Example 2: Unconditional Polynomial Slopes Models for Change
(complete data, syntax, and output available for SAS, SPSS, and STATA electronically)

This example comes from Hoffman (2015) chapter 6. These data are from a short-term longitudinal study of six occasions over 2 weeks for 101 adults age 65–80 years. We will see how performance on this processing speed task (called "number match 3"), as measured by response time in milliseconds, declines (improves) over the 6 practice sessions. In this example we will examine polynomial models describe individual change; for additional models (piecewise, exponential, and log-transformed time) see CLDP 944 Example 6.

SAS Code for Data Manipulation:

```
* Define global variable for file location -- CHANGE THIS TO YOUR DIRECTORY;
%LET example= C:\Dropbox\Workshop_Illinois_2018\Download\SAS;
LIBNAME example "&example.";
* Import data into work library and center predictors for analysis;
DATA work.Example23; SET example.Example23;
  * Center time for polynomial models;
    c1sess = session - 1; LABEL c1sess = "c1sess: Session Centered at 1"; RUN;
```

SPSS Code for Data Manipulation:

```
* Define file location -- CHANGE THIS TO YOUR DIRECTORY.
FILE HANDLE example /NAME = "C:\Dropbox\Workshop_Illinois_2018\Download\SPSS".
* Open data and center predictors for analysis.
GET FILE = "example/Example23.sav".
DATASET NAME Example23 WINDOW=FRONT.
* Center time for polynomial models.
COMPUTE c1sess = session - 1.
VARIABLE LABELS c1sess "c1sess: Session Centered at 1".
EXECUTE.
```

STATA Code for Data Manipulation:

```
* Define global variable for file location -- CHANGE THIS TO YOUR DIRECTORY
global example "C:\Dropbox\Workshop_Illinois_2018\Download\STATA"
* Open file used in example -- CHANGE THIS
use "$example\Example23.dta", clear
* Center time for polynomial models (and make quadratic version)
gen c1sess = session - 1
label variable c1sess  "c1sess: Session Centered at 1"
gen c1sess2 = c1sess * c1sess
label variable c1sess2 "c1sess2: Quadratic Session Centered at 1"
```

Model 0. Most Liberal Baseline—Saturated Means, Unstructured Variances (Model Answer Key)

```
TITLE1 "SAS Model 0: Saturated Means, Unstructured Variances";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
     REPEATED session / R RCORR TYPE=UN SUBJECT=ID;
     LSMEANS session /; RUN; TITLE1;

ECHO "SPSS Model 0: Saturated Means, Unstructured Variances".
MIXED nm3rt BY ID session
      /METHOD  = REML
      /PRINT   = SOLUTION TESTCOV R
      /FIXED   = session
      /REPEATED = session | SUBJECT(ID) COVTYPE(UN)
      /EMMEANS   = TABLES(session).

* STATA Model 0: Saturated Means, Unstructured Variances
mixed nm3rt ib(last).session, || id: , noconstant ///
         variance reml residuals(unstructured, t(session)) dfmethod(satterthwaite),
```

Placing *session* on the CLASS/BY statements and in the FIXED/MODEL statements treats it as a categorical predictor. So this is an ANOVA means model. No RANDOM statements mean no random effects.

i. indicates categorical predictor of *session* (ref=last to match others)
noconstant = no random intercept (just **R** matrix)

```
estat ic, n(101)                          // AIC and BIC
estat wcorrelation, covariance            // R covariance matrix
estat wcorrelation                        // R correlation matrix
estat df, method(satterthwaite)           // print DDF for fixed effects
contrast session, small                   // omnibus test of mean differences
margins i.session                         // observed means per session
marginsplot                               // plot observed means
```

SAS output:

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	235659	217994	202607	192154	195360
2	235659	**259150**	230217	213232	202092	193268
3	217994	230217	**233368**	205209	196919	188604
4	202607	213232	205209	**217544**	193676	185321
5	192154	202092	196919	193676	**212098**	187840
6	195360	193268	188604	185321	187840	**196733**

This Unstructured **R matrix** estimates all variances and covariances separately. THIS IS THE DATA we are trying to duplicate with our model for the variance

Estimated R Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8424	0.8212	0.7905	0.7593	0.8015
2	0.8424	1.0000	0.9361	0.8981	0.8620	0.8559
3	0.8212	0.9361	1.0000	0.9108	0.8851	0.8802
4	0.7905	0.8981	0.9108	1.0000	0.9016	0.8958
5	0.7593	0.8620	0.8851	0.9016	1.0000	0.9196
6	0.8015	0.8559	0.8802	0.8958	0.9196	1.0000

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8229.8	21	8271.8	8273.4	8294.0	8326.7	8347.7

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		1672.14	44.1345	100	37.89	<.0001
Session	1	289.76	32.7000	100	8.86	<.0001
Session	2	143.04	26.2031	100	5.46	<.0001
Session	3	77.8986	22.8842	100	3.40	0.0010
Session	4	45.6604	20.7853	100	2.20	0.0303
Session	5	35.0397	18.1168	100	1.93	0.0559
Session	6	0	.	.	.	.

Mean diffs relative to session 6 (which is the intercept given that it is the highest value)

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Session	5	100	16.72	<.0001

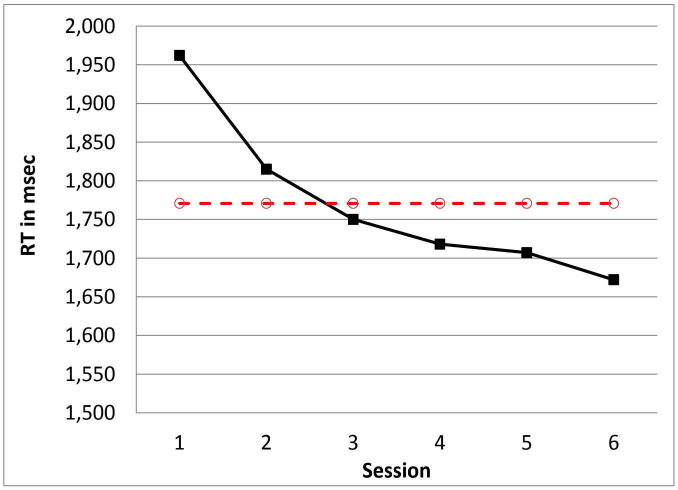
This is the omnibus test of mean differences across 6 sessions.

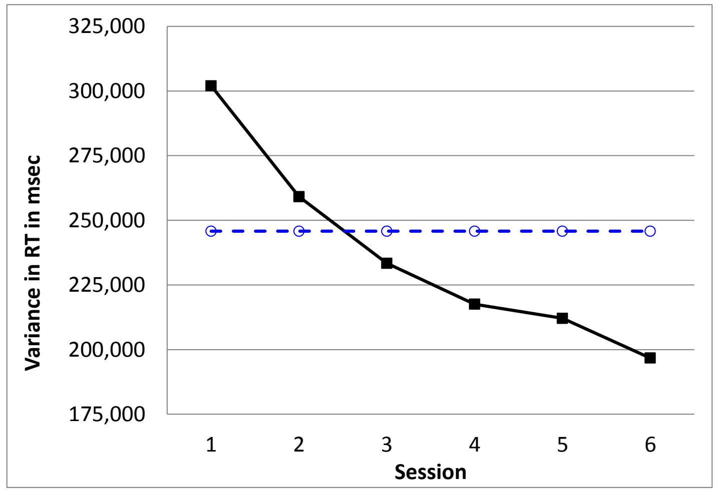
Least Squares Means

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Session	1	1961.89	54.6805	100	35.88	<.0001
Session	2	1815.17	50.6541	100	35.83	<.0001
Session	3	1750.03	48.0684	100	36.41	<.0001
Session	4	1717.80	46.4101	100	37.01	<.0001
Session	5	1707.18	45.8255	100	37.25	<.0001
Session	6	1672.14	44.1345	100	37.89	<.0001

These are the means per session that the fixed effects will be trying to reproduce.

So here is what are we trying to model—the black lines are means and variances from model 0, the data:





So where did the dashed (and flat) red and blue lines come from? Model 1a, up next.

Model 1a. Most Conservative Baseline—Empty Means, Random Intercept

Level 1: $y_{ti} = \beta_{0i} + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

METHOD = ML or REML (default)
CLASS = categorical predictors, nesting
MODEL dv = fixed effects / print solution
RANDOM = person variances in **G**
REPEATED = residuals in **R** matrix

```
TITLE1 "SAS Model 1a: Empty Means, Random Intercept";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
     REPEATED session / R TYPE=VC SUBJECT=ID;
     ODS OUTPUT CovParms=CovEmpty; * CovParms saves estimates to calculate pseudo-R2;
RUN; TITLE1;
```

```
ECHO "SPSS Model 1a: Empty Means, Random Intercept".
MIXED nm3rt BY ID session
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  =
      /RANDOM = INTERCEPT | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).
```

MIXED dv BY categorical predictors
WITH continuous predictors
/METHOD = REML or ML
/PRINT = regression solution
/FIXED = predictors for means model
/RANDOM = person variances in **G**

```
* STATA Model 1a: Empty Means, Random Intercept
mixed nm3rt ,  || id:  ,  ///
         variance reml covariance(unstructured) residuals(independent,t(session)) ///
         dfmethod(satterthwaite),
       estat icc            // get ICC
       estat ic, n(101)     // AIC and BIC
       estat recovariance, relevel(id) // G matrix
```

DV = nm3rt, random part after ||
Level 2 ID is PersonID, random intercept by default
Print variances instead of SD, use reml
covariance(unstructured) refers to G matrix
residuals(independent) → type of R matrix by session
estat ic → Print IC given N = 101 persons

SAS output:

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	44900					
2		44900				
3			44900			
4				44900		
5					44900	
6						44900

This level-1 **R** matrix (with equal variance over time, no covariance of any kind, known as VC or independence) will be used repeatedly as we add fixed

```
          Estimated G Matrix
                        Participant
Row     Effect          ID                   Col1
  1     Intercept           101            200883
```

This is the level-2 **G** matrix, just a random intercept variance so far.

```
                         Estimated V Matrix for ID 101
Row         Col1         Col2         Col3         Col4         Col5         Col6
  1       245783       200883       200883       200883       200883       200883
  2       200883       245783       200883       200883       200883       200883
  3       200883       200883       245783       200883       200883       200883
  4       200883       200883       200883       245783       200883       200883
  5       200883       200883       200883       200883       245783       200883
  6       200883       200883       200883       200883       200883       245783
```

