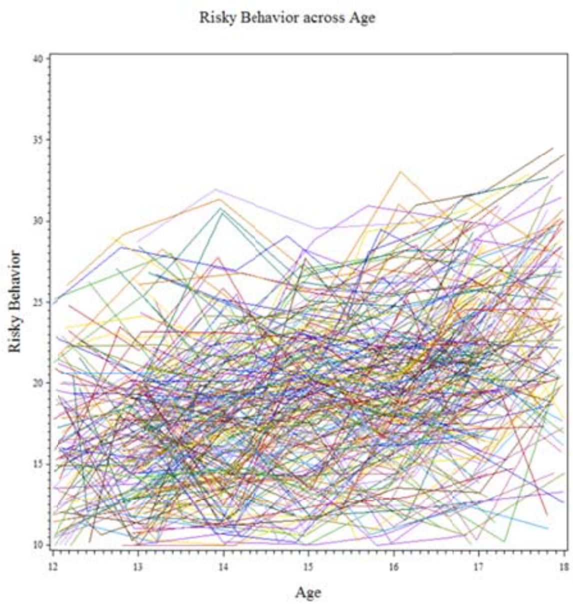
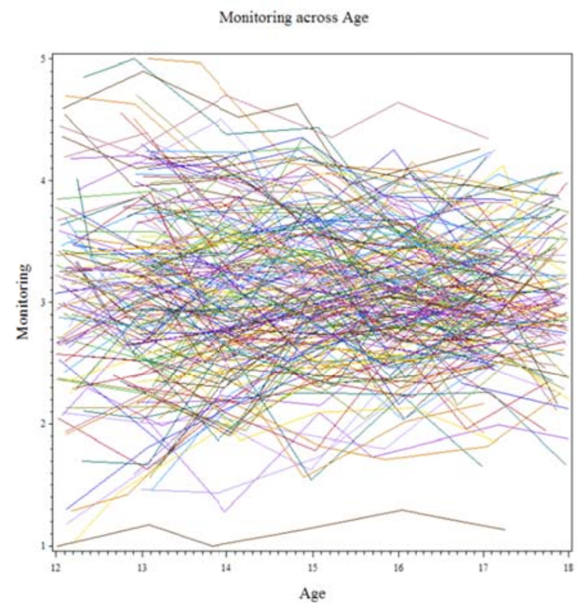
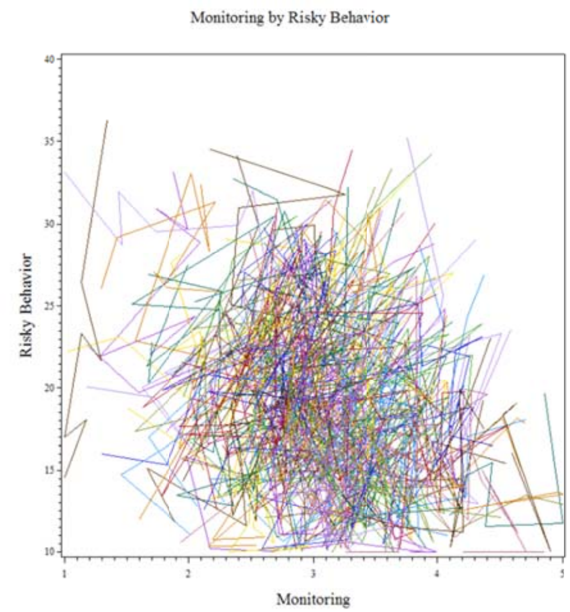
# Seven Ways of Estimating Multivariate Change in SAS PROC MIXED, SAS PROC NLMIXED, and Mplus v. 7.11

These simulated data are from Hoffman chapter 9, and include 200 girls measured approximately annually from ages 12–18 (time 0 = age 18) on their risky behavior (the outcome, a sum ranging from 10 to 50) and the extent to which their mothers monitored their activities (the time-varying predictor, a mean ranging from 1 to 5, centered at 3). A time-invariant predictor of the conservativeness of mothers' attitudes about the smoking and drinking (a mean ranging from 1 to 5, centered at 4) was also collected at the age 12 occasion. Here are the individual growth trajectories for risky behavior and monitoring:




**Multivariate Growth Model**

**Level 1:**

$$y_{tid} = dvR\left[\beta_{0iR} + \beta_{1iR}\left(Age_{tiR} - 18\right) + \beta_{2iR}\left(Age_{tiR} - 18\right)^2 + e_{tiR}\right] +$$

$$dvM\left[\beta_{0iM} + \beta_{1iM}\left(Age_{tiM} - 18\right) + e_{tiM}\right]$$

**Level 2:**

Risky Intercept: $\beta_{0iR} = \gamma_{00R} + \gamma_{01R}\left(Attitudes12_i - 4\right) + U_{0iR}$

Risky Age: $\beta_{1iR} = \gamma_{10R} + \gamma_{11R}\left(Attitudes12_i - 4\right) + U_{1iR}$

Risky Age$^2$: $\beta_{2iR} = \gamma_{20R}$

Monitor Intercept: $\beta_{0iM} = \gamma_{00M} + U_{0iM}$

Monitor Age: $\beta_{1iM} = \gamma_{10M} + U_{1iM}$

The best-fitting unconditional longitudinal model included fixed quadratic and random linear effects of age for risky behavior, but a random linear effect of age for monitoring (although the fixed linear age slope was nonsignificant). In addition, mother's attitudes significantly predicted the intercept and linear age slope for risky behavior, but did not significantly predict monitoring.

Chapter 9 began with person-mean-centering and baseline-centering of monitoring of a time-varying predictor of risky behavior. Both were shown to be inadequate because they do not properly distinguish the intercept, linear age slope, and residual variance contained in the monitoring predictor, each of which could potentially relate to those of risky behavior. So the purpose of this example is to demonstrate multilevel, SEM, and "multilevel SEM" methods of estimating models of multivariate change. In total, we will examine six models, some of which will be statistically equivalent, some of which will not, so that you can decide what will be most optimal for your own data.

## Multivariate Growth Model for Risky Behavior and Monitoring in SAS, controlling risky behavior for attitudes (Model #1):

```
* Stack longitudinal data into multivariate longitudinal;
DATA RiskyStacked2; SET RiskyStacked;
  DV="1risky  "; dvR=1; dvM=0; outcome=risky;    OUTPUT;
  DV="2monitor"; dvR=0; dvM=1; outcome=monitor3; OUTPUT;
RUN;

TITLE1 "Multivariate Model at Age 18 = Time 0";
PROC MIXED DATA=RiskyStacked2 NOITPRINT NOCLPRINT COVTEST
IC NOINFO NAMELEN=100 METHOD=ML;
CLASS FamilyID occasion DV;

MODEL outcome = dvR dvM dvR*agec18 dvM*agec18
                dvR*agec18*agec18 dvR*att4 dvR*att4*agec18
       / NOINT SOLUTION DDFM=Satterthwaite NOTEST;

RANDOM dvR dvM dvR*agec18 dvM*agec18
      / G GCORR TYPE=UN SUBJECT=FamilyID;

REPEATED DV / R RCORR TYPE=UN SUBJECT=occasion*FamilyID;
RUN; TITLE1;

* Sending original longitudinal data to Mplus;
DATA Mplus; SET RiskyStacked;
agesq=agec18*agec18; mon3=monitor-3;
KEEP FamilyID occasion age monitor risky agec18 att4
     PMmon3 WPmon age18Mon3 Change18Mon agesq mon3;
RUN;

* Export to .csv for use in Mplus;
PROC EXPORT DATA=work.Mplus
OUTFILE= "&example.\chapter9v3.csv"
DBMS=CSV REPLACE; PUTNAMES=NO; RUN;
```

## Mplus results start here: This is the same model…

```
MODEL FIT INFORMATION
Number of Free Parameters                        20
Loglikelihood
          H0 Value                        -4392.253
Information Criteria

          Akaike (AIC)                     8824.506
          Bayesian (BIC)                   8929.390
          Sample-Size Adjusted BIC         8865.858
            (n* = (n + 2) / 24)
```

