Unconditional Polynomial Models for Practice Effects in Number Match 3 Response Times
COMPLETED VERSION

These data (in "Example35" data files) come from a short-term longitudinal study of 6 observations over 2 weeks for 101 adults age 65-80. The goal is to see how performance on this processing speed task ("number match 3"), as measured by response time in milliseconds, declines over the 6 practice sessions.

SAS Code for Data Manipulation:

```
* Location for original SAS files for these models;
%LET filepath = F:\12_ICPSR\ICPSR_2012_Download\SAS; LIBNAME filepath "&filepath.";

* SAS code to import data, center time for polynomial models;
DATA work.example35; SET filepath.example35;
      c1sess = session - 1;      LABEL c1sess = "c1sess: Session Centered at 1";
      c6sess = session - 6;      LABEL c6sess = "c6sess: Session Centered at 6"; RUN;
```

SPSS Code for Data Manipulation:

```
* Define locations of files used in examples -- CHANGE THIS.
FILE HANDLE example /NAME = "F:\12_ICPSR\ICPSR_2012_Download\SPSS".

* SPSS code to import data, center time for polynomial models.
GET FILE = "example/Example35.sav".
DATASET NAME example35 WINDOW=FRONT.
COMPUTE c1sess = session - 1.
COMPUTE c6sess = session - 6.
VARIABLE LABELS     c1sess "c1sess: Session Centered at 1"
                    c6sess "c6sess: Session Centered at 6".
```

STATA Code for Data Manipulation:

```
 * STATA code to center time for polynomial models (and make quadratic versions)
gen c1sess = session - 1
gen c6sess = session – 6
gen c1sess2 = c1sess * c1sess
gen c6sess2 = c6sess * c6sess
label variable c1sess  "c1sess: Session Centered at 1"
label variable c6sess  "c6sess: Session Centered at 6"
label variable c1sess2 "c1sess2: Quadratic Session Centered at 1"
label variable c6sess2 "c6sess2: Quadratic Session Centered at 6"
```

Model 1a. Most Conservative Baseline – Empty Means, Random Intercept

```
TITLE1 "SAS Model 1a: Empty Means, Random Intercept Only";
PROC MIXED DATA=work.example35 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
      CLASS ID session;
      MODEL nm3rt = / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID; RUN;

TITLE "SPSS Model 1a: Empty Means, Random Intercept".
MIXED nm3rt BY ID session
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  =
      /RANDOM = INTERCEPT | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).

* STATA Model 1a: Empty Means, Random Intercept
xtmixed nm3rt ,  || id:  ,  ///
      variance reml covariance(unstructured) residuals(independent,t(session)),
      estat ic, n(101)
      estat recovariance, level(id)
```

SAS output (SPSS and STATA versions available electronically):

Dimensions

Covariance Parameters	2
Columns in X	1
Columns in Z Per Subject	1
Subjects	101
Max Obs Per Subject	6

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	44900					
2		44900				
3			44900			
4				44900		
5					44900	
6						44900

This **R matrix** VC structure (equal variance over time, no covariance of any kind) will be used repeatedly as we add fixed and random effects to the model.

Estimated G Matrix

Row	Effect	Participant ID	Col1
1	Intercept	101	200883

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**245783**	200883	200883	200883	200883	200883
2	200883	**245783**	200883	200883	200883	200883
3	200883	200883	**245783**	200883	200883	200883
4	200883	200883	200883	**245783**	200883	200883
5	200883	200883	200883	200883	**245783**	200883
6	200883	200883	200883	200883	200883	**245783**

The **V** matrix is the total variance-covariance matrix that results from combining the level-2 **G** and level-1 **R** matrices.

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8173	0.8173	0.8173	0.8173	0.8173
2	**0.8173**	1.0000	0.8173	0.8173	0.8173	0.8173
3	0.8173	0.8173	1.0000	0.8173	0.8173	0.8173
4	0.8173	0.8173	0.8173	1.0000	0.8173	0.8173
5	0.8173	0.8173	0.8173	0.8173	1.0000	0.8173
6	0.8173	0.8173	0.8173	0.8173	0.8173	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	200883	29471	6.82	<.0001
Session	ID	44900	2825.63	15.89	<.0001

Calculate the ICC for the Number Match 3 outcome:

200883 / (200883 + 44900) = .82

Fit Statistics

-2 Res Log Likelihood	8536.9
AIC (smaller is better)	8540.9
AICC (smaller is better)	8540.9
BIC (smaller is better)	8546.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1770.70	45.4206	100	38.98	<.0001

