

On the Distinction of Between-Person versus Within-Person Relations: Essential or Semantic?

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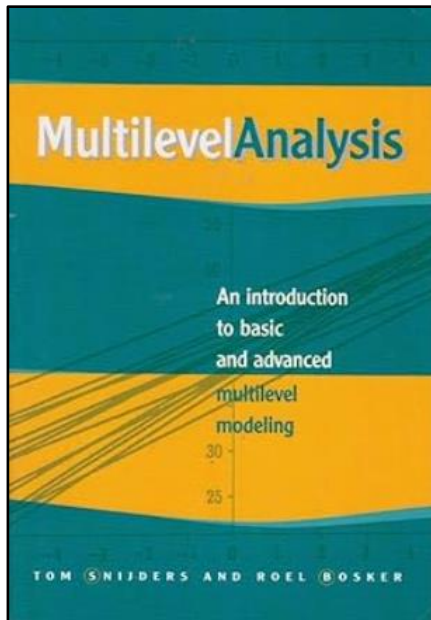
Slides available at: <https://www.lesahoffman.com/Workshops/index.html>

Prologue

Setting: University of Kansas

Dr. Janet Marquis to 2002-Me:
"I need you to learn growth modeling"

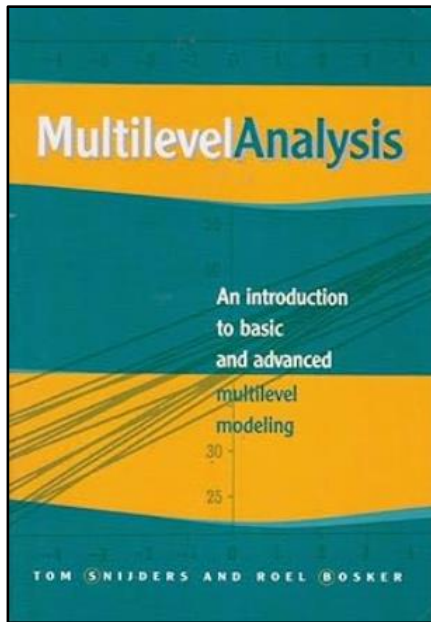
2002-Me: *"Great, how?"*
2002-Janet: ***"Read this"***



**2004-Dr. Me:
"Thank you!"**

Prologue

- Setting: 2002 Cognitive Aging Conference
Presenters: Dr. Scott Hofer and Dr. Marty Sliwinski
2002-Me: *"Great talk! How do I learn to do that?"*
2002-Them: *"Read Snijders & Bosker"*
2006-Me: ***"Thank you both!" (still!)***



My Origin of Between vs. Within

S & B: Level-1 persons (i) in Level-2 clusters (j)

What NOT to Do—Assume no Contextual Effect:

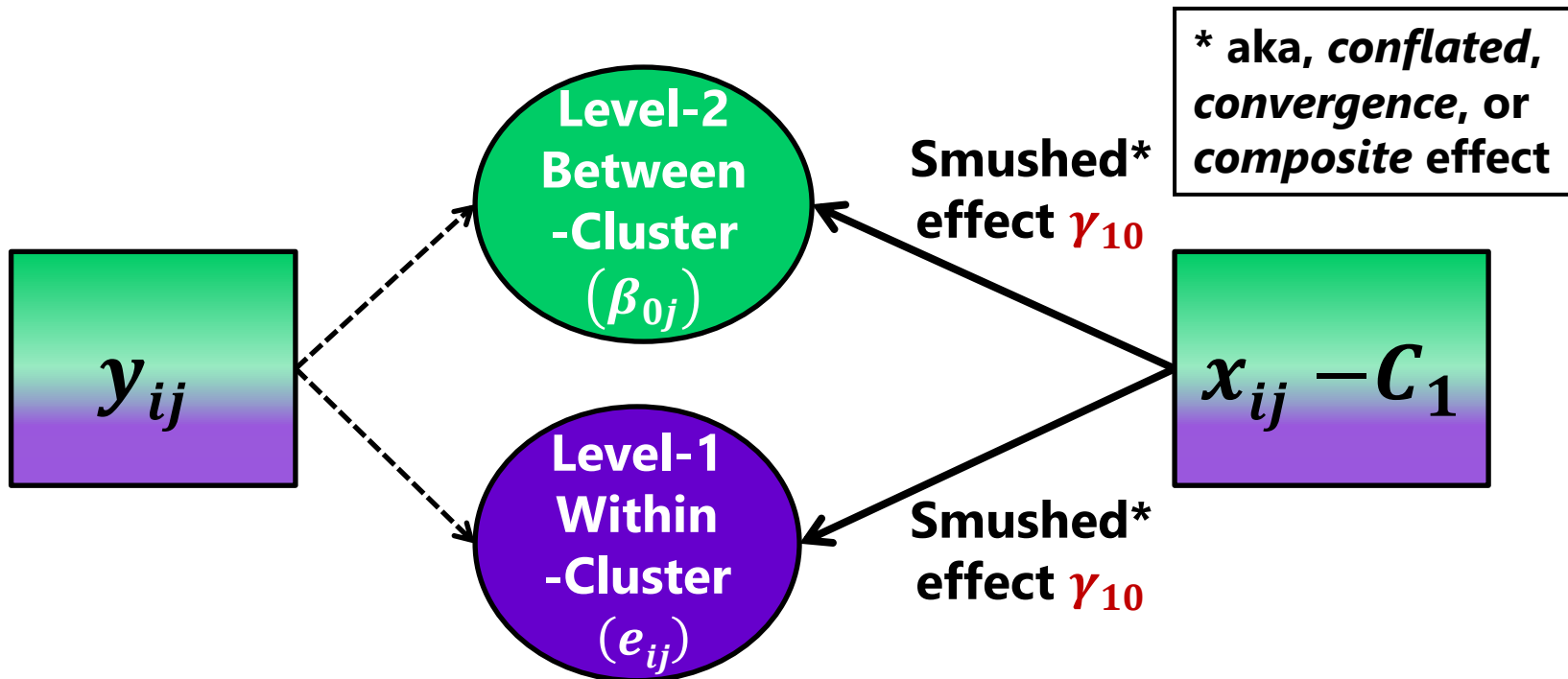
L1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij} - C_1) + e_{ij}$

L2 Intercept: $\beta_{0j} = \gamma_{00} + U_{0j}$

L2 Slope of x_{ij} : $\beta_{1j} = \gamma_{10}$

Combined:

$$y_{ij} = \gamma_{00} + \gamma_{10}(x_{ij} - C_1) + U_{0j} + e_{ij}$$



Separating Between from Within

What to Do Instead—Cluster-Mean-Centered* Version:

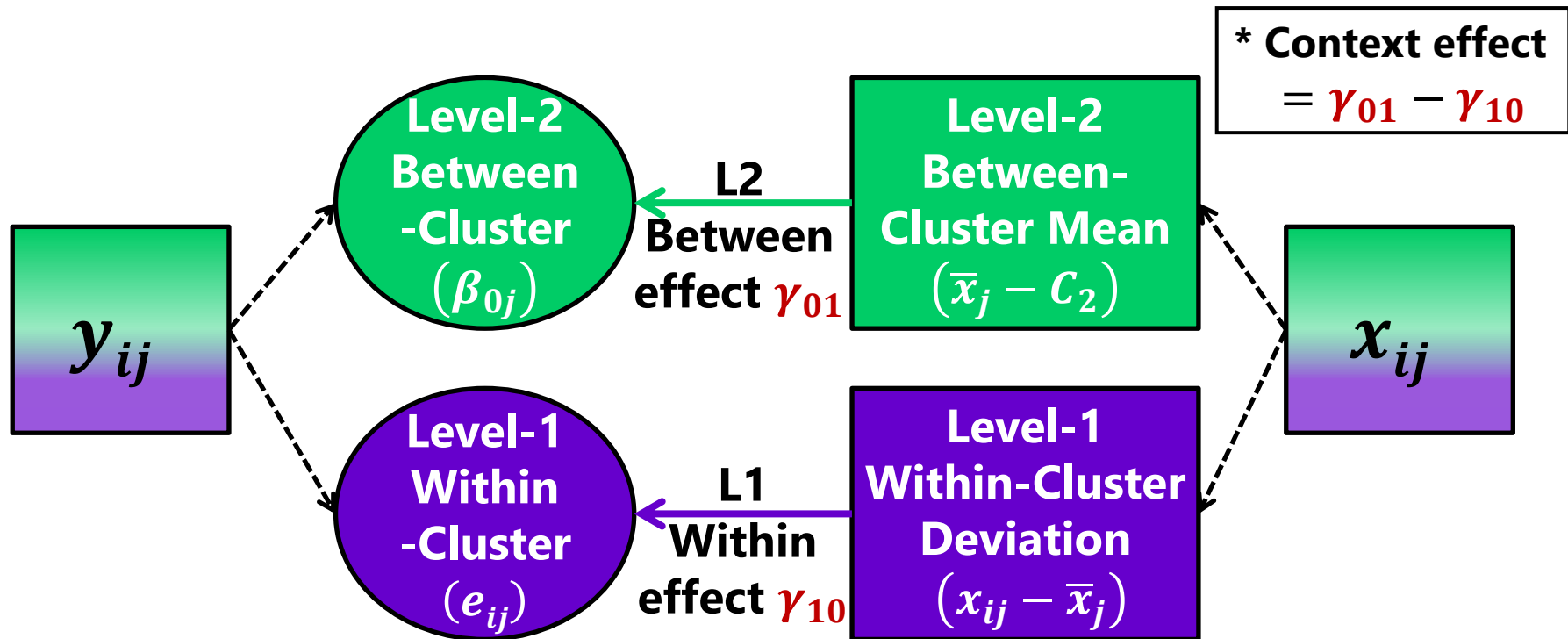
L1: $y_{ij} = \beta_{0j} + \beta_{1j}(x_{ij} - \bar{x}_j) + e_{ij}$

L2 Intercept: $\beta_{0j} = \gamma_{00} + \gamma_{01}(\bar{x}_j - C_2) + U_{0j}$

L2 Within X Slope: $\beta_{1j} = \gamma_{10}$

Combined:

$$y_{ij} = \gamma_{00} + \gamma_{10}(x_{ij} - \bar{x}_j) + \gamma_{01}(\bar{x}_j - C_2) + U_{0j} + e_{ij}$$



From Clustered to Longitudinal: Part 1

Within-Person Fluctuation—Person-Mean-Centered Predictor:

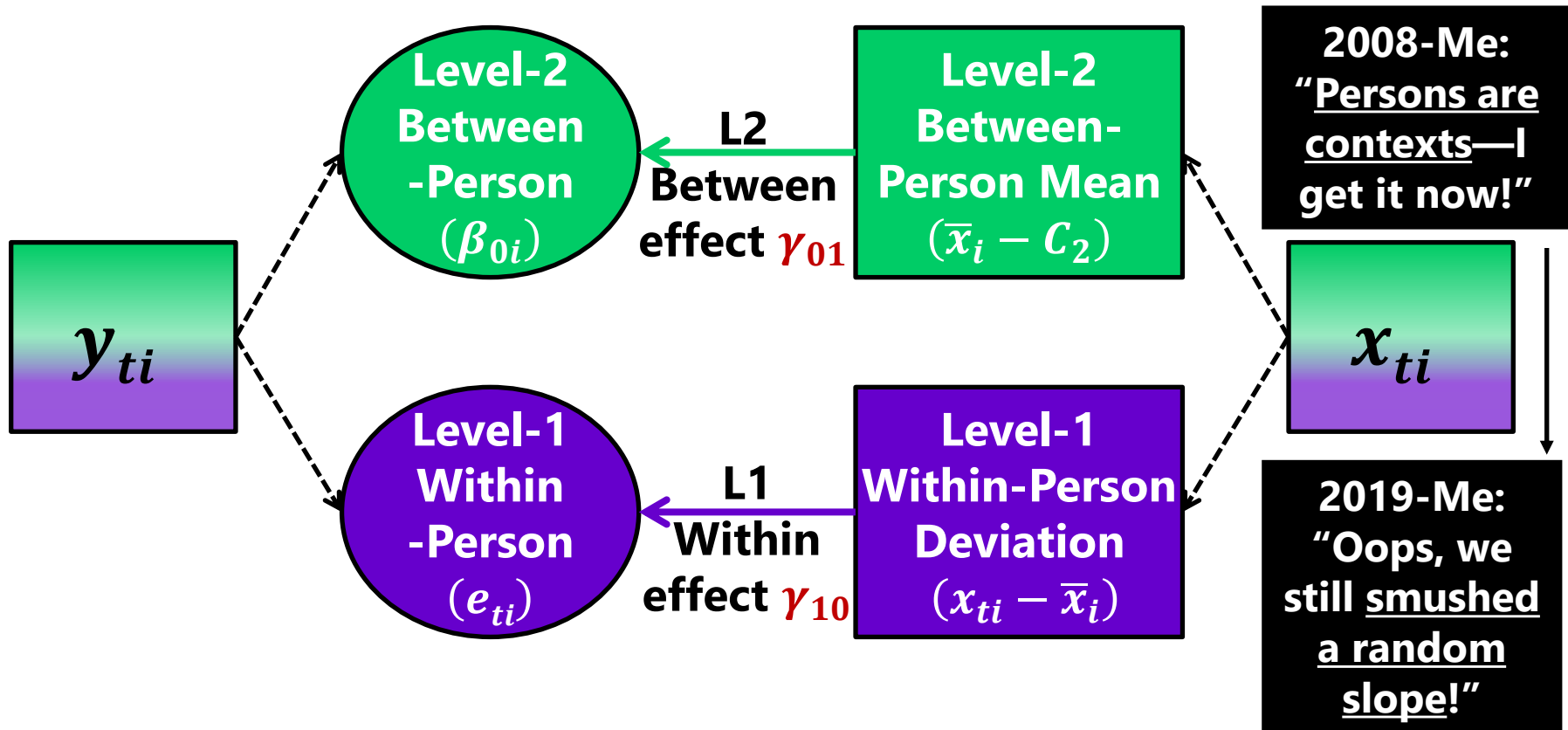
L1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - \bar{x}_i) + e_{ti}$

