Lecture 4: Describing Other Kinds of Within-Person Change and Fluctuation

- **Piecewise Models of Change**
- Negative Exponential Models of Change
- Alternative Covariance Structure Models
- Choosing Among Unconditional Longitudinal Models

Longitudinal Data: Describing Within-Person Change

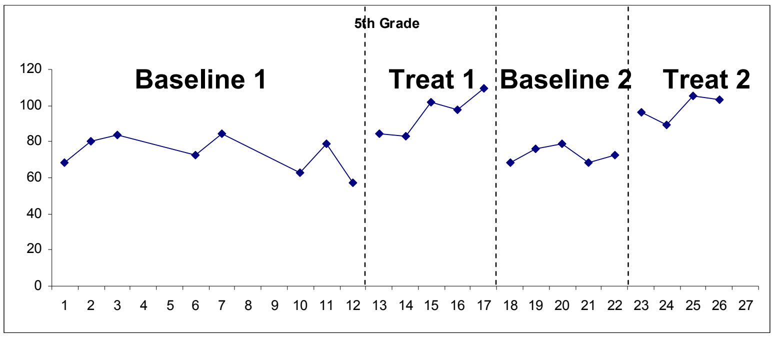
- We have two tasks in describing the effects of "time":

1. **Choose a Model for the Means**
   - What kind of change in the outcome do we have on average?
   - What kind of and how many parameters (fixed effects) do we need to represent that change as parsimoniously but accurately as possible?

2. **Choose a Model for the Variances**
   - Which fixed effects of change should be allowed to vary over persons?
   - This translates into: What kind of pattern do the variances and covariances of the outcome show over time?
   - What kind of and how many parameters (random effects) do we need to represent that pattern as parsimoniously but accurately as possible?

Other Families of Random Effects Models of Change

- **Piecewise models**: Discrete slopes for discrete phases of time
  - Separate terms describe sections of overall trajectories
  - Useful for examining change in intercepts and slopes before/after discrete events (changes in policy, interventions)
  - **Must know where the break point is ahead of time!**



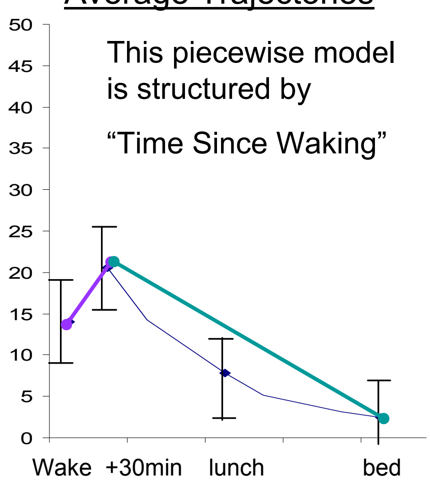
Piecewise Model:

4 slopes
(one per phase)

3 "jumps"
(shift in intercept between phases)

Example of Daily Cortisol Fluctuation: Morning Rise and Afternoon Fall

Average Trajectories



SAS Code to create two piecewise slopes from continuous time of day in stacked data:

```
IF occasion=1 THEN DO;
    P1=0;               P2=0; END;
IF occasion=2 THEN DO;
    P1= time2-time1; P2=0; END;
IF occasion=3 THEN DO;
    P1= time2-time1; P2=time3-time2; END;
IF occasion=4 THEN DO;
    P1= time2-time1; P2=time4-time2; END;
```

Note that a quadratic slope may be necessary for the afternoon fall slope!

