Multivariate Longitudinal Models (in SAS and Mplus)

- Topics:
  - **Time-varying predictors that change over time**
  - Multivariate relations of change
  - Multivariate hypotheses about fixed effects
  - Multivariate longitudinal model specification
  - Time-varying predictors that change over time, revisited
  - SEM for modeling time-varying predictors and outcomes

Baseline Centering for Time-Varying Predictors that Change over Time

- Although using the person mean of the time-varying predictor at level-2 (PMx_i) is the most common way to represent the effect of between-person differences, other options can sometimes can be more useful

- **Level-2 → X at centering point of time (e.g., x_{ti} at time 0)**
  - Useful if x_{ti} at specific time point conveys useful information, such as baseline level of a predictor in an intervention
  - Useful if x_{ti} is expected to change systematically over time, too

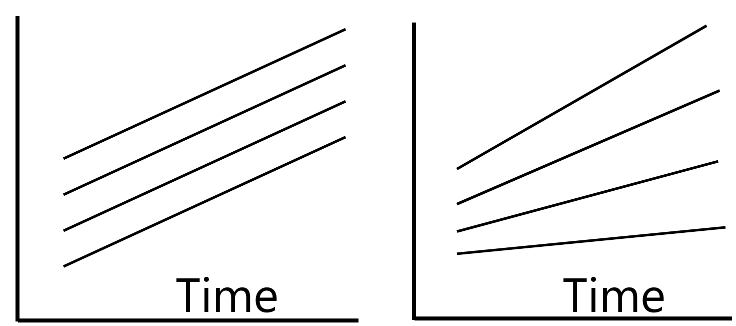
- Create predictors using a variant of PMC → **baseline centering**:
  - Level 1 = **$Motivation_{ti}$** – **$MotivationTime0_i$** → longitudinal effect
    - L1 represents *change from baseline*, not deviation from own mean
  - Level 2 = **$MotivationTime0_i$ – C** → cross-sectional effect
    - L2 represents effect of *baseline level*, not effect of mean level averaged over time

Time-Varying Predictors that Change

- Either centering should be ok if the time-varying predictor shows *fixed* change only (and if fixed effects of time are already in the model for Y)
  - Person-mean-centering: Level 2 = $\text{PersonMeanMotivation}_i - C$
    Level 1 = $\text{Motivation}_{ti} - \text{PersonMeanMotivation}_i$
  - Baseline centering: Level 2 = $\text{PersonMeanMotivationTime0}_i - C$
    Level 1 = $\text{Motivation}_{ti} - \text{MotivationTime0}_i$

- But if the time-varying predictor shows *individual* differences in change, a complete separation of its BP and WP variance is not obtained:
  - Not fitting a model for that change—no separation of true change from error
  - The level-1 predictor has both individual differences in change (U_{1i}) and residual deviations from change (e_{ti}), which should each have their own relationship to Y
  - Accordingly, there are at least two “kinds” of BP variance to be concerned with: intercept and time slope (and possibly more for other kinds of change)

If people change differently, then BP differences between people must depend on time!



Time-Varying Predictors and Effect Direction

- Direction of prediction is less clear for some time-varying predictors—which should be X and which should be Y?
  - Clear for time-varying age → outcome, but less clear in other cases (e.g., smoking frequency and # friends who smoke)
  - Could examine lagged predictive effects
    - If X precedes Y in time, you would have a better leg to stand on regarding directionality of the effects (but still can't claim "causality")

- Or don't choose → treat X as another outcome instead
  - Can still examine BP and WP relationships between X and Y, but it's done via covariances in multivariate longitudinal models instead of fixed effects in univariate longitudinal models
  - Each approach has some pros and cons, which we'll consider after we examine how multivariate models work

What do I mean by “Multivariate Longitudinal Models”?

- “Multivariate”:
  - Multiple outcomes from one level-2 unit (e.g., person, group)
- “Longitudinal”:
  - Two dimensions of sampling → time within person

- What are they used for?
  - Can examine **relations among multivariate outcomes** at different levels of analysis (mostly through the model for the variances)
  - Examine **differences in effect size of predictors** across outcomes
  - As an alternative approach to modeling time-varying predictors
  - As an alternative to difference score models

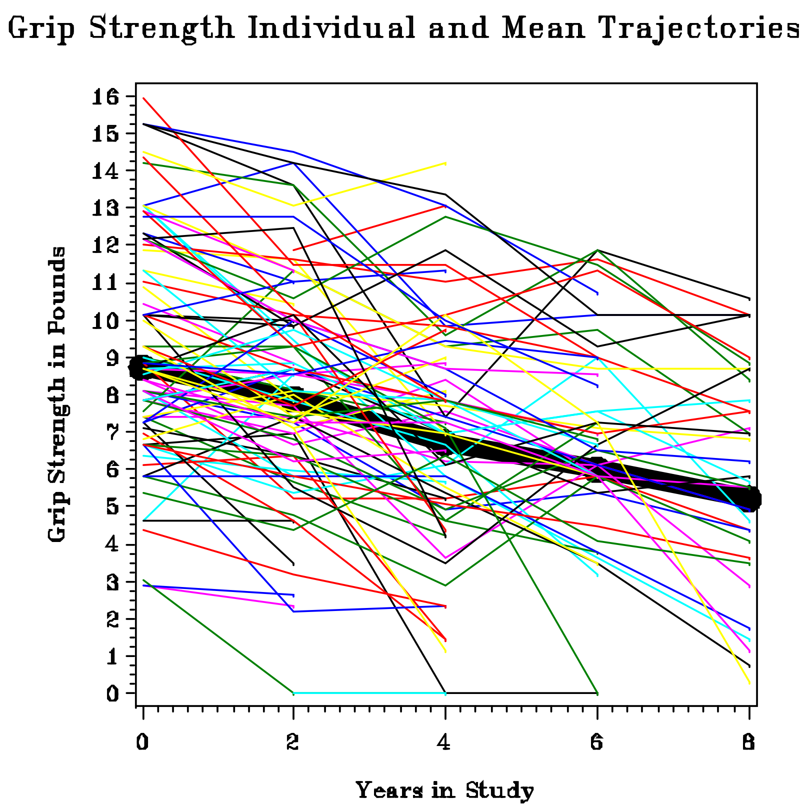
Multivariate Relations of Change: BP

- Multivariate questions about **fixed effects**:
  Does change appear similar **on average** across DVs?
  - Are the fixed effects for the overall sample heading in the same direction or of the same magnitude?
  - Tells us about average change, but says nothing about individuals

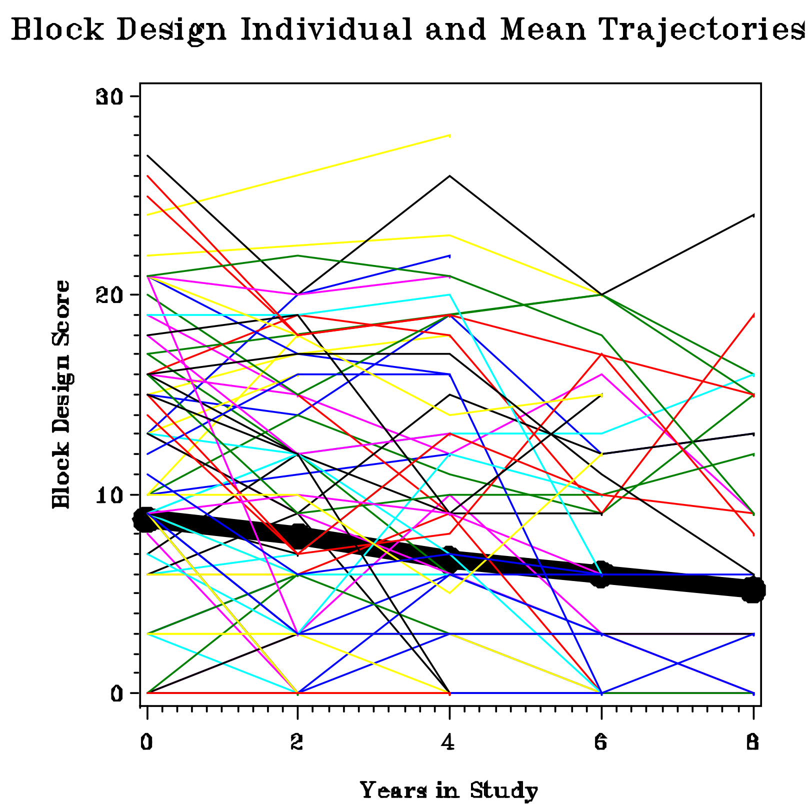
- Multivariate questions about **random effects**:
  Are **individual differences** in change related across DVs?
  - Is level (intercept) on one DV related to level (intercept) on another DV (at the centering point)?
  - Is magnitude of change (slope) on one DV related to magnitude of change (slope) on another DV?
  - These are **Between-Person** relations, relative to other people

