPx_{ti}}$ is negative? Below your own mean → "less than usual"

Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 x_{ti}**

→ WP and BP slopes directly through **separate** parameters

x_{ti} is person-mean-centered into WPx_{ti}, with PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$

$WPx_{ti} = x_{ti} - \bar{X}_i$ → it has only Level-1 WP variation

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$PMx_i = \bar{X}_i - C_2$ → it has only Level-2 BP variation

γ_{10} = WP main effect of having *more x_{ti} than usual*

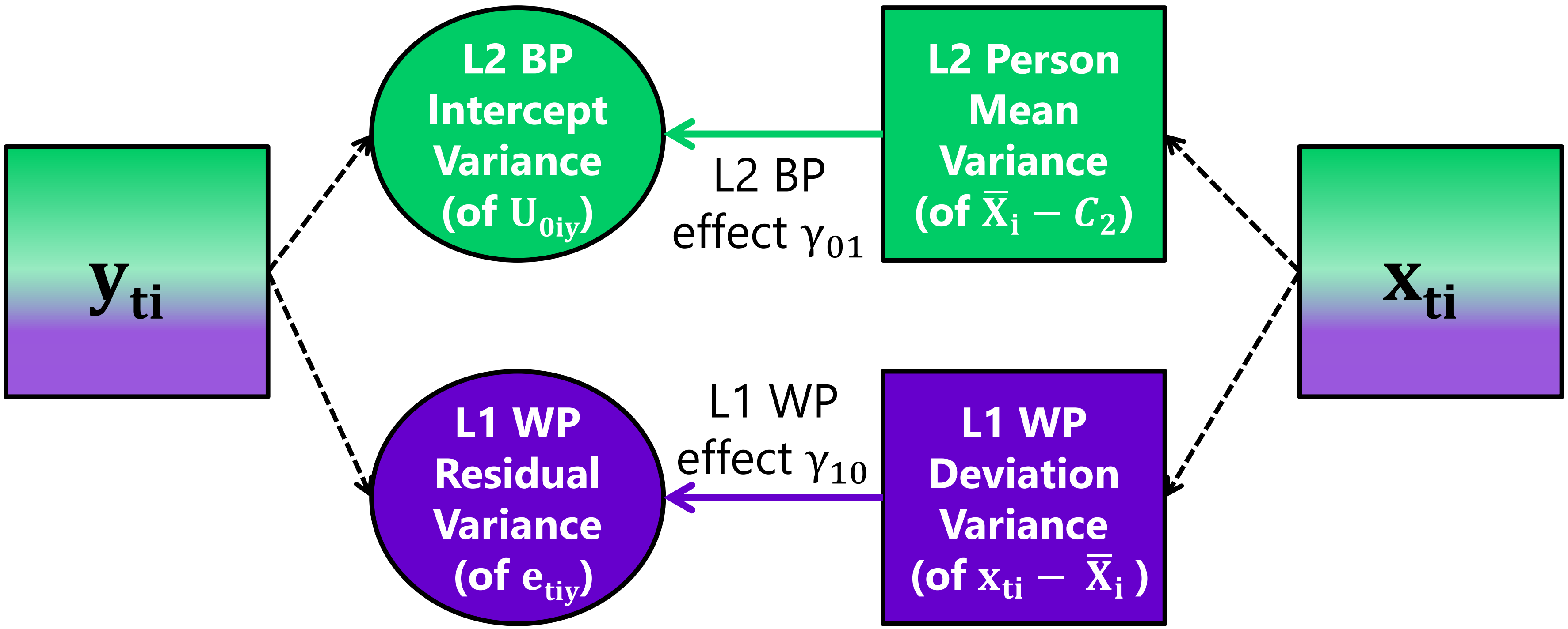
γ_{01} = BP main effect of having *more $\bar{X}_i$ than other people*

Because WPx_{ti} and PMx_i are uncorrelated, each gets the total effect for its level (WP=L1, BP=L2)

Univariate MLM: Variable-Centering

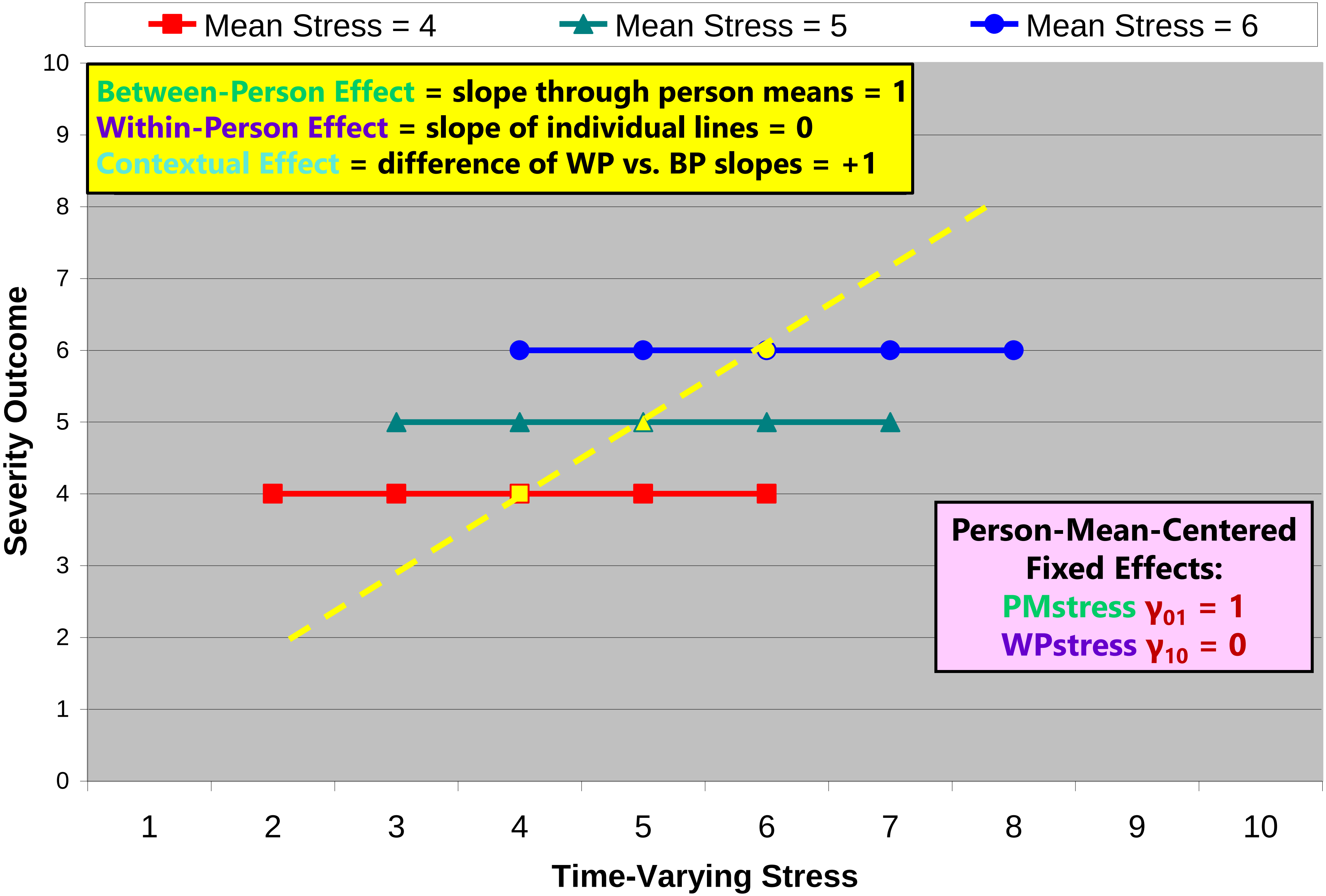
Model-based partitioning of level-1 y_{ti} outcome variance into estimated **variance components**:

Brute-force partitioning of level-1 x_{ti} predictor variance into **observed variables**:

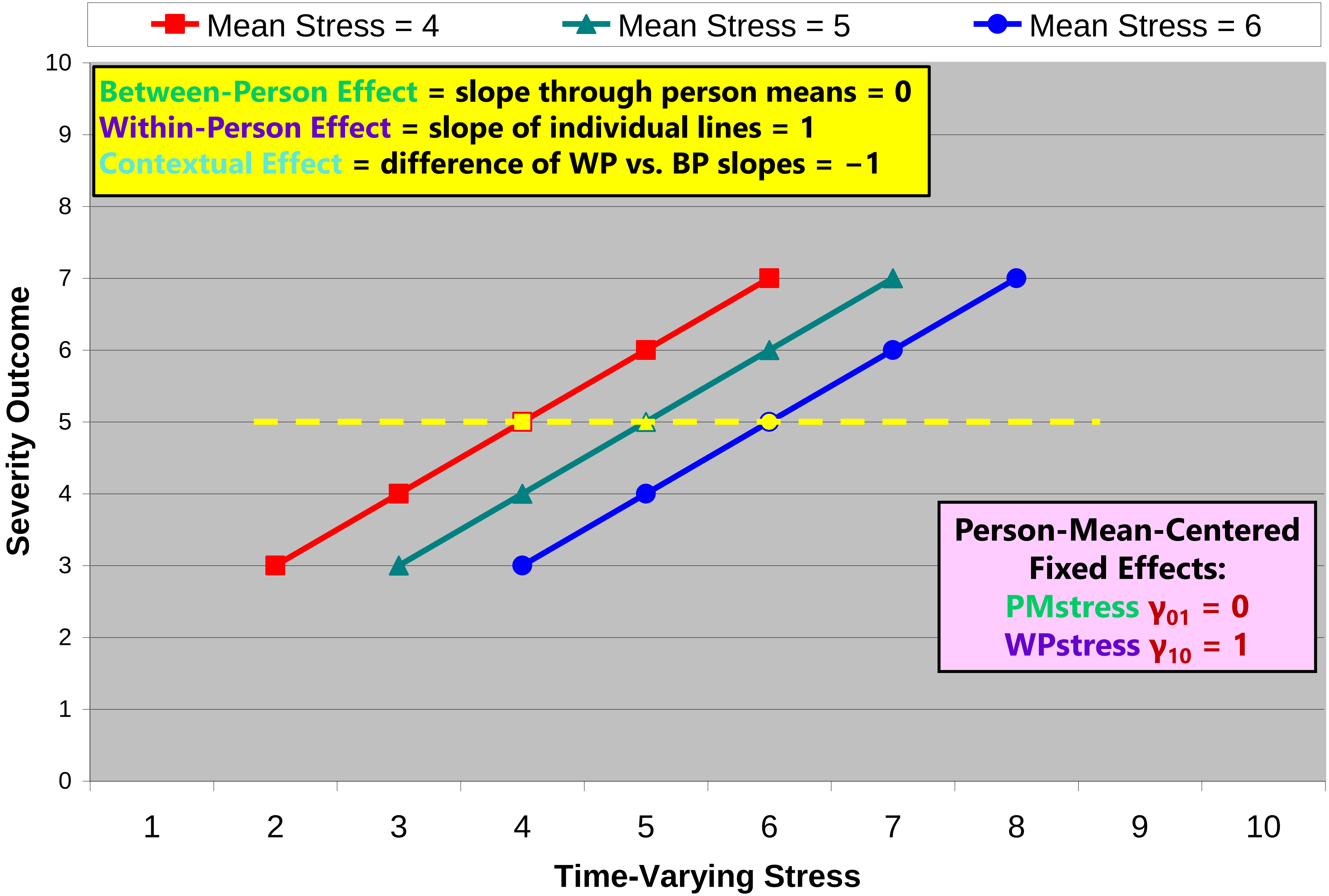


Why not let the model estimate variance components for x_{ti}, too?
This is the basis of multivariate MLM (or "multilevel SEM" = M-SEM).

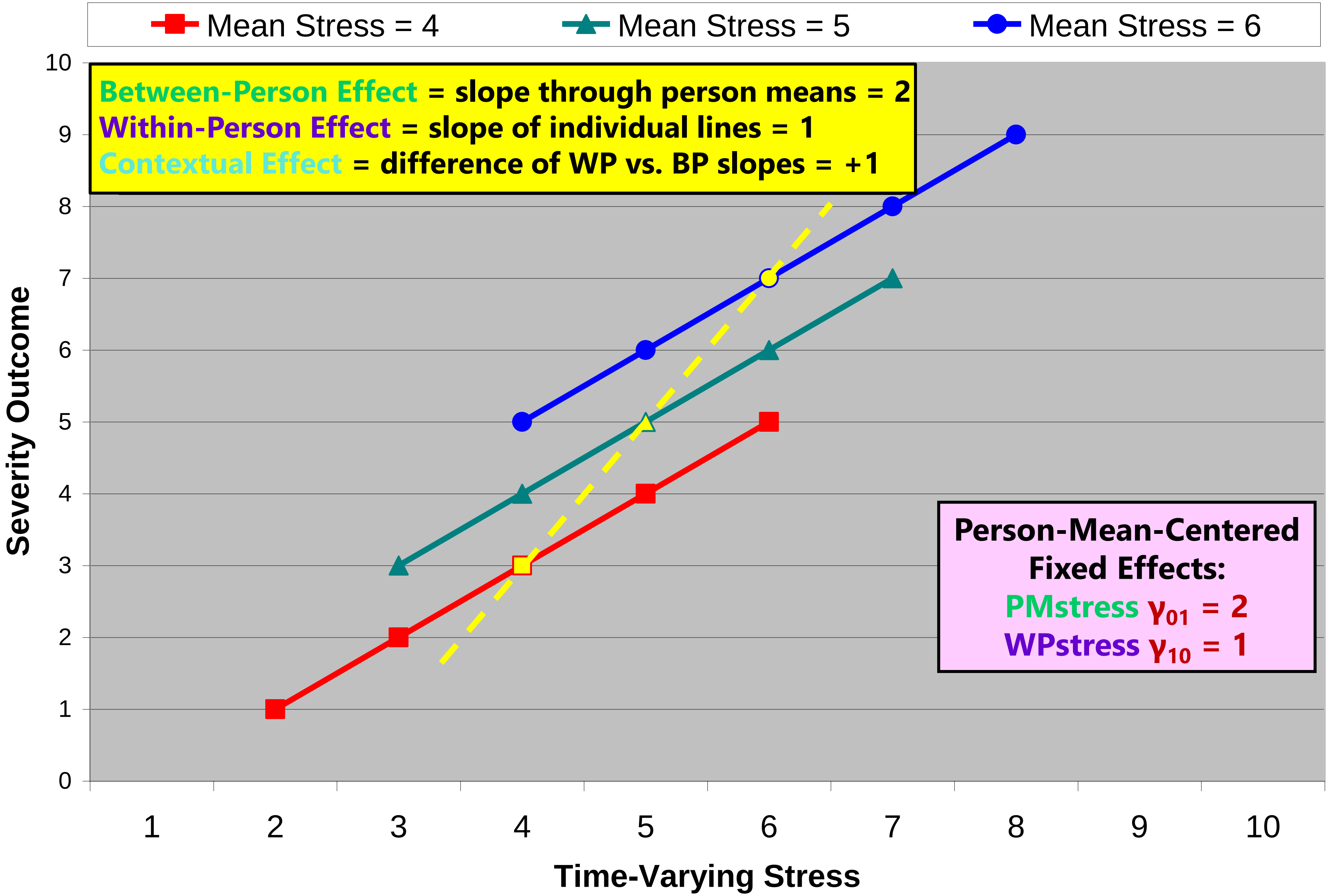
ALL Between-Person Effect, NO Within-Person Effect