Example: Random 2-Piece Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}Slope1_{ti} + \beta_{2i}Slope2_{ti} + e_{ti}$

Level 2 Equations (one per β):

$\beta_{0i} = \gamma_{00} + U_{0i}$

- β_{0i}: **Intercept** person i
- γ_{00}: Mean **Intercept**
- U_{0i}: Random **Intercept** Deviation

$\beta_{1i} = \gamma_{10} + U_{1i}$

- β_{1i}: **Slope1** person i
- γ_{10}: Mean **Slope1**
- U_{1i}: Random **Slope1** Deviation

$\beta_{2i} = \gamma_{20} + U_{2i}$

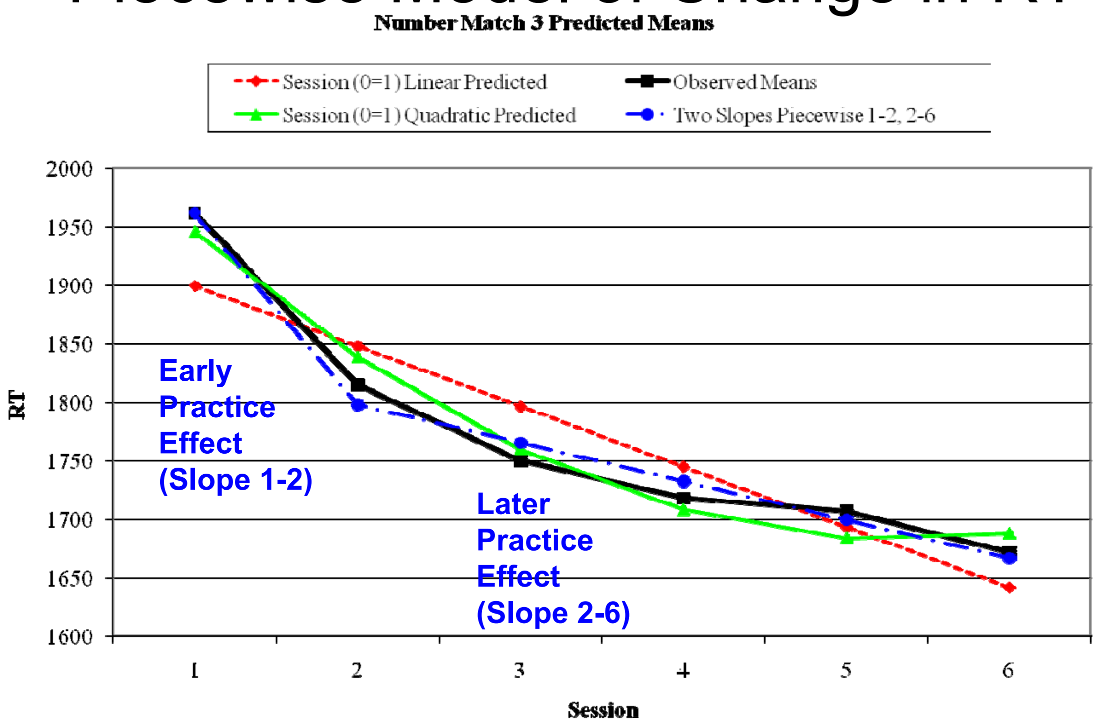
- β_{2i}: **Slope2** person i
- γ_{20}: Mean **Slope2**
- U_{2i}: Random **Slope2** Deviation

