Heterogeneous Variance Models from Hoffman Chapter 8:
Predicting Differential Variation in Fluctuation of Daily Physical Symptoms from Sex, Age, and Daily Negative Mood (PMC) and Stressor Days (GMC)

Same baseline homogeneous variance model via SAS PROC MIXED or via SAS PROC NLMIXED:

```
TITLE1 "Baseline Homogeneous Variance Model with just Fixed Effects";
PROC MIXED DATA=example COVTEST NOCLPRINT NOITPRINT NOINFO NAMELEN=50 METHOD=ML;
CLASS ID;
MODEL symptom5 = female age80 female*age80 PMmood2 PMstress40 female*PMstress40 PMmood2*PMmood2
                 WPmood PMmood2*WPmood TVstress0 female*TVstress0 / SOLUTION DDFM=SATTERTHWAITE;
RANDOM INTERCEPT / TYPE=UN SUBJECT=ID;
REPEATED / TYPE=VC SUBJECT=ID; RUN; TITLE1;


TITLE1 "Baseline Homogeneous Variance Model with just Fixed Effects";
PROC NLMIXED DATA=example METHOD=GAUSS TECH=NEWRAP GCONV=1e-12;
* Must define all parameters to be estimated and provide start values;
* First line is fixed effects, second line is variances;
PARMS  fint=2.45 fsex=-.33 fage=.07 fsexage=-.09
       fbpmood=-.34 fwpmood=.26 fcstress=1.93 fwpstress=.22
       f2sexstress=-1.33 f1sexstress=-.18 fbpmood2=-1.78 fbpwpmood=.26
       vU0int=-.47 varEint=-.50;
* Setting up level-2 equations;
       b0i = fint + fsex*female + fage*age80 + fsexage*female*age80
             + fbpmood*PMmood2 + fcstress*PMstress40
             + f2sexstress*PMstress40*female + fbpmood2*PMmood2*PMmood2 + U0i;
       b1i = fwpmood + fbpwpmood*PMmood2;
       b2i = fwpstress + f1sexstress*female;
* Setting up level-1 equation WITHOUT level-1 residual;
       y = (b0i) + (b1i*WPmood) + (b2i*TVstress0);
* Model for heterogeneous variance at level-2 and level-1;
       vU0 = EXP(vU0int);
       varE = EXP(varEint);
* Telling it which DV, defining level-1 residual;
* symptom5 is normally distributed with a mean of "y" and a variance of "VarE";
       MODEL symptom5 ~ normal(y,varE);
* Defining random effects: normally distributed with means and variances;
       RANDOM U0i ~ normal([0],[vU0]) SUBJECT=ID;
* Requesting additional model-implied estimates;
       ESTIMATE "Actual Random Intercept Variance" EXP(vU0int);
       ESTIMATE "Actual Residual Variance"         EXP(varEint);
RUN; TITLE1;
```

Level 1 :

$$\text{Symptoms}_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Mood}_{ti} - \overline{\text{Mood}}_i\right) + \beta_{2i}\left(\text{Stressor}_{ti}\right) + e_{ti}$$

Residual Variance: $\sigma^2_{e_{ti}} = \exp\left[\eta_{0i}\right]$

Level 2 :

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Women}_i\right) + \gamma_{02}\left(\text{Age}_i - 80\right) + \gamma_{03}\left(\text{Women}_i\right)\left(\text{Age}_i - 80\right) + \gamma_{04}\left(\overline{\text{Mood}}_i - 2\right)$

$$+ \gamma_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{09}\left(\text{Women}_i\right)\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{0,16}\left(\overline{\text{Mood}}_i - 2\right)^2 + U_{0i}$$

Within-Person Mood: $\beta_{1i} = \gamma_{10} + \gamma_{14}\left(\overline{\text{Mood}}_i - 2\right)$

Within-Person Stressor: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Women}_i\right)$

Random Intercept Variance $\tau^2_{U_{0i}} = \exp\left[\upsilon_{00}\right]$

Residual Variance: $\eta_{0i} = \varepsilon_{00}$

Example 2 Within-Peron Fluctuation 1 of 9

PROC MIXED OUTPUT:

```
                Covariance Parameter Estimates
                            Standard      Z
Cov Parm    Subject   Estimate    Error    Value    Pr > Z
UN(1,1)     id         0.6230    0.1044     5.96    <.0001   Random Intercept Variance
Residual    id         0.6093    0.04292   14.19    <.0001   Residual Variance


                    Information Criteria
Neg2LogLike   Parms     AIC      AICC     HQIC      BIC      CAIC
   1379.2      14     1407.2    1408.1   1422.3   1444.4    1458.4


                Solution for Fixed Effects
                        Standard
Effect              Estimate     Error    DF   t Value   Pr > |t|
Intercept            2.4473     0.3474   117     7.05    <.0001
female              -0.3277     0.2140   146    -1.53     0.1278
age80                0.07311    0.03019  109     2.42     0.0171
female*age80        -0.09127    0.03381  108    -2.70     0.0081
PMmood2             -0.3396     0.7999   110    -0.42     0.6720
PMstress40           1.9322     0.6005   124     3.22     0.0016
female*PMstress40   -1.3281     0.6617   128    -2.01     0.0468
PMmood2*PMmood2     -1.7784     0.7131   108    -2.49     0.0141
WPmood               0.2634     0.2213   405     1.19     0.2346
PMmood2*WPmood       0.2591     0.3962   405     0.65     0.5135
TVstress0            0.2192     0.1710   404     1.28     0.2007
female*TVstress0    -0.1812     0.2053   404    -0.88     0.3780
```