The **V** matrix is the total variance-covariance matrix after combining the level-2 **G** and level-1 **R** matrices.

```
                    Estimated V Correlation Matrix for ID 101
Row         Col1         Col2         Col3         Col4         Col5         Col6
  1       1.0000       0.8173       0.8173       0.8173       0.8173       0.8173
  2       0.8173       1.0000       0.8173       0.8173       0.8173       0.8173
  3       0.8173       0.8173       1.0000       0.8173       0.8173       0.8173
  4       0.8173       0.8173       0.8173       1.0000       0.8173       0.8173
  5       0.8173       0.8173       0.8173       0.8173       1.0000       0.8173
  6       0.8173       0.8173       0.8173       0.8173       0.8173       1.0000

                  Covariance Parameter Estimates
                                   tandard          Z
Cov Parm     Subject    Estimate      Error      Value        Pr Z
UN(1,1)      ID           200883      29471       6.82      <.0001   L2 int
Session      ID            44900    2825.63      15.89      <.0001   L1 res

  Null Model Likelihood Ratio Test
    DF    Chi-Square      Pr > ChiSq
     1        691.74          <.0001
```

Calculate the ICC for the Number Match 3 outcome:

$$\text{ICC} = \frac{200883}{200883 + 44900} = .82$$

This null model LRT tells us that the random intercept variance is significantly greater than 0, and thus so is the ICC.

```
                              Information Criteria
Neg2LogLike     Parms         AIC        AICC        HQIC         BIC        CAIC
     8536.9         2      8540.9      8540.9      8543.0      8546.1      8548.1
```

REML only counts the # parameters in the model for the variance (not fixed effects).

```
                    Solution for Fixed Effects
                              Standard
Effect         Estimate          Error       DF    t Value    Pr > |t|
Intercept       1770.70        45.4206      100      38.98      <.0001
```

This is the fixed intercept, gamma00 (the grand mean of the person means so far).

Model 2a. Fixed Linear Time, Random Intercept

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + e_{ti}$$
$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$
$$\text{Linear Session: } \beta_{1i} = \gamma_{10}$$

The predictor of *c1sess* will be treated as continuous given that it is not on the CLASS statement (SAS), it is on WITH (SPSS), and uses c. (STATA). *Session* is still a categorical predictor used as an ID variable in creating the **R** matrix.

```
TITLE1 "SAS Model 2a: Fixed Linear Time, Random Intercept";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = c1sess / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
     REPEATED session / R TYPE=VC SUBJECT=ID;
     * CovParms will be used for pseudo-R2, InfoCrit for LRT against next model;
     ODS OUTPUT CovParms=CovFixLin InfoCrit=FitFixLin; RUN; TITLE1;
```

```
ECHO "SPSS Model 2a: Fixed Linear Time, Random Intercept".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess
      /RANDOM = INTERCEPT | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).

* STATA Model 2a: Fixed Linear Time, Random Intercept
mixed nm3rt c.c1sess,  || id:  ,  ///
        variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
      estat ic, n(101)                  // AIC and BIC
      estat recovariance, relevel(id)   // G matrix
      estat df, method(satterthwaite)   // print DDF for fixed effects
      estimates store FixLin            // store for LRT
```

DV = nm3rt, c. means continuous fixed slope for *c1sess*
Level 2 ID is id, random intercept by default
estimates → save results as “FixLin” for next LRT

SAS output:

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	238084	202422	202422	202422	202422	202422
2	202422	238084	202422	202422	202422	202422
3	202422	202422	238084	202422	202422	202422
4	202422	202422	202422	238084	202422	202422
5	202422	202422	202422	202422	238084	202422
6	202422	202422	202422	202422	202422	238084

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8502	0.8502	0.8502	0.8502	0.8502
2	0.8502	1.0000	0.8502	0.8502	0.8502	0.8502
3	0.8502	0.8502	1.0000	0.8502	0.8502	0.8502
4	0.8502	0.8502	0.8502	1.0000	0.8502	0.8502
5	0.8502	0.8502	0.8502	0.8502	1.0000	0.8502
6	0.8502	0.8502	0.8502	0.8502	0.8502	1.0000

The predicted **V** matrix still has a compound symmetry pattern because we have not yet added to the model for the variance (still a random intercept variance only in **G**).

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	202422	29470	6.87	<.0001
Session	ID	35662	2246.48	15.87	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8414.7	2	8418.7	8418.7	8420.8	8423.9	8425.9

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1899.63	46.7882	113	40.60	<.0001
c1sess	-51.5719	4.4918	504	-11.48	<.0001

The fixed linear effect of *c1sess* is significant according to the Wald test (*p*-value for fixed effect).

```
* Call macro to calculate pseudo R2;
%PseudoR2(NCov=2, CovFewer=CovEmpty, CovMore=CovFixLin);
```

PsuedoR2 (% Reduction) for CovEmpty vs. CovFixLin

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovEmpty	UN(1,1)	PersonID	200883	29471	6.82	<.0001	.
CovEmpty	session	PersonID	44900	2825.63	15.89	<.0001	.
CovFixLin	UN(1,1)	PersonID	202422	29470	6.87	<.0001	-0.00766
CovFixLin	session	PersonID	35662	2246.48	15.87	<.0001	0.20575

Relative to the empty means, random intercept model 1a, the fixed linear effect of *c1sess* explained ~21% of the residual variance (which made the random intercept variance increase by 0.7% due to its smaller correction factor).

Model 2b. Random Linear Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear Session: $\beta_{1i} = \gamma_{10} + U_{1i}$

```
TITLE1 "SAS Model 2b: Random Linear Time";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = c1sess / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT c1sess / G V VCORR TYPE=UN SUBJECT=ID;
     REPEATED session / R TYPE=VC SUBJECT=ID;
     * CovParms will be used for pseudo-R2, InfoCrit for LRT against next model;
     ODS OUTPUT CovParms=CovRandLin InfoCrit=FitRandLin; RUN; TITLE1;
```

Now there are 2 random effects: intercept and linear slope, given by *c1sess* on the RANDOM statements.

```
ECHO "SPSS Model 2b: Random Linear Time".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess
      /RANDOM = INTERCEPT c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).
```

DV = nm3rt, c. means continuous fixed slope for c1sess
Level 2 ID is id, random intercept and c1sess now estimates → save results as “RandLin” for LRT

```
* STATA Model 2b: Random Linear Time
mixed nm3rt c.c1sess,  || id: c1sess,  ///
        variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
      estat ic, n(101)                                   // AIC and BIC
      estat recovariance, relevel(id)                    // G matrix
      estat recovariance, relevel(id) correlation        // GCORR matrix
      estat df, method(satterthwaite)                    // print DDF for fixed effects
      estimates store RandLin                            // store for LRT
      lrtest RandLin FixLin                              // print LRT
```

SAS output:

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	27905					
2		27905				
3			27905			
4				27905		
5					27905	
6						27905

Estimated G Matrix

Row	Effect	Participant ID	Col1	Col2
1	Intercept	101	253258	-12701
2	c1sess	101	-12701	2233.83

Estimated G Correlation Matrix

Row	Effect	Participant ID	Col1	Col2
1	Intercept	101	1.0000	-0.5340
2	c1sess	101	-0.5340	1.0000

GCORR shows the correlation among random effects.

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**281163**	240557	227856	215155	202455	189754
2	240557	**257995**	219623	209156	198689	188222
3	227856	219623	**239295**	203157	194924	186691
4	215155	209156	203157	**225063**	191158	185159
5	202455	198689	194924	191158	**215298**	183627
6	189754	188222	186691	185159	183627	**210001**

The **V** matrix is the total variance-covariance matrix after combining the level-2 **G** and level-1 **R** matrices. Now the variances and covariances are predicted to change based on time.

How the V matrix variances and covariances get calculated in a random linear time model:

$$\mathbf{V}_i \text{ matrix: Variance}\left[y_{time}\right] = \tau^2_{U_0} + \left[(\text{Session}-1)^2\, \tau^2_{U_1}\right] + \left[2(\text{Session}-1)\,\tau_{U_{01}}\right] + \sigma^2_e$$

$$\mathbf{V}_i \text{ matrix: Covariance}\left[y_A, y_B\right] = \tau^2_{U_0} + \left[(A+B)\,\tau_{U_{01}}\right] + \left[(AB)\,\tau^2_{U_1}\right]$$

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8932	0.8784	0.8553	0.8229	0.7809
2	0.8932	1.0000	0.8839	0.8680	0.8430	0.8086
3	0.8784	0.8839	1.0000	0.8754	0.8588	0.8328
4	0.8553	0.8680	0.8754	1.0000	0.8684	0.8517
5	0.8229	0.8430	0.8588	0.8684	1.0000	0.8636
6	0.7809	0.8086	0.8328	0.8517	0.8636	1.0000

The **VCORR** matrix is the correlation version. The ICC is now predicted to change over time, too (and conditional on linear time).

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	253258	37897	6.68	<.0001
UN(2,1)	ID	-12701	3621.98	-3.51	0.0005
UN(2,2)	ID	2233.83	552.92	4.04	<.0001
Session	ID	27905	1963.42	14.21	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8372.1	4	8380.1	8380.2	8384.3	8390.6	8394.6

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1899.63	51.4998	100	36.89	<.0001
c1sess	-51.5719	6.1567	100	-8.38	<.0001

```
* Call macro to calculate LRT for nested models;
%FitTest(FitFewer=FitFixLin, FitMore=FitRandLin);
```

Is the random linear time model (2b) better than the fixed linear time, random intercept model (2a)?
Yep, −2ΔLL= 43, which is bigger than the critical value of 5.99ish on df = ~2ish

Likelihood Ratio Test for FitFixLin vs. FitRandLin

Name	Neg2Log Like	Parms	AIC	BIC	DevDiff	DFdiff	Pvalue
FitFixLin	8414.7	2	8418.7	8423.9	.	.	.
FitRandLin	8372.1	4	8380.1	8390.6	42.5856	2	5.6579E-10

We will not calculate pseudo-R^2 for this random linear time slope model relative to the previous fixed linear time slope, random intercept model because random effects *do not* explain variable—they partition it instead.

