Longitudinal Invariance CFA (using ML) Example in Mplus v. 7.11 (*N* = 151; 6 indicators over 3 occasions)

These data measuring a latent trait of social functioning were collected at a Psychiatric Rehabilitation center, in which time 1 was admittance, and times 2 and 3 were collected at six-month intervals. There were 6 subscales that were completed by the hospital staff for each patient, including positively-oriented measures of Social Competence, Social Interest, and Personal Neatness, and negatively-oriented measures of Psychoticism, Motor Retardation, and Irritability. The negatively-oriented subscales were reflect (*−1) prior to analysis. Initial models examined the fit of one-factor versus two-factor models given the two valences of the subscales, but the fit of the two-factor model was not a significant improvement, and thus a one-factor model with all 6 indicators was used here.

Mplus Code to Read in Data:

```
TITLE:     Longitudinal Invariance
DATA:      FILE IS CAF.dat;                ! Don't need path if in same folder
           FORMAT IS free;                 ! Default
           TYPE IS INDIVIDUAL;             ! Default

VARIABLE:  NAMES ARE REID                  ! Every variable in data set
                  VAR1T1 VAR1T2 VAR1T3
                  VAR2T1 VAR2T2 VAR2T3
                  VAR3T1 VAR3T2 VAR3T3
                  VAR4T1 VAR4T2 VAR4T3
                  VAR5T1 VAR5T2 VAR5T3
                  VAR6T1 VAR6T2 VAR6T3;
           USEVARIABLES ARE                ! Every variable in MODEL
                  VAR1T1 VAR1T2 VAR1T3
                  VAR2T1 VAR2T2 VAR2T3
                  VAR3T1 VAR3T2 VAR3T3
                  VAR4T1 VAR4T2 VAR4T3
                  VAR5T1 VAR5T2 VAR5T3
                  VAR6T1 VAR6T2 VAR6T3;
            MISSING ARE ALL (9999);        ! Make sure to specify all missing values

ANALYSIS:   ESTIMATOR IS ML;         ! Default if TYPE=GENERAL and all continuous items

OUTPUT:     SAMPSTAT                 ! Sample descriptives to verify data
            MODINDICES (6)           ! Voodoo to improve model (list if χ2>6 for p<.01)
            STDYX                    ! Requests fully standardized solution
            RESIDUAL;                ! Requests standardized and normalized residuals

MODEL:      (model syntax goes here, to be changed for each model)
```

Model 1. Configural Longitudinal Invariance Model (Everything separate across time)

```
MODEL:
! Model 1: Configural Longitudinal Invariance
! Factor Loadings - second set to 1 for identification
    FACTOR1 BY VAR1T1*
               VAR2T1@1
               VAR3T1*
               VAR4T1*
               VAR5T1*
               VAR6T1*;
    FACTOR2 BY VAR1T2*
               VAR2T2@1
               VAR3T2*
               VAR4T2*
               VAR5T2*
               VAR6T2*;
    FACTOR3 BY VAR1T3*
               VAR2T3@1
               VAR3T3*
               VAR4T3*
               VAR5T3*
               VAR6T3*;
! Intercepts - second set to 0 for identification
    [VAR1T1*  VAR1T2*  VAR1T3* ];
    [VAR2T1@0 VAR2T2@0 VAR2T3@0];
    [VAR3T1*  VAR3T2*  VAR3T3* ];
    [VAR4T1*  VAR4T2*  VAR4T3* ];
    [VAR5T1*  VAR5T2*  VAR5T3* ];
    [VAR6T1*  VAR6T2*  VAR6T3* ];
! Residual variances - all free now
     VAR1T1* VAR1T2* VAR1T3*;
     VAR2T1* VAR2T2* VAR2T3*;
     VAR3T1* VAR3T2* VAR3T3*;
     VAR4T1* VAR4T2* VAR4T3*;
     VAR5T1* VAR5T2* VAR5T3*;
     VAR6T1* VAR6T2* VAR6T3*;
! Factor variances and covariances (all free now)
    FACTOR1* FACTOR2* FACTOR3*;
    FACTOR1 WITH FACTOR2*;
    FACTOR1 WITH FACTOR3*;
    FACTOR2 WITH FACTOR3*;
! Factor means (all free now)
    [FACTOR1* FACTOR2* FACTOR3*];
! Residual covariances for same indicator across time
    VAR1T1 WITH VAR1T2* VAR1T3*; VAR1T2 WITH VAR1T3*;
    VAR2T1 WITH VAR2T2* VAR2T3*; VAR2T2 WITH VAR2T3*;
    VAR3T1 WITH VAR3T2* VAR3T3*; VAR3T2 WITH VAR3T3*;
    VAR4T1 WITH VAR4T2* VAR4T3*; VAR4T2 WITH VAR4T3*;
    VAR5T1 WITH VAR5T2* VAR5T3*; VAR5T2 WITH VAR5T3*;
    VAR6T1 WITH VAR6T2* VAR6T3*; VAR6T2 WITH VAR6T3*;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       75

Loglikelihood

          H0 Value                       -4430.302
          H1 Value                       -4284.045

Information Criteria

          Akaike (AIC)                    9010.604
          Bayesian (BIC)                  9236.900
          Sample-Size Adjusted BIC        8999.533
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            292.514
          Degrees of Freedom                   114
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.102
          90 Percent C.I.                    0.088  0.116
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                0.912
          TLI                                0.883

Chi-Square Test of Model Fit for the Baseline Model

          Value                           2192.158
          Degrees of Freedom                   153
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.089
```

Although the fit is not great, attempts to improve it logically were unsuccessful, so we proceed from here with this as the configural invariance model…

UNSTANDARDIZED MODEL RESULTS – NOTE ALL PARAMETERS (EXCEPT MARKER LOADINGS AND MARKER INTERCEPTS) DIFFER ACROSS TIME

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS PER OCCASION				
FACTOR1 BY				
VAR1T1	1.682	0.222	7.577	0.000
VAR2T1	1.000	0.000	999.000	999.000
VAR3T1	1.086	0.153	7.096	0.000
VAR4T1	1.031	0.169	6.108	0.000
VAR5T1	0.486	0.078	6.265	0.000
VAR6T1	0.753	0.108	6.941	0.000
FACTOR2 BY				
VAR1T2	1.382	0.141	9.831	0.000
VAR2T2	1.000	0.000	999.000	999.000
VAR3T2	1.029	0.110	9.318	0.000
VAR4T2	1.013	0.132	7.693	0.000
VAR5T2	0.567	0.083	6.793	0.000
VAR6T2	0.730	0.079	9.233	0.000
FACTOR3 BY				
VAR1T3	1.301	0.151	8.591	0.000
VAR2T3	1.000	0.000	999.000	999.000
VAR3T3	0.893	0.117	7.656	0.000
VAR4T3	0.856	0.144	5.937	0.000
VAR5T3	0.521	0.086	6.075	0.000
VAR6T3	0.777	0.091	8.549	0.000
FACTOR1 WITH – **FACTOR COVARIANCES**				
FACTOR2	3.118	0.735	4.244	0.000
FACTOR3	2.656	0.642	4.135	0.000
FACTOR2 WITH				
FACTOR3	2.726	0.624	4.369	0.000
***** Residual covariances among same item across time ******				
VAR1T1 WITH				
VAR1T2	-0.214	0.205	-1.042	0.298
VAR1T3	-0.004	0.206	-0.019	0.984
VAR1T2 WITH				
VAR1T3	0.113	0.171	0.656	0.512
.....				
Variances – ARE VAR2 ITEM VARIANCE DUE TO FACTOR				
FACTOR1	3.667	1.040	3.525	0.000
FACTOR2	4.292	0.963	4.455	0.000
FACTOR3	3.845	0.974	3.947	0.000