L2 Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\bar{x}_i - C_2) + U_{0i}$

L2 Within X Slope: $\beta_{1i} = \gamma_{10}$

Combined:

$$y_{ti} = \gamma_{00} + \gamma_{10}(x_{ti} - \bar{x}_i) + \gamma_{01}(\bar{x}_i - C_2) + U_{0i} + e_{ti}$$



From Clustered to Longitudinal: Part 2

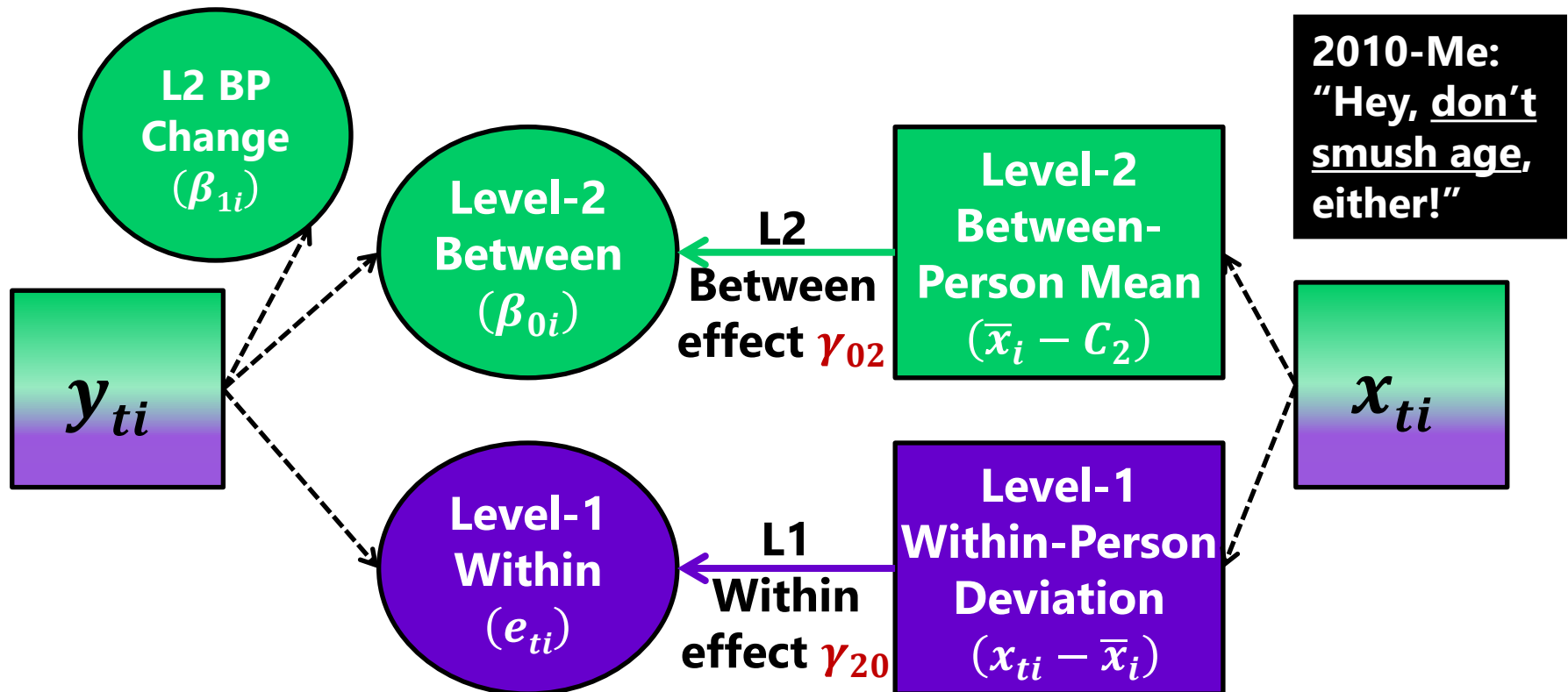
Within-Person CHANGE—Person-Mean-Centered Predictor:

$$\text{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(x_{ti} - \bar{x}_i) + e_{ti}$$

$$\text{L2 Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}(\text{AgeT0}_i) + \gamma_{02}(\bar{x}_i - C_2) + U_{0i}$$

$$\text{L2 Time Slope: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{L2 Within X Slope: } \beta_{2i} = \gamma_{20}$$



From Clustered to Longitudinal: Part 2

Within-Person Change—Person-Mean-Centered Predictor:

$$\text{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(x_{ti} - \bar{x}_i) + e_{ti}$$

$$\text{L2 Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}T0_i) + \gamma_{02}(\bar{x}_i - C_2) + U_{0i}$$

$$\text{L2 Time Slope: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{L2 Within X Slope: } \beta_{2i} = \gamma_{20}$$

- **Level-1 Within-Person (WP) model**

- γ_{10} : Does y_{ti} change on average over time since baseline?
- γ_{20} : Does higher x_{ti} *than usual* \rightarrow higher y_{ti} *than usual*?

- **Level-2 Between-Person (BP) model**

- γ_{01} : Does being older *than others* \rightarrow higher y_i *than others*?
- γ_{02} : Does higher on \bar{x}_i *than others* \rightarrow higher y_i *than others*?
- U_{1i} : Do some people change more over time in y_{ti} than others?

From Clustered to Longitudinal: Part 2

Within-Person Change—Person-Mean-Centered Predictor:

$$\text{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(x_{ti} - \bar{x}_i) + e_{ti}$$

$$\text{L2 Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}(\text{AgeT0}_i) + \gamma_{02}(\bar{x}_i - C_2) + U_{0i}$$

$$\text{L2 Time Slope: } \beta_{1i} = \gamma_{10} + U_{1i}$$

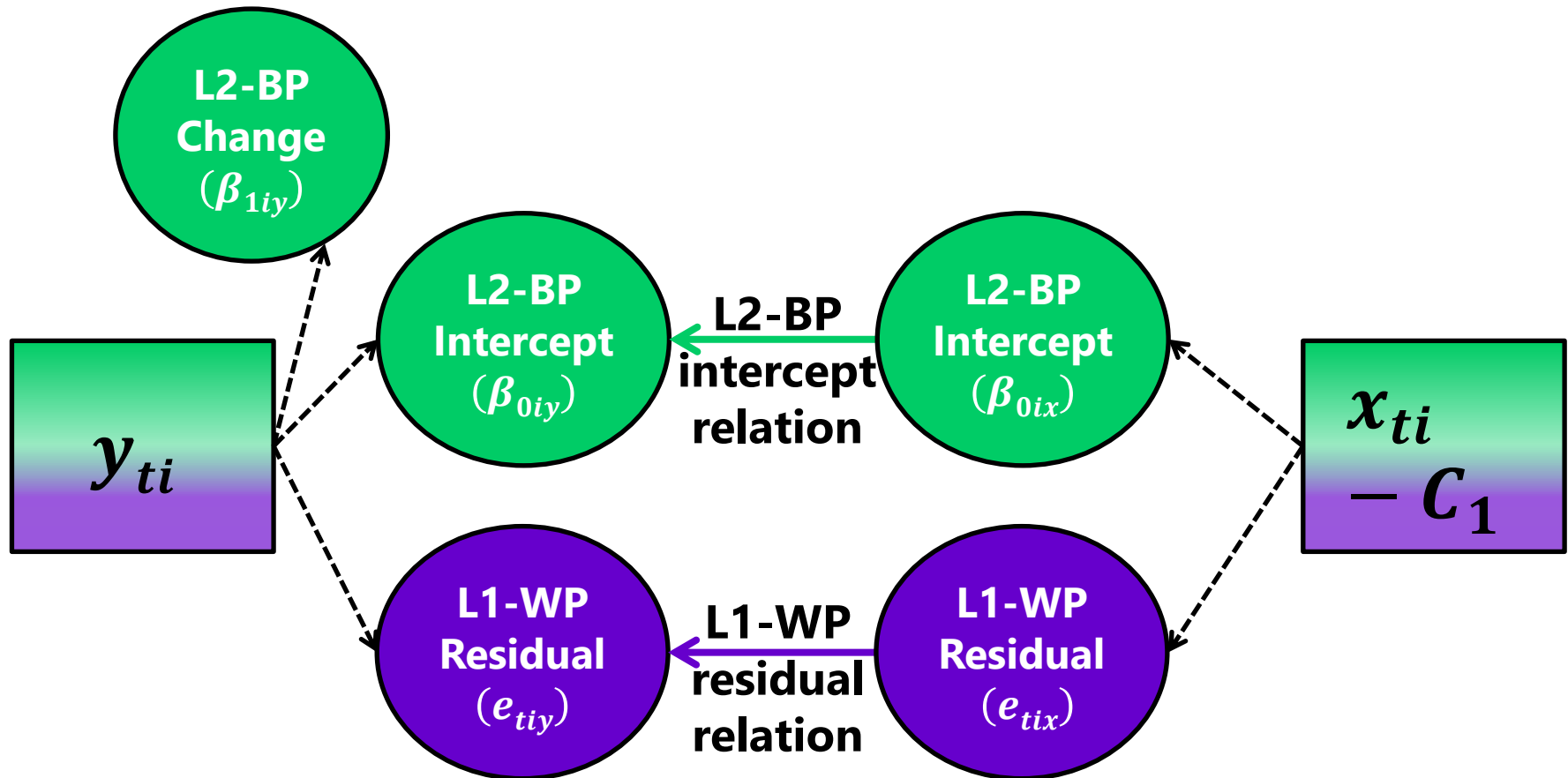
$$\text{L2 Within X Slope: } \beta_{2i} = \gamma_{20}$$

- But would **change over time in x_{ti}** influence $x_{ti} \rightarrow y_{ti}$??
 - If x_{ti} has fixed change only (same across persons), then NO
 - Fixed time slope $\gamma_{10} \rightarrow y_{ti}$ controls for same type of fixed time effect in how L1-WP x_{ti} slope predicts y_{ti}
 - If x_{ti} has random change (differs across persons), then YES
 - Random time slope $U_{1i} \rightarrow y_{ti}$ does NOT control for unmodeled random time slope needed for x_{ti}
- How to fix it?

