Interactions among Categorical Predictors

- Today's Class:
  - Reviewing significance tests
  - Manual contrasts for categorical predictors
  - Program-created contrasts for categorical predictors

Testing Significance of Fixed Effects in the Model for the Means

- Any **single (df=1) fixed effect** has 4-5 pieces of output:
  - **Estimate** = best guess for the fixed effect from our data
  - **Standard Error** = precision of fixed effect estimate (quality of most likely estimate)
  - ***t*-value or *z*-value** = Estimate / Standard Error → **Wald test**
  - ***p*-value** = probability that fixed effect estimate is ≠ 0
  - **95% Confidence Interval** = Estimate ± 1.96*SE = range in which true (population) value of estimate is expected to fall 95% of the time

- Compare Wald test statistic to critical value at chosen level of significance (known as alpha)
- Whether the *p*-value is based on *t* or *z* varies by program…

Evaluating Significance of Fixed Effects

Fixed effects can be tested via **Wald** tests: the ratio of its estimate/SE forms a statistic we compare to a distribution

	Denominator DF is assumed infinite	Denominator DF is estimated instead
Numerator DF = 1 *(Univariate Wald Test)*	use ***z*** distribution (Mplus, STATA)	use ***t*** distribution (SAS, SPSS)
Numerator DF > 1 *(test 2+ effects at once)* *(Multivariate Wald Test)* *("Omnibus F-Test"*	use χ^2 distribution (Mplus, STATA)	use ***F*** distribution (SAS, SPSS)

F * df = χ^2

Multivariate Wald Tests: F or χ^2

- Tests of more than effect at once (numerator df>1) are seen in many different contexts:
  - Test of significance of Model R^2 from 0 → test of whether all regression coefficients are 0 simultaneously
  - Test of significance of change in Model R^2 → test of whether all *new* regression coefficients are 0 simultaneously
  - "Omnibus" ANOVA Test of whether there are any differences (in main effects or interactions) across 3+ groups
    - Provided by default for predictors designated as categorical, but can also be requested for any combination of predictors (and their main effects and interactions) via SPSS TEST, SAS CONTRAST, and STATA TEST (for c. predictors) or CONTRAST (for i. predictors)
    - WILL NOT BE USEFUL WHEN INTERACTIONS ARE PRESENT!

Categorical Predictors (3+ Groups)

- Two alternatives for how to include categorical predictors

1. Create and include manual dummy-coded contrasts
   - Need $g-1$ contrasts for g groups, added all at once, treated as continued (WITH in SPSS, by default in SAS, c. in STATA)
   - Corresponds more directly to linear more representation
   - Easier to set own reference group and contrasts of interest

2. Let the program create and include contrasts for you
   - Treat as categorical: BY in SPSS, CLASS in SAS, i. in STATA
     - SPSS and SAS: reference = highest/last group; STATA: reference = lowest/first group
   - More convenient if you have many groups, want many contrasts, or have interactions among categorical predictors
   - Program marginalizes over these effects when estimating other effects

Categorical Predictors: Manual Coding

- Model: $y_i = \beta_0 + \beta_1(dA_i) + \beta_2(dB_i) + \beta_3(dC_i) + e_i$
  - "Treatgroup" variable: Control=1, TreatA=2, TreatB=3, TreatC=4
  - New variables to be created for the model:
    dA= 0, 1, 0, 0 → difference between Control and TA
    dB= 0, 0, 1, 0 → difference between Control and TB
    dC= 0, 0, 0, 1 → difference between Control and TC
- How does the model give us **all possible group differences**? By determining each group's mean, and then the difference…

Control Mean (Reference)	Treatment A Mean	Treatment B Mean	Treatment C Mean
β_0	$\beta_0+\beta_1 dA_i$	$\beta_0+\beta_2 dB_i$	$\beta_0+\beta_3 dC_i$

- The model for the 4 groups directly provides 3 differences (control vs. each treatment), and indirectly provides another 3 differences (differences between treatments)

Group Differences from Dummy Codes

- Model: $y_i = \beta_0 + \beta_1(dA_i) + \beta_2(dB_i) + \beta_3(dC_i) + e_i$

Control Mean (Reference)	Treatment A Mean	Treatment B Mean	Treatment C Mean
$\boldsymbol{\beta_0}$	$\boldsymbol{\beta_0 + \beta_1 dA_i}$	$\boldsymbol{\beta_0 + \beta_2 dB_i}$	$\boldsymbol{\beta_0 + \beta_3 dC_i}$

	Alt Group	Ref Group	Difference
• Control vs. TA =	$(\beta_0+\beta_1)$	$-\ (\beta_0)$	$= \beta_1$
• Control vs. TB =	$(\beta_0+\beta_2)$	$-\ (\beta_0)$	$= \beta_2$
• Control vs. TC =	$(\beta_0+\beta_3)$	$-\ (\beta_0)$	$= \beta_3$
• TA vs. TB =	$(\beta_0+\beta_2)$	$-\ (\beta_0+\beta_1)$	$= \beta_2 - \beta_1$
• TA vs. TC =	$(\beta_0+\beta_3)$	$-\ (\beta_0+\beta_1)$	$= \beta_3 - \beta_1$
• TB vs. TC =	$(\beta_0+\beta_3)$	$-\ (\beta_0+\beta_2)$	$= \beta_3 - \beta_2$

