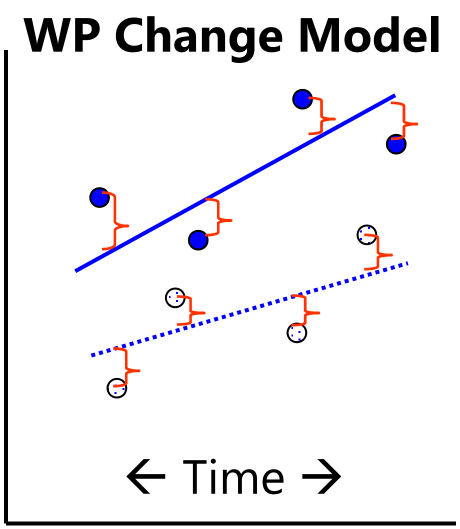
Time-Varying Predictors in Models of Within-Person Fluctuation

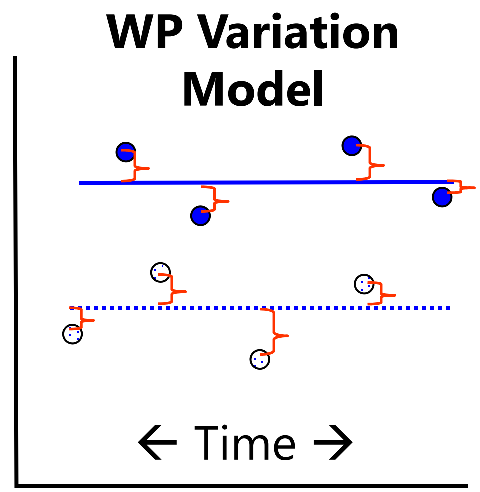
- Today's Class:
  - Effects of Time-Varying Predictors
  - Person-Mean-Centering (PMC)
  - Grand-Mean-Centering (GMC)
  - Model Extensions under PMC vs. GMC

The Joy of Time-Varying Predictors

- TV predictors predict leftover **WP (residual) variation**:



If model for time works, then residuals should look like this →



- Modeling time-varying predictors is complicated because they represent an **aggregated effect**:
  - Effect of the *between-person* variation in the predictor x_{ti} on Y
  - Effect of the *within-person* variation in the predictor x_{ti} on Y
  - Here we are assuming the predictor x_{ti} only **fluctuates** over time…
    - *We will need a different model if x_{ti} changes systematically over time…*

The Joy of Time-Varying Predictors

- Time-varying (TV) predictors usually carry 2 kinds of effects because they are really 2 predictor variables, not 1

- Example: Stress measured daily
  - Some days are worse than others:
    - **WP variation in stress** *(represented as deviation from own mean)*
  - Some people just have more stress than others all the time:
    - **BP variation in stress** *(represented as person mean predictor over time)*

- Can quantify each source of variation with an ICC
  - ICC = (BP variance) / (BP variance + WP variance)
  - ICC > 0? TV predictor has BP variation (so it *could* have a BP effect)
  - ICC < 1? TV predictor has WP variation (so it *could* have a WP effect)

Between-Person vs. Within-Person Effects

- Between-person and within-person effects in <u>SAME</u> direction
  - Stress → Health?
    - **BP: People with more chronic stress than other people may have <u>worse</u> general health than people with less chronic stress**
    - **WP: People may feel <u>worse</u> than usual when they are currently under more stress than usual (regardless of what "usual" is)**
- Between-person and within-person effects in <u>OPPOSITE</u> directions
  - Exercise → Blood pressure?
    - **BP: People who exercise more often generally have <u>lower</u> blood pressure than people who are more sedentary**
    - **WP: During exercise, blood pressure is <u>higher</u> than during rest**
- Variables have different **meanings** at different levels!
- Variables have different **scales** at different levels

3 Kinds of Effects for TV Predictors

- **Is the Between-Person (BP) effect significant?**
  - Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
- **Is the Within-Person (WP) effect significant?**
  - If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ^2_e)?
- **Are the BP and WP effects different sizes: Is there a contextual effect?**
  - After controlling for the absolute value of TV predictor at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond)?
  - If there is no contextual effect, then the BP and WP effects of the TV predictor show *convergence*, such that their effects are of equivalent magnitude

Modeling TV Predictors (labeled as x_{ti})

- **Level-2 effect of x_{ti}:**
  - The level-2 effect of x_{ti} is usually represented by the person's mean of time-varying x_{ti} across time (labeled as **PMx$_i$** or $\mathbf{\bar{X}_i}$)
  - PMx$_i$ should be centered at a CONSTANT (grand mean or other) so that 0 is meaningful, just like any other time-invariant predictor

- **Level-1 effect of x_{ti} can be included two different ways:**
  - "**Group-mean-centering**" → "**person-mean-centering**" in longitudinal, in which level-1 predictors are centered using a level-2 VARIABLE
  - "**Grand-mean-centering**" → level-1 predictors are centered using a CONSTANT (not necessarily the grand mean; it's just called that)
  - Note that these 2 choices do NOT apply to the level-2 effect of x_{ti}!
    - But the interpretation of the level-2 effect of x_{ti} WILL DIFFER based on which centering method you choose for the level-1 effect of x_{ti}!