2015-Me: "No, you can't always use person-mean-centering, please go read chapter 9"

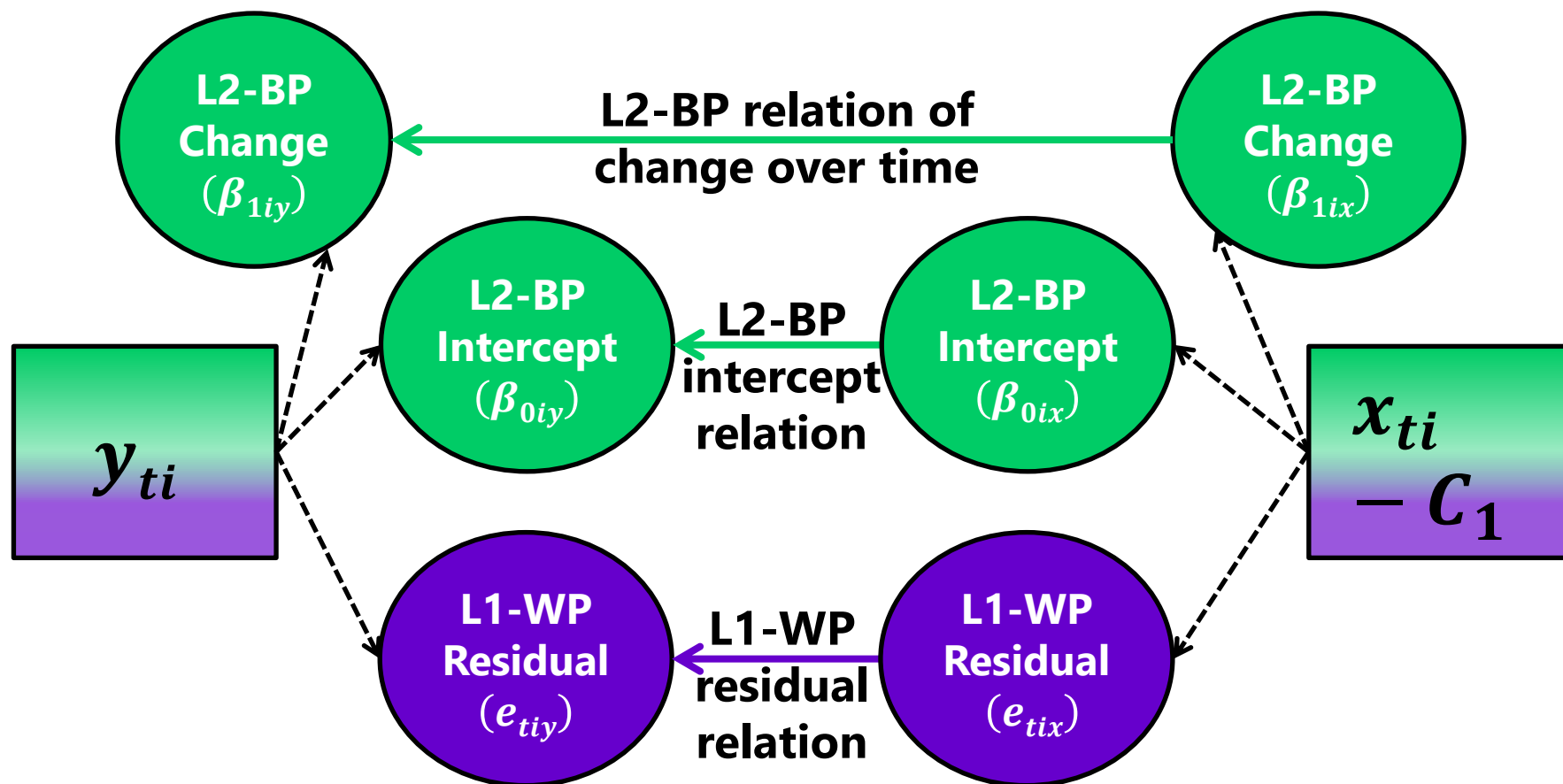
Step 1: “Latent Centering” for x_{ti}

Level-2 Between: Person Mean $\bar{x}_i \rightarrow$ Random Intercept β_{0ix}
Level-1 Within: Within Deviation $x_{ti} - \bar{x}_i \rightarrow$ Residual e_{tix}



Step 2: L2 Random Change in x_{ti}

Now we can distinguish 2 kinds of L2-BP relationships!



Example Longitudinal Associations

- e.g., **Long-term relations** of health (x) with cognition (y) in which there is WP change over time in each variable
 - People who are healthier (*than others at time 0*) may have better cognition → **L2-BP relation of intercepts** (not “means”)
 - People whose health declines less over time (*than others*) may decline less in cognition → **L2-BP relation** of **L1-WP time slopes**
 - When a person feels relatively better (*than predicted by their time trend*), they may then also have relatively better cognition
 - **WP relation of time-specific residuals** (that can differ BP)
 - Feel better *next time* instead? **WP “lagged” relation** (that can differ BP)

Example Longitudinal Associations

- “Change over time” includes **ALL kinds of time trends**, each of which can also show between-person variation
- e.g., **Short-term relations** of health (x) with bad mood (y)
 - People who tend to be less healthy (*than others*) may tend to be grumpier (*than others*) → **L2-BP relation of person means**
 - When people feel worse (*than usual*), they may also be grumpier (*than usual*) → **L1-WP relation of mean deviations**
- How about a **Monday effect***? It may need **L1-WP** slope, too!
 - If some people are more adversely affected by Mondays (*than others*), then that **L1-WP Monday slope** has **L2-BP variation**!
 - People who feel even worse on Mondays (*than others*) may be even grumpier on Mondays → **L2-BP relation** of **L1-WP time slopes**

* See *Office Space* movie: “Case of the Mondays” <https://www.youtube.com/watch?v=2AB9zPfXqQQ>

Confounded Associations?

- No matter the time scale, **any variable measured over time** has the potential for **three distinct sources of (co)variation**:
 - **L2-BP** in a measure of overall level (usually mean or intercept)
 - **L2-BP** differences in **L1-WP** slopes for time and time-varying predictors (including slopes for auto-regressive or “inertia” effects)
 - **L1-WP** time-specific deviations from BP-predicted trajectory
- **But common practice has two common problems**:
 - Time-varying “outcomes” are treated differently than “predictors”
 - “Time” may not be considered adequately in short-term studies
- **Missing L2-BP relation of time slopes will create bias!**
 - **Cue demo via simulation...!**

2020-Me: “I should try to write a real paper about this...”

Simulation Data Generation

- **2 variables** (x and y) with no missing data for **100 persons** (L2 i) over **5 occasions** (L1 t), indexed as $Time = (0,1,2,3,4)^*$

	Unconditional Model for Change		Variances
Level 1 Occasions:	$x_{tix} = \beta_{0ix} + \beta_{1ix}(Time_{tix}) + e_{tix}$ $y_{tiy} = \beta_{0iy} + \beta_{1iy}(Time_{tiy}) + e_{tiy}$		$\sigma_{e_x}^2 = .40$ $\sigma_{e_y}^2 = .40$
Level 2 Intercepts:	$\beta_{0ix} = \gamma_{00x} + U_{0ix}$ $\beta_{0iy} = \gamma_{00y} + U_{0iy}$	$\gamma_{00x} = 0$ $\gamma_{00y} = 0$	$\tau_{U_{0x}}^2 = .60$ $\tau_{U_{0y}}^2 = .60$
Level 2 Time Slopes:	$\beta_{1ix} = \gamma_{10x} + U_{1ix}$ $\beta_{1iy} = \gamma_{10y} + U_{1iy}$	$\gamma_{10x} = ?$ $\gamma_{10y} = ?$	$\tau_{U_{1x}}^2 = .06$ $\tau_{U_{1y}}^2 = .06$

- Total variance set to 1 at $time = 0$, so that:
 - Conditional ICC = .60 → Intercept variance for U_{0ix} and U_{0iy}
 - Slope Reliability = .60 → Time slope variance for U_{1ix} and U_{1iy}

Simulation Manipulations

- Fixed time effects (γ_{10x} absent or present) collapsed here
 - Didn't matter because $Time_{ti}$ was always a predictor of y_{ti}
- Key manipulation: **match across 3 types of relations**
- **Level-2 BP random effects** ($U_{0ix}, U_{0iy}, U_{1ix}, U_{1iy}$) drawn from a MVN distribution with 4 conditions:
 - **Intercept correlations:** $r(U_{0ix}, U_{0iy}) = 0$ or $.3$
 - **Time slope correlations:** $r(U_{1ix}, U_{1iy}) = 0$ or $.3$
 - All other Intercept–Time slope pairs of correlations = 0
- **Level-1 WP residuals** drawn from a separate MVN distribution with 2 conditions: $r(e_{tix}, e_{tiy}) = 0$ or $.3$

Univariate Longitudinal Options

MLMs: L2 BP and L1 WP Effects of x_{ti} as observed predictors

L1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$

L2 Time Slope: $\beta_{1i} = \gamma_{10} + U_{1i}$

L2 x_{ti} Slope: $\beta_{2i} = \gamma_{20}$

Person-Mean (PM) Centering: $+ \beta_{2i}(x_{ti} - \bar{x}_i)$

L2 Int: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\bar{x}_i) + U_{0i}$

Baseline (BL) Centering: $+ \beta_{2i}(x_{ti} - x_{0i})$

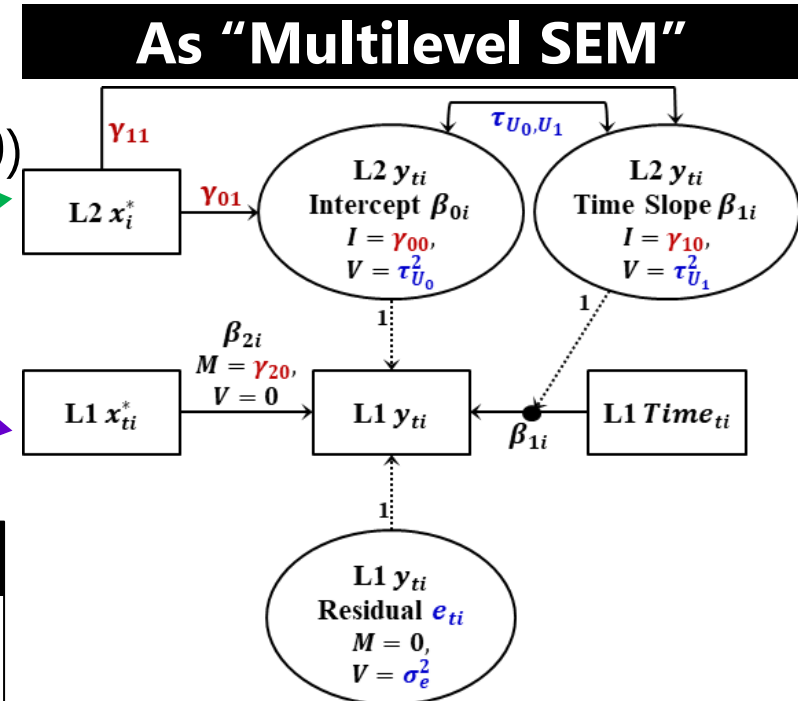
L2 Int: $\beta_{0i} = \gamma_{00} + \gamma_{01}(x_{0i}) + U_{0i}$

- Either should yield: $\gamma_{20} \rightarrow$ **L1-WP effect**; $\gamma_{01} \rightarrow$ **L2-BP effect**
- **L2 PM** \bar{x}_i uses all occasions so L1 residuals should cancel...
 - ...**But timing is off:** L2 *average* x_{ti} predicts L2 y_{ti} *time 0* intercept
- **L2 BL** x_{0i} matches timing to create L2 relation at *time 0*...
 - ...**But still has L1 residual:** Is *actual* x_{0i} , not *predicted* x_{ti} at time 0

Simulation Results: Univ Options

- How well did centering with the person mean (\bar{x}_i) or baseline ($x0_i$) **recover the 3 relations of x_{ti} with y_{ti} ?**

- L2 x_i^* by $Time_{ti}$ slope $\gamma_{11} = 0$
- No L2 x_{ti} random time (slope $r \rightarrow 0$)
- L2 intercept $r \rightarrow$ L2-BP x_i^* slope γ_{01}
- L1 residual $r \rightarrow$ L1-WP x_{ti}^* slope γ_{20}



$$\mathbf{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + \beta_{2i}(x_{ti}^*) + e_{ti}$$

$$\mathbf{L2 Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}(x_i^*) + U_{0i}$$

$$\mathbf{L2 Time Slope: } \beta_{1i} = \gamma_{10} + \gamma_{11}(x_i^*) + U_{1i}$$

