Higher-Order Models (CFA with MLR and IFA with WLSMV) in Mplus version 7.11

Example data: 1336 college students self-reporting on 49 items (measuring 5 factors) assessing childhood maltreatment: Items are answered on a 1-5 scale: 1=Strongly Disagree, 2=Disagree, 3=Neutral, 4=Agree, 5=Strongly Agree. The items are not normally distributed, so we'll use both CFA with MLR and IFA with WLSMV as 2 options to examine the fit of these models (as an example of how to do each, but NOT to compare between estimators).

1. Spurning: Verbal and nonverbal caregiver acts that reject and degrade a child
2. Terrorizing: Caregiver behaviors that threaten or are likely to physically hurt, kill, abandon, or place the child or the child's loved ones or objects in recognizably dangerous situations.
3. Isolating: Caregiver acts that consistently deny the child opportunities to meet needs for interacting or communicating with peers or adults inside or outside the home.
4. Corrupting: Caregiver acts that encourage the child to develop inappropriate behaviors (self-destructive, antisocial, criminal, deviant, or other maladaptive behaviors).
5. Ignoring: Emotional unresponsiveness includes caregiver acts that ignore the child's attempts and needs to interact (failing to express affection, caring, and love for the child) and show no emotion in interactions with the child

Here are the results from fitting the factor separately to ensure their individual fit FIRST:

ASESSMENT OF MODEL FIT USING MLR

Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
MLR Spurning	12	90	36	224.797	1.401	54	<.0001	0.959	0.049	0.042	0.055	0.619
MLR Terror	9	54	27	189.815	1.588	27	<.0001	0.918	0.067	0.058	0.076	0.001
MLR Isolate	6	27	18	80.354	1.494	9	<.0001	0.916	0.077	0.062	0.093	0.002
MLR Corrupt	7	35	21	54.964	1.908	14	<.0001	0.934	0.047	0.034	0.060	0.633
MLR Ignore	15	135	45	484.291	1.792	90	<.0001	0.932	0.057	0.052	0.062	0.008

ASESSMENT OF MODEL FIT USING WLSMV

Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
WLSMV Spurning	12	126	60	294.707		54	<.0001	0.983	0.058	0.051	0.064	0.023
WLSMV Terror	9	81	45	263.155		27	<.0001	0.966	0.081	0.072	0.090	<.0001
WLSMV Isolate	6	45	30	129.827		9	<.0001	0.962	0.100	0.085	0.116	<.0001
WLSMV Corrupt	7	56	35	87.488		14	<.0001	0.976	0.063	0.055	0.076	0.044
WLSMV Ignore	15	180	75	897.691		90	<.0001	0.976	0.082	0.077	0.087	<.0001

Here are the standardized factor loadings for each item under each estimation method. Note that the WLSMV factor loadings are higher in this case – probably because of range restriction in the original data and thus the implausibility of a linear model.

MLR Spurning	WLSMV Spurning	MLR Terror	WLSMV Terror	MLR Isolate	WLSMV Isolate	MLR Corrupt	WLSMV Corrupt	MLR Ignore	WLSMV Ignore
0.599	0.660	0.512	0.617	0.521	0.696	0.589	0.739	0.672	0.813
0.457	0.528	0.673	0.771	0.550	0.630	0.545	0.713	0.654	0.749
0.769	0.837	0.451	0.713	0.545	0.685	0.375	0.523	0.657	0.748
0.526	0.597	0.612	0.721	0.540	0.629	0.545	0.854	0.724	0.801
0.607	0.677	0.571	0.787	0.563	0.726	0.631	0.826	0.445	0.540
0.816	0.865	0.554	0.617	0.752	0.822	0.580	0.708	0.745	0.833
0.835	0.907	0.685	0.805			0.646	0.840	0.847	0.913
0.465	0.538	0.643	0.743					0.713	0.813
0.516	0.728	0.732	0.815					0.808	0.891
0.655	0.744							0.749	0.845
0.674	0.756							0.656	0.795
0.610	0.680							0.830	0.904
								0.712	0.806
								0.739	0.815
								0.825	0.918