Model 3a. Fixed Quadratic, Random Linear Time

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti}-1) + \beta_{2i}(\text{Session}_{ti}-1)^2 + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear Session: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Quadratic Session: } \beta_{2i} = \gamma_{20}$$

Interactions can be defined on the fly in SAS and SPSS using *. Interactions can be defined on the fly in STATA using # for fixed effects, but not for random effects.

```
TITLE1 "SAS Model 3a: Fixed Quadratic, Random Linear Time";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;                                  * OUTPM saves fixed-predicted DV;
     MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite OUTPM=PredTime;
     RANDOM INTERCEPT c1sess / G V VCORR TYPE=UN SUBJECT=ID;
     REPEATED session / R TYPE=VC SUBJECT=ID;
     * CovParms will be used for pseudo-R2, InfoCrit for LRT against next model;
     ODS OUTPUT CovParms=CovFixQuad InfoCrit=FitFixQuad; RUN; TITLE1;

ECHO "SPSS Model 3a: Fixed Quadratic, Random Linear Time".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess
      /RANDOM = INTERCEPT c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predtime).
CORRELATIONS predtime nm3rt.
```

The OUTPM in SAS, /SAVE in SPSS, and predict in STATA calculate outcomes predicted by the fixed effects. We can then correlate the predicted and actual outcomes to get total R^2 (actual variance explained).

```
* STATA Model 3a: Fixed Quadratic, Random Linear Time
mixed nm3rt c.c1sess c.c1sess#c.c1sess,   || id: c1sess,  ///
         variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
      estat ic, n(101)                                // AIC and BIC
      estat recovariance, relevel(id)                 // G matrix
      estat recovariance, relevel(id) correlation     // GCORR matrix
      estat df, method(satterthwaite)                 // print DDF for fixed effects
      estimates store FixQuad                        // store for LRT
      predict predtime                 // save fixed-effect predicted outcomes
corr predtime nm3rt                    // get total r to make r2
```

SAS output:

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	26176					
2		26176				
3			26176			
4				26176		
5					26176	
6						26176

Estimated G Matrix

Row	Effect	Participant ID	Col1	Col2
1	Intercept	101	254164	-12948
2	c1sess	101	-12948	2332.67

GCORR shows intercept–slope correlation $r = -.53$

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	280339	241216	228268	215320	202372	189424
2	241216	256776	219985	209370	198755	188140
3	228268	219985	237879	203420	195138	186855
4	215320	209370	203420	223646	191521	185571
5	202372	198755	195138	191521	214079	184286
6	189424	188140	186855	185571	184286	209178

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8991	0.8839	0.8599	0.8261	0.7822
2	0.8991	1.0000	0.8901	0.8737	0.8477	0.8118
3	0.8839	0.8901	1.0000	0.8819	0.8647	0.8377
4	0.8599	0.8737	0.8819	1.0000	0.8753	0.8580
5	0.8261	0.8477	0.8647	0.8753	1.0000	0.8709
6	0.7822	0.8118	0.8377	0.8580	0.8709	1.0000

```
                Covariance Parameter Estimates
                                 Standard         Z
Cov Parm     Subject    Estimate     Error    Value        Pr Z
UN(1,1)      ID           254164     37896     6.71      <.0001
UN(2,1)      ID           -12948   3620.70    -3.58      0.0003
UN(2,2)      ID          2332.67    551.58     4.23      <.0001
session      ID            26176   1844.01    14.20      <.0001

                       Information Criteria
Neg2LogLike     Parms       AIC        AICC       HQIC       BIC       CAIC
     8341.5         4    8349.5      8349.5     8353.7    8359.9     8363.9

                    Solution for Fixed Effects
                              Standard
Effect            Estimate       Error      DF    t Value    Pr > |t|
Intercept          1945.85     52.2433     106      37.25      <.0001
c1sess             -120.90     14.5415     502      -8.31      <.0001
c1sess*c1sess      13.8656      2.6348     403       5.26      <.0001
```

The fixed quadratic effect of *c1sess* is significant according to the Wald test (*p*-value for fixed effect). The linear slope changes by twice the quadratic coefficient per unit time.

```
* Call macro to calculate pseudo R2;
%PseudoR2(NCov=4, CovFewer=CovRandLin, CovMore=CovFixQuad);

PsuedoR2 (% Reduction) for CovRandLin vs. CovFixQuad
   Name        CovParm    Subject     Estimate     StdErr    ZValue    ProbZ     PseudoR2
CovRandLin     UN(1,1)    PersonID      253258      37897      6.68    <.0001        .
CovRandLin     UN(2,2)    PersonID     2233.83     552.92      4.04    <.0001        .
CovRandLin     session    PersonID       27905    1963.42     14.21    <.0001        .
CovFixQuad     UN(1,1)    PersonID      254164      37896      6.71    <.0001    -0.003577
CovFixQuad     UN(2,2)    PersonID     2332.67     551.58      4.23    <.0001    -0.044244
CovFixQuad     session    PersonID       26176    1844.01     14.20    <.0001     0.061980
```

Relative to the random linear time model 2b, the fixed quadratic effect of session explained another ~6% of the residual variance (which made the random intercept variance increase due to its residual variance correction factor).

```
* Get total R2 from fixed linear and fixed quadratic time;
PROC CORR NOSIMPLE DATA=PredTime; VAR pred; WITH nm3rt; RUN;

Pearson Correlation Coefficients, N = 606
        Prob > |r| under H0: Rho=0
                              Pred
nm3rt                      0.19167
Number Match 3 RT           <.0001
```

r = .19167 → **TOTAL R^2 = .0367**
~ 4% of total nm3rt variance is accounted for by linear and quadratic effects of session

---

Model 3b. Random Quadratic Time (and an example of ESTIMATE/TEST/LINCOM statements)

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear Session: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Quadratic Session: } \beta_{2i} = \gamma_{20} + U_{2i}$$

```
TITLE1 "SAS Model 3b: Random Quadratic Time";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT c1sess c1sess*c1sess / G V VCORR TYPE=UN SUBJECT=ID;
     REPEATED session / R TYPE=VC SUBJECT=ID;
     ODS OUTPUT CovParms=CovTime InfoCrit=FitRandQuad; * Save for pseudo-R2, LRT;
```

```
    * Predicting intercept at each session;
    ESTIMATE "Intercept at Session 1"   intercept 1 c1sess 0   c1sess*c1sess 0;
    ESTIMATE "Intercept at Session 2"   intercept 1 c1sess 1   c1sess*c1sess 1;
    ESTIMATE "Intercept at Session 3"   intercept 1 c1sess 2   c1sess*c1sess 4;
    ESTIMATE "Intercept at Session 4"   intercept 1 c1sess 3   c1sess*c1sess 9;
    ESTIMATE "Intercept at Session 5"   intercept 1 c1sess 4   c1sess*c1sess 16;
    ESTIMATE "Intercept at Session 6"   intercept 1 c1sess 5   c1sess*c1sess 25;
    * Predicting linear rate of change at each session (linear changes by 2*quad);
    ESTIMATE "Linear Slope at Session 1"      c1sess 1   c1sess*c1sess 0;
    ESTIMATE "Linear Slope at Session 2"      c1sess 1   c1sess*c1sess 2;
    ESTIMATE "Linear Slope at Session 3"      c1sess 1   c1sess*c1sess 4;
    ESTIMATE "Linear Slope at Session 4"      c1sess 1   c1sess*c1sess 6;
    ESTIMATE "Linear Slope at Session 5"      c1sess 1   c1sess*c1sess 8;
    ESTIMATE "Linear Slope at Session 6"      c1sess 1   c1sess*c1sess 10;
RUN; TITLE1;

ECHO "SPSS Model 3b: Random Quadratic Time".
MIXED nm3rt BY ID session WITH c1sess
     /METHOD = REML
     /PRINT  = SOLUTION TESTCOV G R
     /FIXED  = c1sess c1sess*c1sess
     /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
     /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
     /TEST = "Intercept at Session 1"    intercept 1 c1sess 0   c1sess*c1sess 0
     /TEST = "Intercept at Session 2"    intercept 1 c1sess 1   c1sess*c1sess 1
     /TEST = "Intercept at Session 3"    intercept 1 c1sess 2   c1sess*c1sess 4
     /TEST = "Intercept at Session 4"    intercept 1 c1sess 3   c1sess*c1sess 9
     /TEST = "Intercept at Session 5"    intercept 1 c1sess 4   c1sess*c1sess 16
     /TEST = "Intercept at Session 6"    intercept 1 c1sess 5   c1sess*c1sess 25
     /TEST = "Linear Slope at Session 1"      c1sess 1    c1sess*c1sess 0
     /TEST = "Linear Slope at Session 2"      c1sess 1    c1sess*c1sess 2
     /TEST = "Linear Slope at Session 3"      c1sess 1    c1sess*c1sess 4
     /TEST = "Linear Slope at Session 4"      c1sess 1    c1sess*c1sess 6
     /TEST = "Linear Slope at Session 5"      c1sess 1    c1sess*c1sess 8
     /TEST = "Linear Slope at Session 6"      c1sess 1    c1sess*c1sess 10.

* STATA Model 3b: Random Quadratic Time
mixed nm3rt c.c1sess c.c1sess#c.c1sess,  || id: c1sess c1sess2,  ///
        variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
     estat ic, n(101)                                 // AIC and BIC
     estat recovariance, relevel(id)                  // G matrix
     estat recovariance, relevel(id) correlation      // GCORR matrix
     estat df, method(satterthwaite),   // print DDF for fixed effects
     estimates store RandQuad           // store for LRT
     lrtest RandQuad FixQuad            // do LRT
     margins, at(c.c1sess=(0(1)5)) vsquish                  // intercepts per session
     marginsplot, name(predicted_means, replace)            // plot intercepts
     margins, at(c.c1sess=(0(1)5)) dydx(c.c1sess) vsquish  // linear slope per session
     marginsplot                                            // plot quadratic effect
```

This random statement will not accept interaction terms, so we are using *c1sess2*.