## In Mplus, doing the same thing as a multivariate multilevel model (#2):

```
TITLE:  Multivariate Growth Model as MLM
DATA:   FILE = chapter9v3.csv;   ! Syntax in same folder as data
VARIABLE:
! List of variables in data file
  NAMES = FamilyID occasion age monitor risky age18 att4
          PMmon3 WPmon Age18M3 C18Mon agesq mon3;
! Variables to be analyzed in this model
  USEVARIABLE = age18 agesq att4 risky mon3;
  MISSING ARE ALL (-999);   ! Missing data identifier
! MLM options
  CLUSTER = FamilyID;        ! Level-2 ID
  BETWEEN = att4;            ! Observed ONLY level-2 predictors
  WITHIN  = age18 agesq;     ! Observed ONLY level-1 predictors

ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;
MODEL:  ! R = risky behavior, M = monitoring
%WITHIN%
 risky* mon3* (Rresvar Mresvar);  ! Residual variances
 Rslp  | risky ON age18*;         ! R linear age slope
 Rquad | risky ON agesq*;         ! R quadratic age slope
 Mslp  | mon3  ON age18*;         ! M linear age slope
 risky WITH mon3* (ResCov);       ! Residual covariance

%BETWEEN%
 risky* mon3*  (Rintvar Mintvar);  ! Intercept variances
 Rslp* Mslp*   (Rslpvar Mslpvar);  ! Linear age slope variances
 Rquad@0;                          ! No quadratic age slope variance
 [risky* mon3*];                   ! Fixed intercepts
 [Rquad* Rslp* Mslp*];             ! Fixed age slopes
 risky Rslp ON att4*;              ! Att --> R int, linear age slope
 risky WITH Rslp*  (RIntSlp);      ! R Int-slope covariance
 mon3  WITH Mslp*  (MIntSlp);      ! M Int-slope covariance

 risky WITH mon3*  (IntCov);   ! Intercept covariance
 Rslp  WITH Mslp*  (SlpCov);   ! Slope covariance
 mon3  WITH Rslp*  (Int2Slp);  ! M int, R slope covariance
 Mslp  WITH risky* (Slp2Int);  ! M slope, R int covariance

MODEL CONSTRAINT:  ! Estimating correlations
NEW(ResCor IntCor SlpCor RIScor MIScor I2SCor S2ICor);
  ResCor = ResCov  / (SQRT(Rresvar)*SQRT(Mresvar));
  IntCor = IntCov  / (SQRT(Rintvar)*SQRT(Mintvar));
  SlpCor = SlpCov  / (SQRT(Rslpvar)*SQRT(Mslpvar));
  RIScor = RIntSlp / (SQRT(Rintvar)*SQRT(Rslpvar));
  MIScor = MIntSlp / (SQRT(Mintvar)*SQRT(Mslpvar));
  I2Scor = Int2Slp / (SQRT(Mintvar)*SQRT(Rslpvar));
  S2Icor = Slp2Int / (SQRT(Mslpvar)*SQRT(Rintvar));
```

## SAS multivariate MLM Results:

Estimated R Matrix for FamilyID*occasion 1 12

| Row | Col1 | Col2 |
|---|---|---|
| 1 | 8.3538 | 0.2874 |
| 2 | 0.2874 | 0.08077 |

Estimated R Correlation Matrix for FamilyID*occasion 1 12

| Row | Col1 | Col2 |
|---|---|---|
| 1 | 1.0000 | 0.3499 |
| 2 | 0.3499 | 1.0000 |

Estimated G Matrix

| Row | Effect | Family ID | Col1 | Col2 | Col3 | Col4 |
|---|---|---|---|---|---|---|
| 1 | dvR | 1 | 18.0644 | -0.8554 | 1.8829 | 0.04072 |
| 2 | dvM | 1 | -0.8554 | 0.1953 | -0.1064 | -0.00047 |
| 3 | dvR*agec18 | 1 | 1.8829 | -0.1064 | 0.4883 | -0.01815 |
| 4 | dvM*agec18 | 1 | 0.04072 | -0.00047 | -0.01815 | 0.01049 |

Estimated G Correlation Matrix

| Row | Effect | Family ID | Col1 | Col2 | Col3 | Col4 |
|---|---|---|---|---|---|---|
| 1 | dvR | 1 | 1.0000 | -0.4554 | 0.6340 | 0.09356 |
| 2 | dvM | 1 | -0.4554 | 1.0000 | -0.3446 | -0.01043 |
| 3 | dvR*agec18 | 1 | 0.6340 | -0.3446 | 1.0000 | -0.2537 |
| 4 | dvM*agec18 | 1 | 0.09356 | -0.01043 | -0.2537 | 1.0000 |

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr Z |
|---|---|---|---|---|---|
| UN(2,1) | FamilyID | -0.8554 | 0.1685 | -5.08 | <.0001 |
| UN(3,1) | FamilyID | 1.8829 | 0.3564 | 5.28 | <.0001 |
| UN(3,2) | FamilyID | -0.1064 | 0.03086 | -3.45 | 0.0006 |
| UN(4,1) | FamilyID | 0.04072 | 0.03879 | 1.05 | 0.2939 |
| UN(4,2) | FamilyID | -0.00047 | 0.004005 | -0.12 | 0.9062 |
| UN(4,3) | FamilyID | -0.01815 | 0.007344 | -2.47 | 0.0135 |
| UN(2,1) | FamilyID*occasion | 0.2874 | 0.02753 | 10.44 | <.0001 |

Information Criteria

| Neg2LogLike | Parms | AIC | AICC | HQIC | BIC | CAIC |
|---|---|---|---|---|---|---|
| 8784.5 | 20 | 8824.5 | 8824.8 | 8851.2 | 8890.5 | 8910.5 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| dvR | 23.3138 | 0.3477 | 239 | 67.06 | <.0001 |
| dvM | 0.06505 | 0.03412 | 200 | 1.91 | 0.0580 |
| dvR*agec18 | 1.9743 | 0.1386 | 1185 | 14.25 | <.0001 |
| dvM*agec18 | -0.00328 | 0.008176 | 200 | -0.40 | 0.6884 |
| dvR*agec18*agec18 | 0.1466 | 0.02058 | 1010 | 7.12 | <.0001 |
| dvR*Att4 | -3.3328 | 0.5126 | 199 | -6.50 | <.0001 |
| dvR*agec18*Att4 | -0.5298 | 0.1025 | 199 | -5.17 | <.0001 |

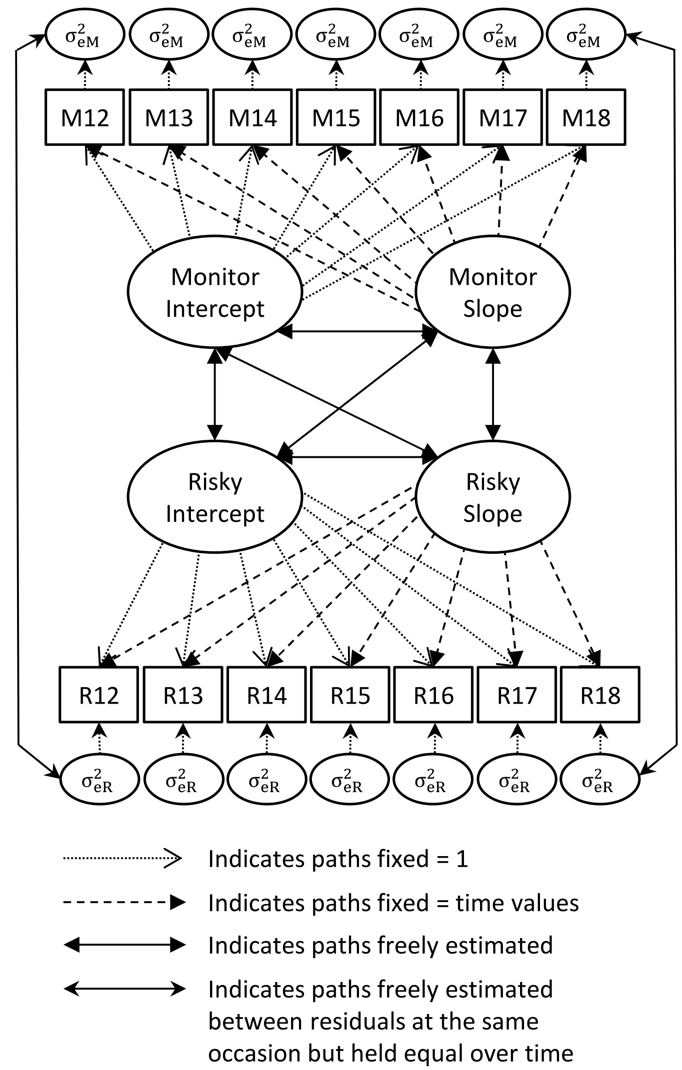
## Mplus results continue: This is the same model…

```
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Within Level

RISKY    WITH
    MON3               0.287      0.028     10.441      0.000
 Residual Variances
    MON3               0.081      0.004     22.354      0.000
    RISKY              8.352      0.374     22.351      0.000

Between Level

 RSLP       ON
    ATT4              -0.530      0.103     -5.161      0.000
 RISKY      ON
    ATT4              -3.333      0.514     -6.491      0.000
 RISKY    WITH
    RSLP               1.879      0.356      5.272      0.000
 MON3     WITH
    MSLP               0.000      0.004     -0.118      0.906
    RSLP              -0.106      0.031     -3.445      0.001
 RSLP     WITH
    MSLP              -0.018      0.007     -2.475      0.013
 MSLP     WITH
    RISKY              0.041      0.039      1.049      0.294
 RISKY    WITH
    MON3              -0.855      0.168     -5.076      0.000

 Means
    MON3               0.065      0.034      1.907      0.057
    RQUAD              0.147      0.021      7.117      0.000
    MSLP              -0.003      0.008     -0.402      0.688

 Intercepts
    RISKY             23.314      0.348     67.062      0.000
    RSLP               1.974      0.138     14.255      0.000

 Variances
    MON3               0.195      0.023      8.376      0.000
    RQUAD              0.000      0.000    999.000    999.000
    MSLP               0.010      0.001      7.803      0.000

 Residual Variances
    RISKY             18.060      2.204      8.195      0.000
    RSLP               0.485      0.080      6.071      0.000

 New/Additional Parameters
    RESCOR             0.350      0.028     12.607      0.000
    INTCOR            -0.455      0.074     -6.124      0.000
    SLPCOR            -0.255      0.103     -2.480      0.013

    RISCOR             0.635      0.057     11.087      0.000
    MISCOR            -0.010      0.089     -0.117      0.906
    I2SCOR            -0.346      0.095     -3.642      0.000
    S2ICOR             0.094      0.087      1.071      0.284
```