Fixed Effect Subscripts:
1st = which L1 term
2nd = which L2 term

Also need 4 occasions to fit a random 2-piece model

2 Fixed slopes (n – 1)
2 Random slopes (n – 2)

Piecewise Model of Change in RT



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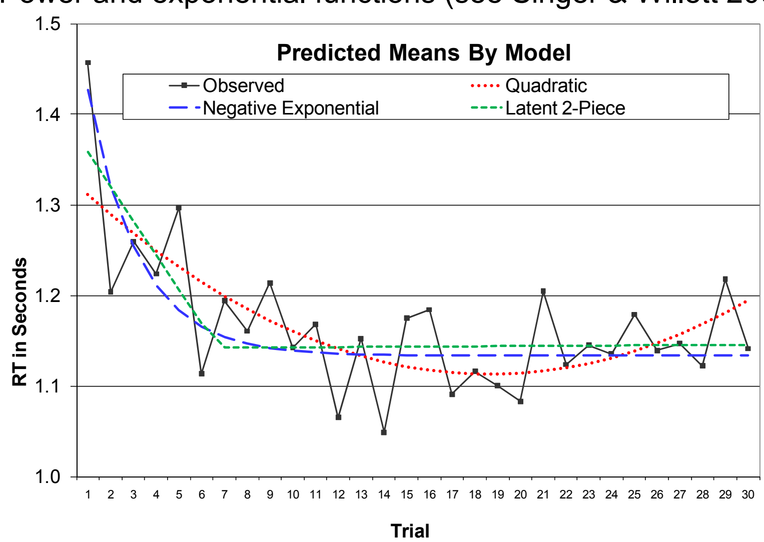
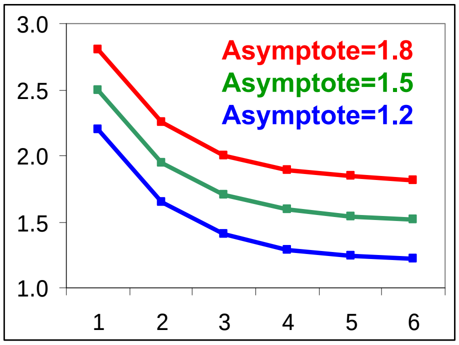
Other Families of Random Effects Models of Change

- **Truly nonlinear models**: Non-additive terms to describe change
  - Models can include asymptotes (so change can "shut off")
  - Power and exponential functions (see Singer & Willett 2003 ch. 6)



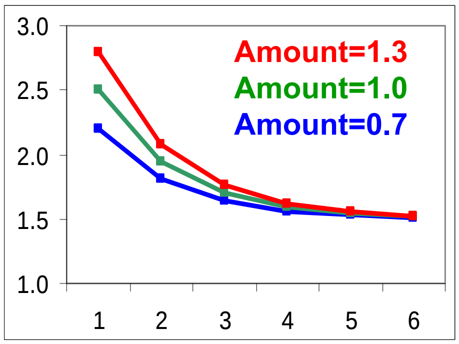
Negative Exponential Model

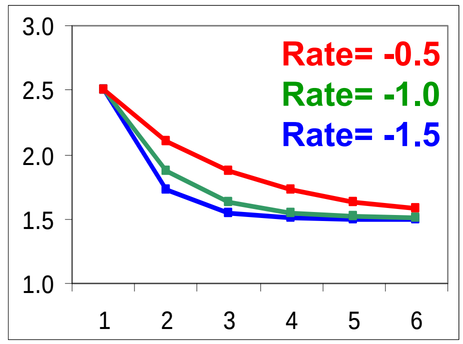
1)


1) Different Asymptotes, same amount and rate

2) Different Amounts, same asymptote and rate

3) Different Rates, same asymptote and amount

2)


3)


Example: Negative Exponential Model
(3 Random Effects)

<u>Level 1</u>: $\mathbf{y_{ti} = \beta_{0i} + \beta_{1i}{*}exp(\beta_{2i}{*}Time_{ti}) + e_{ti}}$

<u>Level 2 Equations (one per β):</u>

$\beta_{0i} = \gamma_{00} + U_{0i}$

Asymptote person i — Mean **Asymptote** — Random **Asymptote** Deviation

$\beta_{1i} = \gamma_{10} + U_{1i}$

Amount person i — Mean **Amount** — Random **Amount** Deviation

$\beta_{2i} = \gamma_{20} + U_{2i}$

Rate person i — Mean **Rate** — Random **Rate** Deviation

Fixed Effect Subscripts:
1st = which L1 term
2nd = which L2 term

Also need 4 occasions to fit this random exp model

2 Fixed slopes (*n* – 1)
2 Random slopes (*n* – 2)

(Likely need way more to get a good U_{2i}, though)

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- **Alternative Covariance Structure Models**
- Choosing Among Unconditional Longitudinal Models

What if I don't have change?

