Time-Invariant Predictors in Longitudinal Models

- Today's Class:
  - Summary of steps in building unconditional models for time
  - What happens to missing predictors
  - Effects of time-invariant predictors
  - Fixed vs. systematically varying vs. random effects
  - Model building strategies and assessing significance

Summary of Steps in Unconditional Longitudinal Modeling

For all outcomes:

1. Empty Model; Calculate ICC
2. Decide on a metric of time
3. Decide on a centering point
4. Estimate means model and plot individual trajectories

If your outcome shows systematic change:

5. Evaluate fixed and random effects of time
6. Still consider possible alternative models for the residuals (**R** matrix)

If your outcome does NOT show ANY systematic change:

5. Evaluate alternative models for the variances (**G**+**R**, or **R**)

Back to the Big Picture…

- Unconditional Longitudinal **Models for the Means**:
  - Describe the average pattern of change over time (if any)
    - Linear or non-linear? Continuous or discontinuous?
    - This is what the fixed effects of time are for

- Unconditional Longitudinal **Models for the Variance**:
  - Describe the pattern of variation and covariation of residuals across occasions and persons
  - Most simple: Random Intercept Only or CS (Univar. RM ANOVA)
  - Most complex: Unstructured R (Multivariate RM ANOVA)
  - Multilevel models offer two families of intermediate alternatives:
    - Random effects models ("multilevel" models)
    - Alternative covariance structures

1. Empty Means, Random Intercept Model

- Not really predictive, but is a useful statistical baseline model
  - Baseline model fit
  - Partitions variance into between- and within-person variance

- Calculate **ICC** = between / (between + within variance)
  - = Average correlation between occasions
  - = Proportion of variance that is between persons
  - Effect size for amount of person dependency due to mean differences

- Tells you where the action will be:
  - If most of the variance is **between-persons in the random intercept (at level 2)**, you will need **person-level** predictors to reduce that variance (i.e., to account for inter-individual differences)
  - If most of the variance is **within-persons in the residual(at level 1)**, you will need **time-level** predictors to reduce that variance (i.e., to account for intra-individual differences)

2. Decide on the Metric of Time

- "Occasion of Study" as Time:
  - Can be used generically for many purposes
  - Include age, time to event as predictors of change
- "Age" as Time:
  - Is equivalent to time-in-study if same age at beginning of study
  - Implies age convergence → that people only differ in age regardless of when they came into the study (BP effects = WP effects)
- "Distance to/from an Event" as Time:
  - Is appropriate if a distinct process is responsible for changes
  - Also implies convergence (BP effects = WP effects)
  - Only includes people that have experienced the event
- Make sure to use exact time regardless of which "time" used

3. Decide on a Centering Point

- How to choose: At what occasion would you like a snap-shot of inter-individual differences?
  - Intercept variance represents inter-individual differences at that particular time point (that you can later predict!)

- Where do you want your intercept?
  - Re-code time such that the centering point = 0
  - Multiple variants could be used (e.g., moving snapshots)

- Different versions of time = 0 will produce statistically equivalent models with re-arranged parameters
  - i.e., conditional level and rate of change at time 0

4. Plot Saturated Means and Individuals

- If time is balanced across persons:
  - Estimate a saturated means model to generate means

- If time is NOT balanced across persons:
  - Create a rounded time variable to estimate means model ONLY
  - Still use exact time/age variable for analysis!

- Plot the means – what kind of trajectory do you see?

- Please note: ML/REML estimated means per occasion may NOT be the same as the observed means (i.e., as given by PROC MEANS). The estimated means are what would have been obtained *had your data been complete* (assuming MAR), whereas observed means are not adjusted to reflect any missing data (MCAR). Report the ML/REML estimated means.

What if I have no change?

