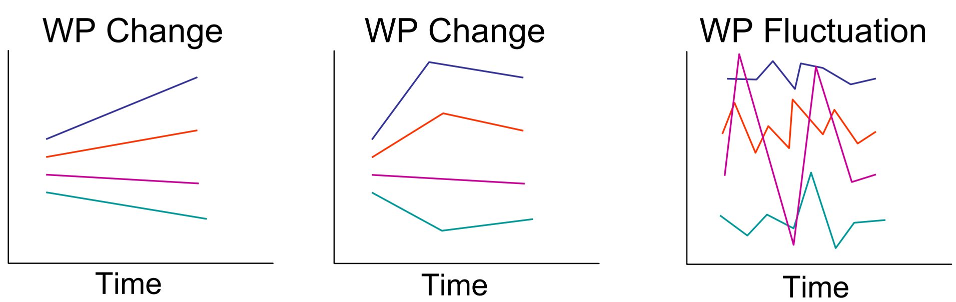
Lecture 3: Describing Within-Person Change via Polynomial Models

- **Big Picture: Modeling Means and Variances**
- Polynomial Fixed and Random Effects Models
- Fun with Model Comparisons and Effect Size
- Unconditional Polynomial Models in SAS, SPSS, and STATA

Modeling Within-Person Change vs. Within-Person Fluctuation



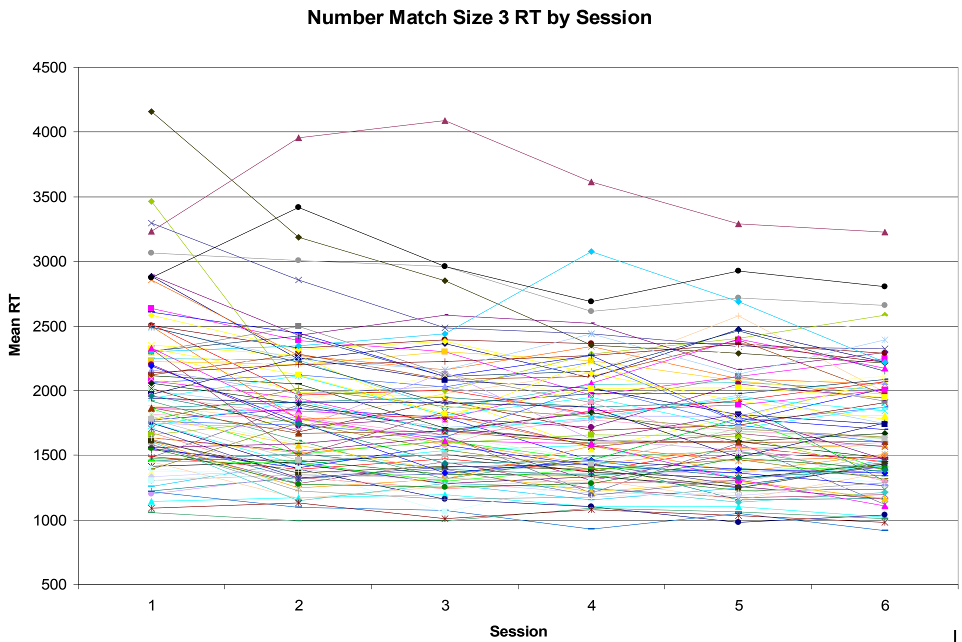
Model for the Means:

- **WP Change** → describe pattern of *average* change (over "time")
- WP Fluctuation → *may* not need anything (if no systematic change)

Model for the Variances:

- **WP Change** → describe *individual differences* in change (random effects)
  → this allows variances and covariances to differ over time
- WP Fluctuation → describe pattern of variances and covariances over time

Example Data Individual Observed Trajectories (N = 101, n = 6)



Describing Within-Person Change: *Model for the Means (Fixed Effects)*

- What kind of change occurs on average?
  - **What is the most appropriate metric of time?**
    - Time in study *(with predictors for BP differences in 'time')*?
    - Time since birth *(age)*? Time to event *(time since diagnosis)*?
    - Measurement occasions need not be the same across persons or equally spaced (use exact time when possible)
  - **What kind of population model generated the observed trajectories?**
    - Linear or nonlinear? Continuous or discontinuous?
    - Many options: Polynomial, piecewise, & nonlinear families

Big Picture Modeling Framework: *Choices for Modeling Means*

- **What kind of change on average (in the means)?**
  - "**Empty**" refers to model for the means with no predictors (just fixed intercept for **grand mean** outcome over time)
  - "**Saturated**" refers to model for means with all possible means estimated (#parameters = #occasions) → **THIS IS ANOVA**
    - Is a DESCRIPTION of the means, not a predictive MODEL