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – FACTOR MEANS ARE APPROXIMATELY MEANS OF VAR2 OVER TIME				
FACTOR1	8.672	0.297	29.163	0.000
FACTOR2	9.981	0.267	37.414	0.000
FACTOR3	10.442	0.279	37.399	0.000
Intercepts – ARE EXPECTED OUTCOME WHEN FACTOR IS AT 0				
VAR1T1	1.488	1.986	0.749	0.454
VAR1T2	3.432	1.434	2.392	0.017
VAR1T3	4.175	1.586	2.633	0.008
VAR2T1	0.000	0.000	999.000	999.000
VAR2T2	0.000	0.000	999.000	999.000
VAR2T3	0.000	0.000	999.000	999.000
VAR3T1	2.551	1.372	1.859	0.063
VAR3T2	2.193	1.131	1.939	0.053
VAR3T3	3.706	1.226	3.022	0.003
VAR4T1	-11.979	1.514	-7.913	0.000
VAR4T2	-13.317	1.350	-9.865	0.000
VAR4T3	-11.673	1.515	-7.707	0.000
VAR5T1	-5.499	0.697	-7.885	0.000
VAR5T2	-7.325	0.861	-8.507	0.000
VAR5T3	-6.683	0.905	-7.383	0.000
VAR6T1	-9.398	0.972	-9.665	0.000
VAR6T2	-9.696	0.809	-11.989	0.000
VAR6T3	-10.187	0.955	-10.666	0.000
Residual Variances – VARIANCE PER ITEM THAT IS NOT THE FACTOR				
VAR1T1	0.241	0.345	0.699	0.485
VAR1T2	0.511	0.248	2.059	0.039
VAR1T3	0.523	0.238	2.196	0.028
VAR2T1	8.672	1.073	8.084	0.000
VAR2T2	5.913	0.743	7.954	0.000
VAR2T3	5.142	0.766	6.710	0.000
VAR3T1	2.413	0.367	6.583	0.000
VAR3T2	2.202	0.309	7.120	0.000
VAR3T3	2.381	0.366	6.510	0.000
VAR4T1	7.199	0.893	8.065	0.000
VAR4T2	6.765	0.841	8.048	0.000
VAR4T3	6.456	0.895	7.215	0.000
VAR5T1	1.824	0.218	8.369	0.000
VAR5T2	4.676	0.559	8.364	0.000
VAR5T3	2.944	0.373	7.902	0.000
VAR6T1	1.694	0.221	7.646	0.000
VAR6T2	1.103	0.146	7.575	0.000
VAR6T3	0.751	0.129	5.830	0.000

Model 2a. Metric Invariance Model (ALL loadings held equal across time – identified model using Factor1 Variance = 1)

```
MODEL:
! Model 2a: Metric Invariance – factor loadings constrained
    FACTOR1 BY VAR1T1* (L1)
               VAR2T1* (L2)
               VAR3T1* (L3)
               VAR4T1* (L4)
               VAR5T1* (L5)
               VAR6T1* (L6);
    FACTOR2 BY VAR1T2* (L1)
               VAR2T2* (L2)
               VAR3T2* (L3)
               VAR4T2* (L4)
               VAR5T2* (L5)
               VAR6T2* (L6);
    FACTOR3 BY VAR1T3* (L1)
               VAR2T3* (L2)
               VAR3T3* (L3)
               VAR4T3* (L4)
               VAR5T3* (L5)
               VAR6T3* (L6);
! Intercepts - second set to 0 for identification
    [VAR1T1*  VAR1T2*  VAR1T3* ];
    [VAR2T1@0 VAR2T2@0 VAR2T3@0];
    [VAR3T1*  VAR3T2*  VAR3T3* ];
    [VAR4T1*  VAR4T2*  VAR4T3* ];
    [VAR5T1*  VAR5T2*  VAR5T3* ];
    [VAR6T1*  VAR6T2*  VAR6T3* ];
! Residual variances - all free now
     VAR1T1* VAR1T2* VAR1T3*;
     VAR2T1* VAR2T2* VAR2T3*;
     VAR3T1* VAR3T2* VAR3T3*;
     VAR4T1* VAR4T2* VAR4T3*;
     VAR5T1* VAR5T2* VAR5T3*;
     VAR6T1* VAR6T2* VAR6T3*;
! Factor variances (first fixed=1 for identification)
    FACTOR1@1 FACTOR2* FACTOR3*;
! Factor covariances (still all free)
    FACTOR1 WITH FACTOR2*;
    FACTOR1 WITH FACTOR3*;
    FACTOR2 WITH FACTOR3*;
! Factor means (all free now)
    [FACTOR1* FACTOR2* FACTOR3*];
! Residual covariances for same indicator across time
    VAR1T1 WITH VAR1T2* VAR1T3*; VAR1T2 WITH VAR1T3*;
    VAR2T1 WITH VAR2T2* VAR2T3*; VAR2T2 WITH VAR2T3*;
    VAR3T1 WITH VAR3T2* VAR3T3*; VAR3T2 WITH VAR3T3*;
    VAR4T1 WITH VAR4T2* VAR4T3*; VAR4T2 WITH VAR4T3*;
    VAR5T1 WITH VAR5T2* VAR5T3*; VAR5T2 WITH VAR5T3*;
    VAR6T1 WITH VAR6T2* VAR6T3*; VAR6T2 WITH VAR6T3*;
```

```
Number of Free Parameters                       65

Loglikelihood
          H0 Value                       -4442.401
          H1 Value                       -4284.045

Information Criteria
          Akaike (AIC)                    9014.803
          Bayesian (BIC)                  9210.926
          Sample-Size Adjusted BIC        9005.208
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            316.712
          Degrees of Freedom                   124
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.101
          90 Percent C.I.                    0.088  0.115
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.905
          TLI                                0.883

SRMR (Standardized Root Mean Square Residual)

          Value                              0.094
```

Did model fit get significantly worse relative to the configural model (1)?

```
MODEL MODIFICATION INDICES
BY Statements
                        M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
BY Statements
FACTOR1  BY VAR1T1     10.912     0.182      0.182        0.058
FACTOR1  BY VAR5T1      6.374    -0.054     -0.054       -0.033
FACTOR3  BY VAR6T3      7.994     0.201      0.175        0.105
```

Modification indices suggest that freeing the loading for Var1 at Time 1 would help, and that matches our observations, so let's try that.

1 UNSTANDARDIZED METRIC MODEL RESULTS – NOW ALL FACTOR LOADINGS ARE HELD EQUAL (VARIANCE OF FACTOR1=1 FOR IDENTIFICAITON)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR1 BY				
VAR1T1	3.004	0.203	14.794	0.000
VAR2T1	2.117	0.214	9.915	0.000
VAR3T1	2.139	0.168	12.722	0.000
VAR4T1	2.018	0.209	9.652	0.000
VAR5T1	0.981	0.106	9.241	0.000
VAR6T1	1.611	0.125	12.843	0.000
FACTOR2 BY				
VAR1T2	3.004	0.203	14.794	0.000
VAR2T2	2.117	0.214	9.915	0.000
VAR3T2	2.139	0.168	12.722	0.000
VAR4T2	2.018	0.209	9.652	0.000
VAR5T2	0.981	0.106	9.241	0.000
VAR6T2	1.611	0.125	12.843	0.000
FACTOR3 BY				
VAR1T3	3.004	0.203	14.794	0.000
VAR2T3	2.117	0.214	9.915	0.000
VAR3T3	2.139	0.168	12.722	0.000
VAR4T3	2.018	0.209	9.652	0.000
VAR5T3	0.981	0.106	9.241	0.000
VAR6T3	1.611	0.125	12.843	0.000
FACTOR1 WITH				
FACTOR2	0.769	0.059	12.937	0.000
FACTOR3	0.626	0.069	9.117	0.000
FACTOR2 WITH				
FACTOR3	0.573	0.084	6.848	0.000
***** Residual covariances among same item across time ******				
VAR1T1 WITH				
VAR1T2	-0.208	0.204	-1.018	0.309
VAR1T3	0.035	0.211	0.168	0.867
VAR1T2 WITH				
VAR1T3	0.096	0.182	0.529	0.597
.....				
Variances OF FACTORS				
FACTOR1	1.000	0.000	999.000	999.000
FACTOR2	0.956	0.120	7.982	0.000
FACTOR3	0.761	0.113	6.709	0.000

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – OF FACTOR – HAVE CHANGED SCALE, BUT RELATIVE STILL HOLDS				
FACTOR1	4.092	0.439	9.329	0.000
FACTOR2	4.716	0.492	9.581	0.000
FACTOR3	4.927	0.512	9.623	0.000
Intercepts – SCALED SO SHOULD BE EQUAL ACROSS TIME				
VAR1T1	3.794	1.073	3.537	0.000
VAR1T2	3.063	1.200	2.551	0.011
VAR1T3	2.966	1.250	2.373	0.018
VAR2T1	0.000	0.000	999.000	999.000
VAR2T2	0.000	0.000	999.000	999.000
VAR2T3	0.000	0.000	999.000	999.000
VAR3T1	3.214	0.813	3.954	0.000
VAR3T2	2.385	0.910	2.621	0.009
VAR3T3	2.503	0.949	2.636	0.008
VAR4T1	-11.298	0.992	-11.383	0.000
VAR4T2	-12.722	1.118	-11.382	0.000
VAR4T3	-12.664	1.164	-10.878	0.000
VAR5T1	-5.306	0.511	-10.383	0.000
VAR5T2	-6.290	0.593	-10.606	0.000
VAR5T3	-6.079	0.608	-9.995	0.000
VAR6T1	-9.469	0.612	-15.469	0.000
VAR6T2	-10.010	0.682	-14.670	0.000
VAR6T3	-10.024	0.710	-14.124	0.000
Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR				
VAR1T1	0.729	0.316	2.307	0.021
VAR1T2	0.455	0.242	1.879	0.060
VAR1T3	0.452	0.237	1.908	0.056
VAR2T1	8.485	1.063	7.984	0.000
VAR2T2	5.963	0.747	7.980	0.000
VAR2T3	5.187	0.767	6.760	0.000
VAR3T1	2.190	0.327	6.708	0.000
VAR3T2	2.235	0.310	7.200	0.000
VAR3T3	2.361	0.366	6.444	0.000
VAR4T1	7.281	0.912	7.980	0.000
VAR4T2	6.745	0.827	8.152	0.000
VAR4T3	6.522	0.897	7.273	0.000
VAR5T1	1.819	0.220	8.258	0.000
VAR5T2	4.694	0.559	8.396	0.000
VAR5T3	2.958	0.374	7.918	0.000
VAR6T1	1.635	0.223	7.341	0.000
VAR6T2	1.082	0.144	7.531	0.000
VAR6T3	0.796	0.129	6.171	0.000