- Longitudinal studies are not always designed to examine systematic change (e.g., daily diary studies)
- In reality, there is a continuum of fluctuation to change:

Design	Time is irrelevant	Time was not supposed to be relevant (but is)	Time is the study purpose
Result	Pure WP Fluctuation	Hybrid	Pure WP Change
Means Model for Time	Empty	Fixed slopes of time	Fixed Slopes of time
Variance Model for Time	Alternative Covariance Structures	Alternative Covariance Structures?	Random Slopes of time

5. and 6. for **Systematic Change**: Evaluate Fixed and Random Effects of Time

Model for the Means:

- What kind of fixed effects of time are needed to parsimoniously represent the observed means across time points?
  - Linear or nonlinear? Continuous or discontinuous?
  - Polynomials? Pieces? Nonlinear curves?
  - How many parameters do you need to Name That Trajectory?
  - Use obtained *p*-values to test significance of fixed effects

Model for the Variance (focus primarily on G):

- What kind of random effects of time are needed:
  - To account for individual differences in aspects of change?
  - To describe the variances and covariances over time?
  - Do the residuals show any pattern after accounting for random effects?
  - Use REML $-2\Delta LL$ tests to test significance of new effects (or ML if big *N*)

Random Effects Variance Models

- Each source of correlation or dependency goes into a new variance component (or pile of variance) until each source meets the usual assumptions of GLM: normality, independence, constant variance
- **Example 2-level longitudinal model:**

Level 1 (one source of) <u>Within-Person Variation</u>: gets accounted for by time-level predictors

Residual Variance (σ_e^2)

FIXED effects make variance go away (explain variance).

RANDOM effects just make a new pile of variance.

Level 2 (two sources of) <u>Between-Person Variation</u>: gets accounted for by person-level predictors

BP Int Variance ($\tau_{U_0}^2$)

BP Slope Variance ($\tau_{U_1}^2$)

$\tau_{U_{01}}$ **covariance**

Now we get to add predictors to account for each pile!

5. for **NO Systematic Change**:
Evaluate Alternative Covariance Structures

Model for the Means:

- Be sure you don't need any terms for systematic effects of time
- If not, keep a fixed intercept only

Model for the Variance (focus primarily on R):

- How many parameters do you need to Name… that… Structure?
- I recommend the hybrid: Random Intercept in **G** + Structure in **R**
  - Separates BP and WP variance
  - Likely more parsimonious than just **R**-only model
- Compare alternative models with the same fixed effects
  - Nested? REML −2ΔLL test for significance
  - Non-nested? REML AIC and BIC for "supporting evidence"

Alternative Covariance Structure Models

- Models for fluctuation typically include only a covariance structure, and at most a random intercept (random slopes for time won't help in the absence of systematic change)

Between-Person Random Intercept in G + Within-Person Structure in R

Level 1 (one source of) Within-Person Variation:

Gets accounted for by time-level predictors

Residual Variance (σ^2_e)

Level 2 (one sources of) Between-Person Variation:

Gets accounted for by person-level predictors

BP Int Variance ($\tau^2_{U_0}$)

TOTAL Structure in R

All sources of variation and covariation are held in one matrix, but if dependency is predicted accurately then it's ok.

Total Variance (σ^2_T)

Why spend so much effort on unconditional models of time? Here is the reasoning…

- The fixed effects of time are what the random effects of time are varying around…
- The random effects of time form the variances that the person-level predictors will account for…
- The effects of person-level predictors are specified as a function of the time effect already in the model…
- The effects of time-varying predictors are supposed to account for variance not accounted for by the model for time…
- What fixed and random time effects of time you include in the model dictate what is to be predicted.
- There is little point in trying to predict individual differences in change (and intraindividual deviation from predicted change) when it's possible that those individual differences (and deviations) only exist because the model for change is mis-specified.
  Make sure to get time right first!

What happens to missing predictors?

- Incomplete data patterns in longitudinal study
  - Sparse missingness (within occasion)
  - Differential attrition (monotonic dropout)
  - Measurements obtained at different intervals (“unbalanced data”)
  - “Planned” missing data (no really, you can do this on purpose)
  - Often unrecognized selection bias at beginning of all studies, too

- Goal: To make valid inferences about population parameters despite bias introduced by attrition
  - The goal is ***not*** to recover the missing data values

- Methods used to do analyses in the presence of missing data require assumptions about the causes associated with the missingness process as well as the variables distributions

Methods of Analysis Given Missing Data

- **What not to do:**
- Listwise deletion (all available whole people)
- Pairwise deletion (all available cases)
- Single mean replacement or regression imputation