**In Mplus again, doing the same thing as SEM. Note that the original multivariate data file (one row per person) is used. Age12–Age18 are exact age variables centered so time 0 = age 18. (Model #3)**

```
TITLE:  Multivariate Growth Model as SEM
DATA:   FILE = ch9multiv.csv;   ! Syntax in same folder as data
VARIABLE:
! List of variables in data file
  NAMES = FamilyID att4 risky12-risky18 age12-age18 mon12-mon18;
! Variables to be analyzed in this model
  USEVARIABLE = att4 age12-age18 mon12-mon18 risky12-risky18;
  MISSING ARE ALL (-999);    ! Missing data identifier
  TSCORES = age12-age18;     ! Exact time indicator
ANALYSIS:   TYPE = RANDOM; ESTIMATOR = ML;
MODEL:   ! R = risky behavior, M = monitoring
! Risky behavior growth model using exact age as loadings
Rint Rslp Rquad | risky12-risky18 AT age12-age18;
Rint* Rslp* (Rintvar Rslpvar);  ! R Int and slope variances
Rquad@0;                        ! No R random quadratic variance
Rint WITH Rslp* (RIntSlp);      ! R Int-slope covariance
[Rint* Rslp* Rquad*];           ! R fixed growth effects
[risky12-risky18@0];            ! R Intercepts fixed to 0
risky12-risky18* (Rresvar);     ! R Residual var held equal
Rint Rslp ON att4*;             ! Attitudes --> risky int, slope

! Monitoring growth model using exact age as loadings
Mint Mslp | mon12-mon18 AT age12-age18;
Mint* Mslp* (Mintvar Mslpvar);  ! M Int and slope variances
Mint WITH Mslp* (MIntSlp);      ! M Int-slope covariance
[Mint* Mslp*];                  ! M fixed growth effects
[mon12-mon18@0];                ! M Intercepts fixed to 0
mon12-mon18* (Mresvar);         ! M Residual var held equal

! Covariances between outcomes
Rint WITH Mint* (IntCov);       ! Intercept covariance
Rslp WITH Mslp* (SlpCov);       ! Slope covariance
Mint WITH Rslp* (Int2Slp);      ! M int, R slope covariance
Mslp WITH Rint* (Slp2Int);      ! M slope, R int covariance
! Residual cov between same ages, held equal across age
mon12-mon18 PWITH risky12-risky18* (ResCov);

MODEL CONSTRAINT:  ! Estimating correlations
NEW(ResCor IntCor SlpCor RIScor MIScor I2SCor S2ICor);
  ResCor = ResCov  / (SQRT(Rresvar)*SQRT(Mresvar));
  IntCor = IntCov  / (SQRT(Rintvar)*SQRT(Mintvar));
  SlpCor = SlpCov  / (SQRT(Rslpvar)*SQRT(Mslpvar));
  RIScor = RIntSlp / (SQRT(Rintvar)*SQRT(Rslpvar));
  MIScor = MIntSlp / (SQRT(Mintvar)*SQRT(Mslpvar));
  I2Scor = Int2Slp / (SQRT(Mintvar)*SQRT(Rslpvar));
  S2Icor = Slp2Int / (SQRT(Mslpvar)*SQRT(Rintvar));
```

**Here is my “latent growth curve structural equation model”:**




```
If you have balanced time, the growth model would become this:

Mint Mslp | mon12@-6 mon13@-5 mon14@-4 mon15@-3
            mon16@-2 mon17@-1 mon18@0;
```

**Mplus results for SEM with correlated growth curves:**

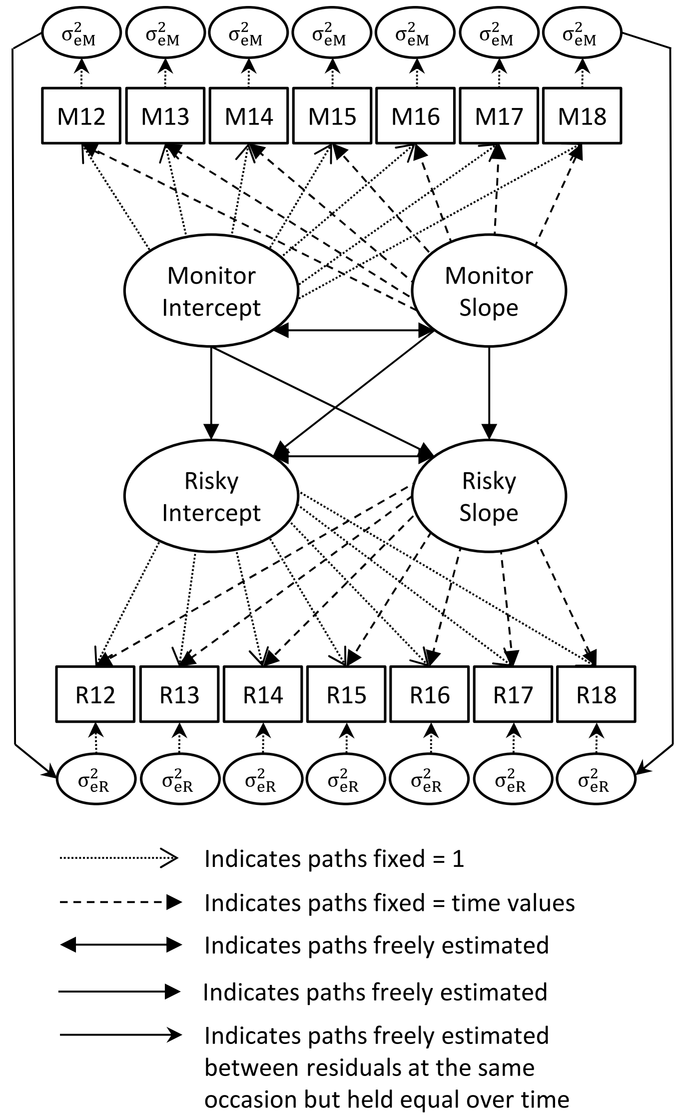
```
MODEL FIT INFORMATION
Number of Free Parameters                       20

Loglikelihood
          H0 Value                       -4392.253
Information Criteria
          Akaike (AIC)                    8824.506
          Bayesian (BIC)                  8890.472
          Sample-Size Adjusted BIC        8827.110
            (n* = (n + 2) / 24)

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 RINT       ON
    ATT4              -3.333      0.514     -6.491      0.000

 RSLP       ON
    ATT4              -0.530      0.103     -5.161      0.000

 RINT     WITH
    RSLP               1.879      0.356      5.272      0.000
    MINT              -0.855      0.168     -5.076      0.000

 MINT     WITH
    MSLP               0.000      0.004     -0.118      0.906
    RSLP              -0.106      0.031     -3.445      0.001

 RSLP     WITH
    MSLP              -0.018      0.007     -2.475      0.013

 MSLP     WITH
    RINT               0.041      0.039      1.049      0.294

 MON12    WITH
    RISKY12            0.287      0.028     10.441      0.000

 MON13    WITH
    RISKY13            0.287      0.028     10.441      0.000

 MON14    WITH
    RISKY14            0.287      0.028     10.441      0.000

 MON15    WITH
    RISKY15            0.287      0.028     10.441      0.000

 MON16    WITH
    RISKY16            0.287      0.028     10.441      0.000

 MON17    WITH
    RISKY17            0.287      0.028     10.441      0.000

 MON18    WITH
    RISKY18            0.287      0.028     10.441      0.000
```

```
Means
    RQUAD              0.147      0.021      7.117      0.000
    MINT               0.065      0.034      1.906      0.057
    MSLP              -0.003      0.008     -0.402      0.688

 Intercepts
    MON12              0.000      0.000    999.000    999.000
    MON13              0.000      0.000    999.000    999.000
    MON14              0.000      0.000    999.000    999.000
    MON15              0.000      0.000    999.000    999.000
    MON16              0.000      0.000    999.000    999.000
    MON17              0.000      0.000    999.000    999.000
    MON18              0.000      0.000    999.000    999.000
    RISKY12            0.000      0.000    999.000    999.000
    RISKY13            0.000      0.000    999.000    999.000
    RISKY14            0.000      0.000    999.000    999.000
    RISKY15            0.000      0.000    999.000    999.000
    RISKY16            0.000      0.000    999.000    999.000
    RISKY17            0.000      0.000    999.000    999.000
    RISKY18            0.000      0.000    999.000    999.000
    RINT              23.314      0.348     67.062      0.000
    RSLP               1.974      0.138     14.255      0.000

 Variances
    RQUAD              0.000      0.000    999.000    999.000
    MINT               0.195      0.023      8.376      0.000
    MSLP               0.010      0.001      7.803      0.000

 Residual Variances
    MON12              0.081      0.004     22.354      0.000
    MON13              0.081      0.004     22.354      0.000
    MON14              0.081      0.004     22.354      0.000
    MON15              0.081      0.004     22.354      0.000
    MON16              0.081      0.004     22.354      0.000
    MON17              0.081      0.004     22.354      0.000
    MON18              0.081      0.004     22.354      0.000
    RISKY12            8.352      0.374     22.351      0.000
    RISKY13            8.352      0.374     22.351      0.000
    RISKY14            8.352      0.374     22.351      0.000
    RISKY15            8.352      0.374     22.351      0.000
    RISKY16            8.352      0.374     22.351      0.000
    RISKY17            8.352      0.374     22.351      0.000
    RISKY18            8.352      0.374     22.351      0.000
    RINT              18.060      2.204      8.195      0.000
    RSLP               0.485      0.080      6.071      0.000

 New/Additional Parameters
    RESCOR             0.350      0.028     12.607      0.000
    INTCOR            -0.455      0.074     -6.124      0.000
    SLPCOR            -0.255      0.103     -2.480      0.013

    RISCOR             0.635      0.057     11.087      0.000
    MISCOR            -0.010      0.089     -0.117      0.907
    I2SCOR            -0.346      0.095     -3.642      0.000
    S2ICOR             0.094      0.087      1.071      0.284
```

