Considerations in Selecting Amongst Alternative Metrics of Time

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Goals of Longitudinal* Modeling

- 5 rationales of longitudinal research
  - Baltes & Nesselroade, 1979
    - Chapter 1: *Longitudinal Research in the Study of Behavior and Development*
- 7 levels of longitudinal analysis
  - Hofer & Sliwinski, 2006
    - Chapter 2: *Handbook of the Psychology of Aging (6th edition)*
- 7+ steps in longitudinal modeling
  - Singer & Willett, 2003
    - Chapter 4: *Applied Longitudinal Data Analysis*

*Applicable to both the MLM and SEM analytic frameworks

Steps in Longitudinal Analysis

1. Decompose **BP and WP variation**—Intraclass Correlation
   - **ICC** = proportion of outcome variance that is *constant* over time, and that results from *cross-sectional* differences

3. Describe pattern of average change over time **(fixed effects)** and individual differences therein **(random effects)**
   - Piecewise slopes models—Phases of discontinuous change
   - Polynomial models—Approximate nonlinear continuous change
   - Truly nonlinear models—Exponential or logistic change
   - Latent basis models—Estimate shape of nonlinear change

Steps in Longitudinal Analysis

4. Predict **inter-individual differences** in change
   - *Why do people need their own intercepts and slopes?*
5. Predict **intra-individual variation** from predicted change
   - *Why are you off your line today (time-specific influences)?*
6. Examine **multivariate relationships**
   - *Between-person correlations among intercepts and slopes*
   - *Within-person covariation of residuals (or lead-lag associations)*
7. Examine other sources of underlying **heterogeneity**
   - *Mixture models for discrete types of individual differences*
   - *Predict individual differences in within-person variability*

Road Map

- Steps in longitudinal analysis
- **The missing step #2**
- Example: Alternative metrics of "time"
- What about just time?
- What else contributes to "time"?

The Missing Step 2

- **Summary across steps**: The goal of creating statistical models of change is to describe the overall pattern of and predict individual differences in **change over time**.

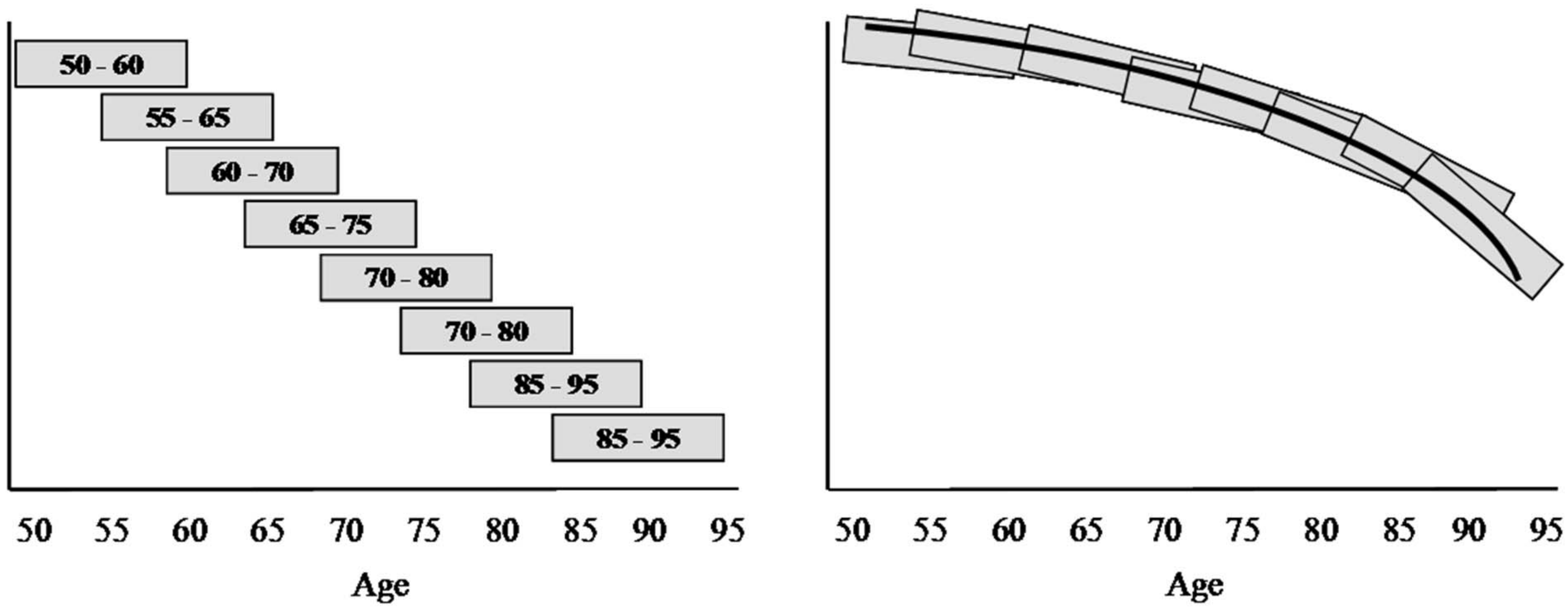
- These models employ an often unrecognized assumption **that we know exactly what "time" should be**.

- So the missing Step 2 is a pre-cursor to every other step in longitudinal analysis, and involves 2 related concerns:
  - **What should "time" be?**
  - **How should "time" be modeled when people differ in "time"?**
  - Concerns apply specifically to *accelerated longitudinal designs*

Accelerated Longitudinal Designs

Want to do a longitudinal study but just don't have the time?

Accelerate: Model trajectories over a wider span of time than would be possible using only the observed longitudinal info…



The Missing Step 2

- **First: What should "time" be?**
  - Which **metric of time** best matches the **causal process** thought to be responsible for observed change?
  - Do **alternative metrics of time** for **multiple processes** create different pictures of change and individual differences therein?
  - Relevant for aligning different persons onto same time trajectory, but this distinction is **not relevant within persons**

- **Second: What do we do when people differ in "time"?**
  - **How should "time" be modeled in accelerated designs?**
  - When people begin a study at different points in time, how should we distinguish effects of ***between-person differences*** in time from effects of ***within-person changes*** in time?

Road Map

- Steps in longitudinal analysis
- The missing step #2
- **Example: Alternative metrics of "time"**
- What about just time?
- What else contributes to "time"?

Example Data: *Octogenarian (Twin) Study of Aging*

- **173 persons (65% women)**
  - Measured up to **5 occasions** over 8 years
  - **Known** dates of birth and death
  - **Estimated** dates of dementia diagnosis (91 Alz., 50 Vas., 32 Mixed)

Correlation	**Age**	**Death**
Death	.23	
Dementia	.17	.52

- **Baseline occasion "time" variability:**
  - 79 to 100 years of age (M = 84, SD = 3)
  - −16 to 0 years from death (M = −6, SD = 4)
  - −12 to 18 years from diagnosis (M = 0, SD = 5)

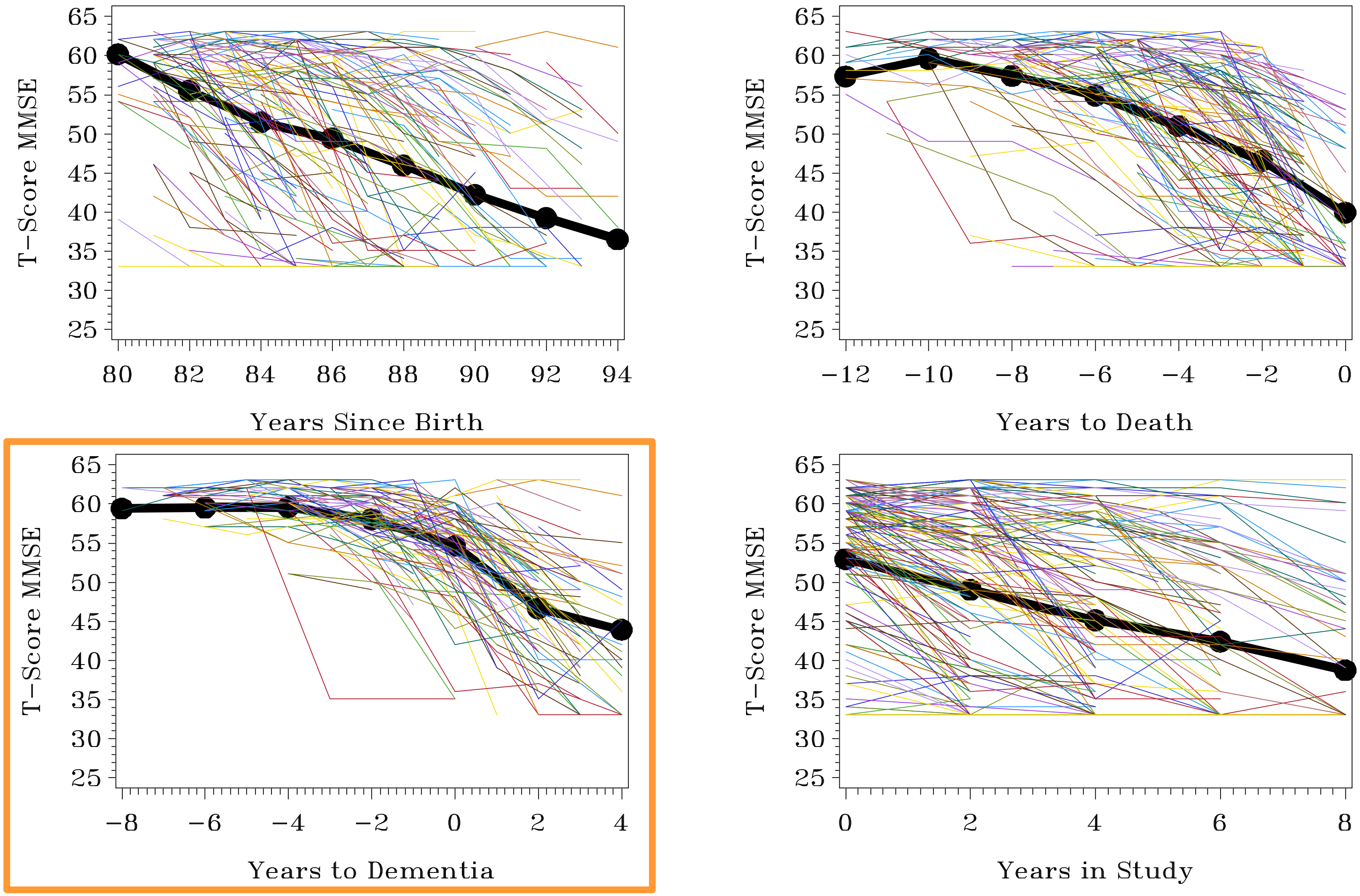
- **Cognition outcomes (each T-scored):**
  - General: Mini-Mental Status Exam
  - Memory: Object Recall
  - Spatial Reasoning: Block Design

#Persons per #Occasions				
1	**2**	**3**	**4**	**5**
28	37	36	36	35
29	31	39	29	18
37	32	31	22	19