Parsimony

<u>**Empty Means:**</u>
Predicts NO change over time
1 Fixed Effect

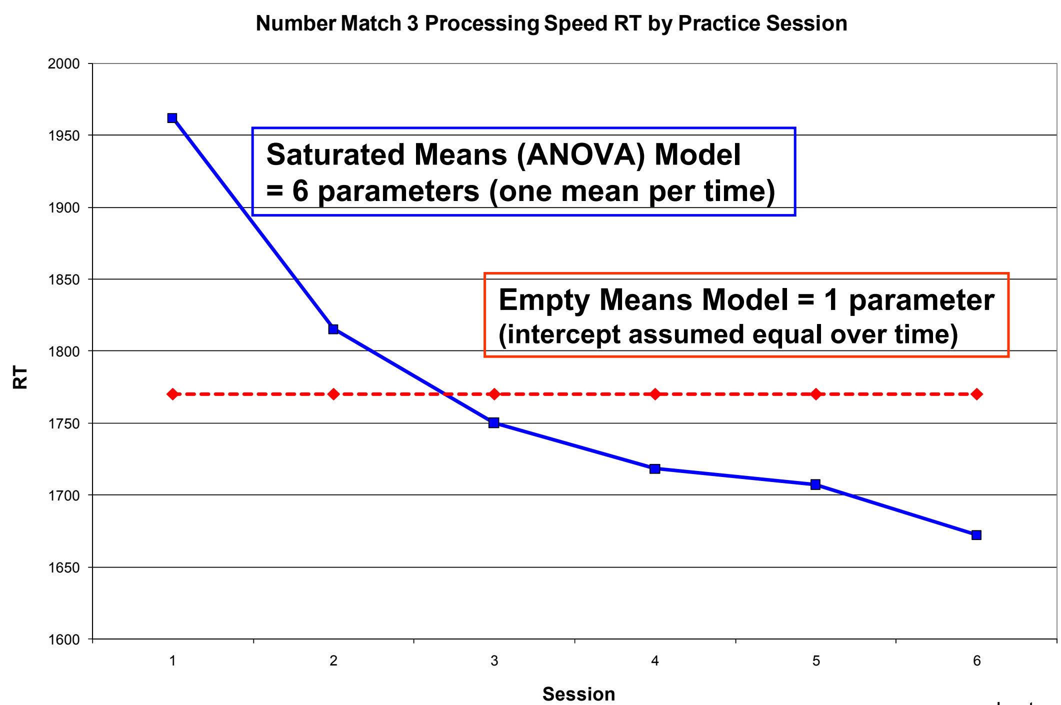
<u>In-between options:</u>
polynomial slopes, piecewise slopes, nonlinear models…

<u>**Saturated Means:**</u>
Reproduces mean at each occasion
 # Fixed Effects = # Occasions

Good fit

Name… that… Trajectory!

Baseline Fixed Effects Models



Describing Within-Person Change:
Model for the Variance (Random Effects)

- ***From a substantive perspective:***
  Are there individual differences in change?
  – Individual differences in the level of an outcome?
    - At what time point are individual differences in outcome level important for your hypotheses (beginning, middle, end)?

  – Individual differences in magnitude of change?
    - Each aspect of change (e.g., linear change, quadratic change) can potentially exhibit individual differences (data permitting)

- ***From a statistical perspective:* What kind of pattern do the variances and covariances exhibit over time?**
  – Do the variances increase or decrease over time?
  – Are the covariances for closer occasions more related?

Choices for Modeling Variance

- The partitioning of variance into piles…
  – Level 2 = BP → **G** matrix of random effects variances/covariances
  – Level 1 = WP → **R** matrix of residual variances/covariances
  – **G** and **R** combine to create **V** matrix of total variances/covariances
  – Many flexible options that allows the variances and covariances to vary in a time-dependent way that better matches the actual data

Compound Symmetry (CS)

$$\begin{bmatrix} \sigma_e^2+\tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \sigma_e^2+\tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \tau_{u_0}^2 & \sigma_e^2+\tau_{u_0}^2 & \tau_{u_0}^2 \\ \tau_{u_0}^2 & \tau_{u_0}^2 & \tau_{u_0}^2 & \sigma_e^2+\tau_{u_0}^2 \end{bmatrix}$$

MLM uses **G** and **R** to predict something in-between!

Unstructured (UN)

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}^2 & \sigma_{43} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44}^2 \end{bmatrix}$$

Parsimony ←→ **Good fit**

Univariate RM ANOVA

NAME …THAT … STRUCTURE!

Multivariate RM ANOVA

Building **V**: Two Major Families of Models for the Variances

- **Random Effects ("Multilevel") models:**
  - Useful for studies of within-person **change** ("growth curves")
  - Focus is on how many BP variance piles (intercepts, slopes) should go into **G** so that **R** is diagonal (e_{ti} residuals are uncorrelated with homogeneous variance over time)
  - Usually, we augment **G**, but **R** is diagonal (but that is testable)

- **Alternative Covariance Structure (ACS) models:**
  - Useful for studies of within-person **fluctuation**
    - If there is no change, then random time slopes won't be useful
    - BUT – variances and covariances may still differ over time
  - Traditionally, focus is on how to structure **R** (**G** is not used)
    - However, I often recommend **R** + random intercept in **G** instead

Programming Models for Variance

- Does your model include r**andom intercept variance (for U_{0i})** ?
  - Use the **RANDOM** statement → **G matrix** (always unstructured)
  - The random intercept then models level-2, BP differences in mean Y
- What about **residual variance (for e_{ti})** ?
  - Use the **REPEATED** statement → **R matrix**
    - **WITH a RANDOM statement: R is WP variance only**
      → Residual variance and covariances to model WP intraindividual variation
      → **G** and **R** put back together = **V matrix** of total variances and covariances
    - **WITHOUT a RANDOM statement: R is BOTH BP and WP variance still**
      → Total variances and covariances (to model all variation, so **R** = **V**)

- The **REPEATED** statement is **always** there implicitly (default = diagonal)
  - Any model always has at least one residual variance in **R** matrix
- But the **RANDOM** statement is **only** there if you write it
  - **G** matrix isn't always necessary (don't always need random effects)

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Longitudinal Data: Modeling Means and Variances

- We have two tasks in describing the effects of “time”:

1. **Choose a Model for the Means**
   - What kind of change in the outcome do we have on average?
   - What kind of and how many parameters do we need to represent that change as parsimoniously but accurately as possible?

