Testing Measurement Invariance across Groups in Item Factor Models in Mplus version 7.11

Example data: 635 older adults (age 80-100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows. We are examining differences between men (N=214) and women (N=420).


Multiple Group IFA Model Syntax and Truncated Output:

```plaintext
TITLE: Assess polytomous IADL items
DATA: FILE IS ADL2.dat;
VARIABLE: NAMES ARE case female cia1-cia7;
USEVARIABLES ARE cia1-cia7;
CATEGORICAL ARE cia1-cia7;
GROUPING IS female (0=Men 1=Women);
IDVARIABLE IS case;
MISSING ARE .;

ANALYSIS: ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
SAVEDATA: DIFFTEST=Configural.dat; ! Save configural info
OUTPUT: STDYX MODINDICES (3.84); !Constraints to drop p<.05

!!! CONFIGURAL MODEL FOR MEN REFERENCE GROUP MODEL:
! Factor loadings all estimated
IADL BY cia1-cia7*;
! Item thresholds all free
[cia1$1-cia7$1*];
[cia1$2-cia7$2*];
[cia1$3-cia7$3*];
! Item residual variances all fixed=1
cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

!!! CONFIGURAL MODEL FOR WOMEN ALTERNATIVE GROUP MODEL WOMEN:
! Factor loadings all estimated
IADL BY cia1-cia7*;
! Item thresholds all free
[cia1$1-cia7$1*];
[cia1$2-cia7$2*];
[cia1$3-cia7$3*];
! Item residual variances all fixed=1
cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

MODEL FIT INFORMATION
Number of Free Parameters 56
Chi-Square Test of Model Fit
Value 72.920*
Degrees of Freedom 28
P-Value 0.0000
Chi-Square Contributions From Each Group
MEN 24.977
WOMEN 47.943

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLMV, MLR and WLSMV chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
Estimate 0.071
90 Percent C.I. 0.051 0.091
Probability RMSEA <= .05 0.040

CFI/TLI
CFI 0.999
TLI 0.998

This will serve as our baseline configural model.

56 parameters estimated = 2*[7 loadings + 21 thresholds] = 56

Possible parameters = 2* ([7*(7+1)] / 2] + 21 thresholds) = 98
DF = 98 – 56 – 14 "residuals" = 28

IFA Multiple Group Invariance 1 of 18
Model 1. Configural Invariance Model
(Everything separate across groups *except* for parameters needed to be constrained for identification)

<table>
<thead>
<tr>
<th>UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)</th>
<th>UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)</th>
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<td><strong>Residual Variances (ALL FIXED=1)</strong></td>
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Model 2a. Metric Invariance Model (IFA loadings held equal across groups – IRT discriminations can still vary via factor variances)

**ANALYSIS:** ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;  
DIFFTEST=Configural.dat; ! Compare against configural  
SAVEDATA: DIFFTEST=MetricA.dat; ! Save metric info  

!!! METRIC MODEL FOR MEN REFERENCE GROUP  
MODEL:  
! Factor loadings all estimated  
IADL BY cia1* (L1)  
cia2* (L2)  
cia3* (L3)  
cia4* (L4)  
cia5* (L5)  
cia6* (L6)  
cia7* (L7);  
! Item thresholds all free  
[cia1$1-cia7$1*];  
[cia1$2-cia7$2*];  
[cia1$3-cia7$3*];  
! Item residual variances all fixed=1  
cia1-cia7@1;  
! Factor mean=0 and variance=1 for identification  
[IADL@0]; IADL@1;  

!!! METRIC MODEL FOR WOMEN ALTERNATIVE GROUP  
MODEL WOMEN:  
! Factor loadings all NOW HELD EQUAL  
IADL BY cia1* (L1)  
cia2* (L2)  
cia3* (L3)  
cia4* (L4)  
cia5* (L5)  
cia6* (L6)  
cia7* (L7);  
! Item thresholds all free  
[cia1$1-cia7$1*];  
[cia1$2-cia7$2*];  
[cia1$3-cia7$3*];  
! Item residual variances STILL FIXED TO 1  
cia1-cia7@1;  
! Factor mean=0 and variance NOW FREE  
[IADL@0]; IADL*;  

Number of Free Parameters 50  
Chi-Square Test of Model Fit  
Value 64.669*  
Degrees of Freedom 34  
P-Value 0.0012  

Chi-Square Contributions From Each Group  
MEN 30.849  
WOMEN 33.820  

**THIS IS THE TEST OF METRIC INVARIANCE**  
Chi-Square Test for Difference Testing  
Value 9.530  
Degrees of Freedom 6  
P-Value 0.367  

RMSEA (Root Mean Square Error Of Approximation)  
Estimate 0.053  
90 Percent C.I. 0.033 0.073  
Probability RMSEA <= .05 0.367  

CFI/TLI  
CFI 0.999  
TLI 0.999  

The DIFFTEST chi-square is nonsignificant, and no modification indices for freeing loadings were indicated, so it looks like metric invariance holds between men and women.  

In addition, the modification indices do not suggest removing any loading constraints, so we can proceed accordingly by testing scalar invariance.