Model 2b. Partial Metric Invariance Model with loading for Var1 at Time 1 free

```
MODEL:
! Model 2b: Partial Metric Invariance without VAR1T1
    FACTOR1 BY VAR1T1*
               VAR2T1* (L2)
               VAR3T1* (L3)
               VAR4T1* (L4)
               VAR5T1* (L5)
               VAR6T1* (L6);
    FACTOR2 BY VAR1T2* (L1)
               VAR2T2* (L2)
               VAR3T2* (L3)
               VAR4T2* (L4)
               VAR5T2* (L5)
               VAR6T2* (L6);
    FACTOR3 BY VAR1T3* (L1)
               VAR2T3* (L2)
               VAR3T3* (L3)
               VAR4T3* (L4)
               VAR5T3* (L5)
               VAR6T3* (L6);
! Intercepts - second set to 0 for identification
    [VAR1T1*  VAR1T2*  VAR1T3* ];
    [VAR2T1@0 VAR2T2@0 VAR2T3@0];
    [VAR3T1*  VAR3T2*  VAR3T3* ];
    [VAR4T1*  VAR4T2*  VAR4T3* ];
    [VAR5T1*  VAR5T2*  VAR5T3* ];
    [VAR6T1*  VAR6T2*  VAR6T3* ];
! Residual variances - all free now
     VAR1T1* VAR1T2* VAR1T3*;
     VAR2T1* VAR2T2* VAR2T3*;
     VAR3T1* VAR3T2* VAR3T3*;
     VAR4T1* VAR4T2* VAR4T3*;
     VAR5T1* VAR5T2* VAR5T3*;
     VAR6T1* VAR6T2* VAR6T3*;
! Factor variances (first fixed=1 for identification)
    FACTOR1@1 FACTOR2* FACTOR3*;
! Factor covariances (still all free)
    FACTOR1 WITH FACTOR2*;
    FACTOR1 WITH FACTOR3*;
    FACTOR2 WITH FACTOR3*;
! Factor means (all free now)
    [FACTOR1* FACTOR2* FACTOR3*];
! Residual covariances for same indicator across time
    VAR1T1 WITH VAR1T2* VAR1T3*; VAR1T2 WITH VAR1T3*;
    VAR2T1 WITH VAR2T2* VAR2T3*; VAR2T2 WITH VAR2T3*;
    VAR3T1 WITH VAR3T2* VAR3T3*; VAR3T2 WITH VAR3T3*;
    VAR4T1 WITH VAR4T2* VAR4T3*; VAR4T2 WITH VAR4T3*;
    VAR5T1 WITH VAR5T2* VAR5T3*; VAR5T2 WITH VAR5T3*;
    VAR6T1 WITH VAR6T2* VAR6T3*; VAR6T2 WITH VAR6T3*;
```

```
Number of Free Parameters                       66

Loglikelihood
          H0 Value                      -4435.669
          H1 Value                      -4284.045

Information Criteria
          Akaike (AIC)                   9003.337
          Bayesian (BIC)                 9202.478
          Sample-Size Adjusted BIC       8993.595
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                           303.247
          Degrees of Freedom                  123
          P-Value                          0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.099
          90 Percent C.I.                    0.085  0.113
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.912
          TLI                                0.890

SRMR (Standardized Root Mean Square Residual)
          Value                              0.091
```

How does this fit compare to the fully metric invariant model (2a)?

How does this fit compare to the configural model (1)?

No invariance-related modification indices were found, so we'll call it good!

2b UNSTANDARDIZED PARTIAL METRIC MODEL RESULTS – ALL FACTOR LOADINGS ARE HELD EQUAL EXCEPT VAR1T1

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR1 BY				
VAR1T1	**3.233**	**0.208**	**15.530**	**0.000**
VAR2T1	1.950	0.204	9.558	0.000
VAR3T1	1.967	0.168	11.744	0.000
VAR4T1	1.899	0.199	9.541	0.000
VAR5T1	0.968	0.102	9.458	0.000
VAR6T1	1.476	0.123	12.004	0.000
FACTOR2 BY				
VAR1T2	**2.644**	**0.201**	**13.163**	**0.000**
VAR2T2	1.950	0.204	9.558	0.000
VAR3T2	1.967	0.168	11.744	0.000
VAR4T2	1.899	0.199	9.541	0.000
VAR5T2	0.968	0.102	9.458	0.000
VAR6T2	1.476	0.123	12.004	0.000
FACTOR3 BY				
VAR1T3	**2.644**	**0.201**	**13.163**	**0.000**
VAR2T3	1.950	0.204	9.558	0.000
VAR3T3	1.967	0.168	11.744	0.000
VAR4T3	1.899	0.199	9.541	0.000
VAR5T3	0.968	0.102	9.458	0.000
VAR6T3	1.476	0.123	12.004	0.000
FACTOR1 WITH				
FACTOR2	0.847	0.068	12.472	0.000
FACTOR3	0.682	0.077	8.841	0.000
FACTOR2 WITH				
FACTOR3	0.699	0.110	6.354	0.000
***** Residual covariances among same item across time ******				
VAR1T1 WITH				
VAR1T2	-0.225	0.202	-1.111	0.266
VAR1T3	-0.012	0.205	-0.057	0.955
VAR1T2 WITH				
VAR1T3	0.132	0.176	0.749	0.454
.....				
Variances				
FACTOR1	1.000	0.000	999.000	999.000
FACTOR2	1.162	0.159	7.292	0.000
FACTOR3	0.941	0.152	6.172	0.000

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – OF FACTOR – RELATIVE STILL HOLDS				
FACTOR1	4.446	0.492	9.044	0.000
FACTOR2	5.118	0.553	9.256	0.000
FACTOR3	5.350	0.576	9.293	0.000
Intercepts – SCALED SO SHOULD BE EQUAL ACROSS TIME				
VAR1T1	1.705	1.366	1.248	0.212
VAR1T2	3.697	1.155	3.200	0.001
VAR1T3	3.612	1.202	3.005	0.003
VAR2T1	0.000	0.000	999.000	999.000
VAR2T2	0.000	0.000	999.000	999.000
VAR2T3	0.000	0.000	999.000	999.000
VAR3T1	3.231	0.815	3.963	0.000
VAR3T2	2.401	0.910	2.640	0.008
VAR3T3	2.517	0.949	2.652	0.008
VAR4T1	-11.479	0.998	-11.501	0.000
VAR4T2	-12.929	1.123	-11.512	0.000
VAR4T3	-12.881	1.170	-11.007	0.000
VAR5T1	-5.592	0.535	-10.443	0.000
VAR5T2	-6.616	0.619	-10.686	0.000
VAR5T3	-6.425	0.636	-10.094	0.000
VAR6T1	-9.434	0.607	-15.552	0.000
VAR6T2	-9.967	0.675	-14.774	0.000
VAR6T3	-9.983	0.702	-14.228	0.000
Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR				
VAR1T1	0.170	0.334	0.509	0.611
VAR1T2	0.548	0.243	2.257	0.024
VAR1T3	0.509	0.228	2.230	0.026
VAR2T1	8.702	1.075	8.097	0.000
VAR2T2	5.895	0.739	7.976	0.000
VAR2T3	5.177	0.764	6.776	0.000
VAR3T1	2.502	0.364	6.874	0.000
VAR3T2	2.178	0.303	7.180	0.000
VAR3T3	2.309	0.358	6.455	0.000
VAR4T1	7.172	0.882	8.131	0.000
VAR4T2	6.759	0.834	8.106	0.000
VAR4T3	6.613	0.918	7.205	0.000
VAR5T1	1.829	0.218	8.376	0.000
VAR5T2	4.678	0.557	8.393	0.000
VAR5T3	2.944	0.372	7.919	0.000
VAR6T1	1.707	0.223	7.656	0.000
VAR6T2	1.090	0.146	7.465	0.000
VAR6T3	0.784	0.128	6.146	0.000