2. **Choose a Model for the Variances**
   - What kind of pattern do the variances and covariances of the outcome show over time?
   - What kind of and how many parameters do we need to represent that pattern as parsimoniously but accurately as possible?

Name that Trajectory with Polynomial Fixed Effects of Time

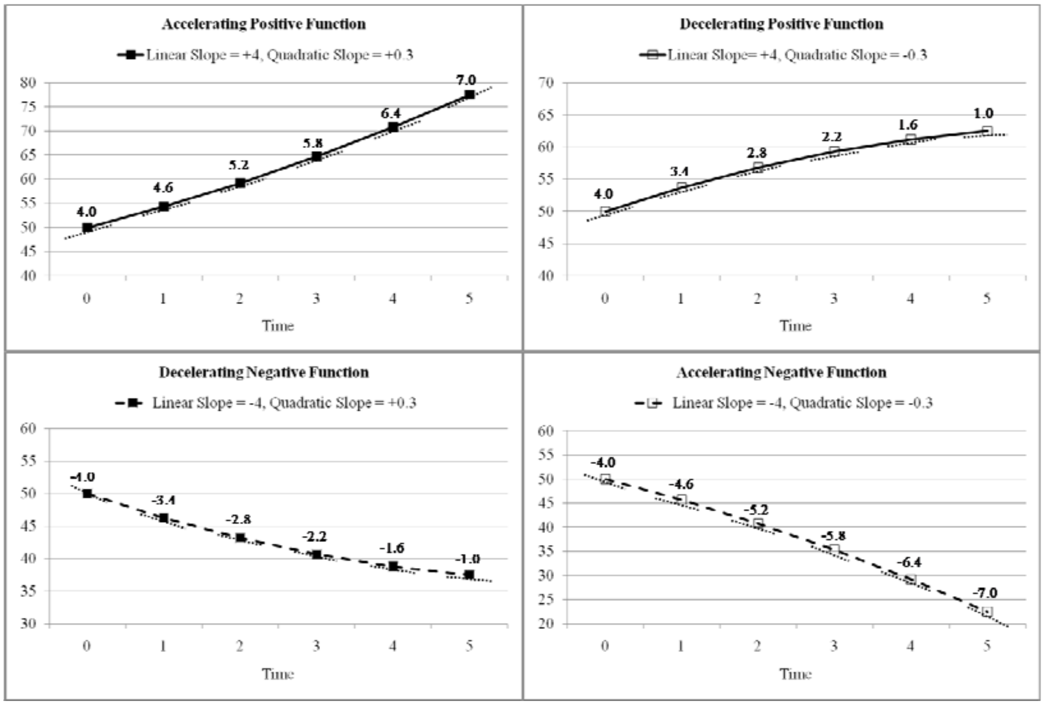
- Represent **mean** patterns of change with polynomial fixed effects:
  - Linear → *constant* amount of change (up or down)
  - Quadratic → *change* in linear rate of change (acceleration/deceleration)
  - Cubic → *change* in acceleration/deceleration in linear rate of change
  - Terms work together to describe curved trajectories
- Can have polynomial **fixed** slopes UP TO: # occasions – 1*
  - 3 occasions = 2nd order = Fixed Quadratic or less
  - 4 occasions = 3rd order = Fixed Cubic or less
- Interpretable polynomials past cubic are rarely seen in practice – just because you *could* estimate higher orders doesn't mean you *should*!
- *This rule can be broken in unbalanced data (but cautiously)

Interpreting Quadratic Coefficients

A Quadratic effect is a two-way interaction: time*time

- Fixed quadratic = **half** the rate of acceleration/deceleration
- **So twice the quadratic coefficient is how the linear time slope itself changes per unit time** (→ more/less positive/negative)
- If linear trend = 4 at time 0, with quadratic = 0.3?
  - Instantaneous linear rate of Δ at time 0 = 4.0, at time 1 = 4.6…
- The "twice" part comes from the derivatives with respect to time:
  - Original function → $y = 50 + 4(\text{time}) + .3(\text{time})^2$
  - Linear rate of Δ = 1st derivative → $y' = \_\_ + 4 + \mathbf{.6}(\text{time})$
  - Slope of rate of Δ = 2nd derivative → $y'' = \_\_ + \_\_ + \mathbf{.6}$
- Put another way: Because time is interacting with itself, there is no second 'main effect' in the model. So the quadratic effect gets applied to time twice, given that there is only one main effect of time.