**In Mplus again, doing the same thing as a directed path SEM (Model #4).**

```
TITLE:  Multivariate Growth Model as Directed SEM
DATA:   FILE = ch9multiv.csv;   ! Syntax in same folder as data
VARIABLE:
! List of variables in data file
  NAMES = FamilyID att4 risky12-risky18 age12-age18 mon12-mon18;
! Variables to be analyzed in this model
  USEVARIABLE = att4 age12-age18 mon12-mon18 risky12-risky18;
  MISSING ARE ALL (-999);    ! Missing data identifier
  TSCORES = age12-age18;     ! Exact time indicator
ANALYSIS:   TYPE = RANDOM; ESTIMATOR = ML;
MODEL:  ! R = risky behavior, M = monitoring
! Risky behavior growth model using exact age as loadings
Rint Rslp Rquad | risky12-risky18 AT age12-age18;
Rint* Rslp* (Rintvar Rslpvar);  ! R Int and slope variances
Rquad@0;                        ! No R random quadratic variance
Rint WITH Rslp* (RIntSlp);      ! R Int-slope covariance
[Rint* Rslp* Rquad*];           ! R fixed growth effects
[risky12-risky18@0];            ! R Intercepts fixed to 0
risky12-risky18* (Rresvar);     ! R Residual var held equal
Rint Rslp ON att4*;             ! Attitudes --> risky int, slope

! Monitoring growth model using exact age as loadings
Mint Mslp | mon12-mon18 AT age12-age18;
Mint* Mslp* (Mintvar Mslpvar);  ! M Int and slope variances
Mint WITH Mslp* (MIntSlp);      ! M Int-slope covariance
[Mint* Mslp*];                  ! M fixed growth effects
[mon12-mon18@0];                ! M Intercepts fixed to 0
mon12-mon18* (Mresvar);         ! M Residual var held equal

! Regressions between outcomes
Rint ON Mint* (IntCont);        ! Intercept contextual effect
Rslp ON Mslp* (SlpCont);        ! Slope contextual effect
Rslp ON Mint* (Int2Slp);        ! M int --> R slope between effect
Rint ON Mslp* (Slp2Int);        ! M slope --> R int between effect
! Residual WP effect between same ages, held equal across age
risky12-risky18 PON mon12-mon18* (ResEff);

MODEL CONSTRAINT:
NEW(IntEff SlpEff ResStd IntStd SlpStd RIScor MIScor I2Sstd S2Istd);
IntEff = ResEff + IntCont;    ! Between intercept effect
SlpEff = ResEff + SlpCont;    ! Between slope effect

ResStd = ResEff  * SQRT(Mresvar)/SQRT(Rresvar);   ! STD WP effect
IntStd = IntEff  * SQRT(Mintvar)/SQRT(Rintvar);   ! STD Int effect
SlpStd = SlpEff  * SQRT(Mslpvar)/SQRT(Rslpvar);   ! STD Slope effect
I2Sstd = Int2Slp * SQRT(Mintvar)/SQRT(Rslpvar);   ! STD M int --> R slope
S2Istd = Slp2Int * SQRT(Mslpvar)/SQRT(Rintvar);   ! STD M slope --> R int
RIScor = RIntSlp /(SQRT(Rintvar)*SQRT(Rslpvar));  ! R Int-slope correlation
MIScor = MIntSlp /(SQRT(Mintvar)*SQRT(Mslpvar));  ! M Int-slope correlation
```

**Here is my "latent growth curve structural equation model" using directed paths from monitoring to risky behavior. Can you spot the differences?**



## Mplus results for <u>directed path</u> SEM:

```
MODEL FIT INFORMATION
Number of Free Parameters                       20

Loglikelihood
          H0 Value                       -4392.253
Information Criteria
          Akaike (AIC)                    8824.506
          Bayesian (BIC)                  8890.472
          Sample-Size Adjusted BIC        8827.110
            (n* = (n + 2) / 24)

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

RINT       ON
    MINT              -7.928      0.861     -9.211      0.000
    MSLP               3.685      3.494      1.055      0.292

 RSLP       ON
    MSLP              -5.316      0.816     -6.517      0.000
    MINT              -0.548      0.160     -3.431      0.001

 RINT       ON
    ATT4              -3.333      0.514     -6.491      0.000

 RSLP       ON
    ATT4              -0.530      0.103     -5.161      0.000

 RISKY12    ON
    MON12              3.559      0.301     11.809      0.000

 RISKY13    ON
    MON13              3.559      0.301     11.809      0.000

 RISKY14    ON
    MON14              3.559      0.301     11.809      0.000

 RISKY15    ON
    MON15              3.559      0.301     11.809      0.000

 RISKY16    ON
    MON16              3.559      0.301     11.809      0.000

 RISKY17    ON
    MON17              3.559      0.301     11.809      0.000

 RISKY18    ON
    MON18              3.559      0.301     11.809      0.000

 RINT     WITH
    RSLP               1.481      0.345      4.291      0.000

 MINT     WITH
    MSLP               0.000      0.004     -0.118      0.906
```

```
Means
    RQUAD              0.147      0.021      7.117      0.000
    MINT               0.065      0.034      1.906      0.057
    MSLP              -0.003      0.008     -0.402      0.688
 Intercepts
    MON12              0.000      0.000    999.000    999.000
    MON13              0.000      0.000    999.000    999.000
    MON14              0.000      0.000    999.000    999.000
    MON15              0.000      0.000    999.000    999.000
    MON16              0.000      0.000    999.000    999.000
    MON17              0.000      0.000    999.000    999.000
    MON18              0.000      0.000    999.000    999.000
    RISKY12            0.000      0.000    999.000    999.000
    RISKY13            0.000      0.000    999.000    999.000
    RISKY14            0.000      0.000    999.000    999.000
    RISKY15            0.000      0.000    999.000    999.000
    RISKY16            0.000      0.000    999.000    999.000
    RISKY17            0.000      0.000    999.000    999.000
    RISKY18            0.000      0.000    999.000    999.000
    RINT              23.610      0.333     70.898      0.000
    RSLP               2.004      0.139     14.405      0.000
 Variances
    RQUAD              0.000      0.000    999.000    999.000
    MINT               0.195      0.023      8.376      0.000
    MSLP               0.010      0.001      7.803      0.000

 Residual Variances
    MON12              0.081      0.004     22.354      0.000
    MON13              0.081      0.004     22.354      0.000
    MON14              0.081      0.004     22.354      0.000
    MON15              0.081      0.004     22.354      0.000
    MON16              0.081      0.004     22.354      0.000
    MON17              0.081      0.004     22.354      0.000
    MON18              0.081      0.004     22.354      0.000
    RISKY12            7.329      0.328     22.353      0.000
    RISKY13            7.329      0.328     22.353      0.000
    RISKY14            7.329      0.328     22.353      0.000
    RISKY15            7.329      0.328     22.353      0.000
    RISKY16            7.329      0.328     22.353      0.000
    RISKY17            7.329      0.328     22.353      0.000
    RISKY18            7.329      0.328     22.353      0.000
    RINT              14.173      1.965      7.213      0.000
    RSLP               0.394      0.082      4.787      0.000

 New/Additional Parameters
    INTEFF            -4.369      0.784     -5.575      0.000
    SLPEFF            -1.758      0.724     -2.429      0.015

    RESSTD             0.374      0.034     11.063      0.000
    INTSTD            -0.513      0.104     -4.928      0.000
    SLPSTD            -0.287      0.126     -2.270      0.023

    RISCOR             0.627      0.068      9.266      0.000
    MISCOR            -0.010      0.089     -0.117      0.907
    I2SSTD            -0.386      0.125     -3.078      0.002
    S2ISTD             0.100      0.095      1.051      0.293
```