NO Between-Person Effect, ALL Within-Person Effect

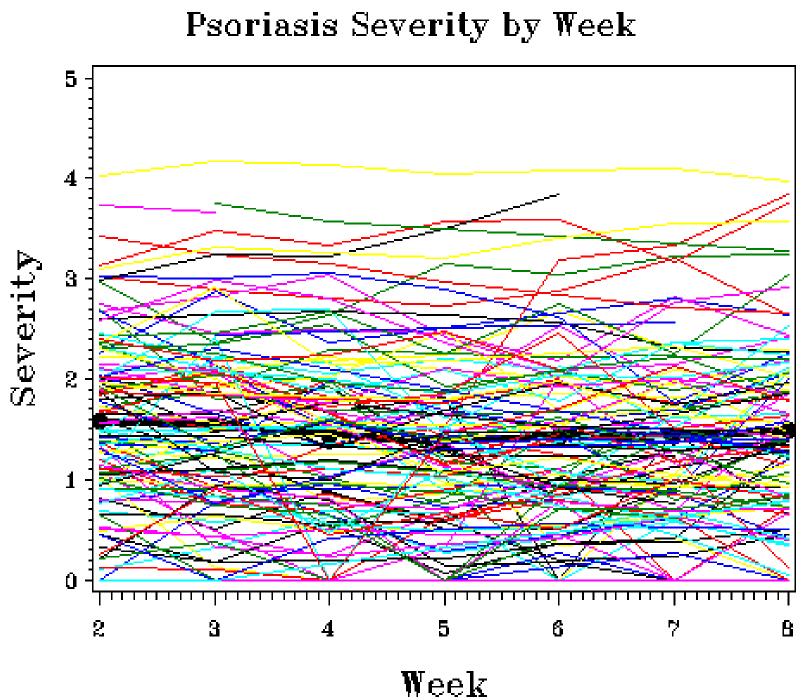


Between-Person Effect > Within-Person Effect



Example 5: Weekly Stress and Psoriasis

- 127 psoriasis patients, 8 weekly assessments (only last 7 used)
- How does perceived stress predict psoriasis severity?
  Is there a time lag for these processes to occur?
- No change in treatment → only fluctuation over time
- Analysis plan:
  - ICCs for stress and severity—how much variance is at each level?
  - Assess pattern of variance and covariance in severity over time
  - Evaluate prediction of severity by stress at lag 0 and lag 1 weeks… without smushing!



Example 5: Weekly Stress and Psoriasis

- Empty means, random intercept model to get ICCs → proportion of total variance due to BP mean differences
  - For each variable: $y_{ti} = \gamma_{00} + U_{0i} + e_{ti}$, $\textbf{ICC} = \dfrac{\tau_{U_0}^2}{\tau_{U_0}^2 + \sigma_e^2} = \dfrac{\textbf{BP}}{\textbf{BP+WP}}$
  - Severity outcome: ICC = .83; stress predictor: ICC = .56

- For the severity outcome, the best-fitting unconditional time model for the variance had a level-2 random intercept (in G), along with heterogeneous level-1 residual variances and a Toeplitz (banded) correlation structure up to lag 3 (in R, below)

Estimated R Correlation Matrix for ID 1 → WP residual correlation

Row	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1.0000	0.5115	0.3566	0.1112			
2	0.5115	1.0000	0.5115	0.3566	0.1112		
3	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112	
4	0.1112	0.3566	0.5115	1.0000	0.5115	0.3566	0.1112
5		0.1112	0.3566	0.5115	1.0000	0.5115	0.3566
6			0.1112	0.3566	0.5115	1.0000	0.5115
7				0.1112	0.3566	0.5115	1.0000

Example 5: Weekly Stress and Psoriasis

Level 1: $\text{severity}_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{WPstressL0}_{ti})$
$+ \beta_{2i}(\mathbf{WPstressL1}_{ti}) + \mathbf{e}_{ti}$

Level 2: $\beta_{0i} = \boldsymbol{\gamma}_{00} + \boldsymbol{\gamma}_{01}(\mathbf{PMstress}_i - 2) + \mathbf{U}_{0i}$

$\beta_{1i} = \boldsymbol{\gamma}_{10}$

$\beta_{2i} = \boldsymbol{\gamma}_{20}$

WP effects are <u>fixed</u> (no random slopes) → same for everyone

$\mathbf{WPx}_{ti} = \mathbf{x}_{ti} - \bar{\mathbf{X}}_i$ **→ it has only Level-1 WP variation**

$\mathbf{PMx}_i = \bar{\mathbf{X}}_i - C_2$ **→ it has only Level-2 BP variation**

Model for the Means:

- γ_{00} → expected severity for someone with person mean stress = 2, and who had severity = 2 last week and currently
- γ_{01} → BP difference in *average* severity per unit person mean stress
- γ_{10} and γ_{20} → WP change in *current* severity per unit more stress than usual this week (lag 0) and last week (lag 1)

Example 5: Weekly Stress and Psoriasis

Level 1: $\text{severity}_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{WPstressL0}_{ti})$
$+ \beta_{2i}(\mathbf{WPstressL1}_{ti}) + \mathbf{e}_{ti}$

Level 2: $\beta_{0i} = \mathbf{1.96} + \mathbf{0.48{*}}(\mathbf{PMstress}_i - 2) + \mathbf{U}_{0i}$

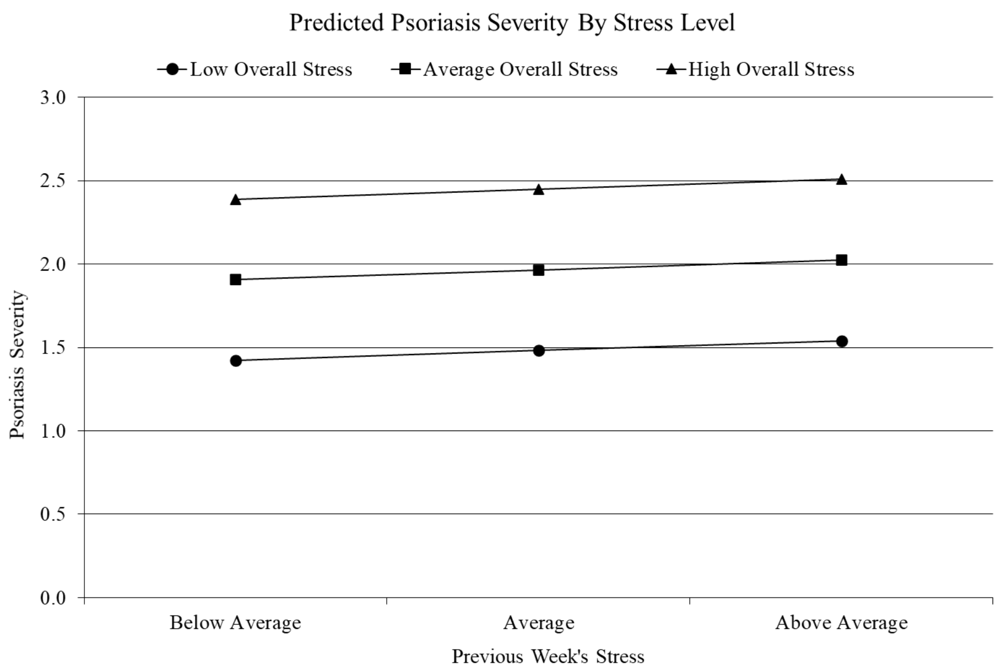
$\beta_{1i} = \mathbf{0.02}$

$\beta_{2i} = \mathbf{0.06{*}}$

WP effects are fixed (no random slopes) → same for everyone

$\mathbf{WPx}_{ti} = \mathbf{x}_{ti} - \bar{\mathbf{X}}_i$ **→ it has only Level-1 WP variation**

$\mathbf{PMx}_i = \bar{\mathbf{X}}_i - C_2$ **→ it has only Level-2 BP variation**



Example 5: Syntax by Univariate MLM Program (Stacked Data)

SAS:

```
PROC MIXED DATA=work.Example5 COVTEST METHOD=REML;
     CLASS ID;
     MODEL severity = PMstress WPstressL0 WPstressL1 / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / TYPE=UN SUBJECT=ID;
     REPEATED week / RCORR TYPE=TOEPH(4) SUBJECT=ID;
RUN;
```

R (lmer from lme4 package)—using lmertest package, which does provide correct denominator DF, but custom R matrix structures are not available (might be possible using gls from nlme instead), so RI only here:

```
model5 = lmer(data=Example5, REML=TRUE,
         formula=severity~1+PMstress+WPstressL0+WPstressL1+(1+|ID))
summary(model5, ddf="Satterthwaite")
```

STATA—don't think custom Toeplitz structure with heterogeneous residual variances is possible, so used RI + a homogeneous residual variance version here:

```
mixed severity PMstress WPstressL0 WPstressL1, || ID: , ///
      variance reml covariance(un) residuals(toeplitz3,t(week)) ///
      dfmethod(satterthwaite) dftable(pvalue)
```

SPSS—don't think custom Toeplitz structure with heterogeneous variances is possible, so RI only here :

```
MIXED severity BY ID WITH PMstress WPstressL0 WPstressL1
      /METHOD = REML
      /PRINT = SOLUTION TESTCOV
      /FIXED = PMstress WPstressL0 WPstressL1
      /RANDOM = INTERCEPT | COVTYPE(UN) SUBJECT(ID).
```

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3 Kinds of Fixed Slopes for TV Predictors

- **First 2 slopes Person-Mean-Centering tells us <u>directly</u>:**

- **Is the Level-2 Between-Person (BP) effect significant?**
  - Are people with higher predictor values <u>than other people</u> (*on average over time*) also higher on Y <u>than other people</u> (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
  - This would be indicated by a significant fixed slope of $\mathbf{PMx_i}$
  - Note: this is NOT controlling for the original value of x_{ti} at each occasion

- **Is the Level-1 Within-Person (WP) effect significant?**
  - If you have higher predictor values <u>than usual</u> (*at this occasion*), do you also have higher outcomes values <u>than usual</u> (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ^2_e)?
  - This would be indicated by a significant fixed slope of $\mathbf{WPx_{ti}}$
  - Note: this is represented by the <u>relative</u> value of x_{ti}, NOT the <u>original</u> value of x_{ti}

3rd Kind of Slope for TV Predictors

- **What Person-Mean-Centering DOES NOT tell us <u>directly</u>:**

- **Do the BP and WP slopes differ: Is there a level-2 contextual effect?**
  - After controlling for the original value of the TV predictor at that occasion, is there still <u>an incremental contribution from having a higher person mean of the TV predictor</u> (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond just the time-specific value of the predictor)?
  - If there is no contextual effect, then the TV predictor's BP and WP slopes show <u>***convergence***</u>, such that their effects are of equivalent magnitude

- **To answer this question about the level-2 contextual effect for the incremental contribution of the person mean, we have two options:**
  - Ask for the contextual effect as a linear combination (via SAS ESTIMATE, R glht, SPSS TEST, STATA LINCOM, or Mplus NEW), like this: $\mathbf{WPx_{ti}}$ **−1** $\mathbf{PMx_{i}}$ **1**
  - Use "<u>**constant-centering**</u>" for time-varying x_{ti} instead: $\mathbf{TVx_{ti} = x_{ti} - \boldsymbol{C_1}}$
    → **centered at CONSTANT $\boldsymbol{C_1}$, NOT A LEVEL-2 VARIABLE**
    - Which constant only matters for the reference point; it could be the grand mean or any (even 0)

Why the Difference in the Level-2 Effect? Remember Regular Old Regression…

- In this model: $y_i = \beta_0 + \beta_1(x1_i) + \beta_2(x2_i) + e_i$
- If $x1_i$ and $x2_i$ **ARE NOT** correlated:
  - β_1 carries **ALL the relationship** between $x1_i$ and y_i
  - β_2 carries **ALL the relationship** between $x2_i$ and y_i

- If $x1_i$ and $x2_i$ **ARE** correlated:
  - β_1 is **different than** the bivariate relationship between $x1_i$ and y_i
    - "Unique" effect of $x1_i$ *controlling for* $x2_i$ (or *holding* $x2_i$ *constant)*
  - β_2 is **different than** the bivariate relationship between $x2_i$ and y_i
    - "Unique" effect of $x2_i$ *controlling for* $x1_i$ (or *holding* $x1_i$ *constant)*

- Hang onto that idea…

Person-MC vs. Grand-MC: Variable vs. Constant-Centering for TV Predictors

Level 2		Original	Person-MC Level 1	Grand-MC Level 1
$\bar{X}_i$	$PMx_i = \bar{X}_i - 5$	x_{ti}	$WPx_{ti} = x_{ti} - \bar{X}_i$	$TVx_{ti} = x_{ti} - 5$
3	−2	2	−1	−3
3	−2	4	1	−1
7	2	6	−1	1
7	2	8	1	3

Same PMx_i goes into the model using either way of centering the level-1 variable x_{ti}

Using **Person-MC**, WPx_{ti} has NO level-2 BP variation, so it is not correlated with PMx_i

Using **Grand-MC**, TVx_{ti} STILL has level-2 BP variation, so it is STILL CORRELATED with PMx_i

So the effects of PMx_i and TVx_{ti} when included together under Grand-MC will be different than their effects would be if they were by themselves…

Within-Person Fluctuation Model with **Constant-Centered Level-1 x_{ti}**

→ Model tests difference of WP vs. BP slopes (it's been fixed!)