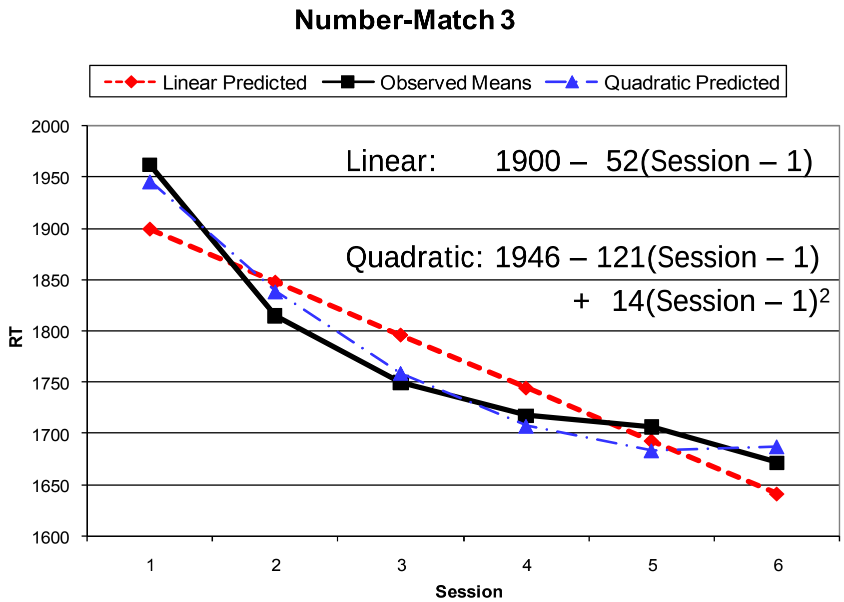
Examples of Fixed Quadratic Models



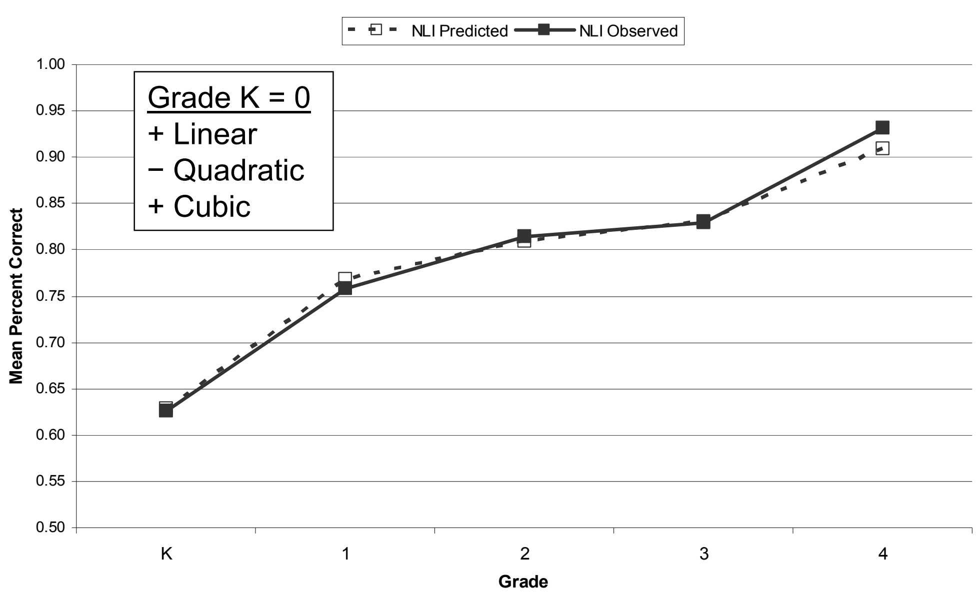
Lower-Order *Fixed* Effects are Conditional on Higher-Order *Fixed* Effects

- You can put the intercept (set time=0) anywhere you want, and you will get the *same model fit and predicted values*
- Different centerings of time will yield models with different estimates and interpretations for the *lower-order terms*, however:
  - ***Fixed Intercept Only?***
    - Fixed Intercept = predicted mean of Y *for any occasion (= grand mean)*
  - ***Add Fixed Linear?***
    - Fixed Intercept = **now** predicted mean of Y from linear time *at time=0*
    - Fixed Linear = mean linear rate of change *across all occasions*
  - ***Add Fixed Quadratic?***
    - Fixed Intercept = still predicted mean of Y *at time=0* (but now as predicted from quadratic time instead of linear time)
    - Fixed Linear = **now** mean linear rate of change *at time=0*
    - Fixed Quadratic = half the mean rate of acceleration or deceleration of change *across all occasions* (i.e., the linear slope changes the same over time)

Example Data:
Observed vs. Model-Predicted Means



Real-Life Example: Fixed Cubic Model
Growth of Grammar Skills in Kids with Language Impairments



Polynomial Fixed vs. Random Effects

- Polynomial **fixed effects** combine to describe **sample mean trajectories** over time (***fixed*** **slopes up to #occasions – 1**):
  - Fixed Intercept = Predicted **mean** level (at time 0)
  - Fixed Linear = **Mean** linear rate of change (at time 0)
  - Fixed Quadratic = Half of **Mean** acceleration/deceleration in linear rate of change (2*quad is how the linear time slope changes per unit time)

- Polynomial **random effects** (individual deviations from the sample mean for each change parameter) describe **individual differences** in those change parameters (***random*** **slopes up to #occasions – 2**):
  - Random Intercept = **BP Variance** (indiv diffs) in level (at time 0)
  - Random Linear = **BP Variance** (indiv diffs) in linear time slope (at time 0)
  - Random Quadratic = **BP Variance** (indiv diffs) in half the rate of acceleration/deceleration of linear time slope