Model 1b. Most Liberal Baseline – Saturated Means, Unstructured Variances (Model Answer Key)

```
TITLE1 "SAS Model 1b: Saturated Means, Unstructured Variances";
PROC MIXED DATA=work.example35 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
      CLASS ID session;
      MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
      REPEATED session / R RCORR TYPE=UN SUBJECT=ID;
      LSMEANS session /; RUN;

TITLE "SPSS Model 1b: Saturated Means, Unstructured Variances".
MIXED nm3rt BY ID session
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV R
      /FIXED  = session
      /REPEATED = session | SUBJECT(ID) COVTYPE(UN)
      /EMMEANS  = TABLES(session).

 * STATA Model 1b: Saturated Means, Unstructured Variances
xtmixed nm3rt ib(last).session, || id: , noconstant ///
      variance reml residuals(unstructured, t(session)),
      estat ic, n(101),
      margins i.session
```

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	235659	217994	202607	192154	195360
2	235659	**259150**	230217	213232	202092	193268
3	217994	230217	**233368**	205209	196919	188604
4	202607	213232	205209	**217544**	193676	185321
5	192154	202092	196919	193676	**212098**	187840
6	195360	193268	188604	185321	187840	**196733**

This **R matrix** UN structure lets all variances and covariances be what they want. THIS IS THE DATA we are trying to duplicate with our model for the variances.

Estimated R Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8424	0.8212	0.7905	0.7593	0.8015
2	0.8424	1.0000	0.9361	0.8981	0.8620	0.8559
3	0.8212	0.9361	1.0000	0.9108	0.8851	0.8802
4	0.7905	0.8981	0.9108	1.0000	0.9016	0.8958
5	0.7593	0.8620	0.8851	0.9016	1.0000	0.9196
6	0.8015	0.8559	0.8802	0.8958	0.9196	1.0000

Fit Statistics

-2 Res Log Likelihood	8229.8
AIC (smaller is better)	8271.8
AICC (smaller is better)	8273.4
BIC (smaller is better)	8326.7

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		1672.14	44.1345	100	37.89	<.0001
Session	1	289.76	32.7000	100	8.86	<.0001
Session	2	143.04	26.2031	100	5.46	<.0001
Session	3	77.8986	22.8842	100	3.40	0.0010
Session	4	45.6604	20.7853	100	2.20	0.0303
Session	5	35.0397	18.1168	100	1.93	0.0559
Session	6	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Session	5	100	16.72	<.0001

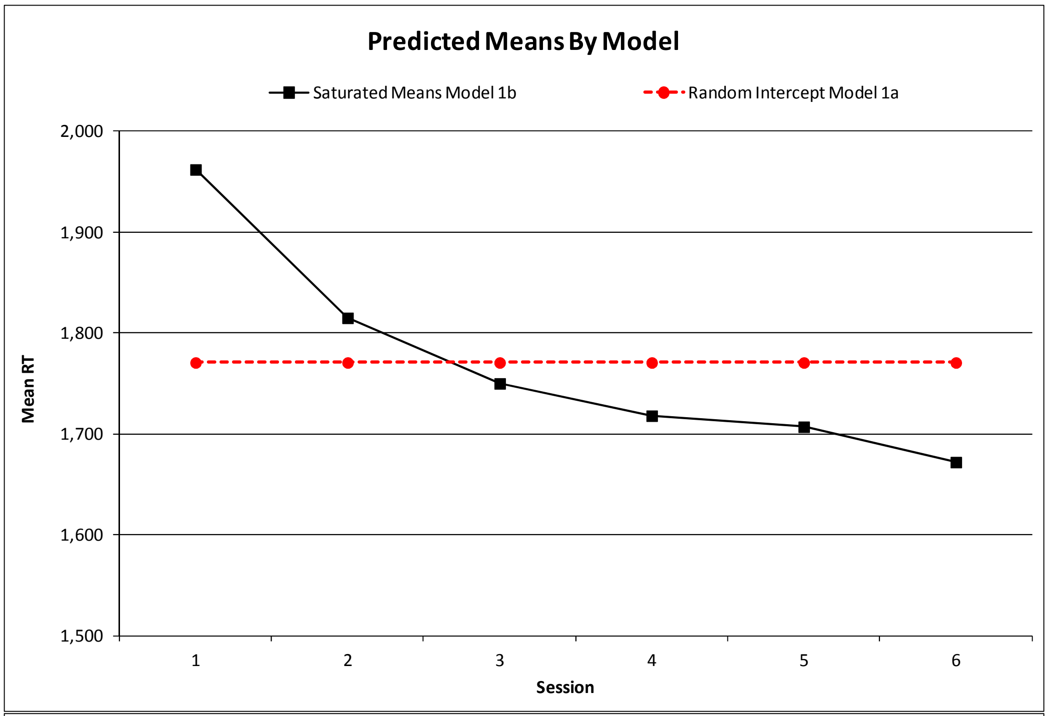
What does the intercept represent? Mean at session 6 (that's why there are dots for that line)

