Testing Measurement Invariance across Groups in Item Factor Models in Mplus version 7.11

Example data: 635 older adults (age 80-100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows. We are examining differences between men (*N*=214) and women (*N*=420).

1. Housework (cleaning and laundry), 2. Bedmaking, 3. Cooking, 4. Everyday shopping, 5. Getting to places outside of walking distance, 6. Handling banking and other business, 7. Using the telephone

Multiple Group IFA Model Syntax and Truncated Output:

```
TITLE:  Assess polytomous IADL items
DATA:   FILE IS ADL2.dat;
VARIABLE:   NAMES ARE case female cia1-cia7;
            USEVARIABLES ARE cia1-cia7;
            CATEGORICAL ARE cia1-cia7;
            GROUPING IS female (0=Men 1=Women);
            IDVARIABLE IS case;
            MISSING ARE .;

ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;

SAVEDATA:  DIFFTEST=Configural.dat; ! Save configural info

OUTPUT:    STDYX MODINDICES (3.84); !Constraints to drop p<.05

!!! CONFIGURAL MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7*;
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! CONFIGURAL MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all estimated
    IADL BY cia1-cia7*;
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       56

Chi-Square Test of Model Fit
          Value                             72.920*
          Degrees of Freedom                    28
          P-Value                           0.0000

Chi-Square Contributions From Each Group
          MEN                               24.977
          WOMEN                             47.943

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV
cannot be used for chi-square difference testing in the regular way.
MLM, MLR and WLSM chi-square difference testing is described on the
Mplus website.  MLMV, WLSMV, and ULSMV difference testing is done using
the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.071
          90 Percent C.I.                    0.051  0.091
          Probability RMSEA <= .05           0.040

CFI/TLI
          CFI                                0.999
          TLI                                0.998
```

This will serve as our baseline configural model.

56 parameters estimated = 2*[7 loadings + 21 thresholds] = 56

Possible parameters = 2* ([7*(7+1)] / 2] + 21 thresholds) = 98
DF = 98 – 56 – 14 “residuals” = 28

Model 1. Configural Invariance Model
(Everything separate across groups *except* for parameters needed to be constrained for identification)

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group MEN

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA				
IADL BY				
CIA1	5.876	1.474	3.986	0.000
CIA2	3.186	0.549	5.801	0.000
CIA3	3.090	0.445	6.946	0.000
CIA4	4.137	0.830	4.986	0.000
CIA5	2.578	0.375	6.884	0.000
CIA6	2.662	0.495	5.383	0.000
CIA7	1.115	0.218	5.119	0.000
Means: **MEAN OF THETA**				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-7.447	1.856	-4.012	0.000
CIA1$2	-5.134	1.374	-3.737	0.000
CIA1$3	-0.832	0.662	-1.257	0.209
CIA2$1	-4.485	0.639	-7.015	0.000
CIA2$2	-3.851	0.588	-6.548	0.000
CIA2$3	-2.265	0.489	-4.633	0.000
CIA3$1	-3.880	0.484	-8.025	0.000
CIA3$2	-2.810	0.451	-6.227	0.000
CIA3$3	-0.565	0.325	-1.737	0.082
CIA4$1	-5.182	0.925	-5.605	0.000
CIA4$2	-4.191	0.757	-5.537	0.000
CIA4$3	-2.175	0.567	-3.836	0.000
CIA5$1	-4.615	0.574	-8.041	0.000
CIA5$2	-2.623	0.375	-7.001	0.000
CIA5$3	-1.191	0.300	-3.972	0.000
CIA6$1	-4.602	0.708	-6.496	0.000
CIA6$2	-3.340	0.569	-5.866	0.000
CIA6$3	-2.232	0.473	-4.714	0.000
CIA7$1	-3.114	0.383	-8.121	0.000
CIA7$2	-2.376	0.279	-8.514	0.000
CIA7$3	-1.780	0.235	-7.591	0.000
Variances: **VARIANCE OF THETA**				
IADL	1.000	0.000	999.000	999.000
Residual Variances (ALL FIXED=1)				

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group WOMEN

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA				
IADL BY				
CIA1	3.084	0.305	10.123	0.000
CIA2	3.416	0.464	7.367	0.000
CIA3	4.883	0.919	5.313	0.000
CIA4	3.284	0.330	9.945	0.000
CIA5	2.202	0.196	11.258	0.000
CIA6	1.741	0.172	10.101	0.000
CIA7	1.056	0.156	6.786	0.000
Means: **MEAN OF THETA**				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-4.581	0.400	-11.450	0.000
CIA1$2	-3.289	0.336	-9.796	0.000
CIA1$3	-0.706	0.221	-3.189	0.001
CIA2$1	-5.425	0.623	-8.712	0.000
CIA2$2	-4.454	0.546	-8.158	0.000
CIA2$3	-2.801	0.442	-6.337	0.000
CIA3$1	-7.200	1.187	-6.064	0.000
CIA3$2	-6.136	1.067	-5.752	0.000
CIA3$3	-3.877	0.831	-4.666	0.000
CIA4$1	-4.479	0.377	-11.866	0.000
CIA4$2	-2.803	0.306	-9.166	0.000
CIA4$3	-0.677	0.237	-2.852	0.004
CIA5$1	-3.746	0.281	-13.345	0.000
CIA5$2	-1.686	0.190	-8.880	0.000
CIA5$3	-0.118	0.151	-0.780	0.435
CIA6$1	-3.202	0.243	-13.152	0.000
CIA6$2	-2.115	0.190	-11.134	0.000
CIA6$3	-1.173	0.162	-7.255	0.000
CIA7$1	-3.408	0.325	-10.500	0.000
CIA7$2	-2.712	0.233	-11.637	0.000
CIA7$3	-1.747	0.165	-10.580	0.000
Variances: **VARIANCE OF THETA**				
IADL	1.000	0.000	999.000	999.000
Residual Variances (ALL FIXED=1)				

Model 2a. Metric Invariance Model (IFA loadings held equal across groups – IRT discriminations can still vary via factor variances)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=Configural.dat;   ! Compare against configural