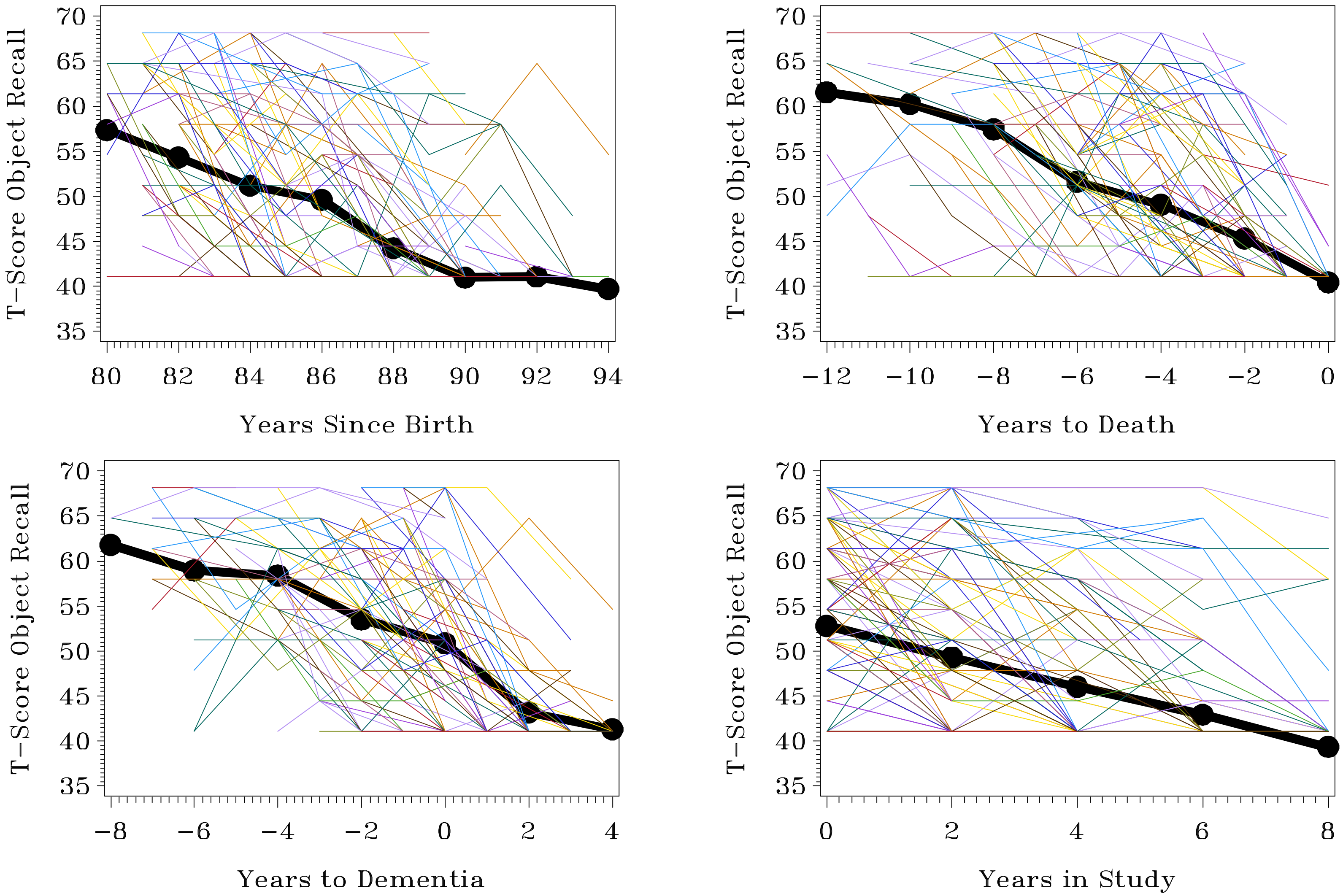
Alternative Metrics of Time (and ICC)

- Chronological Age as Time (47% BP)
  - Individual differences are organized around the mean level for a given **distance from birth** (84 years) and change with distance from birth
- Years to Death as Time (24% BP)
  - Individual differences are organized around the mean level for a given **distance from death** (−7 years) and change with distance from death
- Years to Dementia Diagnosis as Time (70% BP)
  - Individual differences are organized around the mean level for a given **distance from diagnosis** (=0) and change with distance from diagnosis
- Years in Study as Time (0% BP)
  - Individual differences are organized around the mean level for a given **distance from baseline** (=0) and change with distance from baseline

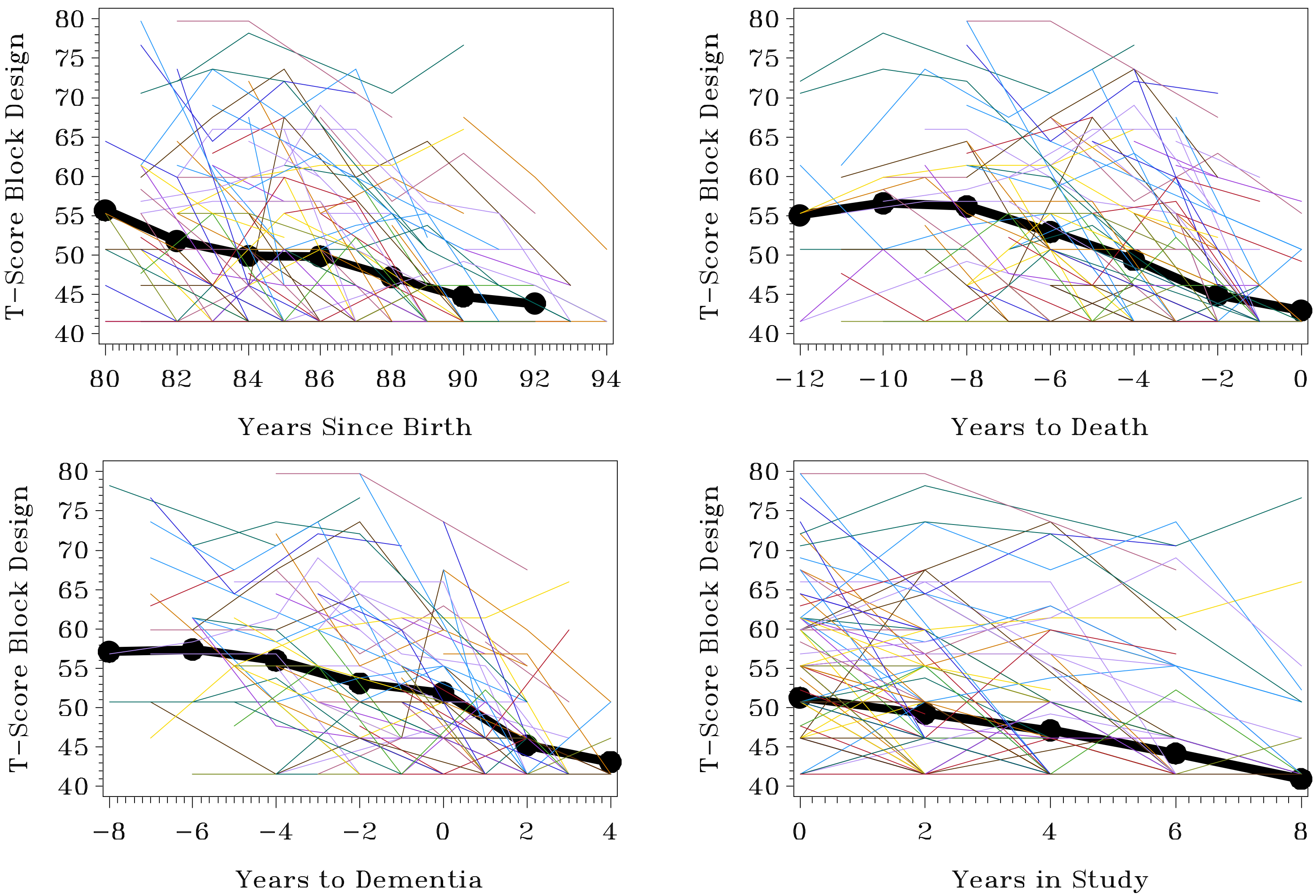
General Cognition: MMSE



Memory: Object Recall



Spatial Reasoning: Block Design



First Option: Age-as-Time

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + \beta_{2i}(Age_{ti} - 84)^2 + e_{ti}$

Level-2 Equations → Fixed and Random Effects:

$\beta_{0i} = \gamma_{00} + U_{0i}$ → predicted Y when age=84

- β_{0i}: **Intercept** for person *i*
- γ_{00}: Fixed **Intercept** (mean)
- U_{0i}: Random **Intercept** Deviation

$\beta_{1i} = \gamma_{10} + U_{1i}$ → rate of Δ when age=84

- β_{1i}: **Linear Slope** for person *i*
- γ_{10}: Fixed **Linear Slope** (mean)
- U_{1i}: Random **Linear Slope** Deviation

$\beta_{2i} = \gamma_{20} + U_{2i}$ → ½ rate of Δ in Δ per year

- β_{2i}: **Quad Slope** for person *i*
- γ_{20}: Fixed **Quad Slope** (mean)
- U_{2i}: Random **Quad Slope** Deviation

First Option: Age-as-Time

- If people differ in initial age, measuring change as a function of age requires assuming **age convergence:**
  - Younger people and older people differ *only* by age
  - Effects of between-person, **cross-sectional age differences** are equivalent to effects of within-person, **longitudinal age changes**

- Age convergence is not likely to hold when:
  - Initial **age range** is large (47% of age is BP here)
  - **Cohort** differences and **selection** effects are large

- Is exactly the same problem as not fully separating the BP and WP effects of **ANY** time-varying predictor
  - *i.e., conflated, convergence, composite, or smushed effect*

Examining Age Convergence Effects

Can use a variant of **grand-mean-centering** to test equivalence of BP and WP age effects empirically

Level-1 **Age-Based** Change:

$$y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + \beta_{2i}(Age_{ti} - 84)^2 + e_{ti}$$

Level-2 Equations:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(AgeT1_i - 84) + U_{0i}$$

$$\beta_{1i} = \gamma_{10} + \gamma_{11}(AgeT1_i - 84) + U_{1i}$$

$$\beta_{2i} = \gamma_{20} + \gamma_{21}(AgeT1_i - 84) + U_{2i}$$

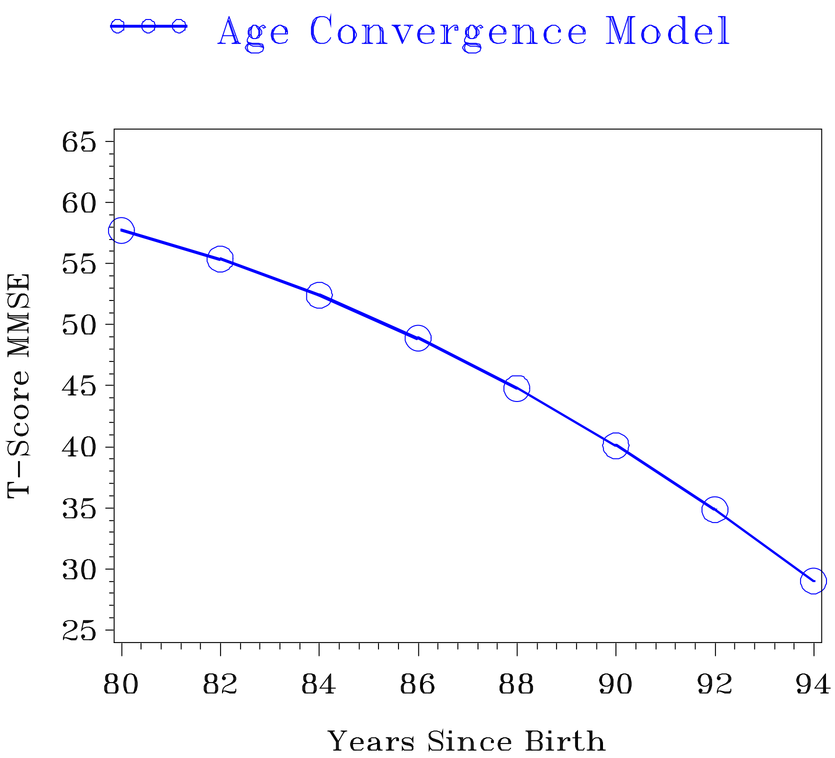
AgeT1 → Incremental effect of cross-sectional age (**cohort**)