- **What to do: FIML or multiple imputation**
  - FIML = Full-information maximum likelihood → uses all the original data in estimating model, not just a summary thereof
  - MIXED and Mplus use FIML by default for missing responses (REML and ML as we know them are both Full-Information)
  - Asymptotically equivalent results given the same missingness model, however, but FIML is more direct than multiple imputation (and is more readily available for not-normal variables)
  - Both of these assume Missing at Random, though…

Categorizations of Missing Data

- If data are missing from some occasions, all is not lost!
- Missingness predictors: Person-level variables, outcomes at other observed occasions:
  - Missing Completely at Random (MCAR): probability of missingness is unrelated to what those missing responses would have been
  - **Missing at Random (MAR):** probability of missingness depends on the persons' predictors or their other observed outcomes, but you can draw correct inferences after including (controlling for) their other data
  - Missing Not At Random (MNAR): probability of missingness is systematic but is not predictable based on the information you have (everything will be wrong)

- You will likely get different estimates from models with complete cases only… so use all the data you have if possible!
- Now, the bad news…

Missing Data in MLM Software

- Common misconceptions about how MLM "handles" missing data
- Most MLM programs (e.g., MIXED) analyze only COMPLETE CASES
  - Does NOT require listwise deletion of *whole persons*
  - DOES delete any incomplete cases (occasions within a person)
- Observations missing predictors OR outcomes are not included!
  - **Time** is (probably) measured for **everyone**
  - **Predictors may NOT be measured for everyone**
  - *N* may change due to missing data for different predictors across models
- You may need to think about what predictors you want to examine PRIOR to model building, and pre-select your sample accordingly
  - Models and model fit statistics −2LL, AIC, and BIC are only **directly comparable** if they include the **exact same observations (LL is sum of each height)**
  - Will have less statistical power as a result of removing incomplete cases

Be Careful of Missing Predictors!

Multivariate (wide) data → stacked (long) data

ID	T1	T2	T3	T4	Person Pred	T1 Pred	T2 Pred	T3 Pred	T4 Pred
100	5	6	8	12	50	4	6	7	.
101	4	7	.	11	.	7	.	4	9

Row	ID	Time	DV	Person Pred	Time Pred
1	**100**	1	5	50	4
2	**100**	2	6	50	6
3	**100**	3	8	50	7
4	**100**	4	12	50	.
5	**101**	1	4	.	7
6	**101**	2	7	.	.
7	**101**	3	.	.	4
8	**101**	4	11	.	9

Only rows with complete data get used – for each model, which rows get used in MIXED?

Model with Time → DV: 1-6, 8

Model with Time, Time Pred → DV: 1-3, 5, 8

Model with Time, Person Pred → DV: 1-4

Model with Time, Time Pred, & Person Pred → DV: 1-3

So what does this mean for missing data in MLM?

- **Missing outcomes are assumed MAR**
  - Because the likelihood function is for predicted Y, just estimated on whatever Y responses a person does have (can be incomplete)
- **Missing time-varying predictors are MAR-to-MCAR ish**
  - Would be MCAR because X is not in the likelihood function (is Y given X instead), but other occasions may have predictors (so MAR-ish)
- **Missing time-invariant predictors are assumed MCAR**
  - Because the predictor would be missing for all occasions, whole people will be deleted (may lead to bias)

- Missingness on predictors can be accommodated:
  - In Multilevel SEM with certain assumptions (≈ outcomes then)
  - Via multilevel multiple imputation in Mplus v 6.0+ (but careful!)
    - Must preserve all effects of potential interest in imputation model, including random effects; $-2\Delta LL$ tests are not done in same way