x_{ti} is constant-centered into TVx_{ti}, <u>WITH</u> PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(TVx_{ti}) + e_{ti}$

$TVx_{ti} = x_{ti} - C_1$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$PMx_i = \bar{X}_i - C_2$ → it has only Level-2 BP variation

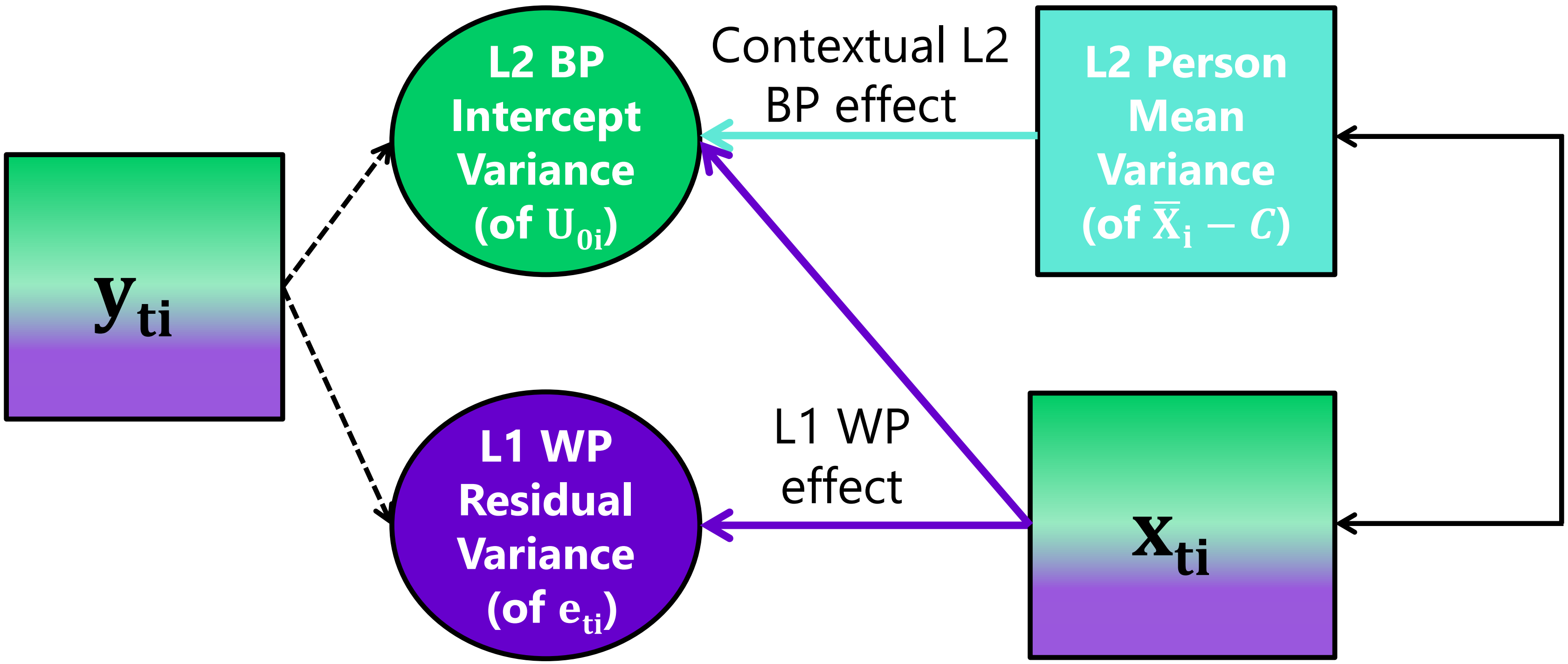
γ_{10} becomes the WP slope → *unique* level-1 effect after controlling for PMx_i

γ_{01} *becomes* the contextual effect that indicates <u>how the BP effect differs from the WP effect</u>
→ *unique* level-2 slope after controlling for TVx_{ti}
→ does usual level matter beyond current level?

Univariate: Constant-Centering WITH Level-2 Predictor = OK NOW!

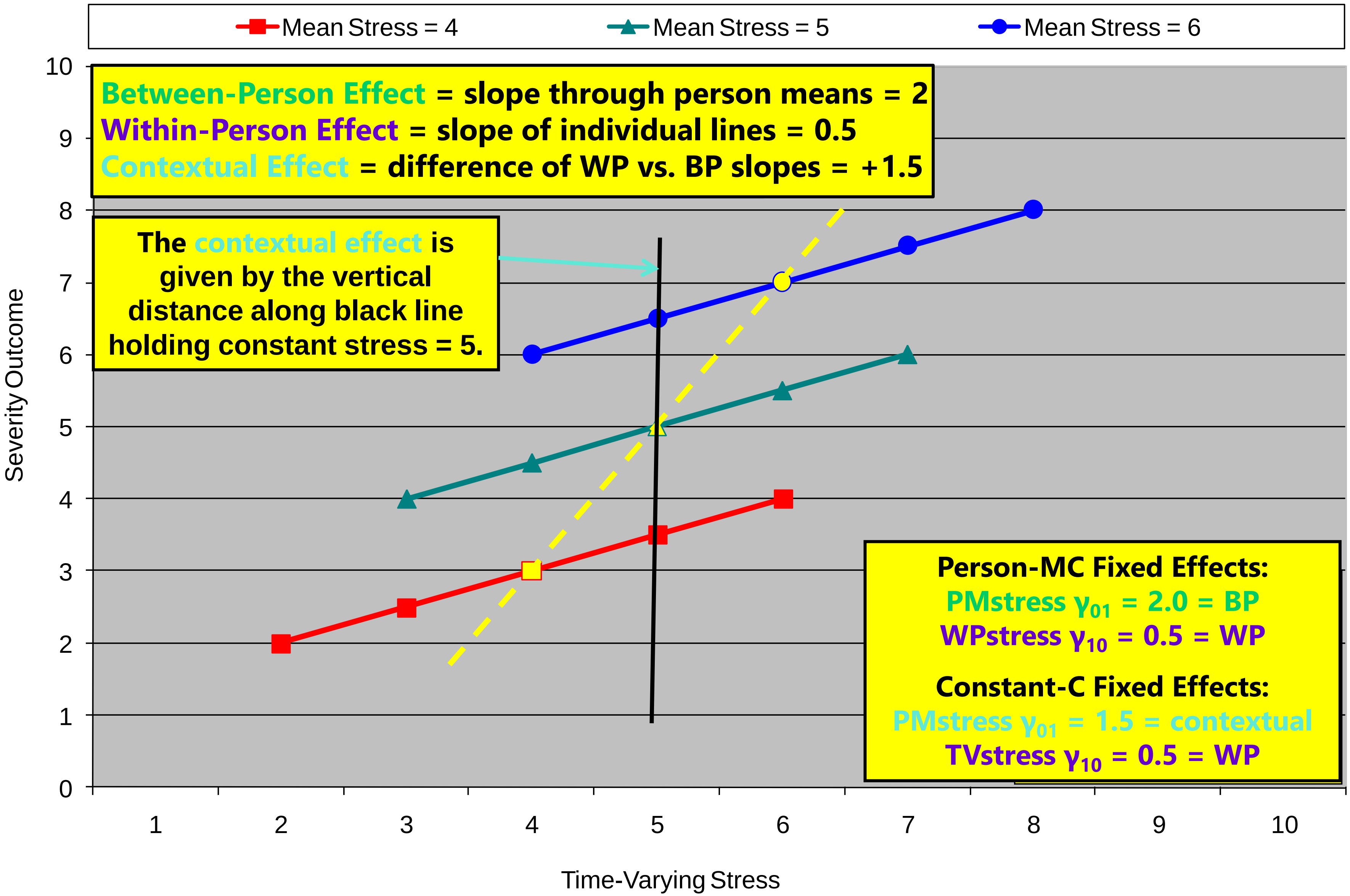
Model-based partitioning of y_{ti} outcome into estimated **variance components**:

Level-1 x_{ti} is still not partitioned, but person mean $\mathbf{\bar{X}_i} - \boldsymbol{C_2}$ is added to allow an extra (different) effect at L2.



Because original x_{ti} still has BP variance, it still carries *part* of the BP effect…

P-MC vs. Constant-C: Interpretation Example



Person-MC and Constant-C Models are Equivalent Given a Fixed Level-1 Main Effect Slope Only

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$

$\rightarrow y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Composite Model:
← In terms of P-MC
← In terms of Const-C

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Effect	P-MC	Const-C
Intercept	γ_{00}	γ_{00}
WP Effect	γ_{10}	γ_{10}
Contextual	$\gamma_{01} - \gamma_{10}$	γ_{01}
BP Effect	γ_{01}	$\gamma_{01} + \gamma_{10}$

When Person-MC ≠ Constant-Centering: Random Slopes of TV Predictors

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to PMx_i is removed from the random slope in Person-MC.

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + U_{1i}(x_{ti} - PMx_i) + e_{ti}$

Constant-C: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

PMx_i variance is still part of the Const-C random slope → smushed random effect! Thus, the level-1 predictor to be given a random slope should be P-MC to prevent this problem.

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + U_{1i}(x_{ti}) + e_{ti}$

Modeling Time-Varying Categorical Predictors

- Person-MC and Constant-C usually refer to *quantitative* TV predictors, but the need to consider BP and WP effects applies to *categorical* TV predictors too
- Binary level-1 predictors do not lend themselves intuitively to Person-MC
  - e.g., x_{ti} = 0 or 1 per occasion, person mean = .50 across occasions → impossible values (if x_{ti} = 0, then WPx_{ti} = 0 − .50 = − 0.50; if x_{ti} = 1, then WPx_{ti} = 1 − .50 = 0.50)
  - Better: Leave x_{ti} uncentered in estimating its fixed slope and include person mean as level-2 predictor so that results ~ Const-C (but still use P-MC in estimating its random slope)
- For >2 categories, person means of multiple dummy codes starts to break down, but we can think about types of people, and code BP effects accordingly

- <u>Example: Dementia present/not at each time point?</u>
  - **BP effects →** Ever diagnosed with dementia (no, yes)?
    - People who will eventually be diagnosed may differ prior to diagnosis (a BP effect)
  - **TV effect →** Diagnosed with dementia at each time point (no, yes)?
    - Acute differences of before/after diagnosis logically can only exist in the "ever" people

- Other examples: Mentor status, father absence, type of shift work (AM/PM)

Summary: Univariate MLM for Specifying Effects of Time-Varying Predictors

- "Univariate" approach to MLM is appropriate for time-varying predictors that *fluctuate* over time (and lower-level predictors with only mean differences across higher levels in general)

- Level-1 predictor can be created two different ways:
  - Easier to understand is variable-centering: $\mathbf{WPx_{ti}} = \mathbf{x_{ti}} - \mathbf{\bar{X}_i}$
    - Directly isolates level-1 within variance, so $\mathbf{WPx_{ti}}$ → within effects
  - More common is constant-centering: $\mathbf{TVx_{ti}} = \mathbf{x_{ti}} - \boldsymbol{C_1}$
    - Does NOT isolate level-1 within variance, so $\mathbf{TVx_{ti}}$ will have smushed between/within effects unless it is paired with level-2 predictor analog

- Level-2 predictor is always constant-centered: $\mathbf{PMx_i} = \mathbf{\bar{X}_i} - \boldsymbol{C_2}$
  - $\mathbf{PMx_i}$ indicates **between** effect when paired with $\mathbf{WPx_{ti}}$
  - $\mathbf{PMx_i}$ indicates **contextual** effect when paired with $\mathbf{TVx_{ti}}$
    - Within + Contextual = Between; Between − Within = Contextual

I Prefer Variable-Centering…

- …because constant-centering is much easier to screw up! ☺
- See Table 1 from: Hoffman, L., & Walters, R. W. (2022). Catching up on multilevel modeling. *Annual Review of Psychology, 73*, 629-658.

Table 1 Predictor effect type by model specification

Centering strategy for level-1 predictor (constant-centered level-2 predictor)	Fixed effect type by predictors included: Level-1 only	Level-2 only	Both levels
Variable-centered level-1			
Level-1 predictor: $L1x_{wb} = x_{wb} - \bar{x}_b$	Within	(= 0)	Within
Level-2 predictor: $L2x_b = \bar{x}_b - C_2$	(= 0)	Between	Between
Constant-centered level-1			
Level-1 predictor: $L1x_{wb} = x_{wb} - C_1$	Smushed	(= 0)	Within
Level-2 predictor: $L2x_{wb} = \bar{x}_b - C_2$	(= Within)	Between	Contextual

Abbreviations: w, within; b, between; C_1, level-1 centering constant; C_2, level-2 centering constant. Parentheses indicate assumptions about the fixed slopes of omitted predictors.

Variance Accounted For By Level-1 Predictors

- **Fixed effects of level 1 predictors *by themselves*:**
  - Level-1 (WP) main effects reduce Level-1 (WP) residual variance
  - Level-1 (WP) interactions also reduce Level-1 (WP) residual variance

- **What happens at level 2 depends on what kind of variance the level-1 predictor has:**
  - If the level-1 predictor ALSO has level-2 variance (e.g., Constant-C predictors), then its level-2 variance will also likely reduce level-2 random intercept variance
  - If the level-1 predictor DOES NOT have level-2 variance (e.g., Person-MC predictors), then its reduction in the level-1 residual variance will cause an INCREASE in level-2 random intercept variance
    - Same thing happens with constant-C level-1 predictors, but you don't generally see it
  - It's just an artifact that the estimate of true random intercept variance is:

    True $\tau^2_{U_0}$ = observed $\tau^2_{U_0} - \frac{\sigma^2_e}{n}$ → so if only σ^2_e decreases, $\tau^2_{U_0}$ increases

The Joy of Interactions Involving Time-Varying Predictors

- **<u>Must consider interactions with both its BP and WP parts:</u>**
- Example: Does time-varying stress (x_{ti}) interact with sex (Sex_i)?