PROC NLMIXED OUTPUT:

```
            Dimensions
Observations Used           509
Observations Not Used         0
Total Observations          509
Subjects                    105
Max Obs Per Subject           5
Parameters                   14
Quadrature Points             1
```

```
                    Specifications
Data Set                            WORK.EXAMPLE
Dependent Variable                  symptom5
Distribution for Dependent Variable Normal
Random Effects                      UOi
Distribution for Random Effects     Normal
Subject Variable                    id
Optimization Technique              Newton-Raphson
Integration Method                  Adaptive Gaussian Quadrature
```

```
        NOTE: ABSGCONV convergence criterion satisfied.


            Fit Statistics
-2 Log Likelihood           1379.2
AIC (smaller is better)     1407.2
AICC (smaller is better)    1408.1
BIC (smaller is better)     1444.4
```

Make sure the gradient (partial first derivative with respect to each parameter) is as close to 0 as possible. Otherwise, the estimation really hasn't finished on the "best" estimates.

```
                            Parameter Estimates
                    Standard
Parameter  Estimate   Error     DF   t Value  Pr > |t|  Alpha    Lower     Upper     Gradient
fint        2.4473    0.3474    104    7.05    <.0001    0.05    1.7585    3.1362    -6.09E-8
fsex       -0.3277    0.2140    104   -1.53     0.1286   0.05   -0.7520    0.09659    3.344E-8
fage        0.07311   0.03019   104    2.42     0.0172   0.05    0.01324   0.1330     5.738E-6
fsexage    -0.09127   0.03381   104   -2.70     0.0081   0.05   -0.1583   -0.02423    3.875E-6
fbpmood    -0.3396    0.7999    104   -0.42     0.6720   0.05   -1.9257    1.2466     1.156E-7
fwpmood     0.2634    0.2213    104    1.19     0.2366   0.05   -0.1754    0.7022    -1.09E-8
fcstress    1.9322    0.6005    104    3.22     0.0017   0.05    0.7415    3.1230     1.034E-8
fwpstress   0.2192    0.1710    104    1.28     0.2028   0.05   -0.1199    0.5584    -6.11E-9
f2sexstress -1.3281   0.6617    104   -2.01     0.0473   0.05   -2.6402   -0.01600    1.161E-8
f1sexstress -0.1812   0.2053    104   -0.88     0.3796   0.05   -0.5882    0.2259     3.322E-8
fbpmood2   -1.7784    0.7131    104   -2.49     0.0142   0.05   -3.1926   -0.3643    -9.03E-8
fbpwpmood   0.2591    0.3962    104    0.65     0.5145   0.05   -0.5266    1.0449     7.878E-9
vUOint     -0.4733    0.1677    104   -2.82     0.0057   0.05   -0.8057   -0.1408    -5.58E-7
varEint    -0.4955    0.07045   104   -7.03    <.0001    0.05   -0.6352   -0.3558    -7.07E-7


                            Additional Estimates
                        Standard
Label                           Estimate   Error    DF   t Value  Pr > |t|  Alpha   Lower    Upper
Actual Random Intercept Variance  0.6230   0.1044   104    5.96    <.0001    0.05   0.4159   0.8301
Actual Residual Variance          0.6093   0.04292  104   14.19    <.0001    0.05   0.5241   0.6944
```

Example 2 Within-Peron Fluctuation 2 of 9

Testing a ω_{0i} scale factor for random individual differences in within-person variation:

Level 1:

$$\text{Symptoms}_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Mood}_{ti} - \overline{\text{Mood}}_i\right) + \beta_{2i}\left(\text{Stressor}_{ti}\right) + e_{ti}$$

Residual Variance: $\sigma^2_{e_{ti}} = \exp\left[\eta_{0i}\right]$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Women}_i\right) + \gamma_{02}\left(\text{Age}_i - 80\right) + \gamma_{03}\left(\text{Women}_i\right)\left(\text{Age}_i - 80\right) + \gamma_{04}\left(\overline{\text{Mood}}_i - 2\right)$

$$+ \gamma_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{09}\left(\text{Women}_i\right)\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{0,16}\left(\overline{\text{Mood}}_i - 2\right)^2 + U_{0i}$$

Within-Person Mood: $\beta_{1i} = \gamma_{10} + \gamma_{14}\left(\overline{\text{Mood}}_i - 2\right)$

Within-Person Stressor: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Women}_i\right)$