Modeling Time-Invariant Predictors

<u>What independent variables can be time-invariant predictors?</u>

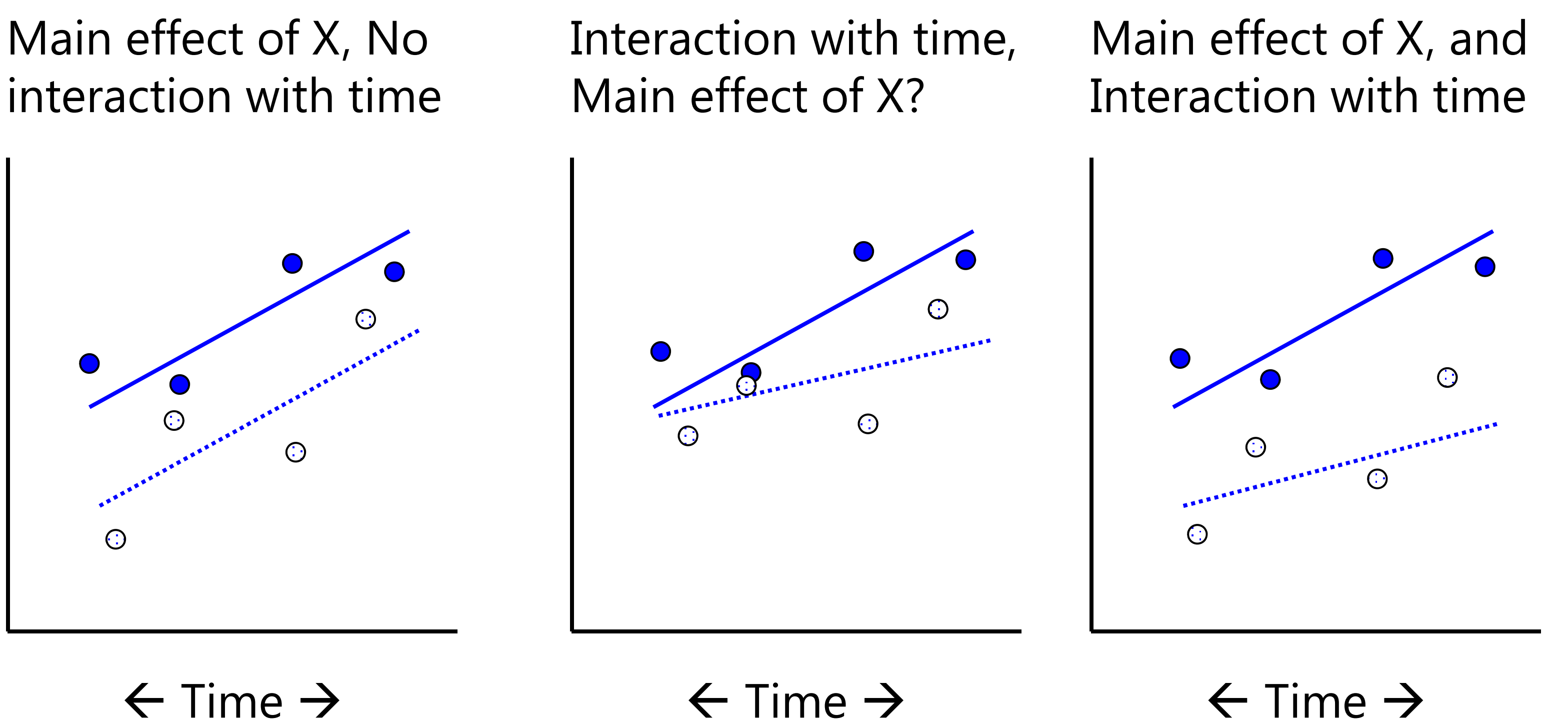
- Also known as “person-level” or “level-2” predictors
- Include substantive predictors, controls, and predictors of missingness
- Can be anything that **does not change across time** (e.g., Biological Sex)
- Can be anything that **is not likely to change across the study,** but you may have to argue for this (e.g., Parenting Strategies, SES)
- Can be anything that **does change across the study**…
  - But you have **only measured once**
    - Limit conclusions to variable’s status at time of measurement
    - e.g., “Parenting Strategies at age 10”
  - Or **is perfectly correlated with time** (age, time to event)
    - Would use Age at Baseline, or Time to Event *from Baseline* instead

Centering Time-Invariant Predictors

- Very useful to center all predictors such that 0 is a meaningful value:
  - Same significance level of main effect, different interpretation of intercept
  - Different (more interpretable) main effects within higher-order interactions
    - With interactions, main effects = simple effects when other predictor = 0
- Choices for centering **continuous** predictors:
  - At Mean: Reference point is *average level of predictor within the sample*
    - Useful if predictor is on arbitrary metric (e.g., unfamiliar test)
  - Better → At Meaningful Point: Reference point is *chosen level of predictor*
    - Useful if predictor is already on a meaningful metric (e.g., age, education)
- Choices for centering **categorical** predictors:
  - Re-code group so that your chosen reference group = **highest category!** (which is the default in SAS and SPSS mixed models)
  - I do not recommend mean-centering categorical predictors (because who is at the mean of a categorical variable ?!?)