Syntax for CFA model with MLR including all 5 correlated factors ("biggest model" for comparison):

```
TITLE:  5-factor model: 5 correlated factors
DATA:   FILE IS abuse.dat;

VARIABLE:
NAMES ARE ID   ! All variables in DATA SET
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES ARE  ! All variables in MODEL
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

MISSING ARE .;
IDVARIABLE IS ID;

ANALYSIS:   ESTIMATOR IS MLR;         ! Robust estimator

OUTPUT:     STDYX         ! Standardized solution
            MODINDICES    ! Voodoo for fixing the model
            RESIDUAL      ! Local fit info
            TECH4;        ! Factor correlation matrix

SAVEDATA:   SAVE = FSCORES;           ! Save factor scores
            FILE IS Abuse_Thetas.dat; ! File of factor scores

PLOT:   TYPE IS PLOT1 PLOT2 PLOT3;
```

```
MODEL:
! 5 Lower-Order Factors (loadings for first item fixed =1)

! 12-Item Spurning
Spurn   BY  p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
Terror  BY  p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
Isolate BY  p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
Corrupt BY  p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
Ignore  BY  p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free)
Spurn* Terror* Isolate* Corrupt* Ignore*;

! Factor Means (all fixed = 0 by default)
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Factor Covariance (all free by default if predictors)
Spurn  Terror  Isolate  Corrupt  Ignore  WITH
Spurn* Terror* Isolate* Corrupt* Ignore*;
```

NOTE: With respect to fit of the structural model, letting the separate factors be correlated is as good as it gets. This will be our "larger model" baseline with which to compare the fit of a single higher-order factor model ("smaller model").

Output for CFA model with MLR including all 5 correlated factors ("biggest model" for comparison):

```
Number of Free Parameters                       157

Loglikelihood
          H0 Value                      -69027.431
          H0 Scaling Correction Factor      2.5033
            for MLR
          H1 Value                      -65787.405
          H1 Scaling Correction Factor      1.5925
            for MLR

Information Criteria
          Akaike (AIC)                  138368.862
          Bayesian (BIC)                139184.860
          Sample-Size Adjusted BIC      138686.140
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                           4424.700*
          Degrees of Freedom                 1117
          P-Value                          0.0000
          Scaling Correction Factor          1.4645
            for MLR

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and
WLSMV cannot be used for chi-square difference testing in the
regular way.  MLM, MLR and WLSM chi-square difference testing
is described on the Mplus website.  MLMV, WLSMV, and ULSMV
difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.047
          90 Percent C.I.              0.046  0.049
          Probability RMSEA <= .05           1.000

CFI/TLI
          CFI                                0.847
          TLI                                0.839

Chi-Square Test of Model Fit for the Baseline Model
          Value                         22801.852
          Degrees of Freedom                 1176
          P-Value                          0.0000

SRMR (Standardized Root Mean Square Residual)
          Value                              0.057
```

		SPURN	TERROR	ISOLATE	CORRUPT
Latent Variable Correlations	SPURN				
	TERROR	0.929			
	ISOLATE	0.898	0.876		
	CORRUPT	0.689	0.792	0.658	
	IGNORE	0.830	0.767	0.828	0.630
Factor	**SPURN**	**TERROR**	**ISOLATE**	**CORRUPT**	**IGNORE**
Variance	0.493	0.231	0.129	0.129	0.212
Standardized Loadings for Items	0.583	0.532	0.493	0.601	0.681
	0.444	0.678	0.606	0.535	0.653
	0.764	0.462	0.601	0.365	0.650
	0.524	0.596	0.585	0.500	0.717
	0.593	0.587	0.497	0.627	0.474
	0.796	0.592	0.683	0.611	0.743
	0.824	0.674		0.654	0.842
	0.515	0.626			0.708
	0.562	0.706			0.807
	0.663				0.757
	0.677				0.670
	0.629				0.822
					0.700
					0.754
					0.822

Note: #free parameters = 157 = 44 loadings + 49 intercepts + 49 residuals + 5 factor variances + 10 factor covariances = 157 parameters USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 – 157 = 1117

Now we can test the fit of a constrained structural model that posits a single higher-order "General Abuse" factor to account for the correlations among these 5 latent factors.