Random Intercept Variance $\tau^2_{U_{0i}} = \exp\left[\upsilon_{00}\right]$

Residual Variance: $\eta_{0i} = \varepsilon_{00} + \omega_{0i}$

```
TITLE1 "Baseline Model with just Fixed Effects";
TITLE2 "Test Scale Factor";
PROC NLMIXED DATA=example METHOD=GAUSS TECH=NEWRAP GCONV=1e-12;  *METHOD=FIRO;
* Must define all parameters to be estimated and provide start values;
* First line is fixed effects, second line is variances;
PARMS  fint=2.45 fsex=-.33 fage=.07 fsexage=-.09
       fbpmood=-.34 fwpmood=.26 fcstress=1.93 fwpstress=.22
       f2sexstress=-1.33 f1sexstress=-.18 fbpmood2=-1.78 fbpwpmood=.26
       varEint=-.50 vU0int=-.47 vscale=-.22 cU0scale=.00;
* Setting up level-2 equations;
       b0i = fint + fsex*female + fage*age80 + fsexage*female*age80
             + fbpmood*PMmood2 + fcstress*PMstress40
             + f2sexstress*PMstress40*female + fbpmood2*PMmood2*PMmood2 + U0i;
       b1i = fwpmood + fbpwpmood*PMmood2;
       b2i = fwpstress + f1sexstress*female;
* Setting up level-1 equation WITHOUT level-1 residual;
       y = (b0i) + (b1i*WPmood) + (b2i*TVstress0);
* Model for heterogeneous variance at level-2 and level-1;
       vU0 = EXP(vU0int);
       varE = EXP(varEint + Scalei);
* Telling it which DV, defining level-1 residual;
* symptom5 is normally distributed with a mean of "y" and a variance of "VarE";
       MODEL symptom5 ~ normal(y,varE);
* Defining random effects: normally distributed with means and variances;
       RANDOM U0i Scalei ~ normal([0,0],[vU0,cU0scale,vscale]) SUBJECT=ID;
RUN; TITLE1; TITLE2;
```

PROC NLMIXED OUTPUT:

```
                    Specifications
Data Set                         WORK.EXAMPLE
Dependent Variable               symptom5
Distribution for Dependent Variable   Normal
Random Effects                   UOi Scalei
Distribution for Random Effects  Normal
Subject Variable                 id
Optimization Technique           Newton-Raphson
Integration Method               Adaptive Gaussian Quadrature
```

That's all we get – SAS could not find an estimate for the scale factor variance, even after switching to less intensive estimation routines (e.g., METHOD=FIRO), so we must proceed assuming we don't have one.

Example 2 Within-Peron Fluctuation 3 of 9

Examining predictors of differential level-2 random intercept variance across persons:

Level 1 :

$$\text{Symptoms}_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Mood}_{ti} - \overline{\text{Mood}}_i\right) + \beta_{2i}\left(\text{Stressor}_{ti}\right) + e_{ti}$$

Residual Variance: $\sigma^2_{e_{ti}} = \exp\left[\eta_{0i}\right]$

Level 2 :

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Women}_i\right) + \gamma_{02}\left(\text{Age}_i - 80\right) + \gamma_{03}\left(\text{Women}_i\right)\left(\text{Age}_i - 80\right) + \gamma_{04}\left(\overline{\text{Mood}}_i - 2\right)$

$$+ \gamma_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{09}\left(\text{Women}_i\right)\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{0,16}\left(\overline{\text{Mood}}_i - 2\right)^2 + U_{0i}$$

Within-Person Mood: $\beta_{1i} = \gamma_{10} + \gamma_{14}\left(\overline{\text{Mood}}_i - 2\right)$

Within-Person Stressor: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Women}_i\right)$

Random Intercept Variance $\tau^2_{U_{0i}} = \exp\begin{bmatrix} \upsilon_{00} + \upsilon_{01}(\text{Women}_i) + \upsilon_{02}(\text{Age}_i - 80) \\ + \upsilon_{01}(\overline{\text{Mood}}_i - 2) + \upsilon_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) \end{bmatrix}$

Residual Variance: $\eta_{0i} = \varepsilon_{00}$

```
TITLE1 "Add Level 2 Predictors for Sex, Age, Mood, and Stress";
PROC NLMIXED DATA=example METHOD=GAUSS TECH=NEWRAP GCONV=1e-12;
* Must define all parameters to be estimated and provide start values;
* First line is fixed effects, second line is variances;
PARMS  fint=2.45 fsex=-.33 fage=.07 fsexage=-.09
       fbpmood=-.34 fwpmood=.26 fcstress=1.93 fwpstress=.22
       f2sexstress=-1.33 f1sexstress=-.18 fbpmood2=-1.78 fbpwpmood=.26
       vU0int=-.47 vU0sex=0 vU0age=0 vU0bpmood=0 vU0bpstress=0 varEint=-.50;
* Setting up level-2 equations;
       b0i = fint + fsex*female + fage*age80 + fsexage*female*age80
             + fbpmood*PMmood2 + fcstress*PMstress40
             + f2sexstress*PMstress40*female + fbpmood2*PMmood2*PMmood2 + U0i;
       b1i = fwpmood + fbpwpmood*PMmood2;
       b2i = fwpstress + f1sexstress*female;
* Setting up level-1 equation WITHOUT level-1 residual;
       y = (b0i) + (b1i*WPmood) + (b2i*TVstress0);
* Model for heterogeneous variance at level-2 and level-1;
   vU0 = EXP(vU0int + vU0sex*female + vU0age*age80 + vU0bpmood*PMmood2 + vU0bpstress*PMstress40);
   varE = EXP(varEint);
* Telling it which DV, defining level-1 residual;
* symptom5 is normally distributed with a mean of "y" and a variance of "VarE";
       MODEL symptom5 ~ normal(y,varE);
* Defining random effects: normally distributed with means and variances;
       RANDOM U0i ~ normal([0],[vU0]) SUBJECT=ID;
RUN; TITLE1;
```

Dimensions		Specifications	
Observations Used	509	Data Set	WORK.EXAMPLE
Observations Not Used	0	Dependent Variable	symptom5
Total Observations	509	Distribution for Dependent Variable	Normal
Subjects	105	Random Effects	U0i
Max Obs Per Subject	5	Distribution for Random Effects	Normal
Parameters	**18**	Subject Variable	id
Quadrature Points	1	Optimization Technique	Newton-Raphson
		Integration Method	Adaptive Gaussian Quadrature

NOTE: GCONV convergence criterion satisfied.