The Role of Time-Invariant Predictors in the **Model for the Means**

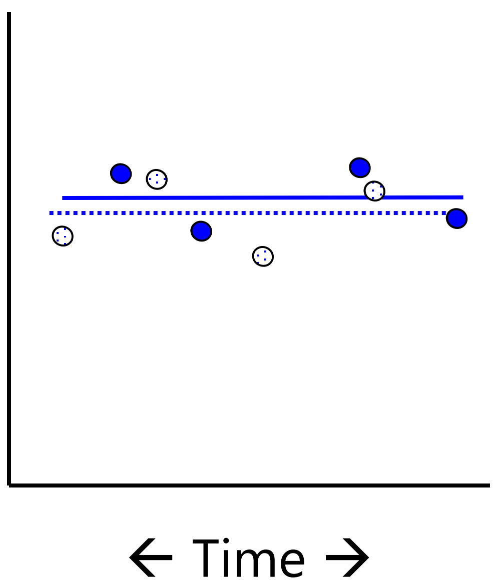
- **In Within-Person Change Models** → Adjust growth curve



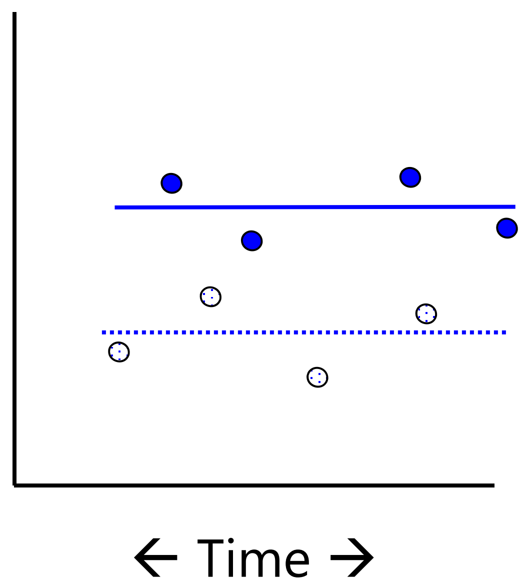
The Role of Time-Invariant Predictors in the **Model for the Means**

- In **Within-Person Fluctuation Models** → Adjust mean level

No main effect of X



Main effect of X

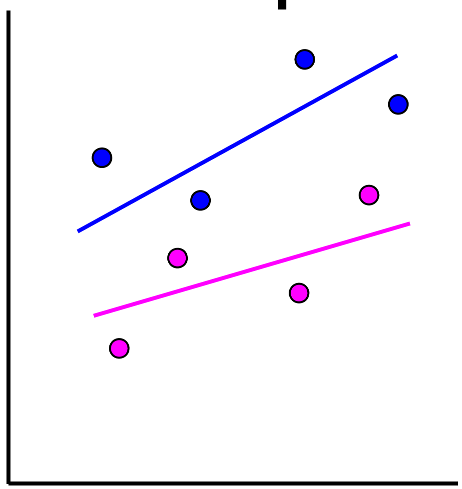


The Role of Time-Invariant Predictors in the **Model for the Variance**

- In addition to fixed effects in the model for the means, time-invariant predictors can allow be used to allow **heterogeneity of variance** at their level or below
- e.g., Sex as a predictor of heterogeneity of variance:
  - **At level 2**: amount of individual differences in intercepts/slopes differs between boys and girls (i.e., one group is more variable)
  - **At level 1**: amount of within-person residual variation differs between boys and girls
    - In within-person **fluctuation** model: differential fluctuation over time
    - In within-person **change** model: differential fluctuation/variation remaining after controlling for fixed and random effects of time
- These models are harder to estimate (i.e., use NLMIXED)