### Full metric invariance solution (factor loadings constrained)

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<th>P-Value</th>
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**Means**

| IADL | 0.000 | 0.000 | 999.000 | 999.000 |

**Thresholds**

| CIA1$1 | -5.052 | 0.492 | -10.262 | 0.000 |
| CIA1$2 | -3.481 | 0.485 | -7.181 | 0.000 |
| CIA1$3 | -0.564 | 0.395 | -1.429 | 0.153 |
| CIA2$1 | -5.566 | 0.611 | -8.051 | 0.000 |
| CIA2$2 | -4.775 | 0.656 | -7.279 | 0.000 |
| CIA2$3 | -2.809 | 0.536 | -5.237 | 0.000 |
| CIA3$1 | -5.822 | 0.765 | -7.612 | 0.000 |
| CIA3$2 | -4.225 | 0.699 | -6.045 | 0.000 |
| CIA3$3 | -0.850 | 0.487 | -1.744 | 0.081 |
| CIA4$1 | -5.085 | 0.575 | -8.847 | 0.000 |
| CIA4$2 | -4.110 | 0.530 | -7.754 | 0.000 |
| CIA4$3 | -2.138 | 0.459 | -4.655 | 0.000 |
| CIA5$1 | -4.653 | 0.454 | -10.242 | 0.000 |
| CIA5$2 | -2.644 | 0.330 | -8.018 | 0.000 |
| CIA5$3 | -1.201 | 0.277 | -4.332 | 0.000 |
| CIA6$1 | -4.011 | 0.399 | -10.055 | 0.000 |
| CIA6$2 | -2.911 | 0.325 | -8.966 | 0.000 |
| CIA6$3 | -1.945 | 0.292 | -6.657 | 0.000 |
| CIA7$1 | -3.263 | 0.341 | -9.556 | 0.000 |
| CIA7$2 | -2.489 | 0.243 | -10.251 | 0.000 |
| CIA7$3 | -1.865 | 0.210 | -8.898 | 0.000 |

**Variances**

| IADL | 1.000 | 0.000 | 999.000 | 999.000 |

**Residual Variances**

| CIA1 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA2 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA3 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA4 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA5 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA6 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA7 | 1.000 | 0.000 | 999.000 | 999.000 |

<table>
<thead>
<tr>
<th>Group</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WOMEN</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IADL</td>
<td>3.915</td>
<td>0.397</td>
<td>9.868</td>
<td>0.000</td>
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<td>CIA1</td>
<td>4.019</td>
<td>0.566</td>
<td>7.096</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA2</td>
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<td>0.697</td>
<td>6.847</td>
<td>0.000</td>
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<tr>
<td>CIA3</td>
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<td>8.289</td>
<td>0.000</td>
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<td>CIA4</td>
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<td>0.283</td>
<td>9.191</td>
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<tr>
<td>CIA5</td>
<td>2.269</td>
<td>0.250</td>
<td>9.063</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA6</td>
<td>1.210</td>
<td>0.166</td>
<td>7.292</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Means**

| IADL | 0.000 | 0.000 | 999.000 | 999.000 |

**Thresholds**

| CIA1$1 | -4.819 | 0.407 | -11.840 | 0.000 |
| CIA1$2 | -3.458 | 0.344 | -10.063 | 0.000 |
| CIA1$3 | -0.744 | 0.231 | -3.213 | 0.001 |
| CIA2$1 | -5.324 | 0.529 | -10.069 | 0.000 |
| CIA2$2 | -4.372 | 0.471 | -9.289 | 0.000 |
| CIA2$3 | -2.748 | 0.393 | -6.997 | 0.000 |
| CIA3$1 | -5.913 | 0.620 | -9.530 | 0.000 |
| CIA3$2 | -5.042 | 0.576 | -8.761 | 0.000 |
| CIA3$3 | -3.186 | 0.485 | -6.663 | 0.000 |
| CIA4$1 | -4.595 | 0.385 | -11.923 | 0.000 |
| CIA4$2 | -2.877 | 0.315 | -9.136 | 0.000 |
| CIA4$3 | -0.696 | 0.242 | -2.873 | 0.004 |
| CIA5$1 | -3.696 | 0.263 | -14.036 | 0.000 |
| CIA5$2 | -1.664 | 0.182 | -9.164 | 0.000 |
| CIA5$3 | -0.116 | 0.149 | -0.782 | 0.434 |
| CIA6$1 | -3.010 | 0.264 | -11.703 | 0.000 |
| CIA6$2 | -2.251 | 0.203 | -11.707 | 0.000 |
| CIA6$3 | -1.248 | 0.172 | -7.278 | 0.000 |
| CIA7$1 | -3.326 | 0.291 | -11.444 | 0.000 |
| CIA7$2 | -2.647 | 0.204 | -13.001 | 0.000 |
| CIA7$3 | -1.705 | 0.145 | -11.790 | 0.000 |

**Variances**

| IADL | 0.693 | 0.154 | 4.492 | 0.000 |

**Residual Variances**

| CIA1 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA2 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA3 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA4 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA5 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA6 | 1.000 | 0.000 | 999.000 | 999.000 |
| CIA7 | 1.000 | 0.000 | 999.000 | 999.000 |
Model 3a. Full Threshold Invariance Model (IFA thresholds held equal across groups – IRT difficulties can still vary via factor diffs)

ANALYSIS:
- ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
- DIFFTEST=MetricA.dat;

SAVEDATA:
- DIFFTEST=ScalarA.dat; ! Save full scalar info

!! FULL SCALAR MODEL FOR MEN REFERENCE GROUP

MODEL:
- Factor loadings all estimated
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- Item thresholds all free
  - [cia1$1-cia7$1*];
  - [cia1$2-cia7$2*];
  - [cia1$3-cia7$3*];
- Item residual variances all fixed=1
  - cia1-cia7@1;
- Factor mean=0 and variance=1 for identification
  - [IADL@0]; IADL@1;

!! FULL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP

MODEL WOMEN:
- Factor loadings all STILL HELD EQUAL
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- Item thresholds NOW HELD EQUAL IF LEFT OFF (LESS TYPING)
- Item residual variances STILL FIXED to 1
  - cia1-cia7@1;
- Factor mean NOW FREE and variance STILL FREE
  - [IADL*]; IADL*;

The DIFFTEST chi-square is significant, and the modification indices suggest that item 3 threshold 3 is the biggest problem. Let’s see what happens when we free the item 3 threshold 3 between groups.