Fit Statistics	
-2 Log Likelihood	**1371.0**
AIC (smaller is better)	1407.0
AICC (smaller is better)	1408.4
BIC (smaller is better)	1454.8

The previous model was $-2\text{LL}(14) = 1379.2$. The current model is $-2\text{LL}(18) = 1371.0$, so $-2\Delta\text{LL}(4) = 8.2$, $p = .085$, although only one new effect is significant individually…

Example 2 Within-Peron Fluctuation 4 of 9

```
                                  Parameter Estimates
                         Standard
Parameter      Estimate     Error    DF    t Value   Pr > |t|   Alpha    Lower     Upper    Gradient
fint            2.4208     0.3426    104     7.07     <.0001     0.05    1.7415    3.1001   -474E-12
fsex           -0.3364     0.2099    104    -1.60     0.1121     0.05   -0.7527    0.07992  -505E-12
fage            0.07753    0.03087   104     2.51     0.0136     0.05    0.01631   0.1387    4.143E-9
fsexage        -0.08882    0.03427   104    -2.59     0.0109     0.05   -0.1568   -0.02086   3.719E-9
fbpmood        -0.2178     0.7821    104    -0.28     0.7812     0.05   -1.7688    1.3332    4.97E-10
fwpmood         0.2616     0.2212    104     1.18     0.2396     0.05   -0.1770    0.7002    2.09E-12
fcstress        1.8901     0.6084    104     3.11     0.0024     0.05    0.6837    3.0965    1.18E-10
fwpstress       0.2233     0.1710    104     1.31     0.1945     0.05   -0.1157    0.5623   -112E-12
f2sexstress    -1.2330     0.6696    104    -1.84     0.0684     0.05   -2.5608    0.09487   1.08E-10
f1sexstress    -0.1850     0.2052    104    -0.90     0.3692     0.05   -0.5919    0.2218   -107E-12
fbpmood2       -1.5905     0.6760    104    -2.35     0.0205     0.05   -2.9310   -0.2501   -372E-12
fbpwpmood       0.2564     0.3960    104     0.65     0.5188     0.05   -0.5289    1.0416    1.77E-13
vU0int         -1.1082     0.7895    104    -1.40     0.1634     0.05   -2.6739    0.4575   -215E-12

vU0sex         -0.1760     0.3852    104    -0.46     0.6487     0.05   -0.9398    0.5878   -211E-12
vU0age          0.03867    0.03293   104     1.17     0.2430     0.05   -0.02663   0.1040    6.8E-10
vU0bpmood      -0.7150     0.8469    104    -0.84     0.4004     0.05   -2.3944    0.9644    1.71E-10
vU0bpstress     1.5833     0.6031    104     2.63     0.0100     0.05    0.3874    2.7792    2.87E-11

varEint        -0.4968     0.07034   104    -7.06     <.0001     0.05   -0.6363   -0.3573   -133E-12
```

The effect of PMstressor40$_i$ on the amount of random intercept variance indicates that for every unit higher proportion of stressor days, the log of the random intercept variance is predicted to be higher by 1.58. In other words, there is significantly greater person-to-person variation in mean daily reported physical symptoms among people who report experiencing a greater proportion of stressor days.

Examining predictors of differential level-1 residual variance across persons (retaining all effects of heterogeneity on the level-2 random intercept variance for now):

Level 1 :

$$\text{Symptoms}_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Mood}_{ti} - \overline{\text{Mood}}_i\right) + \beta_{2i}\left(\text{Stressor}_{ti}\right) + e_{ti}$$

Residual Variance: $\sigma^2_{e_{ti}} = \exp\left[\eta_{0i} + \underline{\eta_{1i}\left(\text{Mood}_{ti} - \overline{\text{Mood}}_i\right)} + \underline{\eta_{2i}\left(\text{Stressor}_{ti}\right)}\right]$

Level 2 :

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Women}_i\right) + \gamma_{02}\left(\text{Age}_i - 80\right) + \gamma_{03}\left(\text{Women}_i\right)\left(\text{Age}_i - 80\right) + \gamma_{04}\left(\overline{\text{Mood}}_i - 2\right)$

$\qquad + \gamma_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{09}\left(\text{Women}_i\right)\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{0,16}\left(\overline{\text{Mood}}_i - 2\right)^2 + U_{0i}$

Within-Person Mood: $\beta_{1i} = \gamma_{10} + \gamma_{14}\left(\overline{\text{Mood}}_i - 2\right)$

Within-Person Stressor: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Women}_i\right)$