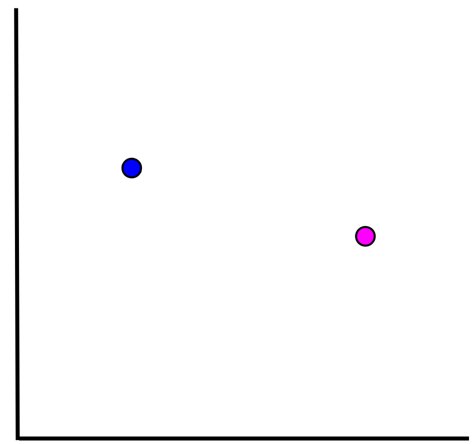
Why Level-2 Predictors Cannot Have Random Effects in 2-Level Models

Random Slopes for Time



Time
(or Any Level-1 Predictor)

Random Slopes for Sex?



Sex
(or any Level-2 Predictor)

You cannot make a line out of a dot, so level-2 effects cannot vary randomly over persons.

Education as a Time-Invariant Predictor: Example using a Random Quadratic Time Model

- Main Effect of Education = Education*Intercept Interaction
  - Moderates the intercept → Increase or decrease in expected outcome at time 0 for every year of education

- Effect of Education on Linear Time = Education*Time Interaction
  - Moderates the linear time slope → Increase or decrease in expected rate of change at time 0 for every year of education

- Effect of Education on Quadratic Time = Education*Time^2 Interaction
  - Moderates the quadratic time slope → Increase or decrease in half of expected acceleration/deceleration of linear rate of change for every year of education

Education (12 years = 0) as a Time-Invariant Predictor: Example using a Random Quadratic Time Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}Time_{ti} + \beta_{2i}Time_{ti}^2 + e_{ti}$

Level 2 Equations (one per β):

$\beta_{0i} = \gamma_{00} + \gamma_{01}Ed_i + U_{0i}$

- β_{0i}: **Intercept** for person *i*
- γ_{00}: **Fixed Intercept when Time=0 and Ed=12**
- γ_{01}: **Δ in Intercept per unit Δ in Ed**
- U_{0i}: **Random (Deviation) Intercept after controlling for Ed**

$\beta_{1i} = \gamma_{10} + \gamma_{11}Ed_i + U_{1i}$

- β_{1i}: **Linear Slope** for person *i*
- γ_{10}: **Fixed Linear Time Slope when Time=0 and Ed=12**
- γ_{11}: **Δ in Linear Time Slope per unit Δ in Ed (=Ed*time)**
- U_{1i}: **Random (Deviation) Linear Time Slope after controlling for Ed**

$\beta_{2i} = \gamma_{20} + \gamma_{21}Ed_i + U_{2i}$

- β_{2i}: **Quad Slope** for person *i*
- γ_{20}: **Fixed Quad Time Slope when Ed = 12**
- γ_{21}: **Δ in Quad Time Slope per unit Δ in Ed (=Ed*time^2)**
- U_{2i}: **Random (Deviation) Quad Time Slope after controlling for Ed**

Education (12 years = 0) as a Time-Invariant Predictor: Example using a Random Quadratic Time Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}Time_{ti} + \beta_{2i}Time_{ti}^2 + e_{ti}$

Level 2 Equations (one per β):

$$\beta_{0i} = \gamma_{00} + \gamma_{01}Ed_i + U_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}Ed_i + U_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21}Ed_i + U_{2i}$$

γ_{11} and γ_{21} are known as "**cross-level**" interactions (level-1 predictor by level-2 predictor)

- Composite equation:
- $y_{ti} = (\gamma_{00} + \gamma_{01}Ed_i + U_{0i}) + (\gamma_{10} + \gamma_{11}Ed_i + U_{1i})Time_{ti} + (\gamma_{20} + \gamma_{21}Ed_i + U_{2i})Time_{ti}^2 + e_{ti}$