Model 3a. Scalar Invariance Model (all intercepts held equal across over time except VAR1T1); identified by Factor1 mean=0

```
MODEL: !Model 3a: Scalar Invariance-intercepts (no VAR1T1)
    FACTOR1 BY VAR1T1*
               VAR2T1* (L2)
               VAR3T1* (L3)
               VAR4T1* (L4)
               VAR5T1* (L5)
               VAR6T1* (L6);
    FACTOR2 BY VAR1T2* (L1)
               VAR2T2* (L2)
               VAR3T2* (L3)
               VAR4T2* (L4)
               VAR5T2* (L5)
               VAR6T2* (L6);
    FACTOR3 BY VAR1T3* (L1)
               VAR2T3* (L2)
               VAR3T3* (L3)
               VAR4T3* (L4)
               VAR5T3* (L5)
               VAR6T3* (L6);
! All intercepts now constrained except VAR1T1
    [VAR1T1*];
             [VAR1T2*  VAR1T3* ] (I1);
    [VAR2T1*  VAR2T2*  VAR2T3* ] (I2);
    [VAR3T1*  VAR3T2*  VAR3T3* ] (I3);
    [VAR4T1*  VAR4T2*  VAR4T3* ] (I4);
    [VAR5T1*  VAR5T2*  VAR5T3* ] (I5);
    [VAR6T1*  VAR6T2*  VAR6T3* ] (I6);
! Residual variances - all free now
     VAR1T1* VAR1T2* VAR1T3*;
     VAR2T1* VAR2T2* VAR2T3*;
     VAR3T1* VAR3T2* VAR3T3*;
     VAR4T1* VAR4T2* VAR4T3*;
     VAR5T1* VAR5T2* VAR5T3*;
     VAR6T1* VAR6T2* VAR6T3*;
! Factor variances (first fixed=1 for identification)
    FACTOR1@1 FACTOR2* FACTOR3*;
! Factor covariances (still all free)
    FACTOR1 WITH FACTOR2*;
    FACTOR1 WITH FACTOR3*;
    FACTOR2 WITH FACTOR3*;
! Factor means (first=0 for identification)
    [FACTOR1@0 FACTOR2* FACTOR3*];
! Residual covariances for same indicator across time
    VAR1T1 WITH VAR1T2* VAR1T3*; VAR1T2 WITH VAR1T3*;
    VAR2T1 WITH VAR2T2* VAR2T3*; VAR2T2 WITH VAR2T3*;
    VAR3T1 WITH VAR3T2* VAR3T3*; VAR3T2 WITH VAR3T3*;
    VAR4T1 WITH VAR4T2* VAR4T3*; VAR4T2 WITH VAR4T3*;
    VAR5T1 WITH VAR5T2* VAR5T3*; VAR5T2 WITH VAR5T3*;
    VAR6T1 WITH VAR6T2* VAR6T3*; VAR6T2 WITH VAR6T3*;
```

```
Number of Free Parameters                       57
Loglikelihood
          H0 Value                       -4461.842
          H1 Value                       -4284.045

Information Criteria
          Akaike (AIC)                    9037.685
          Bayesian (BIC)                  9209.670
          Sample-Size Adjusted BIC        9029.271
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            355.594
          Degrees of Freedom                   132
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.106
          90 Percent C.I.                    0.093  0.119
          Probability RMSEA <= .05           0.000
CFI/TLI
          CFI                                0.890
          TLI                                0.873
SRMR (Standardized Root Mean Square Residual)
          Value                              0.093
```

Is model fit significantly worse relative to the partial metric invariance model (2b)?

```
MODEL MODIFICATION INDICES
Means/Intercepts/Thresholds

                    M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

[ VAR2T1   ]          15.324    -0.696     -0.696       -0.189
[ VAR4T1   ]          10.797     0.366      0.366        0.113
[ VAR5T1   ]          21.688    -0.027     -0.027       -0.017
[ VAR5T2   ]          14.732    -0.596     -0.596       -0.241
```

Modification indices suggest that freeing these intercepts would help, so let's try VAR5T1 first (biggest χ^2 change suggested).

3a UNSTANDARDIZED SCALAR MODEL RESULTS – NOW FACTOR LOADINGS AND INTERCEPTS ARE HELD EQUAL (EXCEPT VAR1T1)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR1 BY				
VAR1T1	3.223	0.210	15.368	0.000
VAR2T1	2.072	0.210	9.850	0.000
VAR3T1	2.023	0.169	11.983	0.000
VAR4T1	1.794	0.194	9.244	0.000
VAR5T1	0.907	0.103	8.834	0.000
VAR6T1	1.516	0.125	12.154	0.000
FACTOR2 BY				
VAR1T2	2.662	0.203	13.102	0.000
VAR2T2	2.072	0.210	9.850	0.000
VAR3T2	2.023	0.169	11.983	0.000
VAR4T2	1.794	0.194	9.244	0.000
VAR5T2	0.907	0.103	8.834	0.000
VAR6T2	1.516	0.125	12.154	0.000
FACTOR3 BY				
VAR1T3	2.662	0.203	13.102	0.000
VAR2T3	2.072	0.210	9.850	0.000
VAR3T3	2.023	0.169	11.983	0.000
VAR4T3	1.794	0.194	9.244	0.000
VAR5T3	0.907	0.103	8.834	0.000
VAR6T3	1.516	0.125	12.154	0.000
FACTOR1 WITH				
FACTOR2	0.837	0.067	12.435	0.000
FACTOR3	0.678	0.076	8.883	0.000
FACTOR2 WITH				
FACTOR3	0.689	0.108	6.365	0.000
Variances				
FACTOR1	1.000	0.000	999.000	999.000
FACTOR2	1.133	0.156	7.270	0.000
FACTOR3	0.923	0.150	6.170	0.000

***** Residual covariances among same item across time still there but not shown ******

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – NOW FACTOR MEAN DIFFERENCES (SCALE HAS CHANGED SO THAT ZERO IS MEAN AT TIME 1)				
FACTOR1	0.000	0.000	999.000	999.000
FACTOR2	0.228	0.073	3.131	0.002
FACTOR3	0.452	0.089	5.052	0.000
Intercepts				
VAR1T1	**16.089**	**0.273**	**58.973**	**0.000**
VAR1T2	16.589	0.252	65.753	0.000
VAR1T3	16.589	0.252	65.753	0.000
VAR2T1	9.368	0.258	36.248	0.000
VAR2T2	9.368	0.258	36.248	0.000
VAR2T3	9.368	0.258	36.248	0.000
VAR3T1	12.026	0.205	58.581	0.000
VAR3T2	12.026	0.205	58.581	0.000
VAR3T3	12.026	0.205	58.581	0.000
VAR4T1	-3.388	0.252	-13.456	0.000
VAR4T2	-3.388	0.252	-13.456	0.000
VAR4T3	-3.388	0.252	-13.456	0.000
VAR5T1	-1.189	0.135	-8.784	0.000
VAR5T2	-1.189	0.135	-8.784	0.000
VAR5T3	-1.189	0.135	-8.784	0.000
VAR6T1	-2.775	0.156	-17.774	0.000
VAR6T2	-2.775	0.156	-17.774	0.000
VAR6T3	-2.775	0.156	-17.774	0.000
Residual Variances				
VAR1T1	0.247	0.348	0.711	0.477
VAR1T2	0.603	0.244	2.475	0.013
VAR1T3	0.560	0.229	2.440	0.015
VAR2T1	9.256	1.185	7.813	0.000
VAR2T2	5.874	0.743	7.911	0.000
VAR2T3	5.060	0.747	6.775	0.000
VAR3T1	2.443	0.367	6.659	0.000
VAR3T2	2.145	0.301	7.125	0.000
VAR3T3	2.284	0.355	6.429	0.000
VAR4T1	7.291	0.912	7.996	0.000
VAR4T2	6.715	0.827	8.122	0.000
VAR4T3	6.512	0.900	7.238	0.000
VAR5T1	1.846	0.222	8.315	0.000
VAR5T2	5.196	0.638	8.139	0.000
VAR5T3	3.193	0.412	7.748	0.000
VAR6T1	1.692	0.226	7.488	0.000
VAR6T2	1.073	0.146	7.366	0.000
VAR6T3	0.757	0.126	6.022	0.000

Model 3b. Partial Scalar Invariance Model (all intercepts held equal across over time except VAR1T1 and VAR5T1)

```
MODEL: !Model 3b: Partial Scalar Invariance (no VAR1T1 or
VAR5T1)
(rest of code before is same)

! All intercepts now constrained except VAR1T1 and VAR5T1
    [VAR1T1*];
             [VAR1T2*  VAR1T3* ] (I1);
    [VAR2T1*  VAR2T2*  VAR2T3* ] (I2);
    [VAR3T1*  VAR3T2*  VAR3T3* ] (I3);
    [VAR4T1*  VAR4T2*  VAR4T3* ] (I4);
    [VAR5T1*];
             [VAR5T2*  VAR5T3* ] (I5);
    [VAR6T1*  VAR6T2*  VAR6T3* ] (I6);
! Factor means (first=0 for identification)
    [FACTOR1@0 FACTOR2* FACTOR3*];

(rest of code after is same)
```

```
Number of Free Parameters                       58
Loglikelihood
          H0 Value                       -4450.001
          H1 Value                       -4284.045

Information Criteria
          Akaike (AIC)                    9016.001
          Bayesian (BIC)                  9191.004
          Sample-Size Adjusted BIC        9007.440
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            331.911
          Degrees of Freedom                   131
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.101
          90 Percent C.I.                    0.087  0.114
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.901
          TLI                                0.885
SRMR (Standardized Root Mean Square Residual)
          Value                              0.086
```

How does this fit compare to the full scalar invariance model (3a)?