SAVEDATA:   DIFFTEST=MetricA.dat;      ! Save metric info

!!! METRIC MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! METRIC MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all NOW HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances STILL FIXED TO 1
    cia1-cia7@1;
! Factor mean=0 and variance NOW FREE
    [IADL@0]; IADL*;
```

```
Number of Free Parameters                       50

Chi-Square Test of Model Fit
          Value                             64.669*
          Degrees of Freedom                    34
          P-Value                           0.0012

Chi-Square Contributions From Each Group
          MEN                               30.849
          WOMEN                             33.820

THIS IS THE TEST OF METRIC INVARIANCE
Chi-Square Test for Difference Testing
          Value                              9.530
          Degrees of Freedom                     6
          P-Value                           0.1459

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.053
          90 Percent C.I.                    0.033  0.073
          Probability RMSEA <= .05           0.367

CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

The DIFFTEST chi-square is nonsignificant, and no modification indices for freeing loadings were indicated, so it looks like metric invariance holds between men and women.

In addition, the modification indices do not suggest removing any loading constraints, so we can proceed accordingly by testing scalar invariance.

Full metric invariance solution (factor loadings constrained)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group MEN				
IADL BY – CHANGE IN PROBIT FOR ONE-UNIT CHANGE IN THETA				
CIA1	3.915	0.397	9.868	0.000
CIA2	4.019	0.566	7.096	0.000
CIA3	4.771	0.697	6.847	0.000
CIA4	4.055	0.489	8.289	0.000
CIA5	2.601	0.283	9.191	0.000
CIA6	2.269	0.250	9.063	0.000
CIA7	1.210	0.166	7.292	0.000
Means				
IADL	0.000	0.000	999.000	999.000
Thresholds – EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-5.052	0.492	-10.262	0.000
CIA1$2	-3.481	0.485	-7.181	0.000
CIA1$3	-0.564	0.395	-1.429	0.153
CIA2$1	-5.566	0.691	-8.051	0.000
CIA2$2	-4.775	0.656	-7.279	0.000
CIA2$3	-2.809	0.536	-5.237	0.000
CIA3$1	-5.822	0.765	-7.612	0.000
CIA3$2	-4.225	0.699	-6.045	0.000
CIA3$3	-0.850	0.487	-1.744	0.081
CIA4$1	-5.085	0.575	-8.847	0.000
CIA4$2	-4.110	0.530	-7.754	0.000
CIA4$3	-2.138	0.459	-4.655	0.000
CIA5$1	-4.653	0.454	-10.242	0.000
CIA5$2	-2.644	0.330	-8.018	0.000
CIA5$3	-1.201	0.277	-4.332	0.000
CIA6$1	-4.011	0.399	-10.055	0.000
CIA6$2	-2.911	0.325	-8.966	0.000
CIA6$3	-1.945	0.292	-6.657	0.000
CIA7$1	-3.263	0.341	-9.556	0.000
CIA7$2	-2.489	0.243	-10.251	0.000
CIA7$3	-1.865	0.210	-8.898	0.000
Variances				
IADL	1.000	0.000	999.000	999.000
Residual Variances				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group WOMEN				
IADL BY				
CIA1	3.915	0.397	9.868	0.000
CIA2	4.019	0.566	7.096	0.000
CIA3	4.771	0.697	6.847	0.000
CIA4	4.055	0.489	8.289	0.000
CIA5	2.601	0.283	9.191	0.000
CIA6	2.269	0.250	9.063	0.000
CIA7	1.210	0.166	7.292	0.000
Means				
IADL	0.000	0.000	999.000	999.000
Thresholds				
CIA1$1	-4.819	0.407	-11.840	0.000
CIA1$2	-3.458	0.344	-10.063	0.000
CIA1$3	-0.744	0.231	-3.213	0.001
CIA2$1	-5.324	0.529	-10.069	0.000
CIA2$2	-4.372	0.471	-9.289	0.000
CIA2$3	-2.748	0.393	-6.997	0.000
CIA3$1	-5.913	0.620	-9.530	0.000
CIA3$2	-5.042	0.576	-8.761	0.000
CIA3$3	-3.186	0.485	-6.563	0.000
CIA4$1	-4.595	0.385	-11.923	0.000
CIA4$2	-2.877	0.315	-9.136	0.000
CIA4$3	-0.696	0.242	-2.873	0.004
CIA5$1	-3.696	0.263	-14.036	0.000
CIA5$2	-1.664	0.182	-9.164	0.000
CIA5$3	-0.116	0.149	-0.782	0.434
CIA6$1	-3.409	0.264	-12.910	0.000
CIA6$2	-2.251	0.203	-11.073	0.000
CIA6$3	-1.248	0.172	-7.278	0.000
CIA7$1	-3.326	0.291	-11.444	0.000
CIA7$2	-2.647	0.204	-13.001	0.000
CIA7$3	-1.705	0.145	-11.790	0.000
Variances				
IADL	0.693	0.154	4.492	0.000
Residual Variances				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

Model 3a. Full Threshold Invariance Model (IFA thresholds held equal across groups – IRT difficulties can still vary via factor diffs)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;

SAVEDATA:   DIFFTEST=ScalarA.dat;      ! Save full scalar info

!!! FULL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! FULL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds NOW HELD EQUAL IF LEFT OFF (LESS TYPING)



! Item residual variances STILL FIXED to 1
    cia1-cia7@1;
! Factor mean NOW FREE and variance STILL FREE
    [IADL*]; IADL*;
```

The DIFFTEST chi-square is significant, and the modification indices suggest that item 3 threshold 3 is the biggest problem. Let's see what happens when we free the item 3 threshold 3 between groups.