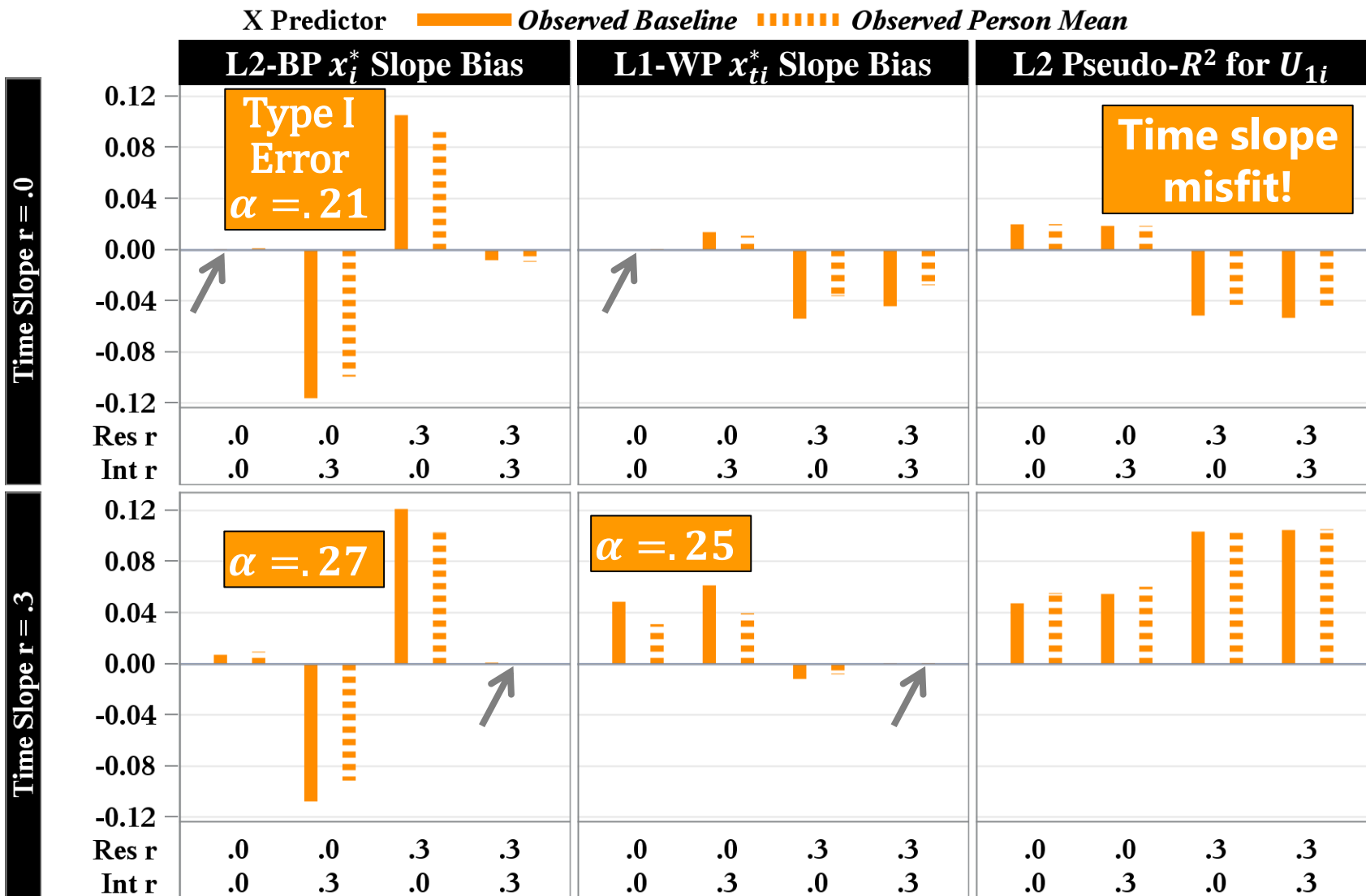
$$\mathbf{L2 } x_{ti} \mathbf{ Slope: } \beta_{2i} = \gamma_{20}$$

$$\mathbf{L2 } x_i^* = \bar{x}_i \mathbf{ or } x0_i$$

$$\mathbf{L1 } x_{ti}^* = (x_{ti} - \bar{x}_i)$$

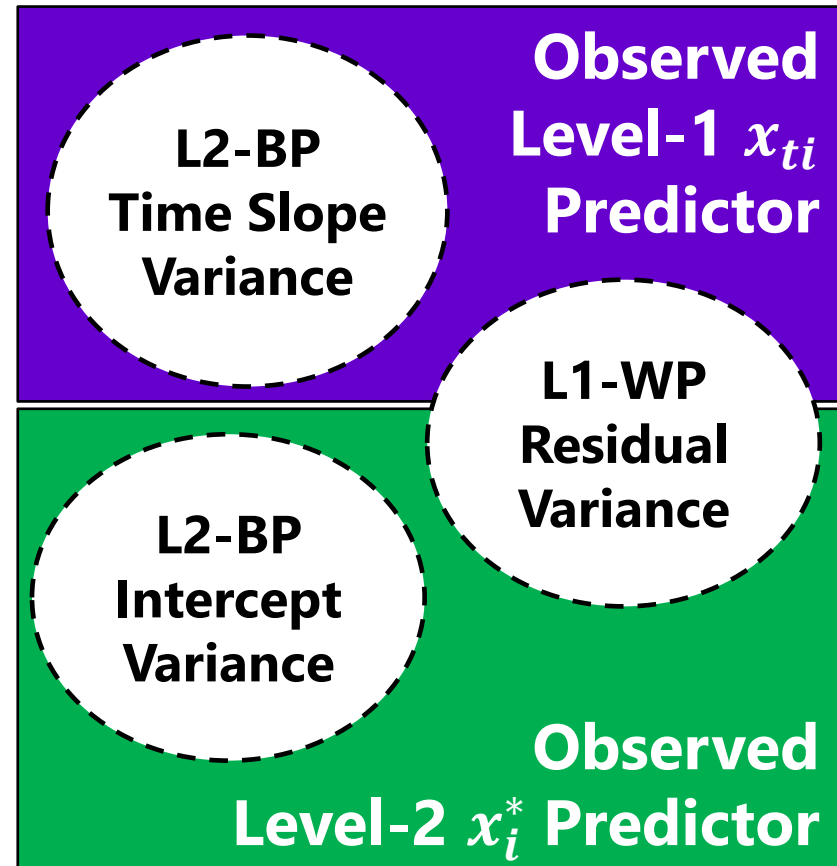
$$\mathbf{or } (x_{ti} - x0_i)$$

Univ Results: Time-Smushing Bias!



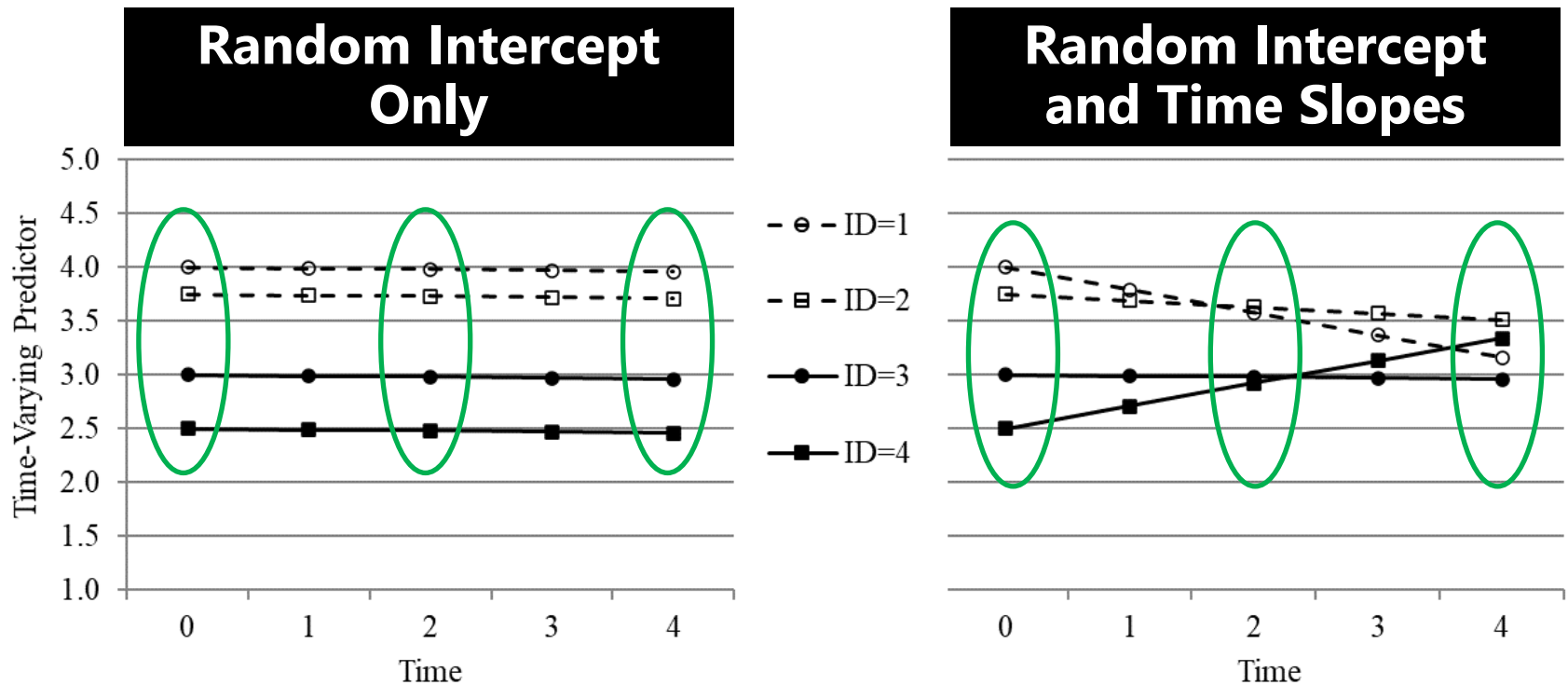
Why Time-Smushing Bias Happens

- **Ignoring L2-BP relationships between the time slopes of longitudinal variables can contaminate their other relations:**
 - Top: if the **L1-WP** x_{ti} still contains unmodeled L2-BP variance in time slopes, **the L1-WP effect will be smushed with the missing L2-BP time slope effect!**
 - Different than well-known problems of **intercept-smushed L1 WP effects** OR **bias from using observed mean** (bottom)



Why Level-2 BP Slopes are Affected

- **Ignoring L2-BP relationships between the time slopes of longitudinal variables can contaminate their other relations:**
 - Also in the **L2-BP Intercept**—because it must change over time!



Fixing Level-1 Bias... Univariately

- “**Detrended residuals**” is a univariate strategy designed to remove time-related variance from the level-1 x_{ti} predictor
- Is a **two-stage approach** analogous to “**slopes-as-outcomes**”:
 - Fit **separate regression model** to each person’s data
 - Save time-specific x_{ti} **residuals** to use as **level-1 x_{ti}^***
 - Save **fixed intercept at $time_{ti} = 0$** to use as **level-2 x_i^***

As “Univariate MLM”

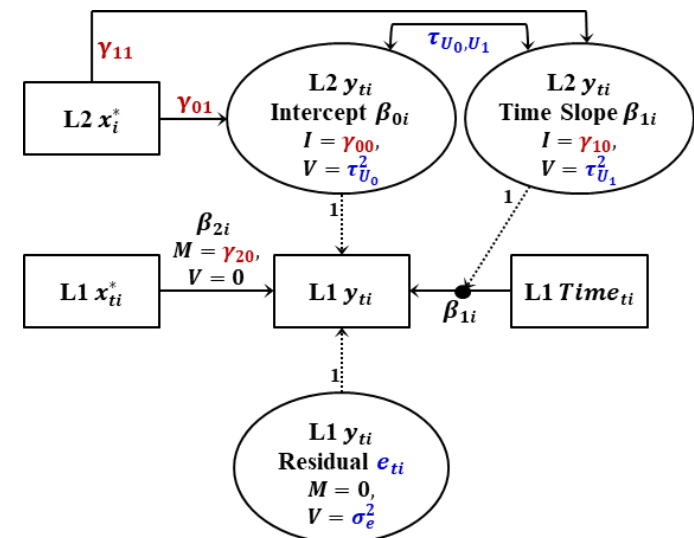
$$\text{L1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Time}_{ti}) + \beta_{2i}(x_{ti}^*) + e_{ti}$$

$$\text{L2 Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}(x_i^*) + U_{0i}$$