TESTs when using dummy codes

	Alt Group	Ref Group	Difference
• Control vs. TA =	$(\beta_0+\beta_1)$	$-\ (\beta_0)$	$= \beta_1$
• Control vs. TB =	$(\beta_0+\beta_2)$	$-\ (\beta_0)$	$= \beta_2$
• Control vs. TC =	$(\beta_0+\beta_3)$	$-\ (\beta_0)$	$= \beta_3$
• TA vs. TB =	$(\beta_0+\beta_2)$	$-\ (\beta_0+\beta_1)$	$= \beta_2 - \beta_1$
• TA vs. TC =	$(\beta_0+\beta_3)$	$-\ (\beta_0+\beta_1)$	$= \beta_3 - \beta_1$
• TB vs. TC =	$(\beta_0+\beta_3)$	$-\ (\beta_0+\beta_2)$	$= \beta_3 - \beta_2$

Note the order of the equations: the reference group mean *is subtracted from* the alternative group mean.

In SAS ESTIMATE statements (or SPSS TEST or STATA LINCOM), the variables refer to their betas; the numbers refer to the operations of their betas.

```
ECHO 'Differences among 4 groups'.
MIXED y WITH dA dB dC /METHOD = REML /PRINT = SOLUTION TESTCOV
/FIXED = dA dB dC
/TEST = "Omnibus F-test" dA 1; dB 1; dC 1
/TEST = "Control Mean" intercept 1 dA 0 dB 0 dC 0
/TEST = "TA Mean"      intercept 1 dA 1 dB 0 dC 0
/TEST = "TB Mean"      intercept 1 dA 0 dB 1 dC 0
/TEST = "TC Mean"      intercept 1 dA 0 dB 0 dC 1
/TEST = "Mean: Control vs. TA" dA  1 dB  0 dC 0
/TEST = "Mean: Control vs. TB" dA  0 dB  1 dC 0
/TEST = "Mean: Control vs. TC" dA  0 dB  0 dC 1
/TEST = "Mean: TA vs. TB"      dA -1 dB  1 dC 0
/TEST = "Mean: TA vs. TC"      dA -1 dB  0 dC 1
/TEST = "Mean: TB vs. TC"      dA  0 dB -1 dC 1.
```

Intercepts are used only in predicted outcomes.

Positive values indicate addition; negative values indicate subtraction.

Interactions with manual group differences

- When doing manual contrasts, interactions have to be specified with each group contrast as well
- For example, adding interaction with age (0=85):

$$y_i = \beta_0 + \beta_1(dA_i) + \beta_2(dB_i) + \beta_3(dC_i) + \beta_4(Age_i - 85) + \beta_5(dA_i)(Age_i - 85) + \beta_7(dB_i)(Age_i - 85) + \beta_8(dC_i)(Age_i - 85) + e_i$$

```
ECHO 'Group by Age (0=85)'.
MIXED y WITH dA dB dC age /METHOD = REML /PRINT = SOLUTION TESTCOV
/FIXED = dA dB dC age dA*age dB*age dC*age
/TEST = "Omnibus main effect F-test" dA 1; dB 1; dC 1
/TEST = "Omnibus interaction F-test" dA*age 1; dB*age 1; dC*age 1
/TEST = "Age Slope for Control"            age 1 dA*age 0 dB*dage 0 dC*age 0
/TEST = "Age Slope for Treat A"            age 1 dA*age 1 dB*dage 0 dC*age 0
/TEST = "Age Slope for Treat B"            age 1 dA*age 0 dB*dage 1 dC*age 0
/TEST = "Age Slope for Treat C"            age 1 dA*age 0 dB*dage 0 dC*age 1
/TEST = "Age Slope: Control vs. Treat A"  dA*age  1 dB*dage  0 dC*age  0
/TEST = "Age Slope: Control vs. Treat B"  dA*age  0 dB*dage  1 dC*age  0
/TEST = "Age Slope: Control vs. Treat C"  dA*age  0 dB*dage  0 dC*age  1
/TEST = "Age Slope: Treat A vs. Treat B"  dA*age -1 dB*dage  1 dC*age  0
/TEST = "Age Slope: Treat A vs. Treat C"  dA*age -1 dB*dage  0 dC*age  1
/TEST = "Age Slope: Treat B vs. Treat C"  dA*age  0 dB*dage -1 dC*age  1.
```

Using BY/CLASS/i. statements instead

- Designate as "categorical" in program syntax
  - If you let SAS/SPSS do the dummy coding via CLASS/BY, then the **highest/last group is default reference**
    - Hard to change reference group (must re-code variable)
    - "Type III test of fixed effects" provide omnibus tests by default
    - LSMEANS/EMMEANS can be used to get all means and comparisons without specifying each individual contrast
  - If you let STATA do the dummy coding via i.group, then the **lowest/first group is reference**
    - Easy to change reference group, e.g., last = ref → ib(last).group
    - CONTRAST used to get omnibus tests instead of TEST
    - MARGINS can be used to get all means and comparisons with much less code than describing each individual contrast