```
MODEL FIT INFORMATION
Number of Free Parameters                       30

Chi-Square Test of Model Fit
          Value                             153.911*
          Degrees of Freedom                    54
          P-Value                           0.0000

Chi-Square Contributions From Each Group
          MEN                                97.304
          WOMEN                              56.607

THIS IS THE TEST OF FULL METRIC VS. FULL SCALAR A INVARIANCE
Chi-Square Test for Difference Testing
          Value                             119.661
          Degrees of Freedom                    20
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.076
          90 Percent C.I.                    0.062  0.091
          Probability RMSEA <= .05           0.001

CFI/TLI
          CFI                                0.997
          TLI                                0.998


MODEL MODIFICATION INDICES
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA1     ]                 4.797    -0.711     -0.711       -0.174
[ CIA3     ]                46.520    -2.718     -2.718       -0.540
[ CIA4     ]                 9.640     1.037      1.037        0.244
[ CIA5     ]                24.673     1.118      1.118        0.396
[ CIA6     ]                 8.398     0.606      0.606        0.243
[ CIA1$3   ]                 4.082     0.504      0.504        0.123
[ CIA3$2   ]                 6.986     1.021      1.021        0.203
[ CIA3$3   ]                46.123     2.136      2.136        0.424
[ CIA4$3   ]                 7.420    -0.708     -0.708       -0.167
[ CIA5$2   ]                 7.205    -0.525     -0.525       -0.186
[ CIA5$3   ]                11.782    -0.561     -0.561       -0.199
[ CIA6$3   ]                 3.882    -0.319     -0.319       -0.128
```

Model 3b. Partial Threshold Invariance Model (freeing item 3 threshold 3 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;      ! Compare against metric

SAVEDATA:   DIFFTEST=ScalarB.dat;      ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 3 NOW FREE between groups
    [cia3$3*];


! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=FREE and variance STILL FREE
    [IADL*]; IADL*;
```

Threshold for Men…

CIA3$1	-6.680	0.798	-8.373	0.000
CIA3$2	-5.568	0.748	-7.448	0.000
CIA3$3	-0.891	0.512	-1.739	0.082

Threshold for Women…

CIA3$1	-6.680	0.798	-8.373	0.000
CIA3$2	-5.568	0.748	-7.448	0.000
CIA3$3	-4.146	0.683	-6.068	0.000

```
MODEL FIT INFORMATION
Number of Free Parameters                       31

Chi-Square Test of Model Fit
          Value                             115.561*
          Degrees of Freedom                    53
          P-Value                           0.0000

Chi-Square Contributions From Each Group
          MEN                                68.946
          WOMEN                              46.615

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR B INVARIANCE
Chi-Square Test for Difference Testing
          Value                              63.048
          Degrees of Freedom                     19
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.061
          90 Percent C.I.                    0.046  0.076
          Probability RMSEA <= .05           0.111

CFI/TLI
          CFI                                0.998
          TLI                                0.999

MODEL MODIFICATION INDICES
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA1     ]                 9.581    -1.016     -1.016       -0.247
[ CIA3     ]                10.724    -1.682     -1.682       -0.329
[ CIA4     ]                 4.838     0.748      0.748        0.173
[ CIA5     ]                16.673     0.934      0.934        0.326
[ CIA6     ]                 4.734     0.459      0.459        0.182
[ IADL     ]               999.000     0.000      0.000        0.000
[ CIA1$3   ]                 7.206     0.673      0.673        0.164
[ CIA3$2   ]                 9.284     1.200      1.200        0.235
[ CIA4$3   ]                 4.317    -0.546     -0.546       -0.127
[ CIA5$2   ]                 4.783    -0.434     -0.434       -0.152
[ CIA5$3   ]                 7.638    -0.456     -0.456       -0.159
```

The DIFFTEST chi-square is still significant, and the modification indices suggest that the other thresholds of item 3 are a problem. Let's see what happens when we free the item 3 threshold 2 between groups.

Model 3c. Partial Threshold Invariance Model (also freeing item 3 threshold 2 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;      ! Compare against metric

SAVEDATA:   DIFFTEST=ScalarC.dat;      ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3 FREE between groups
    [cia3$2* cia3$3*];



! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=FREE and variance STILL FREE
    [IADL*]; IADL*;
```

Thresholds for Men…

CIA3$1	-6.686	0.790	-8.465	0.000
CIA3$2	-4.374	0.726	-6.026	0.000
CIA3$3	-0.879	0.506	-1.735	0.083

Different Thresholds for Women…

CIA3$2	-6.060	0.761	-7.966	0.000
CIA3$3	-4.194	0.679	-6.179	0.000

MODEL FIT INFORMATION

Number of Free Parameters	32
Chi-Square Test of Model Fit	
Value	106.996*
Degrees of Freedom	52
P-Value	0.0000
Chi-Square Contributions From Each Group	
MEN	62.255
WOMEN	44.741

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR C INVARIANCE

Chi-Square Test for Difference Testing	
Value	51.190
Degrees of Freedom	18
P-Value	0.0000
RMSEA (Root Mean Square Error Of Approximation)	
Estimate	0.058
90 Percent C.I.	0.042 0.073
Probability RMSEA <= .05	0.195
CFI/TLI	
CFI	0.998
TLI	0.999

MODEL MODIFICATION INDICES

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
Group MEN				
Means/Intercepts/Thresholds				
[CIA1]	11.976	-1.145	-1.145	-0.279
[CIA2]	4.308	-0.732	-0.732	-0.172
[CIA5]	14.240	0.868	0.868	0.305
[CIA1$3]	**8.264**	**0.716**	**0.716**	**0.175**
[CIA5$2]	3.980	-0.393	-0.393	-0.138
[CIA5$3]	6.678	-0.423	-0.423	-0.149

The DIFFTEST chi-square is still significant, and the modification indices suggest that item 1 threshold 3 is a problem. Let's see what happens when we free the item 1 threshold 3 between groups.

Model 3d. Partial Threshold Invariance Model (also freeing item 1 threshold 3 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;       ! Compare against metric

SAVEDATA:   DIFFTEST=ScalarD.dat;      ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];


! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=FREE and variance STILL FREE
    [IADL*]; IADL*;
```