Person-Mean-Centering (P-MC)

- In P-MC, we decompose the TV predictor x_{ti} into 2 variables that directly represent its BP (level-2) and WP (level-1) sources of variation, and include those variables as the predictors instead:

- **Level-2, PM predictor = person mean of $\mathbf{x_{ti}}$**
  - $\mathbf{PMx_i = \overline{X}_i - \boldsymbol{C}}$
  - $\mathrm{PMx_i}$ is centered at a constant C, chosen so 0 is meaningful
  - $\mathrm{PMx_i}$ is positive? Above sample mean → "more than other people"
  - $\mathrm{PMx_i}$ is negative? Below sample mean → "less than other people"

- **Level-1, WP predictor = deviation from person mean of $\mathbf{x_{ti}}$**
  - $\mathbf{WPx_{ti} = x_{ti} - \overline{X}_i}$ *(note: uncentered person mean $\boldsymbol{\overline{X}_i}$ is used to center x_{ti})*
  - $\mathrm{WPx_{ti}}$ is NOT centered at a constant; is centered at a VARIABLE
  - $\mathrm{WPx_{ti}}$ is positive? Above your own mean → "more than usual"
  - $\mathrm{WPx_{ti}}$ is negative? Below your own mean → "less than usual"

Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 x_{ti}**

→ WP and BP Effects directly through **separate** parameters

x_{ti} is person-mean-centered into WPx_{ti}, with PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{WPx_{ti}}) + \mathbf{e_{ti}}$

$\mathrm{WPx_{ti}} = \mathrm{x_{ti}} - \bar{\mathrm{X}}_\mathrm{i}$ → it has only Level-1 WP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \boldsymbol{\gamma_{01}}(\mathbf{PMx_i}) + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}}$

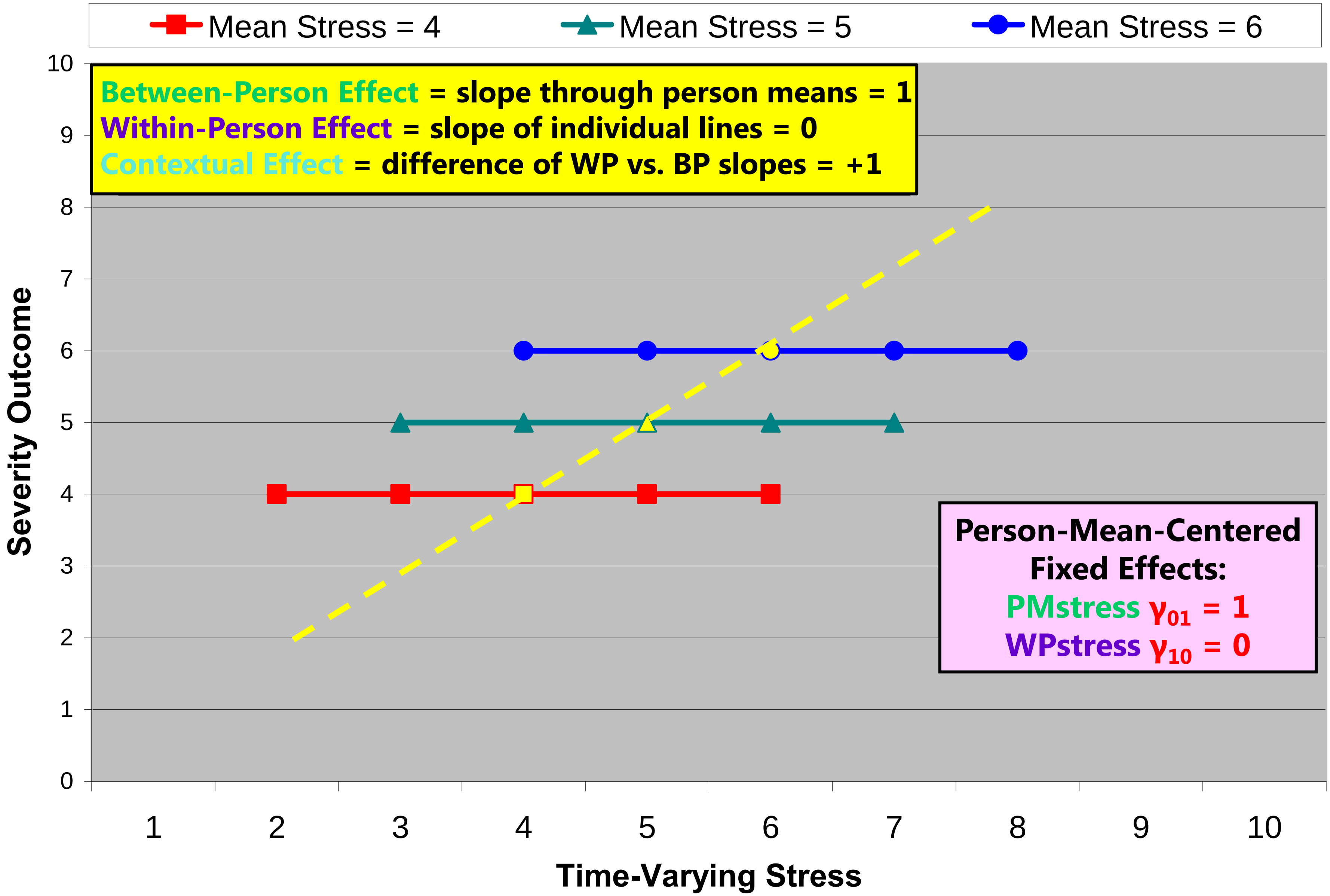
$\mathrm{PMx_i} = \bar{\mathrm{X}}_\mathrm{i} - C$ → it has only Level-2 BP variation