SAS output:

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	20298					
2		20298				
3			20298			
4				20298		
5					20298	
6						20298

Estimated G Matrix

Row	Effect	Participant ID	Col1	Col2	Col3
1	Intercept	101	276206	-35734	3901.96
2	c1sess	101	-35734	25840	-3903.32
3	c1sess*c1sess	101	3901.96	-3903.32	634.47

Estimated G Correlation Matrix

Row	Effect	ID: Participant ID	Col1	Col2	Col3
1	Intercept	101	1.0000	-0.4230	0.2948
2	c1sess	101	-0.4230	1.0000	-0.9640
3	c1sess*c1sess	101	0.2948	-0.9640	1.0000

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**296504**	244374	220346	204122	195702	195085
2	244374	**251508**	219312	208680	199315	191215
3	220346	219312	**235842**	209043	199808	187840
4	204122	208680	209043	**225508**	197182	184958
5	195702	199315	199808	197182	**211735**	182571
6	195085	191215	187840	184958	182571	**200977**

The **V** matrix is the total variance-covariance matrix after combining the level-2 **G** and level-1 **R** matrices. The variances and covariances are predicted to change based on time, but differently than before.

How the V matrix variances and covariances get calculated in a random quadratic time model:

Predicted Variance at Time *T*:

$$\text{Var}(y_T) = \sigma_e^2 + \tau_{U_0}^2 + 2*T*\tau_{U_{01}} + T^2*\tau_{U_1}^2 + 2*T^2*\tau_{U_{02}} + 2*T^3*\tau_{U_{12}} + T^4*\tau_{U_2}^2$$

Predicted Covariance between Time *A* and *B*:

$$\text{Cov}(y_A, y_B) = \tau_{U_0}^2 + (A+B)*\tau_{U_{01}} + (AB)*\tau_{U_1}^2 + (A^2+B^2)*\tau_{U_{02}} + (AB^2)+(A^2B)*\tau_{U_{12}} + (A^2B^2)*\tau_{U_2}^2$$

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8949	0.8333	0.7894	0.7811	0.7992
2	0.8949	1.0000	0.9005	0.8762	0.8637	0.8505
3	0.8333	0.9005	1.0000	0.9064	0.8941	0.8628
4	0.7894	0.8762	0.9064	1.0000	0.9024	0.8688
5	0.7811	0.8637	0.8941	0.9024	1.0000	0.8850
6	0.7992	0.8505	0.8628	0.8688	0.8850	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	276206	41442	6.66	<.0001
UN(2,1)	ID	-35734	11941	-2.99	0.0028
UN(2,2)	ID	25840	5864.41	4.41	<.0001
UN(3,1)	ID	3901.96	1949.06	2.00	0.0453
UN(3,2)	ID	-3903.32	982.61	-3.97	<.0001
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8302.7	7	8316.7	8316.9	8324.2	8335.1	8342.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	53.8497	100	36.13	<.0001
c1sess	-120.90	20.0476	100	-6.03	<.0001
c1sess*c1sess	13.8656	3.4154	100	4.06	<.0001

```
* Call macro to calculate LRT for nested models;
%FitTest(FitFewer=FitFixQuad, FitMore=FitRandQuad);
```

Is the random quadratic model (3b) better than the fixed quadratic, random linear model (3a)?
Yep, −2ΔLL= 39, which is bigger than the critical value of 7.82ish on df= ~3ish

Likelihood Ratio Test for FitFixQuad vs. FitRandQuad

Name	Neg2Log Like	Parms	AIC	BIC	DevDiff	DFdiff	Pvalue
FitFixQuad	8341.5	4	8349.5	8359.9	.	.	.
FitRandQuad	8302.7	7	8316.7	8335.1	38.7316	3	1.9784E-8

We will not calculate pseudo-R^2 for this random linear time slope model relative to the previous fixed linear time slope, random intercept model because random effects *do not* explain variable—they partition it instead.

Computing random effects confidence intervals for each random effect:

$$\text{Random Effect 95\% CI} = \text{fixed effect} \pm \left(1.96^*\sqrt{\text{Random Variance}}\right)$$

$$\text{Intercept 95\% CI} = \gamma_{00} \pm \left(1.96^*\sqrt{\tau^2_{U_0}}\right) \rightarrow 1{,}945.9 \pm \left(1.96^*\sqrt{276{,}209}\right) = 916 \text{ to } 2{,}976$$

$$\text{Linear Time Slope 95\% CI} = \gamma_{10} \pm \left(1.96^*\sqrt{\tau^2_{U_1}}\right) \rightarrow -120.9 \pm \left(1.96^*\sqrt{25{,}840}\right) = -436 \text{ to } 194$$

$$\text{Quadratic Time Slope 95\% CI} = \gamma_{20} \pm \left(1.96^*\sqrt{\tau^2_{U_2}}\right) \rightarrow 13.9 \pm \left(1.96^*\sqrt{634}\right) = -36 \text{ to } 63$$

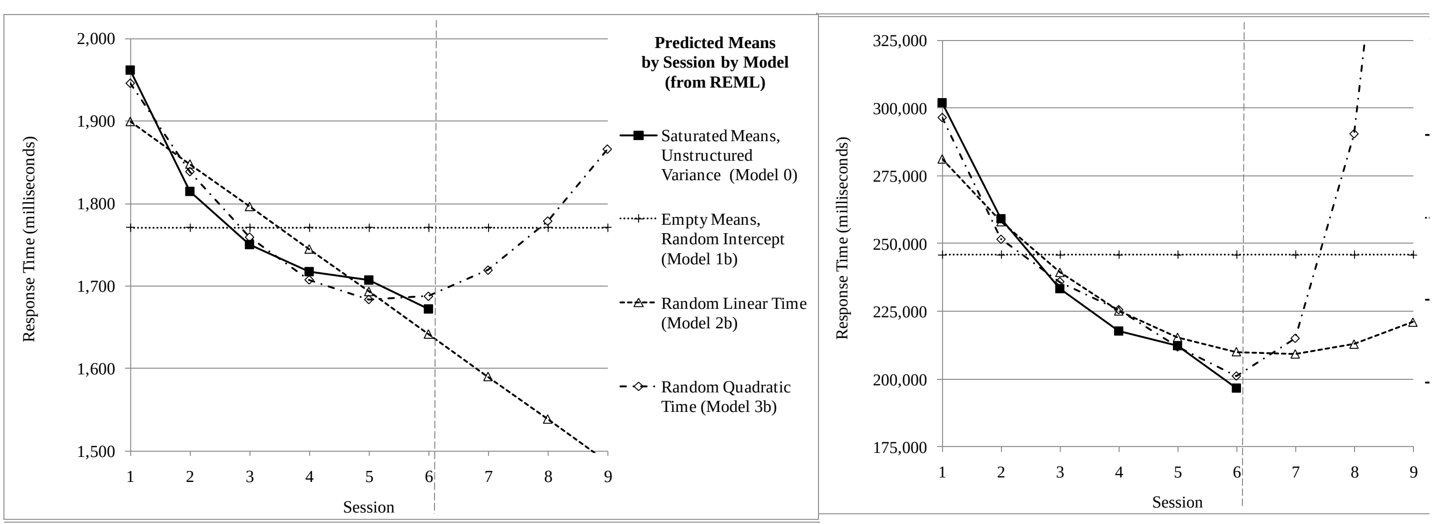
Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept at Session 1	1945.85	53.8497	100	36.13	<.0001
Intercept at Session 2	1838.82	48.4864	100	37.92	<.0001
Intercept at Session 3	1759.51	46.9973	100	37.44	<.0001
Intercept at Session 4	1707.94	45.8959	100	37.21	<.0001
Intercept at Session 5	1684.10	44.2395	100	38.07	<.0001
Intercept at Session 6	1687.99	44.2038	100	38.19	<.0001
Linear Slope at Session 1	-120.90	20.0476	100	-6.03	<.0001
Linear Slope at Session 2	-93.1687	13.6497	100	-6.83	<.0001
Linear Slope at Session 3	-65.4375	8.0028	100	-8.18	<.0001
Linear Slope at Session 4	-37.7062	5.9242	100	-6.36	<.0001
Linear Slope at Session 5	-9.9750	9.9733	100	-1.00	0.3196
Linear Slope at Session 6	17.7562	16.0362	100	1.11	0.2708

These are the quadratic-model-predicted means (intercepts) per session.

These are the instantaneous linear slopes at each session. Note how the SEs narrow towards the middle sessions.

How well do the predicted means (left) and variances (right) from the random quadratic model (3b) match the original data means and variances from the saturated means model (0)?



See chapter 6 for more options for modeling change and an example write-up for these models.

Example 3: Time-Invariant Predictors of Polynomial Individual Change
(complete data, syntax, and output available for SAS, SPSS, and STATA electronically)

This example is not found in the Hoffman (2015) text, but it continues with the example from chapter 6. Here we will examine how three time-invariant predictors collected at baseline—age, abstract reasoning (a measure of working memory), and education—predict individual differences in polynomial change.

SAS Code for Data Manipulation:

```
* Center level-2 predictor variables for analysis;
DATA work.Example23; SET work.Example23;
  age80  = age - 80;               * 80 = convenient value;
  reas22 = absreas - 22;   * 22 = near sample mean;
  LABEL age80  = "age80: Age Centered (0=80)"
        reas22 = "reas22: Abstract Reasoning Centered (0=22)";
* Make education a grouping variable for purpose of demonstration only;
       IF educyrs LE 12                   THEN educgrp=1;
  ELSE IF educyrs GT 12 AND EducYrs LE 16 THEN educgrp=2;
  ELSE IF educyrs GT 16                   THEN educgrp=3;
  ELSE IF educyrs = .                     THEN educGrp=.;
  LABEL educgrp = "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)";
* Remove cases with missing predictors;
  IF NMISS(age80, reas22, educgrp)>0 THEN DELETE;
RUN;
```

SPSS Code for Data Manipulation:

```
* Center level-2 predictor variables for analysis.
DATASET ACTIVATE Example23 WINDOW=FRONT.
COMPUTE age80  = age - 80.
COMPUTE reas22 = absreas - 22.
VARIABLE LABELS age80  "age80: Age Centered (0=80)"
                reas22 "reas22: Abstract Reasoning Centered (0=22)".
* Make education a grouping variable for purpose of demonstration only.
IF educyrs LE 12                  educgrp=1.
IF educyrs GT 12 AND educyrs LE 16 educgrp=2.
IF educyrs GT 16                  educgrp=3.
VARIABLE LABELS educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)".
* Remove cases with missing predictors.
SELECT IF (NVALID(age80, reas22, educgrp)=3).
EXECUTE.
```