How does this fit compare to the partial metric invariance model (2b)?

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Means - OF FACTORS (0=MEAN AT TIME 1)
    FACTOR1            0.000      0.000    999.000    999.000
    FACTOR2            0.285      0.075      3.802      0.000
    FACTOR3            0.514      0.093      5.555      0.000

Intercepts
    VAR1T1            16.090      0.273     58.974      0.000
    VAR1T2            16.444      0.251     65.421      0.000
    VAR1T3            16.444      0.251     65.421      0.000
    VAR2T1             9.271      0.257     36.038      0.000
    VAR2T2             9.271      0.257     36.038      0.000
    VAR2T3             9.271      0.257     36.038      0.000
    VAR3T1            11.957      0.203     58.999      0.000
    VAR3T2            11.957      0.203     58.999      0.000
    VAR3T3            11.957      0.203     58.999      0.000
    VAR4T1            -3.447      0.251    -13.717      0.000
    VAR4T2            -3.447      0.251    -13.717      0.000
    VAR4T3            -3.447      0.251    -13.717      0.000
    VAR5T1            -1.216      0.132     -9.229      0.000
    VAR5T2            -1.794      0.179    -10.002      0.000
    VAR5T3            -1.794      0.179    -10.002      0.000
    VAR6T1            -2.847      0.154    -18.432      0.000
    VAR6T2            -2.847      0.154    -18.432      0.000
    VAR6T3            -2.847      0.154    -18.432      0.000


MODEL MODIFICATION INDICES
Means/Intercepts/Thresholds

                      M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.

[ VAR2T1   ]         12.033    -0.599      -0.599       -0.164
[ VAR4T1   ]         14.398     0.425       0.425        0.132
[ VAR4T2   ]          6.677    -0.306      -0.306       -0.096
```

Modification indices still suggest that freeing these intercepts would help, so let's try VAR4T1 next (biggest χ^2 change suggested).

Model 3c. Partial Scalar Invariance Model (all intercepts held equal across over time except VAR1T1, VAR5T1, and VAR4T1)

```
MODEL: ! Model 3c: Partial Scalar (no VAR1T1, VAR5T1, or
VAR4T1)
(rest of code before is same)

! All intercepts now constrained except VAR1T1, VAR5T1, VAR4T1
    [VAR1T1*];
             [VAR1T2*  VAR1T3* ] (I1);
    [VAR2T1*  VAR2T2*  VAR2T3* ] (I2);
    [VAR3T1*  VAR3T2*  VAR3T3* ] (I3);
    [VAR4T1*];
             [VAR4T2*  VAR4T3* ] (I4);
    [VAR5T1*];
             [VAR5T2*  VAR5T3* ] (I5);
    [VAR6T1*  VAR6T2*  VAR6T3* ] (I6);
! Factor means (first=0 for identification)
    [FACTOR1@0 FACTOR2* FACTOR3*];

(rest of code after is same)
```

```
Number of Free Parameters                       59
Loglikelihood
          H0 Value                       -4442.214
          H1 Value                       -4284.045

Information Criteria
          Akaike (AIC)                    9002.427
          Bayesian (BIC)                  9180.447
          Sample-Size Adjusted BIC        8993.718
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            316.337
          Degrees of Freedom                   130
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.097
          90 Percent C.I.                    0.084  0.111
          Probability RMSEA <= .05           0.000
CFI/TLI
          CFI                                0.909
          TLI                                0.892
SRMR (Standardized Root Mean Square Residual)
          Value                              0.092
```

How does this fit compare to the partially scalar invariant model (3b)?

How does this fit compare to the partial metric invariance model (2b)?

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – OF FACTORS (0=MEAN AT TIME 1)				
FACTOR1	0.000	0.000	999.000	999.000
FACTOR2	0.346	0.078	4.425	0.000
FACTOR3	0.573	0.096	5.969	0.000
Intercepts				
VAR1T1	**16.090**	**0.273**	**58.984**	**0.000**
VAR1T2	16.286	0.256	63.738	0.000
VAR1T3	16.286	0.256	63.738	0.000
VAR2T1	9.171	0.258	35.595	0.000
VAR2T2	9.171	0.258	35.595	0.000
VAR2T3	9.171	0.258	35.595	0.000
VAR3T1	11.885	0.202	58.933	0.000
VAR3T2	11.885	0.202	58.933	0.000
VAR3T3	11.885	0.202	58.933	0.000
VAR4T1	**-3.023**	**0.274**	**-11.045**	**0.000**
VAR4T2	-3.845	0.278	-13.832	0.000
VAR4T3	-3.845	0.278	-13.832	0.000
VAR5T1	**-1.214**	**0.132**	**-9.211**	**0.000**
VAR5T2	-1.854	0.181	-10.220	0.000
VAR5T3	-1.854	0.181	-10.220	0.000
VAR6T1	-2.919	0.154	-18.941	0.000
VAR6T2	-2.919	0.154	-18.941	0.000
VAR6T3	-2.919	0.154	-18.941	0.000

MODEL MODIFICATION INDICES
Means/Intercepts/Thresholds

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
[VAR2T1]	**8.891**	**-0.497**	**-0.497**	**-0.137**

Modification indices still suggest that freeing this intercept would help, so let's try VAR2T1 next (last χ^2 change over 6).

Model 3d. Partial Scalar Invariance Model (all intercepts held equal across over time except VAR1T1, VAR5T1, VAR4T1, VAR2T1)

```
MODEL: ! Model 3d: Partial Scalar
       !(no VAR1T1, VAR5T1, VAR4T1, VAR2T1)

(rest of code before and after is same)

! All intercepts now constrained except
! VAR1T1, VAR5T1, VAR4T1, VAR2T1
    [VAR1T1*];
             [VAR1T2*  VAR1T3* ] (I1);
    [VAR2T1*];
             [VAR2T2*  VAR2T3* ] (I2);
    [VAR3T1*  VAR3T2*  VAR3T3* ] (I3);
    [VAR4T1*];
             [VAR4T2*  VAR4T3* ] (I4);
    [VAR5T1*];
             [VAR5T2*  VAR5T3* ] (I5);
    [VAR6T1*  VAR6T2*  VAR6T3* ] (I6);

! Factor means (first=0 for identification)
    [FACTOR1@0 FACTOR2* FACTOR3*];
```

```
Number of Free Parameters                       60
Loglikelihood
          H0 Value                       -4437.665
          H1 Value                       -4284.045

Information Criteria
          Akaike (AIC)                    8995.330
          Bayesian (BIC)                  9176.366
          Sample-Size Adjusted BIC        8986.473
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                            307.239
          Degrees of Freedom                   129
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.096
          90 Percent C.I.                    0.082  0.109
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.913
          TLI                                0.896

SRMR (Standardized Root Mean Square Residual)
          Value                              0.091
```

How does this fit compare to the partial scalar invariance model (3c)?

How does this fit compare to the partial metric invariance model (2b)?

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – OF FACTORS (0=MEAN AT TIME 1)				
FACTOR1	0.000	0.000	999.000	999.000
FACTOR2	0.293	0.078	3.738	0.000
FACTOR3	0.521	0.095	5.479	0.000
Intercepts				
VAR1T1	**16.090**	**0.273**	**58.988**	**0.000**
VAR1T2	16.425	0.257	63.862	0.000
VAR1T3	16.425	0.257	63.862	0.000
VAR2T1	**8.674**	**0.295**	**29.376**	**0.000**
VAR2T2	9.413	0.258	36.418	0.000
VAR2T3	9.413	0.258	36.418	0.000
VAR3T1	11.950	0.204	58.683	0.000
VAR3T2	11.950	0.204	58.683	0.000
VAR3T3	11.950	0.204	58.683	0.000
VAR4T1	**-3.024**	**0.274**	**-11.047**	**0.000**
VAR4T2	-3.744	0.277	-13.512	0.000
VAR4T3	-3.744	0.277	-13.512	0.000
VAR5T1	**-1.215**	**0.132**	**-9.219**	**0.000**
VAR5T2	-1.802	0.181	-9.970	0.000
VAR5T3	-1.802	0.181	-9.970	0.000
VAR6T1	-2.854	0.156	-18.306	0.000
VAR6T2	-2.854	0.156	-18.306	0.000
VAR6T3	-2.854	0.156	-18.306	0.000

For whatever reason, even after controlling for factor level, the responses to indicators 1 and 2 at time 1 are lower than what is predicted; responses to indicators 4 and 5 are higher than what is predicted.

No invariance-related modification indices were found, so we'll call it good!

Model 4a. Residual Variance Invariance Model (error variances held equal for all except non-invariant items)