- <u>Person-Mean-Centering:</u>
  - $\mathbf{WPx_{ti}} * \mathbf{Sex_i}$ → Does the WP stress effect differ between men and women?
  - $\mathbf{PMx_i} * \mathbf{Sex_i}$ → Does the BP stress effect differ between men and women?
    - Not controlling for current levels of stress
    - If forgotten, then $\mathbf{Sex_i}$ moderates the stress effect only at level 1 (WP, not BP)

- <u>Grand-Mean-Centering:</u>
  - $\mathbf{TVx_{ti}} * \mathbf{Sex_i}$ → Does the WP stress effect differ between men and women?
  - $\mathbf{PMx_i} * \mathbf{Sex_i}$ → Does the *contextual* stress effect differ b/t men and women?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of $\mathbf{PMx_i}$, the interaction of $\mathbf{TVx_{ti}} * \mathbf{Sex_i}$ would still be smushed

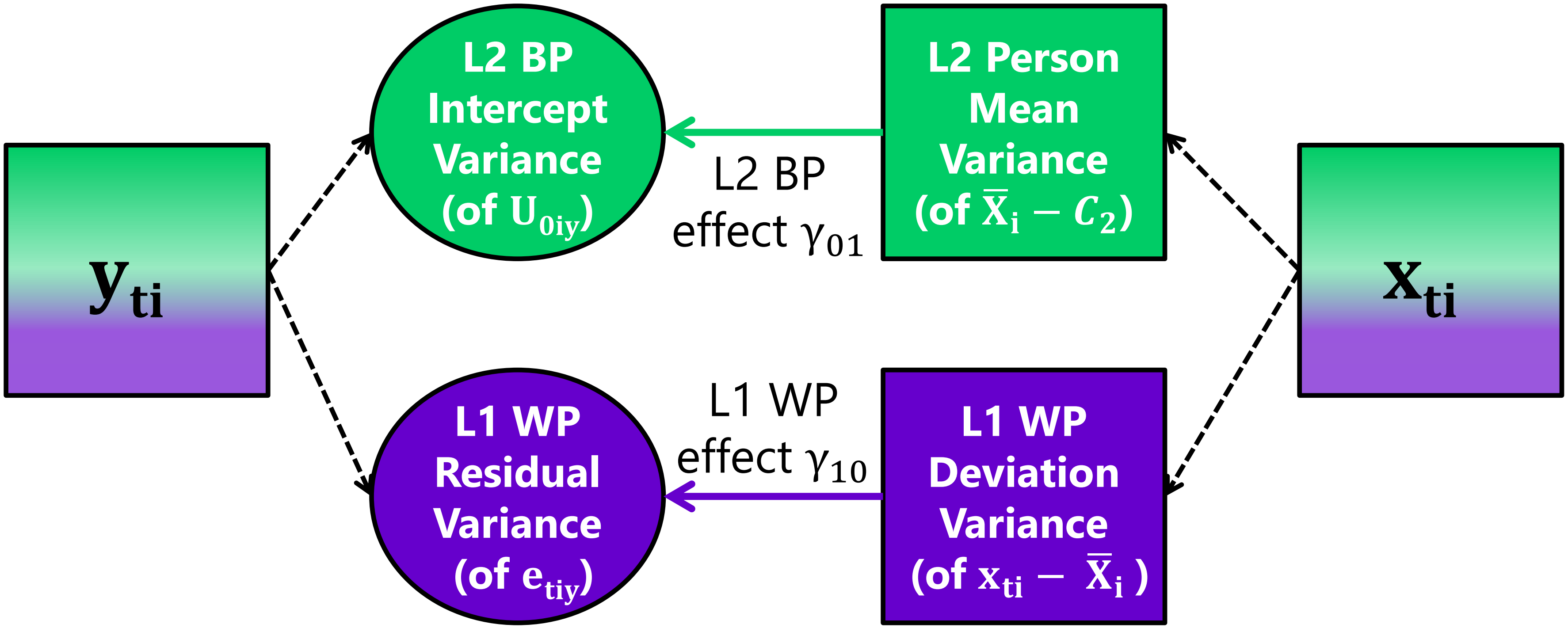
Time-Varying Predictors and Their Levels of Relations in Longitudinal Models

- Topics:
  - Time-varying predictors in models of fluctuation
    - Concepts and what NOT to do in MLMs
    - Univariate MLM: Person-(group)-mean-centering (Example 5)
    - Univariate MLM: Grand-mean-(constant)-centering
  - **Multivariate MLM (via SEM and M-SEM): Latent centering of time-varying predictors and models of change**
  - Also what not to do: path models for longitudinal data with smushed effects (and how to fix it)

Univariate MLM: Variable-Centering

Model-based partitioning of level-1 y_{ti} outcome variance into estimated **variance components**:

Brute-force partitioning of level-1 x_{ti} predictor variance into **observed variables**:

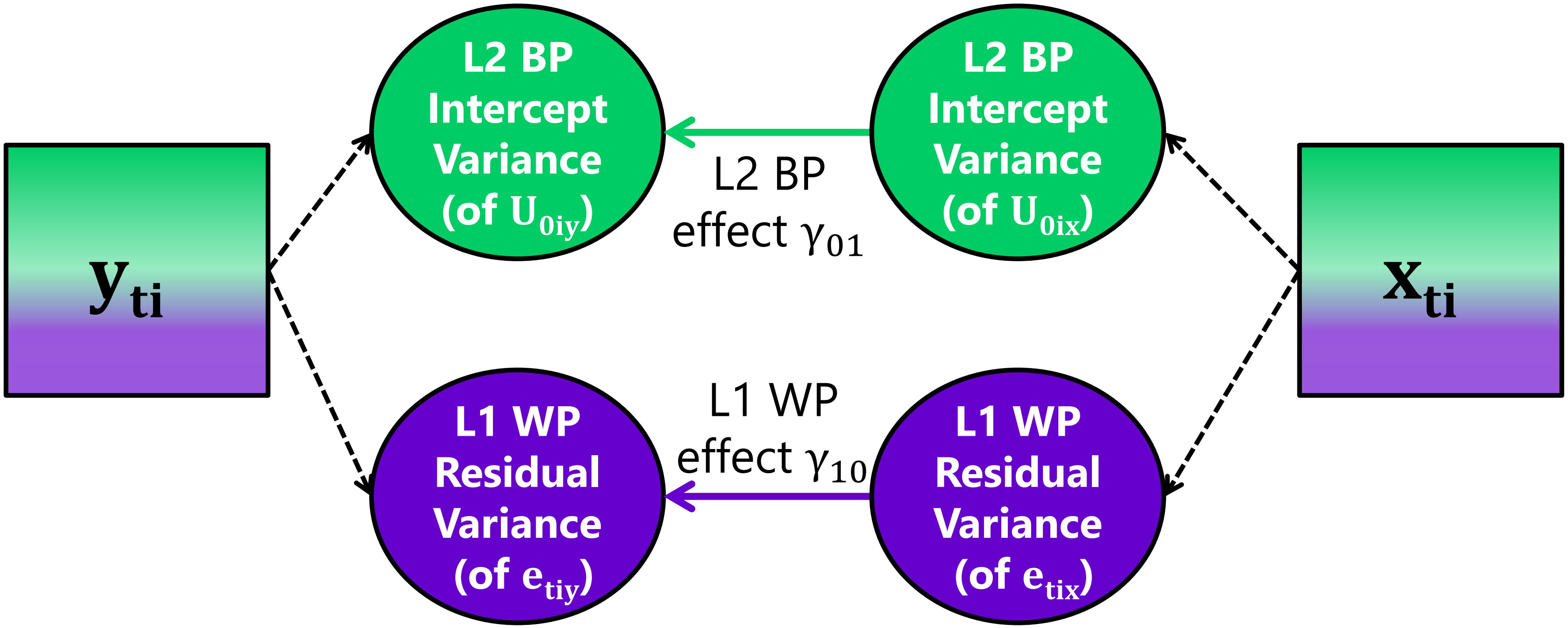


Why not let the model estimate variance components for x_{ti}, too?
This is the basis of multivariate MLM (or "multilevel SEM" = M-SEM).

Multivariate MLM: Latent-Centering

Model-based partitioning of level-1 y_{ti} outcome variance into estimated **variance components**:

Model-based partitioning of level-1 x_{ti} "predictor" variance into estimate **variance components**:



Univariate MLM software can do multivariate MLM if the relationships between X and Y at each level are phrased as covariances, but if you want directed regressions (or moderators thereof), you need **"M-SEM"**

Univariate vs. Truly Multivariate MLM (M-SEM)

- If your time-varying predictors have only BP intercept variance, their piles of variance can be reasonably approximated in univariate MLM OR by truly multivariate MLMs (so-called Multilevel SEM, or M-SEM)
  - It's called "SEM" because random effects = latent variables, but there is no latent variable measurement model as in traditional SEM, which is why I don't like the term M-SEM, and prefer "(Truly) Multivariate MLM" (where "truly" distinguishes which software is used)

- Pros of Truly Multivariate MLMs (M-SEM):
  - Univariate MLM uses observed variables for variance in X, but fits a model for the variance in Y; truly multivariate MLMs fit a model for both X and Y, which makes more sense
  - Simulations suggest that the L2 fixed slopes in M-SEM are less biased (because person means are not perfectly reliable as assumed), but the L2 fixed slopes also less precise, particularly for variables with lower ICCs (little intercept info) and small level-1 *n*

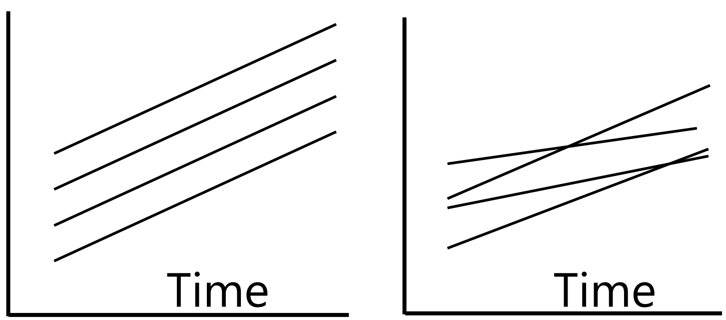
- Cons of Truly Multivariate MLMs (M-SEM):
  - Current software does not have REML or denominator DF → not good for small samples
  - Interactions among what used to be person means in univariate MLM instead become interactions among latent variables (random effects) in multivariate MLM (hard to estimate)
  - Whether your level-2 slopes are between or contextual varies by software used, syntax specification, and method of estimation! (see details in Hoffman 2019, AMPPS)

Time-Varying Predictors that Change **Need** Multivariate MLMs (via SEM or M-SEM)

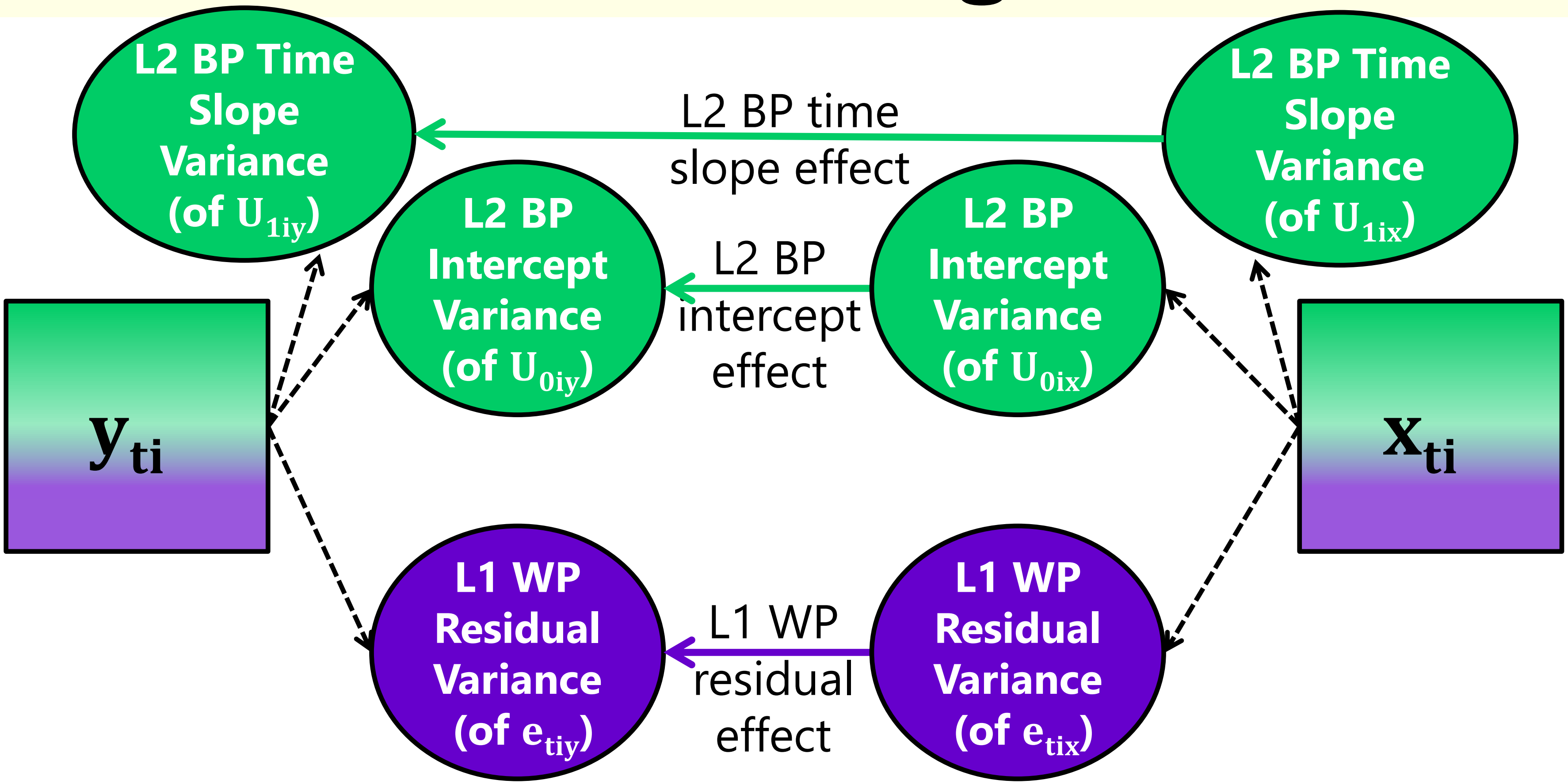
- Univariate MLMs for time-varying predictors can still be reasonable if a time-varying predictor has only a **fixed effect of time**
  - Adding fixed time slopes creates other "unique" effects controlling for time

- But if a time-varying predictor has ***individual differences* in change**, univariate MLM (variable-centering) cannot provide a reasonable separation of its between and within variance:
  - There are then **at least two "kinds" of BP variance** to be concerned with: intercept and time slope (and possibly more for other kinds of change)
  - The level-1 predictor has both individual differences in change (U_{1i}) and residual deviations from change (e_{ti}), which should each have their own relationship to y_{ti}, otherwise they are **smushed into the level-1 WP effect**