γ_{10} = WP main effect of having more x_{ti} than usual

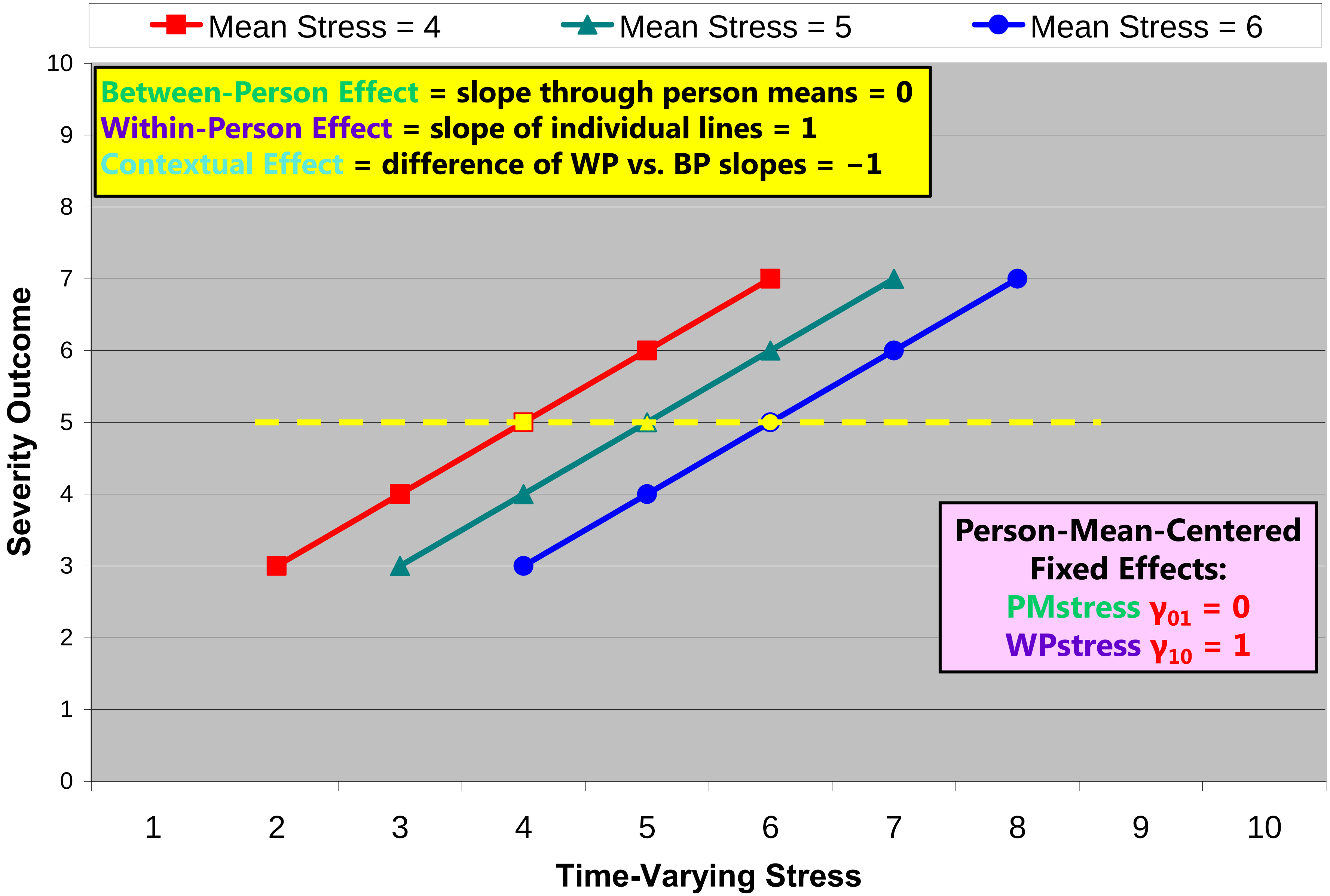
γ_{01} = BP main effect of having more $\bar{X}_i$ than other people

Because WPx_{ti} and PMx_i are uncorrelated, each gets the total effect for its level (WP=L1, BP=L2)

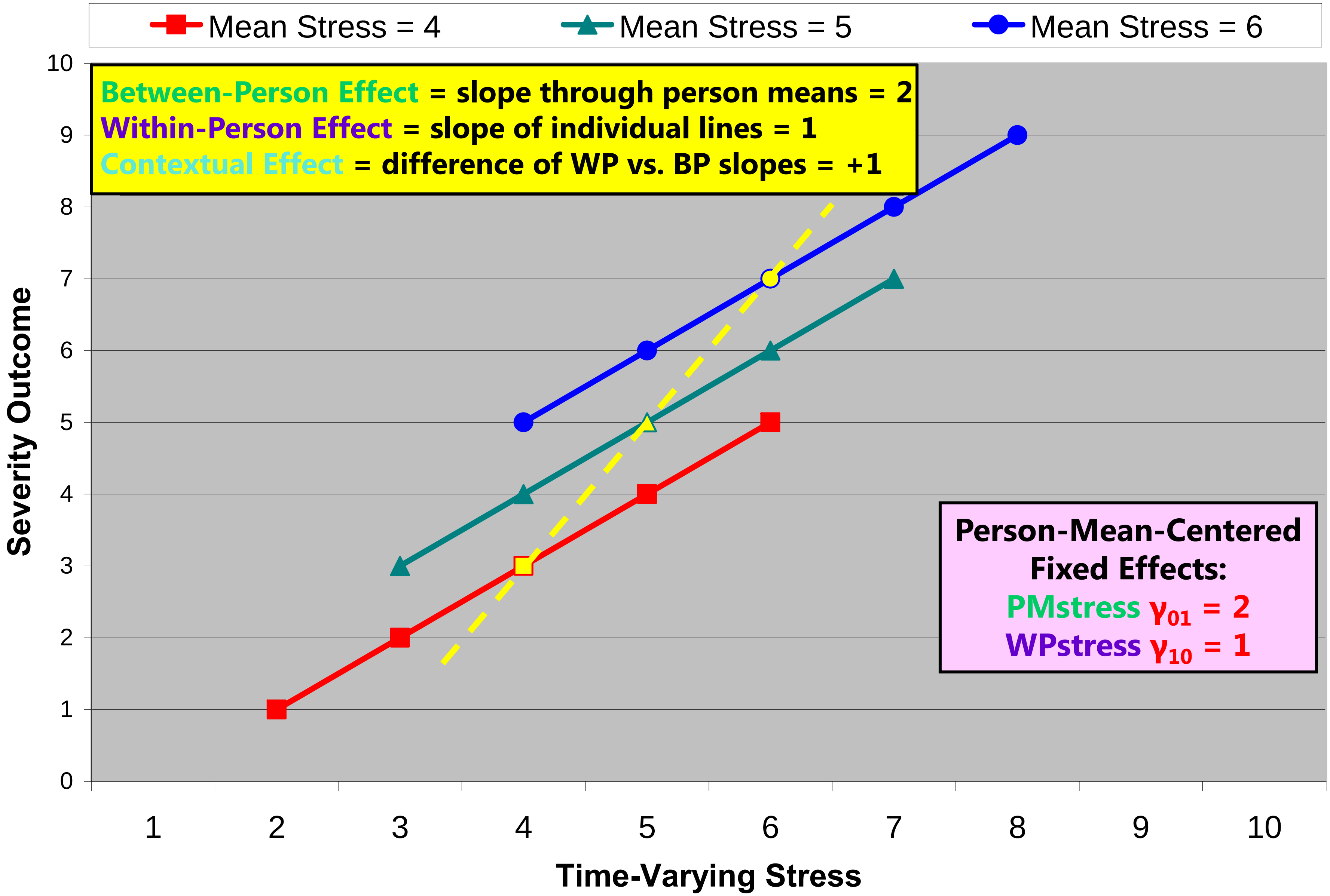
ALL Between-Person Effect, NO Within-Person Effect



