Lecture 8: Multilevel Models for Crossed Observations

- Multilevel models are specified based on the relevant dimensions by which observations differ each other

- Two-level models… (at least) two kinds of variance:
  - Longitudinal Data: Time nested within Person
  - Students nested within Teachers
  - Patients within Doctors

- Three-level models… (at least) three kinds of variance:
  - Time nested within Person within Family
  - Student nested within Teacher within Schools
  - Patients within Doctors within Hospitals

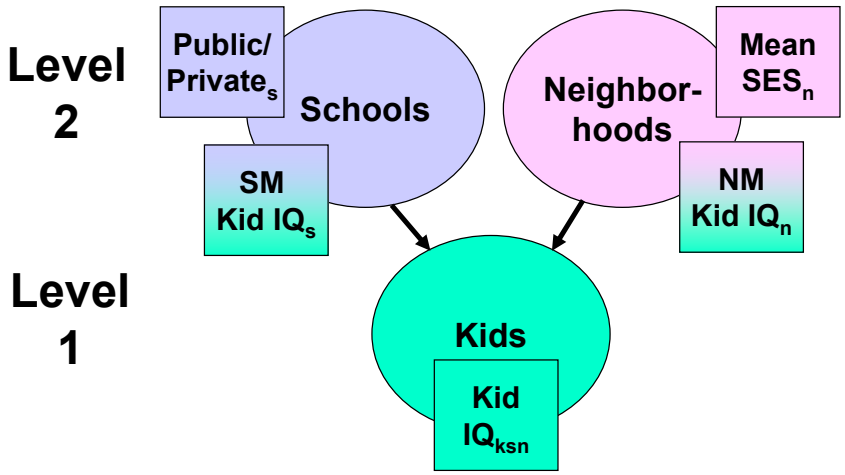
Nested vs. Crossed Groups…

- In other designs, multiple sources of systematic variation may be present, but not strictly nested…
  - They may be **crossed** instead of nested
  - Crossed sources of systematic variation (i.e., dependency in the residuals) still need to be accounted for in the model somehow...
    - Otherwise will have the same problems as when not properly accounting for nesting (messed up SE's and *p*-values)

- Same idea as crossed random effects (i.e., as we had for persons and items), but these are known as "cross-classified" models in the clustered data world
  - Here are a few examples…

Kids, Schools, and Neighborhoods

- Kids are nested within schools AND within neighborhoods
- Not all kids from same neighborhood live in same school, so schools and neighborhoods are **crossed** at level 2
- Can include predictors for each source of variation



Kids, Schools, and Neighborhoods

The composite equation for that model:

$$y_{ksn} = \gamma_{000} \quad \rightarrow \textit{fixed intercept (all x's = 0)}$$
$$+ \gamma_{010}(\text{Private}_s) + \gamma_{020}(\text{SMIQ}_s) \quad \rightarrow \textit{school effects}$$
$$+ \gamma_{001}(\text{SES}_n) + \gamma_{002}(\text{NMIQ}_n) \quad \rightarrow \textit{neighborhood effects}$$
$$+ \gamma_{100}(\text{KidIQ}_{ksn}) \quad \rightarrow \textit{kid effects}$$
$$+ U_{0s0} \quad \rightarrow \textit{random effect of school}$$
$$+ U_{00n} \quad \rightarrow \textit{random effect of neighborhood}$$
$$+ e_{ksn} \quad \rightarrow \textit{residual kid-to-kid variation}$$

Time, Kids, and Classrooms

- If kids move into different classrooms across time…
  - Time at level 1 is nested within Kid and Classroom, Kid is crossed with Classroom at level 2
- How to model a time-varying random classroom effect?
  - This is the basis of so-called "value-added models"

- (At least) Two options:
  - **Temporary classroom effect:** Random effect for classroom that operates only at the point when the kid is in that classroom
    - e.g., Classroom effect ← teacher bias
    - Once out of classroom, effect is no longer present
  - **Cumulative classroom effect:** Random effect for classroom that operates at the point when the kid is in that classroom forwards
    - e.g., Classroom effect ← differential learning
    - Effect stays with the kid in the future

More on Cross-Classified Models

- In crossed models one can have random slopes of lower-level predictors over higher levels AND random slopes of the other crossed factor at the same level
  - Example: Kids, Schools, and Neighborhoods (data permitting)
    - Kid effects could vary over schools AND/OR neighborhoods
    - School effects could vary over neighborhoods (both level 2)
    - Neighborhood effects could vary over schools (both level 2)

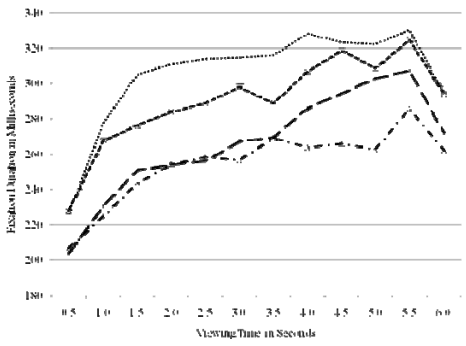
- Concerns about smushing still apply over both level-2's
  - Separate contextual effects of kid predictors for schools and neighborhoods (e.g., after controlling for how smart you are, it matters incrementally whether you go to a smart school AND if you live in a neighborhood with smart kids)

Repeated Measures Designs: *ANOVA works well when…*

- Experimental stimuli are ***controlled*** and ***exchangeable***
  - **Controlled** → Constructed, not sampled from a population
  - **Exchangeable** → Stimuli vary only in dimensions of interest
  - …What to do with non-exchangeable stimuli (e.g., words, scenes)?
- Experimental manipulations create ***discrete*** conditions
  - e.g., set size of 3 vs. 6 vs. 9 items
  - e.g., response compatible vs. incompatible distractors
  - …What to do with continuous item variables (e.g., time, salience)?
- One has ***complete data***
  - e.g., if outcome is RT and accuracy is near ceiling
  - e.g., if responses are missing for no systematic reason
  - …What if data are not missing completely at random (e.g., inaccuracy)?