```
MODEL: ! Model 4a: Residual Variances
       ! (except for non-invariant items)

(rest of code before and after is same)

! All residual variances constrained
! except for non-invariant items
    VAR1T1*;
           VAR1T2* VAR1T3* (E1);
    VAR2T1*;
           VAR2T2* VAR2T3* (E2);
    VAR3T1* VAR3T2* VAR3T3* (E3);
    VAR4T1*;
           VAR4T2* VAR4T3* (E4);
    VAR5T1*;
           VAR5T2* VAR5T3* (E5);
    VAR6T1* VAR6T2* VAR6T3* (E6);
```

Number of Free Parameters	52	
Loglikelihood		
H0 Value	-4454.592	
H1 Value	-4284.045	
Information Criteria		
Akaike (AIC)	9013.185	
Bayesian (BIC)	9170.083	
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	9005.509	
Chi-Square Test of Model Fit		
Value	341.094	
Degrees of Freedom	137	
P-Value	0.0000	
RMSEA (Root Mean Square Error Of Approximation)		
Estimate	0.099	
90 Percent C.I.	0.086	0.113
Probability RMSEA <= .05	0.000	
CFI/TLI		
CFI	0.900	
TLI	0.888	
SRMR (Standardized Root Mean Square Residual)		
Value	0.095	

How does this fit compare to the partial scalar invariance model (3d)?

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
PREVIOUS MODEL (3D) Residual Variances				
VAR1T1	0.186	0.331	0.560	0.575
VAR1T2	0.577	0.242	2.379	0.017
VAR1T3	0.521	0.227	2.297	0.022
VAR2T1	8.692	1.074	8.096	0.000
VAR2T2	5.868	0.736	7.973	0.000
VAR2T3	5.162	0.758	6.806	0.000
VAR3T1	2.490	0.362	6.882	0.000
VAR3T2	2.169	0.303	7.158	0.000
VAR3T3	2.300	0.356	6.453	0.000
VAR4T1	7.174	0.883	8.124	0.000
VAR4T2	6.758	0.835	8.095	0.000
VAR4T3	6.616	0.919	7.197	0.000
VAR5T1	1.836	0.220	8.340	0.000
VAR5T2	4.696	0.562	8.352	0.000
VAR5T3	2.930	0.370	7.925	0.000
VAR6T1	1.704	0.223	7.657	0.000
VAR6T2	1.089	0.146	7.441	0.000
VAR6T3	0.780	0.127	6.150	0.000
CURRENT MODEL (4A) Residual Variances				
VAR1T1	**0.520**	**0.314**	**1.658**	**0.097**
VAR1T2	0.604	0.215	2.806	0.005
VAR1T3	0.604	0.215	2.806	0.005
VAR2T1	**8.389**	**1.034**	**8.111**	**0.000**
VAR2T2	5.536	0.567	9.771	0.000
VAR2T3	5.536	0.567	9.771	0.000
VAR3T1	2.238	0.234	9.553	0.000
VAR3T2	2.238	0.234	9.553	0.000
VAR3T3	2.238	0.234	9.553	0.000
VAR4T1	**7.204**	**0.890**	**8.096**	**0.000**
VAR4T2	6.608	0.731	9.042	0.000
VAR4T3	6.608	0.731	9.042	0.000
VAR5T1	**1.657**	**0.189**	**8.777**	**0.000**
VAR5T2	3.868	0.410	9.430	0.000
VAR5T3	3.868	0.410	9.430	0.000
VAR6T1	1.218	0.124	9.817	0.000
VAR6T2	1.218	0.124	9.817	0.000
VAR6T3	1.218	0.124	9.817	0.000

MODEL MODIFICATION INDICES
Variances/Residual Variances

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
VAR5T2	**13.652**	**0.755**	**0.755**	**0.153**
VAR5T3	**13.653**	**-1.125**	**-1.125**	**-0.238**
VAR6T1	14.725	0.421	0.421	0.124
VAR6T3	8.375	-0.393	-0.393	-0.124

Model 4b. Partial Residual Variance Invariance Model (error variances held equal for all except non-invariant items and Var5T2 from Var5T3)

```
MODEL: ! Model 4b: Residual Variances
       ! (no Var5T2 and Var5T3)
(rest of code before and after is same)

! All residual variances constrained
! except for non-invariant items
    VAR1T1*;
           VAR1T2* VAR1T3* (E1);
    VAR2T1*;
           VAR2T2* VAR2T3* (E2);
    VAR3T1* VAR3T2* VAR3T3* (E3);
    VAR4T1*;
           VAR4T2* VAR4T3* (E4);
    VAR5T1*;
           VAR5T2*;
                  VAR5T3*;
    VAR6T1* VAR6T2* VAR6T3* (E6);
```

```
Number of Free Parameters                       53
Loglikelihood
          H0 Value                       -4447.259
          H1 Value                       -4284.045
Information Criteria
          Akaike (AIC)                    9000.518
          Bayesian (BIC)                  9160.434
          Sample-Size Adjusted BIC        8992.694
            (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
          Value                            326.427
          Degrees of Freedom                   136
          P-Value                           0.0000
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                          0.096
          90 Percent C.I.                   0.083  0.110
          Probability RMSEA <= .05          0.000
CFI/TLI
          CFI                               0.907
          TLI                               0.895
SRMR (Standardized Root Mean Square Residual)
          Value                             0.093
```

How does this fit compare to the full residual invariance model (4a)?

How does this fit compare to the partial scalar invariance model (3d)?

```
                                                    Two-Tailed
                   Estimate       S.E.  Est./S.E.    P-Value
CURRENT MODEL (4b) Residual Variances
Residual Variances
    VAR1T1            0.515      0.314      1.643      0.100
    VAR1T2            0.601      0.215      2.794      0.005
    VAR1T3            0.601      0.215      2.794      0.005
    VAR2T1            8.396      1.034      8.119      0.000
    VAR2T2            5.552      0.566      9.804      0.000
    VAR2T3            5.552      0.566      9.804      0.000
    VAR3T1            2.235      0.234      9.537      0.000
    VAR3T2            2.235      0.234      9.537      0.000
    VAR3T3            2.235      0.234      9.537      0.000
    VAR4T1            7.189      0.887      8.102      0.000
    VAR4T2            6.604      0.730      9.041      0.000
    VAR4T3            6.604      0.730      9.041      0.000
    VAR5T1            1.840      0.222      8.278      0.000
    VAR5T2            4.674      0.561      8.338      0.000
    VAR5T3            2.876      0.364      7.901      0.000
    VAR6T1            1.222      0.124      9.840      0.000
    VAR6T2            1.222      0.124      9.840      0.000
    VAR6T3            1.222      0.124      9.840      0.000

MODEL MODIFICATION INDICES
Variances/Residual Variances
                     M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
VAR6T1              14.531     0.419      0.419        0.125
VAR6T3               7.543    -0.373     -0.373       -0.118
```

After freeing Var6T1 (Model 4c):

