

CFA Example Using Forgiveness of Situations (N = 1103) using SAS MIXED (See Example 4 for corresponding Mplus syntax and output)

SAS Code to Read in Mplus Data:

```

* Import data from Mplus, becomes var1-var23 without names at top;
PROC IMPORT OUT=work.Situation DATAFILE= "&example.\Study2.dat" DBMS=TAB REPLACE;
  GETNAMES=NO; DATAROW=1; RUN;

* Rename variables;
DATA Situation; SET Situation;
  ARRAY old(23) var1-var23;
  ARRAY new(23) PersonID Self1 Self2r Self3 Self4r Self5 Self6r
    Other1r Other2 Other3r Other4 Other5r Other6
    Sit1r Sit2 Sit3r Sit4 Sit5r Sit6
    Selfsub Othsub Sitsub HFSsum;
  DO i=1 TO 23; new(i)=old(i); IF new(i)=99999 THEN new(i)=.; END;
  DROP i var1-var23; RUN;

* Stack situation items;
DATA SituationStacked; SET Situation;
  ARRAY aitem(6) Sit1r Sit2 Sit3r Sit4 Sit5r Sit6;
  DO i=1 TO 6; itemnum=i; response=aitem(i); OUTPUT; END; DROP i; RUN;

```

Independence (Null) Baseline Model: Item means and variances, but NO covariances

```

TITLE "Independence (Null) CFA Model in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
  CLASS PersonID itemnum;
  MODEL response = itemnum / SOLUTION NOINT NOTEST;
  REPEATED itemnum / TYPE=TOEPH(1) SUBJECT=PersonID R; RUN;

```

TYPE=TOEPH(1) predicts a diagonal matrix (would be the same as TYPE=UN(1).

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493					
2		1.9028				
3			2.5431			
4				1.9672		
5					2.9451	
6						2.3412

The R matrix shows the unconditional variances per item—repeated in the next piece of output as Var(item). Note that this independence model predicts no covariances between items.

Covariance Parameter Estimates

Cov	Subject	Estimate	Standard Error	Z	Pr > Z
Var(1)	PersonID	3.0493	0.1298	23.48	<.0001
Var(2)	PersonID	1.9028	0.08102	23.48	<.0001
Var(3)	PersonID	2.5431	0.1083	23.48	<.0001
Var(4)	PersonID	1.9672	0.08377	23.48	<.0001
Var(5)	PersonID	2.9451	0.1254	23.48	<.0001
Var(6)	PersonID	2.3412	0.09969	23.48	<.0001

Model fit is given as -2LL rather than LL (but otherwise is the same as given from Mplus).

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
24625.9	12	24649.9	24650.0	24672.6	24710.0	24722.0

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects show the unconditional means per item.

Saturated (Unstructured) Baseline Model: Item means, variances, and covariances in original data

```
TITLE "Saturated (Unstructured) CFA Model in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
CLASS PersonID itemnum;
MODEL response = itemnum / SOLUTION NOINT NOTEST;
REPEATED itemnum / TYPE=UN(6) SUBJECT=PersonID R RCORR; RUN;
```

TYPE=UN(6) predicts a fully-estimated matrix without any constraints whatsoever.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493	0.5772	1.8022	0.7339	1.3583	0.7946
2	0.5772	1.9028	0.6974	1.1029	0.6043	0.9652
3	1.8022	0.6974	2.5431	0.8244	1.3191	0.8676
4	0.7339	1.1029	0.8244	1.9672	0.6947	0.9618
5	1.3583	0.6043	1.3191	0.6947	2.9451	0.7982
6	0.7946	0.9652	0.8676	0.9618	0.7982	2.3412

The R matrix shows the unconditional variances and covariances for the items.

RCORR is the unconditional correlation matrix.

Note THIS IS THE DATA—the only discrepancies you'd see relative to descriptive statistics would be from missing data, as these are ML estimates (that assume MAR rather than MCAR as in listwise deletion).