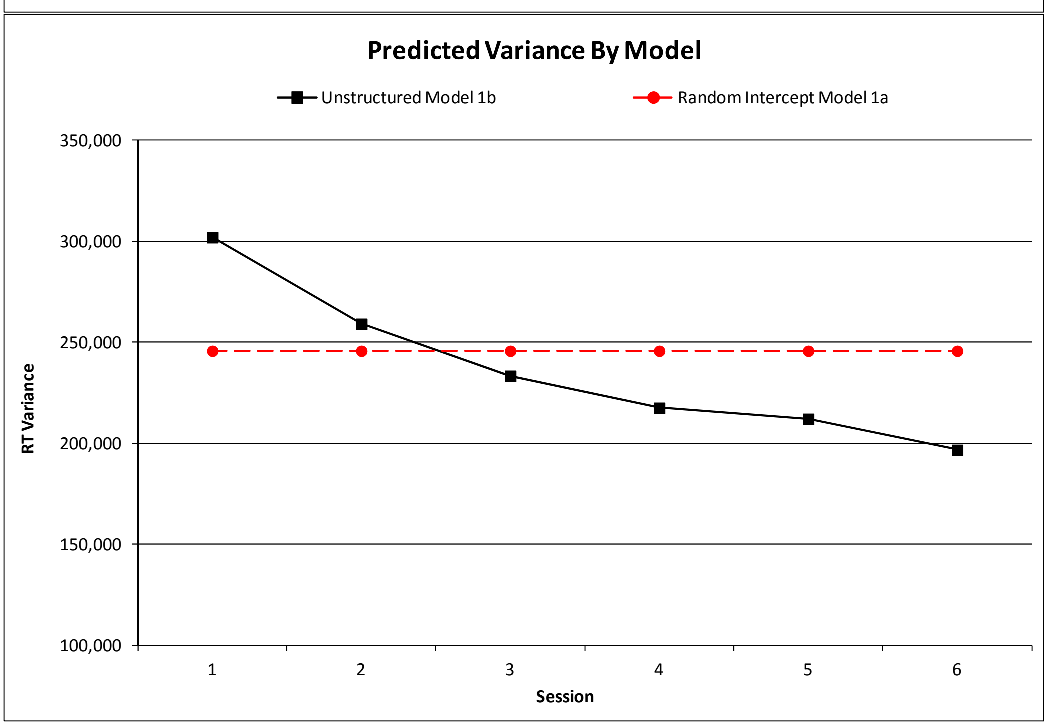
What do the effects for sessions 1-5 represent? Differences between session 6 and each session

What does the F-test mean? There are differences across the 6 sessions.

```
     Least Squares Means → What we are trying to duplicate with fixed effects
                                         Standard
Effect       Session #         Estimate       Error       DF     t Value     Pr > |t|
Session              1          1961.89     54.6805      100       35.88       <.0001
Session              2          1815.17     50.6541      100       35.83       <.0001
Session              3          1750.03     48.0684      100       36.41       <.0001
Session              4          1717.80     46.4101      100       37.01       <.0001
Session              5          1707.18     45.8255      100       37.25       <.0001
Session              6          1672.14     44.1345      100       37.89       <.0001
```

So here is what are we trying to model—means and variances, where model 1b is the data:





Model 2a. Fixed Linear Time, Random Intercept

```
TITLE1 "SAS Model 2a: Fixed Linear Time, Random Intercept";
PROC MIXED DATA=work.example35 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
      CLASS ID session;
      MODEL nm3rt = c1sess / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID; RUN;
```

```
TITLE "SPSS Model 2a: Fixed Linear Time, Random Intercept".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess
      /RANDOM = INTERCEPT | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).
```

```
 * STATA Model 2a: Fixed Linear Time, Random Intercept
xtmixed nm3rt c.c1sess,  || id:  ,  ///
      variance reml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estimates store FixLin
```

DV = nm3rt, continuous fixed slope for c1sess
Level 2 ID is id, random intercept by default
Print variances instead of SD, use reml
residuals → R matrix is diagonal
estat ic → Print IC given N = 101 persons
estimates → save results as “FixLin” for comparison

Dimensions

Covariance Parameters	2
Columns in X	2
Columns in Z Per Subject	1
Subjects	101
Max Obs Per Subject	6

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	202422	29470	6.87	<.0001
Session	ID	35662	2246.48	15.87	<.0001

Relative to the empty means, random intercept model 1a, the fixed linear effect of session explained ~21% of the residual variance (which made the random intercept variance increase due to its residual variance correction factor).

Fit Statistics

-2 Res Log Likelihood	8414.7
AIC (smaller is better)	8418.7
AICC (smaller is better)	8418.7
BIC (smaller is better)	8423.9

Is the fixed linear time, random intercept model (2a) better than the empty means, random intercept model (1a)?