- Longitudinal studies are not always designed to examine systematic change (e.g., daily diary studies)
- In reality, there is a continuum of *fluctuation* to *change*:

Design	Time is irrelevant	Time was not supposed to be relevant (but is)	Time is the study purpose
Result	**Pure WP Fluctuation**	**Hybrid**	**Pure WP Change**
Means Model for Time	Empty	Fixed Slopes of Time	Fixed Slopes of Time
Variance Model for Time	Alternative Covariance Structures	Alternative Covariance Structures?	Random Slopes of time

Big Picture Modeling Framework:
Choices for Modeling Variance

Compound Symmetry (CS)

$$\begin{bmatrix} \sigma_e^2+\tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \sigma_e^2+\tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \tau_{u_0}^2 & \sigma_e^2+\tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 & \sigma_e^2+\tau_{u_0}^2 \end{bmatrix}$$

Most useful model: likely somewhere in between!

Unstructured (UN)

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}^2 & \sigma_{43} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44}^2 \end{bmatrix}$$

Parsimony ⟷ Good fit

Univariate RM ANOVA

NAME ...THAT … STRUCTURE!

Multivariate RM ANOVA

What is the pattern of variance and covariance over time?

CS and UN are just two of the many, many options available within MLM, including ***random effects models*** (for change) and ***alternative covariance structure models*** (for fluctuation).

About ACS Models

- Useful in predicting patterns of variance and covariance due to the outcome fluctuation across time:
  - **Variances**: Same (homogeneous) or different (heterogeneous)?
  - **Covariances**: Same or different? What kind of different?
  - Often don't need any fixed effects for time in the model for the means (although this is always an empirical question)
  - Can be used with or without a random intercept variance in **G**
- Limitations for most of these models:
  - Require **equal-interval** occasions (based on idea of "time lag")
  - Require **balanced** time across persons (no intermediate values)
  - But do not require complete data (unlike when estimated via least squares instead of ML/REML as in ANOVA)

R-Only Alternative Covariance Structures: **Autoregressive R Matrix**

1st Order Auto-Regressive = AR(1)

- 2 parms = 1 total variance, 1 correlation
- Correlation for t-1 diagonal
- Correlation squared for t-2 diagonal
- Correlation cubed for t-3 diagonal

$$\begin{bmatrix} \sigma_T^2 & & & \\ r^1 & \sigma_T^2 & & \\ r^2 & r^1 & \sigma_T^2 & \\ r^3 & r^2 & r^1 & \sigma_T^2 \end{bmatrix}$$

1st Order Auto-Regressive Heterogeneous = ARH(1)

- Each total variance estimated separately
- Correlations estimated same as above

$$\begin{bmatrix} \sigma_{11}^2 & & & \\ r^1 & \sigma_{22}^2 & & \\ r^2 & r^1 & \sigma_{33}^2 & \\ r^3 & r^2 & r^1 & \sigma_{44}^2 \end{bmatrix}$$

***Note: Use TYPE=SP(POW)(time) to get AR(1) for unbalanced time*

R-Only Alternative Covariance Structures: **Toeplitz (# Bands) R Matrix**

Toeplitz = TOEP(4) → 1 var + 3 lags

- 4 parms = 1 total variance, then:
  - Covariance for t-1 diagonal
  - Another covariance for t-2 diagonal
  - Another covariance for t-3 diagonal
  - Off-diagonal given as covariances

$$\begin{bmatrix} \sigma_T^2 & & & \\ c_1 & \sigma_T^2 & & \\ c_2 & c_1 & \sigma_T^2 & \\ c_3 & c_2 & c_1 & \sigma_T^2 \end{bmatrix}$$

Toeplitz Heterogeneous = TOEPH(4)

- Each total variance estimated separately
- Off-diagonal given as correlations instead (because covariance changes if variances change over time, but correlation doesn't)

$$\begin{bmatrix} \sigma_{11}^2 & & & \\ r_1 & \sigma_{22}^2 & & \\ r_2 & r_1 & \sigma_{33}^2 & \\ r_3 & r_2 & r_1 & \sigma_{44}^2 \end{bmatrix}$$

Alternative Covariance Structures that Combine **G** and **R** into **V**

- Modeling WP variation traditionally involves using the **R** matrix only (no **G**) → Total BP + WP variance shoved into just the **R** (=**V**) matrix
  - Correlations would still be expected even at distant lags because of constant individual differences (i.e., the BP random intercept)
  - Resulting model may need lots of parameters in it to account for that (e.g., 8 time points? Pry need a 7-lag Toeplitz model)