Estimated R Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.2396	0.6472	0.2997	0.4533	0.2974
2	0.2396	1.0000	0.3170	0.5700	0.2553	0.4573
3	0.6472	0.3170	1.0000	0.3686	0.4820	0.3555
4	0.2997	0.5700	0.3686	1.0000	0.2886	0.4482
5	0.4533	0.2553	0.4820	0.2886	1.0000	0.3040
6	0.2974	0.4573	0.3555	0.4482	0.3040	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	PersonID	3.0493	0.1298	23.48	<.0001
UN(2,1)	PersonID	0.5772	0.07458	7.74	<.0001
UN(2,2)	PersonID	1.9028	0.08102	23.48	<.0001
UN(3,1)	PersonID	1.8022	0.09988	18.04	<.0001
UN(3,2)	PersonID	0.6974	0.06948	10.04	<.0001
UN(3,3)	PersonID	2.5431	0.1083	23.48	<.0001
UN(4,1)	PersonID	0.7339	0.07699	9.53	<.0001
UN(4,2)	PersonID	1.1029	0.06705	16.45	<.0001
UN(4,3)	PersonID	0.8244	0.07178	11.49	<.0001
UN(4,4)	PersonID	1.9672	0.08377	23.48	<.0001
UN(5,1)	PersonID	1.3583	0.09907	13.71	<.0001
UN(5,2)	PersonID	0.6043	0.07356	8.21	<.0001
UN(5,3)	PersonID	1.3191	0.09148	14.42	<.0001
UN(5,4)	PersonID	0.6947	0.07543	9.21	<.0001
UN(5,5)	PersonID	2.9451	0.1254	23.48	<.0001
UN(6,1)	PersonID	0.7946	0.08393	9.47	<.0001
UN(6,2)	PersonID	0.9652	0.06988	13.81	<.0001
UN(6,3)	PersonID	0.8676	0.07798	11.13	<.0001
UN(6,4)	PersonID	0.9618	0.07081	13.58	<.0001
UN(6,5)	PersonID	0.7982	0.08264	9.66	<.0001
UN(6,6)	PersonID	2.3412	0.09969	23.48	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
22644.9	27	22698.9	22699.1	22750.0	22834.0	22861.0

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects again show the unconditional means per item.

**Model 1. Fully Z-Scored Factor Model Identification
(Factor Variance = 1, Factor Mean = 0, All Loadings and Intercepts Estimated)**

```
TITLE "Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
  CLASS PersonID itemnum;
  MODEL response = itemnum / SOLUTION NOINT NOTEST;
  REPEATED itemnum / TYPE=FA(1) SUBJECT=PersonID R RCORR;
RUN;
```

TYPE=FA(1) creates the covariance matrix that would be predicted by a single-factor model.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	3.0493	0.8670	1.5313	0.9682	1.2626	1.0108
2	0.8670	1.9028	0.8716	0.5511	0.7187	0.5753
3	1.5313	0.8716	2.5431	0.9733	1.2692	1.0161
4	0.9682	0.5511	0.9733	1.9672	0.8025	0.6424
5	1.2626	0.7187	1.2692	0.8025	2.9451	0.8378
6	1.0108	0.5753	1.0161	0.6424	0.8378	2.3412

The R matrix shows the predicted variances and covariances for the items.

RCORR is the single-factor predicted correlation matrix.

THIS IS NO LONGER THE DATA. So the objective is to see how close this predicted covariance matrix is from the one given by the saturated model (which was the data).