STATA Code for Data Manipulation:

```
* Center level-2 predictor variables for analysis
gen age80 = age - 80
gen reas22 = absreas - 22
label variable age80 "age80: Age Centered (0=80 years)"
label variable reas22 "reas22: Abstract Reasoning Centered (0=22)"
* Make education a grouping variable for purpose of demonstration only
gen educgrp=.
replace educgrp=1 if (educyrs <= 12)
replace educgrp=2 if (educyrs > 12 & educyrs <= 16)
replace educgrp=3 if (educyrs > 16)
label variable educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)"
* Create new variable to hold number of missing cases
* Then drop cases with incomplete predictors
egen nummiss = rowmiss(age80 reas22 educgrp)
drop if nummiss>0
```

Model 4a. Add Age as Predictor of Intercept, Linear, and Quadratic Time Slopes

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$$

Level 2:

$$\text{Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + U_{0i}$$

$$\text{Linear: } \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + U_{1i}$$

$$\text{Quadratic: } \beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$$

```
TITLE1 "SAS Model 4a: Add Age as Predictor of Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                     / SOLUTION DDFM=Satterthwaite OUTPM=work.PredAge;
     RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
     REPEATED session / TYPE=VC SUBJECT=ID;
     ODS OUTPUT CovParms=CovAge; * Save for pseudo-R2;
* Multivariate Wald test for multiple fixed effects (analog to LRT when using REML);
CONTRAST "DF=3 Wald Test for Age Effects" Age80 1, Age80*c1sess 1, Age80*c1sess*c1sess 1 / CHISQ;
     * Requesting additional effects for age;
     ESTIMATE "Age Effect at Session 1" age80 1 c1sess*age80 0 c1sess*c1sess*age80 0;
     ESTIMATE "Age Effect at Session 2" age80 1 c1sess*age80 1 c1sess*c1sess*age80 1;
     ESTIMATE "Age Effect at Session 3" age80 1 c1sess*age80 2 c1sess*c1sess*age80 4;
     ESTIMATE "Age Effect at Session 4" age80 1 c1sess*age80 3 c1sess*c1sess*age80 9;
     ESTIMATE "Age Effect at Session 5" age80 1 c1sess*age80 4 c1sess*c1sess*age80 16;
     ESTIMATE "Age Effect at Session 6" age80 1 c1sess*age80 5 c1sess*c1sess*age80 25;
RUN; TITLE1;

ECHO "SPSS Model 4a: Add Age as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session WITH c1sess age80
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /TEST = "DF=3 Wald Test for Age Effects" age80 1; c1sess*age80 1; c1sess*c1sess*age80 1
      /SAVE = FIXPRED (predage)
      /TEST = "Age Effect at Session 1" age80 1 c1sess*age80 0 c1sess*c1sess*age80 0
      /TEST = "Age Effect at Session 2" age80 1 c1sess*age80 1 c1sess*c1sess*age80 1
      /TEST = "Age Effect at Session 3" age80 1 c1sess*age80 2 c1sess*c1sess*age80 4
      /TEST = "Age Effect at Session 4" age80 1 c1sess*age80 3 c1sess*c1sess*age80 9
      /TEST = "Age Effect at Session 5" age80 1 c1sess*age80 4 c1sess*c1sess*age80 16
      /TEST = "Age Effect at Session 6" age80 1 c1sess*age80 5 c1sess*c1sess*age80 25.
CORRELATIONS predage nm3rt.

* STATA Model 4a: Add Age as Predictor of Intercept, Linear, and Quadratic
mixed nm3rt c.c1sess c.c1sess#c.c1sess                          ///
        c.age80 c.c1sess#c.age80 c.c1sess#c.c1sess#c.age80,     ///
        || id: c1sess c1sess2,                                  ///
        variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
      estat ic, n(101)                              // AIC and BIC
      estat recovariance, relevel(id)               // G matrix
      estat recovariance, relevel(id) correlation   // GCORR matrix
      estat df, method(satterthwaite)               // print DDF for fixed effects
      test (c.age80=0) (c.c1sess#c.age80=0) (c.c1sess#c.c1sess#c.age80=0) // DF=3 Wald test
      predict predage                               // save fixed-effect predicted outcomes
      margins, at(c.c1sess=(0(1)5)) dydx(c.age80) vsquish      // age slope per session
      margins, at(c.c1sess=(0(1)5) c.age80=(-5 0 5)) vsquish   // predictions per session
      marginsplot                                              // plot age predictions
corr predage nm3rt                                             // get total r to make r2
```

SAS output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	ID	247691	37599	6.59	<.0001	intercept var
UN(2,1)	ID	-30154	11191	-2.69	0.0070	int-lin covar
UN(2,2)	ID	25083	5787.37	4.33	<.0001	linear var
UN(3,1)	ID	3232.78	1847.12	1.75	0.0801	int-quad covar
UN(3,2)	ID	-3830.21	976.76	-3.92	<.0001	lin-quad covar
UN(3,3)	ID	629.58	172.51	3.65	0.0001	quad var
session	ID	20298	1649.11	12.31	<.0001	residual var

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8283.2	7	8297.2	8297.3	8304.6	8315.5	8322.5

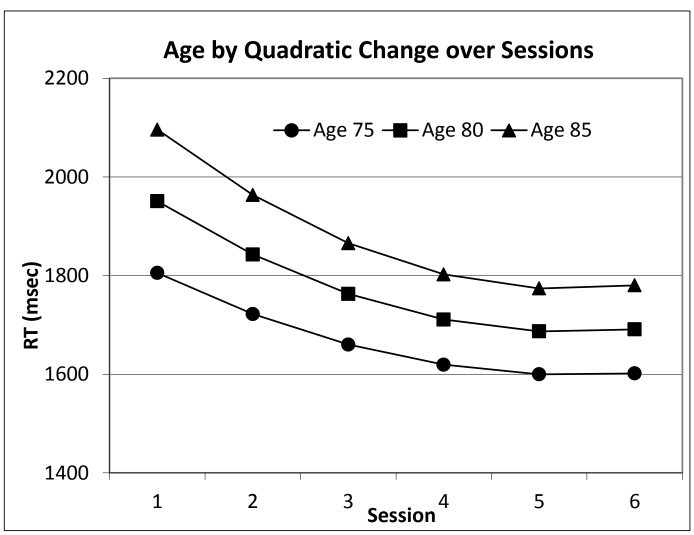
Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1950.69	51.1806	99	38.11	<.0001
c1sess	-121.83	19.8672	99	-6.13	<.0001
c1sess*c1sess	13.9774	3.4096	99	4.10	<.0001
age80	29.0495	8.4616	99	3.43	0.0009
c1sess*age80	-5.5946	3.2846	99	-1.70	0.0916
c1sess*c1sess*age80	0.6709	0.5637	99	1.19	0.2368

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Age Effect at Session 1	29.0495	8.4616	99	3.43	0.0009
Age Effect at Session 2	24.1258	7.6862	99	3.14	0.0022
Age Effect at Session 3	20.5439	7.5343	99	2.73	0.0076
Age Effect at Session 4	18.3038	7.4038	99	2.47	0.0151
Age Effect at Session 5	17.4056	7.1425	99	2.44	0.0166
Age Effect at Session 6	17.8492	7.1254	99	2.51	0.0139

These are the simple slopes for the effect of age per session.



The pattern of the interaction, shown here, can be interpreted as how the pattern of decline in RT differs by age, or by how the simple slopes of age change across sessions.

Is the age by quadratic model (4a) better than the unconditional quadratic growth model (3b)? Yep:

Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
DF=3 Wald Test for Effects of Age	3	99	12.01	4.00	0.0073	0.0098

```
* Calculate Total R2 change relative to unconditional model;
%TotalR2(DV=nm3rt, PredFewer=PredTime, PredMore=PredAge);
```

```
Total R2 (% Reduction) for PredTime vs. PredAge
              Pred                  Total
  Name        Corr      TotalR2     R2Diff
PredTime    0.19167    0.03674        .
PredAge     0.32688    0.10685    0.070114
```

The fixed effects of time before accounted for ~3.7% of the variance in nm3rt, so there is a net increase of ~7% due to age.

```
* Calculate PseudoR2 relative to unconditional time model;
%PseudoR2(NCov=7, CovFewer=CovTime, CovMore=CovAge);
```

```
PsuedoR2 (% Reduction) for CovTime vs. CovAge
 Name      CovParm    Subject    Estimate      StdErr    ZValue     ProbZ       Pseudo R2
CovTime    UN(1,1)      ID         276206       41442      6.66     <.0001       .
CovTime    UN(2,2)      ID          25840     5864.41      4.41     <.0001       .
CovTime    UN(3,3)      ID         634.47      172.37      3.68     0.0001       .
CovTime    session      ID          20298     1649.11     12.31     <.0001       .
CovAge     UN(1,1)      ID         247691       37599      6.59     <.0001     0.10324 for L2 intercept
CovAge     UN(2,2)      ID          25083     5787.37      4.33     <.0001     0.02931 for L2 linear
CovAge     UN(3,3)      ID         629.58      172.51      3.65     0.0001     0.00770 for L2 quad
CovAge     session      ID          20298     1649.11     12.31     <.0001     0.00000 for L1 residual
```

Model 5a. Add Abstract Reasoning as Predictor of Intercept, Linear, and Quadratic Time Slopes