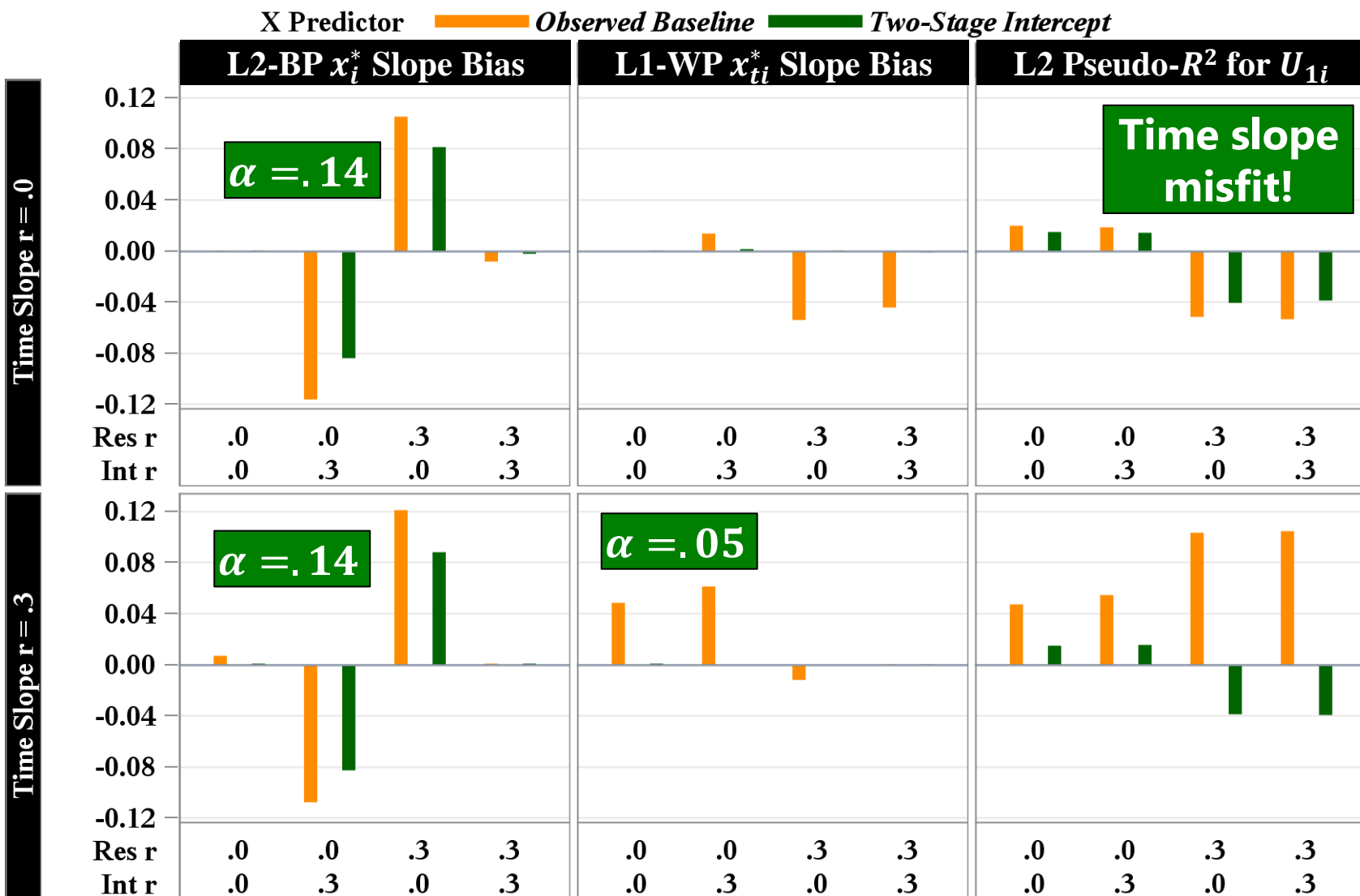
$$\text{L2 Time Slope: } \beta_{1i} = \gamma_{10} + \gamma_{11}(x_i^*) + U_{1i}$$

$$\text{L2 } x_{ti} \text{ Slope: } \beta_{2i} = \gamma_{20}$$

As “Multilevel SEM”



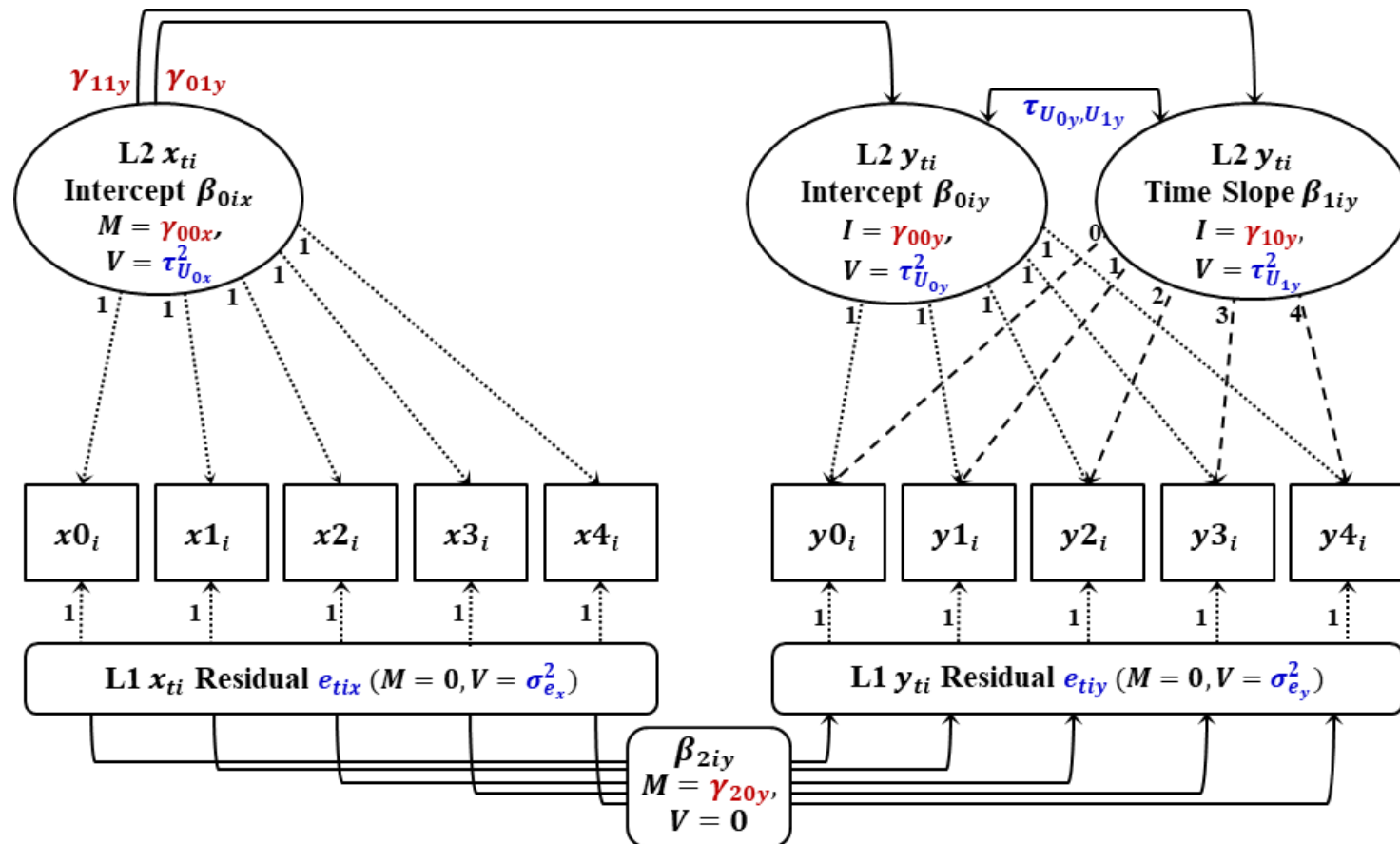
Univ Results: A Partial Fix!



2 Longitudinal Modeling Families

- **Univariate models: observed x_{ti} predictors $\rightarrow y_{ti}$**
 - As multilevel models (MLMs) using person-mean-centered, baseline-centered, or detrended-residual observed predictors
 - Multivariate relations for outcomes x_{ti} and y_{ti} \rightarrow covariances only
- **Multivariate models: latent x_{ti} predictors $\rightarrow y_{ti}$**
 - Both x_{ti} and y_{ti} are modeled as multilevel outcomes
 - As a single-level SEM (wide data) with latent variables $\rightarrow x_{ti}$
 - As a multilevel SEM (long data) with “latent centering” for x_{ti}
- **Let’s first see what happens when including a random intercept for x_{ti} without its random change slope...**

Multivariate MLM via Single-Level SEM: Only Random Intercept for x_{ti}



L1-WP effect between x_{ti} and y_{ti} **structured residuals** to get **L2-between effects** instead of L2-contextual effects

Multivariate MLM via Multilevel SEM: Only Random Intercept for x_{ti}

Fixed Effects from Intercept and Residual of Latent x_{ti} Only:

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$
 $y_{tix} = \beta_{0iy} + yw_{tix}$

w indicates a **L1** *within* variable

L1: $xw_{tix} = e_{tix}$

$yw_{tix} = \beta_{1iy}(Time_{tix}) + \beta_{2iy}(xw_{tix}) + e_{tix}$ $\beta_{2iy} = \gamma_{20y}$

L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$

$\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + U_{0iy}$

L2 Time Slopes: (β_{1ix} doesn't exist yet)

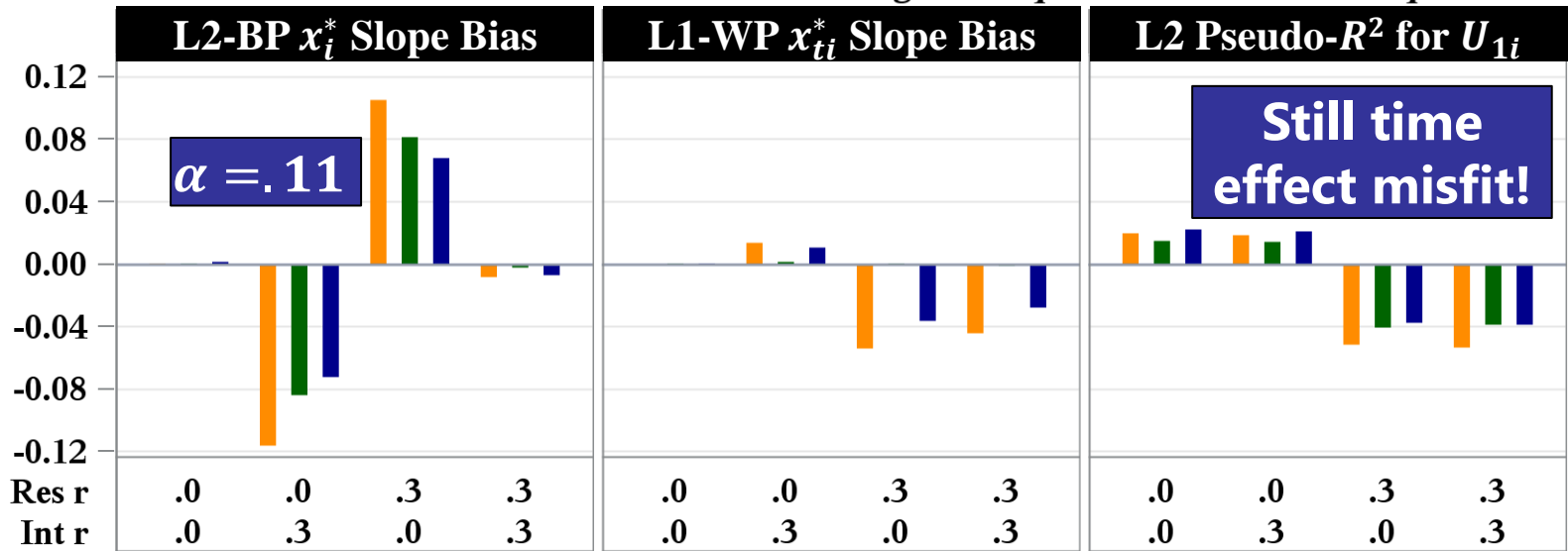
$\beta_{1iy} = \gamma_{10y} + \gamma_{11y}(\beta_{0ix}) + U_{1iy}$

So how does using **latent x_{ti} predictors** compare with **observed x_{ti} predictors** (baseline or two-stage intercept)?

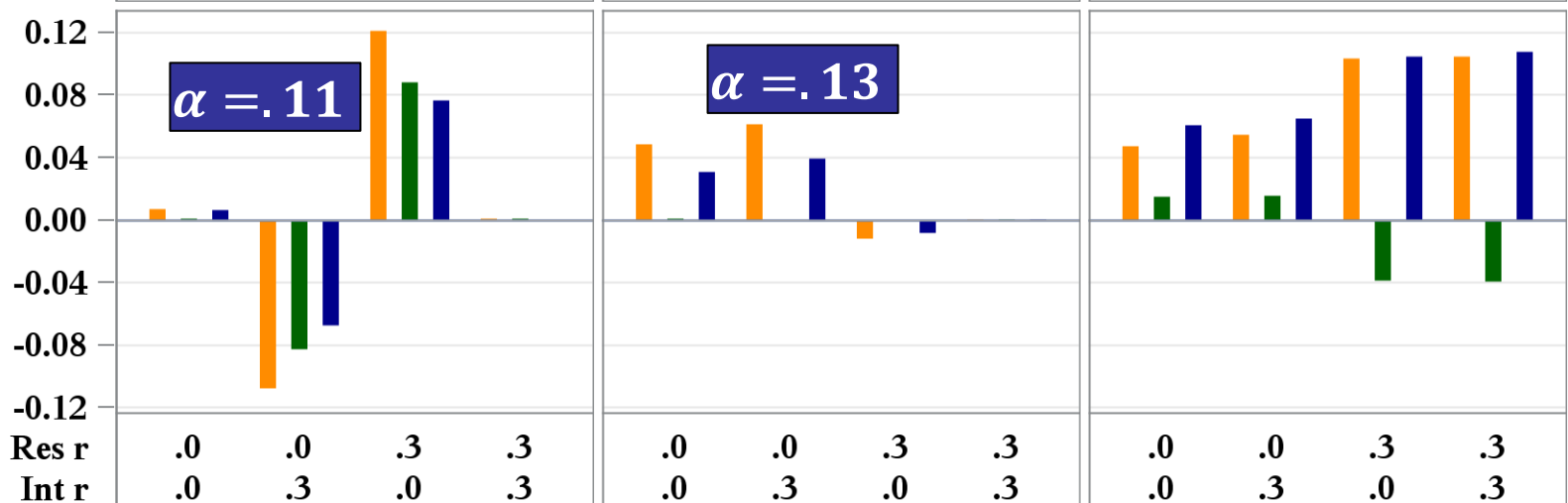
Latent x_{ti} \rightarrow Less Bias? Not yet...