Estimated R Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3600	0.5499	0.3953	0.4213	0.3783
2	0.3600	1.0000	0.3962	0.2848	0.3036	0.2726
3	0.5499	0.3962	1.0000	0.4351	0.4638	0.4164
4	0.3953	0.2848	0.4351	1.0000	0.3334	0.2994
5	0.4213	0.3036	0.4638	0.3334	1.0000	0.3191
6	0.3783	0.2726	0.4164	0.2994	0.3191	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z	Pr > Z
FA(1)	PersonID	1.5259	0.09440	16.16	<.0001
FA(2)	PersonID	1.4093	0.07096	19.86	<.0001
FA(3)	PersonID	1.0038	0.07755	12.94	<.0001
FA(4)	PersonID	1.3518	0.07071	19.12	<.0001
FA(5)	PersonID	1.8986	0.09312	20.39	<.0001
FA(6)	PersonID	1.6706	0.08330	20.05	<.0001
FA(1,1)	PersonID	1.2342	0.05332	23.15	<.0001
FA(2,1)	PersonID	0.7025	0.04720	14.88	<.0001
FA(3,1)	PersonID	1.2407	0.04783	25.94	<.0001
FA(4,1)	PersonID	0.7845	0.04679	16.76	<.0001
FA(5,1)	PersonID	1.0230	0.05202	19.67	<.0001
FA(6,1)	PersonID	0.8190	0.05019	16.32	<.0001

The FA(item) terms are the item residual variances. The FA(item, factor) terms are the item factor loadings.

So the total variance per item is given by: $\text{loading}^2(1) + \text{error variance}$, as shown in the R matrix above.

Item 1 = $1.2342^2 + 1.5259 = 3.0493$

The covariance between items is given by their loadings multiplied together.

Item 1 and 2 cov = $1.2342 * 0.7025 = 0.8670$

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23072.8	18	23108.8	23108.9	23142.9	23198.9	23216.9

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05258	5509	86.47	<.0001
itemnum	2	5.2892	0.04153	5509	127.35	<.0001
itemnum	3	4.8957	0.04802	5509	101.96	<.0001
itemnum	4	5.3590	0.04223	5509	126.90	<.0001
itemnum	5	4.8604	0.05167	5509	94.06	<.0001
itemnum	6	5.3209	0.04607	5509	115.49	<.0001

The fixed effects now show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

**Tau-Equivalent Items Single-Factor Model with Marker Item Factor Model Identification
(Factor Variance = ?, Factor Mean = 0, All Loadings Equal at 1)**

```
TITLE "Tau-Equivalent Items Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
CLASS PersonID itemnum;
MODEL response = itemnum / SOLUTION NOINT NOTEST;
RANDOM INTERCEPT / TYPE=UN SUBJECT=PersonID G V VCORR;
REPEATED itemnum / TYPE=TOEPH(1) SUBJECT=PersonID R; RUN;
```

A random intercept creates a constant source of covariance across all items.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.0017					
2		1.1357				
3			1.4550			
4				1.0866		
5					2.0552	
6						1.4565

The R matrix shows the item residual variances.

The G matrix shows the variance due to the factor for all items.

V is the predicted covariance matrix from putting G and R back together, and VCORR is the predicted correlation matrix.

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	1	0.9127

Estimated V Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.9143	0.9127	0.9127	0.9127	0.9127	0.9127
2	0.9127	2.0483	0.9127	0.9127	0.9127	0.9127
3	0.9127	0.9127	2.3677	0.9127	0.9127	0.9127
4	0.9127	0.9127	0.9127	1.9993	0.9127	0.9127
5	0.9127	0.9127	0.9127	0.9127	2.9679	0.9127
6	0.9127	0.9127	0.9127	0.9127	0.9127	2.3691

Estimated V Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3735	0.3474	0.3781	0.3103	0.3473
2	0.3735	1.0000	0.4144	0.4510	0.3702	0.4143
3	0.3474	0.4144	1.0000	0.4195	0.3443	0.3853
4	0.3781	0.4510	0.4195	1.0000	0.3747	0.4194
5	0.3103	0.3702	0.3443	0.3747	1.0000	0.3442
6	0.3473	0.4143	0.3853	0.4194	0.3442	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z	Pr > Z
UN(1,1)	PersonID	0.9127	0.04938	18.48	<.0001
Var(1)	PersonID	2.0017	0.09613	20.82	<.0001
Var(2)	PersonID	1.1357	0.05929	19.15	<.0001
Var(3)	PersonID	1.4550	0.07304	19.92	<.0001
Var(4)	PersonID	1.0866	0.05703	19.05	<.0001
Var(5)	PersonID	2.0552	0.09729	21.13	<.0001
Var(6)	PersonID	1.4565	0.07161	20.34	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23131.1	13	23157.1	23157.1	23181.7	23222.2	23235.2