Individual Relations of Functional and Cognitive Change in Old Age

Functional Change

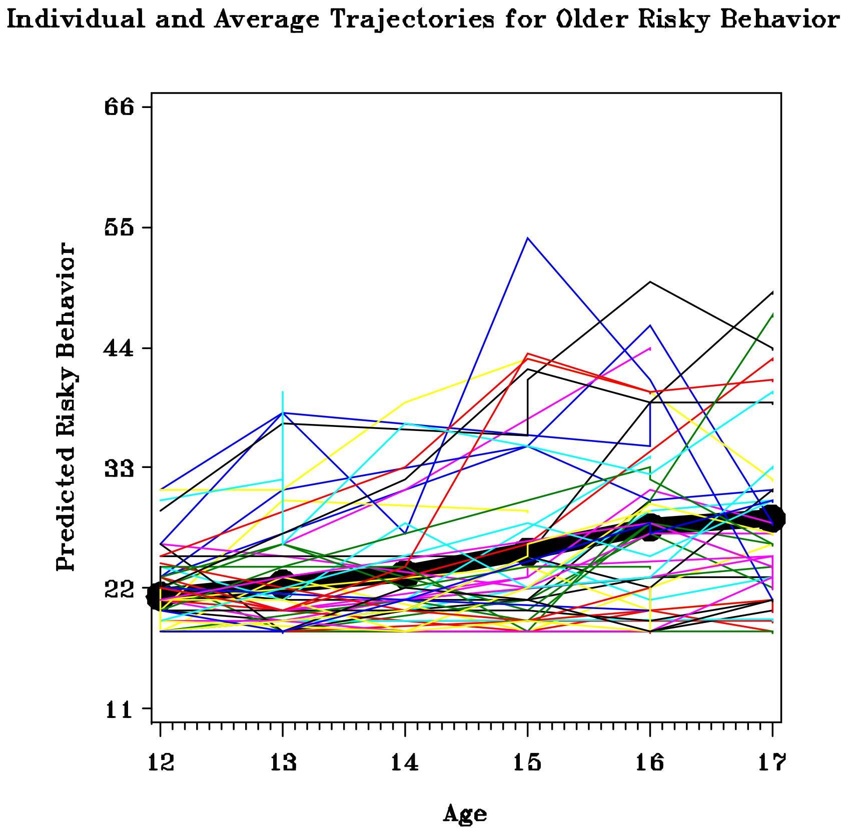


Cognitive Change

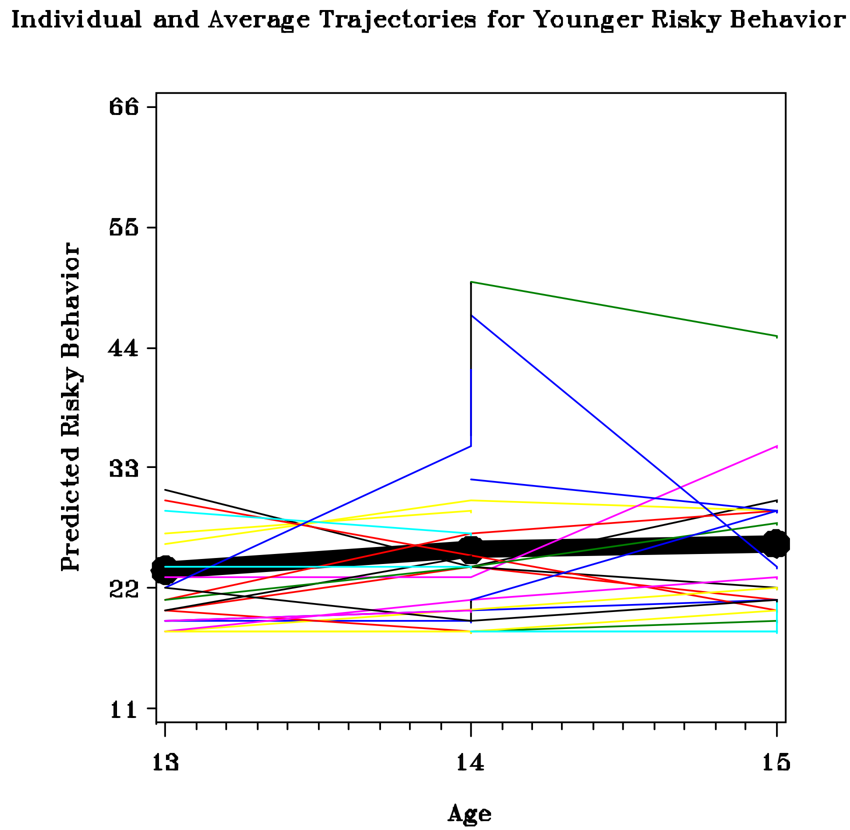


Individual Relations of Change in Risky Behavior Across Siblings

Older Siblings

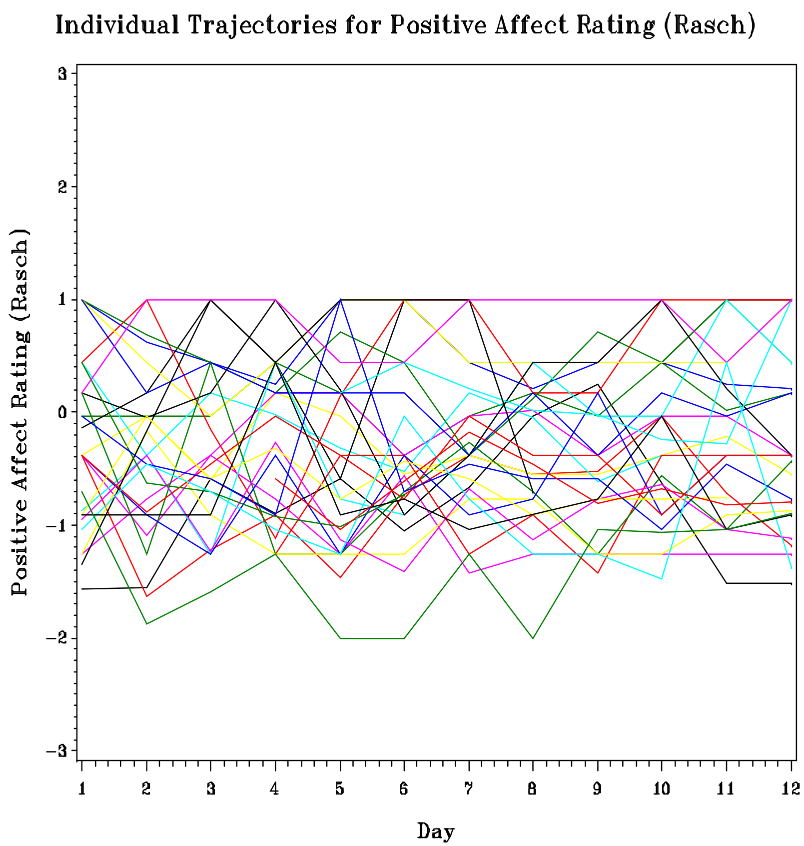


Younger Siblings

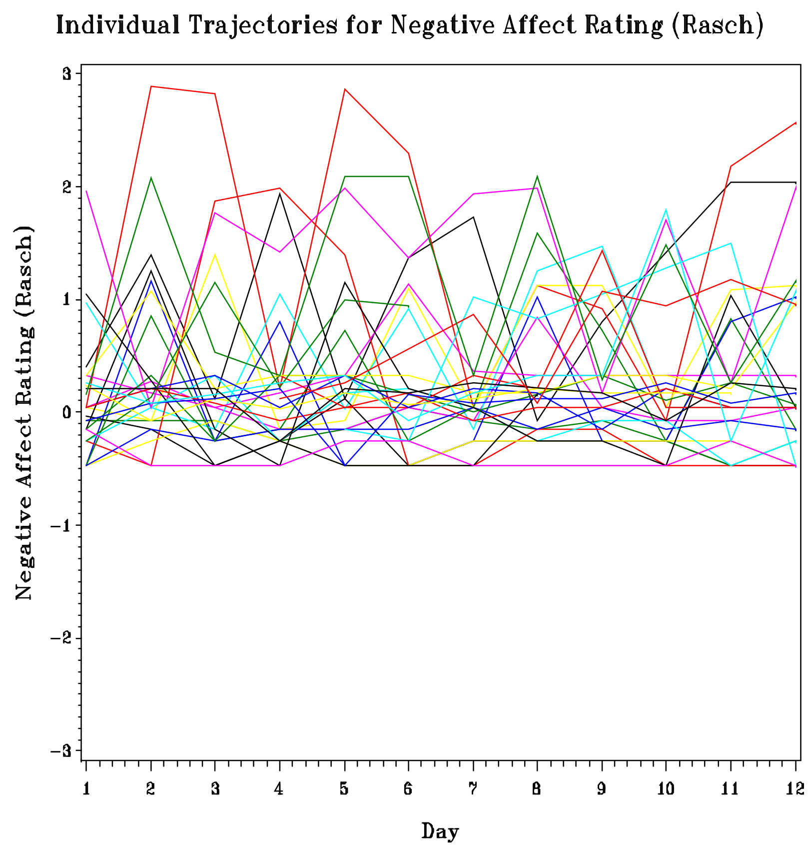


Daily Covariation in Rated Positive and Negative Affect

Rated Positive Affect