NO Between-Person Effect, ALL Within-Person Effect



Between-Person Effect > Within-Person Effect



Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 x_{ti}**

→ WP and BP Effects directly through **separate** parameters

x_{ti} is person-mean-centered into WPx_{ti}, with PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{WPx_{ti}}) + \mathbf{e_{ti}}$

$\mathrm{WPx_{ti}} = \mathrm{x_{ti}} - \bar{\mathrm{X}}_\mathrm{i}$ → it has only Level-1 WP variation

$\mathrm{PMx_i} = \bar{\mathrm{X}}_\mathrm{i} - C$ → it has only Level-2 BP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \boldsymbol{\gamma_{01}}(\mathbf{PMx_i}) + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}} + \boldsymbol{\gamma_{11}}(\mathbf{PMx_i}) + \mathbf{U_{1i}}$

U_{1i} is a random slope for the WP effect of x_{ti}

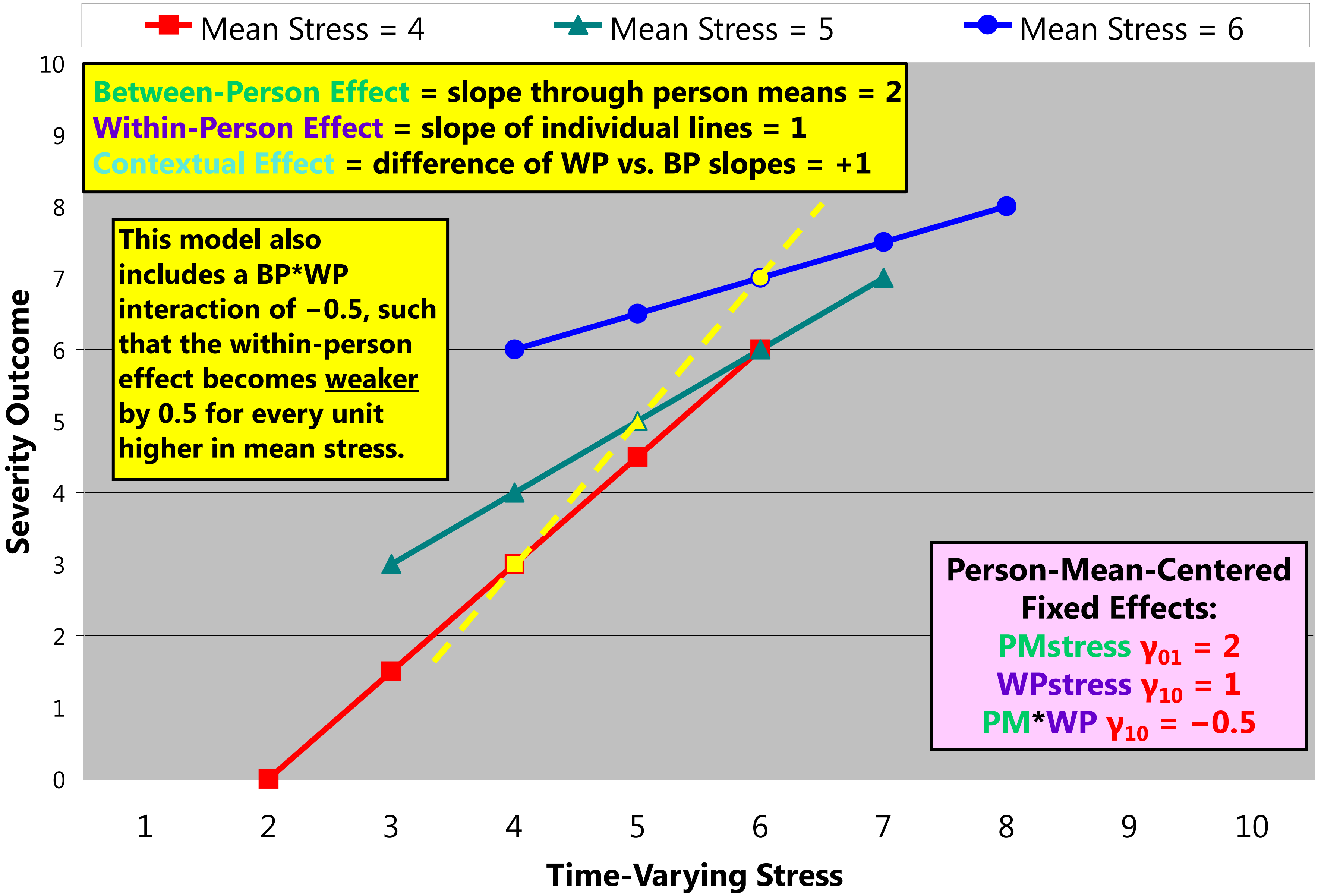
γ_{10} = WP simple main effect of having more x_{ti} than usual *for* $PMx_i = 0$