### Model 3b. Partial Threshold Invariance Model (freeing item 3 threshold 3 between groups)

**ANALYSIS:**
- **ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;**
- **DIFFTEST=MetricA.dat;** ! Compare against metric

**SAVEDATA:**
- **DIFFTEST=ScalarB.dat;** ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP

**MODEL:**
- Factor loadings all estimated
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- Item thresholds all free
  - [cia1$1-cia7$1*];
  - [cia1$2-cia7$2*];
  - [cia1$3-cia7$3*];
- Item residual variances all fixed=1
  - cia1-cia7@1;
- Factor mean=0 and variance=1 for identification
  - [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP

**MODEL WOMEN:**
- Factor loadings all STILL HELD EQUAL
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- Item 3 threshold 3 NOW FREE between groups
  - [cia3$3*];
- Item residual variances all fixed=1
  - cia1-cia7@1;
- Factor mean=FREE and variance STILL FREE
  - [IADL*]; IADL*

**Threshold for Men:**
- CIA3$1 -6.680 0.798 -8.373 0.000
- CIA3$2 -5.568 0.748 -7.448 0.000
- CIA3$3 -4.146 0.683 -6.068 0.000

**Threshold for Women:**
- CIA3$1 -6.680 0.798 -8.373 0.000
- CIA3$2 -5.568 0.748 -7.448 0.000
- CIA3$3 -4.146 0.683 -6.068 0.000

---

**MODEL FIT INFORMATION**

<table>
<thead>
<tr>
<th>Number of Free Parameters</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square Test of Model Fit</td>
<td>Value</td>
</tr>
<tr>
<td></td>
<td>Degrees of Freedom</td>
</tr>
<tr>
<td></td>
<td>P-Value</td>
</tr>
</tbody>
</table>

**Chi-Square Contributions From Each Group**
- **MEN**
  - Value | 68.946 |
  - Degrees of Freedom | 19 |
  - P-Value | 0.0000 |

**RMSEA (Root Mean Square Error Of Approximation)**
- **Estimate** | 0.061 |
- **90 Percent C.I.** | 0.046 0.076 |
- **Probability RMSEA <= .05** | 0.111 |

**CFI/TLI**
- **CFI** | 0.998 |
- **TLI** | 0.999 |

**MODEL MODIFICATION INDICES**

<table>
<thead>
<tr>
<th>MI</th>
<th>E.P.C.</th>
<th>Std E.P.C.</th>
<th>StdYX E.P.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA1</td>
<td>9.581</td>
<td>-1.016</td>
<td>-1.016</td>
</tr>
<tr>
<td>CIA3</td>
<td>10.724</td>
<td>-1.682</td>
<td>-1.682</td>
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<tr>
<td>CIA4</td>
<td>4.838</td>
<td>0.748</td>
<td>0.748</td>
</tr>
<tr>
<td>CIA5</td>
<td>16.673</td>
<td>0.934</td>
<td>0.934</td>
</tr>
<tr>
<td>CIA6</td>
<td>4.734</td>
<td>0.459</td>
<td>0.459</td>
</tr>
<tr>
<td>CIA7</td>
<td>999.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA1$3</td>
<td>7.206</td>
<td>0.673</td>
<td>0.673</td>
</tr>
<tr>
<td>CIA3$2</td>
<td>9.284</td>
<td>1.200</td>
<td>1.200</td>
</tr>
<tr>
<td>CIA3$3</td>
<td>4.317</td>
<td>-0.546</td>
<td>-0.546</td>
</tr>
<tr>
<td>CIA5$3</td>
<td>4.783</td>
<td>-0.434</td>
<td>-0.434</td>
</tr>
<tr>
<td>CIA5$3</td>
<td>7.638</td>
<td>-0.456</td>
<td>-0.456</td>
</tr>
</tbody>
</table>

The DIFFTEST chi-square is still significant, and the modification indices suggest that the other thresholds of item 3 are a problem. Let’s see what happens when we free the item 3 threshold 2 between groups.
Model 3c. Partial Threshold Invariance Model (also freeing item 3 threshold 2 between groups)

**ANALYSIS:**
ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
DIFFTEST=MetricA.dat; ! Compare against metric
SAVEDATA: DIFFTEST=ScalarC.dat; ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP

**MODEL**:
! Factor loadings all estimated
   IADL BY cia1* (L1)
   cia2* (L2)
   cia3* (L3)
   cia4* (L4)
   cia5* (L5)
   cia6* (L6)
   cia7* (L7);
! Item thresholds all free
   [cia1$1-cia7$1*];
   [cia1$2-cia7$2*];
   [cia1$3-cia7$3*];
! Item residual variances all fixed=1
   cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
   [IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP

**MODEL WOMEN**:
! Factor loadings all STILL HELD EQUAL
   IADL BY cia1* (L1)
   cia2* (L2)
   cia3* (L3)
   cia4* (L4)
   cia5* (L5)
   cia6* (L6)
   cia7* (L7);
! Item 3 threshold 2 and 3 FREE between groups
   [cia3$2* cia3$3*];
! Item residual variances all fixed=1
   cia1-cia7@1;
! Factor mean=FREE and variance STILL FREE
   [IADL*]; IADL*;

The DIFFTEST chi-square is still significant, and the modification indices suggest that item 1 threshold 3 is a problem. Let's see what happens when we free the item 1 threshold 3 between groups.
Model 3d. Partial Threshold Invariance Model (also freeing item 1 threshold 3 between groups)

<table>
<thead>
<tr>
<th>ANALYSIS: ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA; DIFFTEST=MetricA.dat; ! Compare against metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAVEDATA: DIFFTEST=ScalarD.dat; ! Save partial scalar info !!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP MODEL:</td>
</tr>
<tr>
<td>! Factor loadings all estimated IADL BY cia1* (L1) cia2* (L2) cia3* (L3) cia4* (L4) cia5* (L5) cia6* (L6) cia7* (L7); ! Item thresholds all free [cia1$1-cia7$1*]; [cia1$2-cia7$2*]; [cia1$3-cia7$3*]; ! Item residual variances all fixed=1 cia1-cia7@1; ! Factor mean=0 and variance=1 for identification [IADL@0]; IADL@1; !!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP MODEL WOMEN:</td>
</tr>
<tr>
<td>! Factor loadings all STILL HELD EQUAL IADL BY cia1* (L1) cia2* (L2) cia3* (L3) cia4* (L4) cia5* (L5) cia6* (L6) cia7* (L7); ! Item 3 threshold 2 and 3, item 1 threshold 3 FREE between groups [cia3$2* cia3$3*]; [cia1$3*]; ! Item residual variances all fixed=1 cia-cia7@1; ! Factor mean=FREE and variance STILL FREE [IADL*]; IADL*;</td>
</tr>
</tbody>
</table>

The DIFFTEST chi-square is still significant. Let's try item 5 threshold 3....