Lower-Order *Random* Effects are Conditional on Higher-Order *Random* Effects

- As with fixed effects, different centerings will yield equivalent models with different estimates and interpretations for the *lower-order terms*:
  - ***Random Intercept Only?***
    - Random Intercept = individual differences *for any occasion* in predicted mean Y (= variance in grand mean because individual lines are parallel)
  - ***Add Random Linear?***
    - Random Intercept = **now** individual differences *at time=0* in predicted mean Y
    - Random Linear = individual differences *across all occasions* in linear rate of change *(would be the same if time was centered elsewhere)*
  - ***Add Random Quadratic?***
    - Random Intercept = still individual differences *at time=0* in predicted mean Y
    - Random Linear = **now** individual differences *at time=0* in linear rate of change *(would be different if time was centered elsewhere)*
    - Random Quadratic = individual differences *across all occasions* in half of accel/decel of change *(would be the same if time was centered elsewhere)*

Example: Random Quadratic Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}Time_{ti} + \beta_{2i}Time_{ti}^2 + e_{ti}$

Level 2 Equations (one per β):

$\beta_{0i} = \gamma_{00} + U_{0i}$

- β_{0i}: **Intercept** person i
- γ_{00}: Mean **Intercept**
- U_{0i}: Random **Intercept** Deviation

$\beta_{1i} = \gamma_{10} + U_{1i}$

- β_{1i}: **Linear Slope** person i
- γ_{10}: Mean **Linear Slope**
- U_{1i}: Random **Linear Slope** Deviation

$\beta_{2i} = \gamma_{20} + U_{2i}$

- β_{2i}: **Quad Slope** person i
- γ_{20}: Mean **Quad Slope**
- U_{2i}: Random **Quad Slope** Deviation

Fixed Effect Subscripts:
1st = which L1 term
2nd = which L2 term

Number of Possible Slopes by Number of Occasions (*n*):

\# Fixed slopes = *n* – 1
\# Random slopes = *n* – 2

Need 4 occasions to fit this random quadratic model

Random Effects Allowed by #Occasions

	V Matrix		G Matrix	R Matrix
2 occasions = **3** unique pieces of information	$\begin{bmatrix} \sigma^2_{11} & \\ \sigma_{21} & \sigma^2_{22} \end{bmatrix}$	$\longrightarrow$	$\begin{pmatrix} \tau^2_{U_0} \end{pmatrix}$ **Random Intercept only**	$\left(\sigma^2_e\right)$ **Diagonal**: constant variance and no covariance across all times and people
3 occasions = **6** unique pieces of information	$\begin{bmatrix} \sigma^2_{11} & & \\ \sigma_{21} & \sigma^2_{22} & \\ \sigma_{31} & \sigma_{32} & \sigma^2_{33} \end{bmatrix}$	$\longrightarrow$	$\begin{pmatrix} \tau^2_{U_0} & \\ \tau_{U_{01}} & \tau^2_{U_1} \end{pmatrix}$ **Up to 1 Random slope**	$\left(\sigma^2_e\right)$ **Diagonal**: constant variance and no covariance across all times and people
4 occasions = **10** unique pieces of information	$\begin{bmatrix} \sigma^2_{11} & & & \\ \sigma_{21} & \sigma^2_{22} & & \\ \sigma_{31} & \sigma_{32} & \sigma^2_{33} & \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma^2_{44} \end{bmatrix}$	$\longrightarrow$	$\begin{pmatrix} \tau^2_{U_0} & & \\ \tau_{U_{01}} & \tau^2_{U_1} & \\ \tau_{U_{02}} & \tau_{U_{12}} & \tau^2_{U_2} \end{pmatrix}$ **Up to 2 Random slopes**	$\left(\sigma^2_e\right)$ **Diagonal**: constant variance and no covariance across all times and people

Predicted **V** Matrix from Polynomial Random Effects Models

- **Random linear model?** Variance has a **quadratic** dependence on time
  - Variance will be at a minimum when time = $-\text{Cov}(U_0,U_1)/\text{Var}(U_1)$, and will increase parabolically and symmetrically over time
  - **Predicted variance** at each occasion and covariance between A and B:

    $\text{Var}(y_{time}) = \text{Var}(e_t) + \text{Var}(U_0) + 2\text{Cov}(U_0,U_1)(\textbf{time}_t) + \text{Var}(U_1)(\textbf{time}_t^2)$

    $\text{Cov}(y_A,y_B) = \text{Var}(U_0) + \text{Cov}(U_0,U_1)(A + B) + \text{Var}(U_1)(AB)$
- **Random quadratic model?** Variance has a **quartic** dependence on time

  $\text{Var}(y_{time}) = \text{Var}(e_t) + \text{Var}(U_0) + 2\text{Cov}(U_0,U_1)(\textbf{time}_t) + \text{Var}(U_1)(\textbf{time}_t^2) + 2\text{Cov}(U_0,U_2)(\textbf{time}_t^2) + 2\text{Cov}(U_1,U_2)(\textbf{time}_t^3) + \text{Var}(U_2)(\textbf{time}_t^4)$