Yep, by the p-value for the fixed linear effect of session

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1899.63	46.7882	113	40.60	<.0001
C1sess	-51.5719	4.4918	504	-11.48	<.0001

Model 2b. Random Linear Time

```
TITLE1 "SAS Model 2b: Random Linear Time";
PROC MIXED DATA=work.example35 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
      CLASS ID session;
      MODEL nm3rt = c1sess / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT c1sess / G V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID; RUN;

TITLE "SPSS Model 2b: Random Linear Time".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess
      /RANDOM = INTERCEPT c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).

 * STATA Model 2b: Random Linear Time
xtmixed nm3rt c.c1sess,  || id: c1sess,  ///
      variance reml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store RandLin,
      lrtest RandLin FixLin
```

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	27905					
2		27905				
3			27905			
4				27905		
5					27905	
6						27905

Estimated G Matrix

Row	Effect	Participant ID	Col1	Col2
1	Intercept	101	253258	-12701
2	C1sess	101	-12701	2233.83

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**281163**	240557	227856	215155	202455	189754
2	240557	**257995**	219623	209156	198689	188222
3	227856	219623	**239295**	203157	194924	186691
4	215155	209156	203157	**225063**	191158	185159
5	202455	198689	194924	191158	**215298**	183627
6	189754	188222	186691	185159	183627	**210001**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8932	0.8784	0.8553	0.8229	0.7809
2	0.8932	1.0000	0.8839	0.8680	0.8430	0.8086
3	0.8784	0.8839	1.0000	0.8754	0.8588	0.8328
4	0.8553	0.8680	0.8754	1.0000	0.8684	0.8517
5	0.8229	0.8430	0.8588	0.8684	1.0000	0.8636
6	0.7809	0.8086	0.8328	0.8517	0.8636	1.0000

```
                 Covariance Parameter Estimates
                                  Standard         Z
Cov Parm     Subject     Estimate      Error     Value        Pr Z
UN(1,1)      ID            253258      37897      6.68      <.0001
UN(2,1)      ID            -12701    3621.98     -3.51      0.0005
UN(2,2)      ID           2233.83     552.92      4.04      <.0001
Session      ID             27905    1963.42     14.21      <.0001
           Fit Statistics
-2 Res Log Likelihood          8372.1
AIC (smaller is better)        8380.1
AICC (smaller is better)       8380.2
BIC (smaller is better)        8390.6
                   Solution for Fixed Effects
                         Standard
Effect        Estimate      Error      DF    t Value     Pr > |t|
Intercept      1899.63    51.4998     100      36.89       <.0001
C1sess        -51.5719     6.1567     100      -8.38       <.0001
```

Is the random linear time model (2b) better than the fixed linear time, random intercept model (2a)?

Yep, -2ΔLL= 43, which is bigger than the critical value of 5.99ish on df =2ish

Model 3a. Fixed Quadratic, Random Linear Time

```
TITLE1 "SAS Model 3a: Fixed Quadratic, Random Linear Time";
PROC MIXED DATA=work.example35 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT c1sess / G V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID; RUN;

TITLE "SPSS Model 3a: Fixed Quadratic, Random Linear Time".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess
      /RANDOM = INTERCEPT c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID).

 * STATA Model 3a: Fixed Quadratic, Random Linear Time
xtmixed nm3rt c.c1sess c.c1sess2,  || id: c1sess,  ///
      variance reml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store FixQuad
```

```
                 Covariance Parameter Estimates
                                  Standard         Z
Cov Parm     Subject     Estimate      Error     Value        Pr Z
UN(1,1)      ID            254164      37896      6.71      <.0001
UN(2,1)      ID            -12948    3620.70     -3.58      0.0003
UN(2,2)      ID           2332.67     551.58      4.23      <.0001
Session      ID             26176    1844.01     14.20      <.0001

           Fit Statistics
-2 Res Log Likelihood          8341.5
AIC (smaller is better)        8349.5
AICC (smaller is better)       8349.5
BIC (smaller is better)        8359.9

                       Solution for Fixed Effects
                                 Standard
Effect            Estimate        Error      DF    t Value     Pr > |t|
Intercept          1945.85      52.2433     106      37.25       <.0001
C1sess             -120.90      14.5415     502      -8.31       <.0001
C1sess*C1sess      13.8656       2.6348     403       5.26       <.0001
```

Relative to the random linear time model 2b, the fixed quadratic effect of session explained another ~6% of the residual variance (which made the random intercept variance increase due to its residual variance correction factor).

Is the fixed quadratic, random linear model (3a) better than the random linear model (2b)?