And, if people change differently over time, then BP differences change over time, too



Multivariate Modeling of Time-Varying Predictors that ***Change*** over Time

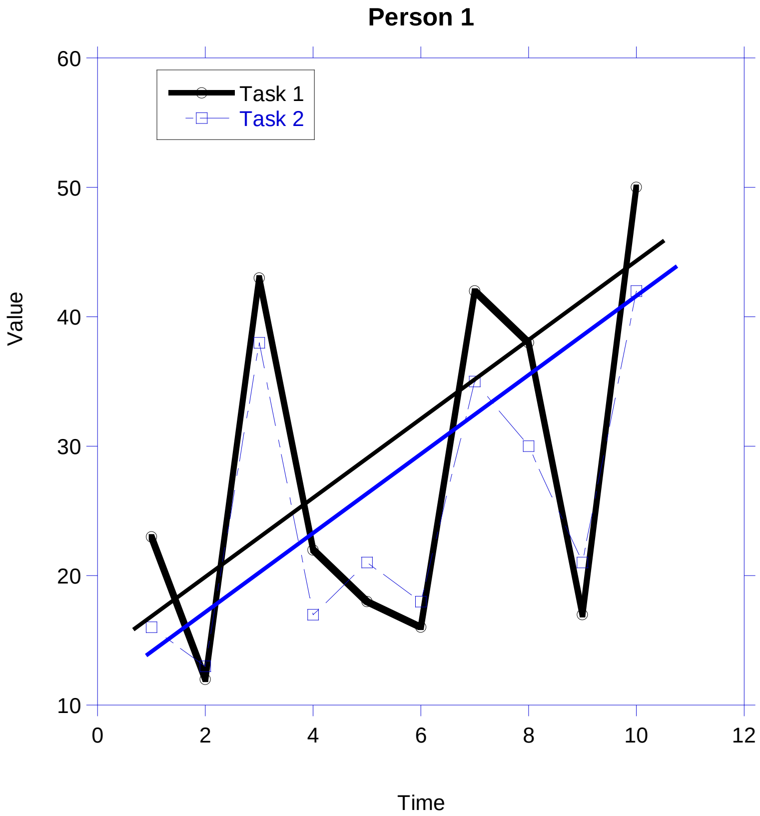


Univariate MLM software can do multivariate MLM if the relationships between X and Y at each level are covariances, but if you want directed regressions (or moderators thereof), you need **SEM** or **"M-SEM"**

Multivariate Relations of Models of Change

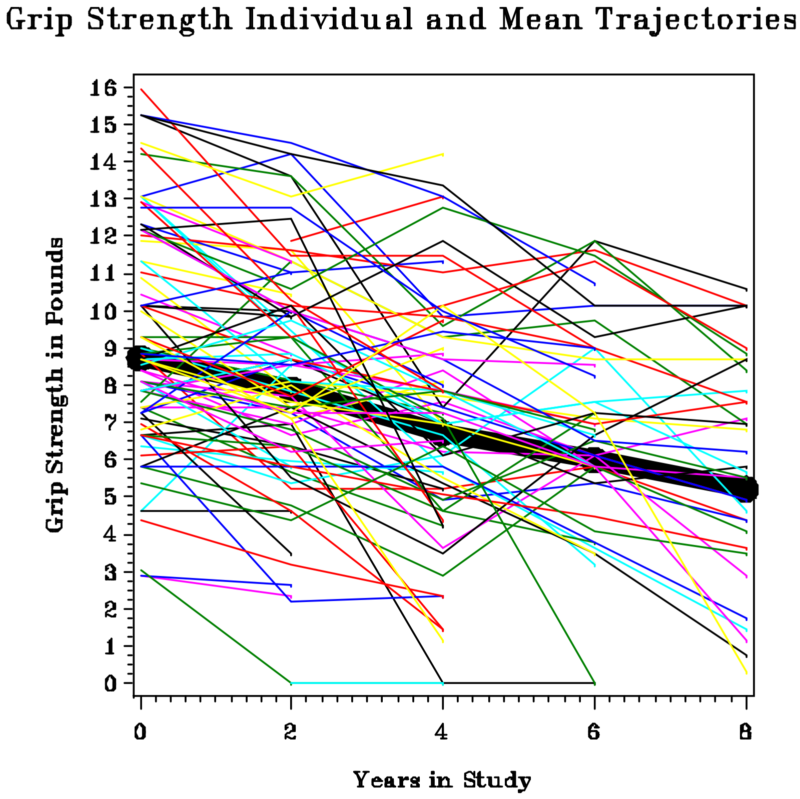
- Relations among **random effects for individual differences**
  - Intercepts: Are the predicted means (at time = 0) of X and Y related?
  - Time Slopes: Are the predicted rates of change of X and Y related?
  - These are **Between-Person** relations → relative to other people

- Relations among **residuals for within-person variation**: If I am higher than my predicted trajectory on x_{ti}, am I also likely higher than predicted on y_{ti} at…
  - Same occasion (concurrent relation)?
  - Next occasion (lagged relation)?
    - Btw, fitting same lagged relation across time only makes sense for equal-interval balanced longitudinal data

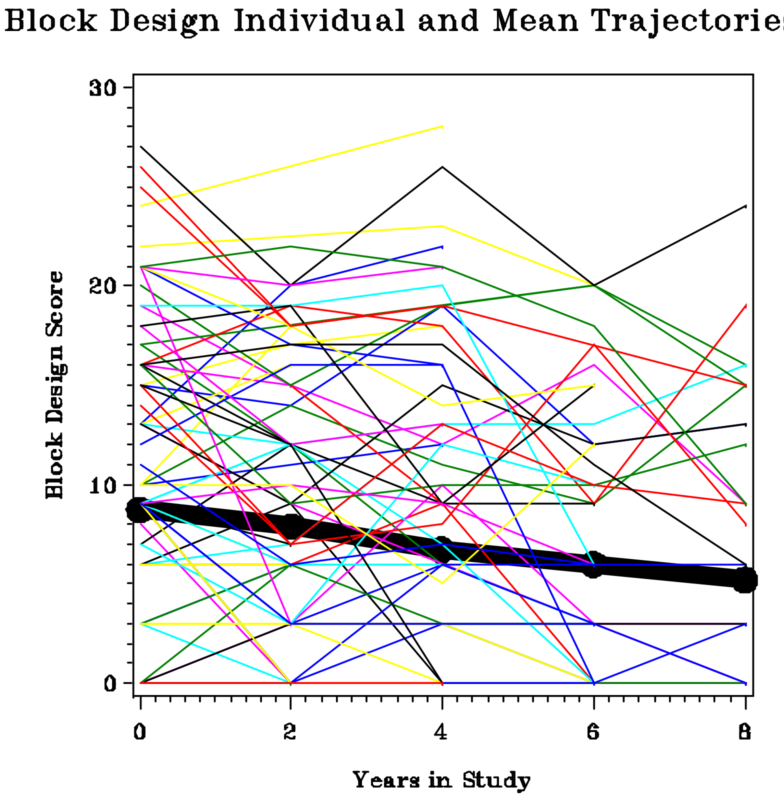


Individual Relations of Functional and Cognitive Change in Old Age

Functional Change

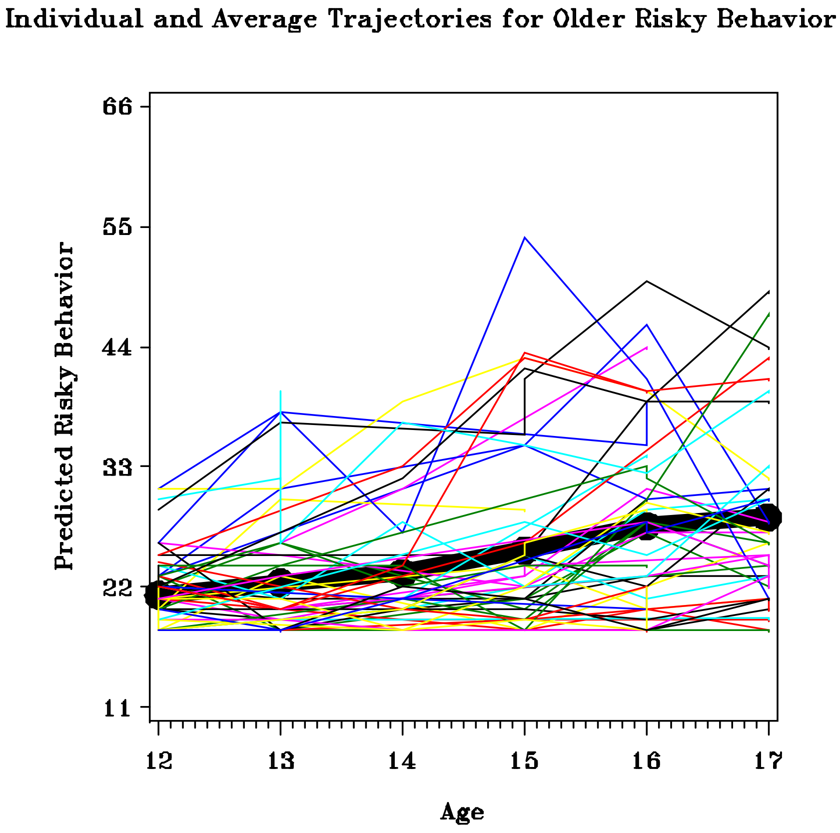


Cognitive Change

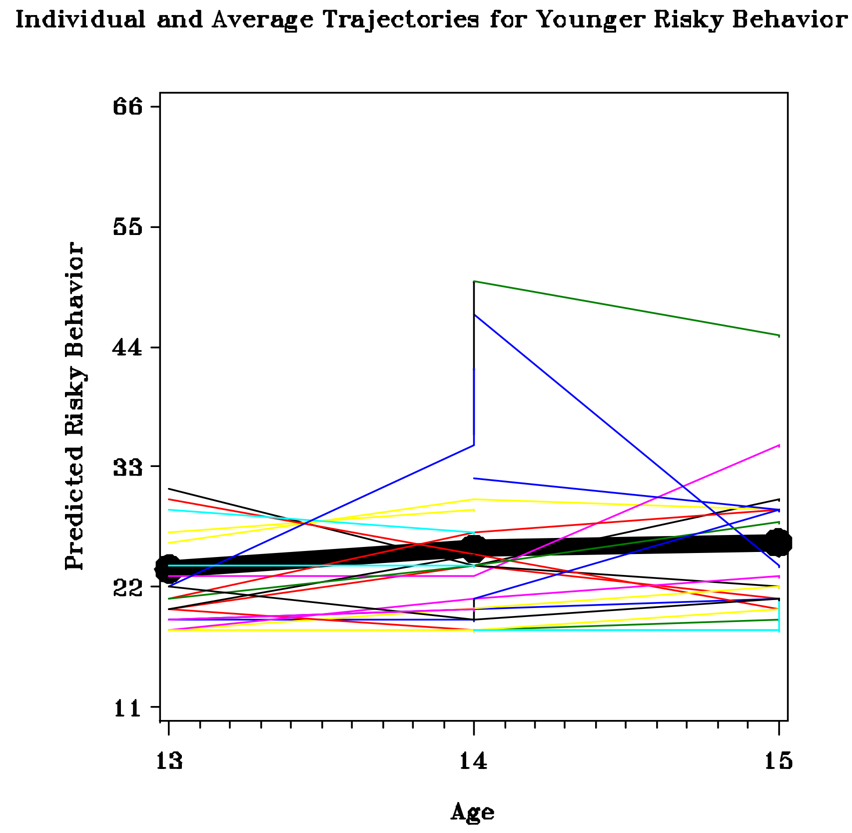


Individual Relations of Change in Risky Behavior Across Siblings

Older Siblings

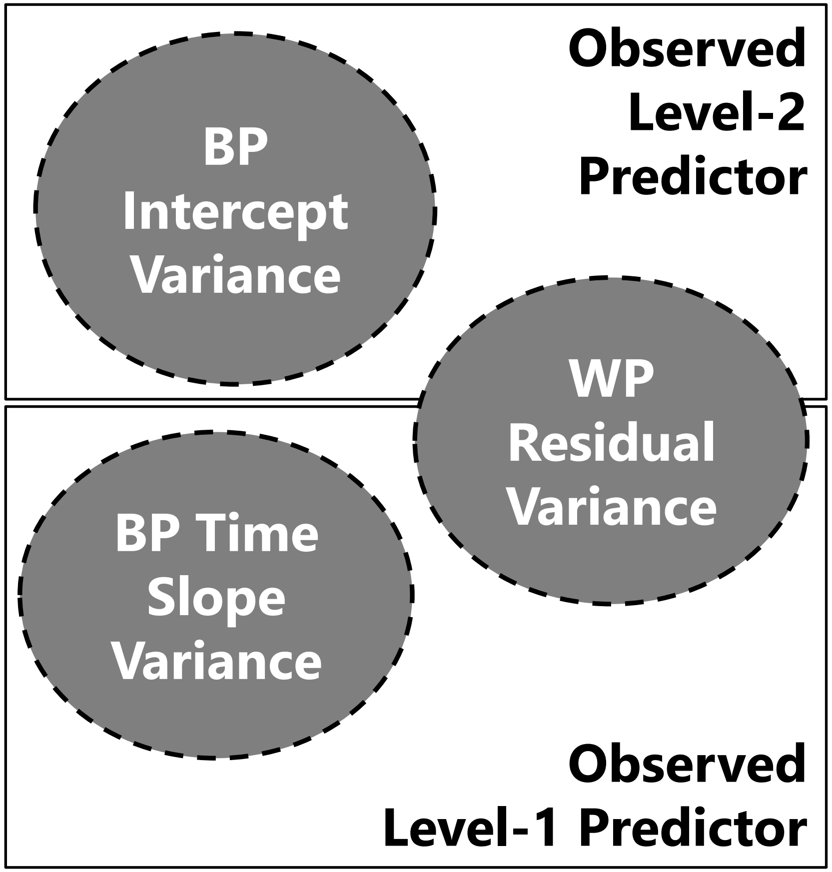


Younger Siblings



Summary: Longitudinal Relations

- **Ignoring relationships between the BP time slopes** of longitudinal variables can contaminate their other relations:
  - If the **WP residual** still contains the unmodeled BP time slope variance, **the level-1 effect will be smushed with the missing L2 time slope effect!** (bottom panel)
  - Different problem than more well-known result of **intercept-smushed L1 effects** (top panel)