Random Intercept Variance $\tau^2_{U_{0i}} = \exp\left[\begin{array}{l} \upsilon_{00} + \upsilon_{01}\left(\text{Women}_i\right) + \upsilon_{02}\left(\text{Age}_i - 80\right) \\ + \upsilon_{04}\left(\overline{\text{Mood}}_i - 2\right) + \upsilon_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) \end{array}\right]$

Residual Variance:

$\quad \eta_{0i} = \varepsilon_{00} + \underline{\varepsilon_{01}\left(\text{Women}_i\right)} + \underline{\varepsilon_{02}\left(\text{Age}_i - 80\right)} + \underline{\varepsilon_{04}\left(\overline{\text{Mood}}_i - 2\right)} + \underline{\varepsilon_{08}\left(\overline{\text{Stressor}}_i - 0.40\right)}$

$\quad \eta_{1i} = \varepsilon_{10}$

$\quad \eta_{2i} = \varepsilon_{20}$

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```
TITLE1 "Level-1 Heterogeneity by L2 for Sex, Age, Mood, and Stress, L1 Mood and Stress";
PROC NLMIXED DATA=example METHOD=GAUSS TECH=NEWRAP GCONV=1e-12;
* Must define all parameters to be estimated and provide start values;
* First line is fixed effects, second line is variances;
PARMS  fint=2.45 fsex=-.33 fage=.07 fsexage=-.09
       fbpmood=-.34 fwpmood=.26 fcstress=1.93 fwpstress=.22
       f2sexstress=-1.33 f1sexstress=-.18 fbpmood2=-1.78 fbpwpmood=.26
       vU0int=-.47 vU0sex=0 vU0age=0 vU0bpmood=0 vU0bpstress=0
       varEint=-.50 varEsex=0 varEage=0 varEbpmood=0 varEbpstress=0 varEwpmood=0 varEwpstress=0;
* Setting up level-2 equations;
       b0i = fint + fsex*female + fage*age80 + fsexage*female*age80
             + fbpmood*PMmood2 + fcstress*PMstress40
             + f2sexstress*PMstress40*female + fbpmood2*PMmood2*PMmood2 + U0i;
       b1i = fwpmood + fbpwpmood*PMmood2;
       b2i = fwpstress + f1sexstress*female;
* Setting up level-1 equation WITHOUT level-1 residual;
       y = (b0i) + (b1i*WPmood) + (b2i*TVstress0);
* Model for heterogeneous variance at level-2 and level-1;
   vU0 = EXP(vU0int + vU0sex*female + vU0age*age80 + vU0bpmood*PMmood2 + vU0bpstress*PMstress40);
   varE = EXP(varEint + varEsex*female + varEage*age80 + varEbpmood*PMmood2
          + varEbpstress*PMstress40 + varEwpmood*WPmood + varEwpstress*TVstress0);
* Telling it which DV, defining level-1 residual;
* symptom5 is normally distributed with a mean of "y" and a variance of "VarE";
       MODEL symptom5 ~ normal(y,varE);
* Defining random effects: normally distributed with means and variances;
       RANDOM U0i ~ normal([0],[vU0]) SUBJECT=ID;
RUN; TITLE1;
```