Yep, by the p-value for the fixed quadratic effect of session

Model 3b. Random Quadratic Time (and an example of ESTIMATE/TEST/LINCOM statements)

```
TITLE1 "SAS Model 3b: Random Quadratic Time";
PROC MIXED DATA=work.example35 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 METHOD=REML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID;
      ESTIMATE "Intercept at Session 1"    intercept 1 c1sess 0   c1sess*c1sess 0;
      ESTIMATE "Intercept at Session 2"    intercept 1 c1sess 1   c1sess*c1sess 1;
      ESTIMATE "Intercept at Session 3"    intercept 1 c1sess 2   c1sess*c1sess 4;
      ESTIMATE "Intercept at Session 4"    intercept 1 c1sess 3   c1sess*c1sess 9;
      ESTIMATE "Intercept at Session 5"    intercept 1 c1sess 4   c1sess*c1sess 16;
      ESTIMATE "Intercept at Session 6"    intercept 1 c1sess 5   c1sess*c1sess 25;
      * Predicting linear rate of change at each session (linear changes by 2*quad);
      ESTIMATE "Linear Slope at Session 1"      c1sess 1   c1sess*c1sess 0;
      ESTIMATE "Linear Slope at Session 2"      c1sess 1   c1sess*c1sess 2;
      ESTIMATE "Linear Slope at Session 3"      c1sess 1   c1sess*c1sess 4;
      ESTIMATE "Linear Slope at Session 4"      c1sess 1   c1sess*c1sess 6;
      ESTIMATE "Linear Slope at Session 5"      c1sess 1   c1sess*c1sess 8;
      ESTIMATE "Linear Slope at Session 6"      c1sess 1   c1sess*c1sess 10;  RUN;

TITLE "SPSS Model 3b: Random Quadratic Time".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD = REML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /TEST = "Intercept at Session 1"      intercept 1 c1sess 0   c1sess*c1sess 0
      /TEST = "Intercept at Session 2"      intercept 1 c1sess 1   c1sess*c1sess 1
      /TEST = "Intercept at Session 3"      intercept 1 c1sess 2   c1sess*c1sess 4
      /TEST = "Intercept at Session 4"      intercept 1 c1sess 3   c1sess*c1sess 9
      /TEST = "Intercept at Session 5"      intercept 1 c1sess 4   c1sess*c1sess 16
      /TEST = "Intercept at Session 6"      intercept 1 c1sess 5   c1sess*c1sess 25
      /TEST = "Linear Slope at Session 1"       c1sess 1    c1sess*c1sess 0
      /TEST = "Linear Slope at Session 2"       c1sess 1    c1sess*c1sess 2
      /TEST = "Linear Slope at Session 3"       c1sess 1    c1sess*c1sess 4
      /TEST = "Linear Slope at Session 4"       c1sess 1    c1sess*c1sess 6
      /TEST = "Linear Slope at Session 5"       c1sess 1    c1sess*c1sess 8
      /TEST = "Linear Slope at Session 6"       c1sess 1    c1sess*c1sess 10.

 * STATA Model 3b: Random Quadratic Time
xtmixed nm3rt c.c1sess c.c1sess2,  || id: c1sess c1sess2,  ///
      variance reml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store RandQuad,
      lrtest RandQuad FixQuad,
      lincom 1*_cons + 0*c1sess   + 0*c1sess2     // intercept at session 1
      lincom 1*_cons + 1*c1sess   + 1*c1sess2     // intercept at session 2
      lincom 1*_cons + 2*c1sess   + 4*c1sess2     // intercept at session 3
      lincom 1*_cons + 3*c1sess   + 9*c1sess2     // intercept at session 4
      lincom 1*_cons + 4*c1sess   + 16*c1sess2    // intercept at session 5
      lincom 1*_cons + 5*c1sess   + 25*c1sess2    // intercept at session 6
      lincom 1*c1sess + 0*c1sess2     // linear slope at session 1
      lincom 1*c1sess + 2*c1sess2     // linear slope at session 2
      lincom 1*c1sess + 4*c1sess2     // linear slope at session 3
      lincom 1*c1sess + 6*c1sess2     // linear slope at session 4
      lincom 1*c1sess + 8*c1sess2     // linear slope at session 5
      lincom 1*c1sess + 10*c1sess2    // linear slope at session 6
```

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	20298					
2		20298				
3			20298			
4				20298		
5					20298	
6						20298

Estimated G Matrix

Row	Effect	Participant ID	Col1	Col2	Col3
1	Intercept	101	276206	-35734	3901.96
2	C1sess	101	-35734	25840	-3903.32
3	C1sess*C1sess	101	3901.96	-3903.32	634.47

Estimated V Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**296504**	244374	220346	204122	195702	195085
2	244374	**251508**	219312	208680	199315	191215
3	220346	219312	**235842**	209043	199808	187840
4	204122	208680	209043	**225508**	197182	184958
5	195702	199315	199808	197182	**211735**	182571
6	195085	191215	187840	184958	182571	**200977**

Estimated V Correlation Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8949	0.8333	0.7894	0.7811	0.7992
2	0.8949	1.0000	0.9005	0.8762	0.8637	0.8505
3	0.8333	0.9005	1.0000	0.9064	0.8941	0.8628
4	0.7894	0.8762	0.9064	1.0000	0.9024	0.8688
5	0.7811	0.8637	0.8941	0.9024	1.0000	0.8850
6	0.7992	0.8505	0.8628	0.8688	0.8850	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	276206	41442	6.66	<.0001
UN(2,1)	ID	-35734	11941	-2.99	0.0028
UN(2,2)	ID	25840	5864.41	4.41	<.0001
UN(3,1)	ID	3901.96	1949.06	2.00	0.0453
UN(3,2)	ID	-3903.32	982.61	-3.97	<.0001
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Fit Statistics

-2 Res Log Likelihood	8302.7
AIC (smaller is better)	8316.7
AICC (smaller is better)	8316.9
BIC (smaller is better)	8335.1

Is the random quadratic model (3b) better than the fixed quadratic, random linear model (3a)?