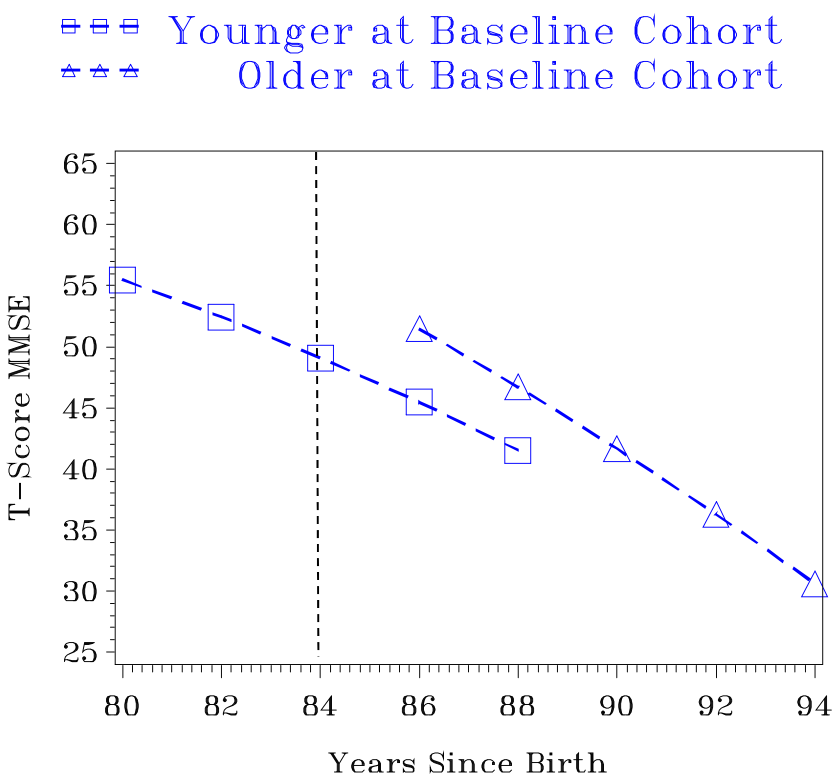
Use **age at time 1** (or birth year) instead of mean age to lessen bias from attrition-related missing data

Significance → Nonconvergence
It matters **WHEN** you were 84

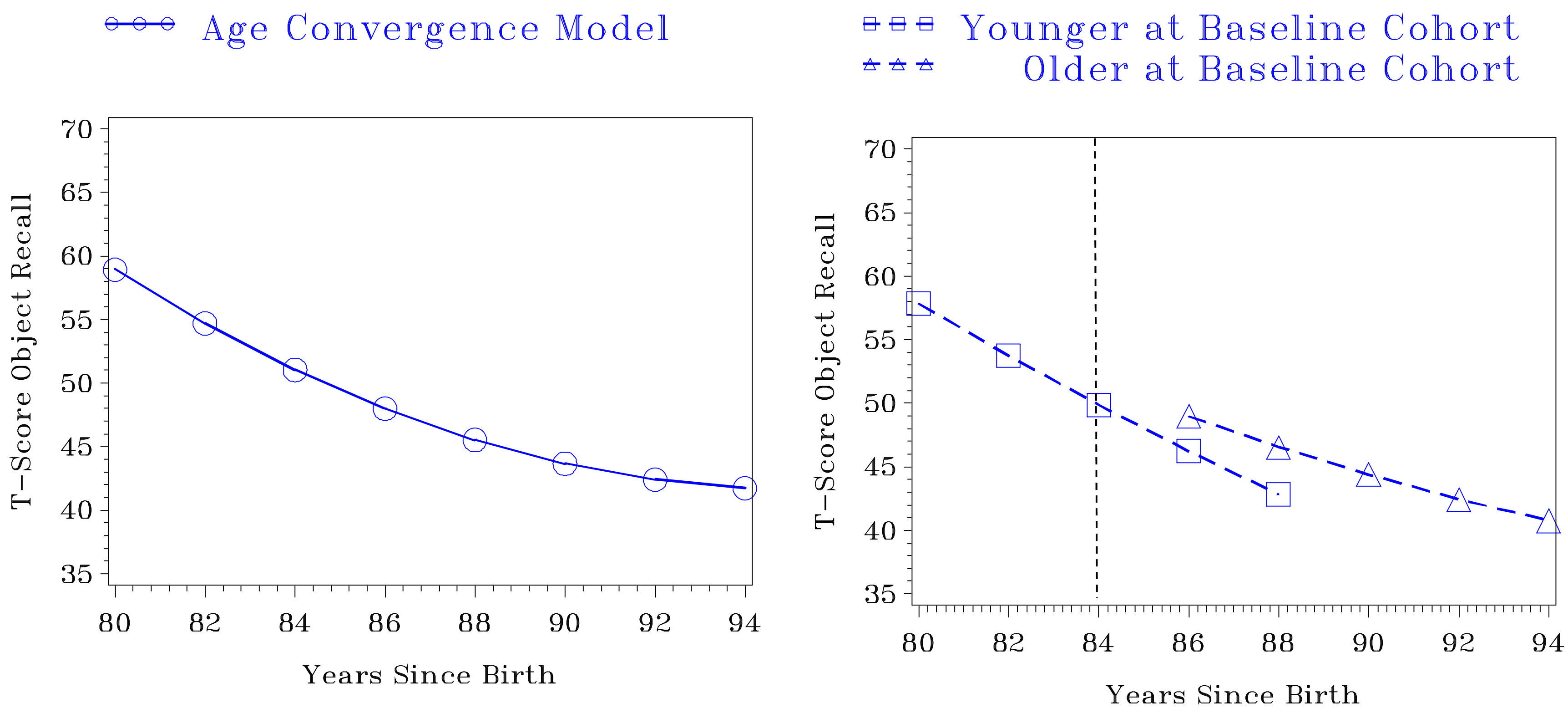
Persons create **contextual effects**

Age-Based Models: MMSE

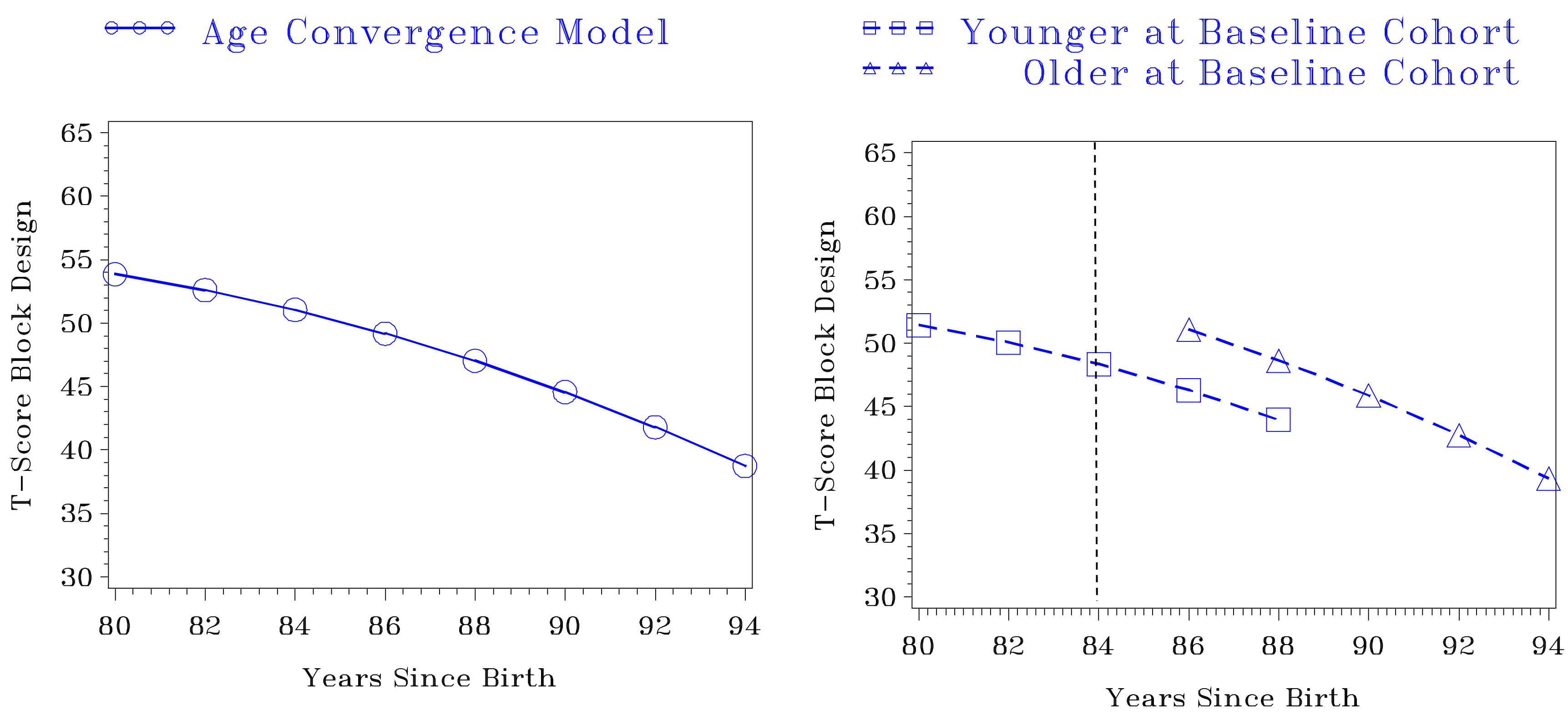




Age-Based Models: Object Recall



Age-Based Models: Spatial Reasoning



So if age is just a time-varying predictor…

- Because **years to death** and **years to diagnosis** also have BP variation (24%, 70%), the same concerns about **testing convergence** apply to them, too

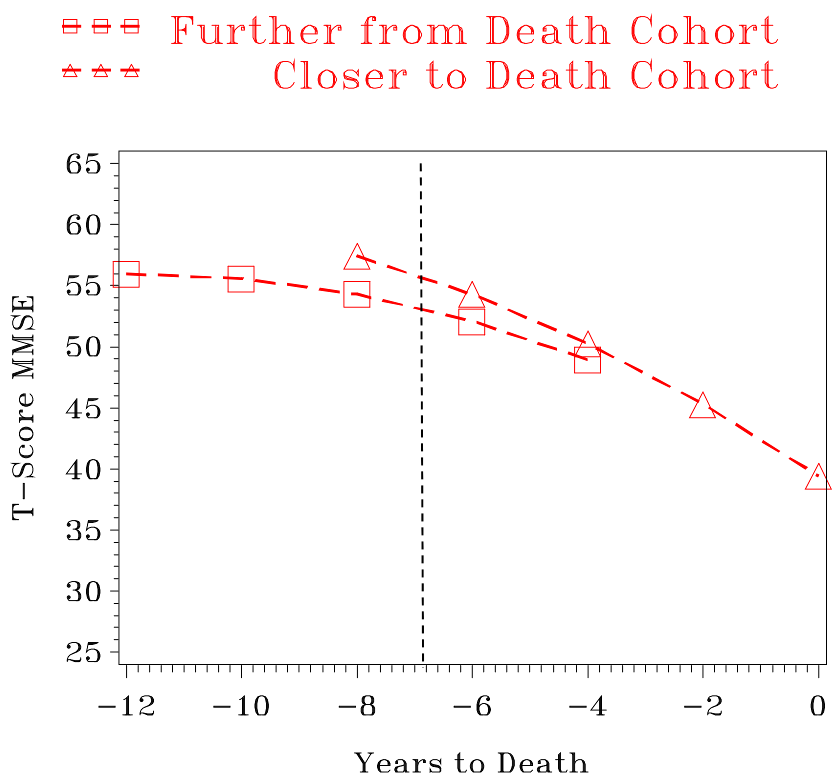
- **Years to death**
  - Level 1: $YTdeath_{ti}$ + 7
  - Level 2: $YTdeathT1_{i}$ + 7