```
             Dimensions
Observations Used               509
Observations Not Used             0
Total Observations              509
Subjects                        105
Max Obs Per Subject               5
Parameters                       24
Quadrature Points                 1
         NOTE: ABSGCONV convergence criterion satisfied.
           Fit Statistics
-2 Log Likelihood             1346.2
AIC (smaller is better)       1394.2
AICC (smaller is better)      1396.7
BIC (smaller is better)       1457.9
```

The previous model was $-2LL(18) = 1371.0$. The current model is $-2LL(24) = 1346.2$, so $-2\Delta LL(6) = 24.8$, $p < .001$, although only two new effects are significant individually…

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
fint	2.4319	0.3651	104	6.66	<.0001	0.05	1.7079	3.1560	-1.49E-7
fsex	-0.3358	0.2079	104	-1.62	0.1093	0.05	-0.7481	0.07647	-1.43E-7
fage	0.07538	0.03087	104	2.44	0.0163	0.05	0.01417	0.1366	8.042E-7
fsexage	-0.08680	0.03438	104	-2.52	0.0131	0.05	-0.1550	-0.01862	7.225E-7
fbpmood	-0.1622	0.8895	104	-0.18	0.8556	0.05	-1.9262	1.6017	1.228E-7
fwpmood	0.1526	0.3170	104	0.48	0.6311	0.05	-0.4759	0.7812	2.418E-8
fcstress	1.9231	0.5954	104	3.23	0.0017	0.05	0.7425	3.1037	2.812E-8
fwpstress	0.2081	0.1610	104	1.29	0.1989	0.05	-0.1111	0.5273	-2.94E-8
f2sexstress	-1.2523	0.6588	104	-1.90	0.0601	0.05	-2.5586	0.05405	2.74E-8
f1sexstress	-0.1748	0.1950	104	-0.90	0.3721	0.05	-0.5616	0.2120	-2.72E-8
fbpmood2	-1.5408	0.7463	104	-2.06	0.0415	0.05	-3.0208	-0.06075	-1.08E-7
fbpwpmood	0.08266	0.5216	104	0.16	0.8744	0.05	-0.9517	1.1170	-2E-9
vU0int	-1.0869	0.8058	104	-1.35	0.1803	0.05	-2.6848	0.5110	-3.49E-8
vU0sex	-0.1516	0.3839	104	-0.39	0.6938	0.05	-0.9129	0.6098	-3.36E-8
vU0age	0.03924	0.03203	104	1.22	0.2234	0.05	-0.02428	0.1028	2.059E-7
vU0bpmood	-0.7041	0.8717	104	-0.81	0.4211	0.05	-2.4327	1.0245	2.874E-8
vU0bpstress	1.4262	0.5871	104	2.43	0.0169	0.05	0.2619	2.5904	6.579E-9
varEint	-0.08445	0.3079	104	-0.27	0.7844	0.05	-0.6949	0.5260	-6.25E-8
varEsex	0.07127	0.1663	104	0.43	0.6691	0.05	-0.2585	0.4010	-4.75E-8
varEage	-0.00337	0.01311	104	-0.26	0.7979	0.05	-0.02937	0.02264	-4.53E-8
varEbpmood	0.6972	0.3368	104	2.07	0.0409	0.05	0.02922	1.3651	2.565E-8
varEbpstress	0.6967	0.3152	104	2.21	0.0293	0.05	0.07160	1.3219	-4.66E-9
varEwpmood	0.02650	0.3095	104	0.09	0.9319	0.05	-0.5873	0.6403	6.281E-9
varEwpstress	-0.00774	0.2177	104	-0.04	0.9717	0.05	-0.4395	0.4241	-2.94E-8

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The effect of PMmood2$_i$ on the amount of residual variance indicates that for every unit higher person mean negative mood, the log of the residual variance is predicted to be higher by 0.6972. Likewise, the effect of PMstressor40$_i$ on the amount of residual variance indicates that for every unit higher proportion of stressor days, the log of the residual variance is predicted to be higher by 0.6967. In other words, there is significantly greater within-person daily fluctuation in reported physical symptoms among people who report greater average levels of negative and who report experiencing a greater proportion of stressor days.

Best model (after trimming any effects that were no longer significant, as shown in Table 8.7):

Level 1:

$$\text{Symptoms}_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Mood}_{ti} - \overline{\text{Mood}}_i\right) + \beta_{2i}\left(\text{Stressor}_{ti}\right) + e_{ti}$$

Residual Variance: $\sigma^2_{e_{ti}} = \exp\left[\eta_{0i}\right]$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Women}_i\right) + \gamma_{02}\left(\text{Age}_i - 80\right) + \gamma_{03}\left(\text{Women}_i\right)\left(\text{Age}_i - 80\right) + \gamma_{04}\left(\overline{\text{Mood}}_i - 2\right)$

$$+ \gamma_{08}\left(\overline{\text{Stressor}}_i - 0.40\right) + \gamma_{09}\left(\text{Women}_i\right)\left(\overline{\text{Stressor}}_i - 0.40\right) + U_{0i}$$

Within-Person Mood: $\beta_{1i} = \gamma_{10}$

Within-Person Stressor: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Women}_i\right)$

Random Intercept Variance $\tau^2_{U_{0i}} = \exp\left[\upsilon_{00} + \upsilon_{08}\left(\overline{\text{Stressor}}_i - 0.40\right)\right]$

Residual Variance:

$$\eta_{0i} = \varepsilon_{00} + \varepsilon_{04}\left(\overline{\text{Mood}}_i - 2\right) + \varepsilon_{08}\left(\overline{\text{Stressor}}_i - 0.40\right)$$

```
TITLE1 "Final Heterogeneous Variance Model without Quad PM Mood and BP*WP Mood";
PROC NLMIXED DATA=example METHOD=GAUSS TECH=NEWRAP GCONV=1e-12;
* Must define all parameters to be estimated and provide start values;
* First line is fixed effects, second line is variances;
PARMS  fint=2.45 fsex=-.33 fage=.07 fsexage=-.09
       fbpmood=1.66 fwpmood=.26
       fcstress=1.93 fwpstress=.22 f2sexstress=-1.33 f1sexstress=-.18
       vU0int=-.47 vU0bpstress=0
       varEint=-.50 varEbpmood=0 varEbpstress=0;

* Setting up level-2 equations;
       b0ifixed = fint + fsex*female + fage*age80 + fsexage*female*age80
                   + fbpmood*PMmood2 + fcstress*PMstress40
                   + f2sexstress*PMstress40*female;
       b1i = fwpmood;
       b2i = fwpstress + f1sexstress*female;

* Setting up level-1 equation WITHOUT level-1 residual;
       yfixed = (b0ifixed) + (b1i*WPmood) + (b2i*TVstress0);
       y = yfixed + U0i;

* Model for heterogeneous variance at level-2 and level-1;
       vU0 = EXP(vU0int  + vU0bpstress*PMstress40);
       varE = EXP(varEint + varEbpmood*PMmood2 + varEbpstress*PMstress40);