Main Effects of "Categorical" Predictors

```
ECHO 'Differences among 4 groups'.
MIXED y BY treatgroup /METHOD = REML /PRINT = SOLUTION TESTCOV
/FIXED   = treatgroup
/EMMEANS = TABLES(treatgroup) COMPARE(treatgroup)
```

OR write all of the below instead of EMMEANS line… note that one value has to be given for each possible level of the categorical predictor

```
/TEST = "Control Mean"  intercept 1 treatgroup 1 0 0 0
/TEST = "T1 Mean"       intercept 1 treatgroup 0 1 0 0
/TEST = "T2 Mean"       intercept 1 treatgroup 0 0 1 0
/TEST = "T3 Mean"       intercept 1 treatgroup 0 0 0 1
```

Here, 1 means for that group only

```
/TEST = "Control vs. T1"      treatgroup  -1  1  0  0
/TEST = "Control vs. T2"      treatgroup  -1  0  1  0
/TEST = "Control vs. T3"      treatgroup  -1  0  0  1
/TEST = "T1 vs. T2"           treatgroup   0 -1  1  0
/TEST = "T1 vs. T3"           treatgroup   0 -1  0  1
/TEST = "T2 vs. T3"           treatgroup   0  0 -1  1
```

Contrasts must sum to 0; here −1 = ref, 1 = alt, and 0 = ignore

Can also make up whatever contrasts you feel like:

```
/TEST = "Treat ABC Mean"    intercept 1 treatgroup  0 1 1 1 | DIVISOR=3
/TEST = "Control vs. Mean of ABC"       treatgroup -3 1 1 1 | DIVISOR=3
```

Interactions with "Categorical" Predictors

```
ECHO 'Group by Age (same model via categorical group)'.
MIXED y BY treatgroup WITH age /METHOD = REML /PRINT = SOLUTION
/FIXED   = treatgroup age treatgroup*age
```

In requesting means, have to specify at what level of the other predictors:

```
/EMMEANS = TABLES(treatgroup) COMPARE(treatgroup) WITH(age=0)
```

In requesting anything, always have to say for what group(s):

```
/TEST = "Age Slope: Control"  age 1 treatgroup*age 1 0 0 0
/TEST = "Age Slope: Treat A"  age 1 treatgroup*age 0 1 0 0
/TEST = "Age Slope: Treat B"  age 1 treatgroup*age 0 0 1 0
/TEST = "Age Slope: Treat C"  age 1 treatgroup*age 0 0 0 1
```

Here, 1 means for that group only, but now it's referring to the age slope

```
/TEST = "Age Slope: Control vs. TA" treatgroup*age  -1  1  0  0
/TEST = "Age Slope: Control vs. TB" treatgroup*age  -1  0  1  0
/TEST = "Age Slope: Control vs. TC" treatgroup*age  -1  0  0  1
/TEST = "Age Slope: TA vs. TB"      treatgroup*age   0 -1  1  0
/TEST = "Age Slope: TA vs. TC"      treatgroup*age   0 -1  0  1
/TEST = "Age Slope: TB vs. TC"      treatgroup*age   0  0 -1  1
```

Contrasts must sum to 0; here −1 = ref, 1 = alt, and 0 = ignore

Can also make up whatever contrasts you feel like:

```
/TEST = "Age Slope: Mean across ABC"   age 1 treatgroup*age  0 1 1 1 | DIVISOR=3

/TEST = "Age Slope: Control vs ABC"          treatgroup*age -3 1 1 1 | DIVISOR=3
```

Categorical Predictors = Marginal Effects

- Letting the program build contrasts for categorical predictors (instead of creating manual dummy codes) does the following:
  - Allows LSMEANS/EMMEANS/MARGINS (for cell means and differences)
  - Provides omnibus (multiple df) group F-tests (or χ^2 tests)
  - **Marginalizes the group effect across interacting predictors**
    → omnibus F-tests represent marginal main effects (instead of simple)
  - e.g., **`/FIXED = Treatgroup Gender Treatgroup*Gender`**
    (in which Treatgroup is always "categorical")

Type 3 Tests of Fixed Effects	Interpretation if gender is "continuous"	Interpretation if gender is "categorical"
Gender	Marginal gender diff	Marginal gender diff
Treatgroup	Group diff if gender=0	Marginal group diff
Treatgroup*Gender	Interaction	Interaction

Interactions: $y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 X_i Z_i + e_i$

- **Interaction = Moderation:** the effect of a predictor depends on the value of the interacting predictor
- Interactions among categorical predictors are commonly evaluated (e.g., ANOVA), but by default:
  - Estimate all possible interactions among categorical predictors
    - Software does this for you; nonsignificant interactions usually still are kept in the model (even if only significant interactions are interpreted)
  - Omnibus marginal main effects are provided
    - But are basically useless if given significant interactions
  - Omnibus interaction effects are provided
    - But are basically useless in actually understanding the interaction
- Let's see how to make software give us more useful info…