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$$

Level 2:

$$\text{Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + U_{0i}$$

$$\text{Linear: } \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + U_{1i}$$

$$\text{Quadratic: } \beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{22}\left(\text{Reason}_i - 22\right) + U_{2i}$$

```
TITLE1 "SAS Model 5a: Add Reasoning as Predictor of Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                   reas22 c1sess*reas22 c1sess*c1sess*reas22
                        / SOLUTION DDFM=Satterthwaite OUTPM=work.PredReasQ;
    RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
    REPEATED session / TYPE=VC SUBJECT=ID;
    ODS OUTPUT CovParms=CovReasQ; * Save for pseudo-R2;
CONTRAST "DF=3 Wald Test for Reas Effects" Reas22 1, Reas22*c1sess 1, Reas22*c1sess*c1sess 1
       / CHISQ; RUN; TITLE1;
```

```
ECHO "SPSS Model 5a: Add Reasoning as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session WITH c1sess age80 reas22
      /METHOD  = REML
      /PRINT   = SOLUTION TESTCOV G R
      /FIXED   = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                 reas22 c1sess*reas22 c1sess*c1sess*reas22
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /TEST = "DF=3 Wald Test for Reas Effects" reas22 1; c1sess*reas22 1; c1sess*c1sess*reas22 1
      /SAVE = FIXPRED (predreasQ).
CORRELATIONS predreasQ nm3rt.
```

```
* STATA Model 5a: Add Reasoning as Predictor of Intercept, Linear, and Quadratic
mixed nm3rt c.c1sess c.c1sess#c.c1sess                                 ///
         c.age80 c.c1sess#c.age80 c.c1sess#c.c1sess#c.age80          ///
```

```
         c.reas22 c.c1sess# c.reas22 c.c1sess#c.c1sess#c.reas22,  ///
         || id: c1sess c1sess2,                                   ///
         variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
    estat ic, n(101),                               // AIC and BIC
    estat recovariance, relevel(id),                // G matrix
    estat recovariance, relevel(id) correlation,    // GCORR matrix
    estat df, method(satterthwaite),                // print DDF for fixed effects
    test (c.reas22=0) (c.c1sess#c.reas22=0) (c.c1sess#c.c1sess#c.reas22=0) // DF=3 Wald test
    predict predreasQ                        // save fixed-effect predicted outcomes
corr predreasQ nm3rt                         // get total r to make r2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	235541	36056	6.53	<.0001
UN(2,1)	ID	-32552	11138	-2.92	0.0035
UN(2,2)	ID	25228	5835.93	4.32	<.0001
UN(3,1)	ID	3918.44	1826.88	2.14	0.0320
UN(3,2)	ID	-3812.99	978.05	-3.90	<.0001
UN(3,3)	ID	614.47	171.25	3.59	0.0002
session	ID	20298	1649.11	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8261.0	7	8275.0	8275.2	8282.4	8293.3	8300.3

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1966.47	50.4203	98	39.00	<.0001
c1sess	-119.74	20.0746	98	-5.96	<.0001
c1sess*c1sess	13.3036	3.4167	98	3.89	0.0002
age80	22.2782	8.7324	98	2.55	0.0123
c1sess*age80	-6.4921	3.4768	98	-1.87	0.0649
c1sess*c1sess*age80	0.9601	0.5917	98	1.62	0.1079
reas22	-27.1004	11.2829	98	-2.40	0.0182
c1sess*reas22	-3.5917	4.4922	98	-0.80	0.4259
c1sess*c1sess*reas22	1.1575	0.7646	98	1.51	0.1333

Is the reasoning by quadratic model (5a) better than the age by quadratic model (4a)? Yep:

Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
DF=3 Wald Test for Reas Effects	3	98	12.88	4.29	0.0049	0.0068

```
* Calculate Total R2 change relative to age model;
%TotalR2(DV=nm3rt, PredFewer=PredAge, PredMore=PredReasQ);
```

Total R2 (% Reduction) for PredAge vs. PredReasQ

Name	Pred Corr	TotalR2	Total R2Diff
PredAge	0.32688	0.10685	.
PredReasQ	0.40108	0.16086	0.054011

R =.4011, so R^2 for time+age+reas = .1609

The fixed effects of time and age before accounted for ~10.7% of the variance in RT, so there is a net increase of ~5.4% due to reasoning.

```
* Calculate PseudoR2 relative to age model;
%PseudoR2(NCov=7, CovFewer=CovAge, CovMore=CovReasQ);
```

PsuedoR2 (% Reduction) for CovAge vs. CovReasQ

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovAge	UN(1,1)	ID	247691	37599	6.59	<.0001	.
CovAge	UN(2,2)	ID	25083	5787.37	4.33	<.0001	.
CovAge	UN(3,3)	ID	629.58	172.51	3.65	0.0001	.

```
CovAge      session     ID        20298     1649.11     12.31    <.0001        .
CovReasQ    UN(1,1)     ID       235541       36056      6.53    <.0001     0.049052 for L2 intercept
CovReasQ    UN(2,2)     ID        25228     5835.93      4.32    <.0001    -0.005808 for L2 linear
CovReasQ    UN(3,3)     ID       614.47      171.25      3.59    0.0002     0.024008 for L2 quad
CovReasQ    session     ID        20298     1649.11     12.31    <.0001    -0.000000 for L1 residual
```

Model 5b. Add Abstract Reasoning as Predictor of Intercept and Linear Time Slope Only

$$
\begin{aligned}
&\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti} \\
&\text{Level 2:} \\
&\text{Intercept: } \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + U_{0i} \\
&\text{Linear: } \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + U_{1i} \\
&\text{Quadratic: } \beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}
\end{aligned}
$$

```
TITLE1 "SAS Model 5b: Add Reasoning as Predictor of Intercept and Linear Only";
PROC MIXED DATA=work.Example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
     CLASS ID session;
     MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                   reas22 c1sess*reas22
                        / SOLUTION DDFM=Satterthwaite OUTPM=work.PredReasL;
    RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
    REPEATED session / TYPE=VC SUBJECT=ID;
    ODS OUTPUT CovParms=CovReasL; * Save for pseudo-R2;
    CONTRAST "DF=2 Wald Test for Reas Effects" Reas22 1, Reas22*c1sess 1 / CHISQ;
    * Requesting additional effects for reasoning instead;
    ESTIMATE "Reasoning Effect at Session 1" reas22 1 c1sess*reas22 0;
    ESTIMATE "Reasoning Effect at Session 2" reas22 1 c1sess*reas22 1;
    ESTIMATE "Reasoning Effect at Session 3" reas22 1 c1sess*reas22 2;
    ESTIMATE "Reasoning Effect at Session 4" reas22 1 c1sess*reas22 3;
    ESTIMATE "Reasoning Effect at Session 5" reas22 1 c1sess*reas22 4;
    ESTIMATE "Reasoning Effect at Session 6" reas22 1 c1sess*reas22 5;
RUN; TITLE1;

ECHO "SPSS Model 5b: Add Reasoning as Predictor of Intercept and Linear Only".
MIXED nm3rt BY ID session WITH c1sess age80 reas22
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80 reas22 c1sess*reas22
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /TEST = "DF=2 Wald Test for Reas Effects" reas22 1; c1sess*reas22 1
      /TEST = "Reasoning Effect at Session 1" reas22 1 c1sess*reas22 0
      /TEST = "Reasoning Effect at Session 2" reas22 1 c1sess*reas22 1
      /TEST = "Reasoning Effect at Session 3" reas22 1 c1sess*reas22 2
      /TEST = "Reasoning Effect at Session 4" reas22 1 c1sess*reas22 3
      /TEST = "Reasoning Effect at Session 5" reas22 1 c1sess*reas22 4
      /TEST = "Reasoning Effect at Session 6" reas22 1 c1sess*reas22 5
      /SAVE = FIXPRED (predreasL).
CORRELATIONS predreasL nm3rt.

* STATA Model 5b: Add Reasoning as Predictor of Intercept and Linear Only
mixed nm3rt c.c1sess c.c1sess#c.c1sess                              ///
         c.age80 c.c1sess#c.age80 c.c1sess#c.c1sess#c.age80          ///
         c.reas22 c.c1sess# c.reas22, || id: c1sess c1sess2,         ///
         variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
      estat ic, n(101)                                  // AIC and BIC
      estat recovariance, relevel(id)                   // G matrix
      estat recovariance, relevel(id) correlation       // GCORR matrix
      estat df, method(satterthwaite)                  // print DDF for fixed effects
      test (c.reas22=0) (c.c1sess#c.reas22=0)          // DF=2 Wald test
      margins, at(c.c1sess=(0(1)5)) dydx(c.reas22) vsquish   // reas slope per session
```

```
      margins, at(c.c1sess=(0(1)5) c.reas22=(-5 0 5)) vsquish  // predictions per session
      marginsplot                                             // plot reas predictions
      predict predreasL                           // save fixed-effect predicted outcomes
corr predreasL nm3rt                              // get total r to make r2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	235909	36153	6.53	<.0001
UN(2,1)	ID	-32972	11262	-2.93	0.0034
UN(2,2)	ID	25707	5883.65	4.37	<.0001
UN(3,1)	ID	3993.04	1848.58	2.16	0.0308
UN(3,2)	ID	-3897.93	985.52	-3.96	<.0001
UN(3,3)	ID	629.52	172.50	3.65	0.0001
session	ID	20298	1649.11	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8264.6	7	8278.6	8278.8	8286.0	8296.9	8303.9

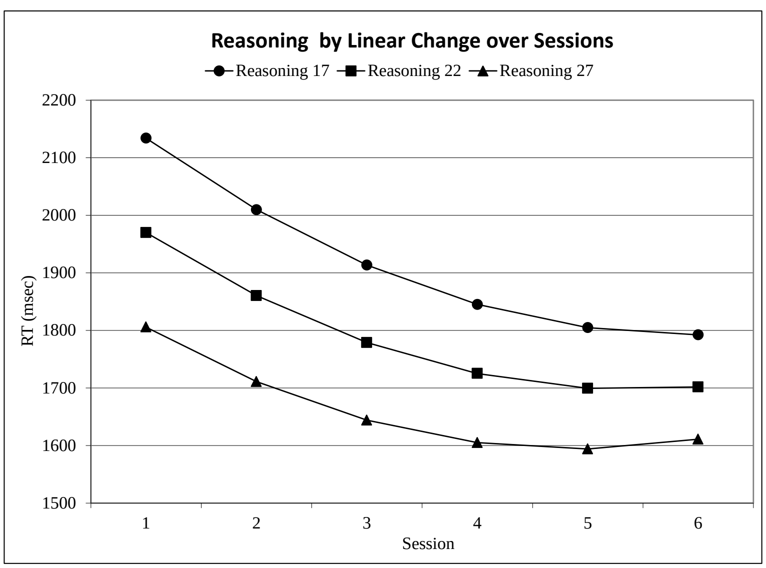
Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1969.80	50.4084	98.1	39.08	<.0001
c1sess	-123.54	20.0358	98.9	-6.17	<.0001
c1sess*c1sess	13.9774	3.4095	99	4.10	<.0001
age80	20.8470	8.6868	99.7	2.40	0.0183
c1sess*age80	-4.8610	3.3252	100	-1.46	0.1469
c1sess*c1sess*age80	0.6709	0.5637	99	1.19	0.2368
reas22	-32.8281	10.6297	98	-3.09	0.0026
c1sess*reas22	2.9362	1.2602	98	2.33	0.0219

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Reasoning Effect at Session 1	-32.8281	10.6297	98	-3.09	0.0026
Reasoning Effect at Session 2	-29.8919	10.1128	98	-2.96	0.0039
Reasoning Effect at Session 3	-26.9557	9.7326	98	-2.77	0.0067
Reasoning Effect at Session 4	-24.0195	9.5055	98	-2.53	0.0131
Reasoning Effect at Session 5	-21.0833	9.4425	98	-2.23	0.0278
Reasoning Effect at Session 6	-18.1471	9.5469	98	-1.90	0.0603

These are the simple slopes for the effect of reasoning per session.