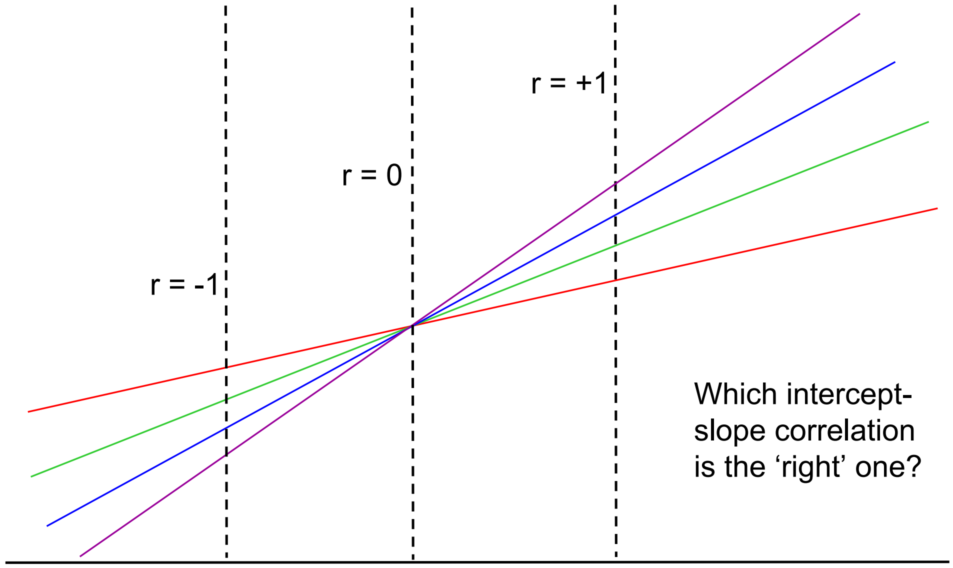
  $\text{Cov}(y_A,y_B) = \text{Var}(U_0) + \text{Cov}(U_0,U_1)(A + B) + \text{Var}(U_1)(AB) + \text{Cov}(U_0,U_2)(A^2 + B^2) + \text{Cov}(U_1,U_2)[(AB^2)+(A^2B)] + \text{Var}(U_2)(A^2B^2)$

- *The point of the story – random effects are a way of allowing the variances and covariances to differ over time in specific, time-dependent patterns (that result from differential change over time).*

More Rules for Polynomial Models
(and in general for fixed and random effects)

- On the same side of the model (means or variances side), lower-order effects stay in EVEN IF NONSIGNIFICANT (for correct interpretation)
  - e.g., Significant fixed quadratic? Keep the fixed linear
  - e.g., Significant random quadratic? Keep the random linear
- Also remember – you can have a significant random effect EVEN IF the corresponding fixed effect is not significant (keep it anyway):
  - e.g., Fixed linear not significant, but random linear is significant?
    → No change *on average*, but significant individual differences in change
- Language: A random effect supersedes a fixed effect:
  - If Fixed = intercept, linear, quad; Random = intercept, linear, quad?
    - Call it a “Random quadratic model” (implies everything beneath those terms)
  - If Fixed = intercept, linear, quad; Random = intercept, linear?
    - Call it a “Fixed quadratic, random linear model" (distinguishes no random quad)
- Intercept-slope correlation depends largely on centering of time

Correlation Between Intercept & Slope



!! Nonparallel lines will eventually cross.

Session Centered at 1:

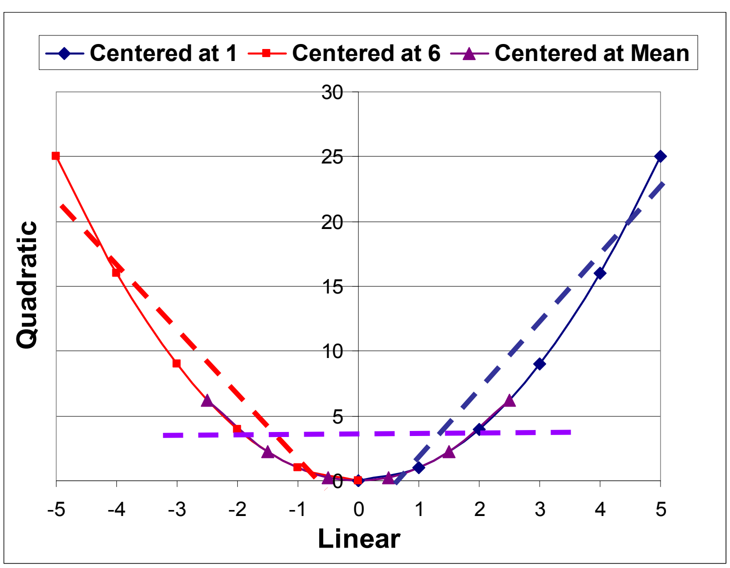
Session	Linear	Quadratic
1	0	0
2	1	1
3	2	4
4	3	9
5	4	16
6	5	25

Session Centered at 6:

Session	Linear	Quadratic
1	-5	25
2	-4	16
3	-3	9
4	-2	4
5	-1	1
6	0	0

Session Centered at Mean:

Session	Linear	Quadratic
1	-2.5	6.25
2	-1.5	2.25
3	-0.5	0.25
4	0.5	0.25
5	1.5	2.25
6	2.5	6.25



Correlations among growth terms can be induced by centering time near the start or near the end.