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.05140	5510	88.45	<.0001
itemnum	2	5.2892	0.04309	5510	122.74	<.0001
itemnum	3	4.8957	0.04633	5510	105.67	<.0001
itemnum	4	5.3590	0.04257	5510	125.87	<.0001
itemnum	5	4.8604	0.05187	5510	93.70	<.0001
itemnum	6	5.3209	0.04635	5510	114.81	<.0001

The fixed effects still show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Parallel Items Single-Factor Model with Marker Item Factor Model Identification (Factor Variance = ?, Factor Mean = 0, All Loadings = 1 and All Error Variances Equal)

```
TITLE "Parallel Items Single-Factor CFA Model (Factor Variance=1, Factor Mean=0) in MIXED";
PROC MIXED DATA=SituationStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
CLASS PersonID itemnum;
MODEL response = itemnum / SOLUTION NOINT NOTEST;
RANDOM INTERCEPT / TYPE=UN SUBJECT=PersonID G V VCORR;
REPEATED itemnum / TYPE=VC SUBJECT=PersonID R; RUN;
```

A random intercept creates a constant source of covariance across all items. A Type=VC R matrix means equal residual variance across items.

Estimated R Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.5180					
2		1.5180				
3			1.5180			
4				1.5180		
5					1.5180	
6						1.5180

Estimated G Matrix

Row	Effect	Person ID	Col1
1	Intercept	1	0.9401

Estimated V Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	2.4581	0.9401	0.9401	0.9401	0.9401	0.9401
2	0.9401	2.4581	0.9401	0.9401	0.9401	0.9401
3	0.9401	0.9401	2.4581	0.9401	0.9401	0.9401
4	0.9401	0.9401	0.9401	2.4581	0.9401	0.9401
5	0.9401	0.9401	0.9401	0.9401	2.4581	0.9401
6	0.9401	0.9401	0.9401	0.9401	0.9401	2.4581

The **R** matrix shows the item residual variances.

The **G** matrix shows the variance due to the factor for all items.

V is the predicted covariance matrix from putting **G** and **R** back together, and **VCORR** is the predicted correlation matrix.

This type of predicted covariance matrix has a special name: **compound symmetry**.

Estimated V Correlation Matrix for PersonID 1

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.3825	0.3825	0.3825	0.3825	0.3825
2	0.3825	1.0000	0.3825	0.3825	0.3825	0.3825
3	0.3825	0.3825	1.0000	0.3825	0.3825	0.3825
4	0.3825	0.3825	0.3825	1.0000	0.3825	0.3825
5	0.3825	0.3825	0.3825	0.3825	1.0000	0.3825
6	0.3825	0.3825	0.3825	0.3825	0.3825	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	PersonID	0.9401	0.05103	18.42	<.0001
itemnum	PersonID	1.5180	0.02891	52.51	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
23254.0	8	23270.0	23270.1	23285.2	23310.1	23318.1

Solution for Fixed Effects

Effect	itemnum	Estimate	Standard Error	DF	t Value	Pr > t
itemnum	1	4.5467	0.04721	5510	96.31	<.0001
itemnum	2	5.2892	0.04721	5510	112.04	<.0001
itemnum	3	4.8957	0.04721	5510	103.71	<.0001
itemnum	4	5.3590	0.04721	5510	113.52	<.0001
itemnum	5	4.8604	0.04721	5510	102.96	<.0001
itemnum	6	5.3209	0.04721	5510	112.71	<.0001

The fixed effects still show the intercepts per item conditional on factor = 0 (which then are equal to the original item means).

Unfortunately, multiple factor models in MIXED appear to be EFA models instead of CFA models, so no examples of two-factor models are given.