**In Mplus still, doing the same thing as a "truly multivariate" MLM instead of SEM (Model #5)**
***This is called "multilevel SEM" by some people…***

```
TITLE:  Multivariate Growth Model as MLM with directed paths
DATA:   FILE = chapter9v3.csv;   ! Syntax in same folder as data
VARIABLE:
! List of variables in data file
  NAMES = FamilyID occasion age monitor risky age18 att4
          PMmon3 WPmon Age18M3 C18Mon agesq mon3;
! Variables to be analyzed in this model
  USEVARIABLE = age18 agesq att4 risky mon3;
  MISSING ARE ALL (-999);   ! Missing data identifier
! MLM options
  CLUSTER = FamilyID;       ! Level-2 ID
  BETWEEN = att4;           ! Observed ONLY level-2 predictors
  WITHIN  = age18 agesq;    ! Observed ONLY level-1 predictors

ANALYSIS:   TYPE = TWOLEVEL RANDOM; ESTIMATOR = ML;
MODEL:  ! R = risky behavior, M = monitoring
%WITHIN%
 risky* mon3* (Rresvar Mresvar);  ! Residual variances
 Rslp  | risky ON age18*;         ! R linear age slope
 Rquad | risky ON agesq*;         ! R quadratic age slope
 Mslp  | mon3  ON age18*;         ! M linear age slope
 WPres | risky ON mon3*;          ! WP effect M --> R

%BETWEEN%
 risky* mon3* (Rintvar Mintvar);  ! Intercept variances
 Rslp*  Mslp* (Rslpvar Mslpvar);  ! Linear age slope variances
 Rquad@0;                         ! No quadratic age slope variance
 [risky* mon3*];                  ! Fixed intercepts
 [Rslp* Rquad* Mslp*];            ! Fixed age slopes
 risky Rslp ON att4*;             ! Att --> R int, linear age slope
 risky WITH Rslp*  (RIntSlp);     ! R Int-slope covariance
 mon3  WITH Mslp*  (MIntSlp);     ! M Int-slope covariance

! Regressions between outcomes
 risky ON mon3* (IntCont);        ! Intercept contextual effect
 Rslp  ON Mslp* (SlpCont);        ! Slope contextual effect
 Rslp  ON mon3* (Int2Slp);        ! M int --> R slope between effect
 risky ON Mslp* (Slp2Int);        ! M slope --> R int between effect

! Residual WP effect between same ages, held equal across age
 [WPres] (ResEff);
  WPres@0;          ! No random WP effect variance

MODEL CONSTRAINT:
NEW(IntEff SlpEff ResStd IntStd SlpStd RIScor MIScor I2Sstd S2Istd);

IntEff = ResEff + IntCont;    ! Between intercept effect
SlpEff = ResEff + SlpCont;    ! Between slope effect

ResStd = ResEff  * SQRT(Mresvar)/SQRT(Rresvar);   ! STD WP effect
IntStd = IntEff  * SQRT(Mintvar)/SQRT(Rintvar);   ! STD Int effect
SlpStd = SlpEff  * SQRT(Mslpvar)/SQRT(Rslpvar);   ! STD Slope effect

I2Sstd = Int2Slp * SQRT(Mintvar)/SQRT(Rslpvar);   ! STD M int --> R slope
S2Istd = Slp2Int * SQRT(Mslpvar)/SQRT(Rintvar);   ! STD M slope --> R int

RIScor = RIntSlp /(SQRT(Rintvar)*SQRT(Rslpvar));  ! R Int-slope correlation
MIScor = MIntSlp /(SQRT(Mintvar)*SQRT(Mslpvar));  ! M Int-slope correlation
```

**From chapter 9:**

Figure 9.5. Multivariate longitudinal model when using truly multivariate software.

**Level 1:** $\text{Monitor}_{ti} - 3 = \beta_{00M} + \beta_{10M}\left(\text{Age}_{tiM} - 18\right) + e_{tiM}$

$$\text{Risky}_{ti} = \beta_{00R} + \beta_{10R}\left(\text{Age}_{tiR} - 18\right) + \beta_{20R}\left(\text{Age}_{tiR} - 18\right)^2 + \beta_{30R}\left(e_{tiM}\right) + e_{tiR}$$

**Level 2:**

Monitor Intercept: $\beta_{00M} = \gamma_{00M} + U_{0iM}$

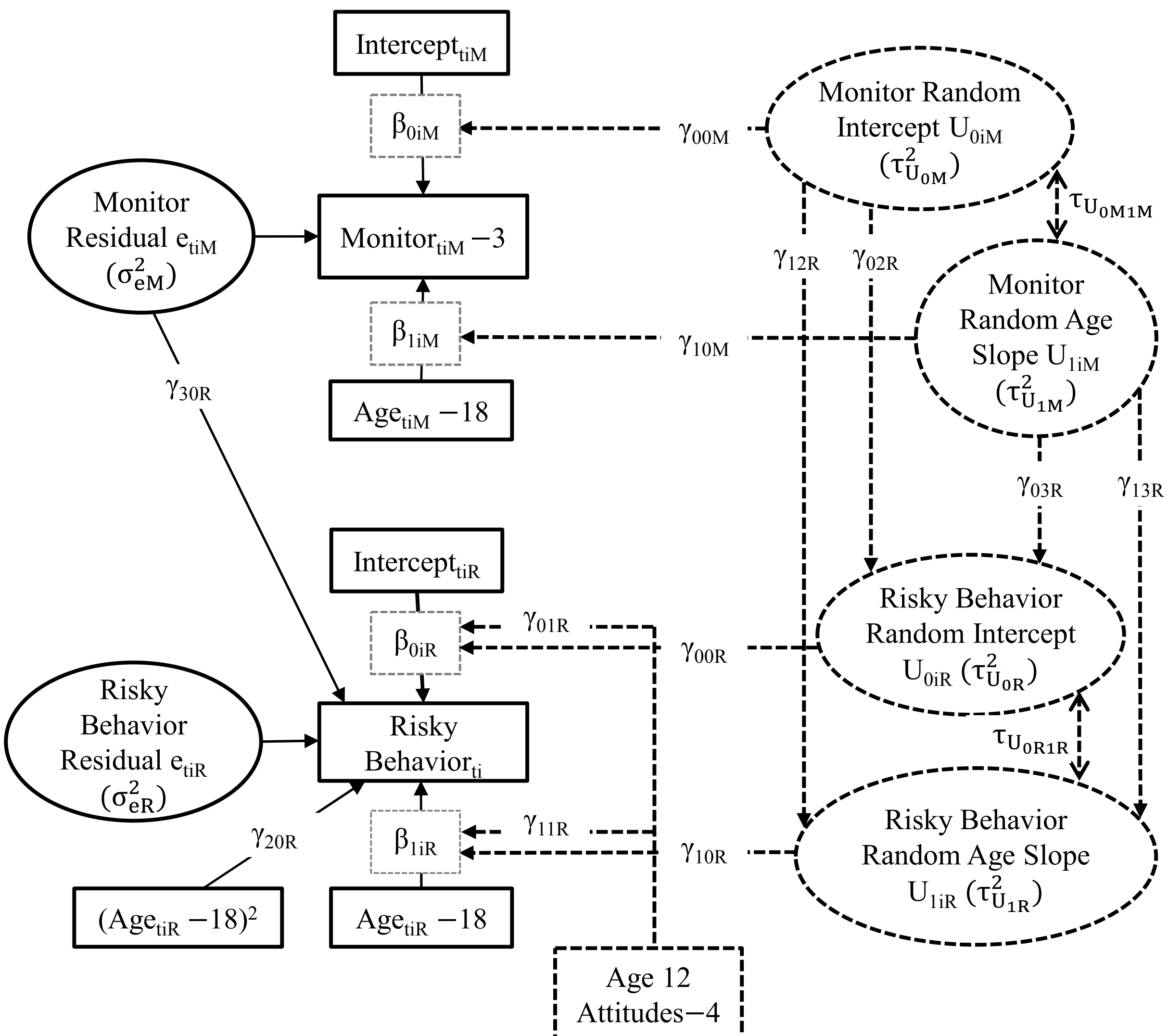
Monitor Age: $\beta_{10M} = \gamma_{10M} + U_{1iM}$