The DIFFTEST chi-square is still significant. Let's try item 5 threshold 3....

```
Number of Free Parameters                       33

Chi-Square Test of Model Fit
          Value                             99.865*
          Degrees of Freedom                   51
          P-Value                           0.0001

Chi-Square Contributions From Each Group
          MEN                               57.085
          WOMEN                             42.780

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR D INVARIANCE
Chi-Square Test for Difference Testing
          Value                             41.247
          Degrees of Freedom                   17
          P-Value                           0.0009

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.055
          90 Percent C.I.                    0.039  0.071
          Probability RMSEA <= .05           0.288

CFI/TLI
          CFI                                0.999
          TLI                                0.999

Thresholds for Men…
    CIA1$1            -5.528      0.494    -11.180      0.000
    CIA1$2            -4.139      0.459     -9.027      0.000
    CIA1$3            -0.579      0.405     -1.429      0.153

    CIA3$1            -6.880      0.823     -8.361      0.000
    CIA3$2            -4.464      0.747     -5.974      0.000
    CIA3$3            -0.895      0.516     -1.733      0.083

Different Thresholds for Women…
    CIA1$3            -1.673      0.408     -4.096      0.000
    CIA3$2            -6.267      0.794     -7.893      0.000
    CIA3$3            -4.399      0.712     -6.177      0.000


MODEL MODIFICATION INDICES
                                 M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA1     ]                    4.520    -0.898     -0.898       -0.217
[ CIA2     ]                    5.852    -0.869     -0.869       -0.201
[ CIA5     ]                   10.959     0.778      0.778        0.270
[ CIA2$3   ]                    4.321     0.584      0.584        0.135
[ CIA5$3   ]                    4.772    -0.360     -0.360       -0.125
```

Model 3e. Partial Threshold Invariance Model (also freeing item 5 threshold 3 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;       ! Compare against metric

SAVEDATA:   DIFFTEST=ScalarE.dat;      ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1,5 threshold 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];
    [cia5$3*];

! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=FREE and variance still FREE
    [IADL*]; IADL*;
```

The DIFFTEST chi-square is still significant. Let's try item 5 threshold 2....

```
Number of Free Parameters                       34

Chi-Square Test of Model Fit
          Value                             95.447*
          Degrees of Freedom                   50
          P-Value                           0.0001

Chi-Square Contributions From Each Group
          MEN                               53.258
          WOMEN                             42.189

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR E INVARIANCE
Chi-Square Test for Difference Testing
          Value                             35.350
          Degrees of Freedom                   16
          P-Value                           0.0036

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.054
          90 Percent C.I.                    0.037  0.070
          Probability RMSEA <= .05           0.341

CFI/TLI
          CFI                                0.999
          TLI                                0.999

Thresholds for Men…
    CIA1$1            -5.443      0.490    -11.110      0.000
    CIA1$2            -4.046      0.454     -8.919      0.000
    CIA1$3            -0.573      0.401     -1.429      0.153

    CIA3$1            -6.699      0.799     -8.382      0.000
    CIA3$2            -4.378      0.731     -5.992      0.000
    CIA3$3            -0.877      0.506     -1.732      0.083

    CIA5$1            -4.334      0.340    -12.752      0.000
    CIA5$2            -2.311      0.287     -8.043      0.000
    CIA5$3            -1.227      0.285     -4.298      0.000

Different Thresholds for Women…
    CIA1$3            -1.541      0.409     -3.769      0.000
    CIA3$2            -6.074      0.770     -7.889      0.000
    CIA3$3            -4.209      0.691     -6.090      0.000
    CIA5$3            -0.658      0.278     -2.370      0.018

MODEL MODIFICATION INDICES
                          M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA2     ]             4.616    -0.773     -0.773       -0.181
[ CIA5     ]             6.245     0.756      0.756        0.266
[ CIA5$2   ]             3.920    -0.383     -0.383       -0.135
```

Model 3f. Partial Threshold Invariance Model (also freeing item 5 threshold 2 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;       ! Compare against metric

SAVEDATA:   DIFFTEST=ScalarF.dat;      ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];
    [cia5$2* cia5$3*];

! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=FREE and variance still FREE
    [IADL*]; IADL*;
```

```
Number of Free Parameters                       35

Chi-Square Test of Model Fit
          Value                             91.604*
          Degrees of Freedom                   49
          P-Value                           0.0002

Chi-Square Contributions From Each Group
          MEN                               50.317
          WOMEN                             41.287

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR F INVARIANCE
Chi-Square Test for Difference Testing
          Value                             29.886
          Degrees of Freedom                   15
          P-Value                           0.0123

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.052
          90 Percent C.I.                    0.035  0.069
          Probability RMSEA <= .05           0.387

CFI/TLI
          CFI                                0.999
          TLI                                0.999


MODEL MODIFICATION INDICES
                            M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA4     ]               4.975     0.802      0.802        0.188
[ CIA6     ]               4.570     0.469      0.469        0.187
[ CIA4$3   ]               4.529    -0.536     -0.536       -0.125
```

The DIFFTEST chi-square is still significant. Let's try item 4 threshold 3....

Model 3g. Partial Threshold Invariance Model (also freeing item 4 threshold 3 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricA.dat;       ! Compare against metric

SAVEDATA:   DIFFTEST=ScalarG.dat;      ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];
    [cia5$2* cia5$3*];
    [cia4$3*];

! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=FREE and variance still FREE
    [IADL*]; IADL*;
```

```
Number of Free Parameters                       36

Chi-Square Test of Model Fit
          Value                             87.391*
          Degrees of Freedom                   48
          P-Value                           0.0004

Chi-Square Contributions From Each Group
          MEN                               46.648
          WOMEN                             40.742

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR G INVARIANCE
Chi-Square Test for Difference Testing
          Value                             24.426
          Degrees of Freedom                   14
          P-Value                           0.0407

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.051
          90 Percent C.I.                    0.033  0.068
          Probability RMSEA <= .05           0.444