* Telling it which DV, defining level-1 residual;
* symptom5 is normally distributed with a mean of "y" and a variance of "VarE";
       MODEL symptom5 ~ normal(y,varE);
* Defining random effects: normally distributed with means and variances;
       RANDOM U0i ~ normal([0],[vU0]) SUBJECT=ID;
```

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```
* Asking for fixed effect predicted values;
      PREDICT yfixed OUT=Predicted;
* Getting all model implied estimates;
      ESTIMATE "Intercept: Men"                fint;
      ESTIMATE "Intercept: Women"              fint + fsex;
      ESTIMATE "Intercept: Sex Diff"           fsex;
      ESTIMATE "Age Slope: Men"                fage;
      ESTIMATE "Age Slope: Women"              fage + fsexage;
      ESTIMATE "Age Slope: Sex Diff"           fsexage;
      ESTIMATE "BP Mood: Linear"               fbpmood;
      ESTIMATE "Contextual Mood: Linear"       fbpmood - fwpmood;
      ESTIMATE "WP Mood"                       fwpmood;
      ESTIMATE "BP Stress: Men"                fcstress + fwpstress;
      ESTIMATE "BP Stress: Women"              fcstress + f2sexstress + fwpstress + f1sexstress;
      ESTIMATE "BP Stress: Sex Diff"           f2sexstress + f1sexstress;
      ESTIMATE "Contextual Stress: Men"        fcstress;
      ESTIMATE "Contextual Stress: Women"      fcstress + f2sexstress;
      ESTIMATE "Contextual Stress: Sex Diff"   f2sexstress;
      ESTIMATE "WP Stress: Men"                fwpstress;
      ESTIMATE "WP Stress: Women"              fwpstress + f1sexstress;
      ESTIMATE "WP Stress: Sex Diff"           f1sexstress;
      ESTIMATE "Contextual Stress per Day: Men"    fcstress/5;
      ESTIMATE "Contextual Stress per Day: Women"(fcstress + f2sexstress)/5;
RUN; PROC CORR DATA=Predicted; VAR symptom5 pred; RUN; TITLE1;
```

Specifications

Data Set	WORK.EXAMPLE
Dependent Variable	symptom5
Distribution for Dependent Variable	Normal
Random Effects	UOi
Distribution for Random Effects	Normal
Subject Variable	id
Optimization Technique	Newton-Raphson
Integration Method	Adaptive Gaussian Quadrature

Dimensions

Observations Used	509
Observations Not Used	0
Total Observations	509
Subjects	105
Max Obs Per Subject	5
Parameters	**15**
Quadrature Points	1

NOTE: ABSGCONV convergence criterion satisfied.

Fit Statistics

-2 Log Likelihood	**1351.8**
AIC (smaller is better)	1381.8
AICC (smaller is better)	1382.7
BIC (smaller is better)	1421.6

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
fint	2.8017	0.3680	104	7.61	<.0001	0.05	2.0719	3.5314	4.992E-6
fsex	-0.3413	0.2059	104	-1.66	0.1004	0.05	-0.7495	0.06699	3.998E-6
fage	0.06876	0.03007	104	2.29	0.0243	0.05	0.009126	0.1284	-1.21E-6
fsexage	-0.08837	0.03327	104	-2.66	0.0092	0.05	-0.1543	-0.02239	4.658E-7
fbpmood	1.6584	0.3974	104	4.17	<.0001	0.05	0.8703	2.4464	-4.42E-6
fwpmood	0.1024	0.1473	104	0.69	0.4887	0.05	-0.1898	0.3945	-2.69E-9
fcstress	2.0496	0.6011	104	3.41	0.0009	0.05	0.8576	3.2415	-1.48E-6
fwpstress	0.2043	0.1647	104	1.24	0.2176	0.05	-0.1223	0.5309	5.092E-7
f2sexstress	-1.3699	0.6707	104	-2.04	0.0436	0.05	-2.6998	-0.03991	-1.24E-6
f1sexstress	-0.1754	0.1980	104	-0.89	0.3776	0.05	-0.5680	0.2171	3.609E-7
vUOint	-0.5724	0.1712	104	-3.34	0.0011	0.05	-0.9119	-0.2330	-1.16E-6
vUObpstress	1.3881	0.5349	104	2.60	0.0108	0.05	0.3275	2.4488	2.933E-7
varEint	-0.01100	0.2766	104	-0.04	0.9684	0.05	-0.5595	0.5375	-9.49E-7
varEbpmood	0.7228	0.3287	104	2.20	0.0301	0.05	0.07095	1.3746	8.15E-7
varEbpstress	0.6775	0.2250	104	3.01	0.0033	0.05	0.2312	1.1238	2.5E-7