Yep, -2ΔLL= 39, which is bigger than the critical value of 7.82ish on df=3ish

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	53.8497	100	36.13	<.0001
C1sess	-120.90	20.0476	100	-6.03	<.0001
C1sess*C1sess	13.8656	3.4154	100	4.06	<.0001

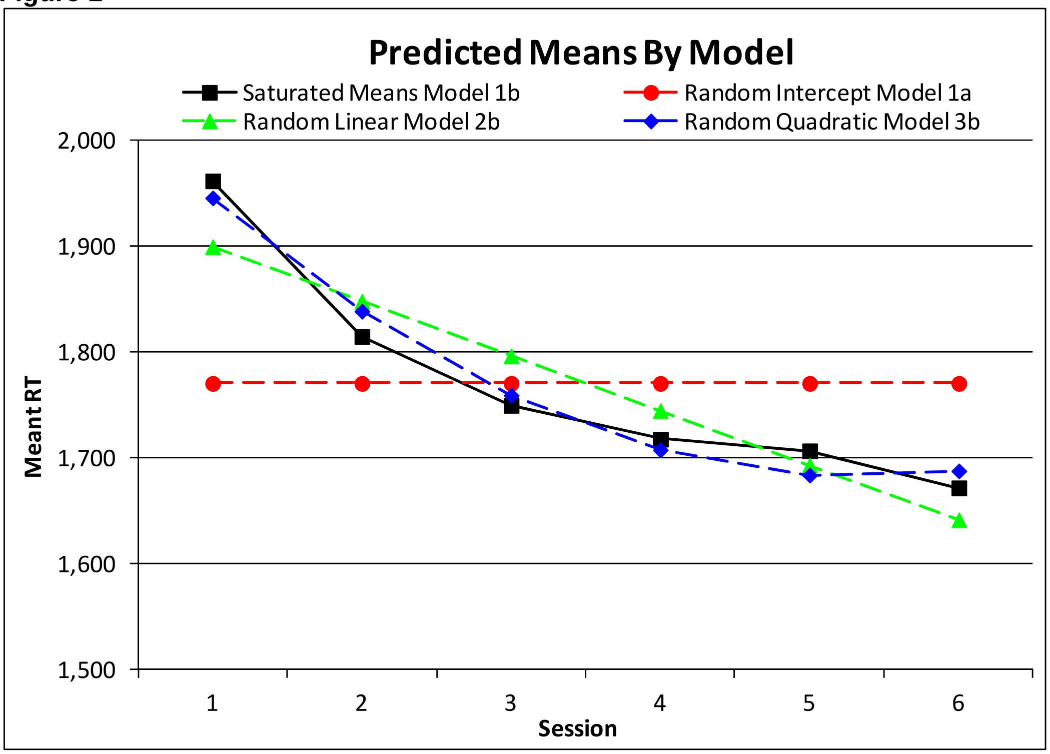
Estimates

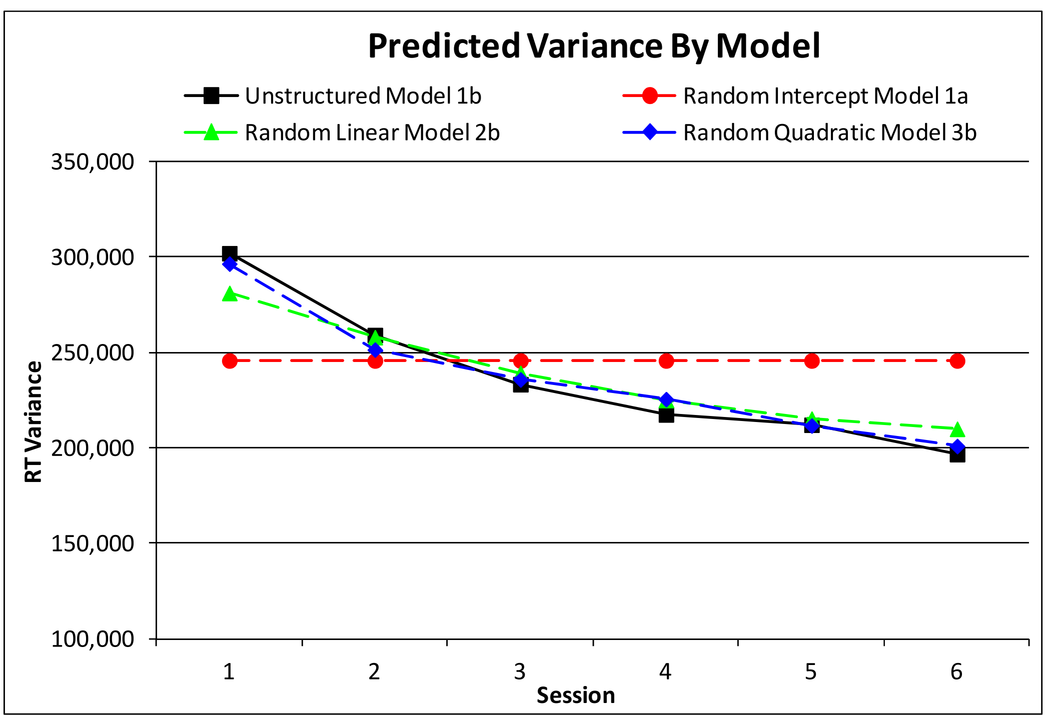
Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept at Session 1	1945.85	53.8497	100	36.13	<.0001
Intercept at Session 2	1838.82	48.4864	100	37.92	<.0001
Intercept at Session 3	1759.51	46.9973	100	37.44	<.0001
Intercept at Session 4	1707.94	45.8959	100	37.21	<.0001
Intercept at Session 5	1684.10	44.2395	100	38.07	<.0001
Intercept at Session 6	1687.99	44.2038	100	38.19	<.0001