Time-Varying Predictors and Their Levels of Relations in Longitudinal Models

- Topics:
  - Time-varying predictors in models of fluctuation
    - Concepts and what NOT to do in MLMs
    - Univariate MLM: Person-(group)-mean-centering (Example 5)
    - Univariate MLM: Grand-mean-(constant)-centering
  - Multivariate MLM (via SEM and M-SEM): Latent centering of time-varying predictors and models of change
  - **Also what not to do: path models for longitudinal data with smushed effects (and how to fix it)**

Modeling Cross-Lagged Relations

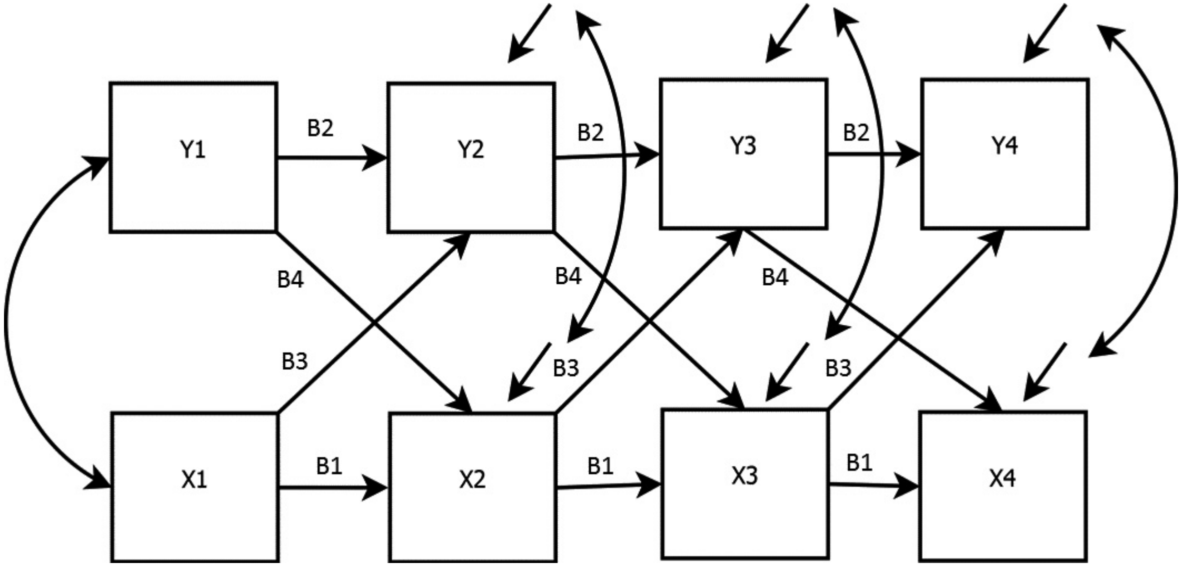
- All the within-person (WP) relations described so far have been concurrent—between x_{ti} and y_{ti} at the same occasion
- Lagged WP relations can be examined in univariate MLM, but:
  - Rows with unpredicted y_{ti} at prior occasions will be dropped by default
  - Relations go in one direction only: observed x_{ti} → latent y_{ti}
- To examine "cross-lagged" reciprocal relations between x_{ti} and y_{ti} at different occasions, the model needs to have access to all the occasions at once (across rows)
  - Although one can create lagged observed WP x_{ti} variables, there are no comparable **observed** WP y_{ti} variables to lag
  - Thus, cross-lagged relations can be easier to examine in wide data using SEM (or Mplus M-SEM using "dynamic" SEM lagging features)
- However, the same issues of using centering to avoid smushed effects are still relevant (even though it's not as obvious)…!
  - Just having "longitudinal" paths (e.g., T1 → T2) is not enough!

What *Not* to Do with Longitudinal Data

- Mis-specified path models (involving observed variables only) for longitudinal data are still far too common
  - These models often examine auto-regressive effects, cross-lagged effects, and observed variable mediation effects, which involve different variables each measured on three or more occasions
  - Next slides give common exemplars to watch out for!

- The problem in each is a lack of differentiation of sources (piles) of variance, and thus what their paths (slopes) mean
  - Big picture: If the path model variables have not been de-trended for person mean differences (AND for any individual change over time), then **all paths reflect smushed BP/WP relations to some degree…**
  - … and this problem will not necessarily be reflected by bad model fit!

A Model that Needs to Go Away*

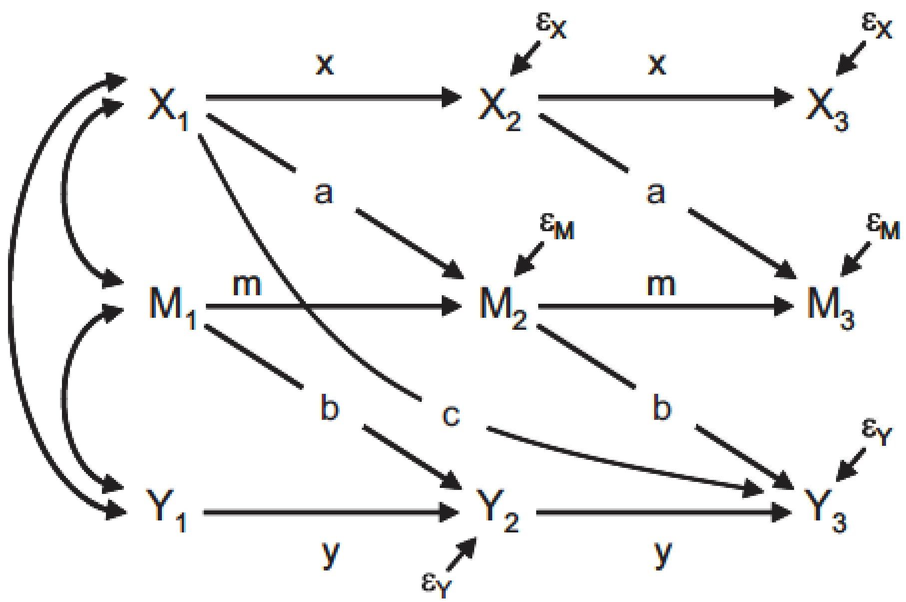


Autoregressive cross-legged panel model

* Emphasis mine, picture provided by [Berry & Willoughby (2017, *Child Development*)]

- Logic: by including **auto-regressive paths** (B1 and B2) to "control" for previous occasions, the **cross-lagged paths** (B3 and B4) then represent effects of "change" on each variable in predicting the other (so they are "longitudinal" predictions of time *t*-1 predicting time *t*)
- Reality: by allowing only one path (usually constrained equal over time), it reflects smushed effects across sources of variance—BP intercept, BP time slope(s), WP residual; autoregressive paths do NOT adequately control for BP differences (they assume an AR(1) correlation over time)

And take this one with it*…



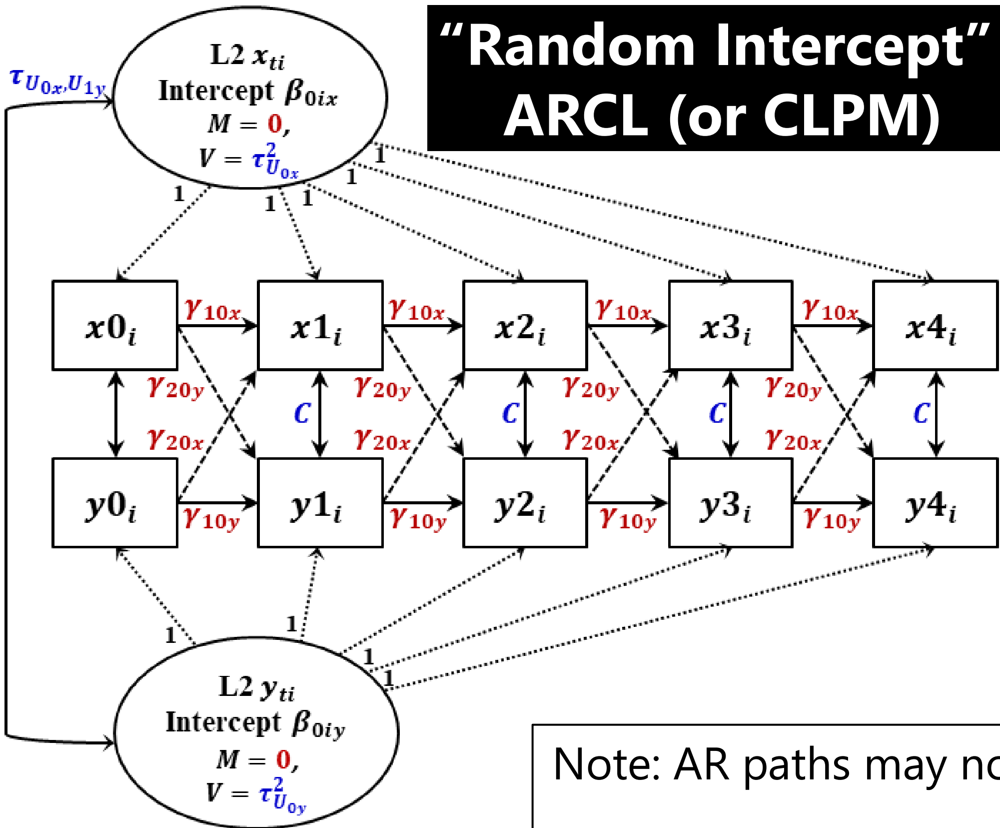
Longitudinal mediation model

X= predictor, M= mediator, Y= outcome

* My point of view only, picture provided by Maxwell & Cole (2007, *Psychological Methods*)

- Logic: mediation should time to occur, so indirect effects should be specified across different occasions (as before, of "change")
- Agreed, but if these variables haven't been de-trended for ALL sources of BP variance, then the *b* and *c* paths are smushed
- And what about BP mediation? Capturing BP variances in the same model would allow examination of that, too…
  - BP intercept mediation, BP time slope mediation, WP residual mediation…

Remedies for Intercept Smushing



"Random Intercept" ARCL (or CLPM)

Many authors have also pointed out the need to distinguish constant BP effects from WP effects via:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + \boxed{U_{0ix}} + e_{tix}$$

$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + \boxed{U_{0iy}} + e_{tiy}$$

Note: AR paths may no longer be needed given RIs!

Given the interest in cross-lagged "which came first" level-1 WP residual paths, the level-2 random intercept relationship is usually specified as a covariance instead of a slope—and whether a slope would capture the between or contextual effects differs by software, estimator, and model specification…

Btw, equal AR and CL paths over time only make sense for equal-interval balanced occasions

Mplus Syntax for RI-(AR)CLPM

Just showing **MODEL** part, which would be preceded by **DATA**, **VARIABLE**, and **ANALYSIS** as usual (estimated using **wide** data)

```
! Factor loadings fixed by @
  IntX BY x1-x5@1;
  IntY BY y1-y5@1;

! Factor intercepts estimated = fixed effects
  [IntX IntY];

! Level-2 factor variances estimated (in G)
  IntX IntY;

! Level-2 factor covariance estimated (in G)
  IntY WITH IntX;

! Per-occasion intercepts fixed to 0
  [x1-x5@0 y1-y5@0];

! Level-1 residual variances (in R)
! held equal if predicted
  x2-x5 (ResVarX); ! x1 resvar is separate
  y2-y5 (ResVarY); ! y1 resvar is separate
```

```
! Level-1 residuals' same-occasion
! Covariances (in R)
  ! Unpredicted occasions covariance
    x1 WITH y1;
  ! Predicted occasions pairwise covariance
    x2-x5 PWITH y2-y5 (ResCov);

! Level-1 auto-regressive paths held
! equal over time (PON = pairwise)
  x2-x5 PON x1-x4 (ARx);
  y2-y5 PON y1-y4 (ARy);

! Level-1 cross-lagged paths held
! equal over time (PON = pairwise)
  x2-x5 PON y1-y4 (CLy2x);
  y2-y5 PON x1-x4 (CLx2y);
```

Btw, this code would result in contextual effects instead of BP level-2 effects if IntX → IntY instead. To fix it, you need structured residuals… stay tuned…

R lavaan Syntax for RI-(AR)CLPM

Estimated using **wide** data

```
RI_CPLM_syntax = "
# Factor loadings fixed by *
IntX =~ 1*x1 + 1*x2 + 1*x3 + 1*x4 + 1*x5
IntY =~ 1*y1 + 1*y2 + 1*y3 + 1*y4 + 1*y5

# Factor intercepts estimated = fixed effects
  IntX ~ 1;  IntY ~ 1;
# Level-2 factor variances estimated (in G)
  IntX ~~ IntX;  IntY ~~ IntY
# Level-2 factor covariance estimated (in G)
  IntX ~~ IntY

# Per-occasion intercepts fixed to 0
  x1 ~ 0; x2 ~ 0; x3 ~ 0; x4 ~ 0; x5 ~ 0
  y1 ~ 0; y2 ~ 0; y3 ~ 0; y4 ~ 0; y5 ~ 0

# Level-1 residual variances (in R)
  x1 ~~ x1;  y1 ~~ y1 # x1 and y1 are separate
# Held equal if predicted
  x2 ~~ (ResVarX)*x2;  x3 ~~ (ResVarX)*x3
  x4 ~~ (ResVarX)*x4;  x5 ~~ (ResVarX)*x5
  y2 ~~ (ResVarY)*y2;  y3 ~~ (ResVarY)*y3
  y4 ~~ (ResVarY)*y4;  y5 ~~ (ResVarY)*y5
```

```
# Level-1 residuals' same occasion
# covariances (in R)
  # Unpredicted occasions covariance
    x1 ~~ y1
  # Predicted occasions pairwise covariance
    x2 ~~ (ResCov)*y2; x3 ~~ (ResCov)*y3
    x4 ~~ (ResCov)*y4; x5 ~~ (ResCov)*y5

# Level-1 auto-regressive paths held
# equal over time
  x2 ~ (ARx)*x1; x3 ~ (ARx)*x2
  x4 ~ (ARx)*x3; x5 ~ (ARx)*x4
  y2 ~ (ARy)*y1; y3 ~ (ARy)*y2
  y4 ~ (ARy)*y3; y5 ~ (ARy)*y4

# Level-1 cross-lagged paths held
# equal over time
  x2 ~ (CLy2x)*y1; x3 ~ (CLy2x)*y2
  x4 ~ (CLy2x)*y3; x5 ~ (CLy2x)*y4
  y2 ~ (CLx2y)*x1; y3 ~ (CLx2y)*x2
  y4 ~ (CLx2y)*x3; y5 ~ (CLx2y)*x4
"
RI_CLPM = lavaan(data=Chapter8wide, model=RI_CPLM_syntax,
                 estimator="ML", mimic="mplus")
summary(RI_CLPM, fit.measures=TRUE, rsquare=TRUE,
        standardized=TRUE)
```