Syntax for CFA model with MLR and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

```
TITLE:  5-factor model: 5 lower-order, 1 higher-order factor
DATA:   FILE IS abuse.dat;

VARIABLE:
NAMES ARE ID   ! All variables in DATA SET
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES ARE  ! All variables in MODEL
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

MISSING ARE .;
IDVARIABLE IS ID;

ANALYSIS:   ESTIMATOR IS MLR;         ! Robust estimator

OUTPUT:     STDYX           ! Standardized solution
            MODINDICES      ! Voodoo for fixing the model
            RESIDUAL;       ! Local fit info

SAVEDATA:   SAVE = FSCORES;           ! Save factor scores (thetas)
            FILE IS Abuse_Thetas.dat; ! File factor scores saved to

PLOT:   TYPE IS PLOT1 PLOT2 PLOT3;
```

```
MODEL:
! 5 Lower-Order Factors (loadings for first item fixed =1)

! 12-Item Spurning
Spurn   BY  p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
Terror  BY  p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
Isolate BY  p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
Corrupt BY  p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
Ignore  BY  p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;


! Factor Variances (all must be free – NOW "DISTURBANCES")
Spurn* Terror* Isolate* Corrupt* Ignore*;


! Factor Means (all fixed = 0 by default)
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];


! Higher-Order Factor (estimate loadings, fix mean=0 & variance=1)
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
Abuse@1;
[Abuse@0];
```

NOTE: With respect to fit of the structural model, we are now fitting a single higher-order factor INSTEAD OF covariances among the 5 factors.

To test the fit against the saturated (all possible factor correlations model), we can do a $-2\Delta LL$ scaled difference test.

Output for CFA model with MLR and a higher-order factor instead of correlations among factors ("smaller/bigger model" for comparison):

```
MODEL FIT INFORMATION
Number of Free Parameters                       152
Loglikelihood
          H0 Value                      -69080.656
          H0 Scaling Correction Factor      2.5109
            for MLR
          H1 Value                      -65787.405
          H1 Scaling Correction Factor      1.5925
            for MLR

Information Criteria
          Akaike (AIC)                  138465.313
          Bayesian (BIC)                139255.323
          Sample-Size Adjusted BIC      138772.486
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                          4486.382*
          Degrees of Freedom                1122
          P-Value                         0.0000
          Scaling Correction Factor         1.4681
            for MLR

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and
WLSMV cannot be used for chi-square difference testing in the
regular way.  MLM, MLR and WLSM chi-square difference testing
is described on the Mplus website.  MLMV, WLSMV, and ULSMV
difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.047
          90 Percent C.I.              0.046  0.049
          Probability RMSEA <= .05           0.999

CFI/TLI
          CFI                                0.844
          TLI                                0.837

SRMR (Standardized Root Mean Square Residual)
          Value                              0.058
```

```
STDYX Standardization
                                                        Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN              0.971      0.010    101.941      0.000
    TERROR             0.952      0.011     88.191      0.000
    ISOLATE            0.933      0.016     59.159      0.000
    CORRUPT            0.745      0.027     27.312      0.000
    IGNORE             0.846      0.018     48.111      0.000

Residual Variances (PROPORTION OF VARIANCE NOT ACCOUNTED FOR)
    SPURN              0.057      0.018      3.107      0.002
    TERROR             0.093      0.021      4.531      0.000
    ISOLATE            0.129      0.029      4.374      0.000
    CORRUPT            0.444      0.041     10.921      0.000
    IGNORE             0.284      0.030      9.557      0.000

    Latent                                              Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value
R-SQUARE (VARIANCE ACCOUNTED FOR BY HIGHER-ORDER FACTOR)
    SPURN              0.943      0.018     50.970      0.000
    TERROR             0.907      0.021     44.096      0.000
    ISOLATE            0.871      0.029     29.580      0.000
    CORRUPT            0.556      0.041     13.656      0.000
    IGNORE             0.716      0.030     24.056      0.000
```

This higher-order factor model uses 5 fewer parameters (5 higher-order loadings to replace the 10 covariances among the factors).