<table>
<thead>
<tr>
<th>Number of Free Parameters</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square Test of Model Fit</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>99.865*</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>51</td>
</tr>
<tr>
<td>P-Value</td>
<td>0.0001</td>
</tr>
<tr>
<td>Chi-Square Contributions From Each Group</td>
<td></td>
</tr>
<tr>
<td>MEN</td>
<td>57.085</td>
</tr>
<tr>
<td>WOMEN</td>
<td>42.780</td>
</tr>
<tr>
<td>THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR D INVARIANCE</td>
<td></td>
</tr>
<tr>
<td>Chi-Square Test for Difference Testing</td>
<td></td>
</tr>
<tr>
<td>Value</td>
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<tr>
<td>Degrees of Freedom</td>
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</tr>
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<td>P-Value</td>
<td>0.288</td>
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<tr>
<td>RMSEA (Root Mean Square Error Of Approximation)</td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>0.055</td>
</tr>
<tr>
<td>90 Percent C.I.</td>
<td>0.039 0.071</td>
</tr>
<tr>
<td>Probability RMSEA &lt;= .05</td>
<td>0.288</td>
</tr>
<tr>
<td>CFI/TLI</td>
<td></td>
</tr>
<tr>
<td>CFI</td>
<td>0.999</td>
</tr>
<tr>
<td>TLI</td>
<td>0.999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thresholds for Men...</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA1$1</td>
</tr>
<tr>
<td>CIA1$2</td>
</tr>
<tr>
<td>CIA1$3</td>
</tr>
<tr>
<td>CIA3$1</td>
</tr>
<tr>
<td>CIA3$2</td>
</tr>
<tr>
<td>CIA3$3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Different Thresholds for Women...</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA1$3</td>
</tr>
<tr>
<td>CIA3$2</td>
</tr>
<tr>
<td>CIA3$3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MODEL MODIFICATION INDICES</th>
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<tbody>
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<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>Group MEN</td>
</tr>
<tr>
<td>Means/Intercepts/Thresholds</td>
</tr>
<tr>
<td>[ CIA1 ]</td>
</tr>
<tr>
<td>[ CIA2 ]</td>
</tr>
<tr>
<td>[ CIA5 ]</td>
</tr>
<tr>
<td>[ CIA2$3 ]</td>
</tr>
<tr>
<td>[ CIA3$3 ]</td>
</tr>
</tbody>
</table>
Model 3e. Partial Threshold Invariance Model (also freeing item 5 threshold 3 between groups)

**ANALYSIS:**
- ESTIMATOR IS WLSMV;
- PARAMETERIZATION=THETA;
- DIFFTEST=MetricA.dat;  
  ! Compare against metric

**SAVEDATA:**
- DIFFTEST=ScalarE.dat;  
  ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP

**MODEL:**
- ! Factor loadings all estimated
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- ! Item thresholds all free
  - [cia1$1-cia7$1*];
  - [cia1$2-cia7$2*];
  - [cia1$3-cia7$3*];
- ! Item residual variances all fixed=1
  - cia1-cia7@1;
- ! Factor mean=0 and variance=1 for identification
  - [IADL@0];  IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP

**MODEL WOMEN:**
- ! Factor loadings all STILL HELD EQUAL
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- ! Item 3 threshold 2 and 3, item 1,5 threshold 3 FREE between groups
  - [cia3$2* cia3$3*];
  - [cia1$3*];
  - [cia5$3*];
- ! Item residual variances all fixed=1
  - cia1-cia7@1;
- ! Factor mean=FREE and variance still FREE
  - [IADL*];  IADL*

The DIFFTEST chi-square is still significant. Let's try item 5 threshold 2....

<table>
<thead>
<tr>
<th>Number of Free Parameters</th>
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</tr>
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<tbody>
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<td></td>
<td>Degrees of Freedom: 50</td>
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<td></td>
<td>P-Value: 0.0001</td>
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</table>

Chi-Square Contributions From Each Group

<table>
<thead>
<tr>
<th>Group</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEN</td>
<td>53.258</td>
</tr>
<tr>
<td>WOMEN</td>
<td>42.189</td>
</tr>
</tbody>
</table>

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR E INVARIANCE

Chi-Square Test for Difference Testing

<table>
<thead>
<tr>
<th>Value</th>
<th>35.350</th>
</tr>
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<tbody>
<tr>
<td>Degrees of Freedom</td>
<td>16</td>
</tr>
<tr>
<td>P-Value</td>
<td>0.0036</td>
</tr>
</tbody>
</table>

RMSEA (Root Mean Square Error Of Approximation)

<table>
<thead>
<tr>
<th>Estimate</th>
<th>RMSEA &lt;= .05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.054</td>
</tr>
<tr>
<td>90 Percent C.I.</td>
<td>0.037 0.070</td>
</tr>
<tr>
<td>Probability</td>
<td>0.341</td>
</tr>
</tbody>
</table>

CFI/TLI

<table>
<thead>
<tr>
<th>CFI</th>
<th>0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>TLI</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Thresholds for Men...

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA1$1*</td>
<td>-5.443</td>
</tr>
<tr>
<td>CIA1$2</td>
<td>-4.046</td>
</tr>
<tr>
<td>CIA1$3*</td>
<td>-0.573</td>
</tr>
<tr>
<td>CIA3$1</td>
<td>-6.699</td>
</tr>
<tr>
<td>CIA3$2*</td>
<td>-4.378</td>
</tr>
<tr>
<td>CIA3$3*</td>
<td>-0.877</td>
</tr>
<tr>
<td>CIA5$1*</td>
<td>-4.334</td>
</tr>
<tr>
<td>CIA5$2</td>
<td>-2.311</td>
</tr>
<tr>
<td>CIA5$3*</td>
<td>-1.227</td>
</tr>
</tbody>
</table>

Different Thresholds for Women...