- **Why not take out the primary reason for the correlation across occasions (the random intercept) and see what's left?**
  - Put a random intercept in **G** → control for individual differences in level
  - THEN structure just the **residual** variance in **R**, not the **total** variance
  - Resulting model may be more parsimonious (e.g., maybe only lag1 or lag2 occasions are still related after removing the random intercept)
  - Has the advantage of still distinguishing BP from WP variance (useful for descriptive purposes and for calculating effect sizes)

New Baseline Model for **G** and **R** Alternative Structures

- **Random Intercept + *n*-1 order Unstructured Model: G and R = V**
  - Because you can't have everything (unstructured R) AND a random intercept, *you have to shut off the highest-lag covariance*
  - For 4 occasions: RI in G (RANDOM), **TYPE=UN(3)** (REPEATED)
  - This is equivalent to no **G**, TYPE=UN in **R** (REPEATED)
  - Random intercept provides covariance for any 0 element in **R**
  - Can see what the **residual** variance and covariance looks like directly

G matrix | **R matrix** | **Total V matrix**

$$\begin{pmatrix} \tau^2_{U_0} \end{pmatrix} \text{ “+” } \boxed{\begin{matrix} \sigma^2_{e11} & & & \\ \sigma_{21} & \sigma^2_{e22} & & \\ \sigma_{31} & \sigma_{32} & \sigma^2_{e33} & \\ 0 & \sigma_{42} & \sigma_{43} & \sigma^2_{e44} \end{matrix}} \longrightarrow \begin{bmatrix} \sigma^2_{11} & & & \\ \sigma_{21} & \sigma^2_{22} & & \\ \sigma_{31} & \sigma_{32} & \sigma^2_{33} & \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma^2_{44} \end{bmatrix}$$

Examples of Random Intercept + Residual Correlation (&/or H) in R

- **Random Intercept + AR1:** ***can also have ARH(1)***

3 parms:
Var(U_{0i})
Var(e_{ti})
AR correlation

G matrix $\begin{pmatrix} \tau^2_{U_0} \end{pmatrix}$ "+" **R matrix** $\begin{bmatrix} \sigma^2_e & & & \\ r^1 & \sigma^2_e & & \\ r^2 & r^1 & \sigma^2_e & \\ r^3 & r^2 & r^1 & \sigma^2_e \end{bmatrix}$

This **RI + AR(1)** model says that the **residual** variance has an auto-correlation, not the **total** variance.

- **Random Intercept + TOEP(*n*-1):** ***can also have TOEPH(n-1)***

4 parms:
Var(U_{0i})
Var(e_{ti})
r_1, r_2

G matrix $\begin{pmatrix} \tau^2_{U_0} \end{pmatrix}$ "+" **R matrix** $\begin{bmatrix} \sigma^2_e & & & \\ r_1 & \sigma^2_e & & \\ r_2 & r_1 & \sigma^2_e & \\ 0 & r_2 & r_1 & \sigma^2_e \end{bmatrix}$