According to the −2ΔLL scaled difference relative to the previous model,

$-2\Delta LL\ (5) = 111.585,\ p < .0001$

trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Based on the factor correlations we examined earlier and the standardized higher-order loadings, I'd guess the issue lies with the "corrupting" factor not being as related to the others.

For the sake of illustration, we can try one more alternative – what if the items were measuring a single factor (i.e., a "total score")? Syntax for CFA model with MLR including a single factor instead of a higher-order factor ("smallest model" for comparison):

```
TITLE:  1 single factor for everything
DATA:   FILE IS abuse.dat;

VARIABLE:
NAMES ARE ID   ! All variables in DATA SET
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES ARE  ! All variables in MODEL
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

MISSING ARE .;
IDVARIABLE IS ID;

ANALYSIS:   ESTIMATOR IS MLR;         ! Robust estimator

OUTPUT:     STDYX           ! Standardized solution
            MODINDICES      ! Voodoo for fixing the model
            RESIDUAL;       ! Local fit info

SAVEDATA:   SAVE = FSCORES;           ! Save factor scores (thetas)
            FILE IS Abuse_Thetas.dat; ! File factor scores saved to

PLOT:   TYPE IS PLOT1 PLOT2 PLOT3;

MODEL:
! Higher-Order Factor
! (estimate loadings and fix mean=0, variance=1)

Abuse BY
p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*
p07* p11* p13* p17* p24* p26* p36* p55* p56*
p01* p18* p19* p23* p39* p43*
p09* p12* p16* p20* p28* p47* p50*
p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
p45* p46* p51* p52* p57*;
Abuse@1; [Abuse@0];
```

```
MODEL FIT INFORMATION
Number of Free Parameters                       147

Loglikelihood
          H0 Value                      -70386.526
          H0 Scaling Correction Factor       2.398
            for MLR
          H1 Value                      -65787.405
          H1 Scaling Correction Factor       1.593
            for MLR

Information Criteria
          Akaike (AIC)                  141067.051
          Bayesian (BIC)                141831.074
          Sample-Size Adjusted BIC      141364.120
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
          Value                           6183.985*
          Degrees of Freedom                 1127
          P-Value                          0.0000
          Scaling Correction Factor          1.487
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.058
          90 Percent C.I.                0.057  0.059
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.766
          TLI                                0.756

SRMR (Standardized Root Mean Square Residual)
          Value                              0.062
```

NOTE: With respect to fit of the structural model, we are now fitting a single factor INSTEAD OF 5 factors and a higher-order factor. This will tell us the extent to which a "total score" is appropriate.

According to the −2ΔLL scaled difference relative to the previous model, −2ΔLL (5) = 448.415, $p < .0001$

Therefore, a single factor fits significantly worse than 5 factors + a higher-order factor, and so one factor does not capture the covariances for these 49 items.

Syntax for IFA model with WLSMV including all 5 correlated factors ("biggest model" for DIFFTEST):

```
TITLE:  5-factor model: 5 correlated factors
DATA:   FILE IS abuse.dat;

VARIABLE:
NAMES ARE ID   ! All variables in DATA SET
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES ARE  ! All variables in MODEL
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

CATEGORICAL ARE  ! All variables for IFA
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

MISSING ARE .;
IDVARIABLE IS ID;

ANALYSIS:   ESTIMATOR IS WLSMV;         ! Limited info estimator
            PARAMETERIZATION IS THETA;

OUTPUT:     STDYX           ! Standardized solution
            MODINDICES      ! Voodoo for fixing the model
            RESIDUAL        ! Local fit info
            TECH4;        ! Factor correlation matrix

SAVEDATA:   DIFFTEST=5factor.dat;     ! Save fit of 5 factor model
            SAVE = FSCORES;           ! Save factor scores (thetas)
            FILE IS Abuse_Thetas.dat; ! File factor scores saved to

PLOT:   TYPE IS PLOT1 PLOT2 PLOT3;
```

```
MODEL:
! 5 Lower-Order Factors (loadings for first item fixed =1)

! 12-Item Spurning
Spurn   BY  p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
Terror  BY  p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
Isolate BY  p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
Corrupt BY  p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
Ignore  BY  p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;


! Factor Variances (all must be free)
Spurn* Terror* Isolate* Corrupt* Ignore*;


! Factor Means (all fixed = 0 by default)
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];


! Factor Covariance (all free by default if predictors)
Spurn  Terror  Isolate  Corrupt  Ignore  WITH
Spurn* Terror* Isolate* Corrupt* Ignore*;
```