```
VAR6T1*;
       VAR6T2* VAR6T3* (E6);
```

```
Chi-Square Test of Model Fit
          Value                            311.852
          Degrees of Freedom                   135
          P-Value                           0.0000
```

How does this fit compare to the partial residual invariance model (4b)?

How does this fit compare to the partial scalar invariance model (3d)?

UNSTANDARDIZED MODEL RESULTS FOR PARTIAL MEASUREMENT INVARIANCE SOLUTION

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR1 BY				
VAR1T1	**3.214**	**0.208**	**15.434**	**0.000**
VAR2T1	1.945	0.202	9.632	0.000
VAR3T1	1.983	0.165	12.042	0.000
VAR4T1	1.913	0.197	9.700	0.000
VAR5T1	0.987	0.102	9.636	0.000
VAR6T1	1.470	0.120	12.220	0.000
FACTOR2 BY				
VAR1T2	2.644	0.199	13.283	0.000
VAR2T2	1.945	0.202	9.632	0.000
VAR3T2	1.983	0.165	12.042	0.000
VAR4T2	1.913	0.197	9.700	0.000
VAR5T2	0.987	0.102	9.636	0.000
VAR6T2	1.470	0.120	12.220	0.000
FACTOR3 BY				
VAR1T3	2.644	0.199	13.283	0.000
VAR2T3	1.945	0.202	9.632	0.000
VAR3T3	1.983	0.165	12.042	0.000
VAR4T3	1.913	0.197	9.700	0.000
VAR5T3	0.987	0.102	9.636	0.000
VAR6T3	1.470	0.120	12.220	0.000
FACTOR1 WITH				
FACTOR2	0.843	0.068	12.417	0.000
FACTOR3	0.683	0.077	8.869	0.000
FACTOR2 WITH				
FACTOR3	0.692	0.109	6.376	0.000
Variances				
FACTOR1	1.000	0.000	999.000	999.000
FACTOR2	1.159	0.158	7.327	0.000
FACTOR3	0.934	0.150	6.235	0.000

***** Residual covariances among same item across time still there but not shown ******

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Means – FACTOR MEAN DIFFERENCES (SCALED SO THAT ZERO IS MEAN AT TIME 1)				
Means				
FACTOR1	0.000	0.000	999.000	999.000
FACTOR2	0.295	0.078	3.759	0.000
FACTOR3	0.520	0.094	5.511	0.000
Intercepts – VAR3 AND VAR6 ARE HOLDING THIS TOGETHER WITH TIME1				
VAR1T1	**16.089**	**0.273**	**59.025**	**0.000**
VAR1T2	16.418	0.258	63.574	0.000
VAR1T3	16.418	0.258	63.574	0.000
VAR2T1	**8.675**	**0.295**	**29.446**	**0.000**
VAR2T2	9.416	0.258	36.525	0.000
VAR2T3	9.416	0.258	36.525	0.000
VAR3T1	11.950	0.203	58.943	0.000
VAR3T2	11.950	0.203	58.943	0.000
VAR3T3	11.950	0.203	58.943	0.000
VAR4T1	**-3.024**	**0.274**	**-11.035**	**0.000**
VAR4T2	-3.750	0.277	-13.520	0.000
VAR4T3	-3.750	0.277	-13.520	0.000
VAR5T1	**-1.213**	**0.132**	**-9.215**	**0.000**
VAR5T2	-1.803	0.181	-9.980	0.000
VAR5T3	-1.803	0.181	-9.980	0.000
VAR6T1	-2.851	0.155	-18.363	0.000
VAR6T2	-2.851	0.155	-18.363	0.000
VAR6T3	-2.851	0.155	-18.363	0.000
Residual Variances				
VAR1T1	**0.284**	**0.312**	**0.912**	**0.362**
VAR1T2	0.539	0.191	2.826	0.005
VAR1T3	0.539	0.191	2.826	0.005
VAR2T1	**8.562**	**1.049**	**8.163**	**0.000**
VAR2T2	5.592	0.566	9.886	0.000
VAR2T3	5.592	0.566	9.886	0.000
VAR3T1	2.312	0.233	9.915	0.000
VAR3T2	2.312	0.233	9.915	0.000
VAR3T3	2.312	0.233	9.915	0.000
VAR4T1	**7.139**	**0.874**	**8.172**	**0.000**
VAR4T2	6.686	0.736	9.080	0.000
VAR4T3	6.686	0.736	9.080	0.000
VAR5T1	**1.829**	**0.220**	**8.332**	**0.000**
VAR5T2	**4.705**	**0.563**	**8.356**	**0.000**
VAR5T3	**2.908**	**0.367**	**7.929**	**0.000**
VAR6T1	**1.664**	**0.217**	**7.687**	**0.000**
VAR6T2	0.957	0.110	8.681	0.000
VAR6T3	0.957	0.110	8.681	0.000

STRUCTURAL INVARIANCE TESTS

Model 5a. Factor Variance Invariance Model

```
MODEL: ! Model 5a: Factor Variance Invariance
(rest of code before and after is same)

! Model 5a: Factor Variance Invariance (all fixed to 1 now)
    FACTOR1@1 FACTOR2@1 FACTOR3@1;
```

Number of Free Parameters	52	
Loglikelihood		
H0 Value	-4441.238	
H1 Value	-4284.045	
Information Criteria		
Akaike (AIC)	8986.475	
Bayesian (BIC)	9143.374	
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	8978.799	
Chi-Square Test of Model Fit		
Value	314.385	
Degrees of Freedom	137	
P-Value	0.0000	
RMSEA (Root Mean Square Error Of Approximation)		
Estimate	0.093	
90 Percent C.I.	0.079	0.106
Probability RMSEA <= .05	0.000	
CFI/TLI		
CFI	0.913	
TLI	0.903	
SRMR (Standardized Root Mean Square Residual)		
Value	0.100	

How does this fit compare to the partial residual invariance model (4c)?

FACTOR2 WITH				
FACTOR1	0.778	0.039	19.950	0.000
FACTOR3 WITH				
FACTOR1	0.713	0.056	12.680	0.000
FACTOR2	0.662	0.057	11.646	0.000

Model 6a. Factor Covariance Invariance Model

```
MODEL: ! Model 6a: Factor Covariance Invariance
(rest of code before and after is same)

! Model 6a: Factor Covariance Invariance
     FACTOR1 WITH FACTOR2* (Fcov);
     FACTOR1 WITH FACTOR3* (Fcov);
     FACTOR2 WITH FACTOR3* (Fcov);
```

Number of Free Parameters	50	
Loglikelihood		
H0 Value	-4443.654	
H1 Value	-4284.045	
Information Criteria		
Akaike (AIC)	8987.308	
Bayesian (BIC)	9138.172	
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	8979.927	
Chi-Square Test of Model Fit		
Value	319.217	
Degrees of Freedom	139	
P-Value	0.0000	
RMSEA (Root Mean Square Error Of Approximation)		
Estimate	0.093	
90 Percent C.I.	0.079	0.106
Probability RMSEA <= .05	0.000	
CFI/TLI		
CFI	0.912	
TLI	0.903	
SRMR (Standardized Root Mean Square Residual)		
Value	0.100	

How does this fit compare to the factor variance invariance model (5a)?

FACTOR COVARIANCES FROM MODEL 6a (REPRESENT CORRELATIONS):

FACTOR1 WITH				
FACTOR2	0.724	0.038	19.297	0.000
FACTOR3	0.724	0.038	19.297	0.000
FACTOR2 WITH				
FACTOR3	0.724	0.038	19.297	0.000

FACTOR MEANS FROM MODEL 5a (REPRESENT MEAN DIFFERENCES):

FACTOR1	0.000	0.000	999.000	999.000
FACTOR2	0.284	0.081	3.532	0.000
FACTOR3	0.520	0.089	5.815	0.000

Model 7a. Factor Mean Invariance Model

```
MODEL: ! Model 7a: Factor Mean Invariance
       ! Testing Diff between Time2 and Time3

(rest of code before and after is same)

! Model 7a: Factor Mean Invariance (time2 and time3 held
equal)
    [FACTOR1@0];
    [FACTOR2* FACTOR3*] (Fmean);
```

```
Number of Free Parameters                       49
Loglikelihood
          H0 Value                      -4448.472
          H1 Value                      -4284.045

Information Criteria
          Akaike (AIC)                   8994.944
          Bayesian (BIC)                 9142.791
          Sample-Size Adjusted BIC       8987.711
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                           328.853
          Degrees of Freedom                  140
          P-Value                          0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.095
          90 Percent C.I.                    0.081  0.108
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.907
          TLI                                0.899
SRMR (Standardized Root Mean Square Residual)
          Value                              0.107
```

How does this fit compare to the factor covariance invariance model (6a)?