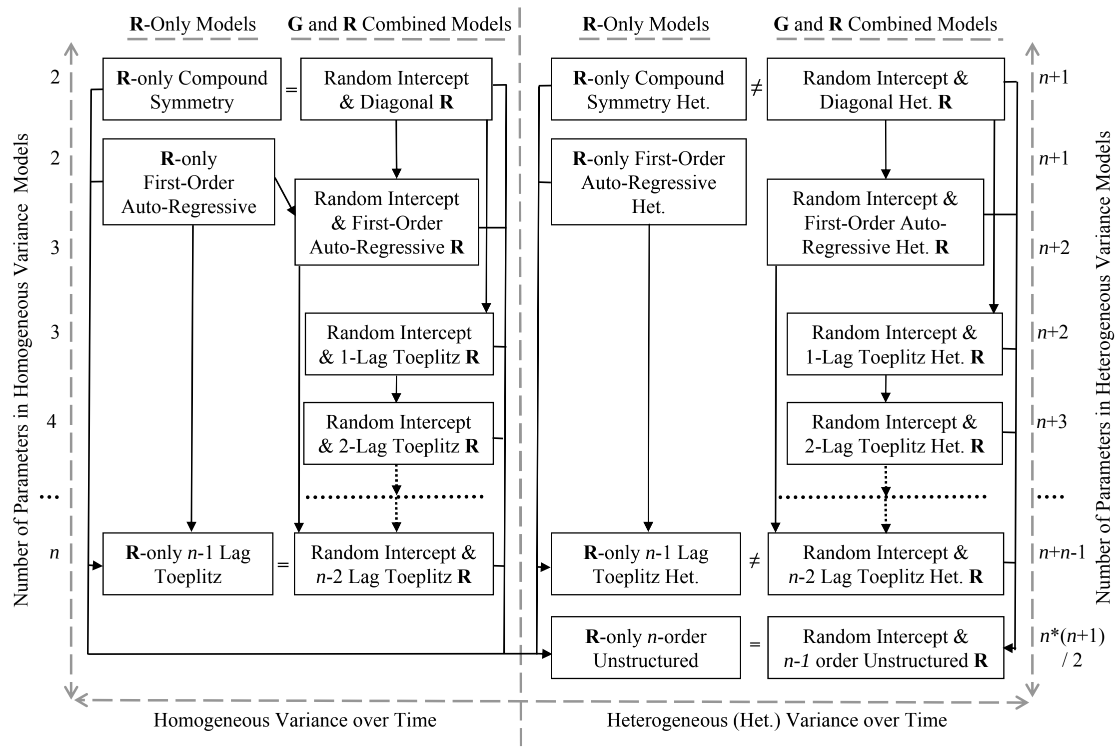
This **RI + TOEP(3)** model is equivalent to **NO RI + TOEP(4)** → must remove last lag (can't estimate r_3 because only one piece of info is available for it).

The reason to use a random intercept + TOEP is that the random intercept might make the more distant lags unnecessary, and thus your model could become more parsimonious (i.e., could include fewer than all possible lags).

Representative Model Sequence

- Optional: test for random intercept first
- **Baseline** models (given below either without **G** / with **G**):
  - Most parsimonious: Compound Symmetry **R** / Random Intercept + VC **R**
  - Best-fitting: *n*-order Unstructured **R** / RI + *n*-1 order Unstructured **R**
- From there, examine the **unstructured** R and RCORR matrices
  - What pattern do the **correlations/residual correlations** show over time?
    - Auto-regressive: Sharp decay across lags?
    - Toeplitz: Each lag gets its own correlation?
    - See SAS manual for a gazillion other kinds (fewer available in SPSS)
  - Do the **variances/residual variances** differ over time?
    - Homogeneous matrix: All variances same over time?
    - Heterogeneous matrix: All variances different over time?
- Try some **alternatives**, examine model fit
  - Goal: Get as close to unstructured **R** using the fewest parameters

Organization of **R**-only and **G** and **R** combined Models



Stuff to Watch Out For…

- **If using a random intercept, don't forget to drop 1 parameter:**
  - ***n*-1 order UN R**: Can't get everything in **R**, plus something else in **G**
  - **TOEP(*n*-1)**: Have to eliminate last lag covariance/correlation
  - No RI + CS **R**: Can't get a constant in **R**, and then another constant in **G**
- If "**time**" is treated as **continuous** in the fixed effects, you will need another variable for **time** that is **categorical** to use in the syntax:
  - "Continuous Time" → on MODEL/FIXED statement
  - "Categorical Time" → on CLASS/BY and REPEATED statements
- Most alternative covariance structure models assume **time is balanced across persons with equal intervals across occasions**
  - If not, holding correlations of same lag equal doesn't make sense
  - Other structures can be used for unbalanced time
    - SP(POW)(time) = AR(1) for unbalanced time
    - See SAS manual REPEATED statement for others