CFI/TLI
          CFI                                0.999
          TLI                                0.999


MODEL MODIFICATION INDICES
                          M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA6     ]              6.640     0.574      0.574        0.231
```

Although the DIFFTEST chi-square is still technically significant, no other modifications to un-constrain the remaining thresholds will improve fit.

I'm calling it done.

Model 4a. Partial Residual Variance Invariance Model (thresholds unconstrained between groups from ScalarG)
This last step for testing measurement invariance proceeds backwards.
Because freeing the residual variances is adding parameters, we must estimate this free residuals model first.

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;

SAVEDATA:   DIFFTEST=ResidualFreeA.dat;  ! Save free residual info

!!! RESIDUAL FIXED MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! RESIDUAL FREE MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];
    [cia5$2* cia5$3*];
    [cia4$3*];

! Item residual variances NOW FREE
    cia1-cia7*;


! Factor mean=FREE and variance still FREE
    [IADL*]; IADL*;
```

```
(Number of Free Parameters                       43

Chi-Square Test of Model Fit
          Value                             80.223*
          Degrees of Freedom                   41
          P-Value                           0.0002

Chi-Square Contributions From Each Group
          MEN                               34.586
          WOMEN                             45.636

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.055
          90 Percent C.I.                    0.037  0.073
          Probability RMSEA <= .05           0.305

CFI/TLI
          CFI                                0.999
          TLI                                0.999

Residual Variances for Men…
    CIA1               1.000      0.000    999.000    999.000
    CIA2               1.000      0.000    999.000    999.000
    CIA3               1.000      0.000    999.000    999.000
    CIA4               1.000      0.000    999.000    999.000
    CIA5               1.000      0.000    999.000    999.000
    CIA6               1.000      0.000    999.000    999.000
    CIA7               1.000      0.000    999.000    999.000

Residual Variances for Women…
    CIA1               2.243      1.286      1.744      0.081
    CIA2               0.538      0.224      2.406      0.016
    CIA3               0.264      0.118      2.239      0.025
    CIA4               1.063      0.540      1.970      0.049
    CIA5               0.940      0.378      2.490      0.013
    CIA6               1.627      0.639      2.546      0.011
    CIA7               0.725      0.192      3.768      0.000
```

Model 4b. Residual Variance Invariance Model (testing all items)

We now constrain the residual variances to be equal between groups and test decrease in fit.

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=ResidualFreeA.dat;  ! Compare to free residual

SAVEDATA:   DIFFTEST=ResidualFixedB.dat;  ! Save fixed residual info

!!! RESIDUAL FIXED MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! RESIDUAL FREE MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];
    [cia5$2* cia5$3*];
    [cia4$3*];
! Item residual variances NOW FIXED
    cia1-cia7@1;

! Factor mean=FREE and variance still FREE
    [IADL*]; IADL*;
```

Because DIFFTEST is nonsignificant, we can constrain the residual variances for the invariant items to be the same. Now we are ready to examine structural invariance (equality of the factor variance and factor mean).

```
(Model fit is same as Scalar G)

TEST OF PARTIAL SCALAR G VS. RESIDUAL VARIANCE INVARIANCE
Chi-Square Test for Difference Testing
          Value                             14.319
          Degrees of Freedom                    7
          P-Value                           0.0458