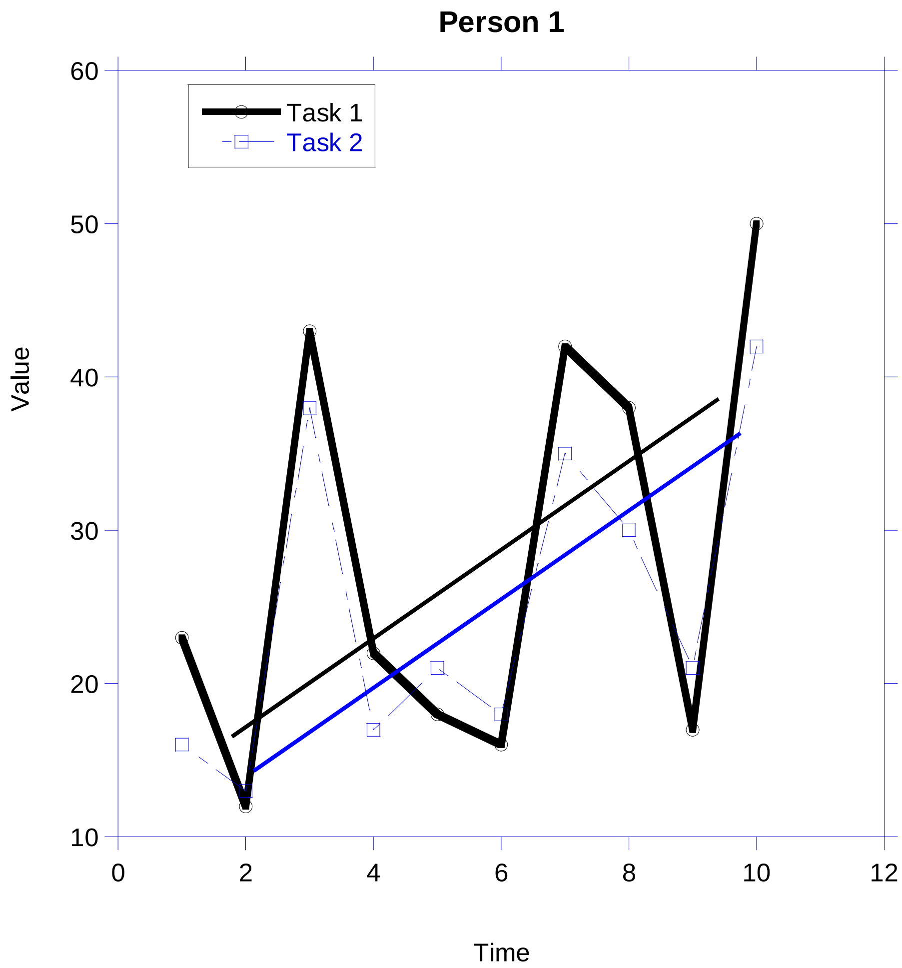


Rated Negative Affect



Multivariate Relations of Change: WP

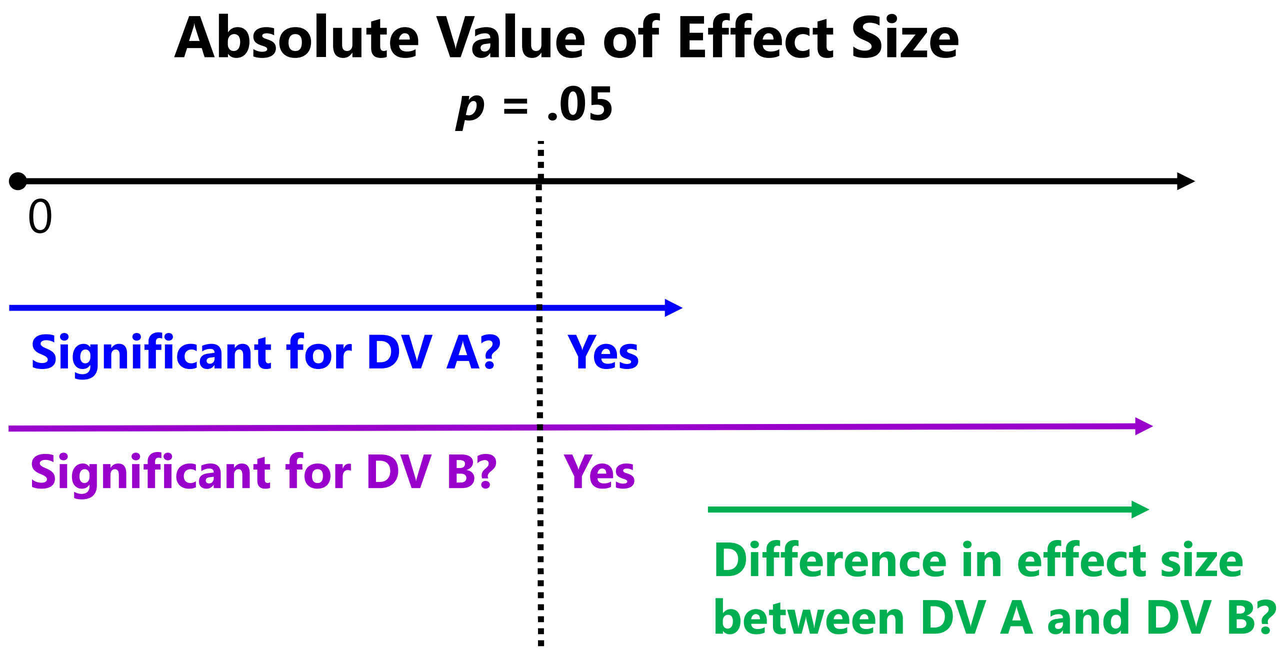
- Outcomes can be related within persons as well
- Correlated (Coupled) Residuals:
  - Do two DVs travel together over time?
  - Are you off your line in the same way for each DV at a given occasion?
    - (Yes, in this picture)



Multivariate models are also really useful in testing multivariate hypotheses about **fixed effects**...