Programming Models for Variances

- Working with **random effects** models?
  - Use the **RANDOM** statement → **G matrix**
  - Random intercepts (and slopes) to model interindividual differences
  - **G** MATRIX IS ALWAYS UNSTRUCTURED → TYPE(UN)
- Working with **alternative covariance structure** models?
  - Use the **REPEATED** statement → **R matrix**
    - WITH a RANDOM statement: **R** is level-1 only
      → Residual variance and covariances to model intraindividual variation
    - WITHOUT a RANDOM statement: **R** is level-2 and level-1 together
      → Total variances and covariances (to model all BP and WP variation together)
- The **REPEATED** statement is **always** there implicitly (default = diagonal)
  - Any model **always** has at least one residual variance in **R** matrix
- But the **RANDOM** statement is only there if you write it
  - **G** matrix isn't always necessary (don't always need random effects)

Comparing Alternative Covariance Structures

- Some models are nested… can do -2ΔLL test
  - Everything is nested under unstructured (going for "not worse")
  - Homogeneous variances nested within heterogeneous variances
  - Different lags: e.g., 5-lag Toeplitz nested within 6-lag Toeplitz
  - CS and AR1 are both nested within Toeplitz
  - **R**-only models nested within **G** and **R** combined models
    - Add random intercept variance? Test using $\chi^2(1) > 2.71$ for $p < .10$
    - Otherwise, test at $p < .05$ (heterogeneity tests are not on boundary)
- Some models are non-nested… compare AIC and BIC
  - CS vs. AR1 is non-nested (different correlation parameter)
- When in doubt, just compare AIC and BIC
  - Still useful even when doing formal -2ΔLL tests

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- **Choosing Among Unconditional Longitudinal Models**

Summary of Steps in Unconditional Longitudinal Modeling

For all outcomes:

1. Empty Model; Calculate ICC
2. Decide on a metric of time
3. Decide on a centering point
4. Estimate means model and plot individual trajectories

If your outcome shows systematic change:

5. Evaluate fixed and random effects of time
6. Still consider possible alternative models for the residuals (**R** matrix)

If your outcome does NOT show ANY systematic change (shows fluctuation instead):

5. Evaluate alternative models for the variances (**G**+**R**, or **R**)

1. Estimate an Empty Means, Random Intercept Only Model

- Not really predictive, but is a useful statistical baseline model
  - Baseline model fit
  - Partitions variance into between- and within-person variance

- Calculate **ICC** = between / (between + within variance)
  = Average correlation between occasions
  = Proportion of variance that is between persons

- Tells you where the action will be:
  - If most of the variance is **between-persons (level 2)**, you will need **person-level predictors** to reduce that variance (i.e., to account for **inter-individual differences**)
  - If most of the variance is **within-persons (level 1)**, you will need **time-level predictors** to reduce that variance (i.e., to account for **intra-individual differences**)

2. Decide on the Metric of *Time*

- "Occasion of Study" as *Time*:
  - Can be used generically for many purposes
  - Include age, time to event as predictors of change
- "Age" as *Time*:
  - Is equivalent to time-in-study if same age at beginning of study
  - Implies age convergence → that people only differ in age regardless of when they came into the study (BP effects = WP effects)
- "Distance to/from an Event" as *Time*:
  - Is appropriate if a distinct process is responsible for changes
  - Also implies convergence (BP effect = WP effects)
  - Only includes people that have experienced the event

- Make sure to **represent actual time metric** if uneven intervals between follow-up occasions and/or variance across persons in distance between follow-up occasions

3. Decide on a Centering Point

- How to choose: At what time point would you like a snap-shot of inter-individual differences?
  - Intercept variance represents inter-individual differences at that particular time point (that you can later predict!)
- Where do you want your intercept?
  - Re-code time such that the centering point = 0
  - Multiple variants could be used (e.g., moving snapshot analysis)
  - Can always request fixed effects evaluated at other points in time
- Different centerings of time will produce statistically equivalent models with re-arranged parameters
  - i.e., expected level and rate of change *at the centering point*

4. Estimate Saturated Means Model and Plot Individuals

- If time is balanced across persons:
  - Estimate a saturated means model to generate means
- If time is NOT balanced across persons (i.e., continuous ages):
  - Create a ‘rounded’ variable (i.e., round age into even intervals, like 9.5 to 10.5 = 10) to estimate means model ONLY
  - Still use exact time/age variable for analysis!
- Plot the means – what kind of trajectory do you see?
- **ML/REML estimated means** per time point may NOT be the same as the **observed means** (i.e., as outputted from DESCRIPTIVES). The estimated means are what would have been obtained had your data been complete, whereas observed means are not adjusted to reflect any missing data. Report the ML/REML estimated means.