MODEL MODIFICATION INDICES
                          M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Variances/Residual Variances
CIA3                     9.421     1.755      1.755        0.072
CIA6                     7.699    -0.670     -0.670       -0.108
```

After freeing residual variance for item 3….

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=ResidualFreeA.dat;  ! Compare to free residual

SAVEDATA:   DIFFTEST=ResidualFixedC.dat;  ! Save fixed residual info

MODEL WOMEN:
! Item residual variances NOW FIXED except for item 3
    cia1-cia7@1;
    cia3*;

Number of Free Parameters                       37
Chi-Square Test of Model Fit
          Value                             79.843*
          Degrees of Freedom                    47
          P-Value                           0.0020

Chi-Square Contributions From Each Group
          MEN                               39.742
          WOMEN                             40.101

TEST OF PARTIAL SCALAR G VS. PARTIAL RESIDUAL VARIANCE INVARIANCE

Chi-Square Test for Difference Testing
          Value                              8.930
          Degrees of Freedom                     6
          P-Value                           0.1776

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.047
          90 Percent C.I.                    0.028  0.064
          Probability RMSEA <= .05           0.591

CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

Final Model: Partial Measurement Invariance (solution from Model 4c)

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group MEN

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA				
IADL BY				
CIA1	4.227	0.463	9.137	0.000
CIA2	4.054	0.561	7.222	0.000
CIA3	3.287	0.488	6.730	0.000
CIA4	4.197	0.509	8.253	0.000
CIA5	2.729	0.304	8.977	0.000
CIA6	2.346	0.263	8.915	0.000
CIA7	1.270	0.176	7.208	0.000
Means: **MEAN OF THETA**				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED PROBIT OF LOWER CATEGORY WHEN THETA=0				
CIA1$1	-5.634	0.572	-9.848	0.000
CIA1$2	-4.185	0.519	-8.065	0.000
CIA1$3	**-0.606**	**0.425**	**-1.426**	**0.154**
CIA2$1	-5.787	0.642	-9.016	0.000
CIA2$2	-4.914	0.600	-8.192	0.000
CIA2$3	-3.261	0.532	-6.131	0.000
CIA3$1	-4.344	0.567	-7.664	0.000
CIA3$2	**-2.975**	**0.487**	**-6.104**	**0.000**
CIA3$3	**-0.598**	**0.347**	**-1.725**	**0.085**
CIA4$1	-5.283	0.543	-9.738	0.000
CIA4$2	-3.755	0.486	-7.719	0.000
CIA4$3	**-2.206**	**0.478**	**-4.619**	**0.000**
CIA5$1	-4.330	0.354	-12.225	0.000
CIA5$2	-2.756	0.353	-7.803	0.000
CIA5$3	-1.252	0.293	-4.269	0.000
CIA6$1	-3.873	0.325	-11.932	0.000
CIA6$2	-2.737	0.287	-9.544	0.000
CIA6$3	-1.754	0.267	-6.566	0.000
CIA7$1	-3.475	0.284	-12.224	0.000
CIA7$2	-2.761	0.222	-12.445	0.000
CIA7$3	-1.934	0.183	-10.562	0.000
Variances: **VARIANCE OF THETA**				
IADL	1.000	0.000	999.000	999.000
Residual Variances (ALL FIXED=1)				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group WOMEN

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA					
IADL BY					
CIA1	4.227	0.463	9.137	0.000	
CIA2	4.054	0.561	7.222	0.000	
CIA3	3.287	0.488	6.730	0.000	
CIA4	4.197	0.509	8.253	0.000	
CIA5	2.729	0.304	8.977	0.000	
CIA6	2.346	0.263	8.915	0.000	
CIA7	1.270	0.176	7.208	0.000	
Means: **MEAN DIFFERENCE OF THETA IN WOMEN**					
IADL	**-0.194**	**0.106**	**-1.833**	**0.067**	
Thresholds: NEGATIVE OF EXPECTED PROBIT WHEN THETA=0					
CIA1$1	-5.634	0.572	-9.848	0.000	
CIA1$2	-4.185	0.519	-8.065	0.000	
CIA1$3	**-1.570**	**0.466**	**-3.369**	**0.001**	**housework**
CIA2$1	-5.787	0.642	-9.016	0.000	
CIA2$2	-4.914	0.600	-8.192	0.000	
CIA2$3	-3.261	0.532	-6.131	0.000	bed making
CIA3$1	-4.344	0.567	-7.664	0.000	
CIA3$2	**-3.866**	**0.534**	**-7.234**	**0.000**	
CIA3$3	**-2.677**	**0.415**	**-6.447**	**0.000**	**cooking**
CIA4$1	-5.283	0.543	-9.738	0.000	
CIA4$2	-3.755	0.486	-7.719	0.000	
CIA4$3	**-1.491**	**0.471**	**-3.164**	**0.002**	**shopping**
CIA5$1	-4.330	0.354	-12.225	0.000	
CIA5$2	**-2.173**	**0.313**	**-6.949**	**0.000**	
CIA5$3	**-0.643**	**0.300**	**-2.144**	**0.032**	**get around**
CIA6$1	-3.873	0.325	-11.932	0.000	
CIA6$2	-2.737	0.287	-9.544	0.000	
CIA6$3	-1.754	0.267	-6.566	0.000	banking
CIA7$1	-3.475	0.284	-12.224	0.000	
CIA7$2	-2.761	0.222	-12.445	0.000	
CIA7$3	-1.934	0.183	-10.562	0.000	telephone
Variances: **VARIANCE OF THETA**					
IADL	0.612	0.137	4.461	0.000	
Residual Variances (NOT ALL FIXED=1)					
CIA1	1.000	0.000	999.000	999.000	
CIA2	1.000	0.000	999.000	999.000	
CIA3	0.267	0.123	2.160	0.031	
CIA4	1.000	0.000	999.000	999.000	
CIA5	1.000	0.000	999.000	999.000	
CIA6	1.000	0.000	999.000	999.000	
CIA7	1.000	0.000	999.000	999.000	

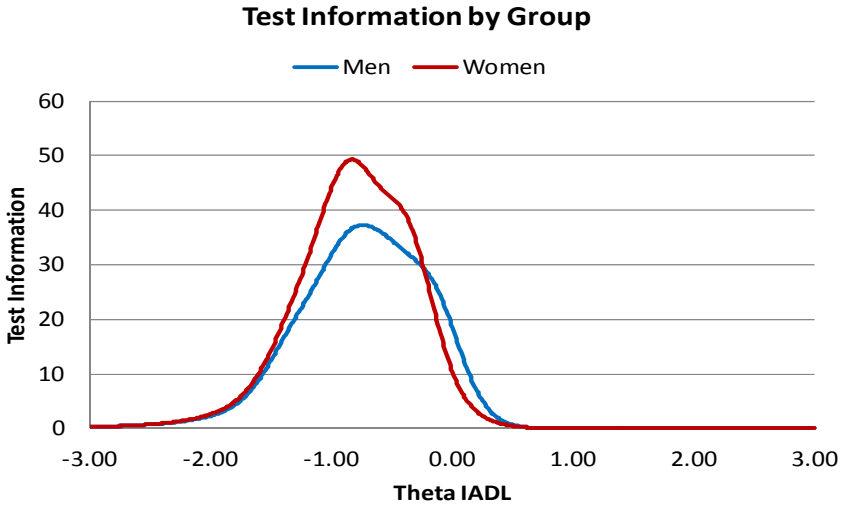
Model 5a. Testing Factor Variance Invariance

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=ResidualFixedC.dat; ! Compare to fixed residual

SAVEDATA:

!!! RESIDUAL FIXED MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! RESIDUAL FREE MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
           cia2* (L2)
           cia3* (L3)
           cia4* (L4)
           cia5* (L5)
           cia6* (L6)
           cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups
    [cia3$2* cia3$3*];
    [cia1$3*];
    [cia5$2* cia5$3*];
    [cia4$3*];

! Item residual variances NOW FIXED except for item 3
    cia1-cia7@1;
    cia3*;
! Factor mean=FREE and variance NOW FIXED
    [IADL*]; IADL@1;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       36

Chi-Square Test of Model Fit

          Value                             90.531*
          Degrees of Freedom                   48
          P-Value                           0.0002

Chi-Square Contributions From Each Group
          MEN                               43.802