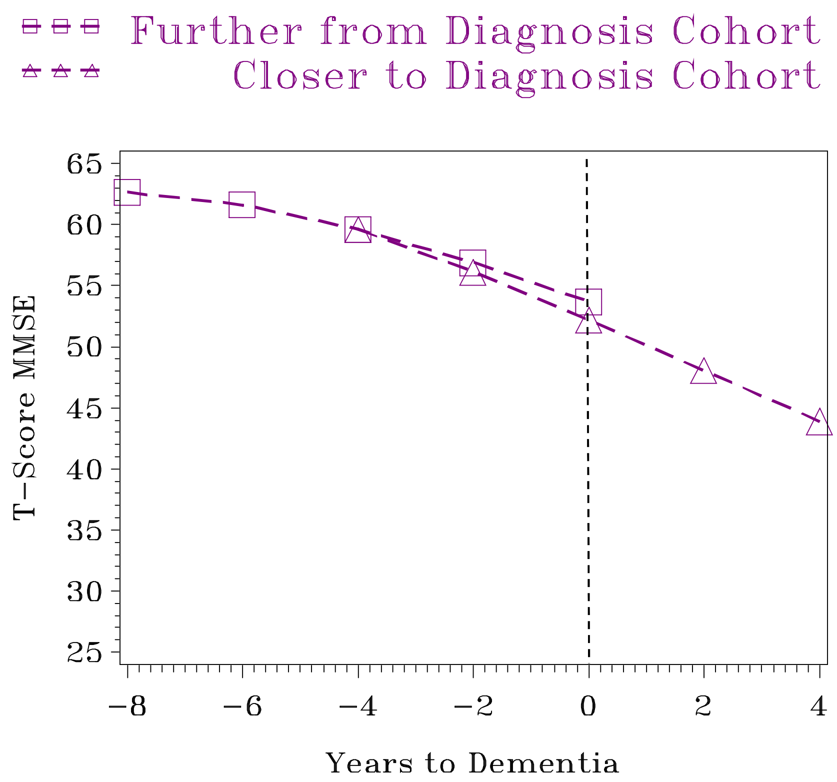
- **Years to diagnosis**
  - Level 1: $YTdem_{ti}$ – 0
  - Level 2: $YTdemT1_{i}$ – 0

- If the level-2 effects in these models are significant, then:
  - **Years to death**: it matters **WHEN** you were 7 years from death
  - **Years to diagnosis**: it matters **WHEN** you were at diagnosis

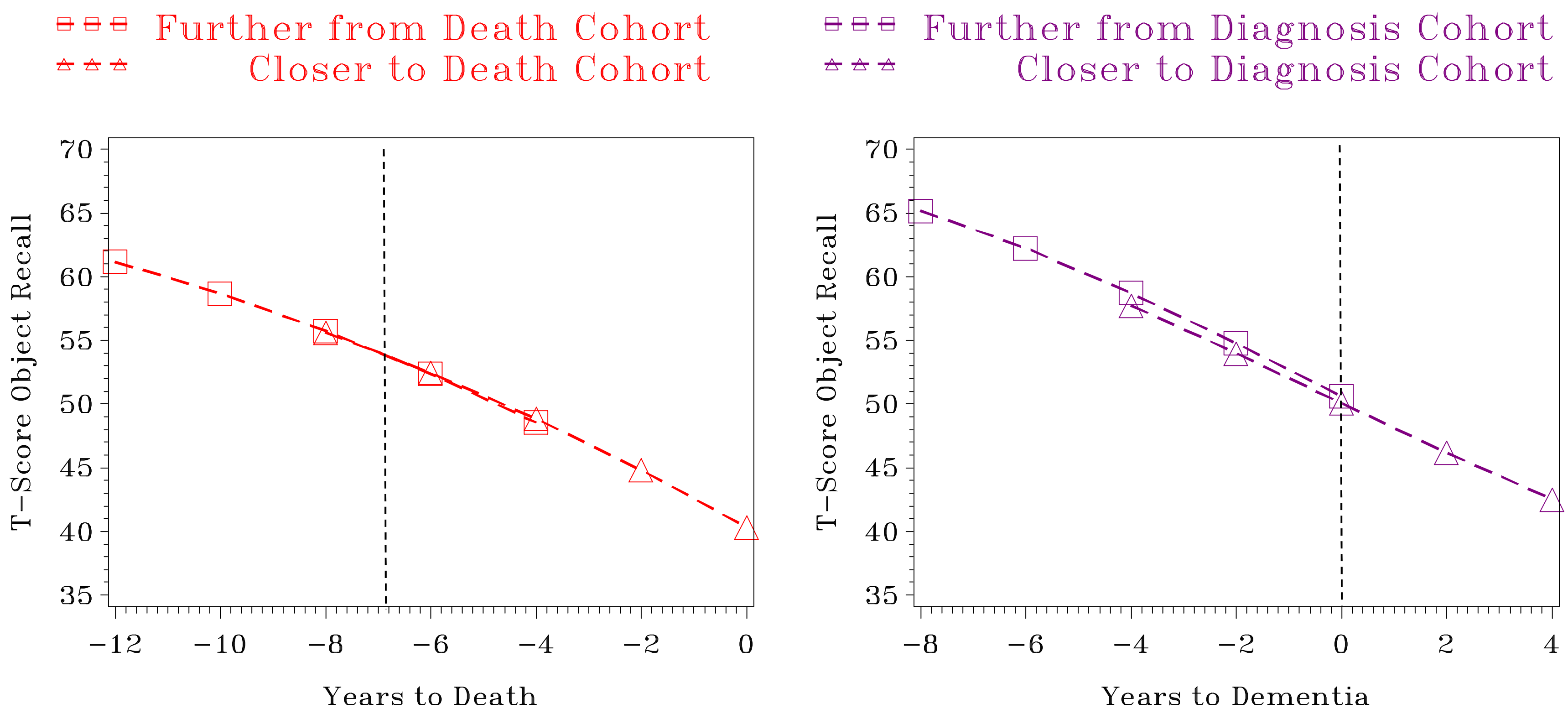
WHEN = cohort difference

Death-Based and Dementia-Based Models: MMSE

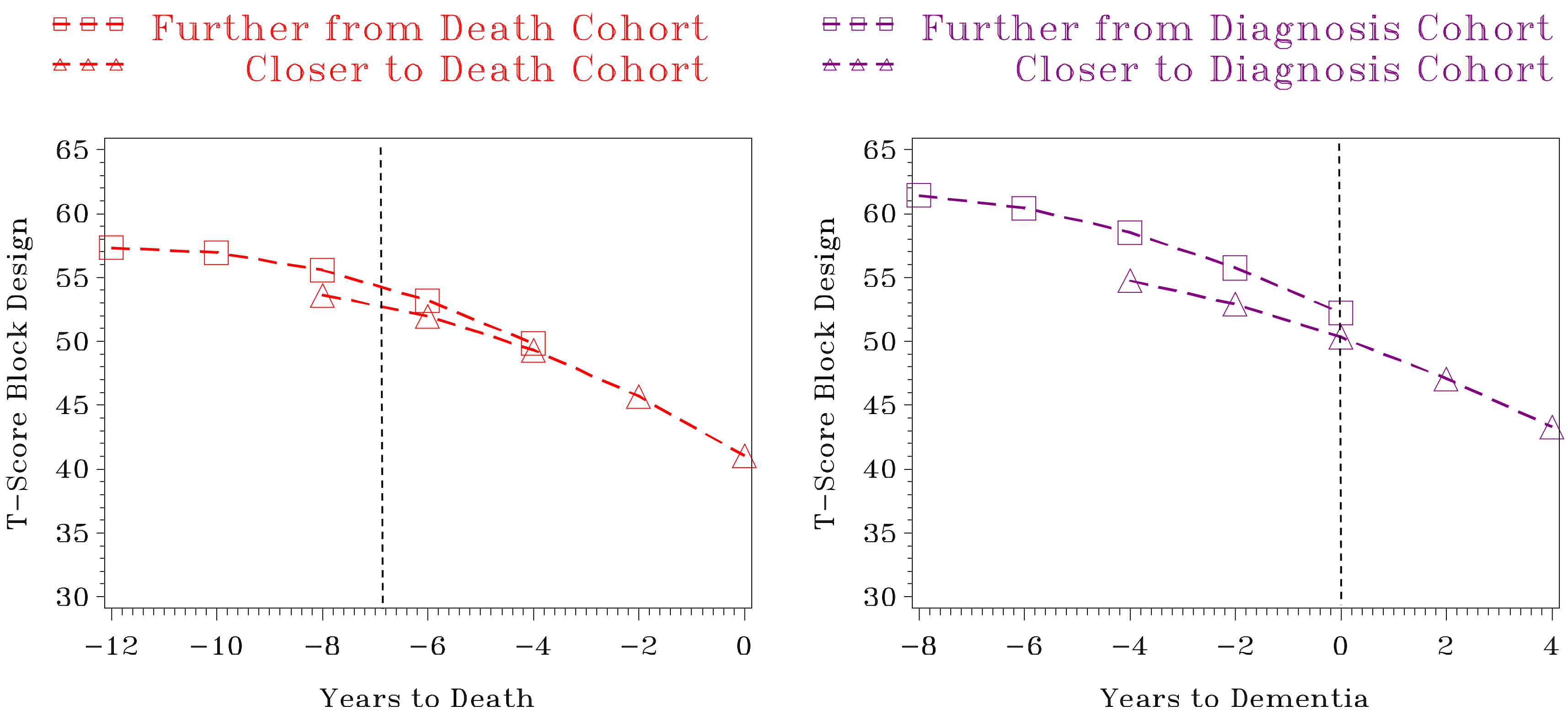




Death-Based and Dementia-Based Models: Object Recall



Death-Based and Dementia-Based Models: Spatial Reasoning



Comparing Models by Fit…

The fit of these alternative metrics of time to the data can be compared using their **information criteria**…

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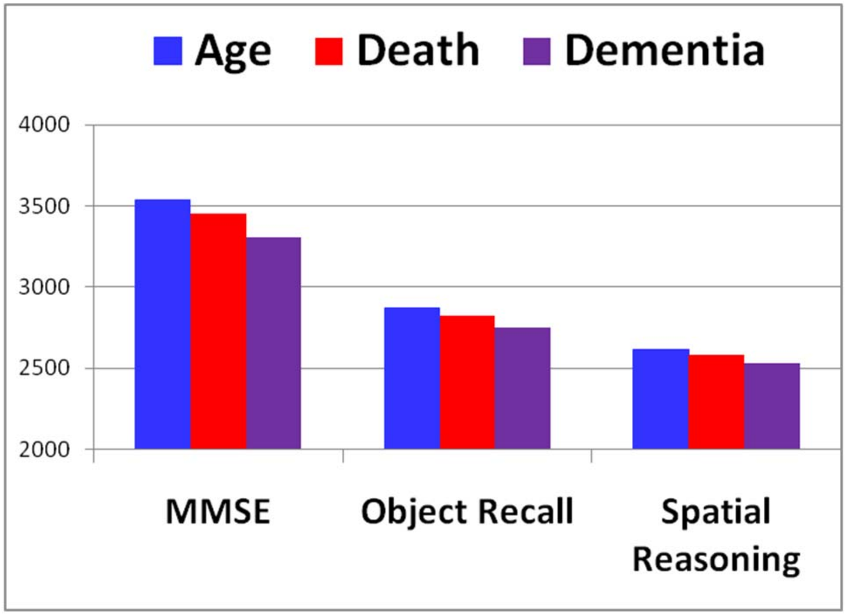
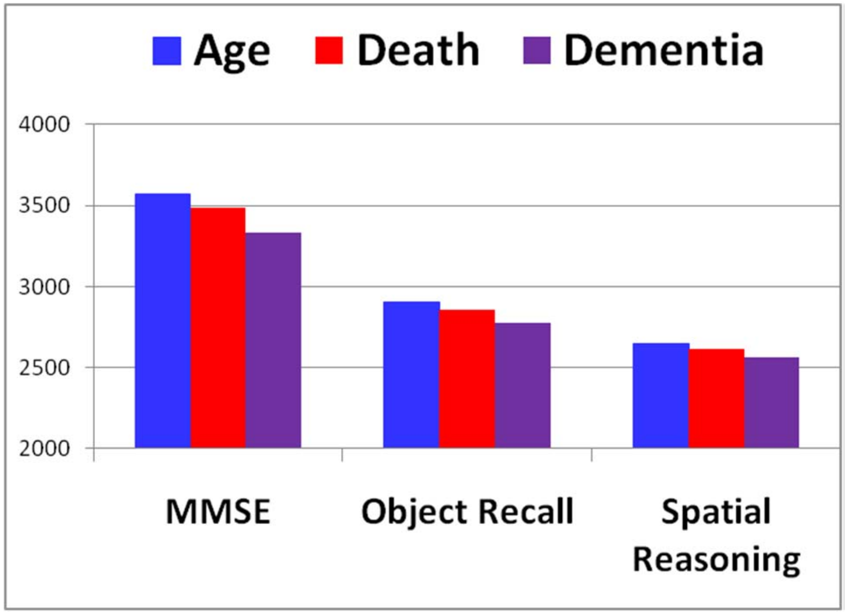