Linear Trend at Session 1	-120.90	20.0476	100	-6.03	<.0001
Linear Trend at Session 2	-93.1687	13.6497	100	-6.83	<.0001
Linear Trend at Session 3	-65.4375	8.0028	100	-8.18	<.0001
Linear Trend at Session 4	-37.7062	5.9242	100	-6.36	<.0001
Linear Trend at Session 5	-9.9750	9.9733	100	-1.00	0.3196
Linear Trend at Session 6	17.7562	16.0362	100	1.11	0.2708

How well do the predicted means, variances, and covariances from the random quadratic model (3b) match the original means, variances, and covariances from the saturated means model (1b)?

Figure 1





What happens to the model parameters if we pick a different centering point?

Fixed Quadratic, Random Linear (Session 1 = 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	254164	37896	6.71	<.0001
UN(2,1)	ID	-12948	3620.70	-3.58	0.0003
UN(2,2)	ID	2332.67	551.58	4.23	<.0001
Session	ID	26176	1844.01	14.20	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	52.2433	106	37.25	<.0001
C1sess	-120.90	14.5415	502	-8.31	<.0001
C1sess*C1sess	13.8656	2.6348	403	5.26	<.0001

Fixed Quadratic, Random Linear (Session 6 = 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	183002	27836	6.57	<.0001
UN(2,1)	ID	-1284.53	2767.79	-0.46	0.6426
UN(2,2)	ID	2332.67	551.58	4.23	<.0001
Session	ID	26176	1844.01	14.20	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1687.99	44.9976	108	37.51	<.0001
C6sess	17.7562	14.5415	502	1.22	0.2226
C6sess*C6sess	13.8656	2.6348	403	5.26	<.0001

Which parameters change and why? The fixed intercept and linear trends refer specifically to session 1 or 6, and thus change. The random intercept variance refers specifically to session 1 or 6, and thus also changes. The fixed quadratic session and random linear variance are unconditional (highest order on their side of the model) and do not change.

Random Quadratic (Session 1 = 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	276206	41442	6.66	<.0001
UN(2,1)	ID	-35734	11941	-2.99	0.0028
UN(2,2)	ID	25840	5864.41	4.41	<.0001
UN(3,1)	ID	3901.96	1949.06	2.00	0.0453
UN(3,2)	ID	-3903.32	982.61	-3.97	<.0001
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1945.85	53.8497	100	36.13	<.0001
C1sess	-120.90	20.0476	100	-6.03	<.0001
C1sess*C1sess	13.8656	3.4154	100	4.06	<.0001

Random Quadratic (Session 6 = 0)

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	180678	27943	6.47	<.0001
UN(2,1)	ID	-1645.67	7298.42	-0.23	0.8216
UN(2,2)	ID	11221	3863.75	2.90	0.0018
UN(3,1)	ID	247.15	1545.72	0.16	0.8730
UN(3,2)	ID	2441.39	788.06	3.10	0.0019
UN(3,3)	ID	634.47	172.37	3.68	0.0001
Session	ID	20298	1649.11	12.31	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	1687.99	44.2038	100	38.19	<.0001
C6sess	17.7562	16.0362	100	1.11	0.2708
C6sess*C6sess	13.8656	3.4154	100	4.06	<.0001

Which parameters change and why? The fixed intercept and linear trends refer specifically to session 1 or 6, and thus change. The random intercept and random linear variances refer specifically to session 1 or 6, and thus also change. The fixed quadratic session and random quadratic variance are unconditional (highest order on their side of the model) and do not change.