NOTE: With respect to fit of the structural model, letting the 5 separate factors be correlated is as good as it gets. This will be our "largest model" baseline with which to compare the fit of a single higher-order factor model ("smaller model").

Output for IFA model with WLSMV including all 5 correlated factors ("biggest model" for DIFFTEST):

```
MODEL FIT INFORMATION

Number of Free Parameters                      255

Chi-Square Test of Model Fit
          Value                          5934.138*
          Degrees of Freedom                1117
          P-Value                         0.0000

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and
WLSMV cannot be used for chi-square difference testing in the
regular way.  MLM, MLR and WLSM chi-square difference testing
is described on the Mplus website.  MLMV, WLSMV, and ULSMV
difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.057
          90 Percent C.I.             0.055  0.058
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.927
          TLI                                0.923

Chi-Square Test of Model Fit for the Baseline Model
          Value                          67288.037
          Degrees of Freedom                1176
          P-Value                         0.0000
```

		SPURN	TERROR	ISOLATE	CORRUPT
Latent Variable Correlations	SPURN				
	TERROR	0.947			
	ISOLATE	0.925	0.885		
	CORRUPT	0.791	0.866	0.776	
	IGNORE	0.882	0.817	0.863	0.729
Factor	**SPURN**	**TERROR**	**ISOLATE**	**CORRUPT**	**IGNORE**
Variance	0.641	0.823	0.895	1.358	2.492
Standardized Loadings for Items					
	0.625	0.672	0.687	0.759	0.845
	0.499	0.778	0.663	0.687	0.738
	0.819	0.713	0.806	0.423	0.717
	0.575	0.687	0.641	0.790	0.781
	0.645	0.796	0.682	0.823	0.676
	0.839	0.692	0.753	0.793	0.822
	0.895	0.795		0.875	0.898
	0.703	0.722			0.807
	0.820	0.762			0.892
	0.731				0.859
	0.754				0.852
	0.693				0.888
					0.763
					0.844
					0.908

Note: #free parameters = 255 = 44 loadings + 49*4=196 thresholds + 5 factor variances + 10 factor covariances = 255 parameters USED or estimated

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 255 – 49 "residuals" = 1117

Now we can test the fit of a constrained structural model that posits a single higher-order "General Abuse" factor to account for the correlations among these 5 latent factors.

Syntax for IFA model with WLSMV including a higher-order factor instead of 5 correlated factors ("smaller/bigger model" for DIFFTEST):

```
TITLE:  5-factor model: 5 lower-order, 1 higher-order factor
DATA:   FILE IS abuse.dat;

VARIABLE:
NAMES ARE ID   ! All variables in DATA SET
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57 total victim;

USEVARIABLES ARE  ! All variables in MODEL
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

CATEGORICAL ARE  ! All variables for IFA
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

MISSING ARE .;
IDVARIABLE IS ID;

ANALYSIS:   ESTIMATOR IS WLSMV;         ! Limited info estimator
            PARAMETERIZATION IS THETA;
            DIFFTEST=5factor.dat;    ! Test fit against saturated

OUTPUT:     STDYX           ! Standardized solution
            MODINDICES      ! Voodoo for fixing the model
            RESIDUAL;       ! Local fit info

SAVEDATA:   DIFFTEST = HigherOrder.dat ! Save fit of higher-order
            SAVE = FSCORES;           ! Save factor scores (thetas)
            FILE IS Abuse_Thetas.dat; ! File factor scores saved to

PLOT:   TYPE IS PLOT1 PLOT2 PLOT3;
```

```
MODEL:
! 5 Lower-Order Factors (loadings for first item fixed =1)

! 12-Item Spurning
Spurn   BY  p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
Terror  BY  p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
Isolate BY  p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
Corrupt BY  p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
Ignore  BY  p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45*  p46* p51* p52* p57*;


! Factor Variances (all must be free – NOW "DISTURBANCES")
Spurn* Terror* Isolate* Corrupt* Ignore*;


! Factor Means (all fixed = 0 by default)
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];


! Higher-Order Factor (estimate loadings, fix mean=0 & variance=1)
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
Abuse@1;
[Abuse@0];
```