Differences in Effect Size across DVs

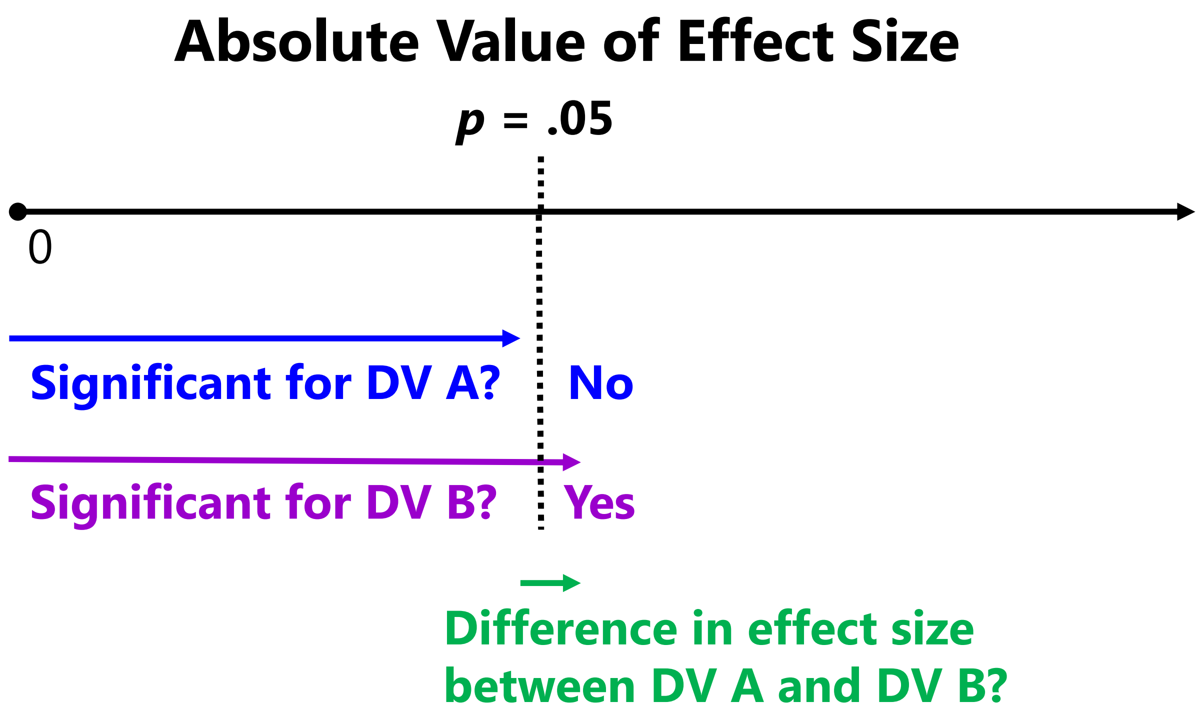
Scenario 1: Fixed effect is significant for both DVs:



Just because a predictor is **significant for both DVs** does not mean it has the **same magnitude** of relationship across DVs!

Differences in Effect Size across DVs

Scenario 1: Fixed effect is significant for DV B only:



Also, just because a predictor is **non-significant for one DV but significant for another DV** does not mean it has **different magnitudes** of relationships across DVs!

Why multivariate models should be used to test hypotheses about differences in effect sizes:

- Testing **differences in effect size of predictors** requires both DVs in the same model!
- But if the effects are the same, you can specify a **single effect** across DVs to reduce the number of estimated parameters.
- Hypotheses about **difference scores** are best tested using the original outcomes that created the difference in a multivariate model so that information about **absolute amount** is also provided.
- If DVs have missing data but are correlated, then tests of fixed effects may have **more power** in a multivariate model.
- Keep in mind that these models test differences in unstandardized fixed effects, so the DVs need to be on the **same scale** (or should be transformed onto the same scale before-hand otherwise).

Multivariate Longitudinal Data Structure: "Double Stacked" into 3 levels

Outcome	DV	dvA	dvB	Wave
Y_{i1a}	A	1	0	1
Y_{i2a}	A	1	0	2
Y_{i3a}	A	1	0	3
Y_{i4a}	A	1	0	4
Y_{i5a}	A	1	0	5
Y_{i6a}	A	1	0	6
Y_{i1b}	B	0	1	1
Y_{i2b}	B	0	1	2
Y_{i3b}	B	0	1	3
Y_{i4b}	B	0	1	4
Y_{i5b}	B	0	1	5
Y_{i6b}	B	0	1	6

1. Double-stack two DVs into a single outcome
2. Create an indicator for which DV is which (e.g., A,B)
3. Create a dummy variable for each dvA= (1,0) dvB= (0,1)
4. Keep all other variables

This shows data for 1 person, 2 outcomes, over 6 waves.

We'll use "DV" to structure the **G** and **R** matrices, and "dvA" and "dvB" to create DV-specific fixed effects in the model for the means.

"Direct Effects" Multivariate Model

Level 1 (Time crossed with DV, Within-Person):

y_{tid} = dvA [$\beta_{0ia} + \beta_{1ia}(timeti_a) + eti_a$] +

dvB [$\beta_{0ib} + \beta_{1ib}(timeti_b) + etib$]

If DV=A, β_{0i1} is awake
If DV=B, β_{0i2} is awake

Level 2 (Between-Person):

$\beta_{0ia} = \gamma_{00a} + \gamma_{01a}(Pred_i) + U_{0ia}$

$\beta_{1ia} = \gamma_{10a} + \gamma_{11a}(Pred_i) + U_{1ia}$

Intercept and slope for DV=A

$\beta_{0ib} = \gamma_{00b} + \gamma_{01b}(Pred_i) + U_{0ib}$

$\beta_{1ib} = \gamma_{10b} + \gamma_{11b}(Pred_i) + U_{1ib}$

Intercept and slope for DV=B

SAS code: MODEL outcome = dvA dvB dvA*time dvB*time dvA*pred dvB*pred dvA*time*pred dvB*time*pred / **NOINT**

Note: there are no "main" effects of predictors (i.e., by themselves)

NOINT shuts off the overall intercept so that dvA = γ_{00a} and dvB = γ_{00b}

Multivariate Model Level-2 **G** Matrix

G Matrix for Between-Person Random Effects Variances:
Estimate intercept and slope variances **per DV** and all covariances

SAS code:

```
RANDOM dvA  dvB  dvA*time dvB*time / TYPE=UN SUBJECT=Person
```

Note: there are no intercepts or slopes by themselves listed here

	Int DV A	Int DV B	Slope DV A	Slope DV B
Int DV A	$\tau^2_{U_{0a}}$			
Int DV B	$\tau_{U_{0b0a}}$	$\tau^2_{U_{0b}}$		
Slope DV A	$\tau_{U_{1a0a}}$	$\tau_{U_{1a0b}}$	$\tau^2_{U_{1a}}$	
Slope DV B	$\tau_{U_{1b0a}}$	$\tau_{U_{1b0b}}$	$\tau_{U_{1b1a}}$	$\tau^2_{U_{1b}}$

Intercept Variances

Slope Variances

Int-Int and Slope-Slope Covariances

Caveats about Correlated Random Effects in Multivariate Longitudinal Models

- If the random effects variances are not significant, a covariance between them is not likely to be estimable
  - Can try it anyway if you do get some variance estimates in the first place (i.e., numbers as opposed to dots)
  - Random effects structure doesn't have to match across DVs but it's helpful if it does

- More random effects → tougher estimation
  - Random effects solution may be unstable: numerically large correlations may not be statistically significant due to large SEs for covariances
  - May need to reduce number of random effects (most examples I've seen use linear slopes only)

Multivariate Model Level-1 **R** Matrix

R Matrix for Within-Person Residual Variances: Estimate residual variance **per DV** and covariance between DVs at same occasion

SAS code:

```
REPEATED DV / TYPE=UN SUBJECT=Wave*Person
```

Categorical version of DV is used to structure the **R** matrix

	Res DV A	Res DV B
Res DV A	$\sigma^2_{e_a}$	
Res DV B	$\sigma_{e_{ab}}$	$\sigma^2_{e_b}$

This assumes equal residual variance with no covariance over time WITHIN EACH DV, but residuals at the same occasion can be correlated across DVs.