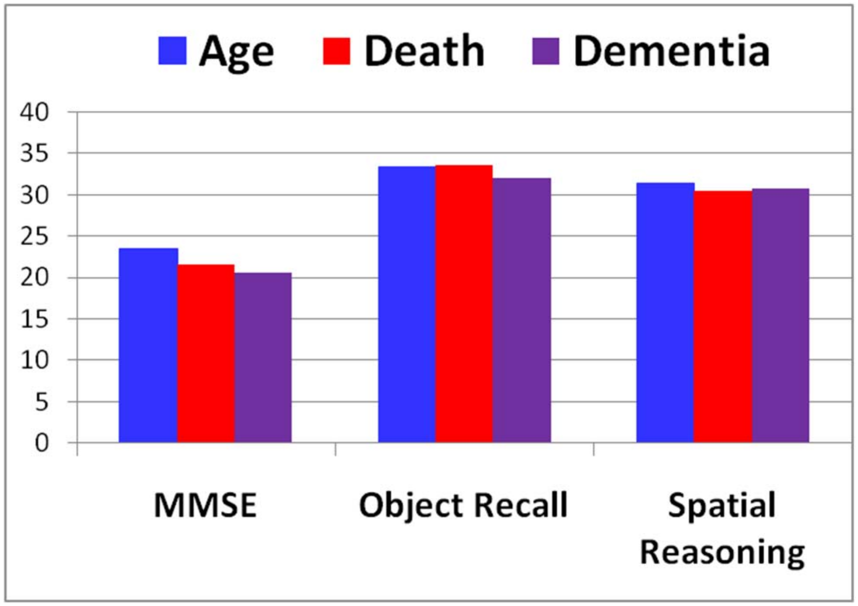
ML BIC



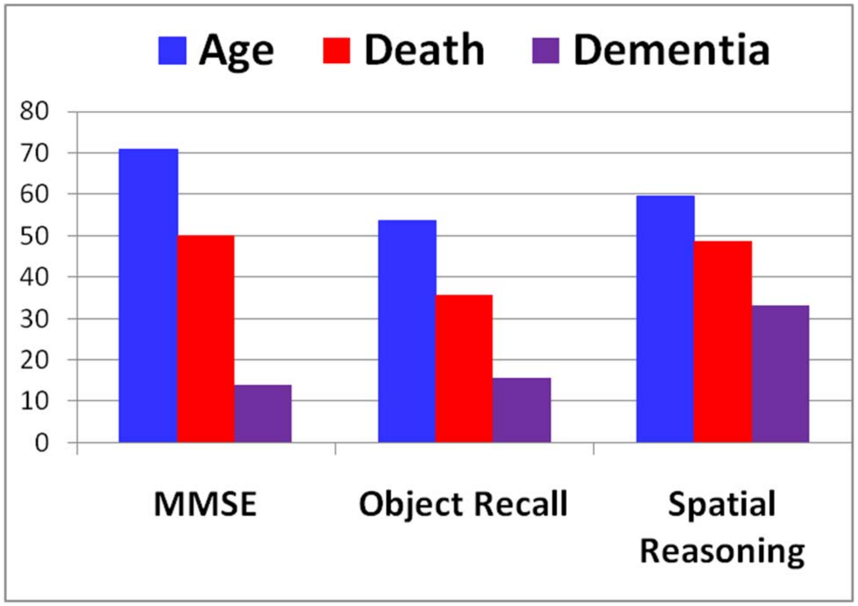
Comparing Models by Variances…

The fit of these alternative metrics of time to the data can also be compared using their **variance** components…

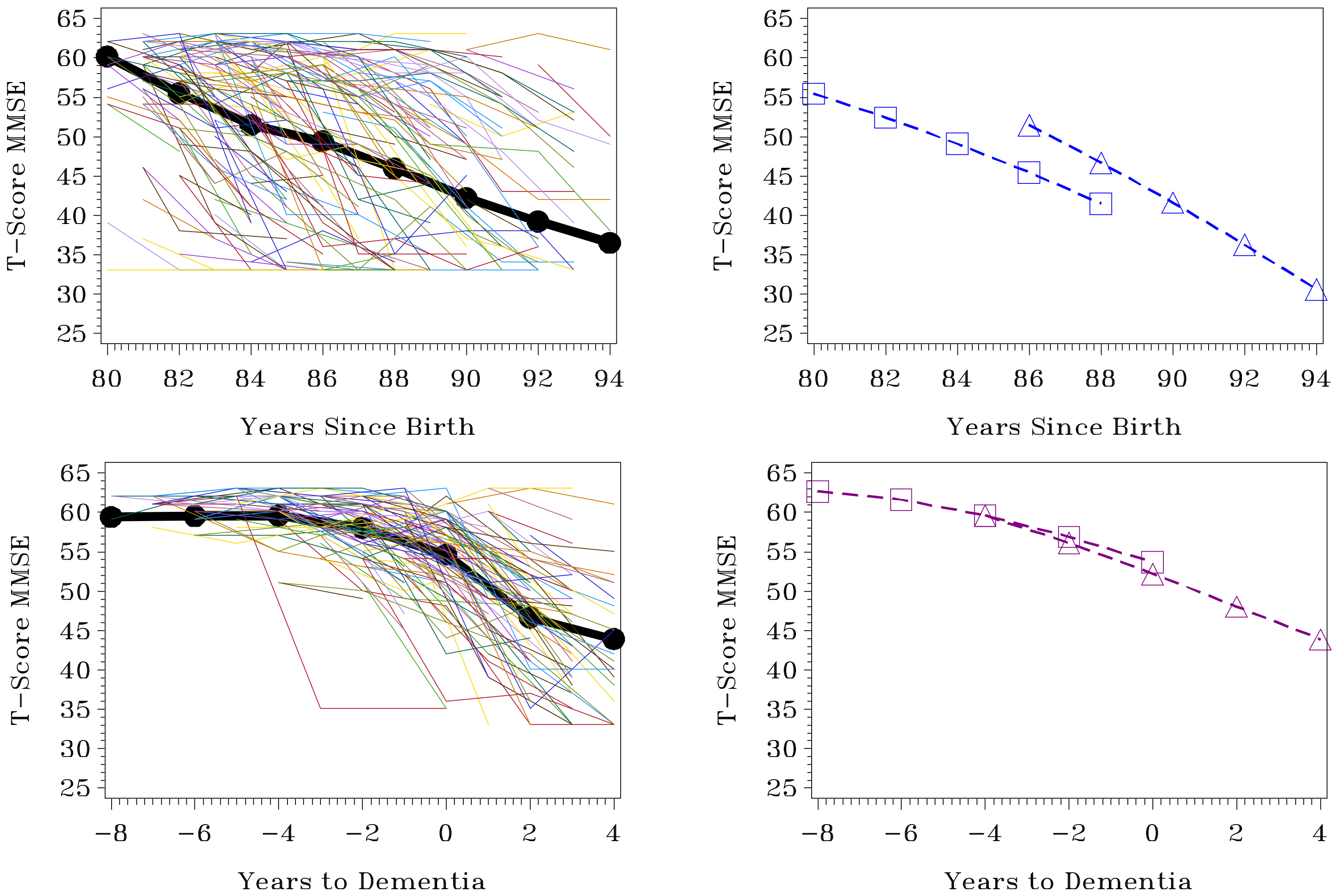
Residual Variance



Intercept Variance



Comparing Models By Data…



Road Map

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- The missing step #2
- Example: Alternative metrics of “time”
- **What about just time?**
- What else contributes to “time”?

What about just time as "Time"?

- When the accelerated time metrics do not show convergence of their BP and WP time effects, an alternative model specification may be more useful

- **Time-in-study models** separate BP and WP effects
  - Accelerated time (age, death…) model → Grand-mean-centering
  - Time-in-study model → **Person/group-mean-centering**

- Time-in-study models **can be made equivalent** to models with accelerated time metric in their fixed effects, but not in their random effects (as shown shortly)

Just Time as "Time"

Original Age
Age_{ti}
80
82
84
80
82
84
84
86
88
84
86
88

Just Time as "Time"

BP Age	Original Age	WP Age
$AgeT1_i$	Age_{ti}	$Age_{ti} - AgeT1_i$
80	80	0
80	82	2
80	84	4
80	80	0
80	82	2
80	84	4
84	84	0
84	86	2
84	88	4
84	84	0
84	86	2
84	88	4

Just Time as "Time"

BP Age $AgeT1_i$	Original Age Age_{ti}	WP Age $Age_{ti} - AgeT1_i$		Original YTD $YTdeath_{ti}$
80	80	0		-12
80	82	2		-10
80	84	4		-8
80	80	0		-8
80	82	2		-6
80	84	4		-4
84	84	0		-12
84	86	2		-10
84	88	4		-8
84	84	0		-8
84	86	2		-6
84	88	4		-4

Just Time as “Time”

BP Age $AgeT1_i$	Original Age Age_{ti}	WP Age $Age_{ti} - AgeT1_i$	BP Years to Death $YTdeathT1_i$	Original YTD $YTdeath_{ti}$	WP Years to Death $YTdeath_{ti} - YTdeathT1_i$
80	80	0	-12	-12	0
80	82	2	-12	-10	2
80	84	4	-12	-8	4
80	80	0	-8	-8	0
80	82	2	-8	-6	2
80	84	4	-8	-4	4
84	84	0	-12	-12	0
84	86	2	-12	-10	2
84	88	4	-12	-8	4
84	84	0	-8	-8	0
84	86	2	-8	-6	2
84	88	4	-8	-4	4

Just Time as "Time"

Time:	BP Age	Original Age	WP Age	BP Years to Death	Original YTD	WP Years to Death
Years in Study	$AgeT1_i$	Age_{ti}	$Age_{ti} - AgeT1_i$	$YTdeathT1_i$	$YTdeath_{ti}$	$YTdeath_{ti} - YTdeathT1_i$
0	80	80	0	-12	-12	0
2	80	82	2	-12	-10	2
4	80	84	4	-12	-8	4
0	80	80	0	-8	-8	0
2	80	82	2	-8	-6	2
4	80	84	4	-8	-4	4
0	84	84	0	-12	-12	0
2	84	86	2	-12	-10	2
4	84	88	4	-12	-8	4
0	84	84	0	-8	-8	0
2	84	86	2	-8	-6	2
4	84	88	4	-8	-4	4

Model Variants Using Age

Level-1 Age-Based (Grand-MC):

$$y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - 84) + e_{ti}$$

Level-1 Time-Based (Person-MC):

$$y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - AgeT1_{i}) + e_{ti}$$

Same pattern would result in any other accelerated time metric (such as years to death)

Same Level-2 Equations:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(AgeT1_{i} - 84) + U_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}(AgeT1_{i} - 84) + U_{1i}$$

Level-2 AgeT1 effects:

Age-Based: ***Incremental*** effect of cross-sectional age (***contextual*** age cohort effect)

Time-Based: ***Total*** effect of cross-sectional age (***between-person*** age effect)

Effect of Age Cohort on Intercept (Fixed Level-1 Linear Age Slope)

Time-in-Study ≈ Person-MC:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti} - AgeT1_i) + e_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(AT1_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(AT1_i) + \gamma_{10}(Age_{ti} - AT1_i) + U_{0i} + e_{ti}$ ← In terms of Time

$\rightarrow y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(AT1_i) + \gamma_{10}(Age_{ti}) + U_{0i} + e_{ti}$ ← In terms of Age

Age-Based ≈ Grand-MC:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(Age_{ti}) + e_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(AT1_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(AT1_i) + \gamma_{10}(Age_{ti}) + U_{0i} + e_{ti}$

Term	Time	Age
Intercept	γ_{00}	γ_{00}
WP Effect	γ_{10}	γ_{10}
Contextual	$\gamma_{01} - \gamma_{10}$	γ_{01}
BP Effect	γ_{01}	$\gamma_{01} + \gamma_{10}$