Correlations are *most* interpretable when centering time at its mean.

Interpreting Random Effects Variances
(see Snijders & Bosker, 1999 p. 48-50)

- -2ΔLL deviance difference tests (stay tuned!) will tell you whether or not a given random effect is significant…
  - In other words, if there are significant individual differences in that random effect (i.e., if everyone needs their own intercept or slope)
- But variance components aren't directly meaningful…
  - "I have a random intercept variance of 273,306.
    It's significant, but what does that actually *mean*? Is 273,306 a lot?"
- Interpret the magnitude of variance components by constructing *95% Random Effects Confidence Intervals*
  - Predicted range for that fixed effect under which 95% of the sample is predicted to fall, using the original metric of the variable

Interpreting Random Effects Variances

- **Random Effects 95% CI = fixed effect ± (1.96 * SQRT(variance))**
  → Predicted values assuming a normal, symmetric $\mathbf{U}_i$ distribution!!
  → If yours isn't normal, these can go out of bounds of the possible data
- Example: fixed intercept = 1946, random intercept variance = 273,306
  = 1946 ± (1.96 * SQRT(273,306)) → 1946 ± 1046 → 900 to 2992
  = 95% of my sample is predicted to have an individual intercept between 900 and 2992
  - Can do the same for each random effect (e.g., intercept, slopes…)
- NOT the same as a typical CI around the point estimate
  - Point estimate CI tells you what the **fixed effect** is predicted to be 95% of the time (describes uncertainty of point estimate for mean effect)
  - Random effects CI tells you the range of the fixed effect for 95% of the **individuals** in your sample (describes variation across people)

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Fun with Model Comparisons

- Relative model fit is indexed by a summary statistic → **-2LL deviance**
    - Log of likelihood (LL) of observing the data given model parameters, * -2 so that the differences between model LL values follow ~chi-square distribution
  - **-2LL Is a measure of BADNESS of fit, so smaller values are better**
  - Models are compared using their deviance values (significance tests)
  - Comes in two estimation flavors (labeled as -2 log likelihood on output):
    - Maximum Likelihood (**ML**) or Restricted (Residual) ML (**REML**)
- Fit is also indexed by **Information Criteria** that reflect deviance AND # parameters used and/or sample size
  - **AIC** = Akaike IC = Deviance + 2 *(#parms)
  - **BIC** = Bayesian IC = Deviance + log(N)*(#parms) → penalty for complexity
  - In ML → #parms = all parameters
  - In REML → #parms = variance model parameters only
  - No significance tests or critical values, just “smaller is better”

3 Decision Points in Conducting Model Comparisons

1. Are the models **nested** or **non-nested**?
   - Nested: have to add OR subtract effects to go from one to other
     - Can conduct significance tests for improvement in fit
   - Non-nested: have to add AND subtract effects
     - No significance tests available for these comparisons

2. Differ in model for the **means**, **variances**, or **both**?
   - Means? Can use ML -2LL tests to compare (or *p*-values of fixed effects)
   - Variances? Use -2LL tests (preferably REML, but ML is ok) to compare
   - Both sides? Can only use ML -2LL tests to compare

3. Models estimated using **ML** or **REML**?
   - ML: All model comparisons are ok
   - REML: Model comparisons ok for the variance parameters only

ML/REML -2ΔLL Deviance Difference Tests

(to-be-compared models must use the same estimator & *N*)

1. Calculate -2ΔLL: $-2LL_{fewer} - -2LL_{more}$
2. Calculate Δdf: $\#\,Parms_{more} - \#\,Parms_{fewer}$
3. Compare -2ΔLL to χ^2 distribution with df = Δdf
   - CHIDIST function in excel will give exact p-values for the difference test

- For fixed effects *p*<.05: -2ΔLL(1)>3.84, -2ΔLL(2)>5.99, -2ΔLL(3)>7.82
- Some controversy about deviance tests when testing random effects
  - χ^2 is not distributed as df because variances can't be negative
  - Is mixture χ^2 of df and df-1 instead, so χ^2 tests for df will be conservative

- Two proposed solutions when testing random effects:
  - Use 1-tailed test (χ^2 for $p < .10$) for a random intercept: -2ΔLL(1)>2.71
  - $\text{Mixture p-value} = 0.5 * \text{prob}(\chi^2_{df-1} > -2\Delta LL) + 0.5 * \text{prob}(\chi^2_{df} > -2\Delta LL)$
  - In practice these are a little suspect… so we'll use the conservative test

ML vs. REML Solutions

	ML	REML
To select, type…	METHOD=ML (-2 log likelihood)	METHOD=REML *default* (-2 res log likelihood)
Because it indexes the fit of the…	Entire model (means + variance)	Variance model only
You can compare models differing in…	Fixed and/or random effects (either/both)	Random effects only (same fixed effects)
However, it treats fixed effects as…	Known (df for fixed effects not factored in)	Unknown (df for fixed effects is factored in)
So, in small samples, variances will be…	Too small (no real diff after N=30-50 or so)	Unbiased

Rules for Comparing Multilevel Models:
ALL OBSERVATIONS MUST BE EXACTLY THE SAME ACROSS MODELS!!