γ_{01} = BP simple main effect of having more $\bar{\mathrm{X}}_\mathrm{i}$ than other people for people at their own mean ($\mathrm{WPx_{ti}} = \mathrm{x_{ti}} - \bar{\mathrm{X}}_\mathrm{i}$ → 0)

γ_{11} = BP*WP interaction: how the effect of having more x_{ti} than usual differs by how much $\bar{\mathrm{X}}_\mathrm{i}$ you have

Note: this model should also test γ_{02} for $\mathrm{PMx_i} * \mathrm{PMxi}$ (stay tuned)

Between-Person x Within-Person Interaction



3 Kinds of Effects for TV Predictors

- **What Person-Mean-Centering tells us <u>directly</u>:**

- **Is the Between-Person (BP) effect significant?**
  - Are people with higher predictor values <u>than other people</u> (*on average over time*) also higher on Y <u>than other people</u> (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
  - This would be indicated by a significant fixed effect of $\mathbf{PMx_i}$
  - Note: this is NOT controlling for the absolute value of x_{ti} at each occasion

- **Is the Within-Person (WP) effect significant?**
  - If you have higher predictor values <u>than usual</u> (*at this occasion*), do you also have higher outcomes values <u>than usual</u> (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ^2_e)?
  - This would be indicated by a significant fixed effect of $\mathbf{WPx_{ti}}$
  - Note: this is represented by the <u>relative</u> value of x_{ti}, NOT the <u>absolute</u> value of x_{ti}

3 Kinds of Effects for TV Predictors

- **What Person-Mean-Centering DOES NOT tell us directly:**

- **Are the BP and WP effects different sizes: Is there a contextual effect?**
  - After controlling for the absolute value of the TV predictor at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond just the time-specific value of the predictor)?
  - If there is no contextual effect, then the BP and WP effects of the TV predictor show ***convergence***, such that their effects are of equivalent magnitude

- **To answer this question about the contextual effect for the incremental contribution of the person mean, we have two options:**
  - Ask for the contextual effect via an ESTIMATE statement in SAS (or TEST in SPSS, or NEW in Mplus, or LINCOM in STATA): $\mathbf{WPx_{ti}}$ -1 $\mathbf{PMx_i}$ 1
  - Use "**grand-mean-centering**" for time-varying x_{ti} instead: $\mathbf{TVx_{ti}} = \mathbf{x_{ti}} - \boldsymbol{C}$
    → **centered at a CONSTANT, NOT A LEVEL-2 VARIABLE**
    - Which constant only matters for what the reference point is; it could be the grand mean or other

Remember Regular Old Regression?

- In this model: $Y_i = \beta_0 + \beta_1(X_{1i}) + \beta_2(X_{2i}) + e_i$

  - If X_{1i} and X_{2i} **ARE NOT** correlated:
    - β_1 is **ALL the relationship** between X_{1i} and Y_i
    - β_2 is **ALL the relationship** between X_{2i} and Y_i

  - If X_{1i} and X_{2i} **ARE** correlated:
    - β_1 is **different than** the full relationship between X_{1i} and Y_i
      - "Unique" effect of X_{1i} *controlling for* X_{2i} or *holding* X_{2i} *constant*
    - β_2 is **different than** the full relationship between X_{2i} and Y_i
      - "Unique" effect of X_{2i} *controlling for* X_{1i} or *holding* X_{1i} *constant*

  - Hang onto that idea…

Person-MC vs. Grand-MC for Time-Varying Predictors

Level 2		Original	Person-MC Level 1	Grand-MC Level 1
$\bar{X}_i$	$PMx_i = \bar{X}_i - 5$	xti	$WPx_{ti} = x_{ti} - \bar{X}_i$	$TVx_{ti} = x_{ti} - 5$
3	−2	2	−1	−3
3	−2	4	1	−1
7	2	6	−1	1
7	2	8	1	3

Same PMx_i goes into the model using either way of centering the level-1 variable x_{ti}

Using **Person-MC**, WPx_{ti} has NO level-2 BP variation, so it is not correlated with PMx_i

Using **Grand-MC**, TVx_{ti} STILL has level-2 BP variation, so it is STILL CORRELATED with PMx_i

So the effects of PMx_i and TVx_{ti} when included together under Grand-MC will be different than their effects would be if they were by themselves…

Within-Person Fluctuation Model with $\mathbf{x_{ti}}$ represented at Level 1 Only:
→ WP and BP Effects are **Smushed Together**

x_{ti} is grand-mean-centered into TVx_{ti}, WITHOUT PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

$TVx_{ti} = x_{ti} - C$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \gamma_{00} + \mathbf{U_{0i}}$

$\beta_{1i} = \gamma_{10}$

γ_{10} = *smushed* WP and BP effects

Because TVx_{ti} still contains its original 2 different kinds of variation (BP and WP), its 1 fixed effect has to do the work of 2 predictors!