Effect of Age Cohort on Level-1 Age Slope (Fixed Level-1 Linear Age Slope)

On left below: Time-in-Study ≈ Person-MC:

Time as Time: $y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti} - AT1_i) + \gamma_{11}(Age_{ti} - AT1_i)(AT1_i) + \gamma_{01}(AT1_i) + \gamma_{02}(AT1_i^2) + U_{0i} + e_{ti}$

Time as Age: $y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti}) + \gamma_{11}(Age_{ti})(AT1_i) + (\gamma_{01} - \gamma_{10})(AT1_i) + (\gamma_{02} - \gamma_{11})(AT1_i^2) + U_{0i} + e_{ti}$

On right below: Age-Based ≈ Grand-MC:

$$y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti}) + \gamma_{11}(Age_{ti})(AT1_i) + \gamma_{01}(AT1_i) + \gamma_{02}(AT1_i)^2 + U_{0i} + e_{ti}$$

Adding $AgeT1^2$ creates equivalence

Intercept: $\gamma_{00} = \gamma_{00}$ **BP Effect:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Cohort:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect: $\gamma_{10} = \gamma_{10}$ **BP² Effect:** $\gamma_{02} = \gamma_{02} + \gamma_{11}$ **Cohort²:** $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BP*WP or Cohort*WP is the same: γ_{11}

Add Quadratic Level-1 Age Slope (Fixed Level-1 Age Slopes)

On left below: Time-in-Study ≈ Person-MC:

Time as Time: $y_{ti}=\gamma_{00}+\gamma_{10}(Age_{ti}-AT1_i)+\gamma_{20}(Age_{ti}-AT1_i)^2$
$+\gamma_{11}(Age_{ti}-AT1_i)(AT1_i)+\gamma_{01}(AT1_i)+\gamma_{02}(AT1_i^2)+U_{0i}+e_{ti}$

Time as Age: $y_{ti}=\gamma_{00}+\gamma_{10}(Age_{ti})+\gamma_{20}(Age_{ti}^2)+(\gamma_{11}-2\gamma_{20})(Age_{ti})(AT1_i)$
$+(\gamma_{01}-\gamma_{10})(AT1_i)+(\gamma_{02}+\gamma_{20}-\gamma_{11})(AT1_i^2)+U_{0i}+e_{ti}$

On right below: Age-Based ≈ Grand-MC:

$y_{ti}=\gamma_{00}+\gamma_{10}(Age_{ti})+\gamma_{20}(Age_{ti})^2+\gamma_{11}(Age_{ti})(AT1_i)$
$+\gamma_{01}(AT1_i)+\gamma_{02}(AT1_i)^2+U_{0i}+e_{ti}$

Intercept: $\gamma_{00}=\gamma_{00}$ **BP Effect:** $\gamma_{01}=\gamma_{01}+\gamma_{10}$ **Cohort:** $\gamma_{01}=\gamma_{01}-\gamma_{10}$

WP Effect: $\gamma_{10}=\gamma_{10}$ **BP² Effect:** $\gamma_{02}=\gamma_{02}+\gamma_{11}+\gamma_{20}$ **Cohort²:** $\gamma_{02}=\gamma_{02}-\gamma_{11}+\gamma_{20}$

WP² Effect: $\gamma_{20}=\gamma_{20}$ **BP*WP:** $\gamma_{11}=\gamma_{11}+2\gamma_{20}$ **Cohort*WP:** $\gamma_{11}=\gamma_{11}-2\gamma_{20}$

Time-in-Study Models so far…

- Specify WP change using only longitudinal information
- Are equivalent within persons across accelerated time metrics
- Because unique information from the alternative time metrics is really only available BP, it only shows up in the level-2 model
- Can (usually) be made equivalent in their fixed effects to models based in alternative accelerated time metrics
- **So why make a distinction? Different random effects…**

Random Slopes Across Models

Time-in-Study ≈ Person-MC:

as Time: $y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti} - AT1_i) + \gamma_{01}(AT1_i)$
$+ U_{0i} + U_{1i}(Age_{ti} - AT1_i) + e_{ti}$

as Age: $y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti}) + (\gamma_{01} - \gamma_{10})(AT1_i)$
$+ U_{0i} + \boxed{U_{1i}(Age_{ti}) - U_{1i}(AT1_i)} + e_{ti}$

Age-Based ≈ Grand-MC:

$y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti}) + \gamma_{01}(AT1_i)$
$+ U_{0i} + \boxed{U_{1i}(Age_{ti})} + e_{ti}$

So which do we choose?

Variance due to $AT1_i$ is still part of the random slope in the age-based model. So the time-based and age-based models cannot be made equivalent in terms of random effects variances.

Random Slopes Across Models

- **Random intercepts** mean different things under each model:
  - **Time: Person-MC** → Individual differences at **time=0** (that everyone has)
  - **Age: Grand-MC** → Individual differences at **age=0** (that not everyone has)

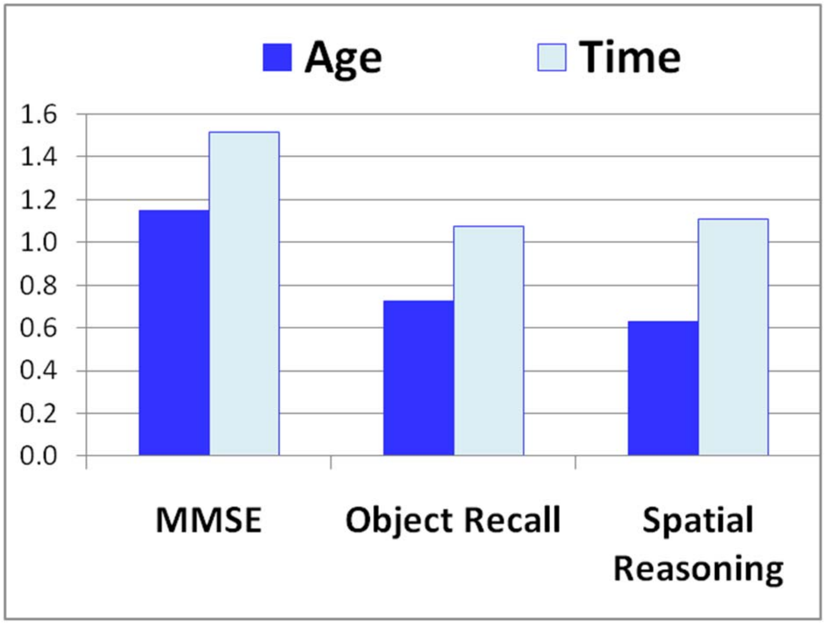
- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
  - **Person-MC** → Won't affect shrinkage of slopes unless highly correlated
  - **Grand-MC** → Will affect shrinkage of slopes due to forced extrapolation

- As a result, the **random slope variance may be smaller** under **grand-MC (age, death…)** than under **person-MC (time)**
  - Problem worsens with greater BP variation in time (more extrapolation)
  - Anecdotal example of downward bias using clustered data was presented in Raudenbush & Bryk (2002; chapter 6), but what about in these data?

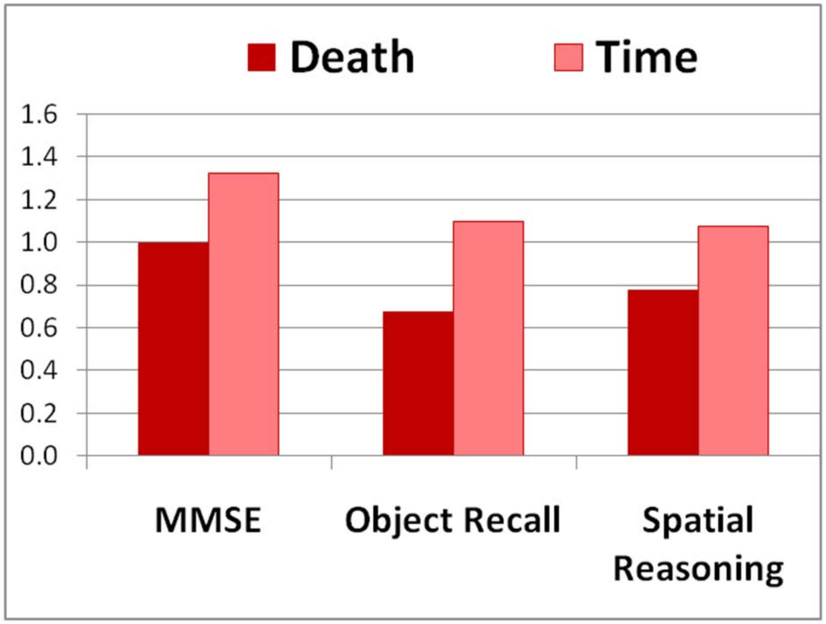
Slope Variance in Example Models

- Slope variance estimate was indeed **33-77% larger** in the time-based model versions across outcomes…

Years-Since-Birth (47% BP)

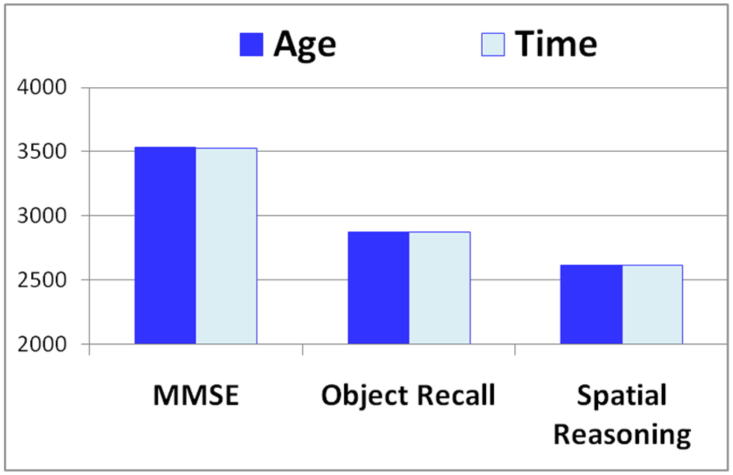


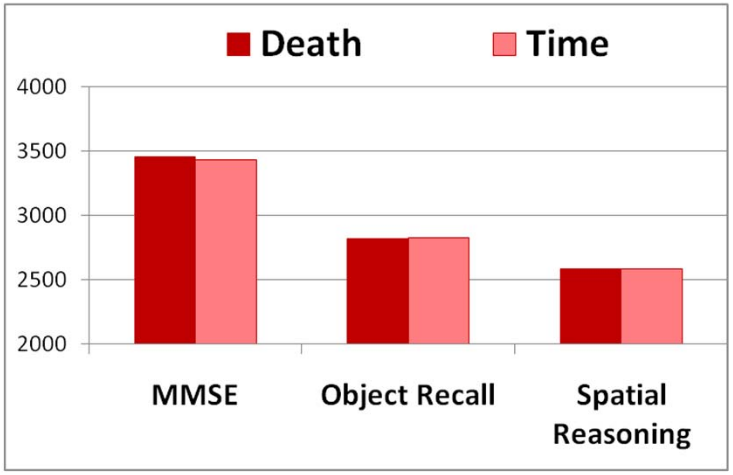
Years-to-Death (24% BP)