X Predictor █ *Observed Baseline* █ *Two-Stage Intercept* █ *Latent Intercept*

Time Slope $r = .0$

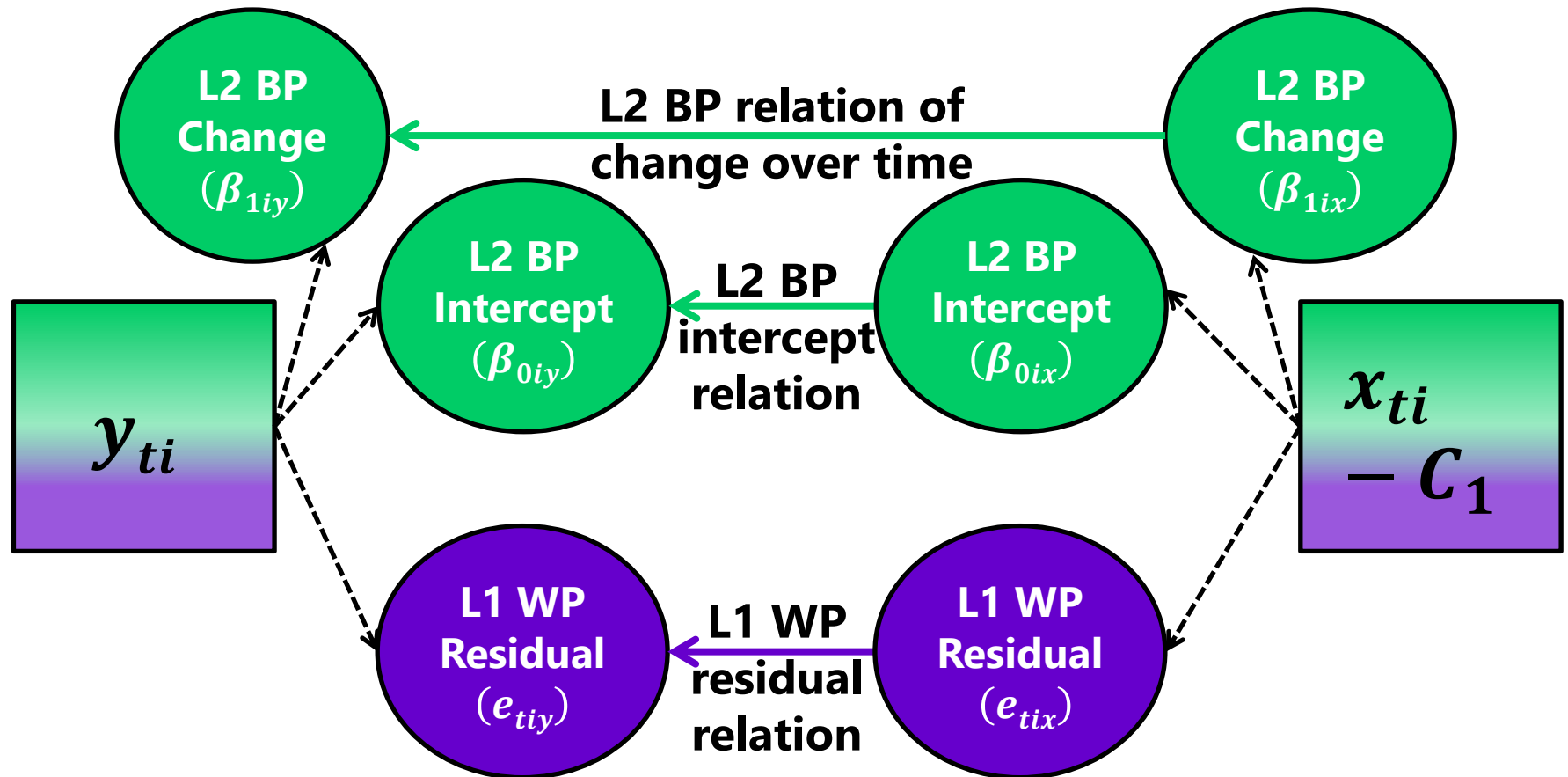


Time Slope $r = .3$



Spoiler Alert

Time-varying predictors with *individual* change over time need to be predicted in a multivariate longitudinal model!



Multivariate MLM via Multilevel SEM: Add Random Time Slope for x_{ti}

Fixed Effects of Intercept, Time Slope, and Residual of Latent x_{ti}

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$ w indicates a **L1** within variable
 $y_{tix} = \beta_{0iy} + yw_{tix}$

L1: $xw_{tix} = \beta_{1ix}(\text{Time}_{tix}) + e_{tix}$
 $yw_{tix} = \beta_{1iy}(\text{Time}_{tix}) + \beta_{2iy}(xw_{tix}) + e_{tix}$ $\beta_{2iy} = \gamma_{20y}$

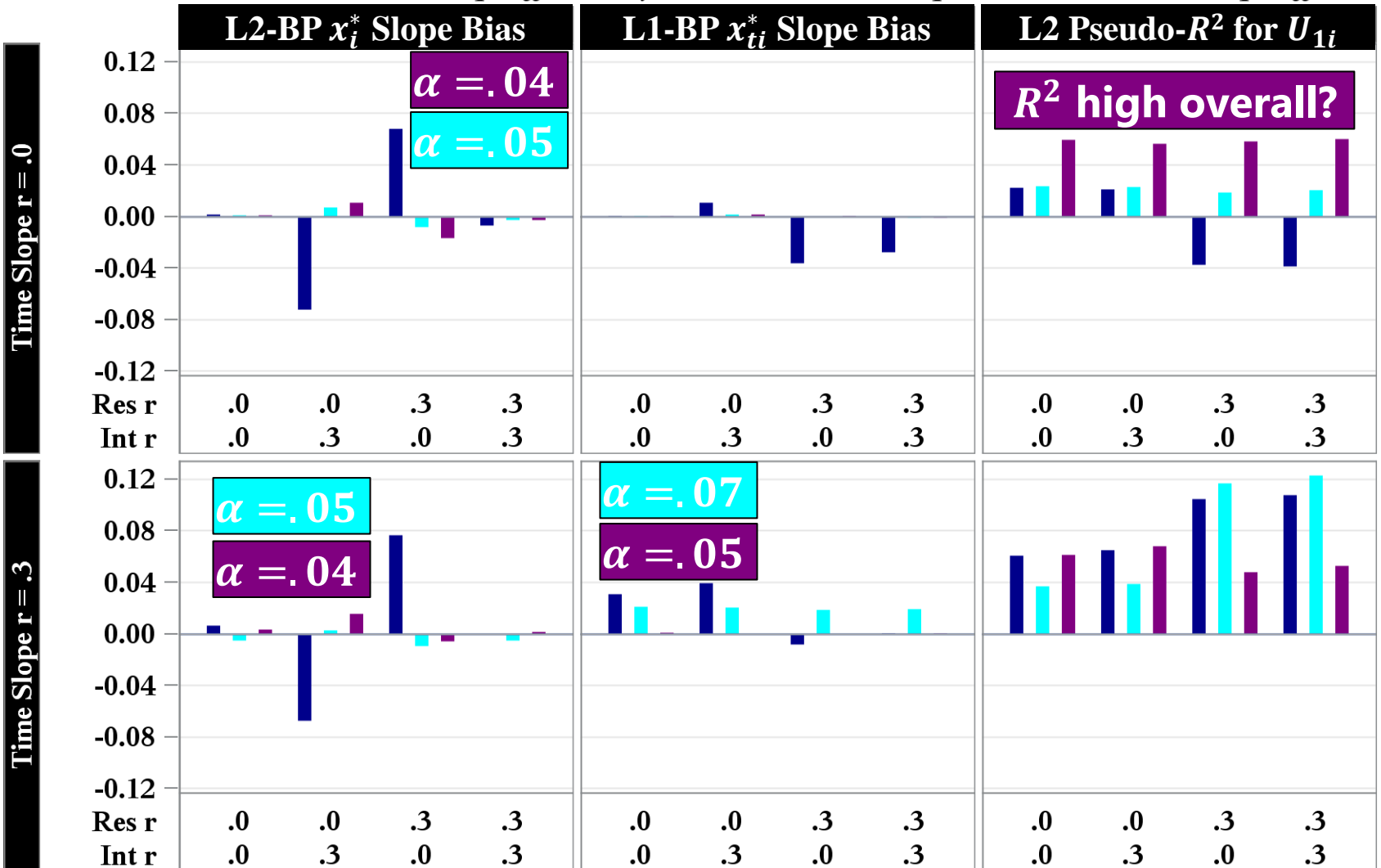
L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$
 $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + \gamma_{02y}(\beta_{1ix}) + U_{0iy}$

L2 Time Slopes: $\beta_{1ix} = \gamma_{10x} + U_{1ix}$
 $\beta_{1iy} = \gamma_{10y} + \gamma_{11y}(\beta_{0ix}) + \gamma_{12y}(\beta_{1ix}) + U_{1iy}$

How well does this "multivariate latent growth curve model with structured residuals" recover the **3 types of relations of x_{ti} with y_{ti}** ?

Results: Better! (But Not Perfect)

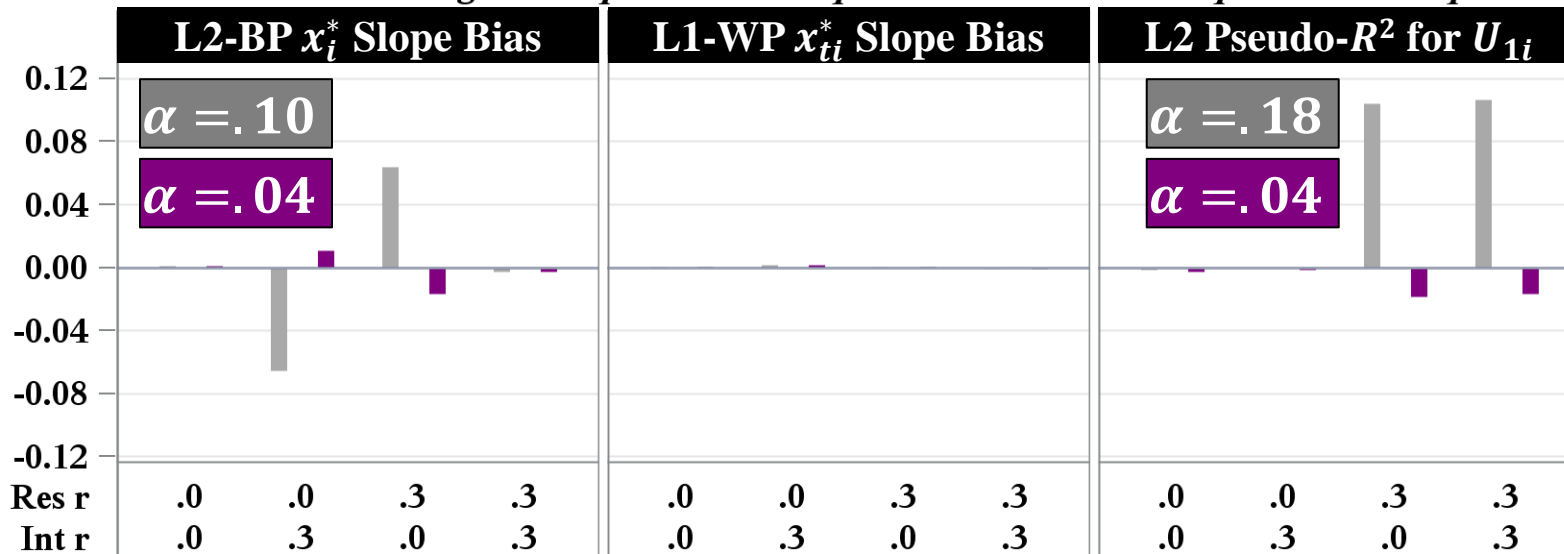
Latent X Predictors ■ *Intercept Effects Only* ■ *Add Time Slope* ■ *Add Time Slope Effects*