Residual variances

Res-Res Covariance: = specific covariance remaining after accounting for the effects of time

What about Multivariate Alternative Covariance Structures Models?

- So far we've only seen multivariate random effects models. Are there multivariate versions of alt structures models?
- Yes, but they are much more limited—3 real options:
  - Direct product structures: TYPE= UN@UN, UN@AR1
    - Assumes equal variances across time
    - Assumes same pattern of autocorrelation holds for each DV!
    - See REPEATED statement in SAS manual for further explanation
  - Completely unstructured multivariate
    - Specify DV*cat_time after REPEATED statement
    - Estimates all possible variances and covariances separately
    - Not terribly informative (no between- and within-person separation)
  - Just specify a random intercept (i.e., assume compound symmetry)
    - Not optimal, but it's the best I can come up with in the software I know

Multivariate Model Specification Options

- So far we've seen a "**direct effects**" fixed effects model:
  - outcome = dvA dvB dvA*time dvB*time dvA*pred dvB*pred / **NOINT**
  - **REMOVE** overall intercept, each effect is specified per DV directly
  - Pro: The model estimates directly provide an intercept and **significance test for each predictor fixed effect** per DV
  - Con: The model does not directly test differences in effect size

G and R always use direct effects either way

- An alternative is the "**difference in effects**" fixed effects model:
  - **KEEP** general intercept, one DV serves as reference
  - Simple main effects are then specifically for reference DV; "interactions" are then *differences* in effects for the interacting DVs
  - outcome = (int) dvB time dvB*time pred dvB*pred /
  - Pros The model "**interactions" directly test differences in effect size**; if removed, the main effect becomes a single effect across DVs
  - Con: The model does not directly provide an intercept and significance test for each predictor fixed effect for the non-reference DVs (but ESTIMATE/TEST/LINCOM can be used to get those)

Time-Varying Predictors vs. Multivariate MLM

Why choose Univariate MLM
(X is time-varying predictor):

- X only fluctuates over time (BP and WP is easy to split)
- You know for sure which is X and which is Y
- X precedes Y in time
- Can test moderators of the X-Y relationship at each level via fixed effects
- Can test random effects of WP X or interactions of WP X*time

Why choose Multivariate MLM
(X is another outcome):

- X changes over time (BP intercept and slope needed)
- Either variable could possibly or logically be X or Y
- X and Y occur at same time
- X-Y relationship is modeled via covariances that cannot differ (except by group maybe)
- WP X effect is constrained equal over persons and time

Multivariate Models via M-SEM

- Person-MC (or baseline centering) is the poor man's version of a model-based decomposition of BP and WP variance, which is necessary when X is treated as a predictor in MLM programs
- Through Multilevel Structural Equation Modeling (M-SEM), it is possible to fit a model for X along with the model for Y
  - It's called SEM because random effects = latent variables, but there is no latent variable measurement model as in traditional uses of SEM
  - Person mean = random intercept variance, WP deviation = residual variance, but can also include random slopes for change over time in X
  - Can directly assess multilevel mediation through simultaneous analysis
  - Some evidence that level-2 effects are less biased (because person mean is not perfectly reliable), but more imprecise (more parameters to estimate)
- What could go wrong? No REML! Good luck fitting interactions!
  - Those involving level-2 effects are modeled as latent variable interactions
  - This requires numeric integration, a very computationally intense way of getting parameter estimates in ML, which may not be possible in all data

Summary: Multivariate models permit…

- Tests of hypotheses about BP relations (among intercepts and slopes) and WP relations (among residuals)
  - BP: Does level on one DV correlate with level on another DV?
  - BP: Does change on one DV correlate with change on another DV?
  - WP: Do two DVs 'travel together' over time within persons?

- Tests about differences in effect size of predictors across DVs
  - Is the effect of the predictor significant per DV?
  - Is the effect of the predictor significantly *different* across DVs?

- Multivariate longitudinal models can be seen as an alternative to univariate longitudinal models with time-varying predictors with pros and cons… and can be estimated using **SEM**…

Translating MLM into SEM…

- "**Random effects**" = "pile of variance" = "variance components"
  - Random effects represent "person*something" interaction terms
  - Random intercept → person*intercept (person "main effect")
  - Random linear slope → person*time interaction
  - Capture **specific patterns of covariation** of unknown origin…
    - *Why do people need their own intercepts and slopes?* We can add person-level predictors to answer these questions

- Random effects can also be seen as **latent variables**
  - Latent variable = unobservable ability or trait
    - Latent variables are created by the common variance across items
    - In longitudinal data, the latent variables can be thought of as "general tendency" and "propensity to change" as created by measuring the same outcome over time

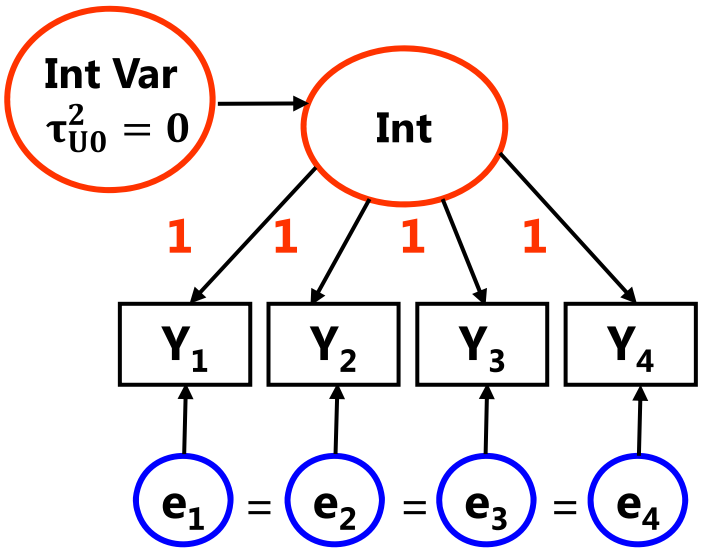
Confirmatory Factor Analysis (CFA)

- **CFA model:** $\mathbf{y_{is} = \mu_i + \lambda_i F_s + e_{is}}$ (SEM is just relations among F's)
  - Observed response for item *i* and subject *s*
    = intercept of item *i* (μ)
    + subject *s*'s latent trait/factor (F), item-weighted by λ
    + error (e) of item *i* and subject *s*

- Two big differences when using two factors for longitudinal data:
  - Usually two factors for "level" and "change" (intercept and slope):
    $\mathbf{y_{ti} = (\gamma_{00} + U_{0i}) + (\gamma_{10} + U_{1i})time_{ti} + e_{ti}}$ → **so the U's are the F's**
  - The **item (outcome)** intercepts $\mathbf{\mu_i}$ cannot be separately identified from the "intercept" factor and therefore must be fixed to 0
  - The factor loadings $\mathbf{\lambda_i}$ for how each outcome relates to the latent factor are usually pre-determined by how much time as passed, and thus usually gets fixed to the difference in time across longitudinal outcomes

Random Effects as Latent Variables

- **BP model: e_{ti}-only model for the variance**
  - **$y_{ti} = \gamma_{00} + e_{ti}$**



Mean of the intercept factor = fixed intercept γ_{00}

Loadings of intercept factor = 1 (all occasions contribute equally)

Item intercepts = 0 (always)

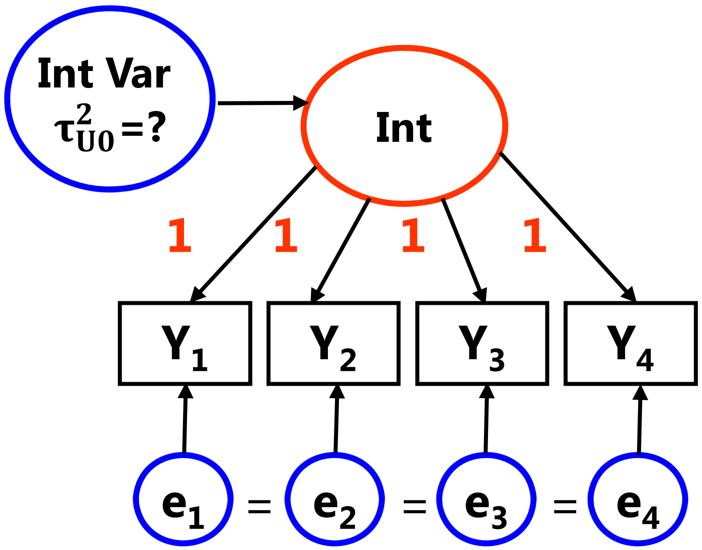
Variance of intercept factor = 0 so far

Residual variance (e) is assumed to be equal across occasions

  - After controlling for the *fixed* intercept, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **+WP model: U_{0i} + e_{ti} model for the variance**
  - **$y_{ti} = \gamma_{00} + U_{0i} + e_{ti}$**