          WOMEN                             46.729

THIS IS THE TEST OF FACTOR VARIANCE INVARIANCE
Chi-Square Test for Difference Testing
          Value                              6.183
          Degrees of Freedom                     1
          P-Value                           0.0129

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.053
          90 Percent C.I.                    0.036  0.069
          Probability RMSEA <= .05           0.369

CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

Because DIFFTEST is significant, this means that the women have significantly less variance in the latent factor than men. Further, we know from the previous model solution that the factor mean for women was marginally different from 0 (the factor mean for men), and thus we don't have to test it anyway. So we call it done, and let the theta mean and variance differ between groups.

Figures from Partial Measurement Invariance Model 4c:



STDYX Solution

MEN: IADL BY				
CIA1	0.973	0.006	172.380	0.000
CIA2	0.971	0.008	125.932	0.000
CIA3	0.957	0.012	79.466	0.000
CIA4	0.973	0.006	153.657	0.000
CIA5	0.939	0.012	75.814	0.000
CIA6	0.920	0.016	57.966	0.000
CIA7	0.786	0.042	18.826	0.000
WOMEN: IADL BY				
CIA1	0.957	0.008	119.985	0.000
CIA2	0.954	0.010	99.084	0.000
CIA3	0.980	0.007	132.080	0.000
CIA4	0.957	0.008	124.012	0.000
CIA5	0.906	0.013	69.084	0.000
CIA6	0.878	0.019	45.773	0.000
CIA7	0.705	0.044	15.901	0.000

Because women have less theta variance, the same unstandardized factor loadings as men imply greater relationship of the items to the latent trait (and thus greater information) in women.

The thresholds predict the probit of y=lower response for something with Theta=0.

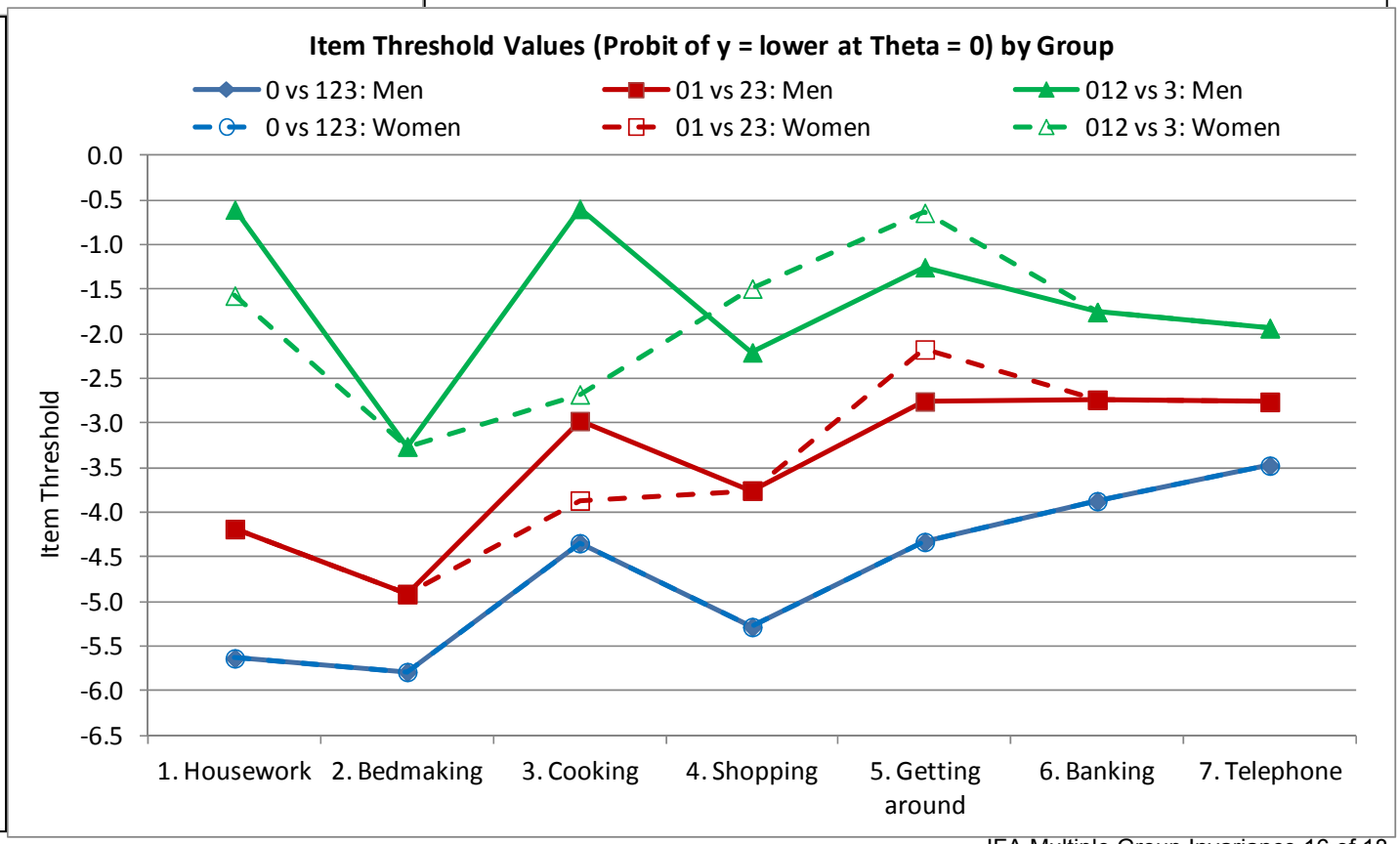
So, holding IADL ability constant at the combined sample factor mean of 0….

Item 1: Women have a **lower** probability of not being able to do housework than men.

Item 3: Women have a **lower** probability of not being able to do cook than men.

Item 4: Women have a **greater** probability of not being able to shop than men.

Item 5: Women have a **greater** probability of not being able to get around than men.



Example write-up of these IFA analyses:

The extent to which am item factor model measuring independent daily living (with seven observed items) exhibited measurement invariance and structural invariance between men and women was examined using Mplus v. 7.11 (Muthén & Muthén, 1998-2012). WLSMV estimation including a probit link and the THETA parameterization was used to estimate all models (Muthén & Muthén, 1998-2012). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items for each group. Nested model comparisons were conducted using the DIFFTEST procedure. A configural invariance model was initially specified in which a single factor was estimated simultaneously in each group. The factor variance was fixed to 1 and the factor mean was fixed to 0 in each group for identification, such that all item factor loadings (one per item) and thresholds (three per item given four response options) were then estimated. The residual variances are not uniquely identified in the configural invariance model and as such were all constrained to 1 in both groups. As shown in Table 1, the configural invariance model had good fit. The analysis proceeded by applying parameter constraints in successive models to examine potential decreases in fit resulting from measurement or structural non-invariance between men and women, with men as the reference group.

Equality of the unstandardized item factor loadings between groups was then examined in a metric invariance model. The factor variance was fixed to 1 in men for identification but was freely estimated in women; the factor mean was fixed to 0 in both groups for identification. All factor loadings were constrained equal across groups, all item thresholds were estimated, and all residual variances were constrained to 1 across groups. The metric invariance model did not fit significantly worse than the configural invariance model, DIFFTEST (6) = 9.53, p = .15. The modification indices did not suggest any points of localized misfit for the constrained loadings. The fact that metric invariance (i.e., “weak invariance”) held indicates that the items were related to the latent factor equivalently across groups, or more simply, that the same latent factor was being measured in each group. However, because the factor variances were permitted to vary across groups (such that women showed less variability in the latent factor), the “a” discrimination parameters from the item response version of the model were not equivalent across groups (i.e., the equality constraints were applied to the factor loadings only).

Equality of the unstandardized item thresholds across groups was then examined in a scalar invariance model. The factor variance and mean were fixed to 1 and 0, respectively, in men for identification, but the factor variance and mean were then estimated for women. All factor loadings and item thresholds were constrained equal across groups; all residual variances were still constrained equal to 1 in both groups. The full scalar invariance model A fit significantly worse than the metric invariance model, DIFFTEST (20) = 119.66, p < .001. The modification indices suggested that threshold 3 of item 3 was the largest source of the misfit and should be freed. After doing so, the partial scalar invariance model B still had significantly worse fit than the full metric invariance model, DIFFTEST (19) = 63.05, p < 001. The modification indices suggested that threshold 2 of item 3 