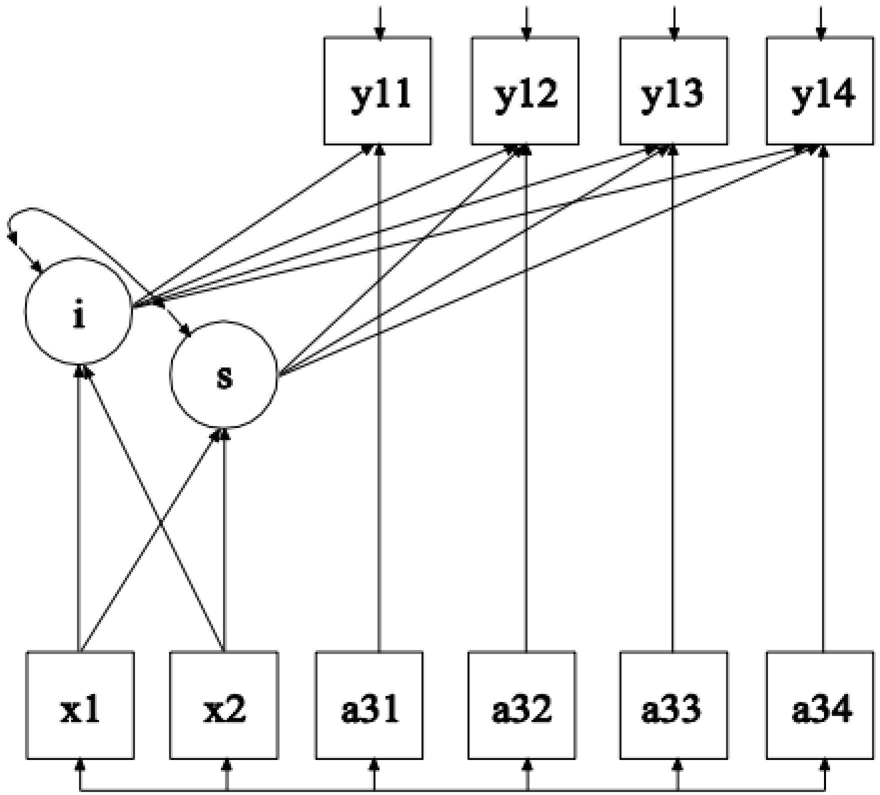
Btw, this code would result in contextual effects instead of BP level-2 effects if IntX → IntY instead. To fix it, you need structured residuals… stay tuned…

What about Change over Time?

- The RI-CLPM is appropriate for longitudinal data that show fluctuation—but not individual change—over time
  - Whether each variable's AR1 paths are still needed after controlling for its random intercept factor is then an empirical question (and they could become covariances instead in single-level SEM)
  - Analysts can decide whether to specify concurrent or lagged paths in one variable predicting another, or covariances (whatever makes sense)

- For outcomes that require a (latent) growth curve model, how to properly specify unsmushed effects of "time-varying predictors" (TVPs) is *still* not well-understood…
  - Big picture: TVPs will usually carry at least one source of BP variance (random intercept for mean differences), possibly more (random time slopes for individual change; random scale factor for volatility)
  - Each source of level-2 variance can have its own set of relations…
  - So let's see how the standard SEM latent growth curve model would needs to adapt to address this… for details, see Example 9c from this advanced (longitudinal) MLM class (to be offered here in Spring 2023!)

Time-Varying Predictors in Single-Level SEM: What *Not* to Do… in Mplus



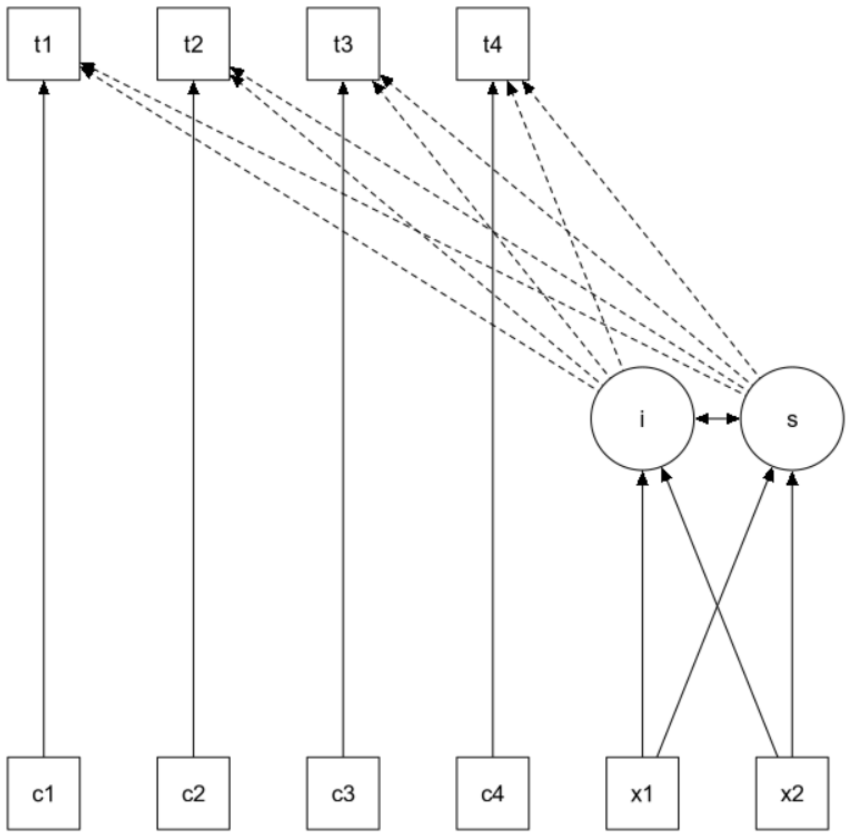

```
TITLE:     this is an example of a linear growth
           model for a continuous outcome with time-
           invariant and time-varying covariates
DATA:      FILE IS ex6.10.dat;
VARIABLE:  NAMES ARE y11-y14 x1 x2 a31-a34;
MODEL:     i s | y11@0 y12@1 y13@2 y14@3;
           i s ON x1 x2;
           y11 ON a31;
           y12 ON a32;
           y13 ON a33;
           y14 ON a34;
```

This diagram is from the (current) Mplus v. 8 Users Guide example 6.10.

Although the y11–y14 outcomes are predicted by latent intercept and time slope factors (separating two kinds of BP variance from WP variance), this is not the case for the a31–a34 TVPs.

Consequently, in the model shown here, the a→y paths will be smushed.

Time-Varying Predictors in Single-Level SEM: What *Not* to Do… in R lavaan



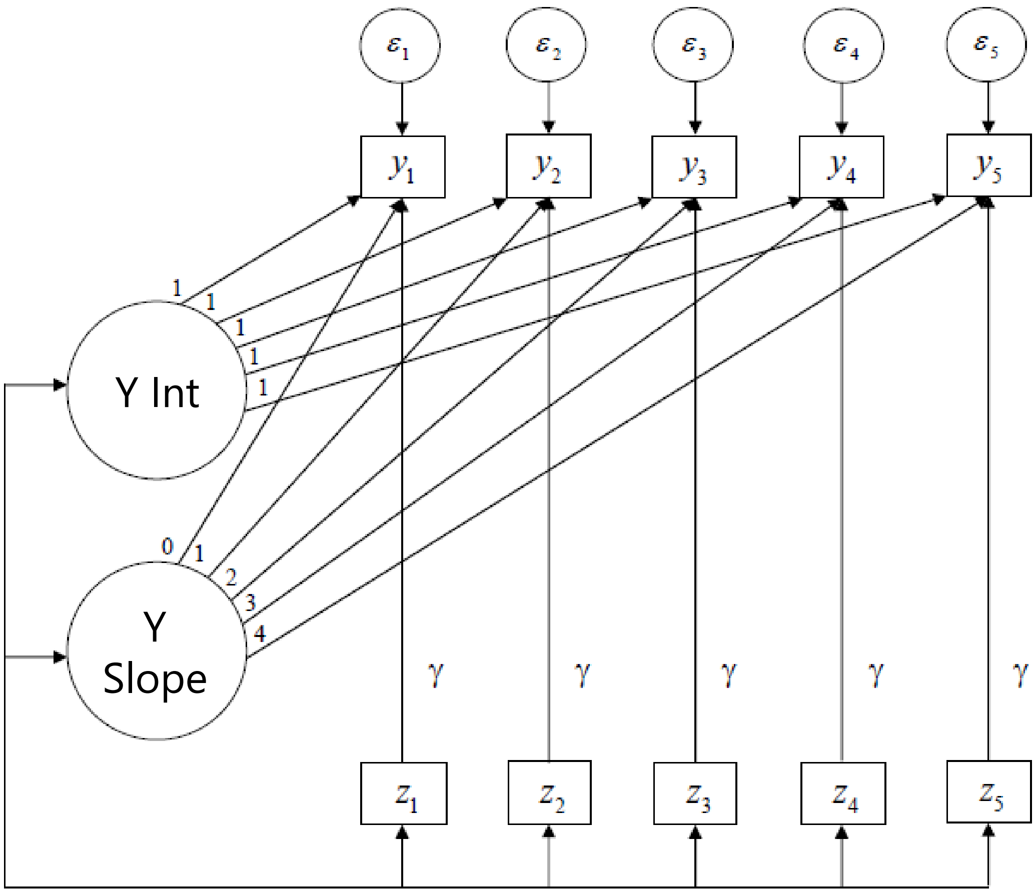

```
# a linear growth model with a time-varying covariate
model <- '
  # intercept and slope with fixed coefficients
    i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
    s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4
  # regressions
    i ~ x1 + x2
    s ~ x1 + x2
  # time-varying covariates
    t1 ~ c1
    t2 ~ c2
    t3 ~ c3
    t4 ~ c4
'
fit <- growth(model, data = Demo.growth)
summary(fit)
```

This diagram is from the (current) lavaan tutorial on growth curves

Although the t1–t4 outcomes are predicted by latent intercept and time slope factors (separating two kinds of BP variance from WP variance), this is not the case for the c1–c4 TVPs.

Consequently, in the model shown here, the c→y paths will be smushed.

Time-Varying Predictors in Single-Level SEM: What *Should* We *Do?*

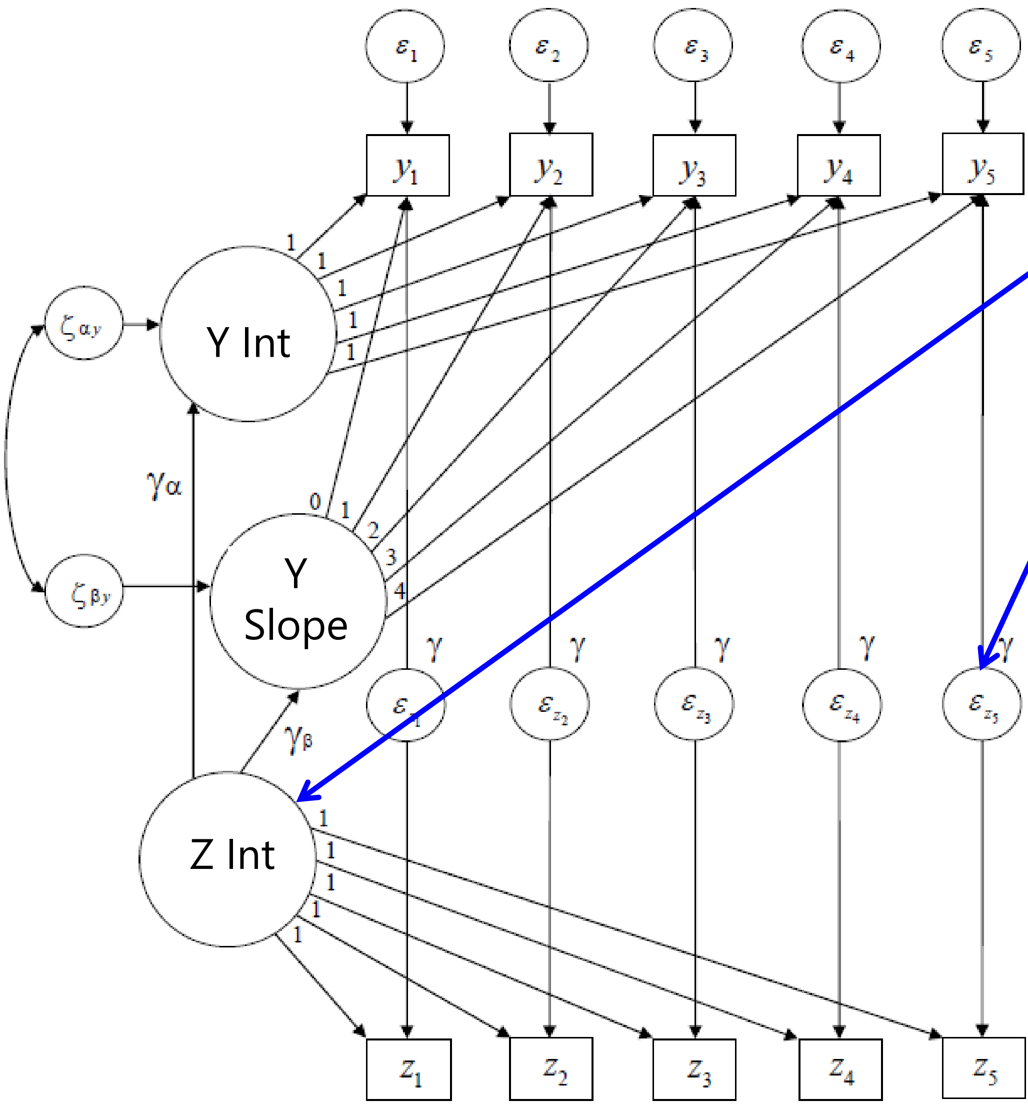


This diagram is from Curran et al. (2012). The time-varying predictors z1–z5 boxes have directed effects onto the y1–y5 outcomes at the same time.