```
FACTOR MEANS FROM MODEL 7a (REPRESENT MEAN DIFFERENCES):
    FACTOR1          0.000      0.000    999.000    999.000
    FACTOR2          0.378      0.077      4.892      0.000
    FACTOR3          0.378      0.077      4.892      0.000
```

Code for model 7a (final):

```
! Factor loadings
FACTOR1 BY VAR1T1*
           VAR2T1* (L2)
           VAR3T1* (L3)
           VAR4T1* (L4)
           VAR5T1* (L5)
           VAR6T1* (L6);
FACTOR2 BY VAR1T2* (L1)
           VAR2T2* (L2)
           VAR3T2* (L3)
           VAR4T2* (L4)
           VAR5T2* (L5)
           VAR6T2* (L6);
FACTOR3 BY VAR1T3* (L1)
           VAR2T3* (L2)
           VAR3T3* (L3)
           VAR4T3* (L4)
           VAR5T3* (L5)
           VAR6T3* (L6);
! Intercepts
[VAR1T1*];
         [VAR1T2*  VAR1T3* ] (I1);
[VAR2T1*];
         [VAR2T2*  VAR2T3* ] (I2);
[VAR3T1*  VAR3T2*  VAR3T3* ] (I3);
[VAR4T1*];
         [VAR4T2*  VAR4T3* ] (I4);
[VAR5T1*];
         [VAR5T2*  VAR5T3* ] (I5);
[VAR6T1*  VAR6T2*  VAR6T3* ] (I6);

! Residual variances
VAR1T1*;
        VAR1T2* VAR1T3* (E1);
VAR2T1*;
        VAR2T2* VAR2T3* (E2);
VAR3T1* VAR3T2* VAR3T3* (E3);
VAR4T1*;
        VAR4T2* VAR4T3* (E4);
VAR5T1*;
        VAR5T2*;
                VAR5T3*;
VAR6T1*;
        VAR6T2* VAR6T3* (E6);
```

```
! Factor variances all fixed to 1
FACTOR1@1 FACTOR2@1 FACTOR3@1;

! Factor covariances all equal
FACTOR1 WITH FACTOR2* (Fcov);
FACTOR1 WITH FACTOR3* (Fcov);
FACTOR2 WITH FACTOR3* (Fcov);

! Factor means all different
[FACTOR1@0];
[FACTOR2* FACTOR3*];
```

Example write-up for these analyses:

The extent to which a confirmatory factor model measuring social functioning (with six observed indicators) exhibited measurement invariance and structural invariance over time (i.e., across three occasions taken at 6-month intervals) was examined using Mplus v. 7.11 (Muthén & Muthén, 1998-2012). Maximum likelihood (ML) estimation was used for all analyses; accordingly, nested model comparisons were conducted using the difference in the model χ^2 values as a function of the difference in model degrees of freedom. A configural invariance model was initially specified in which three correlated factors (i.e., the factor at three occasions) were estimated simultaneously. The second indicator's loading was fixed to 1 and its intercept was fixed to 0 for each factor to identify the model; all factor variances, covariances, and means were then estimated. Residual covariances between the same indicators across occasions were estimated as well. As shown in Table 1, although the configural invariance model had marginal fit, reasonable attempts to improve the level of fit were unsuccessful. Thus, the analysis proceeded by applying parameter constraints in successive models to examine potential decreases in fit resulting from measurement or structural non-invariance over the three occasions.

Equality of the unstandardized indicator factor loadings across occasions was then examined in a metric invariance model. The factor variance was fixed to 1 at time 1 but was freely estimated at times 2 and 3. All factor loadings were constrained equal across time; all intercepts (except for the second item) and residual variances were still permitted to vary across time. Factor covariances and residual covariances were estimated as described previously. The metric invariance model fit significantly worse than the configural invariance model $\Delta\chi^2(10) = 24.20$, $p = .007$. The modification indices suggested that the loading of indicator 1 at time 1 was a source of misfit and should be freed. After doing so, the partial metric invariance model fit significantly better than the full metric invariance model, $\Delta\chi^2(1) = 13.46$, $p < .001$, and the partial metric invariance model did not fit worse than the configural invariance model, $\Delta\chi^2(9) = 10.73$, $p = .29$. The fact that partial metric invariance (i.e., "weak invariance") held indicates that the indicators were related to the latent factor equivalently across time, or more simply, that the same latent factor was being measured at each of occasion (with the exception of indicator 1, which was more related to the factor at time 1 than at time 2 or 3).

Equality of the unstandardized indicator intercepts across time was then examined in a scalar invariance model. The factor variance and mean were fixed to 1 and 0, respectively, at time 1 for identification, but the factor variance and mean were then estimated at times 2 and 3. All factor loadings and indicator intercepts were constrained equal across time (except for indicator 1 at time 1); all residual variances were still permitted to differ across time. Factor covariances and residual covariances were estimated as described previously. The scalar invariance model fit significantly worse than the partial metric invariance model, $\Delta\chi^2(9) = 52.347$, $p < .001$. The modification indices suggested that the intercept of indicator 5 at time 1 was the largest source of the misfit and should be freed. After doing so, although the partial scalar invariance model had significantly better fit than the full scalar invariance model, $\Delta\chi^2(1) = 23.68$, $p < .001$, it still fit worse than the partial metric invariance model, $\Delta\chi^2(8) = 28.66$, $p < 001$. The modification indices suggested that the intercept of indicator 4 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, although the new partial scalar invariance model (with the intercepts for indicators 1, 4, and 5 freed at time 1) fit significantly better than the previous partial scalar invariance model (without the intercept for indicator 4 freed at time 1), $\Delta\chi^2(1) = 15.57$, $p < .001$, it still fit marginally worse than the partial metric invariance model, $\Delta\chi^2(7) = 13.09$, $p = 07$. The modification indices suggested that the intercept of indicator 2 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model (with the intercepts for indicators 1, 2, 4 and 5 freed at time 1) fit significantly better than the previous partial scalar invariance model (without the intercept for indicator 2 freed at time 1), $\Delta\chi^2(1) = 9.91$, $p = .003$, and it did not fit significantly worse than the partial metric invariance model, $\Delta\chi^2(6) = 3.99$, $p = .68$. The factor that partial scalar invariance (i.e., "strong invariance") held indicates that times 2 and 3 have the same expected response for each indicator at the same absolute level of the trait, or more simply, that the observed differences in the indicator means between times 2 and 3 is due to factor mean differences only. However, indicators 1 and 2 had a lower expected indicator response at the same absolute level of social functioning at time 1 than at time 2 or 3, while indicators 4 and 5 had a higher expected response.

Equality of the unstandardized residual variances across time was then examined in a residual variance invariance model. As in the partial scalar invariance model, the factor variance and mean were fixed to 1 and 0, respectively, for identification at time 1, but the factor variance and mean were still estimated at times 2 and 3. All factor loadings (except for indicator 1 at time 1), item intercepts (except for indicators 1, 2, 4, and 5 at time 1), and all residual variances (except for indicators 1, 2, 4, and 5 at time 1) were constrained to be equal across groups. Factor covariances and residual covariances were estimated as described previously. The residual variance invariance model fit significantly worse than the last partial scalar invariance model, $\Delta\chi^2(8) = 33.86$, $p = .001$. The modification indices suggested that the residual variance of indicator 5 at time 2 versus time 3 was the largest remaining source of the misfit and should be freed. After doing so, the partial residual variance invariance model fit significantly better than the residual invariance model, $\Delta\chi^2(1) = 14.67$, $p < .001$, but still fit worse than the last partial scalar invariance model, $\Delta\chi^2(7) = 19.19$, $p = .008$. The modification indices suggested that the residual variance of indicator 6 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial residual variance invariance model (with residual variances for indicators 1, 2, 4, 5, and 6 free at time 1; indicator 5 free at times 2 and 3 also) fit significantly better than the partial residual invariance model (without the residual variance for indicator 6 at time 1 freed), $\Delta\chi^2(1) = 14.58$, $p < .001$, and did not fit worse than the last partial scalar invariance model, $\Delta\chi^2(6) = 4.61$, $p = .59$. The fact that partial residual variance invariance (i.e., “strict invariance”) held indicates that the amount of indicator variance not accounted for by the factor was the same across times 2 and 3 (except for indicator 5, for which there was more residual variance at time 2). However, 5 out of 6 indicators did not have residual variance invariance at time 1 (although this was required because of a lack of metric or scalar invariance for indicators 1, 2, 4, and 5).

After achieving partial measurement invariance as was just described, structural invariance was then tested with 3 additional models. First, the factor variance at times 2 and 3 (which had been estimated freely) was constrained to 1 (i.e., to be equal to the factor variance at time 1), resulting in a nonsignificant decrease in fit relative to the last partial residual invariance model, $\Delta\chi^2(2) = 2.533$, $p = .28$. Thus, equivalent amounts of individual differences in social functioning were found across time. Second, the factor covariances across time were constrained to be equal (which are equal factor correlations given a variance of 1 for each factor across time), resulting in a nonsignificant decrease in fit relative to the factor variance invariance model, $\Delta\chi^2(2) = 4.832$, $p = .09$. Third, the factor means at times 2 and 3 (which had been estimated freely) was constrained to be equal to each other, resulting in a significant decrease in fit relative to the factor covariance invariance model $\Delta\chi^2(1) = 9.636$, $p = .002$, indicating that the factor mean at time 3 was significantly higher than at time 2. The factor mean at time 2 was already significantly different from 0 (the factor mean at time 1), and thus, the three factor means were significantly different, increasing over time.

In conclusion, these analyses showed that partial measurement invariance was obtained over time – that is, the relationships of the indicators to the latent factor of social functioning were equivalent at times 2 and 3, although primarily not equivalent at time 1, as previous described. These analyses also showed that partial structural invariance was obtained over time, such that the same amount of individual differences variance in social functioning was observed with equal covariance over time across occasions (i.e., compound symmetry of the latent factor), although the amount of social functioning on average increased significantly over time. Model parameters from the final model are given in Table 2.

(see excel worksheet for tables 1 and 2)