A *smushed* effect is also referred to as the *convergence*, *conflated*, or *composite* effect

Convergence (Smushed) Effect of a Time-Varying Predictor

$$\text{Convergence Effect: } \gamma_{\text{conv}} \approx \frac{\dfrac{\gamma_{BP}}{SE^2_{BP}} + \dfrac{\gamma_{WP}}{SE^2_{WP}}}{\dfrac{1}{SE^2_{BP}} + \dfrac{1}{SE^2_{WP}}}$$

Adapted from Raudenbush & Bryk (2002, p. 138)

- **The convergence effect will often be closer to the within-person effect** (due to larger level-1 sample size and thus smaller SE)

- **It is the rule, not the exception, that between and within effects differ** (Snijders & Bosker, 1999, p. 52-56, and personal experience!)

- However—when grand-mean-centering a time-varying predictor, **convergence is testable** by including a **contextual effect (carried by the person mean)** for how the **BP effect** differs from the **WP effect**…

Within-Person Fluctuation Model with **Grand-Mean-Centered Level-1 x_{ti}**

→ Model tests difference of WP vs. BP effects (It's been fixed!)

x_{ti} is grand-mean-centered into TVx_{ti}, WITH PMx_i at L2:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

$TVx_{ti} = x_{ti} - C$ → it still has both Level-2 BP and Level-1 WP variation

Level 2: $\beta_{0i} = \boldsymbol{\gamma_{00}} + \boldsymbol{\gamma_{01}}(\mathbf{PMx_i}) + \mathbf{U_{0i}}$

$\beta_{1i} = \boldsymbol{\gamma_{10}}$

$PMx_i = \bar{X}_i - C$ → it has only Level-2 BP variation

γ_{10} becomes the WP effect → *unique* level-1 effect after controlling for PMx_i

γ_{01} *becomes* the contextual effect that indicates how the BP effect differs from the WP effect
→ *unique* level-2 effect after controlling for TVx_{ti}
→ does usual level matter beyond current level?

Person-MC and Grand-MC Models are Equivalent Given a Fixed Level-1 Main Effect Only

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$

$\rightarrow y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Composite Model:
← In terms of P-MC
← In terms of G-MC

Grand-MC: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

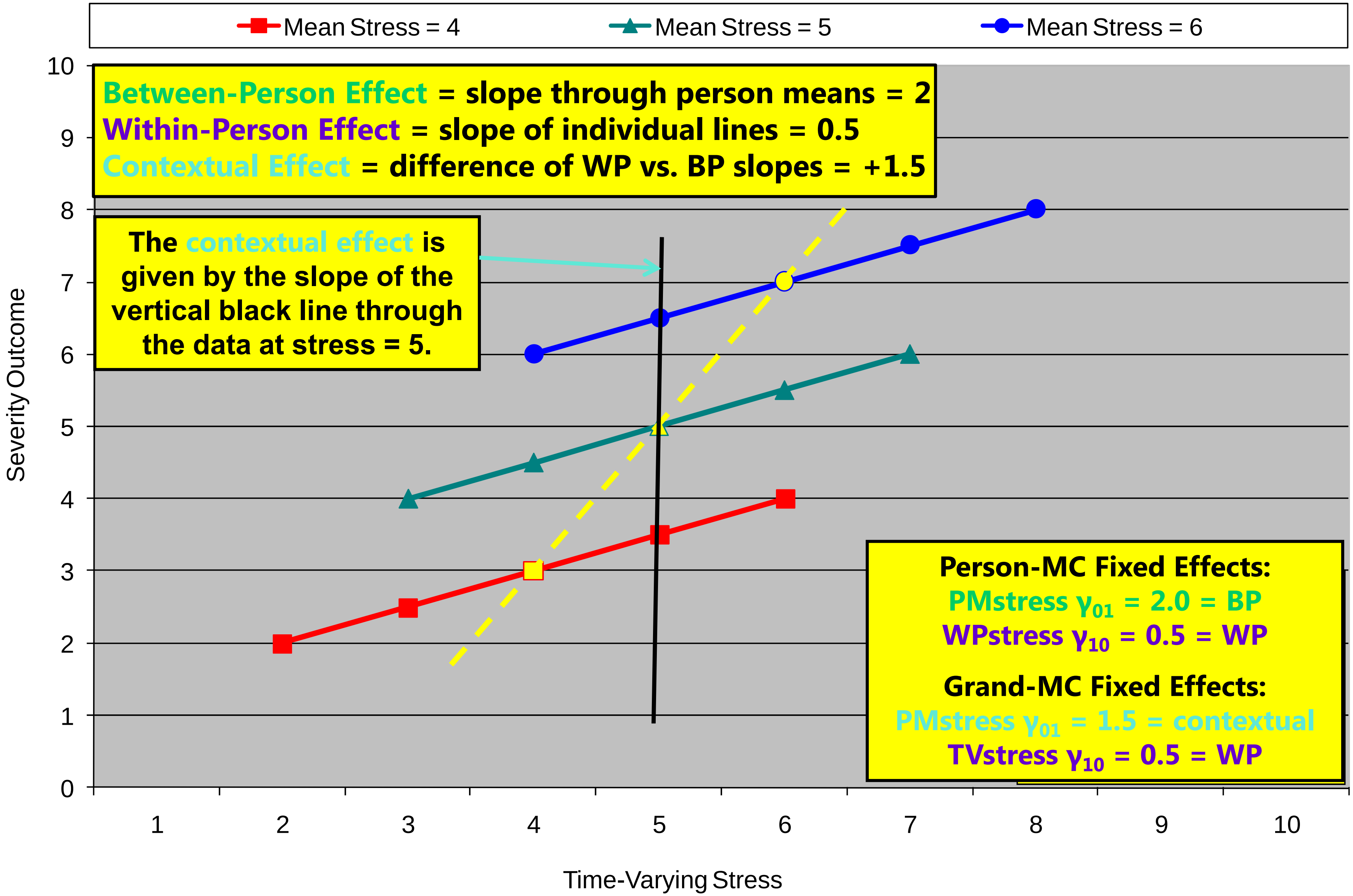
Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