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Additional Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper
Intercept: Men	2.8017	0.3680	104	7.61	<.0001	0.05	2.0719	3.5314
Intercept: Women	2.4604	0.3416	104	7.20	<.0001	0.05	1.7829	3.1379
Intercept: Sex Diff	-0.3413	0.2059	104	-1.66	0.1004	0.05	-0.7495	0.06699
Age Slope: Men	0.06876	0.03007	104	2.29	0.0243	0.05	0.009126	0.1284
Age Slope: Women	-0.01961	0.01506	104	-1.30	0.1955	0.05	-0.04947	0.01024
Age Slope: Sex Diff	-0.08837	0.03327	104	-2.66	0.0092	0.05	-0.1543	-0.02239
BP Mood: Linear	1.6584	0.3974	104	4.17	<.0001	0.05	0.8703	2.4464
Contextual Mood: Linear	1.5560	0.4239	104	3.67	0.0004	0.05	0.7153	2.3967
WP Mood	0.1024	0.1473	104	0.69	0.4887	0.05	-0.1898	0.3945
BP Stress: Men	2.2539	0.5774	104	3.90	0.0002	0.05	1.1089	3.3988
BP Stress: Women	0.7086	0.3137	104	2.26	0.0260	0.05	0.08656	1.3306
BP Stress: Sex Diff	-1.5453	0.6402	104	-2.41	0.0175	0.05	-2.8149	-0.2757
Contextual Stress: Men	2.0496	0.6011	104	3.41	0.0009	0.05	0.8576	3.2415
Contextual Stress: Women	0.6797	0.3329	104	2.04	0.0437	0.05	0.01949	1.3399
Contextual Stress: Sex Diff	-1.3699	0.6707	104	-2.04	0.0436	0.05	-2.6998	-0.03991
WP Stress: Men	0.2043	0.1647	104	1.24	0.2176	0.05	-0.1223	0.5309
WP Stress: Women	0.02886	0.1122	104	0.26	0.7975	0.05	-0.1936	0.2513
WP Stress: Sex Diff	-0.1754	0.1980	104	-0.89	0.3776	0.05	-0.5680	0.2171
Contextual Stress per Day: Men	0.4099	0.1202	104	3.41	0.0009	0.05	0.1715	0.6483
Contextual Stress per Day: Women	0.1359	0.06659	104	2.04	0.0437	0.05	0.003897	0.2680

Pearson Correlation Coefficients, N = 509
Prob > |r| under H0: Rho=0

	symptom5	Pred
symptom5	1.00000	0.52894
symptom5: #Physical Symptoms Truncated at 5		<.0001
Pred	**0.52894**	1.00000
Predicted Value	<.0001	

$R = .5289$, so R^2 for the overall model = .28.

It would be impossible to calculate valid pseudo-R^2 values for each variance component given their heterogeneity across persons and occasions.

Legend: Man, 80% Stressor Days; Woman, 80% Stressor Days; Man, 20% Stressor Days; Woman, 20% Stressor Days

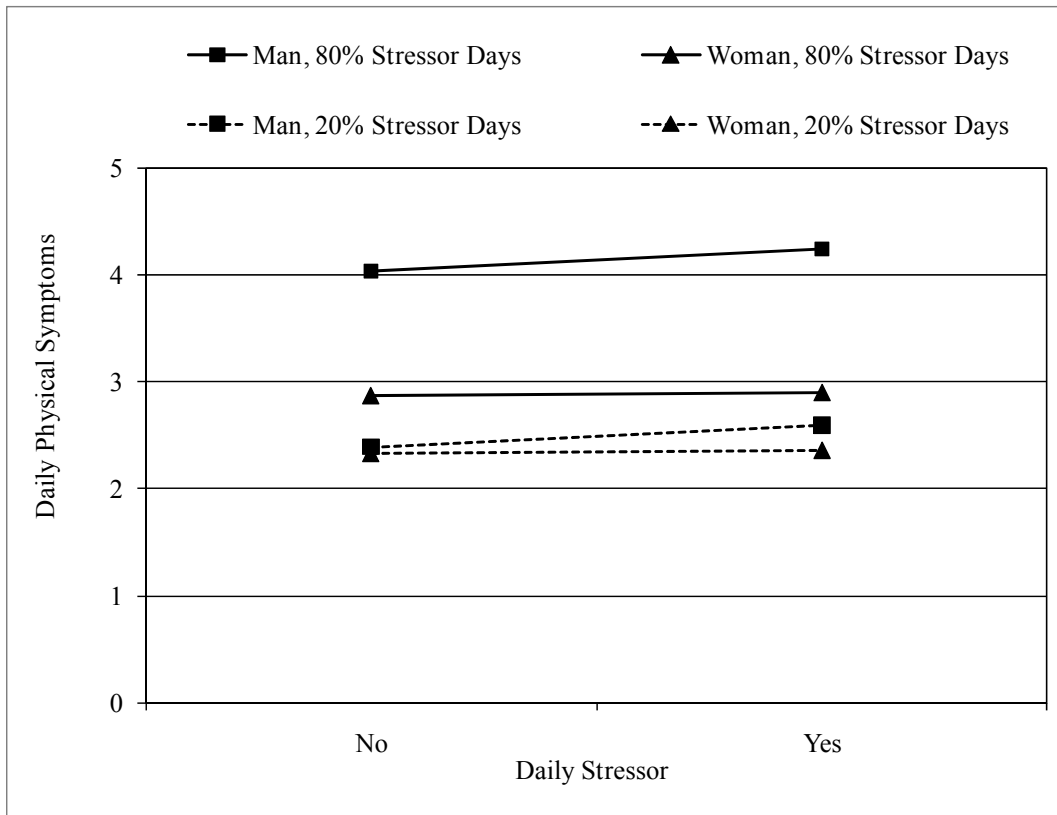X-axis: Daily Stressor (No / Yes); Y-axis: Daily Physical Symptoms

Figure 8.3 illustrates the two interactions of stressor days with sex.

Neither sex has a significant WP stress effect (as shown by the slope of the lines), and this non-effect is the same across men and women.

Both men and women show significant effects of reporting more stressor days, even after controlling for current daily stressors. However, these BP and contextual effects of stress are significantly more positive for men (greater difference between lines).

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