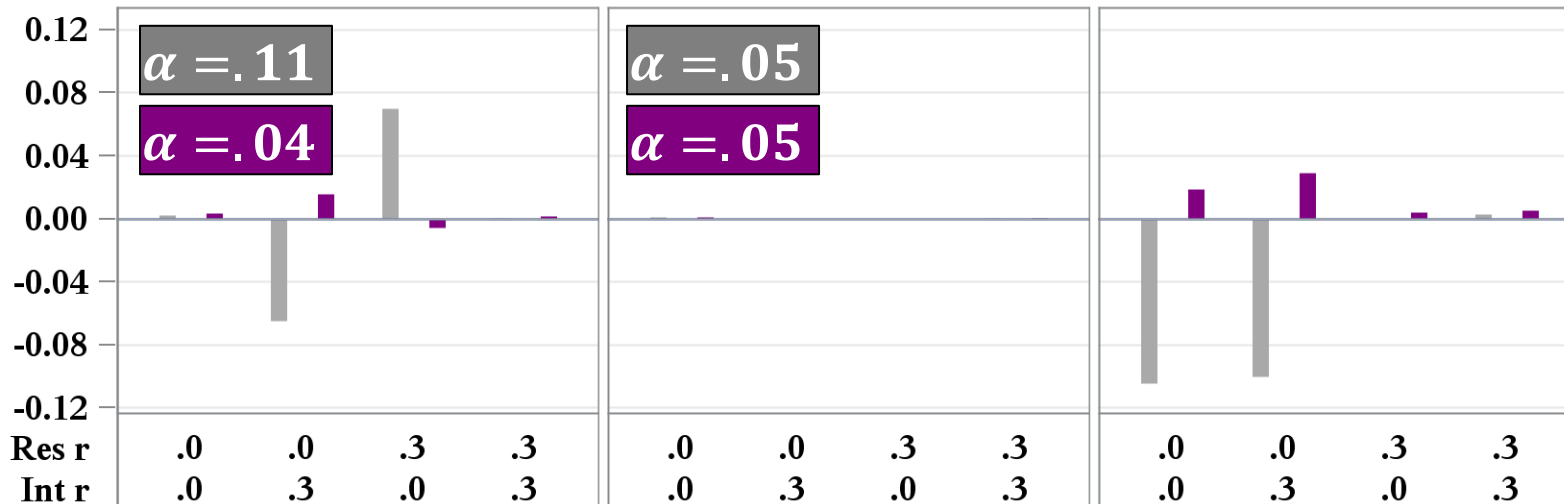
Slopes-as-Outcomes? Still Nope.

X Predictor █ Two-Stage Intercept and Time Slope █ Latent Intercept and Time Slope

Time Slope $r = .0$

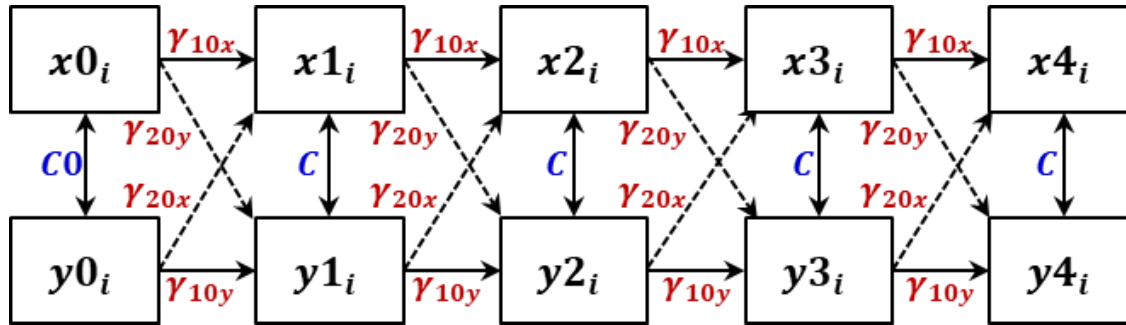


Time Slope $r = .3$



Smushed Effects in Related Models*

Auto-Regressive Cross-Lagged Panel Model (the "ARCL" or "CLPM")



Path model with separate intercepts (and residual variances) per occasion, and lag-1 fixed effects:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + e_{tix}$$

$$y_{t iy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + e_{t iy}$$

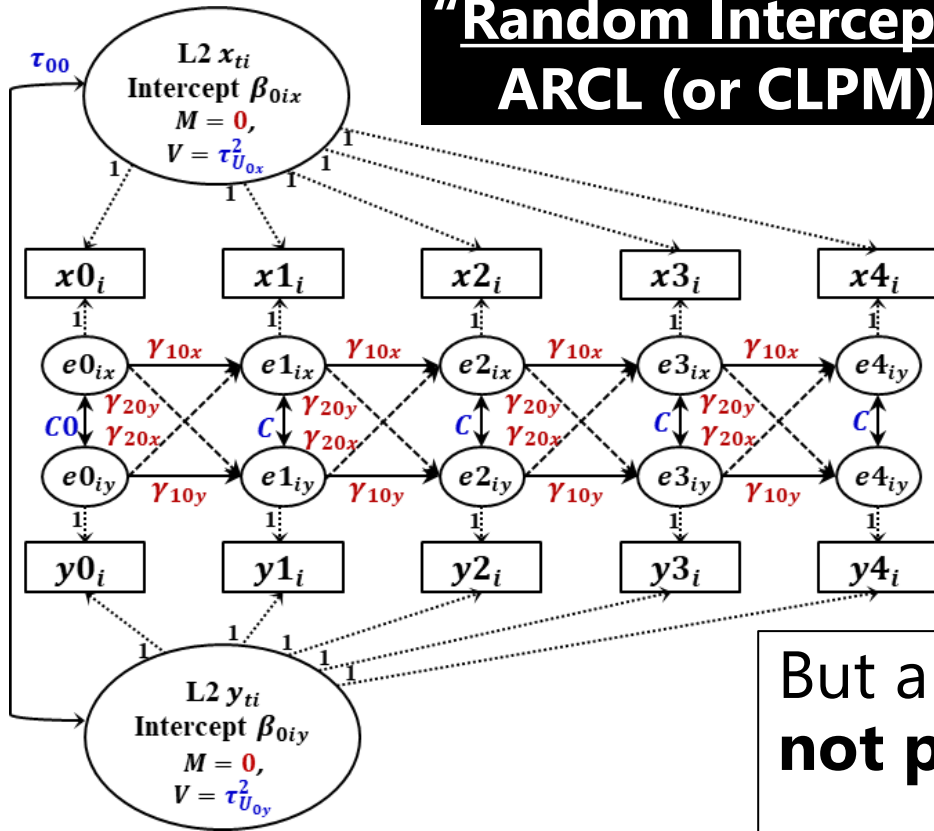
- **CLPM interpretation is problematic:**

- Do the γ_{10} **auto-regressive (AR) effects** "control for stability"?
- Which type of relation is given by γ_{20} **cross-lagged (CL) effects**?
- Which type of relation is the **same-occasion C covariance**?

* *Same problems apply to mediation variants ($X \rightarrow M \rightarrow Y$)*

Remedies for Intercept Smushing

"Random Intercept" ARCL (or CLPM)



Distinguish BP mean effects from WP residual effects:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + U_{0ix} + e_{tix}$$

$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + U_{0iy} + e_{tiy}$$

But a random intercept alone will **not prevent time-smushing...**

Do the **within-variable AR paths** protect against time smushing?

Let's find out!

Simulation: Add CLPM Fixed Effects

Full $X \rightarrow Y$ Model: L2-BP Intercept Effects, L2-BP Time Slope Effects, L1-WP AR Effects, and L1-WP CL Effects

Total: $x_{tix} = \beta_{0ix} + xw_{tix}$
 $y_{tix} = \beta_{0iy} + yw_{tix}$

w indicates a **L1** *within* variable

L1: $xw_{tix} = \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + \beta_{3ix}(\text{Time}_{tix}) + e_{tix}$
 $yw_{tix} = \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + \beta_{3iy}(\text{Time}_{tix}) + e_{tix}$

L2 Intercepts: $\beta_{0ix} = \gamma_{00x} + U_{0ix}$
 $\beta_{0iy} = \gamma_{00y} + \gamma_{01y}(\beta_{0ix}) + U_{0iy}$

Intercept \rightarrow
Intercept

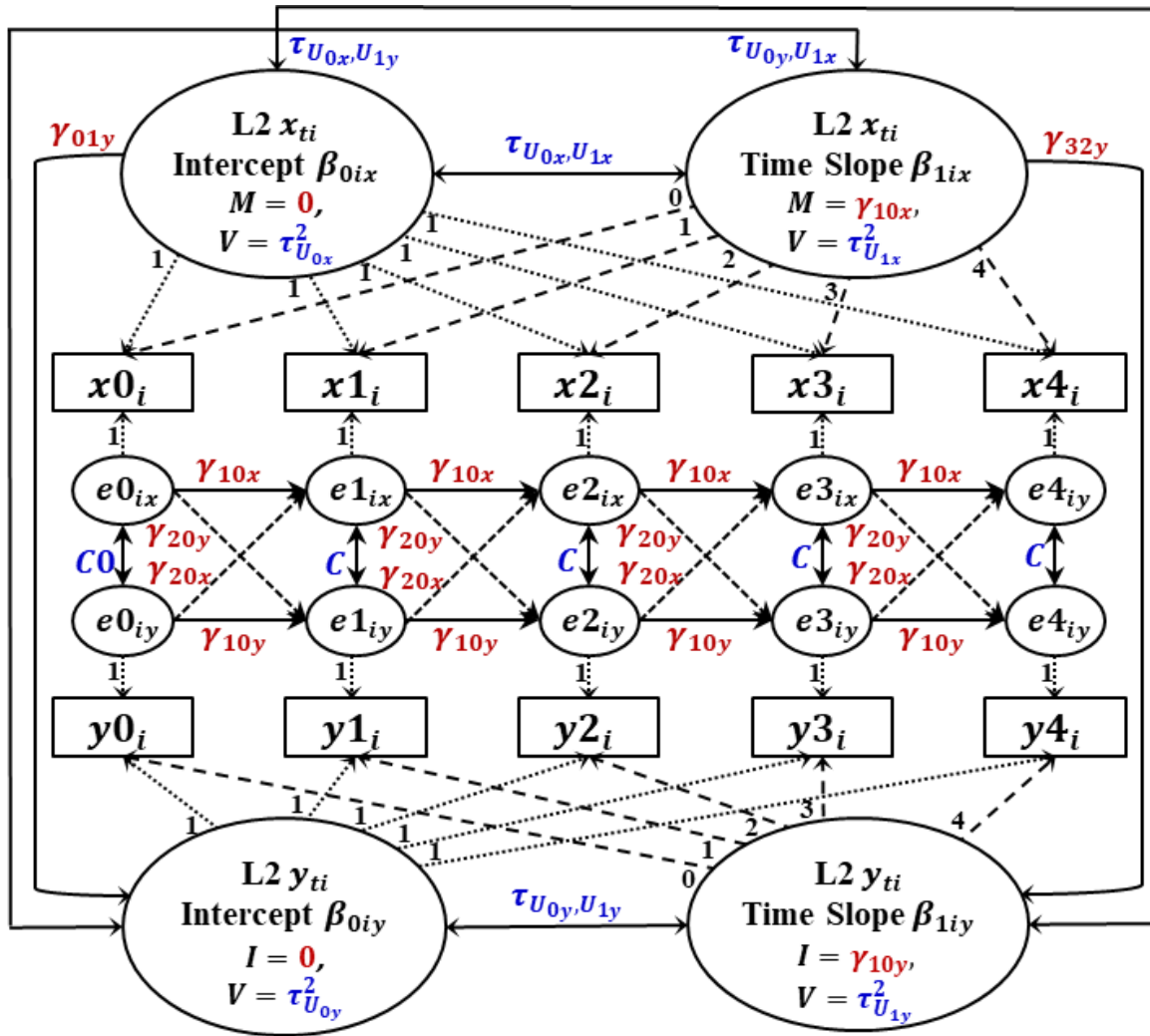
L2 Time Slopes: $\beta_{3ix} = \gamma_{30y} + U_{3iy}$
 $\beta_{3iy} = \gamma_{30y} + \gamma_{32y}(\beta_{1ix}) + U_{3iy}$

Time slope \rightarrow
Time slope

All L1-WP AR and CL Slopes had population values = 0

*Btw, this is also a "latent curve model with structured residuals"

Simulation: Compare Model Variants



Full $X \rightarrow Y$ Model:
L2-BP Intercept Effects,
L2-BP Time Slope Effects,
L1-WP AR Effects, and
L1-WP CL Effects*

Drop Time Slope effect

Drop Time Slope, too

Drop Time Slope effect;
drop L1 AR Slope

Drop Time Slope, too;
drop L1 AR Slope

* *Always included*

Simulation Results: CLPM Fixed Effects

- If a random time slope for x_{ti} was omitted:
 - L1 AR slopes for x_{ti} were very positively biased ($\alpha = .98$)
- If the BP-L2 time slope relation for $x_{ti} \rightarrow y_{ti}$ was omitted:
 - L1 CL slopes for $x_{ti} \rightarrow y_{ti}$ were biased in that direction, even more so when including L1 AR slopes for x_{ti} !
 - L1 CL slopes for $y_{ti} \rightarrow x_{ti}$ had complex patterns of bias
- It seems like WP questions of “which came first” cannot be answered reliably until the BP model is complete
 - Same idea as “detrending” individual time series for time trends *before* looking at time-specific relations across variables
 - **So first check for *random* change in time-varying “predictors”!**

The End...? Not Quite.