Eye Tracking Example: *Subjects Crossed with Items*

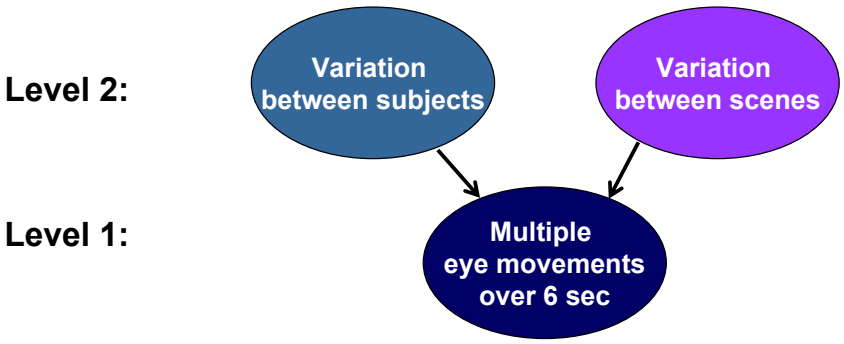
- Do eye movements depend on the purpose of looking at a scene?
  - DVs: Fixation duration in msec, saccadic amplitude in degrees
  - 4 subject groups: Free-view, Memorize, Rate Pleasantness, n/z Search
    Each of the 53 subjects viewed the same 67 scenes for 6 sec
  - Original analysis: Split-plot ANOVA (task by chopped-up viewing time)
    - Average over scenes and average within "time"
    - "time" = 20 conditions



from Mills et al., 2011

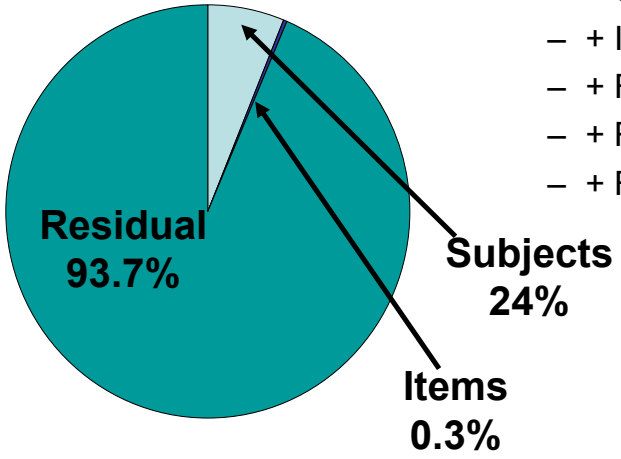
Eye Tracking Example:
Subjects Crossed with Items

- Revised analysis: Growth curve modeling of eye movements
  - Eye movements at level 1 nested within scenes and within subjects
  - Scenes and subjects are crossed random effects at level 2
  - Subject predictor = which task they did, no scene predictors
  - Level-1 predictor = viewing time (with random effects over persons)



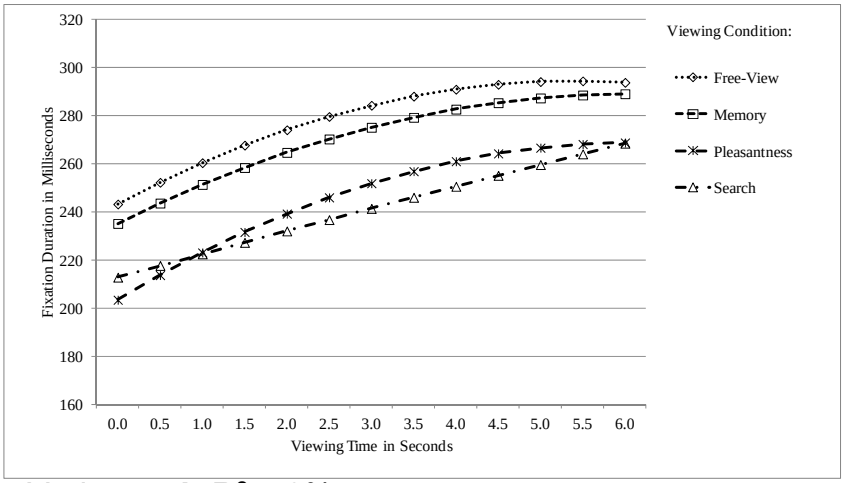
Eye Tracking Example:
Subjects Crossed with Items

Crossed Random Effects Empty Model for Fixation Duration



- Model sequence:
  - Empty (e-only)
  - + Subject Random Intercept
  - + Item Random Intercept
  - + Fixed Effects of Time
  - + Random Effects of Time
  - + Fixed Effects for Task*Time

Eye Tracking Example:
Subjects Crossed with Items



- Items Variance → $R^2 \approx 0\%$
- Subjects Intercept Variance → $R^2 \approx 32\%$
- Subjects Linear Time Slope Variance → $R^2 \approx 3\%$
- Residual Variance → $R^2 \approx 0\%$
- Each source remained significant, however

Summary:
Nested or Crossed Models

- Dimensions of sampling can result in systematic differences (i.e., dependency) that needs to be accounted for in the model for the variances
  - Sometimes this dependency is from nested sampling
  - Sometimes this dependency is from crossed sampling
- Multilevel models that include crossed random effects (or cross-classified models):
  - Can address this dependency (statistical motivation)
  - Can quantify and predict the amount of variation due to each source (substantive motivation)
  - Can include simultaneous hypothesis tests pertaining to each source of variation (substantive motivation)