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIA1$3*</td>
<td>-1.541</td>
</tr>
<tr>
<td>CIA3$2</td>
<td>-6.074</td>
</tr>
<tr>
<td>CIA3$3*</td>
<td>-4.209</td>
</tr>
<tr>
<td>CIA5$3*</td>
<td>-0.658</td>
</tr>
</tbody>
</table>

MODEL MODIFICATION INDICES

<table>
<thead>
<tr>
<th>M.I.</th>
<th>E.F.C.</th>
<th>Std E.F.C.</th>
<th>StdYX E.F.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>MEN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means/Intercepts/Thresholds</td>
<td>CFA2</td>
<td>4.616</td>
<td>-0.773</td>
</tr>
<tr>
<td>CFA5</td>
<td>6.245</td>
<td>0.756</td>
<td>0.756</td>
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<tr>
<td>CFA5$2</td>
<td>3.920</td>
<td>-0.383</td>
<td>-0.383</td>
</tr>
</tbody>
</table>
Model 3f. Partial Threshold Invariance Model (also freeing item 5 threshold 2 between groups)

ANALYSIS: ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
DIFFTEST=MetricA.dat; ! Compare against metric
SAVEDATA: DIFFTEST=ScalarF.dat; ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
IADL BY cia1* (L1)
cia2* (L2)
cia3* (L3)
cia4* (L4)
cia5* (L5)
cia6* (L6)
cia7* (L7);
! Item thresholds all free
[cia1$1-cia7$1*];
[cia1$2-cia7$2*];
[cia1$3-cia7$3*];
! Item residual variances all fixed=1
cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
IADL BY cia1* (L1)
cia2* (L2)
cia3* (L3)
cia4* (L4)
cia5* (L5)
cia6* (L6)
cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3 FREE between groups
[cia3$2* cia3$3*];
[cia1$3*];
[cia5$2* cia5$3*];
! Item residual variances all fixed=1
cia1-cia7@1;
! Factor mean=FREE and variance still FREE
[IADL*]; IADL*;

Number of Free Parameters                       35
Chi-Square Test of Model Fit
   Value                             91.604*           Degrees of Freedom                    49           P-Value                           0.0002
Chi-Square Contributions From Each Group
   MEN                               50.317           WOMEN                             41.287

THIS IS THE TEST OF METRIC VS. PARTIAL SCALAR F INVARIANCE
Chi-Square Test for Difference Testing
   Value                             29.886           Degrees of Freedom                    15           P-Value                           0.0123
RMSEA (Root Mean Square Error Of Approximation)
   Estimate                           0.052           90 Percent C.I.                    0.035  0.069           Probability RMSEA <=.05       0.387
   CFI/TLI
   CFI                             0.999           TLI                             0.999

MODEL MODIFICATION INDICES                           M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C. Group MEN
Means/Intercepts/Thresholds
[CIA4    ]  4.975     0.802      0.802        0.188 [CIA6    ]  4.570     0.469      0.469        0.187 [CIA4$3  ]  4.529    -0.536     -0.536       -0.125
The DIFFTEST chi-square is still significant. Let’s try item 4 threshold 3....

The DIFFTEST chi-square is still significant. Let’s try item 4 threshold 3....
Model 3g. Partial Threshold Invariance Model (also freeing item 4 threshold 3 between groups)

ANALYSIS:
ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
DIFFTEST=MetricA.dat;  ! Compare against metric

SAVEDATA: DIFFTEST=ScalarG.dat;  ! Save partial scalar info

!!! PARTIAL SCALAR MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated
IADL BY cia1* (L1)
cia2* (L2)
cia3* (L3)
cia4* (L4)
cia5* (L5)
cia6* (L6)
cia7* (L7);
! Item thresholds all free
[cia1$1-cia7$1*];
[cia1$2-cia7$2*];
[cia1$3-cia7$3*];
! Item residual variances all fixed=1
cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

!!! PARTIAL SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all STILL HELD EQUAL
IADL BY cia1* (L1)
cia2* (L2)
cia3* (L3)
cia4* (L4)
cia5* (L5)
cia6* (L6)
cia7* (L7);
! Item 3 threshold 2 and 3, item 1 threshold 3,
! item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups
[cia3$2* cia3$3*];
[cia1$3*];
[cia5$2* cia5$3*];
[cia4$3*];
! Item residual variances all fixed=1
cia1-cia7@1;
! Factor mean=FREE and variance still FREE
[IADL*]; IADL*;

Number of Free Parameters 36

Chi-Square Test of Model Fit
Value 87.391*
Degrees of Freedom 48
P-Value 0.0004

Chi-Square Contributions From Each Group
MEN 46.648
WOMEN 40.742

RMSEA (Root Mean Square Error Of Approximation)
Estimate 0.051
90 Percent C.I. 0.033 0.068
Probability RMSEA <= .05 0.444

CFI/TLI
CFI 0.999
TLI 0.999

MODEL MODIFICATION INDICES
M.I.  E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Mean/Intercepts/Thresholds
[ CIA6 ]  6.640  0.574  0.574  0.231

Although the DIFFTEST chi-square is still technically significant, no other modifications to un-constrain the remaining thresholds will improve fit.

I'm calling it done.
This last step for testing measurement invariance proceeds backwards. Because freeing the residual variances is adding parameters, we must estimate this free residuals model first.