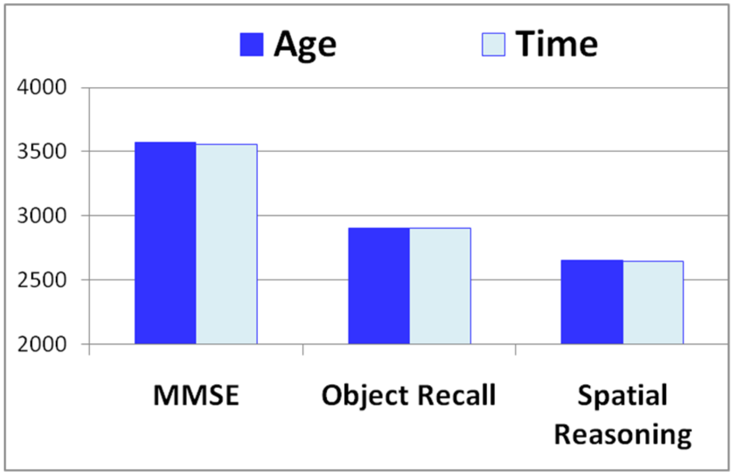
… Although model fit was the same

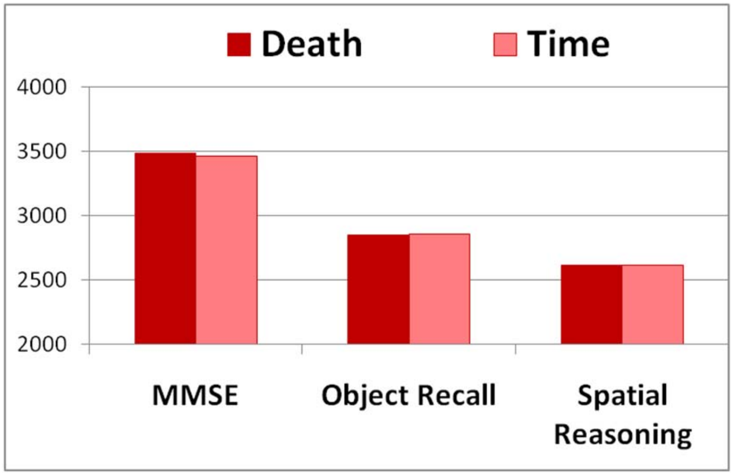
ML AIC



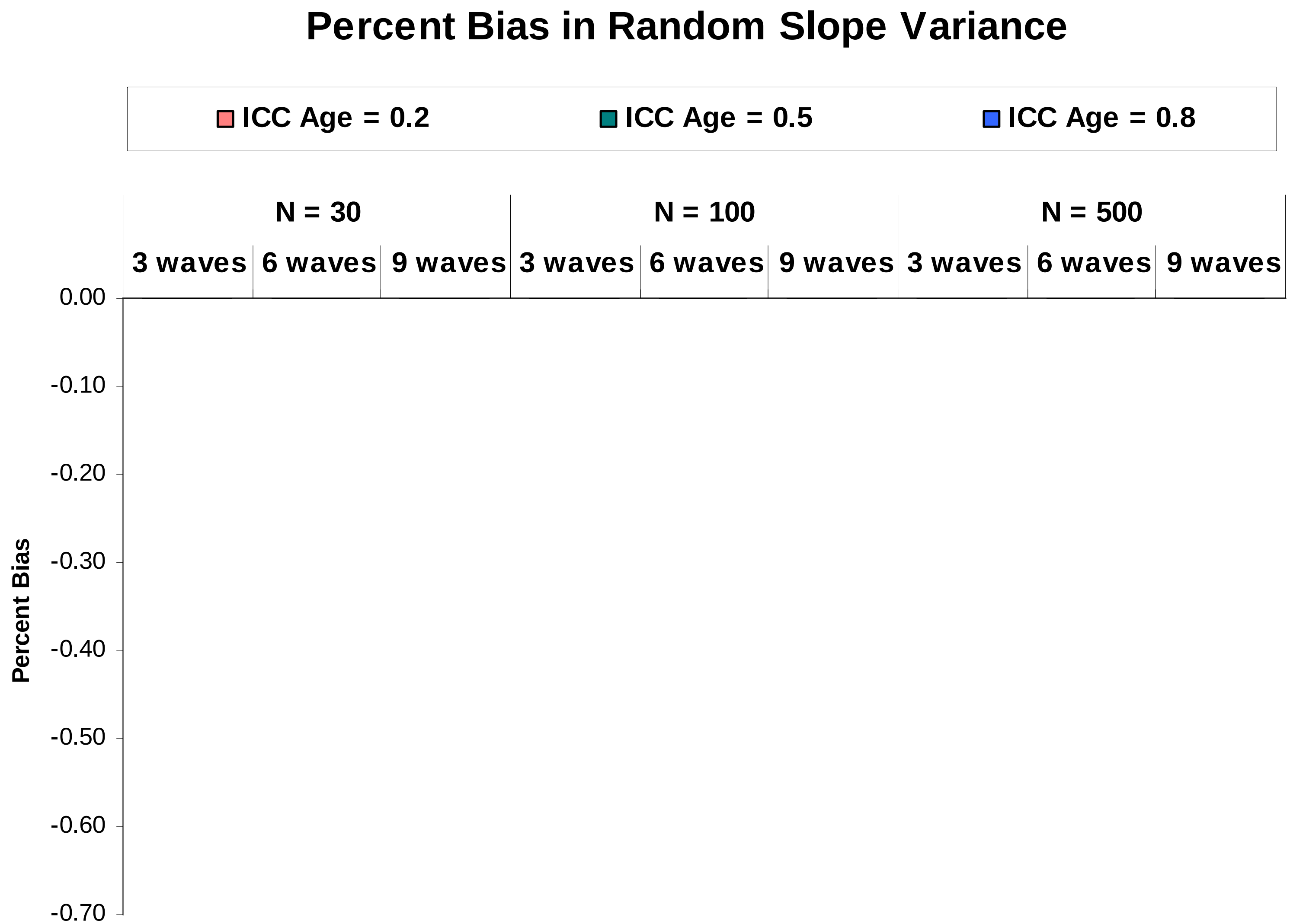


ML BIC





Simulation Study Results
(Generated by Time, Analyzed by Age)



Simulation Study Results
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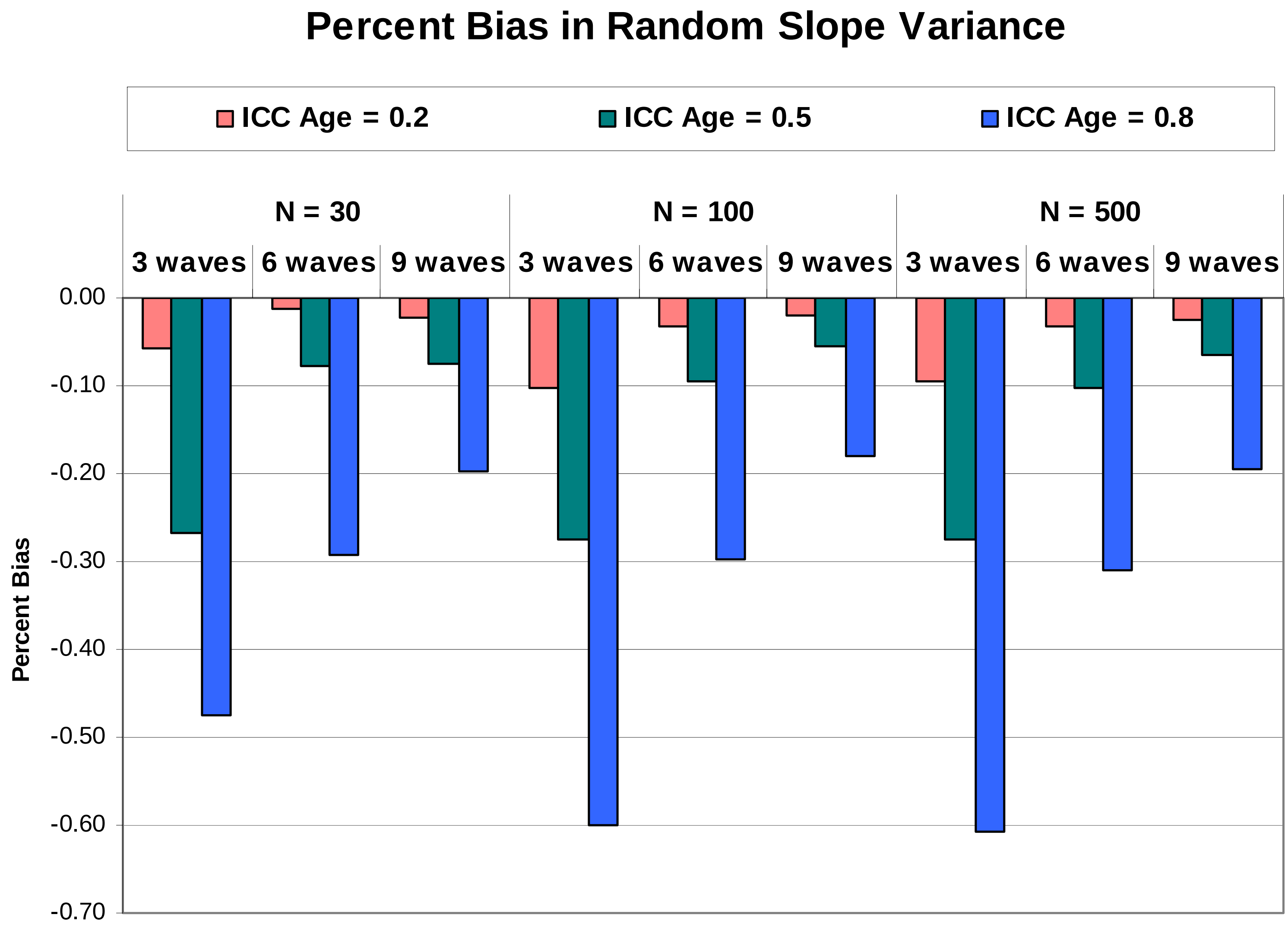


And so the winner is… Time?

- Although seemingly the most non-informative choice, simply tracking **change as a function of study duration**:
  - Represents **WP changes** as directly and parsimoniously as possible
  - Seems to recover **random slope variance** better in accelerated designs
  - Permits inclusion of persons who have not experienced events in an alternative time metric (e.g., death, dementia diagnosis)

- Time-in-study models make no assumptions about processes causing change, so these become **testable hypotheses**
  - Do persons who are older start lower and decline faster?
    - *Age main effect, Age*Time interaction*
  - After considering mortality, do older persons *still* decline faster?
    - *Competing YTdeath*Time and Age*Time interactions*

Road Map

- Steps in longitudinal analysis
- The missing step #2
- Example: Alternative metrics of "time"
- What about just time?
- **What else contributes to "time"?**

What about retest effects?

- Are estimates of age-related change too small without controlling for **practice effects** due to repeated testing?

- Can **time-in-study** index **retest** in age-based models?

$$y_{ti} = \gamma_{00} + \gamma_{10}(Age_{ti}) + \gamma_{20}(Age_{ti})^2 \ldots$$
$$+ e_{ti} + U_{0i} + U_{1i}(Age_{ti}) + \ldots$$
$$+ \gamma_{30}(Retest_{ti})$$
$$+ \gamma_{40}(Retest_{ti})(Age_{ti}) \ldots$$

Individual change due to age

Retest = Time = Difference due to which occasion of measurement

- But not including age cohort assumes **age convergence**… What if age cohort (AT1) and retest effects are BOTH included?
  - Simulation results: missing cohort effects will masquerade as retest effects in the opposite direction—they are **confounded by design**

Conclusions

- When time has both BP and WP variation, one should always **carefully consider what "time" could and should be**
  - Otherwise, aggregate trends may not actually describe any individuals
  - Individual differences can be created artificially through the mis-alignment of different persons onto a single "time" trajectory

- **Multiple processes** may be at work simultaneously, but they have to be **observed independently** to be distinguishable
  - Age vs. Mortality: can be distinguished if not everyone dies at same age
  - But if **aging and retest occur simultaneously within-persons**, retest effects cannot be distinguished from effects of aging and age cohort
  - **Age/Cohort/Time in design → Age/Cohort/Retest in models**

- Considering the effects of time is an important pre-cursor to making informed use of advances in longitudinal models…

Thank you for your time...