Risky Intercept: $\beta_{00R} = \gamma_{00R} + \gamma_{01R}\left(\text{Age12Attitudes}_i - 4\right) + \gamma_{02R}\left(U_{0iM}\right) + \gamma_{03R}\left(U_{1iM}\right) + U_{0iR}$

Risky Age: $\beta_{10R} = \gamma_{10R} + \gamma_{11R}\left(\text{Age12Attitudes}_i - 4\right) + \gamma_{12R}\left(U_{0iM}\right) + \gamma_{13R}\left(U_{1iM}\right) + U_{1iR}$

Risky Age$^2$: $\beta_{20R} = \gamma_{20R}$

Risky WP Monitor: $\beta_{30R} = \gamma_{30R}$



**From chapter 9:**

Table 9.5. Results from multivariate longitudinal model using truly multivariate software.

| | Model Effects | Est | SE | *p* < | STD |
|---|---|---|---|---|---|
| Risky Behavior Model for the Means | | | | | |
| $\gamma_{00R}$ | Intercept | **23.61** | 0.33 | .001 | |
| $\gamma_{10R}$ | Linear Age Slope (0 = 18) | **2.00** | 0.14 | .001 | |
| $\gamma_{20R}$ | Quadratic Age Slope | **0.15** | 0.02 | .001 | |
| $\gamma_{01R}$ | Mothers' Attitudes (0 = 4) | **−3.33** | 0.51 | .001 | |
| $\gamma_{11R}$ | Attitudes by Linear Age | **−0.53** | 0.10 | .001 | |
| Monitoring Model for the Means | | | | | |
| $\gamma_{00M}$ | Intercept | 0.07 | 0.03 | .057 | |
| $\gamma_{10M}$ | Linear Age Slope (0 = 18) | −0.00 | 0.01 | .704 | |
| Risky Behavior Model for the Variance | | | | | |
| $\tau^2_{U_{0R}}$ | Random Intercept Variance | **14.17** | 1.97 | .001 | |
| $\tau^2_{U_{1R}}$ | Linear Age Slope Variance | **0.39** | 0.08 | .001 | |
| $\tau_{U_{0R1R}}$ | Intercept-Age Slope Covariance | **1.48** | 0.35 | .001 | |
| $\sigma^2_{eR}$ | Residual Variance | **7.33** | 0.33 | .001 | |
| Monitoring Model for the Variance | | | | | |
| $\tau^2_{U_{0M}}$ | Random Intercept Variance | **0.20** | 0.02 | .001 | |
| $\tau^2_{U_{1M}}$ | Linear Age Slope Variance | **0.01** | 0.00 | .001 | |
| $\tau_{U_{0M1M}}$ | Intercept-Age Slope Covariance | −0.00 | 0.00 | .916 | |
| $\sigma^2_{eM}$ | Residual Variance | **0.08** | 0.00 | .001 | |
| Cross-Variable Regressions | | | | | |
| | Monitoring Intercept → Risky Behavior Intercept | | | | |
| $\gamma_{02R}$ | Contextual Effect | **−7.93** | 0.86 | .001 | |
| $\gamma_{02R} + \gamma_{30R}$ | Total Effect | **−4.37** | 0.78 | .001 | **−0.51** |
| | Monitoring Age Slope → Risky Behavior Age Slope | | | | |
| $\gamma_{13R}$ | Contextual Effect | **−5.32** | 0.82 | .001 | |
| $\gamma_{13R} + \gamma_{30R}$ | Total Effect | **−1.76** | 0.72 | .015 | **−0.29** |
| $\gamma_{30R}$ | Monitoring Residual → Risky Behavior Residual | **3.56** | 0.30 | .001 | **0.37** |
| $\gamma_{12R}$ | Monitoring Intercept → Risky Behavior Age Slope | **−0.55** | 0.16 | .001 | **−0.39** |
| $\gamma_{03R}$ | Monitoring Age Slope → Risky Behavior Intercept | 3.69 | 3.49 | .291 | 0.10 |

## What Mplus actually gave me for my "truly multivariate" MLM that I used to make Table 9.5:

```
MODEL FIT INFORMATION
Number of Free Parameters                       20

Loglikelihood
          H0 Value                       -4392.253
Information Criteria
          Akaike (AIC)                    8824.506
          Bayesian (BIC)                  8929.390
          Sample-Size Adjusted BIC        8865.858
            (n* = (n + 2) / 24)

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Within Level

 Residual Variances
    MON3               0.081      0.004     22.355      0.000
    RISKY              7.329      0.328     22.353      0.000

Between Level

 RSLP       ON
    MSLP              -5.316      0.816     -6.517      0.000

 RSLP       ON
    ATT4              -0.530      0.103     -5.161      0.000
    MON3              -0.548      0.160     -3.431      0.001

 RISKY      ON
    MSLP               3.685      3.494      1.055      0.292

 RISKY      ON
    ATT4              -3.333      0.514     -6.491      0.000
    MON3              -7.928      0.861     -9.211      0.000

 RISKY    WITH
    RSLP               1.481      0.345      4.291      0.000

 MON3     WITH
    MSLP               0.000      0.004     -0.118      0.906

 Means
    MON3               0.065      0.034      1.906      0.057
    RQUAD              0.147      0.021      7.117      0.000
    MSLP              -0.003      0.008     -0.402      0.688
    WPRES              3.559      0.301     11.810      0.000

 Intercepts
    RISKY             23.610      0.333     70.898      0.000
    RSLP               2.004      0.139     14.405      0.000

 Variances
    MON3               0.195      0.023      8.376      0.000
    RQUAD              0.000      0.000    999.000    999.000
    MSLP               0.010      0.001      7.803      0.000
    WPRES              0.000      0.000    999.000    999.000

 Residual Variances
    RISKY             14.173      1.965      7.213      0.000
    RSLP               0.394      0.082      4.787      0.000

 New/Additional Parameters
    INTEFF            -4.369      0.784     -5.574      0.000
    SLPEFF            -1.758      0.724     -2.428      0.015
    RESSTD             0.374      0.034     11.063      0.000
    INTSTD            -0.513      0.104     -4.928      0.000
    SLPSTD            -0.287      0.126     -2.269      0.023
    RISCOR             0.627      0.068      9.266      0.000
    MISCOR            -0.010      0.089     -0.117      0.907
    I2SSTD            -0.386      0.125     -3.078      0.002
    S2ISTD             0.100      0.095      1.051      0.293
```

### In SAS NLMIXED, where (currently) I can get everything except the residual directed path ☹ (#6)

```
TITLE1 "Truly Multivariate Model (but without residual path)";
PROC NLMIXED DATA=RiskyStacked2 METHOD=GAUSS TECH=QUANEW GCONV=1e-12;
* Starting values for all fixed effects and variance components;
  PARMS FintR=23 FslpR=2 Fslp2R=.15 FattintR=-3 FattslpR=-.5 FintM=0 FslpM=0
        FintMintR=-4 FintMslpR=-.5 FslpMintR=3.7 FslpMslpR=-1.7
        ResVarR=7 IntVarR=14 SlpVarR=.4 CovarR=1.5 ResVarM=.1 IntVarM=.2 SlpVarM=.01 CovarM=0;
* Dont allow variances to go below 0 during estimation;
  BOUNDS ResVarR>0, IntVarR>0, slpVarR>0, ResVarM>0, IntVarM>0, slpVarM>0;
* Level-2 model;
      B0M = FintM + U0M;
      B1M = FslpM + U1M;
      B0R = FintR + FattintR*att4 + FintMintR*U0M + FslpMintR*U1M + U0R;
      B1R = FslpR + FattslpR*att4 + FintMslpR*U0M + FslpMslpR*U1M + U1R;
* Level-1 model;
      predY = dvM*(B0M + B1M*agec18) + dvR*(B0R + B1R*agec18 + Fslp2R*agec18*agec18);
      ResVar = dvR*ResVarR + dvM*ResVarM;
* Put models back together;
  MODEL outcome ~ NORMAL(predY, ResVar);
* Define G matrix;
  RANDOM U0R U1R U0M U1M ~ NORMAL([0,0,0,0],[IntVarR,
                                             CovarR, slpVarR,
                                             0,      0,       IntVarM,
                                             0,      0,       CovarM, slpVarM]) SUBJECT=FamilyID;
* Estimating standardized effects;
  ESTIMATE "STD M --> R Intercept" FintMintR*SQRT(IntVarM)/SQRT(IntVarR);
  ESTIMATE "STD M --> R Slope"     FslpMslpR*SQRT(SlpVarM)/SQRT(SlpVarR);
  ESTIMATE "STD M Int --> R Slope" FintMslpR*SQRT(IntVarM)/SQRT(SlpVarR);
  ESTIMATE "STD M Slope --> R Int" FslpMintR*SQRT(SlpVarM)/SQRT(IntVarR);
RUN;
```