Mean of the intercept factor = fixed intercept γ_{00}

Loadings of intercept factor = 1 (all occasions contribute equally)

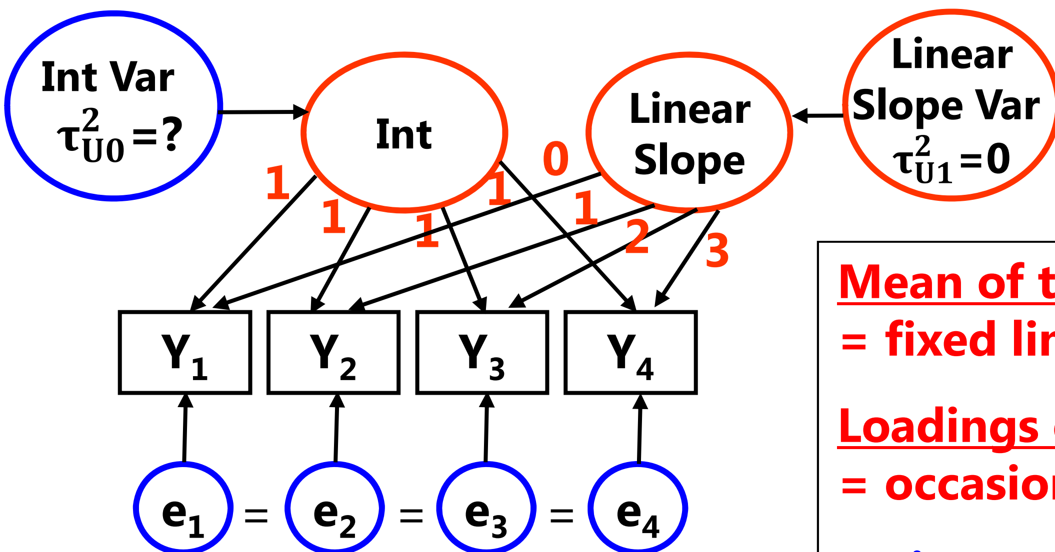
Variance of intercept factor = random intercept variance

Residual variance (e) is assumed to be equal across occasions

  - After controlling for the *random* intercept, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **Fixed linear time, random intercept model:**
  - $y_{ti} = \gamma_{00} + (\gamma_{10}Time_{ti}) + U_{0i} + e_{ti}$



Mean of the linear slope factor
= fixed linear slope γ_{10}

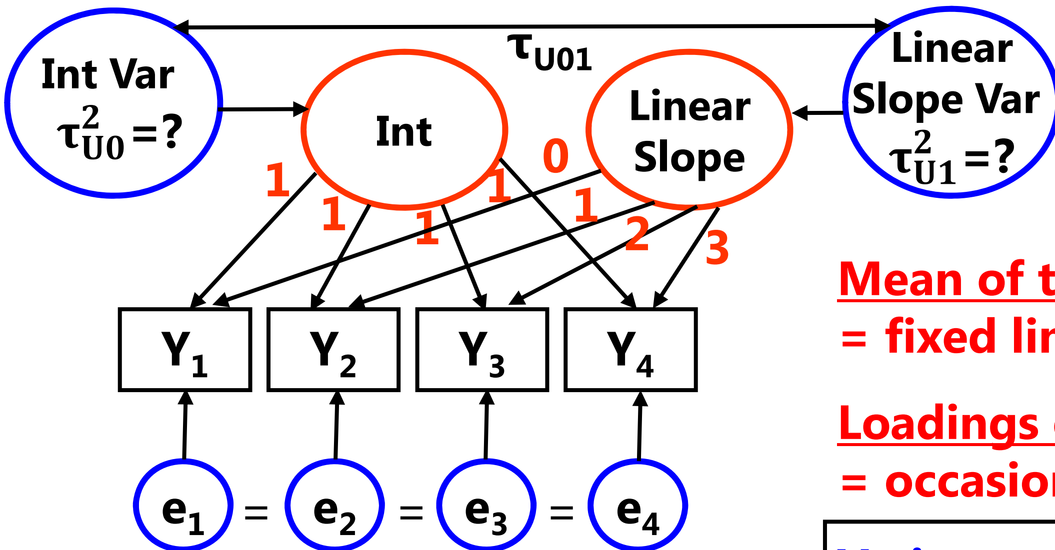
Loadings of linear slope factor
= occasions (keep real time)

Variance of linear slope factor
= 0

  - After controlling for the *fixed linear slope and random intercept*, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **Random linear model:**
  - $y_{ti} = \gamma_{00} + (\gamma_{10}Time_{ti}) + U_{0i} + (U_{1i}Time_{ti}) + e_{ti}$



Mean of the linear slope factor
= fixed linear slope γ_{10}

Loadings of linear slope factor
= occasions (keep real time)

Variance of linear slope factor
= random slope variance

  - After controlling for the *random linear slope and random intercept*, residuals are assumed uncorrelated

Adding Level-2 Predictors

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(Sex_i) + U_{0i}$

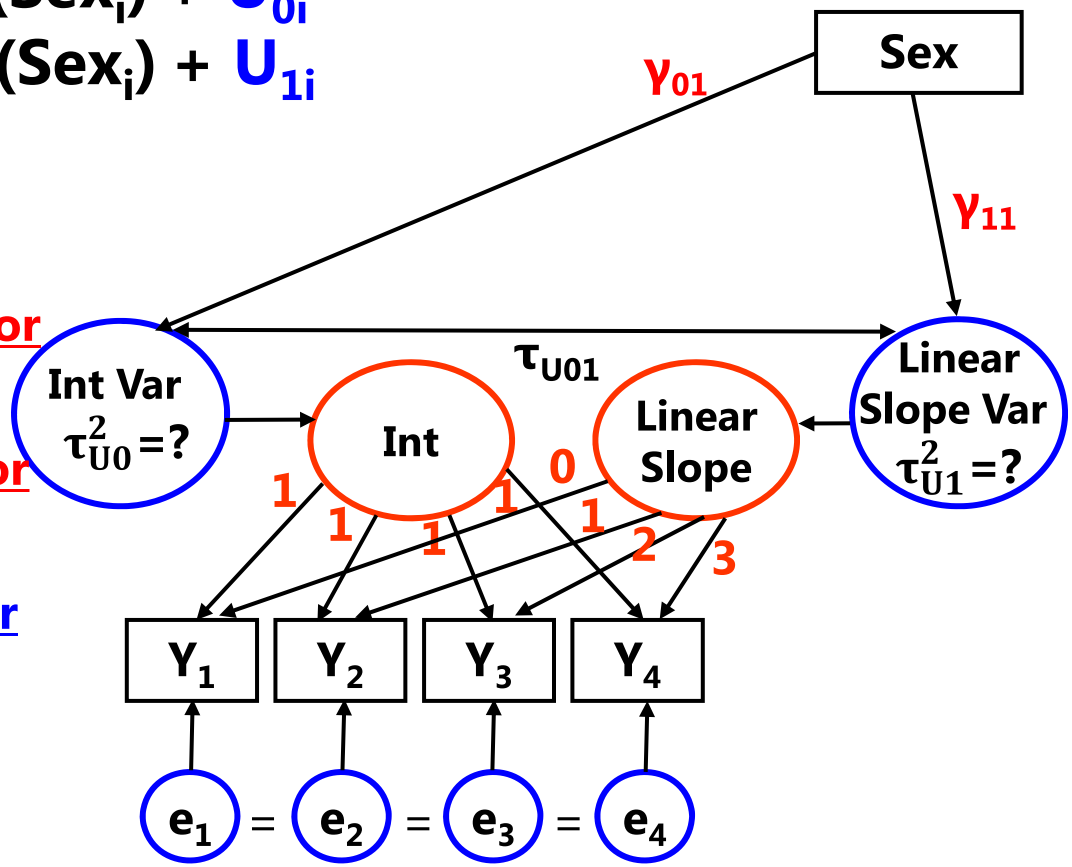
$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i) + U_{1i}$