### ANALYSIS:
```
ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
```

### SAVEDATA:
```
DIFFTEST=ResidualFreeA.dat;  ! Save free residual info
```

---

### !!! RESIDUAL FIXED MODEL FOR MEN REFERENCE GROUP

**MODEL:**

- **Factor loadings all estimated**
  
  IADL BY cia1* (L1)
cia2* (L2)
cia3* (L3)
cia4* (L4)
cia5* (L5)
cia6* (L6)
cia7* (L7);

- **Item thresholds all free**
  
  [cia1$1-cia7$1*];
  [cia1$2-cia7$2*];
  [cia1$3-cia7$3*];

- **Item residual variances all fixed=1**
  
  cia1-cia7@1;

- **Factor mean=0 and variance=1 for identification**
  
  [IADL@0]; IADL@1;

---

### !!! RESIDUAL FREE MODEL FOR WOMEN ALTERNATIVE GROUP

**MODEL WOMEN:**

- **Factor loadings all STILL HELD EQUAL**
  
  IADL BY cia1* (L1)
cia2* (L2)
cia3* (L3)
cia4* (L4)
cia5* (L5)
cia6* (L6)
cia7* (L7);

- **Item 3 threshold 2 and 3, item 1 threshold 3,**

- **item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups**
  
  [cia3$2* cia3$3*];
  [cia1$3*];
  [cia5$2* cia5$3*];
  [cia4$3*];

- **Item residual variances NOW FREE**
  
  cia1-cia7*;

---

### (Number of Free Parameters) 43

### Chi-Square Test of Model Fit

<table>
<thead>
<tr>
<th>Value</th>
<th>Degrees of Freedom</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>80.223*</td>
<td>41</td>
<td>0.0002</td>
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</table>

### Chi-Square Contributions From Each Group

<table>
<thead>
<tr>
<th>Group</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEN</td>
<td>34.586</td>
</tr>
<tr>
<td>WOMEN</td>
<td>45.636</td>
</tr>
</tbody>
</table>

### RMSEA (Root Mean Square Error Of Approximation)

<table>
<thead>
<tr>
<th>Estimate</th>
<th>90 Percent C.I.</th>
<th>Probability RMSEA &lt;= .05</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.055</td>
<td>0.037  0.073</td>
<td>0.005</td>
</tr>
</tbody>
</table>

### CFI/TLI

<table>
<thead>
<tr>
<th>CFI</th>
<th>TLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.999</td>
<td>0.999</td>
</tr>
</tbody>
</table>

### Residual Variances for Men...

| CIA1   | 1.000  | 0.000  | 999.000 | 999.000 |
| CIA2   | 1.000  | 0.000  | 999.000 | 999.000 |
| CIA3   | 1.000  | 0.000  | 999.000 | 999.000 |
| CIA4   | 1.000  | 0.000  | 999.000 | 999.000 |
| CIA5   | 1.000  | 0.000  | 999.000 | 999.000 |
| CIA6   | 1.000  | 0.000  | 999.000 | 999.000 |
| CIA7   | 1.000  | 0.000  | 999.000 | 999.000 |

### Residual Variances for Women...

| CIA1   | 2.243  | 1.286  | 1.744   | 0.081  |
| CIA2   | 0.538  | 0.224  | 2.406   | 0.016  |
| CIA3   | 0.264  | 0.118  | 2.239   | 0.025  |
| CIA4   | 1.063  | 0.540  | 1.970   | 0.049  |
| CIA5   | 0.940  | 0.378  | 2.490   | 0.013  |
| CIA6   | 1.627  | 0.639  | 2.546   | 0.011  |
| CIA7   | 0.725  | 0.192  | 3.768   | 0.000  |
Model 4b. Residual Variance Invariance Model (testing all items)

We now constrain the residual variances to be equal between groups and test decrease in fit.

**ANALYSIS:**
- ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
- DIFFTEST=ResidualFreeA.dat;  ! Compare to free residual
- SAVEDATA: DIFFTEST=ResidualFixedB.dat;  ! Save fixed residual info

!!! RESIDUAL FIXED MODEL FOR MEN REFERENCE GROUP

**MODEL:**

- Factor loadings all estimated
  - IADL BY cia1* (L1);
  - cia2* (L2);
  - cia3* (L3);
  - cia4* (L4);
  - cia5* (L5);
  - cia6* (L6);
  - cia7* (L7);

- Item thresholds all free
  - [cia1$1-cia7$1*];
  - [cia1$2-cia7$2*];
  - [cia1$3-cia7$3*];

- Item residual variances all fixed=1
  - cia1-cia7@1;

- Factor mean=0 and variance=1 for identification
  - [IADL@0]; IADL@1;

!!! RESIDUAL FREE MODEL FOR WOMEN ALTERNATIVE GROUP

**MODEL WOMEN:**

- Factor loadings all STILL HELD EQUAL
  - IADL BY cia1* (L1);
  - cia2* (L2);
  - cia3* (L3);
  - cia4* (L4);
  - cia5* (L5);
  - cia6* (L6);
  - cia7* (L7);

- Item 3 threshold 2 and 3, item 1 threshold 3,
- Item 5 thresholds 2 and 3, item 4 threshold 3 FREE between groups
  - [cia3$2* cia5$3*];
  - [cia5$2* cia5$3*];
  - [cia4$3*];

- Item residual variances NOW FIXED
  - cia1-cia7@1;

- Factor mean=FREE and variance still FREE
  - [IADL*]; IADL*;

Because DIFFTEST is nonsignificant, we can constrain the residual variances for the invariant items to be the same. Now we are ready to examine structural invariance (equality of the factor variance and factor mean).