Fixed Effects of Time-Invariant Predictors

- Question of interest: Why do people change differently?
  - We're trying to predict individual differences in intercepts and slopes (i.e., reduce level-2 random effects variances)
  - So level-2 random effects variances become 'conditional' on predictors → actually random effects variances *left over*

$$\beta_{0i} = \gamma_{00} + U_{0i} \qquad \longrightarrow \qquad \beta_{0i} = \gamma_{00} + \gamma_{01}Ed_i + U_{0i}$$
$$\beta_{1i} = \gamma_{10} + U_{1i} \qquad \qquad \qquad \beta_{1i} = \gamma_{10} + \gamma_{11}Ed_i + U_{1i}$$
$$\beta_{2i} = \gamma_{20} + U_{2i} \qquad \qquad \qquad \beta_{2i} = \gamma_{20} + \gamma_{21}Ed_i + U_{2i}$$

  - Can calculate pseudo-R^2 for each level-2 random effect variance between models with *fewer* versus *more* parameters as:

$$\text{Pseudo R}^2 = \frac{\text{random variance}_{\text{fewer}} - \text{random variance}_{\text{more}}}{\text{random variance}_{\text{fewer}}}$$

Fixed Effects of Time-Invariant Predictors

- What about predicting level-1 effects with no random variance?
  - If the random linear time slope is n.s., can I test interactions with time?

This should be ok to do…

$$\beta_{0i} = \gamma_{00} + \gamma_{01}Ed_i + U_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}Ed_i + U_{1i}$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21}Ed_i + U_{2i}$$

Is this still ok to do?

$$\beta_{0i} = \gamma_{00} + \gamma_{01}Ed_i + U_{0i}$$
$$\beta_{1i} = \gamma_{10} + \gamma_{11}Ed_i$$
$$\beta_{2i} = \gamma_{20} + \gamma_{21}Ed_i$$

  - YES, surprisingly enough….
  - **In theory**, if a level-1 effect does not vary randomly over individuals, then it has "no" variance to predict (so cross-level interactions with that level-1 effect are not necessary)
  - However, because power to detect random effects is often lower than power to detect fixed effects, fixed effects of predictors can still be significant even if there is "no" (≈0) variance for them to predict
  - Small (≈0) random variance → harder to find significant interactions
  - Cue 6-minute SMEP 2011 talk…

3 Types of Effects: Fixed, Random, and Systematically (Non-Randomly) Varying

Let's say we have a significant fixed linear effect of time. What happens after we test a sex*time interaction?

	Non-Significant Sex*Time effect?	Significant Sex*Time effect?
Random time slope initially **not** significant	Linear effect of time is **FIXED**	Linear effect of time is **systematically varying**
Random time initially sig, **not** sig. after sex*time	---	Linear effect of time is **systematically varying**
Random time initially sig, **still** sig. after sex*time	Linear effect of time is **RANDOM**	Linear effect of time is **RANDOM**

The effects of level-1 predictors (time-level) can be fixed, random, or systematically varying. The effects of level-2 predictors (person-level) can only be fixed or systematically varying (nothing to be random over…yet).

Variance Accounted For By Level-2 Time-Invariant Predictors

- **Fixed effects of level 2 predictors *by themselves*:**
  - L2 (BP) main effects (e.g., sex) reduce L2 (BP) random intercept variance
  - L2 (BP) interactions (e.g., sex by ed) also reduce L2 (BP) random intercept variance

- **Fixed effects of *cross-level interactions* (level 1* level 2):**
  - If the interacting level 1 predictor is random, any cross-level interaction with it will reduce its corresponding L2 BP random slope variance
    - e.g., if *time* is random, then sex**time*, ed**time*, and sex*ed**time* can each reduce the random linear time slope variance
  - If the interacting level 1 predictor not random, any cross-level interaction with it will reduce the L1 WP residual variance instead
    - e.g., if $time^2$ is fixed, then sex*$time^2$, ed*$time^2$, and sex*ed*$time^2$ will reduce the L1 (WP) residual variance → Different quadratic slopes from sex and ed will allow better trajectories, reduce the variance around trajectories

Model-Building Strategies

- **Build UP: Start with lowest-level fixed effect**, add higher-order fixed effect interactions IF the lower-lever fixed effects are significant
  - Example: Sex predicting growth over time
    - Start with sex main effect; IF significant, then sex*time, then sex*time^2….
  - Problem: May miss higher-order interactions
    - Example: Even if sex*time^2 is significant, the effects of sex on the intercept and linear time slope may not be significant, and thus you may stop too soon