Mean of the intercept factor
= fixed intercept γ_{00}

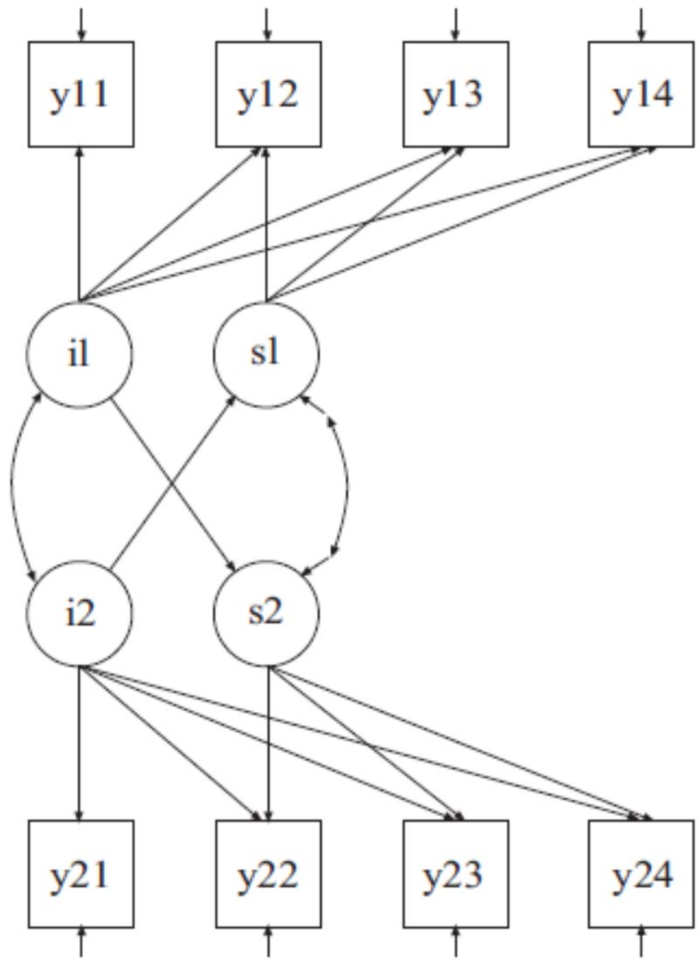
Mean of the linear slope factor
= fixed linear slope γ_{10}

Loadings of linear slope factor
= occasions (keep real time)

Variance of linear slope factor
= random slope variance



Multivariate Growth Model as SEM



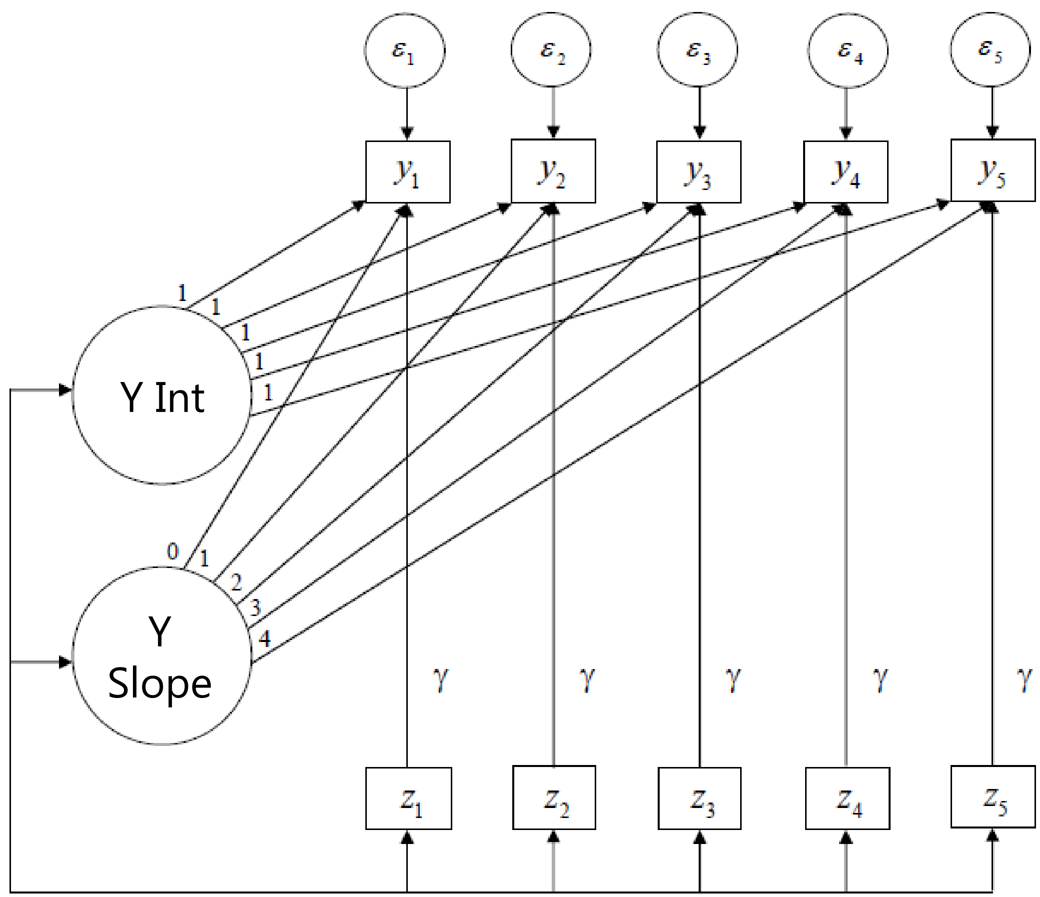
This model is from Mplus v. 7 users guide ex. 6.13.

The means of the factors are the fixed effects, and their variances are the random effects variances. The two-headed arrows between the intercepts and between the slopes are covariances. The single-headed arrows from i1 to s2 and from i2 to s1 are meant to convey directionality.

Random Effects as Latent Variables in General…

- Random effects represent specific sources of covariation among outcomes over time—these are latent variables
  - Mean of latent variable = fixed effect
  - Variance of latent variable = random effect variance
  - Loadings convey time dependency
    - Still don't round time! You can use individually varying time loadings for unbalanced data – in Mplus, this is done via TSCORES
- Longitudinal models can thus be estimated using:
  - "Random effects" = MLM → good for extra kinds of dependency
  - "Latent variables" = SEM → good for measurement models
- For level-2 predictors, MLM = SEM with no real problem
  - This is NOT the case for time-varying predictors