If you constrain these paths to be equal (as γ), you get **a smushed effect** (they call it an "aggregate" effect).

IF you add covariances of the z's with the intercept, then γ becomes **the WP effect**. But the BP effect is not in here! And you cannot add PMz to get it like in MLM because it will be redundant (→ ipsative).

How to Fix It, Part 1 (by Curran et al., 2012)

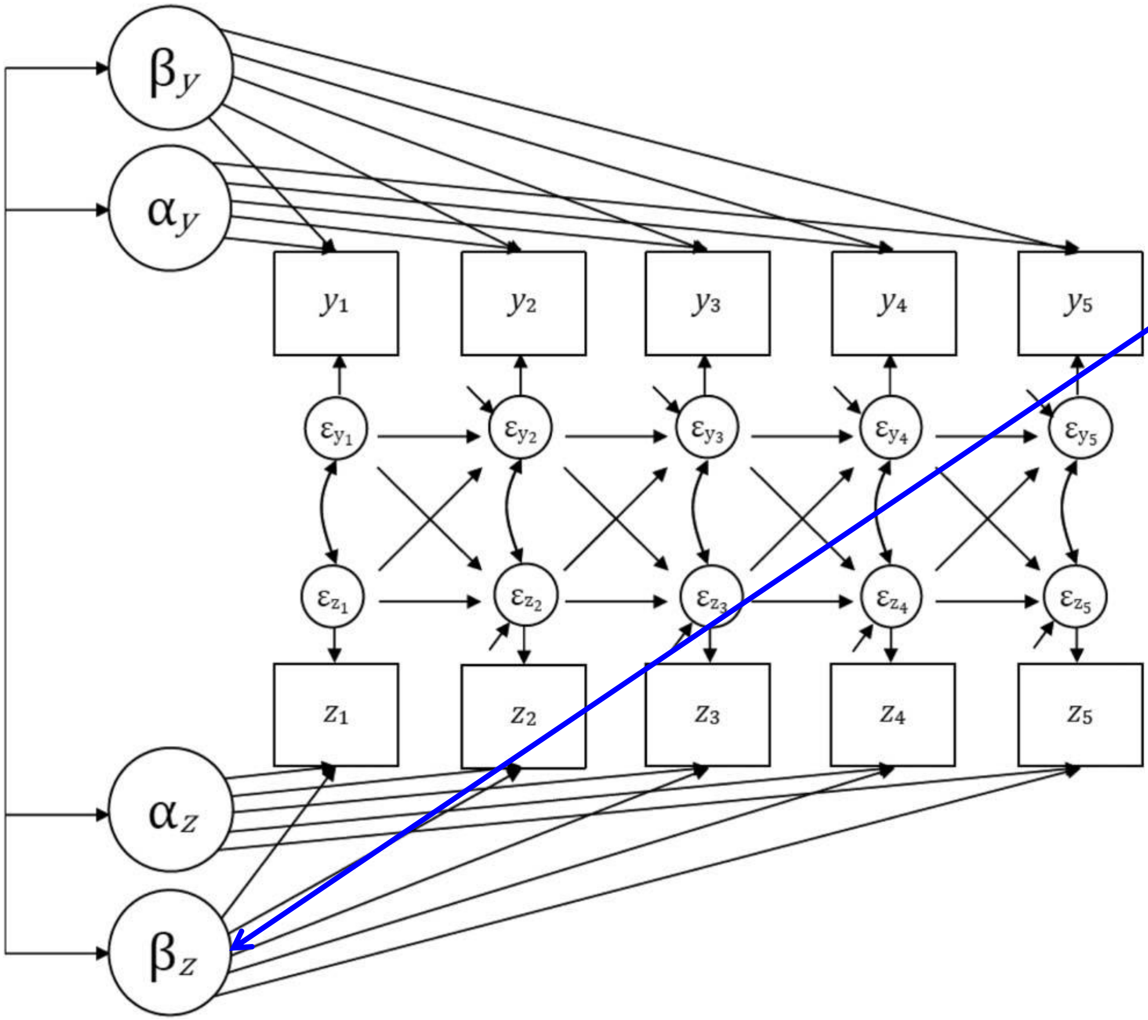


The z1–z5 time-varying predictors now have their own random intercept factor, which directly represents their level-2 BP intercept variance.

The **BP intercept effect of z→y** is given by $\boldsymbol{\gamma_\alpha}$ because of the **structured residuals**: the new ε_z latent variables to which the level-1 residual variances of z1–z5 have been moved. The **WP effect** is now given by **γ** from $\varepsilon_{z1\text{-}z5}$ → y1–y5.

If z1–z5 had predicted y1–y5 directly, the **z→y** intercept path would have held a contextual effect instead of a BP effect.

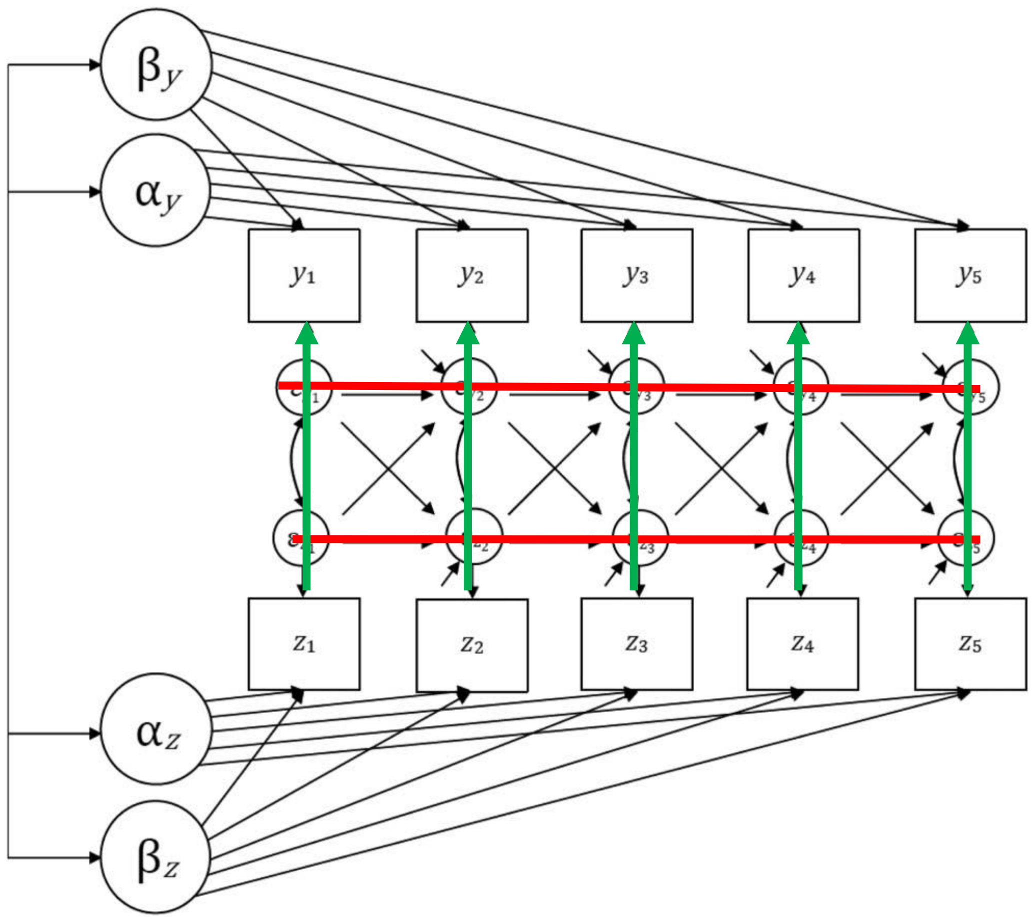
How to Fix It, Part 2 (by Curran et al., 2014)



If z1–z5 has individual differences in change over time instead of just fluctuation, **just add a random time slope factor for z1–z5**—then you'd be back to multivariate multilevel model we began with.

When **using level-1 structured residuals,** all paths among the intercept and slope factors will represent their **total level-2 BP effects**. But structured residuals then don't allow random slopes (or other modifications), at least in ML in Mplus…

How To Fix It Without Structured Residuals



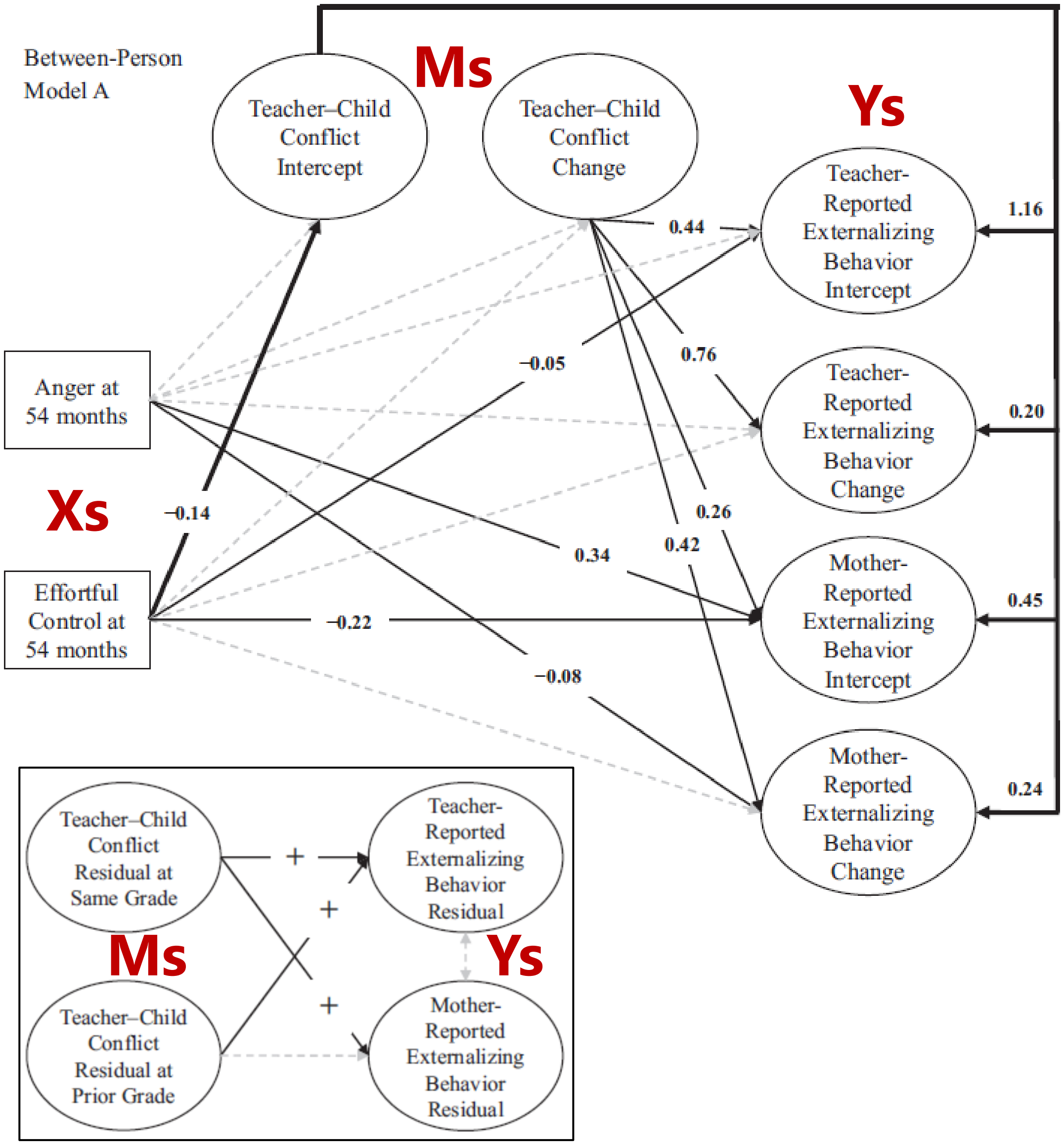
See Hoffman 2019 for more about when level-2 effects become BP or contextual…

IF you predict the y1–y5 residuals directly from z1–z5 (without structured residuals), **that effect is still the level-1 WP effect.**

The problem is that some of the paths among the intercept and slope factors become **BP contextual effects** instead. These include paths for intercept → intercept (and slope → slope), but not for intercept → slope (or slope → intercept).

In either version, you can still get the missing L2 effect (BP or BP contextual) by requesting a linear combination (e.g., in Mplus MODEL CONSTRAINT).

What about "Longitudinal Mediation"?



Mediation cannot be meaningfully examined using smushed effects!

Example from Crockett et al. 2019 *Child Development*—using latent basis change within single-level SEM

Top: Between-Person Model A of direct and indirect effects among level-2 random intercepts and time slopes of 3 longitudinal variables

Bottom: Within-Person Model A of direct effects among level-1 residuals (no indirect effects possible because X = time-invariant)

When to Use Each: Multivariate MLM vs Single-Level SEM

- Models and software are logically separate, but (current) software restrictions may make it so one version is easier than the other for specifying certain types of models

- "Truly" Multivariate MLM / M-SEM (e.g., MLM side of Mplus):
  - Uses stacked data, so level-1 is explicitly separate from level-2, which easily allows for random effects of level-1 predictors, mediation, and/or measurement models at each level of analysis
  - However: be careful of otherwise equivalent Mplus models whose L2 parameters change interpretation with different version of the syntax!

- Single-Level SEM (e.g., SEM side of Mplus):
  - Uses wide data structure, so level-1 parameters must be specified through constraints across multiple observed variables, which assumes balanced time (Mplus Tscores that allows individually varying times for growth models is not relevant for WP fluctuation models)

When to Use Each: Multivariate MLM vs Single-Level SEM

- Models requiring access to level-1 observations *at different occasions* across variables can be easier in single-level SEM

- Single-Level SEM (e.g., SEM side of Mplus):
  - All occasions are accessible at once, which means that patterns of residual covariance over time can be easily included (via constraints)
  - Lagged residual relations across variables can be easily included (e.g., time 1 X → time 2 Y, time 1 Y → time 2 X), just make sure to not smush!

- Multivariate MLM / M-SEM (e.g., MLM side of Mplus):
  - Uses stacked data, so it doesn't have access to previous occasions' information stored on different rows (which needs to be unsmushed)
    - Mplus 8 allows auto-regressive relations, but only as specified as directed paths (not residual covariances) and only by using Bayes MCMC estimation