### Final Model: Partial Measurement Invariance (solution from Model 4c)

#### UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

<table>
<thead>
<tr>
<th>Group</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEN</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FACTOR LOADINGS:</strong> CHAGE IN PROBIT FOR 1-UNIT CHANGE IN THETA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IADL BY CIA1</td>
<td>4.227</td>
<td>0.463</td>
<td>9.137</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA2</td>
<td>4.054</td>
<td>0.561</td>
<td>7.222</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA3</td>
<td>3.287</td>
<td>0.488</td>
<td>6.730</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA4</td>
<td>4.197</td>
<td>0.509</td>
<td>8.253</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA5</td>
<td>2.729</td>
<td>0.304</td>
<td>8.977</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA6</td>
<td>2.346</td>
<td>0.263</td>
<td>8.915</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA7</td>
<td>1.270</td>
<td>0.176</td>
<td>7.208</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>MEANS:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IADL</td>
<td>0.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td><strong>THRESHOLDS:</strong> EXPECTED PROBIT OF LOWER CATEGORY WHEN THETA=0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIA1$1</td>
<td>-5.634</td>
<td>0.572</td>
<td>-9.848</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA1$2</td>
<td>-4.185</td>
<td>0.519</td>
<td>-8.065</td>
<td>0.000</td>
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<tr>
<td>CIA1$3</td>
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<tr>
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</tr>
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<tr>
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</tr>
<tr>
<td>CIA4$1</td>
<td>-5.283</td>
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<td>CIA5$2</td>
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<tr>
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<tr>
<td>CIA7$3</td>
<td>-1.934</td>
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<td>-10.562</td>
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<tr>
<td><strong>VARIANCES:</strong> VARIANCE OF THETA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IADL</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td><strong>Residual Variances (ALL FIXED=1):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIA1</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>CIA2</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>CIA3</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>CIA4</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>CIA5</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>CIA6</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
<tr>
<td>CIA7</td>
<td>1.000</td>
<td>0.000</td>
<td>999.000</td>
<td>999.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Estimate</th>
<th>S.E.</th>
<th>Est./S.E.</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WOMEN</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>FACTOR LOADINGS:</strong> CHAGE IN PROBIT FOR 1-UNIT CHANGE IN THETA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IADL BY CIA1</td>
<td>4.227</td>
<td>0.463</td>
<td>9.137</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA2</td>
<td>4.054</td>
<td>0.561</td>
<td>7.222</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA3</td>
<td>3.287</td>
<td>0.488</td>
<td>6.730</td>
<td>0.000</td>
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<tr>
<td>CIA4</td>
<td>4.197</td>
<td>0.509</td>
<td>8.253</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA5</td>
<td>2.729</td>
<td>0.304</td>
<td>8.977</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA6</td>
<td>2.346</td>
<td>0.263</td>
<td>8.915</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA7</td>
<td>1.270</td>
<td>0.176</td>
<td>7.208</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>MEANS:</strong> MEAN DIFFERENCE OF THETA IN WOMEN</td>
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<td></td>
</tr>
<tr>
<td>IADL</td>
<td>-0.194</td>
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<td>0.067</td>
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<td><strong>THRESHOLDS:</strong> NEGATIVE OF EXPECTED PROBIT WHEN THETA=0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIA1$1</td>
<td>-5.634</td>
<td>0.572</td>
<td>-9.848</td>
<td>0.000</td>
</tr>
<tr>
<td>CIA1$2</td>
<td>-4.185</td>
<td>0.519</td>
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<td>999.000</td>
<td>999.000</td>
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</table>
**Model 5a. Testing Factor Variance Invariance**

**Analysis:**
- Estimator is WLSMV; Parameterization=THETA;
- DIFFTEST=ResidualFixedC.dat; ! Compare to fixed residual

**Savedata:**
- !!!! Residual Fixed Model for Men Reference Group

**Model:**
- ! Factor loadings all estimated
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- ! Item thresholds all free
  - [cia1$1-cia7$1*];
  - [cia1$2-cia7$2*];
  - [cia1$3-cia7$3*];
- ! Item residual variances all fixed=1
  - cia1-cia7@1;
- ! Factor mean=0 and variance=1 for identification
  - [IADL@0]; IADL@1;

- !!!! Residual Free Model for Women Alternative Group

**Model Women:**
- ! Factor loadings all still held equal
  - IADL BY cia1* (L1)
  - cia2* (L2)
  - cia3* (L3)
  - cia4* (L4)
  - cia5* (L5)
  - cia6* (L6)
  - cia7* (L7);
- ! Item 3 threshold 2 and 3, item 1 threshold 3,
- ! item 5 thresholds 2 and 3, item 4 threshold 3 free between groups
  - [cia3$2* cia3$3*];
  - [cia5$2* cia5$3*];
  - [cia4$3*];
- ! Item residual variances now fixed except for item 3
  - cia1-cia7@1;
  - cia3*;
- ! Factor mean=FREE and variance now fixed
  - [IADL*]; IADL@1;

**Model Fit Information**
- Number of Free Parameters 36
- Chi-Square Test of Model Fit
  - Value 90.531*
  - Degrees of Freedom 48
  - P-Value 0.0002
- Chi-Square Contributions From Each Group
  - MEN 43.802
  - WOMEN 46.729

This is the Test of Factor Variance Invariance
- Chi-Square Test for Difference Testing
  - Value 6.183
  - Degrees of Freedom 1
  - P-Value 0.0129
- RMSEA (Root Mean Square Error Of Approximation)
  - Estimate 0.053
  - 90 Percent C.I. 0.036 0.069
  - Probability RMSEA <= .05 0.369
- CFI/TLI
  - CFI 0.999
  - TLI 0.999

Because DIFFTEST is significant, this means that the women have significantly less variance in the latent factor than men. Further, we know from the previous model solution that the factor mean for women was marginally different from 0 (the factor mean for men), and thus we don’t have to test it anyway. So we call it done, and let the theta mean and variance differ between groups.
Because women have less theta variance, the same unstandardized factor loadings as men imply greater relationship of the items to the latent trait (and thus greater information) in women.

The thresholds predict the probit of y=lower response for something with Theta=0.

So, holding IADL ability constant at the combined sample factor mean of 0….

**Item 1:** Women have a lower probability of not being able to do housework than men.

**Item 3:** Women have a lower probability of not being able to do cook than men.

**Item 4:** Women have a greater probability of not being able to shop than men.

**Item 5:** Women have a greater probability of not being able to get around than men.
Example write-up of these IFA analyses:

The extent to which an item factor model measuring independent daily living (with seven observed items) exhibited measurement invariance and structural invariance between men and women was examined using Mplus v. 7.11 (Muthén & Muthén, 1998-2012). WLSMV estimation including a probit link and the THETA parameterization was used to estimate all models (Muthén & Muthén, 1998-2012). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items for each group. Nested model comparisons were conducted using the DIFFTEST procedure. A configural invariance model was initially specified in which a single factor was estimated simultaneously in each group. The factor variance was fixed to 1 and the factor mean was fixed to 0 in each group for identification, such that all item factor loadings (one per item) and thresholds (three per item given four response options) were then estimated. The residual variances are not uniquely identified in the configural invariance model and as such were all constrained to 1 in both groups. As shown in Table 1, the configural invariance model had good fit. The analysis proceeded by applying parameter constraints in successive models to examine potential decreases in fit resulting from measurement or structural non-invariance between men and women, with men as the reference group.