- **Build DOWN: Start with highest-level fixed effect**, drop higher-order fixed effect interactions IF they are not significant
  - Example: Sex predicting growth over time
    - Start with sex*time^2, drop if non-significant, then go to sex*time, drop if non-significant, then go to main effect of sex only (→ sex*intercept)
  - Problem: Where to start?!?
    - Example: 3 predictors in a quadratic growth model: Start with X_1*X_2*X_3*time^2
    - Requires 5 main effects, 10 two-ways, 6 three-ways, 2 four-ways

Evaluating Statistical Significance of New Fixed Effects

Fixed effects can be tested via **Wald** tests: the ratio of its estimate/SE forms a statistic we compare to a distribution

	Denominator DF is assumed infinite	Denominator DF is estimated instead
Numerator DF = 1	use ***z*** distribution (Mplus, STATA)	use ***t*** distribution (SAS, SPSS)
Numerator DF > 1	use χ^2 distribution (Mplus, STATA)	use ***F*** distribution (SAS, SPSS)
Denominator DF (DDFM) options	not applicable, so DDF is not given	SAS: BW and KR SAS and SPSS: Satterthwaite

Denominator DF (DDF) Methods

- **Between-Within** (DDFM=BW in SAS, not in SPSS):
  - Total DDF (T) comes from total number of observations, separated into level-2 for *N* persons and level-1 for *n* occasions
    - **Level-2 DDF** = *N* – #level-2 fixed effects
    - **Level-1 DDF** = Total DDF – Level-2 DDF – #level-1 fixed effects
    - Level-1 effects with random slopes still get level-1 DDF

- **Satterthwaite** (DDFM=Satterthwaite in SAS, default in SPSS):
  - More complicated, but analogous to two-group *t*-test given unequal residual variances and unequal group sizes
  - Incorporates contribution of variance components at each level
    - Level-2 DDF will resemble Level-2 DDF from BW
    - Level-1 DDF will resemble Level-1 DDF from BW if the level-1 effect is not random, but will resemble level-2 DDF if it is random

Denominator DF (DDF) Methods

- **Kenward-Roger** (DDFM=KR in SAS, not in SPSS):
  - Adjusts the sampling covariance matrix of the fixed effects and variance components to reflect the uncertainty introduced by using large-sample techniques of ML/REML in small *N* samples
  - This creates different (larger) SEs for the fixed effects
  - Then uses Satterthwaite DDF, new SEs, and *t* to get *p*-values

- In an unstructured variance model, all effects use level-2 DDF
- Differences in inference not likely to matter often in practice
  - e.g., critical *t*-value at DDF=20 is 2.086, at infinite DDF is 1.960
- When in doubt, use KR (is overkill at worst, becomes Satterthwaite)
  - I used Satterthwaite in the book to maintain comparability across programs

Evaluating Statistical Significance of Multiple New Fixed Effects at Once

- Compare nested models with ML −2ΔLL test
- Useful for 'borderline' cases - example:
  - Ed*time^2 interaction at p = .04, with nonsignificant ed*time and ed*Intercept (main effect of ed) terms?
  - Is it worth keeping a marginal higher-order interaction that requires two (possibly non-significant) lower-order terms?
  - ML −2ΔLL test on df=3: −2ΔLL must be > 7.82
  - **REML is WRONG for −2ΔLL tests for models with different fixed effects, regardless of nested or non-nested**
  - Because of this, it may be more convenient to switch to ML when focusing on modeling fixed effects of predictors

- Compare non-nested models with ML AIC & BIC instead

Wrapping Up…

- MLM uses ONLY rows of data that are COMPLETE – both predictors AND outcomes must be there!
  - Using whatever data you do have for each person will likely lead to better inferences and more statistical power than using only complete persons (listwise deletion)

- Time-invariant predictors modify the level-1 created growth curve → predict individual intercepts and slopes
  - They account for random effect variances (the predictors are the reasons WHY people need their own intercepts and slopes)
  - If a level-1 effect is not random, it can still be moderated by a cross-level interaction with a time-invariant predictor…
    - … but then it will predict L1 residual variance instead