Fit Statistics

| | |
|---|---|
| -2 Log Likelihood | 8915.1 |
| AIC (smaller is better) | 8953.1 |
| AICC (smaller is better) | 8953.4 |
| BIC (smaller is better) | 9015.8 |

Parameter Estimates

| Parameter | Estimate | Standard Error | DF | t Value | Pr > \|t\| | Alpha | Lower | Upper | Gradient |
|---|---|---|---|---|---|---|---|---|---|
| FintR | 23.3053 | 0.3499 | 196 | 66.60 | <.0001 | 0.05 | 22.6152 | 23.9954 | -0.00005 |
| FslpR | 1.9658 | 0.1460 | 196 | 13.46 | <.0001 | 0.05 | 1.6778 | 2.2537 | 0.000057 |
| Fslp2R | 0.1452 | 0.02196 | 196 | 6.61 | <.0001 | 0.05 | 0.1019 | 0.1886 | -0.00034 |
| FattintR | -3.3299 | 0.5138 | 196 | -6.48 | <.0001 | 0.05 | -4.3431 | -2.3166 | -0.00002 |
| FattslpR | -0.5285 | 0.1027 | 196 | -5.15 | <.0001 | 0.05 | -0.7311 | -0.3260 | 0.000042 |
| FintM | 0.06501 | 0.03413 | 196 | 1.91 | 0.0582 | 0.05 | -0.00229 | 0.1323 | 0.000303 |
| FslpM | -0.00330 | 0.008178 | 196 | -0.40 | 0.6866 | 0.05 | -0.01943 | 0.01282 | -0.00017 |
| FintMintR | -3.6741 | 0.7651 | 196 | -4.80 | <.0001 | 0.05 | -5.1831 | -2.1651 | 0.000013 |
| FintMslpR | -0.3891 | 0.1531 | 196 | -2.54 | 0.0118 | 0.05 | -0.6910 | -0.08715 | -0.00003 |
| FslpMintR | 6.7128 | 3.4138 | 196 | 1.97 | 0.0507 | 0.05 | -0.01980 | 13.4454 | 6.237E-6 |
| FslpMslpR | -0.7625 | 0.6843 | 196 | -1.11 | 0.2665 | 0.05 | -2.1121 | 0.5871 | -0.00002 |
| ResVarR | 8.3529 | 0.3736 | 196 | 22.36 | <.0001 | 0.05 | 7.6162 | 9.0897 | 0.000018 |
| IntVarR | 14.9515 | 1.9144 | 196 | 7.81 | <.0001 | 0.05 | 11.1761 | 18.7270 | 0.000021 |
| SlpVarR | 0.4529 | 0.07760 | 196 | 5.84 | <.0001 | 0.05 | 0.2998 | 0.6059 | 0.000043 |
| CovarR | 1.6595 | 0.3295 | 196 | 5.04 | <.0001 | 0.05 | 1.0097 | 2.3093 | -0.00004 |
| ResVarM | 0.08076 | 0.003612 | 196 | 22.36 | <.0001 | 0.05 | 0.07363 | 0.08788 | 0.000718 |
| IntVarM | 0.1954 | 0.02333 | 196 | 8.38 | <.0001 | 0.05 | 0.1494 | 0.2414 | -0.0002 |
| SlpVarM | 0.01049 | 0.001344 | 196 | 7.80 | <.0001 | 0.05 | 0.007838 | 0.01314 | 0.001201 |
| CovarM | -0.00044 | 0.004007 | 196 | -0.11 | 0.9122 | 0.05 | -0.00834 | 0.007460 | -0.0019 |

Additional Estimates → close, but not close enough ☹

| Label | Estimate | Standard Error | DF | t Value | Pr > \|t\| | Alpha | Lower | Upper |
|---|---|---|---|---|---|---|---|---|
| STD M --> R Intercept | -0.4200 | 0.09494 | 196 | -4.42 | <.0001 | 0.05 | -0.6073 | -0.2328 |
| STD M --> R Slope | -0.1160 | 0.1051 | 196 | -1.10 | 0.2707 | 0.05 | -0.3232 | 0.09113 |
| STD M Int --> R Slope | -0.2556 | 0.1045 | 196 | -2.45 | 0.0153 | 0.05 | -0.4616 | -0.04954 |
| STD M Slope --> R Int | 0.1778 | 0.09184 | 196 | 1.94 | 0.0543 | 0.05 | -0.00332 | 0.3589 |

**What if your truly multivariate model won't estimate? Here is an observed variable alternative using SAS PROC MIXED to output per-person random effects and residuals (#7; "slopes-as-outcomes")**

```
TITLE1 "Random Linear Age Model for Parent Monitoring to Output Random Effects and Residuals";
PROC MIXED DATA=RiskyStacked NOITPRINT NOCLPRINT COVTEST IC NAMELEN=100 METHOD=ML;
      CLASS FamilyID occasion;
      MODEL monitor = agec18 / SOLUTION DDFM=Satterthwaite NOTEST OUTP=eUformonitor;
      RANDOM INTERCEPT agec18 / SOLUTION TYPE=UN SUBJECT=FamilyID; * Add solution;
      ODS OUTPUT SolutionR=Uformonitor; * Save random effects to dataset;
RUN; TITLE1;
```

**New "Uformonitor" random effects dataset:**

| | Effect | FamilyID | Estimate | StdErrPred | DF | tValue | Probt |
|---|---|---|---|---|---|---|---|
| 1 | Intercept | 1 | -0.2618 | 0.1669 | 924 | -1.57 | 0.1170 |
| 2 | agec18 | 1 | 0.03944 | 0.04549 | 843 | 0.87 | 0.3861 |
| 3 | Intercept | 2 | 0.4919 | 0.1717 | 889 | 2.86 | 0.0043 |
| 4 | agec18 | 2 | -0.08408 | 0.04531 | 823 | -1.86 | 0.0639 |

```
* Save random intercept centered at 3 to new dataset (have to add fixed effect);
DATA Uint; SET Uformonitor; WHERE INDEX(Effect,"Intercept")>0;
      Uint=Estimate+3.0650-3; KEEP FamilyID Uint; RUN;

* Save random slope to new dataset uncentered since fixed effect is near 0;
DATA Uage; SET Uformonitor; WHERE INDEX(Effect,"agec18")>0;
      Uage=Estimate; KEEP FamilyID Uage; RUN;

* Save residuals to new dataset;
DATA eUage; SET eUformonitor;
      Eres=Resid; KEEP FamilyID occasion Eres; RUN;

* Merge back into original data;
DATA RiskyStacked; MERGE RiskyStacked Uint Uage; BY FamilyID; RUN;
DATA RiskyStacked; MERGE RiskyStacked eUage; BY FamilyID occasion; RUN;
```