5. and 6. for **Systematic Change**:
Evaluate Fixed and Random Effects of Time

Model for the Means:

- What kind of fixed effects of time are needed to parsimoniously represent the observed means across time points?
  - Linear or nonlinear? Continuous or discontinuous? Need to shut off?
  - Polynomials? Pieces? Nonlinear curves?
  - Use obtained *p*-values to test significance of fixed effects

Model for the Variances (focus primarily on G):

- What kind of random effects of time are needed:
  - To account for individual differences in aspects of change?
  - To describe the variances and covariances over time?
  - Do the residuals show any pattern after accounting for random effects?
  - Use (REML) -2ΔLL test to test significance of new effects

Random Effects Variance Models

- Where does the correlation or 'dependency' go?
  Into a new variance component (or 'pile of variance')
- **Example 2-level longitudinal model:**

FIXED effects make variance go away. **RANDOM** effects just make a new pile of variance.

Now we get to add predictors to reduce the piles!

Level 1 (one source of) Within-Person Variation:

Gets accounted for by time-level predictors

Residual Variance (σ_e^2)

Level 2 (two sources of) Between-Person Variation:

Gets accounted for by person-level predictors

BP Int Variance (τ_{U0}^2)

BP Slope Variance (τ_{U1}^2)

τ_{U01} covariance

5. for **NO Systematic Change**:
Evaluate Alternative Covariance Structures

Model for the Means:

- Be sure you don't need any terms for systematic effects of time
- If not, keep a fixed intercept only

Model for the Variances (focus primarily on R):

- How many parameters are needed to predict variances/covariances?
- I recommend the hybrid: Random Intercept in **G** + Structure in **R**
  - Separates BP and WP variance
  - Likely more parsimonious than just an **R** structure by itself
- Compare alternative models with the same fixed effects
  - Nested? (REML) -2ΔLL test for significance
  - Non-nested? (REML) AIC and BIC for "supporting evidence"

Alternative Covariance Structure Models

- Models for variation typically include only a covariance structure, and at most a random intercept (random slopes for time won't help)

Between-Person Random Intercept in G + Within-Person Structure in R

Level 1 (one source of) Within-Person Variation:

Gets accounted for by time-level predictors

Residual Variance (σ_e^2)

Level 2 (one sources of) Between-Person Variation:

Gets accounted for by person-level predictors

BP Int Variance (τ_{U0}^2)

TOTAL Structure in R

All sources of variation and covariation are contained in one matrix, but dependency is still being modeled so it's ok.

Total Variance (σ_T^2)

Why spend so much effort evaluating unconditional models of time? The reasoning…

- The fixed effects of time are what the random effects of time are varying around…
- The random effects of time comprise the variances the person-level predictors are accounting for…
- The effects of person-level predictors are specified as a function of the time parameters already in the model…
- The effects of time-varying predictors are supposed to account for variance not accounted for by the model for time…
- **What fixed and random time effects of time you include in the model constrain what is to be predicted.**
- **There is little point in trying to predict individual differences in change (and intraindividual deviation from predicted change) when it's possible that those individual differences (and deviations) only exist because the model for change is mis-specified. Get *time* right first!**

Conditional Models Up Next…

- Adding predictors to 2-level longitudinal models:
  - Time-Invariant Predictors (Level-2)
  - Time-Varying Predictors (Level-1)
- 2-level clustered models (e.g., people in groups)
  - Group-Level Predictors (Level-2)
  - Person-Level Predictors (Level-1)
  - 2-level cross-classified models (multiple level-2s)
- 3-level clustered longitudinal models
  - Time within Person within Group