- **Questions or comments? Email me: Lesa@unl.edu**
- **Slides available at: http://psych.unl.edu/hoffman/Sheets/Talks.htm**

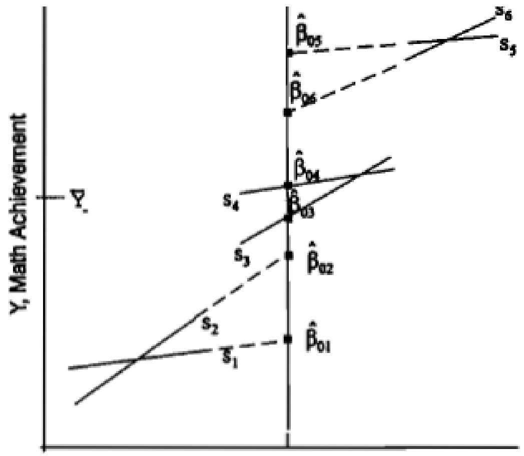
- Works cited:

  - Hoffman, L., Hofer, S. M., & Sliwinski, M. J. (2011). On the confounds among retest gains and age-cohort differences in the estimation of within-person change in longitudinal studies: A simulation study. *Psychology and Aging, 26*(4), 778-791.
  - Hoffman, L. (2012). *Considering alternative metrics of time: Does anybody really know what "time" is*? In J. Harring & G. Hancock (Eds.), *Advances in Longitudinal Methods in the Social and Behavioral Sciences* (pp. 255-287). Charlotte, NC: Information Age Publishing.
  - Hoffman, L., & Templin, J. L. (April, 2008). *The impact of alternative specifications of time on examining individual differences in change*. Poster presented at the Cognitive Aging Conference, Atlanta, GA.

Extra Slides

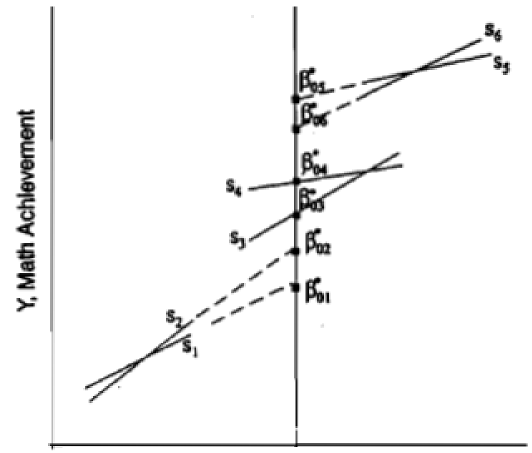
Bias in Random Slope Variance

OLS Per-Person Estimates



Age

EB Shrunken Estimates



Age

Top right: Intercepts and slopes are homogenized in Grand-MC because of intercept extrapolation

Bottom: Downwardly-biased random age slope variance in Grand-MC relative to Person-MC

Unconditional Results	*Conditional Results*
Time: Person-MC	
$\hat{\mathbf{T}} = \begin{bmatrix} 8.68 & 0.05 \\ 0.05 & 0.68 \end{bmatrix}$ $\hat{\sigma}^2 = 36.70$	$\hat{\mathbf{T}} = \begin{bmatrix} 2.38 & 0.19 \\ 0.19 & \boxed{0.15} \end{bmatrix}$ $\hat{\sigma}^2 = 36.70$
Age: Grand-MC	
$\hat{\mathbf{T}} = \begin{bmatrix} 4.83 & -0.15 \\ -0.15 & 0.42 \end{bmatrix}$ $\hat{\sigma}^2 = 36.83$	$\hat{\mathbf{T}} = \begin{bmatrix} 2.41 & 0.19 \\ 0.19 & \boxed{0.06} \end{bmatrix}$ $\hat{\sigma}^2 = 36.74$

Model Variants Using Years to Death

Level 1 Death-Based (Grand-MC):

$$y_{ti} = \beta_{0i} + \beta_{1i}(YTdeath_{ti} + 7) + e_{ti}$$

Level 1 Time-Based (Person-MC):

$$y_{ti} = \beta_{0i} + \beta_{1i}(YTdeath_{ti} - YTdeathT1_{i}) + e_{ti}$$

Same Level 2 Equations:

$$\beta_{0i} = \gamma_{00} + \gamma_{01}(YTdeathT1_{i} + 7) + U_{0i}$$

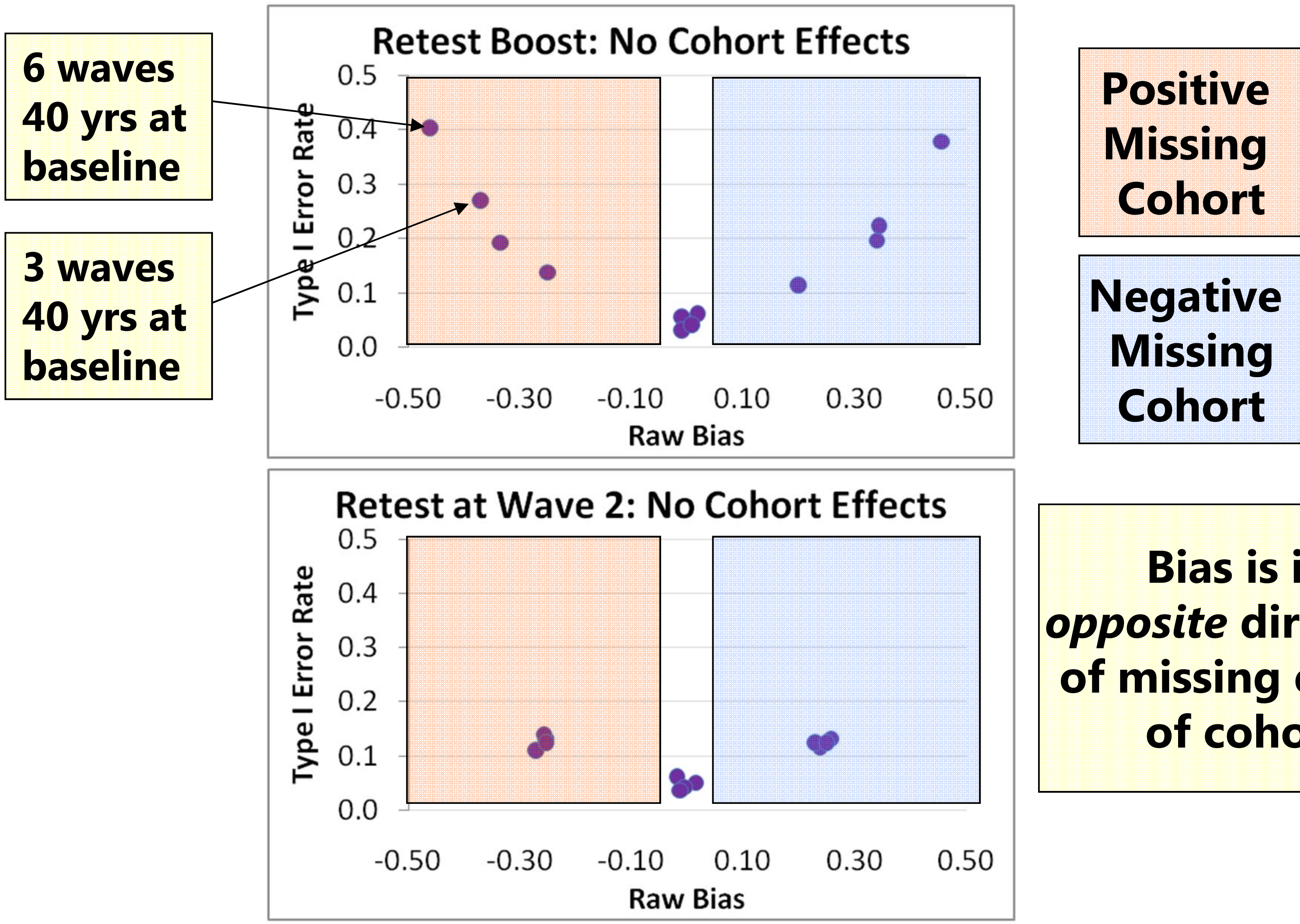
$$\beta_{1i} = \gamma_{10} + \gamma_{11}(YTdeathT1_{i} + 7) + U_{1i}$$

Level-2 YTdeathT1 effects:

Death-Based: *Incremental* effect of cross-sectional YTD (contextual cohort effect)

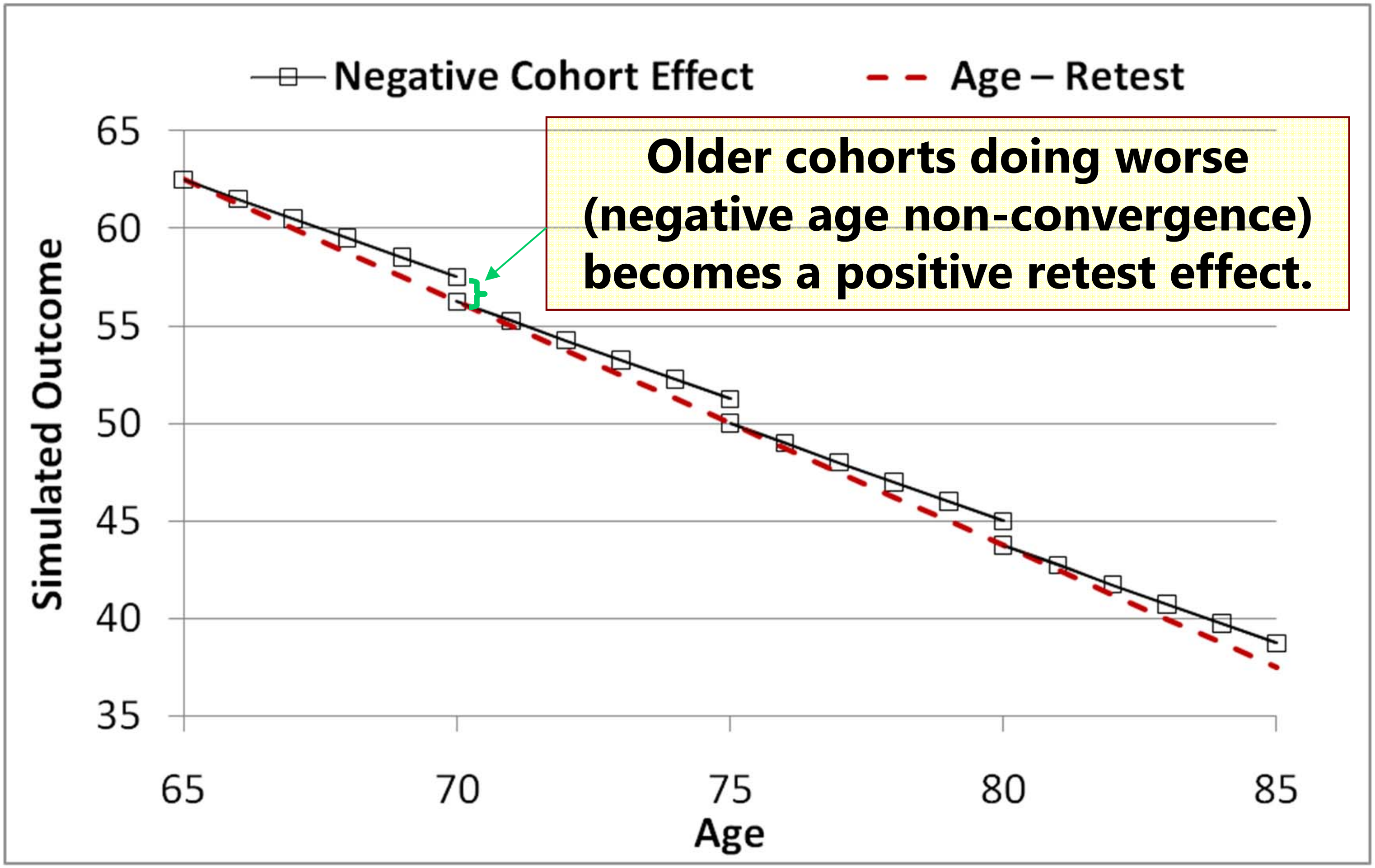
Time-Based: *Total* effect of cross-sectional YTD (between-person effect)

Retest Effects: Bias and Type I Error Rates



Bias is in *opposite* direction of missing effect of cohort

Cohort Effect or Retest Effect?



Older cohorts doing worse (negative age non-convergence) becomes a positive retest effect.

Likewise, missing positive cohort effects resulted in negative retest effects instead—which can't happen.