Table 1

Parameter	Fixed Linear Session=1	Random Linear Session=1	Fixed Quadratic Session=1	Fixed Quadratic Session=6	Random Quadratic Session=1	Random Quadratic Session=6
<u>Fixed Effects:</u>						
Intercept	1899.6	1899.6	1945.9	1688.0	1945.9	1679.0
Linear	-51.6	-51.6	-120.9	17.8	-120.9	17.8
Quadratic			13.9	13.9	13.9	13.9
<u>Variance Components:</u>						
Intercept Variance	202422.0	253258.0	254164.0	183002.0	276206.0	180678.0
Linear Variance		2233.8	2332.7	2332.7	25840.0	11221.0
Quadratic Variance					634.5	634.5
Intercept-Linear Covariance		-12701.0	-12948.0	-1284.5	-35734.0	-1645.7
Intercept-Quadratic Covariance					3902.0	247.2
Linear-Quadratic Covariance					-3903.3	2441.4
Residual Variance	35662.0	27905.0	26176.0	26176.0	20298.0	20298.0
<u>Model Fit:</u>						
REML Deviance	8414.7	8372.1	8341.5	8341.5	8302.7	8302.7
AIC	8418.7	8380.1	8349.5	8349.5	8316.7	8316.7
BIC	8423.9	8390.6	8359.9	8359.9	8335.1	8335.1
Total Number of Parameters	4	6	7	7	10	10

95% Random Effects CIs for Random Quadratic (Session=1):

Intercept: 1946 ± [1.96*SQRT(276206)] = 916 to 2976
95% of sample predicted to have *initial* RTs from 916 to 2976…

Linear: -120.9 ± [1.96*SQRT(25840)] = -436 to 194
… and linear slopes *(at session 1)* from -436 to 194

Quad: 13.9 ± [1.96*SQRT(634)] = -36 to 63
… and quadratic slopes from -36 and 63

95% Random Effects CIs for Random Quadratic (Session=6):

Intercept: 1679 ± [1.96*SQRT(180678)] = 846 to 2512
95% of sample predicted to have *final* RTs from 846 to 2512…

Linear: 17.8 ± [1.96*SQRT(11224)] = -190 to 225
… and linear slopes *(at session 6)* to -190 to 225

Quad: 13.9 ± [1.96*SQRT(634)] = -36 to 63
… and quadratic slopes from -36 and 63

Simple Processing Speed – Example Unconditional Models of Change Results

Model Specification

Linear mixed models were estimated using restricted maximum likelihood (REML) in order to examine the overall pattern of and individual differences in response time over six sessions for a simple processing speed test (number match three). The significance of new fixed effects were evaluated using Wald tests, whereas the significance of new random effects was evaluated using likelihood ratio tests (i.e., −2ΔLL), with degrees of freedom equal to the number of new random effects variances and covariances. Denominator degrees of freedom were estimated using the Satterthwaite method. The 95% confidence interval (CI) for random variation around each fixed effect was calculated as ± 1.96 standard deviations of its accompanying random variance term.

Although the six sessions were held over a period of 6-10 days, given that experience to the test (and not *time* per se) was the most likely reason for changes in response time, session was used as the metric of time (i.e., as opposed to age or day). Session was centered at the first occasion, such that the intercept represented initial status in all models. Observed mean response times (in milliseconds) estimated from a saturated means model (i.e., multivariate analysis of variance) are shown in Figure 1. The intraclass correlation from the unconditional means model (i.e., empty model; random intercept only) was calculated as .82, indicating that over 80% of the variance in number match 3 across sessions occurred between persons in mean RT. Polynomial models were then estimated to approximate the effects of practice across the six sessions, as presented below.

Polynomial Models

Polynomial models were first specified with a random intercept only. A fixed linear effect of session was significant ($p < .001$), such that average response time declined across sessions. The addition of a random linear slope (as well as a covariance between the random intercept and random linear slope) resulted in a significant improvement to the model, −2ΔLL(2) = 43, $p < .001$. However, the magnitude of this linear decline was reduced in later sessions, as indicated by a significant fixed quadratic effect of session (i.e., a decelerating negative trend; $p < .001$). The addition of a random quadratic slope (and its two accompanying covariances with the random intercept and random linear slope) also resulted in a significant improvement in model fit, −2ΔLL(3) = 39, $p < .001$.

The predicted means from the unconditional random quadratic polynomial model for session (i.e., without predictors) are shown in Figure 1, and model parameters using REML estimation are given in Table 1. As shown, the mean predicted response time at session 1 was 1946 ms, with a 95% CI of 916 to 2976 ms. The mean instantaneous linear rate of change at session 1 was −121 ms per session, with a 95% CI of −436 to 194 ms, indicating that not all participants were predicted to improve as evaluated at session 1. Half the mean deceleration in linear rate of change was 14 ms per session, such that the linear rate of change became less negative by 28 ms with each session. The 95% CI for the quadratic effect was of −36 to 63 ms, indicating that not all participants were predicted to decelerate in their rate of improvement across sessions.

Final Model Equation

Level 1: $$y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti} - 1) + \beta_{2i}(\text{Session}_{ti} - 1)^2 + e_{ti}$$

Level 2:
$$\beta_{0i} = \gamma_{00} + U_{0i}$$
$$\beta_{1i} = \gamma_{10} + U_{1i}$$
$$\beta_{2i} = \gamma_{20} + U_{2i}$$