Compare Models Differing In:

Type of Comparison:	**Means Model (Fixed) Only**	**Variance Model (Random, Res.) Only**	**Both Means and Variances Model (Fixed and Random)**
Nested? YES Signif Tests	*p*-values OR -2ΔLL in ML (not REML)	NO *p*-values -2ΔLL in REML (ML ok if big N)	-2ΔLL in ML (not REML)
Non-Nested? NO Signif Tests	ML AIC, BIC	REML AIC, BIC	ML AIC, BIC

Nested = one model is a direct subset of the other

Non-Nested = one model is not a direct subset of the other

Summary: Model Comparisons

- Significance of **fixed effects** can be tested with EITHER their ***p*-values** OR **ML** -2ΔLL deviance difference tests
  - *p*-value → Is EACH of these effects significant? (fine under ML or REML)
  - ML -2ΔLL test → Does this SET of predictors make my model better?
  - *REML deviance difference tests are WRONG for comparing fixed effects*
- Significance of **random effects** can only be tested with **-2ΔLL tests** (preferably using REML; here ML is not wrong, but results in too small variance components and fixed effect SEs in smaller samples)
  - Can get *p*-values as part of output but *shouldn't* use them
  - #parms added (df) should always include the random effect covariances
- My recommended approach to building models:
  - Stay in REML (for best estimates), test new fixed effects with *p*-values
  - THEN add new random effects, testing with the -2ΔLL test

Example Sequence for Testing Fixed and Random Polynomial Effects of Time

Build up fixed and random effects simultaneously:

1. Empty Means, Random Intercept → to calculate ICC
2. Fixed Linear, Random Intercept → check fixed linear *p*-value
3. Random Linear → check -2ΔLL(2) for random linear variance
4. Fixed Quadratic, Random Linear → check fixed quadratic *p*-value
5. Random Quadratic → check -2ΔLL(3) for random quadratic variance
6. …….

*** In general: Can use **REML** for all models, so long as you:

→ Test significance of new **fixed** effects by ***p*-values**

→ Test significance of new **random** effects in separate step by **-2ΔLL**

→ Also see if AIC and BIC are smaller when adding random effects

Effect Size for Unconditional Models in MLM

- Standardized fixed effects do not really exist in MLM
  - No clear basis for standardization, especially across levels
- Most common measure of effect size in MLM is Pseudo-R^2
  - Is supposed to be variance accounted for by predictors
  - Multiple piles of variance means multiple possible values of R^2!
  - Can calculate the proportion reduction in *each variance component*
  - Can calculate the proportion reduction at *each level*
- **Fixed effects** of time (e.g., linear, quadratic) will reduce the level-1 residual e_{ti} variance (σ_e^2) (but also level-2 intercept variance if time varies BP)
- By how much? $\text{Pseudo } R_e^2 = \dfrac{\text{residual variance}_{\text{fewer}} - \text{residual variance}_{\text{more}}}{\text{residual variance}_{\text{fewer}}}$

Effect Size for Unconditional Models

- However – as a consequence of reducing level-1 residual variance, the level-2 random intercept variance **(τ_{U0}^2) will usually INCREASE**
  - This is not really a problem, but here's how it happens:
- Observed **level-2 τ_{U0}^2** is NOT just between-person variance
  - Also has a small part of within variance (**level 1, σ_e^2**) due to sampling
    - → **observed τ_{U0}^2 ≈ true $\tau_{U0}^2 + (\sigma_e^2 / n)$**

      (as *n* occasions goes up, effect of level-1 σ_e^2 is minimized)
  - ML estimate of "true" τ_{U0}^2 uses this as a correction factor:
    - → **true τ_{U0}^2 ≈ observed $\tau_{U0}^2 - (\sigma_e^2 / n)$**
- Example: observed $\tau_{U0}^2 = 4.65$, $\sigma_e^2 = 7.06$, $n = 4$
  - So **true τ_{U0}^2** = 4.65 – (7.06 / 4) = **2.88**
  - Add fixed linear slope, reduce σ_e^2 to 2.17
    - Pseudo-R^2 for σ_e^2 = (7.06 – 2.17)/7.06 = .69, or 69% reduction in σ_e^2
  - But τ_{U0}^2 was not reduced, so **true τ_{U0}^2** = 4.65 – (2.17 / 4) = **4.10** now

Effect Size for Unconditional Models

- What about effect size for **random effects** of time?
  - Does not apply: fixed effects reduce variance, but random effects only **re-partition variance** (random effect = new pile of variance)
    - e.g., Add *fixed* linear time slope? Reduce σ_e^2
    - e.g., Add *random* linear time slope? **CREATE τ_{U1}^2**
    - σ_e^2 will appear to be reduced, but it's not really – just re-partitioned
  - Pseudo-R^2 is only calculable for models with same piles of variance
    - e.g., Add fixed quadratic to random linear model? Reduce σ_e^2
    - Interpret as reduction in residual variance *that was left after accounting for individual differences in linear change*
- How to get around these problems? Calculate **true R^2** instead as proportion of explained total outcome variance (one estimate)
  - Squared correlation of *predicted y from fixed effects* with *actual y*
  - Stay tuned…

Lecture 3: Describing Within-Person Change via Polynomial Models

- Big Picture: Modeling Means and Variances
- Polynomial Fixed and Random Effects Models
- Fun with Model Comparisons and Effect Size
- **Unconditional Polynomial Models in SAS, SPSS, and STATA**