Equality of the unstandardized item factor loadings between groups was then examined in a metric invariance model. The factor variance was fixed to 1 in men for identification but was freely estimated in women; the factor mean was fixed to 0 in both groups for identification. All factor loadings were constrained equal across groups, all item thresholds were estimated, and all residual variances were constrained to 1 across groups. The metric invariance model did not fit significantly worse than the configural invariance model, DIFFTEST (6) = 9.53, \( p = .15 \). The modification indices did not suggest any points of localized misfit for the constrained loadings. The fact that metric invariance (i.e., “weak invariance”) held indicates that the items were related to the latent factor equivalently across groups, or more simply, that the same latent factor was being measured in each group. However, because the factor variances were permitted to vary across groups (such that women showed less variability in the latent factor), the “a” discrimination parameters from the item response version of the model were not equivalent across groups (i.e., the equality constraints were applied to the factor loadings only).

Equality of the unstandardized item thresholds across groups was then examined in a scalar invariance model. The factor variance and mean were fixed to 1 and 0, respectively, in men for identification, but the factor variance and mean were then estimated for women. All factor loadings and item thresholds were constrained equal across groups; all residual variances were still constrained equal to 1 in both groups. The full scalar invariance model A fit significantly worse than the metric invariance model, DIFFTEST (20) = 119.66, \( p < .001 \). The modification indices suggested that threshold 3 of item 3 was the largest source of the misfit and should be freed. After doing so, the partial scalar invariance model B still had significantly worse fit than the full metric invariance model, DIFFTEST (19) = 63.05, \( p < .001 \). The modification indices suggested that threshold 2 of item 3 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model C (with thresholds 2 and 3 for item 3 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (18) = 51.19, \( p < .001 \). The modification indices suggested that threshold 3 of item 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model D (with the thresholds 2 and 3 for item 3 and threshold 3 for item 1 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (17) = 41.247, \( p < .001 \). The modification indices suggested that threshold 3 of item 5 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model E (with the thresholds 2 and 3 for item 3 and threshold 3 for items 1 and 5 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (16) = 35.35, \( p < .004 \). The modification indices suggested that threshold 2 of item 5 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model F (with the thresholds 2 and 3 for items 3 and 5 and threshold 3 for item 1 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (15) = 29.89, \( p = .012 \). The modification indices suggested that threshold 3 of item 4 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model G (with the thresholds 2 and 3 for items 3 and 5 and threshold 3 for items 1, 4 and 5 freed) still fit significantly worse than the full metric invariance model, DIFFTEST (14) =
24.43, \( p = .041 \). However, the modification indices did not indicate any remaining sources of misfit due to constrained thresholds, and thus the partial scalar invariance model G was retained. The factor that partial scalar invariance (i.e., “strong invariance”) held indicates that items 2, 6, and 7 have the same expected response at the same absolute level of the trait, or more simply, that the observed differences in the proportion of responses in each category for those items was due to factor mean differences only. However, at the same absolute level of the IADL factor, threshold 3 for item 1 and thresholds 2 and 3 for item 3 were more difficult for men, indicating that men have a greater probability of not being able to do housework or cook, whereas threshold 3 for item 4 and thresholds 2 and 3 for item 5 were less difficult in men, indicating that men have a lower probability of not being able to shop or get around. Because the factor mean was permitted to vary across groups (such that women were less able overall than men), the “\( b \)” parameters from the item response version of the model were not equivalent across groups (i.e., the equality constraints were applied to the item thresholds only).

Equality of the unstandardized residual variances across groups was then examined in a residual variance invariance model. The model comparison at this step proceeded backwards, such that a model with all residual variances freely estimated in the women was fitted first, and then compared with a model in which all residual variances were fixed to 1 in the women. The residual variances in the men were all fixed to 1 for identification in both models, and the rest of the model parameters were estimated as described for the last partial scalar invariance model G. The model with the residual variances constrained to 1 (to be equal to the men) fit significantly worse than the model with those residual variances freed, \( \text{DIFFTEST (7)} = 14.32, \ p = .046 \). The modification indices suggested that the residual variance for item 3 the largest remaining source of misfit and should be freed. After doing so, the new partial residual variance invariance model B did not fit significantly worse than the partial scalar invariance model G, \( \text{DIFFTEST (6)} = 8.93, \ p = .178 \), indicating that residual variance for item 3 was significantly smaller for women than men. The fact that partial residual variance invariance (i.e., “strict invariance”) held indicates that the amount of item variance not accounted for by the factor was the same across groups in all other items.

After achieving partial measurement invariance as was just described, structural invariance was then tested with one additional model. The factor variance in the women (which had been estimated freely) was constrained to 1 (i.e., to be equal to the factor variance in men), resulting in a significant decrease in fit relative to the last partial residual invariance model B, \( \text{DIFFTEST (1)} = 6.18, \ p = .013 \). Thus, women showed significantly less variability in ability to live independently (factor variance of 0.61) than did men (factor variance of 1.0). The factor mean for women in the partial measurement invariance model was marginally different from 0 (difference = −0.19, SE = 0.11, \( p = .067 \)), indicating that women were marginally less able to live independently than men (factor mean of 0). Test information functions were similar across groups (although slightly larger for women between Theta = −1 and 0), and indicated acceptable reliability (i.e., information > 4) only between a Theta of −2 and 0. Thus, high-functioning individuals will not be measured adequately with these seven items.

In conclusion, these analyses showed that partial measurement invariance was obtained across men and women – that is, the relationships of the items to the latent factor of independent living were equivalent between men women. However, items 1 and 3 (housework and cooking) were systematically more difficult for men than women at the same level of the latent trait, whereas items 4 and 5 (shopping and getting around) was more difficult for women than men. These analyses also showed that structural invariance was not obtained, such that women were less variable and less able on average than men. Model parameters from the final model are given in Table 2.

Table 1 provides fit of each model

Table 2 provides final model parameters