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model C (with thresholds 2 and 3 for item 3 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (18) = 51.19, p < 001. The modification indices suggested that threshold 3 of item 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model D (with the thresholds 2 and 3 for item 3 and threshold 3 for item 1 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (17) = 41.247, p < .001. The modification indices suggested that threshold 3 of item 5 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model E (with the thresholds 2 and 3 for item 3 and threshold 3 for items 1 and 5 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (16) = 35.35, p < .004. The modification indices suggested that threshold 2 of item 5 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model F (with the thresholds 2 and 3 for items 3 and 5 and threshold 3 for item 1 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (15) = 29.89, p = .012. The modification indices suggested that threshold 3 of item 4 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model G (with the thresholds 2 and 3 for items 3 and 5 and threshold 3 for items 1, 4 and 5 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (14) =

24.43, p = .041. However, the modification indices did not indicate any remaining sources of misfit due to constrained thresholds, and thus the partial scalar invariance model G was retained. The factor that partial scalar invariance (i.e., “strong invariance”) held indicates that items 2, 6, and 7 have the same expected response at the same absolute level of the trait, or more simply, that the observed differences in the proportion of responses in each category for those items was due to factor mean differences only. However, at the same absolute level of the IADL factor, threshold 3 for item 1 and thresholds 2 and 3 for item 3 were more difficult for men, indicating that men have a greater probability of not being able to do housework or cook, whereas threshold 3 for item 4 and thresholds 2 and 3 for item 5 were less difficult in men, indicating that men have a lower probability of not being able to shop or get around. Because the factor mean was permitted to vary across groups (such that women were less able overall than men), the “b” parameters from the item response version of the model were not equivalent across groups (i.e., the equality constraints were applied to the item thresholds only).

Equality of the unstandardized residual variances across groups was then examined in a residual variance invariance model. The model comparison at this step proceeded backwards, such that a model with all residual variances freely estimated in the women was fitted first, and then compared with a model in which all residual variances were fixed to 1 in the women. The residual variances in the men were all fixed to 1 for identification in both models, and the rest of the model parameters were estimated as described for the last partial scalar invariance model G. The model with the residual variances constrained to 1 (to be equal to the men) fit significantly worse than the model with those residual variances freed, DIFFTEST (7) = 14.32, p = .046. The modification indices suggested that the residual variance for item 3 the largest remaining source of misfit and should be freed. After doing so, the new partial residual variance invariance model B did not fit significantly worse than the partial scalar invariance model G, DIFFTEST (6) = 8.93, p = .178, indicating that residual variance for item 3 was significantly smaller for women than men. The fact that partial residual variance invariance (i.e., “strict invariance”) held indicates that the amount of item variance not accounted for by the factor was the same across groups in all other items.

After achieving partial measurement invariance as was just described, structural invariance was then tested with one additional model. The factor variance in the women (which had been estimated freely) was constrained to 1 (i.e., to be equal to the factor variance in men), resulting in a significant decrease in fit relative to the last partial residual invariance model B, DIFFTEST (1) = 6.18, p = .013. Thus, women showed significantly less variability in ability to live independently (factor variance of 0.61) than did men (factor variance of 1.0). The factor mean for women in the partial measurement invariance model was marginally different from 0 (difference = −0.19, SE = 0.11, p = .067), indicating that women were marginally less able to live independently than men (factor mean of 0). Test information functions were similar across groups (although slightly larger for women between Theta = −1 and 0), and indicated acceptable reliability (i.e., information > 4) only between a Theta of −2 and 0. Thus, high-functioning individuals will not be measured adequately with these seven items.

In conclusion, these analyses showed that partial measurement invariance was obtained across men and women – that is, the relationships of the items to the latent factor of independent living were equivalent between men women. However, items 1 and 3 (housework and cooking) were systematically more difficult for men than women at the same level of the latent trait, whereas items 4 and 5 (shopping and getting around) was more difficult for women than men. These analyses also showed that structural invariance was not obtained, such that women were less variable and less able on average than men. Model parameters from the final model are given in Table 2.

Table 1 provides fit of each model

Table 2 provides final model parameters