Effect	P-MC	G-MC
Intercept	γ_{00}	γ_{00}
WP Effect	γ_{10}	γ_{10}
Contextual	$\gamma_{01} - \gamma_{10}$	γ_{01}
BP Effect	γ_{01}	$\gamma_{01} + \gamma_{10}$

P-MC vs. G-MC: Interpretation Example



Summary: 3 Effects for TV Predictors

- **Is the Between-Person (BP) effect significant?**
  - Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
  - Given directly by level-2 effect of $\mathrm{PMx_i}$ if using Person-MC for the level-1 predictor (or can be requested via ESTIMATE if using Grand-MC for the level-1 predictor)

- **Is the Within-Person (WP) effect significant?**
  - If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance (σ^2_e)?
  - Given directly by the level-1 effect of $\mathrm{WPx_{ti}}$ if using Person-MC —*OR* — given directly by the level-1 effect of $\mathrm{TVx_{ti}}$ if using Grand-MC and including $\mathrm{PMx_i}$ at level 2 (without $\mathrm{PMx_i}$, the level-1 effect of $\mathrm{TVx_{ti}}$ if using Grand-MC is the smushed effect)

- **Are the BP and WP Effects different sizes: Is there a contextual effect?**
  - After controlling for the absolute value of TV predictor value at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond)?
  - Given directly by level-2 effect of $\mathrm{PMx_i}$ if using Grand-MC for the level-1 predictor (or can be requested via ESTIMATE if using Person-MC for the level-1 predictor)

The Joy of Interactions Involving Time-Varying Predictors

- Must consider interactions with both its BP and WP parts:
- Example: Does time-varying stress (x_{ti}) interact with sex (Sex_i)?

- Person-Mean-Centering:
  - $\mathbf{WPx_{ti}} * \mathbf{Sex_i}$ → Does the WP stress effect differ between men and women?
  - $\mathbf{PMx_i} * \mathbf{Sex_i}$ → Does the BP stress effect differ between men and women?
    - Not controlling for current levels of stress
    - If forgotten, then $\mathbf{Sex_i}$ moderates the stress effect only at level 1 (WP, not BP)

- Grand-Mean-Centering:
  - $\mathbf{TVx_{ti}} * \mathbf{Sex_i}$ → Does the WP stress effect differ between men and women?
  - $\mathbf{PMx_i} * \mathbf{Sex_i}$ → Does the *contextual* stress effect differ b/t men and women?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of $\mathbf{PMx_i}$, the interaction of $\mathbf{TVx_{ti}} * \mathbf{Sex_i}$ would still be smushed

Interactions with Time-Varying Predictors: Example: TV Stress (x_{ti}) by Gender (Sex_i)

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti} - PMx_i)$

Grand-MC: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$

Interactions Involving Time-Varying Predictors Belong at Both Levels of the Model

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti} - PMx_i)$$

← Composite model written as Person-MC

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(Sex_i) + (\gamma_{03} - \gamma_{11})(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$$

← Composite model written as Grand-MC

On the right below → Grand-MC: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$$
$$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$$

After adding an interaction for Sex_i with stress at both levels, then the Person-MC and Grand-MC models are equivalent

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Intercept: $\gamma_{00} = \gamma_{00}$ **BP Effect:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Contextual:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect: $\gamma_{10} = \gamma_{10}$ **BP*Sex Effect:** $\gamma_{03} = \gamma_{03} + \gamma_{11}$ **Contextual*Sex:** $\gamma_{03} = \gamma_{03} - \gamma_{11}$

Sex Effect: $\gamma_{20} = \gamma_{20}$ **BP*WP or Contextual*WP is the same:** $\gamma_{11} = \gamma_{11}$

Intra-variable Interactions

- Still must consider interactions with both its BP and WP parts!
- Example: Interaction of TV stress (x_{ti}) with person mean stress (PMx_i)

- Person-Mean-Centering:
  - $\mathbf{WPx_{ti}} * \mathbf{PMx_i}$ → Does the WP stress effect differ by overall stress level?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the BP stress effect differ by overall stress level?
    - Not controlling for current levels of stress
    - If forgotten, then $\mathbf{PMx_i}$ moderates the stress effect only at level 1 (WP, not BP)

- Grand-Mean-Centering:
  - $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ → Does the WP stress effect differ by overall stress level?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the *contextual* stress effect differ by overall stress?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of $\mathbf{PMx_i}$, the interaction of $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ would still be smushed

Intra-variable Interactions: Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$

Grand-MC: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

Intra-variable Interactions: Example: TV Stress (x_{ti}) by Person Mean Stress (PMx_i)

On the left below → Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti} + \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$$

← Written as Person-MC

$$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} + (\gamma_{02} - \gamma_{11})(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

← Written as Grand-MC

On the right below → Grand-MC: $TVx_{ti} = x_{ti}$

$$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti} + \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$$

After adding an interaction for PMx_i with stress at both levels, then the Person-MC and Grand-MC models are equivalent

---

Intercept: $\gamma_{00} = \gamma_{00}$ **BP Effect:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Contextual:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

WP Effect: $\gamma_{10} = \gamma_{10}$ **BP² Effect:** $\gamma_{02} = \gamma_{02} + \gamma_{11}$ **Contextual²:** $\gamma_{02} = \gamma_{02} - \gamma_{11}$

BP*WP or Contextual*WP is the same: $\gamma_{11} = \gamma_{11}$

When Person-MC ≠ Grand-MC: Random Effects of TV Predictors

Person-MC: $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to PMx_i is removed from the random slope in Person-MC.