The Most Common Way of Including Time-Varying Predictors in SEM…

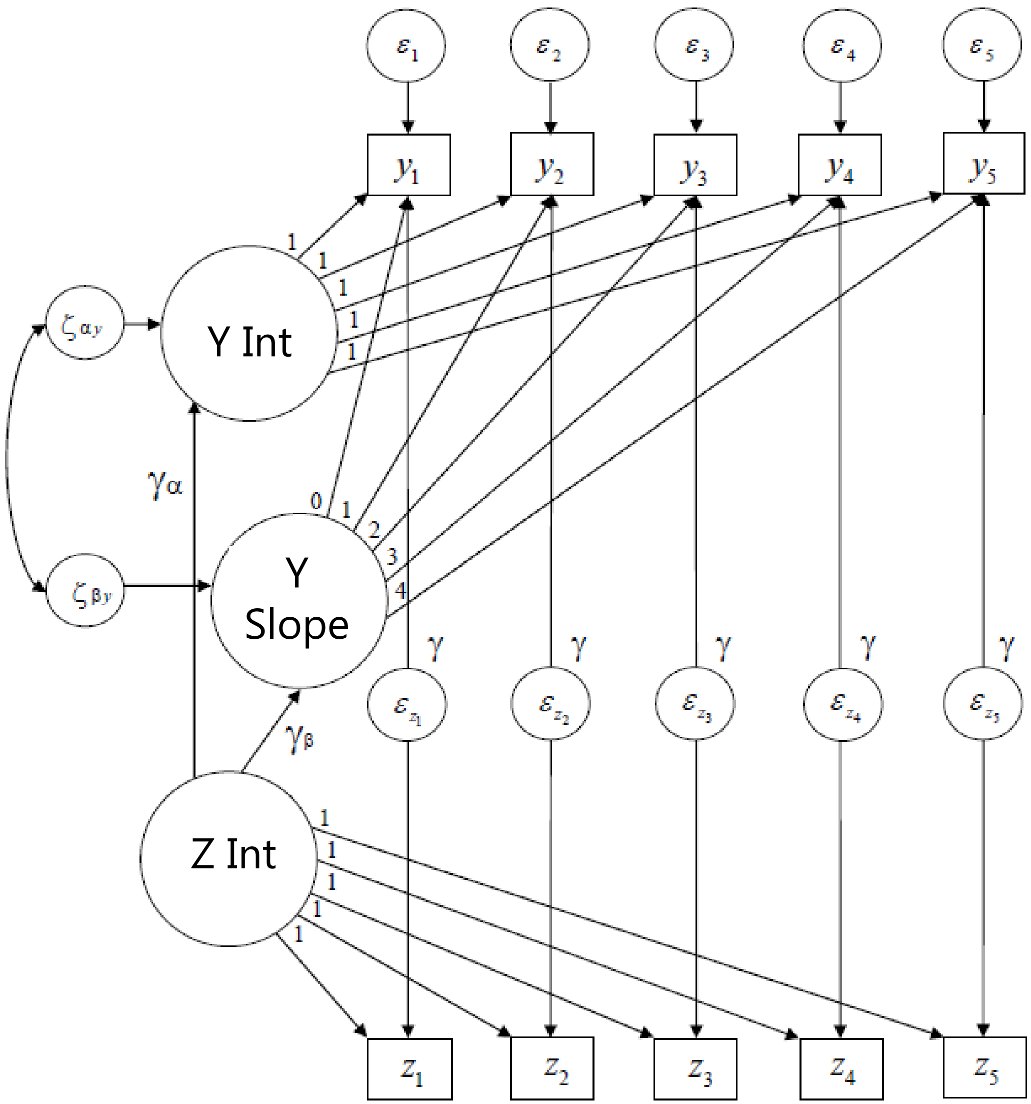


This example is from Curran et al. (2012). The z1-z5 boxes are time-varying predictors that have regression paths onto their outcomes at the same occasion.

If you constrain these paths to be equal (as γ), you get **a smushed effect** (they call it an "aggregate" effect).

If you add covariances of the z's with the intercept, γ then becomes **the WP effect**. But the BP effect is not in here! And you cannot add PMz to get it like in MLM because it will be redundant (→ ipsative).

How to Fix It (Curran et al., 2012)

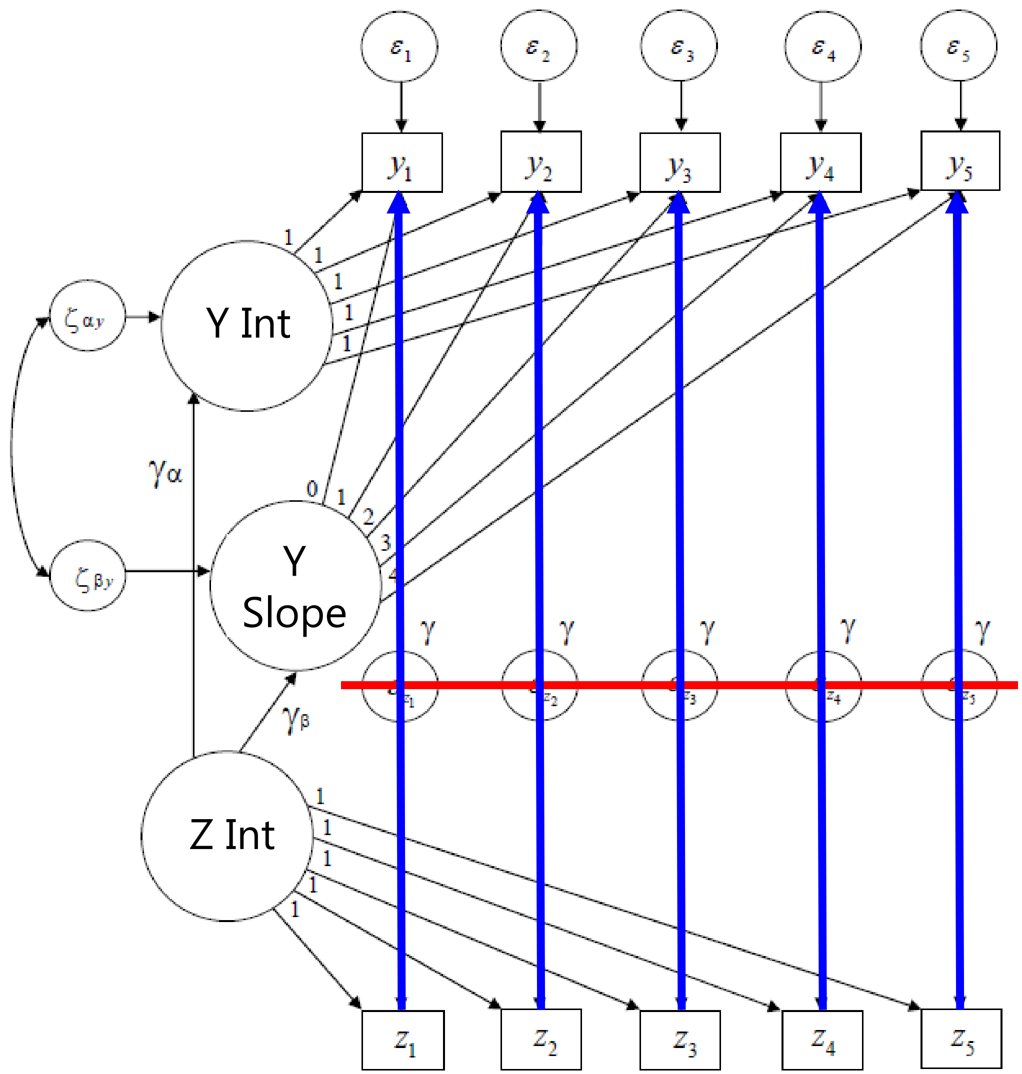


The z1-z5 time-varying predictors now have their own random intercept, which directly represents their between-person variance.

So the **BP effect** is given by $\boldsymbol{\gamma_\alpha}$, and **the WP effect** is now given by $\boldsymbol{\gamma}$ from the residuals of z1-z5 predicting y1-y5.

If your z1-z5 shows change over time instead of just fluctuation, just add a slope factor for it, too—then you'd be back to multivariate growth model we began with.

How to fix it with less code…



The z1-z5 time-varying predictors still have their own random intercept, which directly represents their between-person variance.

However, if you predict the y1-y5 outcomes directly from the z1-z5 variables, that effect is still the **WP effect**, but the **contextual effect** is now given by $\boldsymbol{\gamma_\alpha}$ instead.

In either model, you can get the other implied level-2 effect (BP or contextual) by requesting a NEW effect in MODEL CONSTRAINT.

What could go wrong?

In a word, **interaction terms**:

- In both the regular SEM approach and the multilevel SEM approach to longitudinal analysis, interactions involving level-2 effects must be modeled as latent variable interactions
- This requires numeric integration, a very computationally intense way of getting parameter estimates in ML
  - add ANALYSIS: ALGORITHM = INTEGRATION
- So Mplus may not be able to estimate all the interactions it could (or SAS could) when the predictors are observed variables only (e.g., using observed PMx instead of latent intercept for X)
- So these models can be nice for getting better estimates of level-2 main effects but currently are not as readily extendable to more complex kinds of prediction… (without substituting observed values of some kind)