The effect of reasoning changes linearly over time.

OR, only the intercept and linear slope are moderated by reasoning.

Is the reasoning by linear model (5b) still better than the age by quadratic model (4a)? Yep:

```
                                                   Contrasts
                                       Num      Den
Label                                   DF       DF     Chi-Square    F Value      Pr > ChiSq    Pr > F
DF=2 Wald Test for Reas Effects          2       98          10.59       5.29          0.0050    0.0066
```

```
* Calculate Total R2 change relative to age model;
%TotalR2(DV=nm3rt, PredFewer=PredAge, PredMore=PredReasL);
```

```
Total R2 (% Reduction) for PredAge vs. PredReasL
                 Pred                    Total
  Name           Corr       TotalR2      R2Diff
PredAge        0.32688     0.10685         .
PredReasL      0.40008     0.16006      0.053213
```

R =.4001, so R^2 for time+age+reas = .1601

So ~0.1% of the variance accounted for previously was due to reason*quad

```
* Calculate PseudoR2 relative to age model;
%PseudoR2(NCov=7, CovFewer=CovAge, CovMore=CovReasL);
PsuedoR2 (% Reduction) for CovAge vs. CovReasL
  Name       CovParm     Subject     Estimate       StdErr     ZValue     ProbZ      PseudoR2
CovAge       UN(1,1)      ID          247691         37599       6.59     <.0001        .
CovAge       UN(2,2)      ID           25083       5787.37       4.33     <.0001        .
CovAge       UN(3,3)      ID          629.58        172.51       3.65     0.0001        .
CovAge       session      ID           20298       1649.11      12.31     <.0001        .
CovReasL     UN(1,1)      ID          235909         36153       6.53     <.0001      0.047565 for L2 intercept
CovReasL     UN(2,2)      ID           25707       5883.65       4.37     <.0001     -0.024908 for L2 linear
CovReasL     UN(3,3)      ID          629.52        172.50       3.65     0.0001      0.000095 for L2 quad
CovReasL     session      ID           20298       1649.11      12.31     <.0001     -0.000000 for L1 residual
```

Model 6a. +Education Group on Intercept, Linear, and Quadratic Time Slopes

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti} - 1) + \beta_{2i}(\text{Session}_{ti} - 1)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reason}_i - 22) + \gamma_{03}(\text{Highvs.LowEd}_i) + \gamma_{04}(\text{Highvs.MedEd}_i) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reason}_i - 22) + \gamma_{13}(\text{Highvs.LowEd}_i) + \gamma_{14}(\text{Highvs.MedEd}_i) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{23}(\text{Highvs.LowEd}_i) + \gamma_{24}(\text{Highvs.MedEd}_i) + U_{2i}$

Additional model-implied group differences:

Medium vs. Low education intercept $= (\gamma_{00} + \gamma_{04}) - (\gamma_{00} + \gamma_{03}) = \gamma_{04} - \gamma_{03}$

Medium vs. Low education linear session $= (\gamma_{10} + \gamma_{14}) - (\gamma_{10} + \gamma_{13}) = \gamma_{14} - \gamma_{13}$

Medium vs. Low education quadratic session $= (\gamma_{20} + \gamma_{24}) - (\gamma_{20} + \gamma_{23}) = \gamma_{24} - \gamma_{23}$

```
TITLE1 "SAS Model 6a: Add Education Group on Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.Example34 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=REML;
       CLASS ID session educgrp;
       MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                     reas22 c1sess*reas22 educgrp c1sess*educgrp c1sess*c1sess*educgrp
                          / SOLUTION DDFM=Satterthwaite OUTPM=work.EducPred;
       RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
       REPEATED session / TYPE=VC SUBJECT=ID;
       ODS OUTPUT CovParms=CovEduc; * Save for pseudo-R2;
       CONTRAST "DF=6 Wald Test for Effects of Education" educgrp -1 1 0, educgrp -1 0 1,
                c1sess*educgrp -1 1 0, c1sess*educgrp -1 0 1,
                c1sess*c1sess*educgrp -1 1 0, c1sess*c1sess*educgrp -1 0 1 / CHISQ;
```

```
       * Estimating group means at first and last sessions
       LSMEANS educgrp  / AT (c1sess) = (0) DIFF=ALL;
       LSMEANS educgrp  / AT (c1sess) = (5) DIFF=ALL;
       * Contrasts between groups on intercept, linear, and quadratic slopes
       ESTIMATE "L vs. H Educ for Intercept Main Effect"  educgrp -1  0  1 ;
       ESTIMATE "M vs. H Educ for Intercept Main Effect"  educgrp  0 -1  1 ;
       ESTIMATE "L vs. M Educ for Intercept Main Effect"  educgrp -1  1  0 ;
       ESTIMATE "L vs. H Educ for Linear Session"  c1sess*educgrp -1  0  1 ;
       ESTIMATE "M vs. H Educ for Linear Session"  c1sess*educgrp  0 -1  1 ;
       ESTIMATE "L vs. M Educ for Linear Session"  c1sess*educgrp -1  1  0 ;
       ESTIMATE "L vs. H Educ for Quadratic Session"  c1sess*c1sess*educgrp -1  0  1 ;
       ESTIMATE "M vs. H Educ for Quadratic Session"  c1sess*c1sess*educgrp  0 -1  1 ;
       ESTIMATE "L vs. M Educ for Quadratic Session"  c1sess*c1sess*educgrp -1  1  0 ;
RUN; TITLE1;
```