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + U_{1i}(x_{ti} - PMx_i) + e_{ti}$

Grand-MC: $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to PMx_i is still part of the random slope in Grand-MC. So these models cannot be made equivalent.

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + U_{1i}(x_{ti}) + e_{ti}$

Random Effects of TV Predictors

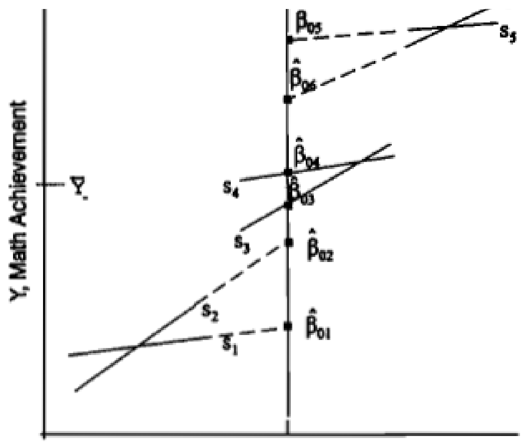
- **Random intercepts** mean different things under each model:
  - **Person-MC** → Individual differences at **WPx_{ti} =0** (that everyone has)
  - **Grand-MC** → Individual differences at **TVx_{ti}=0** (that not everyone has)

- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
  - Person-MC → Won't affect shrinkage of slopes unless highly correlated
  - Grand-MC → Will affect shrinkage of slopes due to forced extrapolation

- As a result, the **random slope variance may be too small** when using Grand-MC rather than Person-MC
  - Problem worsens with greater ICC of TV Predictor (more extrapolation)
  - Anecdotal example using clustered data was presented in Raudenbush & Bryk (2002; chapter 6)

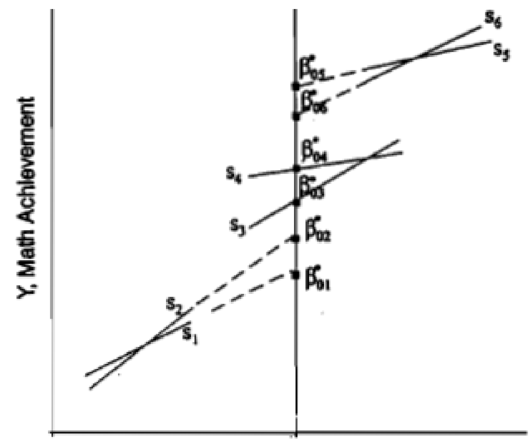
Bias in Random Slope Variance

OLS Per-Person Estimates



Level-1 X

EB Shrunken Estimates



Level-1 X

Top right: Intercepts and slopes are homogenized in Grand-MC because of intercept extrapolation

Bottom: Downwardly-biased random slope variance in Grand-MC relative to Person-MC

Unconditional Results	*Conditional Results*
Person-MC	
$\hat{\mathbf{T}} = \begin{bmatrix} 8.68 & 0.05 \\ 0.05 & 0.68 \end{bmatrix}$	$\hat{\mathbf{T}} = \begin{bmatrix} 2.38 & 0.19 \\ 0.19 & \boxed{0.15} \end{bmatrix}$
$\hat{\sigma}^2 = 36.70$	$\hat{\sigma}^2 = 36.70$
Grand-MC	
$\hat{\mathbf{T}} = \begin{bmatrix} 4.83 & -0.15 \\ -0.15 & 0.42 \end{bmatrix}$	$\hat{\mathbf{T}} = \begin{bmatrix} 2.41 & 0.19 \\ 0.19 & \boxed{0.06} \end{bmatrix}$
$\hat{\sigma}^2 = 36.83$	$\hat{\sigma}^2 = 36.74$

Modeling Time-Varying Categorical Predictors

- Person-MC and Grand-MC really only apply to *continuous* TV predictors, but the need to consider BP and WP effects applies to *categorical* TV predictors too
- Binary level-1 predictors do not lend themselves to Person-MC
  - e.g., x_{ti} = 0 or 1 per occasion, person mean = .50 across occasions → impossible values
  - If $x_{ti} = 0$, then $WPx_{ti} = 0 - .50 = -0.50$; If $x_{ti} = 1$, then $WPx_{ti} = 1 - .50 = 0.50$
  - Better: Leave x_{ti} uncentered and include person mean as level-2 predictor (results ~ Grand-MC)
- For >2 categories, person means of multiple dummy codes starts to break down, but we can think about types of people, and code BP effects accordingly
- Example: Dementia present/not at each time point?
  - **BP effects →** Ever diagnosed with dementia (no, yes)?
    - People who will eventually be diagnosed may differ prior to diagnosis (a BP effect)
  - **TV effect →** Diagnosed with dementia at each time point (no, yes)?
    - Acute differences of before/after diagnosis logically can only exist in the "ever" people
- Other examples: Mentor status, father absence, type of shift work (AM/PM)

Wrapping Up: Person-MC vs. Grand-MC

- Time-varying predictors carry at least two potential effects:
  - Some people are higher/lower than other people → BP, level-2 effect
  - Some occasions are higher/lower than usual → WP, level-1 effect

- BP and WP effects almost always need to be represented by two or more model parameters, using either:
  - *Person-mean-centering* (WPx_{ti} and PMx_i): WP ≠ 0?, BP ≠ 0?
  - *Grand-mean-centering* (TVx_{ti} and PMx_i): WP ≠ 0?, BP ≠ WP?
  - Both yield equivalent models if the level-1 WP effect is fixed, but not if the level-1 WP effect is random
    - Grand MC → *absolute* effect of x_{ti} varies randomly over people
    - Person MC → *relative* effect of x_{ti} varies randomly over people
    - Use prior theory and empirical data (ML AIC, BIC) to decide