**Slopes-as-outcomes for time-varying monitoring predicting risky behavior:**

$$\textbf{Level 1:}\quad \text{Risky Behavior}_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Age}_{ti} - 18\right) + \beta_{2i}\left(\text{Age}_{ti} - 18\right)^2 + \beta_{3i}\left(\text{MonRes}_{ti}\right) + e_{ti}$$

$$\textbf{Level 2:}\quad \text{Intercept:}\quad \beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Attitudes12}_i - 4\right) + \gamma_{02}\left(\text{MonInt}_i - 3\right) + \gamma_{03}\left(\text{MonSlope}_i\right) + U_{0i}$$

$$\text{Age:}\quad \beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Attitudes12}_i - 4\right) + \gamma_{12}\left(\text{MonInt}_i - 3\right) + \gamma_{13}\left(\text{MonSlope}_i\right) + U_{1i}$$

$$\text{Age}^2\text{:}\quad \beta_{2i} = \gamma_{20}$$

$$\text{WP Monitor:}\quad \beta_{3i} = \gamma_{30}$$

$$\textbf{Composite:}$$

$$y_{ti} = \left[\gamma_{00} + \gamma_{01}\left(\text{Attitudes12}_i - 4\right) + \gamma_{02}\left(\text{MonInt}_i - 3\right) + \gamma_{03}\left(\text{MonSlope}_i\right) + U_{0i}\right] + \left[\gamma_{10} + \gamma_{11}\left(\text{Attitudes12}_i - 4\right) + \gamma_{12}\left(\text{MonInt}_i - 3\right) + \gamma_{13}\left(\text{MonSlope}_i\right) + U_{1i}\right]\left(\text{Age}_{ti} - 18\right) + \left[\gamma_{20}\right]\left(\text{Age}_{ti} - 18\right)^2 + \left[\gamma_{30}\right]\left(\text{MonRes}_{ti}\right) + e_{ti}$$

```
TITLE1 "Monitoring Us predicting risky behavior";
PROC MIXED DATA=RiskyStacked NOITPRINT NOCLPRINT COVTEST IC NOINFO NAMELEN=100 METHOD=ML;
      CLASS FamilyID occasion;
      MODEL risky = agec18 agec18*agec18 att4 att4*agec18 Uint Uint*agec18 Uage Uage*agec18 Eres
                    / SOLUTION DDFM=Satterthwaite NOTEST;
      RANDOM INTERCEPT agec18 / TYPE=UN SUBJECT=FamilyID;
      ODS OUTPUT InfoCrit=FixedU; RUN; TITLE1;
```

```
                   Covariance Parameter Estimates
                                      Standard         Z
Cov Parm     Subject     Estimate        Error     Value        Pr Z
UN(1,1)      FamilyID     15.5934       1.9076      8.17      <.0001
UN(2,1)      FamilyID      1.7982       0.3275      5.49      <.0001
UN(2,2)      FamilyID      0.4981      0.07694      6.47      <.0001
Residual                   7.3316       0.3279     22.36      <.0001

                             Information Criteria
Neg2LogLike    Parms        AIC       AICC       HQIC        BIC       CAIC
     7430.8       14     7458.8     7459.1     7477.5     7505.0     7519.0

                      Solution for Fixed Effects
                                    Standard
Effect             Estimate            Error       DF    t Value    Pr > |t|
Intercept           23.5971           0.3295      244      71.62      <.0001
agec18               2.0098           0.1384     1186      14.52      <.0001
agec18*agec18        0.1465          0.02058     1010       7.12      <.0001
att4                -3.3318           0.5137      200      -6.49      <.0001
agec18*att4         -0.5294           0.1027      199      -5.15      <.0001
Uint                -4.3591           0.7652      202      -5.70      <.0001
agec18*Uint         -0.5475           0.1530      201      -3.58      0.0004
Uage                 3.7561           3.4127      202       1.10      0.2724
agec18*Uage         -1.7481           0.6858      206      -2.55      0.0115
Eres                 3.5580           0.3013     1000      11.81      <.0001
```

**So how did we do? The variances of the monitoring predictors from the slopes-as-outcomes model are smaller (shrunken) as expected. Although this may not always be the case, here the results do appear similar, even though the effect sizes are slightly smaller for the slopes-as-outcomes model.**

Table 9.7. Comparison of results for the intercepts (int), age slopes (slope), and residuals (res) for monitoring (M) predicting risky behavior (RB) across models. Bold values are $p < .05$ for the unstandardized fixed effects.

| Monitoring Effects | Multivariate Longitudinal Model with Fixed Effects of Monitoring | | | | Slopes-as-Outcomes Model: Persons as Random Effects | | | |
|---|---|---|---|---|---|---|---|---|
| | Var(X) | Var(Y) | Fixed Effect | STD Effect | Var(X) | Var(Y) | Fixed Effect | STD Effect |
| M int → R int | 0.195 | 14.17 | **−4.37** | −0.51 | 0.168 | 15.59 | **−4.36** | −0.45 |
| M int → R slope | 0.195 | 0.39 | **−0.55** | −0.39 | 0.168 | 0.50 | **−0.55** | −0.32 |
| M slope → R int | 0.010 | 14.17 | 3.69 | 0.10 | 0.008 | 15.59 | 3.76 | 0.09 |
| M slope → R slope | 0.010 | 0.39 | **−1.76** | −0.29 | 0.008 | 0.50 | **−1.75** | −0.23 |
| M res → R res | 0.081 | 7.33 | **3.56** | 0.37 | 0.062 | 7.33 | **3.56** | 0.33 |

**Ok, so this brings up more questions… is the slopes-as-outcomes model still reasonably ok if we add additional effects? For example, what about a WP*age interaction or a random WP effect?**

### Adding a WP*age interaction using Slopes-as-Outcomes in PROC MIXED:

```
TITLE1 "Monitoring Us predicting risky behavior with WP*age interaction";
PROC MIXED DATA=RiskyStacked NOITPRINT NOCLPRINT COVTEST IC NOINFO NAMELEN=100 METHOD=ML;
      CLASS FamilyID occasion;
      MODEL risky = agec18 agec18*agec18 att4 att4*agec18 Uint Uint*agec18 Uage Uage*agec18
                    Eres Eres*agec18 / SOLUTION DDFM=Satterthwaite NOTEST;
      RANDOM INTERCEPT agec18 / TYPE=UN SUBJECT=FamilyID; RUN; TITLE1;
```

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Eres | 5.2010 | 0.6071 | 1095 | 8.57 | <.0001 |
| agec18*Eres | 0.5445 | 0.1749 | 1122 | 3.11 | 0.0019 |

### Adding a WP*age interaction in the Directed Path version of SEM in Mplus:

```
MODEL CONSTRAINT:  ! ResLin = linear change in WP effect across age
NEW(ResLin);
ResLin = ResEff13 - ResEff12;
ResLin = ResEff14 - ResEff13;
ResLin = ResEff15 - ResEff14;
ResLin = ResEff16 - ResEff15;
ResLin = ResEff17 - ResEff16;
ResLin = ResEff18 - ResEff17;
```

```
! Residual WP effect between same ages, now differs by age
risky12-risky18 PON mon12-mon18* (ResEff12-ResEff18);
```

| NEW PARMS | EST | SE | Z | p-value |
|---|---|---|---|---|
| RESLIN | 0.151 | 0.129 | 1.177 | 0.239 |

### Adding a WP*age interaction in the Directed Path version of MLM in Mplus:

```
DEFINE: agemon3 = age18*mon3;  ! Make observed variable interaction term to use as predictor;

%WITHIN% ! WP*age effect
WPage | risky ON agemon3*;

%BETWEEN% ! Fixed WP*age effect only
[WPage*]; WPage@0;
```

| Means | EST | SE | Z | p-value |
|---|---|---|---|---|
| WPAGE | 0.171 | 0.131 | 1.303 | 0.193 |

### What about adding a random WP effect using Slopes-as-Outcomes in PROC MIXED:

```
TITLE1 "Monitoring Us predicting risky behavio with random WP effect";
PROC MIXED DATA=RiskyStacked NOITPRINT NOCLPRINT COVTEST IC NOINFO NAMELEN=100 METHOD=ML;
      CLASS FamilyID occasion;
      MODEL risky = agec18 agec18*agec18 att4 att4*agec18 Uint Uint*agec18 Uage Uage*agec18
                    Eres / SOLUTION DDFM=Satterthwaite NOTEST OUTPM=PredUmon;
      RANDOM INTERCEPT agec18 Eres / TYPE=UN SUBJECT=FamilyID;
      ODS OUTPUT InfoCrit=RandomU; RUN; TITLE1; %FitTest(FitFewer=FixedU, FitMore=RandomU);
```

Likelihood Ratio Test for FixedU vs. RandomU

| Name | Neg2Log Like | Parms | AIC | BIC | DevDiff | DFdiff | Pvalue |
|---|---|---|---|---|---|---|---|
| FixedU | 7430.8 | 14 | 7458.8 | 7505.0 | . | . | . |
| RandomU | 7420.5 | 17 | 7454.5 | 7510.6 | 10.3083 | 3 | 0.016120 |

### Adding a random WP effect in the Directed Path version of MLM in Mplus (not available in SEM):

```
%BETWEEN%
  WPres*; WPres WITH risky* mon3* Rslp* Mslp*; ! Random WP effect variance and covariances added
```

Likelihood Ratio Test for Random Effect in Multivariate Model in Mplus:

| Name | LL | Parms | AIC | BIC | −2ΔLL | DFdiff | Pvalue |
|---|---|---|---|---|---|---|---|
| FixedU | -4392.3 | 20 | 8824.5 | 8929.4 | . | . | . |
| RandomU | -4389.2 | 25 | 8828.4 | 8959.5 | 4.67 | 5 | 0.46 |

**Uh-oh… it looks like the slopes-as-outcomes model may paint too rosy of a picture by treating estimated quantities (random effects and residuals) as perfectly reliable observed variables… …so try to use a multivariate model whenever you can, and proceed with caution otherwise...**