Think of the −1 as the "0" and the "1" as the "1" in a dummy code.

```
ECHO "SPSS Model 6a: Add Education Group as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session educgrp WITH c1sess age80 reas22
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                reas22 c1sess*reas22 educgrp c1sess*educgrp c1sess*c1sess*educgrp
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /TEST = "DF=6 Wald Test for Educ Effects" educgrp -1 1 0; educgrp -1 0 1;
               c1sess*educgrp -1 1 0; c1sess*educgrp -1 0 1;
               c1sess*c1sess*educgrp -1 1 0; c1sess*c1sess*educgrp -1 0 1
      /EMMEANS = TABLES(educgrp) WITH (c1sess=0) COMPARE(educgrp)
      /EMMEANS = TABLES(educgrp) WITH (c1sess=5) COMPARE(educgrp)
      /TEST = "L vs. H Educ for for Main Effect" educgrp -1  0  1
      /TEST = "M vs. H Educ for for Main Effect" educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Main Effect" educgrp -1  1  0
      /TEST = "L vs. H Educ for for Linear Session" c1sess*educgrp -1  0  1
      /TEST = "M vs. H Educ for for Linear Session" c1sess*educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Linear Session" c1sess*educgrp -1  1  0
      /TEST = "L vs. H Educ for for Quadratic Session" c1sess*c1sess*educgrp -1  0  1
      /TEST = "M vs. H Educ for for Quadratic Session" c1sess*c1sess*educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Quadratic Session" c1sess*c1sess*educgrp -1  1  0
      /SAVE = FIXPRED (prededuc).
CORRELATIONS prededuc nm3rt.

* STATA Model 6a: Add Education Group as Predictor of Intercept, Linear, and Quadratic
mixed nm3rt c.c1sess c.c1sess#c.c1sess c.age80 c.c1sess#c.age80        ///
         c.c1sess#c.c1sess#c.age80 c.reas22 c.c1sess#c.reas22          ///
         ib(last).educgrp c.c1sess#ib(last).educgrp                    ///
         c.c1sess#c.c1sess#ib(last).educgrp, || id: c1sess c1sess2,    ///
         variance reml covariance(un) residuals(independent,t(session)) dfmethod(satterthwaite),
       estat ic, n(101)                                 // AIC and BIC
       estat recovariance, relevel(id)                  // G matrix
       estat recovariance, relevel(id) correlation      // GCORR matrix
       estat df, method(satterthwaite)                  // print DDF for fixed effects
       contrast educgrp, small                          // omnibus group diff on intercept
       contrast educgrp#c.c1sess, small                 // omnibus group diff on linear
       contrast educgrp#c.c1sess#c.c1sess, small        // omnibus group diff on quadratic
        * Could not figure out how to get DF=6 overall group contrast
* Estimating group means at first and last sessions
       margins ib(last).educgrp,  at(c.c1sess=(0 5))
* Contrasts between groups on intercept, linear, and quadratic slopes
       test 1.educgrp=3.educgrp, small                          // Low vs. High: Intercept
       test 2.educgrp=3.educgrp, small, small                   // Med vs. High: Intercept
       test 1.educgrp=2.educgrp, small                          // Low vs. Med:  Intercept
       test 1.educgrp#c.c1sess=3.educgrp#c.c1sess, small        // Low vs. High: Linear
       test 2.educgrp#c.c1sess=3.educgrp#c.c1sess, small        // Med vs. High: Linear
       test 1.educgrp#c.c1sess=2.educgrp#c.c1sess, small        // Low vs. Med:  Linear
       test 1.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess, small  // Low vs. High: Quad
       test 2.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess, small  // Med vs. High: Quad
       test 1.educgrp#c.c1sess#c.c1sess=2.educgrp#c.c1sess#c.c1sess, small  // Low vs. Med:  Quad
       margins, at(c.c1sess=(0(1)5) educgrp=(1 2 3)) vsquish  // predictions per session
```

```
      marginsplot                      // plot educ predictions
      predict prededuc                 // save fixed-effect predicted outcomes
corr prededuc nm3rt                    // get total r to make r2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	241027	37339	6.46	<.0001
UN(2,1)	ID	-35271	11645	-3.03	0.0025
UN(2,2)	ID	25772	5956.96	4.33	<.0001
UN(3,1)	ID	4371.57	1907.59	2.29	0.0219
UN(3,2)	ID	-3896.53	995.30	-3.91	<.0001
UN(3,3)	ID	628.15	173.93	3.61	0.0002
session	ID	20298	1649.11	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8211.4	7	8225.4	8225.6	8232.8	8243.7	8250.7

Solution for Fixed Effects

Effect	Education Group (1=HS,2=BA,3=GRAD)	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		1961.89	104.34	95.7	18.80	<.0001
c1sess		-106.50	41.1184	96.7	-2.59	0.0111
c1sess*c1sess		12.4797	6.9879	97	1.79	0.0772
age80		20.2894	8.7750	97.5	2.31	0.0229
c1sess*age80		-4.5759	3.3351	98	-1.37	0.1732
c1sess*c1sess*age80		0.6177	0.5646	97	1.09	0.2767
reas22		-36.6221	11.0407	96	-3.32	0.0013
c1sess*reas22		2.9786	1.3130	96.1	2.27	0.0255
educgrp	1	-51.3792	154.85	96.3	-0.33	0.7408
educgrp	2	37.6426	123.90	95.4	0.30	0.7619
educgrp	3	0	.	.	.	.
c1sess*educgrp	1	-70.2451	60.3032	97.1	-1.16	0.2469
c1sess*educgrp	2	-4.3577	49.1299	96.5	-0.09	0.9295
c1sess*educgrp	3	0	.	.	.	.
c1sess*c1sess*educgrp	1	11.0653	10.2358	97	1.08	0.2824
c1sess*c1sess*educgrp	2	-1.4641	8.3545	97	-0.18	0.8612
c1sess*c1sess*educgrp	3	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
c1sess	1	96.5	35.77	<.0001
c1sess*c1sess	1	97	17.62	<.0001
age80	1	97.5	5.35	0.0229
c1sess*age80	1	98	1.88	0.1732
c1sess*c1sess*age80	1	97	1.20	0.2767
reas22	1	96	11.00	0.0013
c1sess*reas22	1	96.1	5.15	0.0255
educgrp	**2**	**96.1**	**0.23**	**0.7965**
c1sess*educgrp	**2**	**97**	**0.92**	**0.4012**
c1sess*c1sess*educgrp	**2**	**97**	**1.05**	**0.3545**

I normally skip this box if the CLASS statement is not used for predictors, but here the last three entries give us the omnibus (df=2) tests for whether there are any education group differences on the intercept, linear, or quadratic time slopes, not just (some of the possible) pairwise comparisons.

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
L vs. H Educ for Intercept Main Effect	51.3792	154.85	96.3	0.33	0.7408
M vs. H Educ for Intercept Main Effect	-37.6426	123.90	95.4	-0.30	0.7619
L vs. M Educ for Intercept Main Effect	89.0218	134.02	96.8	0.66	0.5081

L vs. H Educ for Linear Session	70.2451	60.3032	97.1	1.16	0.2469
M vs. H Educ for Linear Session	4.3577	49.1299	96.5	0.09	0.9295
L vs. M Educ for Linear Session	65.8874	51.7661	97.4	1.27	0.2061
L vs. H Educ for Quadratic Session	-11.0653	10.2358	97	-1.08	0.2824
M vs. H Educ for Quadratic Session	1.4641	8.3545	97	0.18	0.8612
L vs. M Educ for Quadratic Session	-12.5294	8.7793	97	-1.43	0.1567

Is the education model (6a) better than the reasoning by linear model (5b)? Nope:

Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
DF=6 Wald Test for Effects of Education	6	96.4	4.59	0.76	0.5976	0.5994

Least Squares Means

Effect	Educ Grp	c1sess	Age80	Reas22	Estimate	Standard Error	DF	t Value	Pr > |t|
EducGrp	1	0.00	0.00	0.00	1910.51	112.41	96.1	17.00	<.0001
EducGrp	2	0.00	0.00	0.00	1999.53	69.2506	96.3	28.87	<.0001
EducGrp	3	0.00	0.00	0.00	1961.89	104.34	95.7	18.80	<.0001
EducGrp	1	5.00	0.00	0.00	1615.41	95.7317	96	16.87	<.0001
EducGrp	2	5.00	0.00	0.00	1720.63	59.0105	96.1	29.16	<.0001
EducGrp	3	5.00	0.00	0.00	1741.38	88.7887	95.9	19.61	<.0001

You must specify a value at which to hold each continuous predictor.

Differences of Least Squares Means

Effect	Educ Grp	Educ Grp	c1sess	Age80	Reas22	Estimate	Standard Error	DF	t Value	Pr > |t|
EducGrp	1	2	0.00	0.00	0.00	-89.0218	134.02	96.8	-0.66	0.5081
EducGrp	1	3	0.00	0.00	0.00	-51.3792	154.85	96.3	-0.33	0.7408
EducGrp	2	3	0.00	0.00	0.00	37.6426	123.90	95.4	0.30	0.7619
EducGrp	1	2	5.00	0.00	0.00	-105.22	114.33	96.2	-0.92	0.3597
EducGrp	1	3	5.00	0.00	0.00	-125.97	131.97	96.1	-0.95	0.3422
EducGrp	2	3	5.00	0.00	0.00	-20.7486	105.35	95.9	-0.20	0.8443

```
* Calculate Total R2 change relative to reasL model;
%TotalR2(DV=nm3rt, PredFewer=PredReasL, PredMore=PredEduc);
```

Total R2 (% Reduction) for PredReasL vs. PredEduc

Name	Pred Corr	TotalR2	Total R2Diff
PredReasL	0.40008	0.16006	.
PredEduc	0.41510	0.17231	0.012242

R =.41510, so R^2 for time+age+reas+educ = .172

The fixed effects of time, age, and reasoning before accounted for ~16.0% of the variance in RT, so there is a net increase of 1.2% due to education (which is not significant).

```
* Calculate PseudoR2 relative to reasL model;
%PseudoR2(NCov=7, CovFewer=PredReasL, CovMore=CovEduc);
```

PsuedoR2 (% Reduction) for CovReasL vs. CovEduc

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovReasL	UN(1,1)	ID	235909	36153	6.53	<.0001	.
CovReasL	UN(2,2)	ID	25707	5883.65	4.37	<.0001	.
CovReasL	UN(3,3)	ID	629.52	172.50	3.65	0.0001	.
CovReasL	session	ID	20298	1649.11	12.31	<.0001	.
CovEduc	UN(1,1)	ID	241027	37339	6.46	<.0001	-0.021693 for L2 intercept
CovEduc	UN(2,2)	ID	25772	5956.96	4.33	<.0001	-0.002519 for L2 linear
CovEduc	UN(3,3)	ID	628.15	173.93	3.61	0.0002	0.002185 for L2 quad
CovEduc	session	ID	20298	1649.11	12.31	<.0001	-0.000000 for L1 residual

Based on the lack of significance of the effect of education, I'd say we're done with this model (I had previously tried age*reasoning, and none of those higher-order effects were significant). The age*quadratic interaction could probably be removed, but I choose to leave it in as a control.

Sample Results Section:

The extent to which individual differences in response time (RT) over six sessions for a simple processing speed test (number match three) could be predicted from baseline age, abstract reasoning, and education level was examined in a series of multilevel models (i.e., general linear mixed models) in which the six practice sessions were nested within each participant. Residual maximum likelihood (REML) was used in estimating and reporting all model parameters; denominator degrees of freedom were estimated using the Satterthwaite method. The significance of new fixed effects were evaluated with univariate and multivariate Wald tests. Session (i.e., the index of time) was centered at the first occasion, age was centered at 80 years, abstract reasoning was centered at 22 (near the mean of the predictor scale), and graduate-level education was the reference group for education level (with separate contrasts for high school or less and for bachelor's level education).

The best-fitting unconditional polynomial growth model specified quadratic decline across the six sessions (i.e., a decelerating negative function) with significant individual differences in the intercept, linear, and quadratic time effects. Accordingly, effect size was evaluated via pseduo-R^2 values for the proportion reduction in each random effect variance, as well as with total-R^2, the squared correlation between the actual outcome values and the outcomes predicted by the model fixed effects. In the unconditional growth model, the fixed effects for linear and quadratic change across sessions accounted for approximately 4% of the total variation in RT.

Next, age was added as a predictor of the intercept, linear slope, and quadratic slope. Although the three effects of age together resulted in a significant omnibus effect, $F(3, 99) = 4.00$, $p < .01$, only the fixed effect of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher (slower) by 29.05 ($p < .001$). In terms of pseudo-R^2, age accounted for 10.32% of the level-2 random intercept variance, 2.93% of the level-2 random linear slope variance, and 0.77% of the level-2 random quadratic slope variance. As expected given that baseline age is a time-invariant predictor, the level-1 residual variance was not reduced. The cumulative total-R^2 from session and age was 11%, approximately a 7% increase due to age. Although the interactions of age with the linear and quadratic slopes were not significant, they were retained in the model to fully control for age effects before examining the effects of other predictors.

Abstract reasoning was then added as a predictor of the intercept, linear slope, and quadratic slope. As with the effects of age, although the three effects of abstract reasoning together resulted in a significant omnibus effect, $F(3, 98) = 4.29$, $p < .01$, only the fixed effect of abstract reasoning on the intercept was significant, indicating that for every additional unit of reasoning above 22, RT at the first session was predicted to be significantly lower (faster) by 27.10 ($p < .001$).The nonsignificant effect of reasoning on the quadratic slope was then removed, revealing a significant effect of reasoning on both the intercept and linear slope, $F(2, 98) = 5.29$, $p < .01$, such that for every unit higher reasoning above 22, RT at the first session was expected to be lower by 32.83 and the linear rate of improvement in RT (as evaluated at the first session given the quadratic slope) was expected to be less negative by 2.94 (i.e., faster initial RT with less improvement in persons with greater reasoning). Reasoning accounted for 4.76% of the level-2 random intercept variance but had no measurable reduction of the level-2 random linear and quadratic slope variances. The cumulative total-R^2 from session, age, and reasoning was 16%, approximately a 5% increase due to reasoning.

Education level (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept, linear slope, and quadratic slope. These six effects of education did not significantly improve model fit, $F(6, 96) = 0.76$, $p = .60$. No omnibus main effects of education level on the intercept, linear, or quadratic slopes were significant, and no pairwise comparisons were significant as well. Education accounted for no measurable level-2 random intercept or random linear slope variance, and 2.19% of the random quadratic slope variance. The cumulative total-R^2 from session, age, reasoning, and education was 17%, approximately a 1% increase due to education.

Finally, we examined the interactive effects of age and reasoning in predicting the intercept and each linear slope, although none was significant.
(From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)