Points to Ponder:

- How does “BP/WP” relate to “cross-sectional/longitudinal”?
- When can “static” time trends be reliably distinguished from “dynamics”?
- How do these issues translate in “accumulating” models?

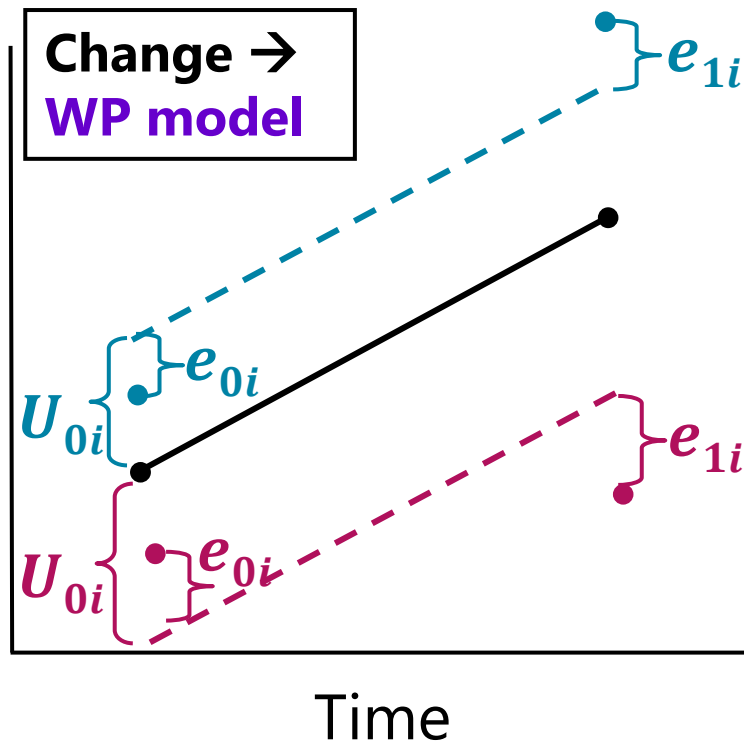
Language: Not so Neat and Tidy

- Applied researchers distinguish “**cross-sectional**” from “**longitudinal**” relations in different ways...
- With respect to **time scale**:
 - “Cross-sectional” → both variables measured at the same time
 - “Longitudinal” → one variable measured after the other
- With respect to **level of inference**:
 - “Cross-sectional” = BP levels → requires > 1 person
 - “Longitudinal” = WP levels → requires > 1 occasion
- So which label should be used for questions like “*Does change in X predict change in Y?*”

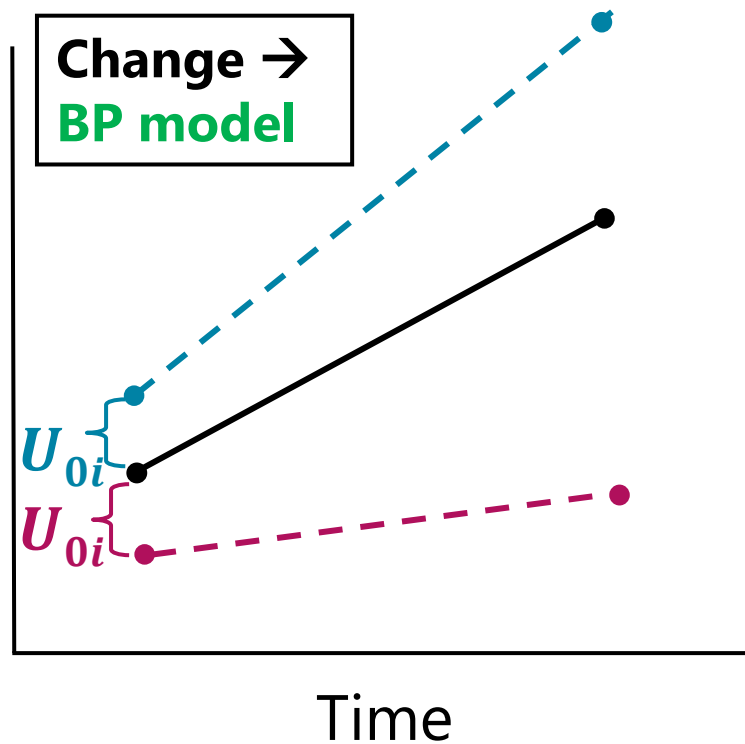
Ambiguity in Two-Occasion Data

This model for the means (fixed effects): $\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(\text{Time}_{ti})$ could have **two equivalent models for the variance**:

U_{0i} & e_{ti}^* → BP differences in intercept + (het) WP residual



U_{0i} & U_{1i} → BP differences in **perfectly measured** intercept and change (no WP residual)!



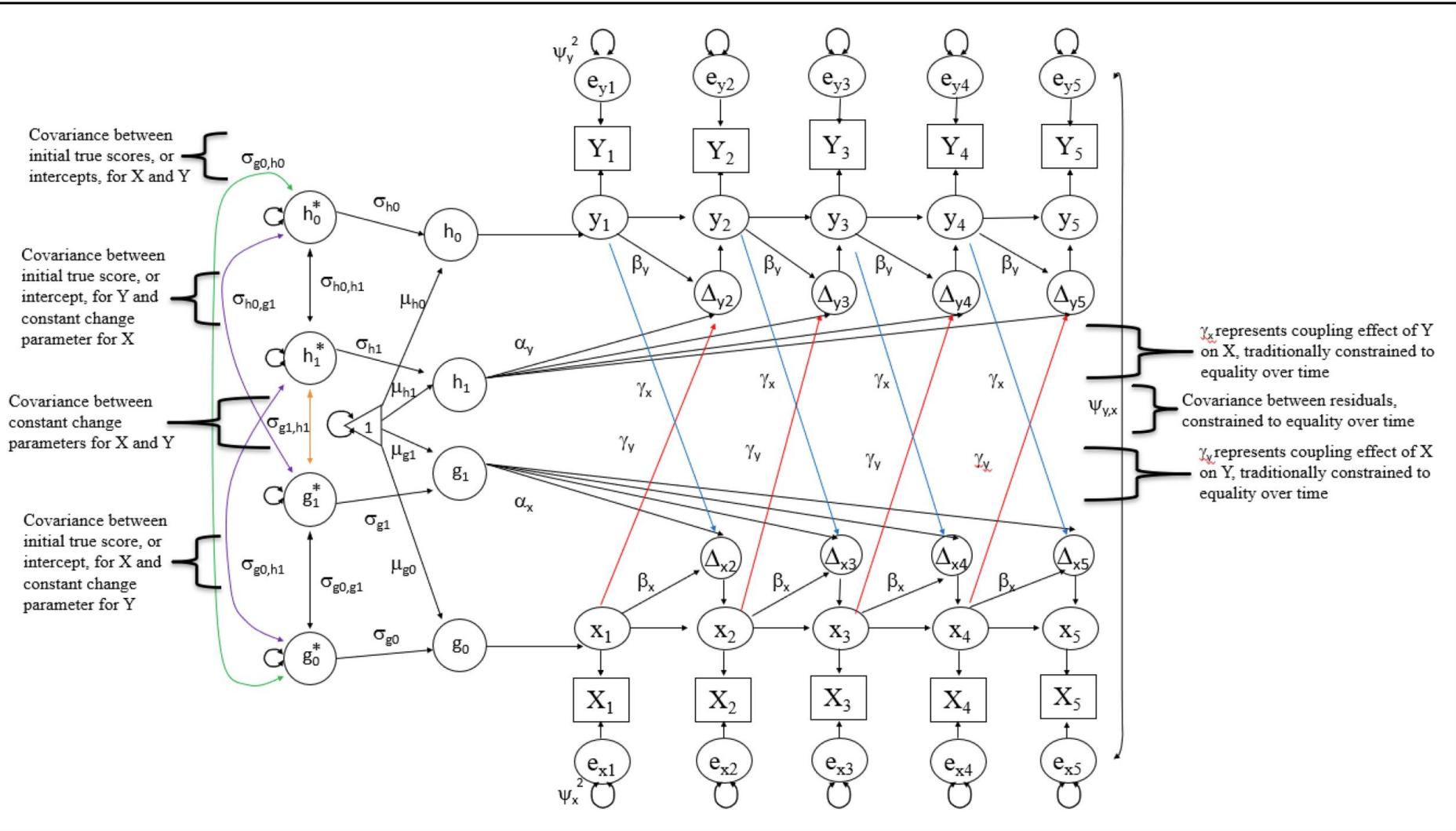
POV: “Static” vs “Dynamic”

- Longitudinal models are sometimes categorized (with continuous or discrete time versions of each type) into:
 - **Static** models capture time trends (e.g., multivariate change models)
 - **Dynamic** models capture past → future (e.g., VAR, CLPM, DSEM)
- **Criticisms of “static” models:**
 - “Time” is not an explanatory variable (no argument here)
 - Do not capture inertia per se or how change → change
- **Criticisms of “dynamic” models:**
 - How can time-invariant and fixed AR or CL slopes be “dynamic”?
 - Not modeling random time trends → lack of stationarity → bias?

Trends + Dynamics: Two Options

- **“Non-accumulating”** models
 - **BP trend relations are partitioned from WP lagged relations** specified through “structured residuals” (SR) of original variables
 - e.g., “latent curve model-SR” (in my second simulation)
 - Continuous “change over time” divided into BP and WP parts
- **“Accumulating”** models
 - **BP trend and WP lagged relations are specified through same variables** → *many* indirect effects and different interpretations
 - e.g., ALT, latent change score (LCS), some “general” CLPM variants
 - In particular: LCS includes baseline and trend factors, but each occasion’s level/change → subsequent level/change as “coupling” without distinguishing their BP and WP parts

Example Latent Change Score Model



Ambiguity in Practice

- Everything works best when fitting the population model, but we don't know what that is in real data!
 - It can be tough to distinguish **compensatory** parameters!
- Difficult to distinguish **WP "dynamics"** from **BP "trends"**:
 - e.g., heterogeneous residual variance vs. random slopes
 - e.g., "simplex" AR models vs. linear growth
 - e.g., AR1 and AR2 slopes vs. random intercepts in CLPMs
- Difficult to distinguish among **types of WP** phenomena:
 - e.g., AR slopes for all occasions vs. day-level random intercept
 - e.g., AR slopes (among variables) vs. MA slopes (among residuals)

In Conclusion

- **Things I am still reasonably convinced about:**
 - BP (mean or intercept) relations \neq WP (residual) relations
 - Different reasons for their variances AND for their covariances
 - Should be distinguished *per longitudinal variable* at a minimum (not common “unit” effects shared across distinct variables)
 - Getting “too much” out of longitudinal data may only be possible through false BP convergence of time-varying info
 - e.g., BP age cohort effects vs. WP retest effects
 - e.g., AR slopes (for variables) vs. MA slopes (for residuals)
 - “Trends” and “dynamics” need a LOT of data to be distinguished
 - Non-accumulating (BP vs WP models) seem to work better at this

In Conclusion

- **Things I am less sure about now than before:**
 - What information should go into “longitudinal” relations?
 - Is it enough to have a random change slope in the model, or do we also need to isolate the BP part of WP change before examining “dynamic” effects?
 - If not, how to interpret results of “accumulating” models—although they can be equivalent to BP vs WP models in some cases, they can suffer from lack of clarity of interpretation...

More from 2004: SMEP-ish in Phoenix

Modeling Developmental Processes in Ecological Context (KU Merrill Center)