NOTE: With respect to fit of the structural model, we are now fitting a single higher-order factor INSTEAD OF covariances among the 5 factors.

To test the fit against the saturated (all possible factor correlations model), we direct DIFFTEST on the ANALYSIS command to use the results from the previous model.

Output for IFA model with WLSMV including a higher-order factor instead of 5 correlated factors ("smaller/bigger model" for DIFFTEST):

```
MODEL FIT INFORMATION

Number of Free Parameters                       250

Chi-Square Test of Model Fit
          Value                             5941.911*
          Degrees of Freedom                 1122
          P-Value                           0.0000

Chi-Square Test for Difference Testing
          Value                             92.518
          Degrees of Freedom                     5
          P-Value                           0.0000

*   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and
WLSMV cannot be used for chi-square difference testing in the
regular way.  MLM, MLR and WLSM chi-square difference testing
is described on the Mplus website.  MLMV, WLSMV, and ULSMV
difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.057
          90 Percent C.I.                0.055  0.058
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.927
          TLI                                0.924
```

This higher-order factor model uses 5 fewer parameters (5 higher-order loadings to replace the 10 covariances among the factors).

```
STDYX Standardization
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN              0.990      0.005    204.055      0.000
    TERROR             0.948      0.007    139.928      0.000
    ISOLATE            0.951      0.009    106.595      0.000
    CORRUPT            0.835      0.014     60.998      0.000
    IGNORE             0.885      0.009     93.999      0.000


Residual Variances (PROPORTION OF VARIANCE NOT ACCOUNTED FOR)
    SPURN              0.020      0.010      2.116      0.034
    TERROR             0.101      0.013      7.878      0.000
    ISOLATE            0.096      0.017      5.634      0.000
    CORRUPT            0.303      0.023     13.286      0.000
    IGNORE             0.218      0.017     13.071      0.000

    Latent                                          Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value
R-SQUARE (VARIANCE ACCOUNTED FOR BY HIGHER-ORDER FACTOR)

    SPURN              0.980      0.010    102.028      0.000
    TERROR             0.899      0.013     69.964      0.000
    ISOLATE            0.904      0.017     53.298      0.000
    CORRUPT            0.697      0.023     30.499      0.000
    IGNORE             0.782      0.017     46.999      0.000
```

According to DIFFTEST, trying to reproduce the 5 factor correlations with a single higher-order factor results in a significant decrease in fit. However, the RMSEA and CFI are reasonably happy with this model, and the higher-order factor accounts for a practically significant amount of variance in each factor.

We can try one more alternative – what if the items were measuring a single factor (i.e., a "total score")?
Syntax and output for IFA model with WLSMV including only a single factor ("smallest model" for DIFFTEST):

```
TITLE:  Saturated 5-factor model: 5 correlated factors
DATA:   FILE IS abuse.dat;

VARIABLE:
NAMES ARE ID   ! All variables in DATA SET
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57 total victim;

USEVARIABLES ARE  ! All variables in MODEL
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

CATEGORICAL ARE ! All variables for IFA
p01 p02 p03 p04     p06 p07     p09 p10
p11 p12 p13 p14     p16 p17 p18 p19 p20
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
p31     p33     p35 p36 p37 p39 p40
        p43 p44 p45 p46 p47 p48 p49 p50
p51 p52 p53 p54 p55 p56 p57;

MISSING ARE .;
IDVARIABLE IS ID;

ANALYSIS:   ESTIMATOR IS WLSMV;         ! Limited info estimator
            PARAMETERIZATION IS THETA;
          DIFFTEST=HigherOrder.dat; ! Test fit against higher-order

OUTPUT:     STDYX           ! Standardized solution
            MODINDICES      ! Voodoo for fixing the model
            RESIDUAL;       ! Local fit info

SAVEDATA:
            SAVE = FSCORES;           ! Save factor scores (thetas)
            FILE IS Abuse_Thetas.dat; ! File factor scores saved to

PLOT:   TYPE IS PLOT1 PLOT2 PLOT3;
```

```
MODEL:
! Higher-Order Factor (estimate loadings and fix variance=1)

Abuse BY
p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*
p07* p11* p13* p17* p24* p26* p36* p55* p56*
p01* p18* p19* p23* p39* p43*
p09* p12* p16* p20* p28* p47* p50*
p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
p45* p46* p51* p52* p57*;
Abuse@1; [Abuse@0];
```

NOTE: With respect to fit of the structural model, we are now fitting a single factor INSTEAD OF 5 factors and a higher-order factor. This will tell us the extent to which a "total score" is appropriate.

To test the fit against the higher-order factor model, we direct DIFFTEST on the ANALYSIS command to use the results from the previous model.

```
MODEL FIT INFORMATION

Number of Free Parameters                      245

Chi-Square Test of Model Fit
          Value                          7563.403*
          Degrees of Freedom                1127
          P-Value                         0.0000

Chi-Square Test for Difference Testing
          Value                           612.219
          Degrees of Freedom                    5
          P-Value                         0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.065
          90 Percent C.I.                0.064  0.067
          Probability RMSEA <= .05           0.000

CFI/TLI
          CFI                                0.903
          TLI                                0.898
```

Nope, we can't fit a single factor instead without hurting fit. This would suggest that a total score (or factor) will not be as useful as 5 separate subscores (or factors).

Example results section for CFA using MLR:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. Each factor was identified by fixing the first item loading on each factor to 1, estimating the factor variance, and then fixing the factor mean to 0, while estimating all possible item intercepts, item residual variances, and remaining item loadings. Robust maximum likelihood (MLR) estimation was used to estimate all higher-order models using Mplus v. 7.11 (Muthén & Muthén, 1998-2012), and differences in fit between nested models were evaluated using −2* rescaled difference in the model log-likelihood values.

As shown in Table 1, the fit of the model with five correlated factors was acceptable by the RMSEA (.047), but not by the CFI (.847). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations of .6 or higher were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was testing by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, $-2\Delta LL(5) = 111.585$, $p < .0001$, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. A further nested model comparison was conducted to examine the extent to which a single factor could describe the covariances among the items rather than five lower-order factors and a single higher-order factor. Fit of the single factor only model was poor, as shown in Table 1, and was significantly worse than the higher-order factor model, $-2\Delta LL(5) = 448.415$, $p < .0001$, indicating that a single “total score” would not be recommended.

Example results section for IFA using WLMSV:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. Each factor was identified by fixing the first item loading on each factor to 1, estimating the factor variance, and then fixing the factor mean to 0, while estimating all possible item thresholds (four for each item given five response options) and remaining item loadings. WLSMV estimation including a probit link and the THETA parameterization (such that all item residual variances were constrained to 1) was used to estimate all higher-order models (Muthén & Muthén, 1998-2010). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items. Nested model comparisons were conducted using the DIFFTEST procedure.

As shown in Table 1, the fit of the model with five correlated factors was acceptable. Item factor analysis parameters (loadings and thresholds) and their corresponding item response model parameters (discriminations and difficulties) are shown in Table 2. Correlations of .7 or higher were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was testing by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained acceptable (see Table 1), a nested model comparison via the DIFFTEST procedure revealed a significant decrease in fit, DIFFTEST(5) = 92.52, $p < .0001$, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. A further nested model comparison was conducted to examine the extent to which a single factor could describe the polychoric correlations among the items rather than five lower-order factors and a single higher-order factor. Fit of the single factor only model was poor, as shown in Table 1, and was significantly worse than the higher-order factor model, DIFFTEST(5) = 612.22, $p < .0001$, indicating that a single “total score” would not be recommended.

Table 1 = table with fit info per model, Table 2 = model parameters