Introduction to Multilevel Models for Longitudinal and Repeated Measures Data

- Today's Class:
  - ➤ Features of longitudinal data
  - ➤ Features of longitudinal models
  - ➤ What can MLM do for you?

What is PSYC 944 about?

- "Longitudinal" data
  - ➤ Same individual units of analysis measured at different occasions (which can range from milliseconds to decades)

- "Repeated measures" data
  - ➤ Same individual units of analysis measured via different items, using different stimuli, or under different conditions

- Both of these fall under a more general category of "multivariate" data of varying complexity
  - ➤ The link between them is the use of **random effects** to describe covariance of outcomes from the same unit

Data Requirements for Our Models

- A useful outcome variable:
  - Has an interval scale*
    - A one-unit difference means the same thing across all scale points
    - In subscales, each contributing item has an equivalent scale

  - Has scores with the same meaning over observations
    - Includes meaning of construct
    - Includes how items relate to the scale
    - Implies measurement invariance

- FANCY MODELS CANNOT SAVE BADLY MEASURED VARIABLES.

* For the models covered in PSYC 944 specifically
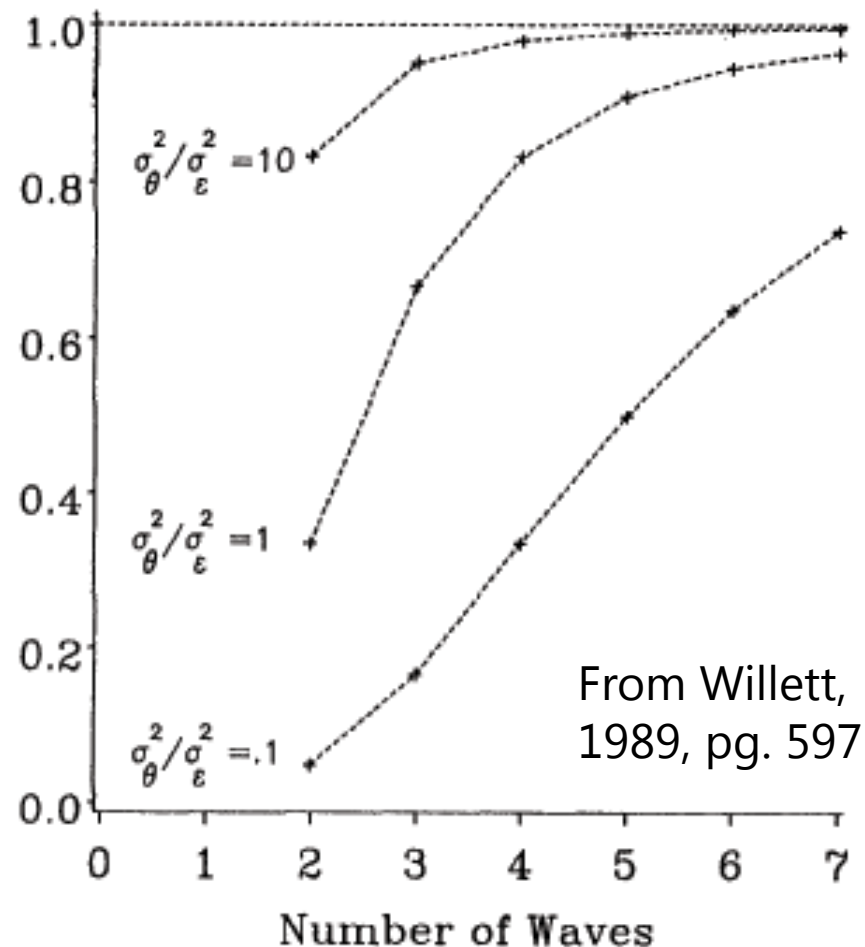
Data Requirements for Our Models

- Multiple <u>OUTCOMES</u> from the same sampling unit!

    - 2 is the minimum, but just 2 can lead to problems:
        - Only 1 kind of change is observable (1 difference)
        - Can't distinguish "real" individual differences in change from error
        - Repeated measures ANOVA is just fine for 2 observations
            - Necessary assumption of "sphericity" is satisfied with only 2 observations even if compound symmetry doesn't hold

    - More data is better (with diminishing returns)
        - More occasions → better description of the form of change
        - More persons → better estimates of amount of individual differences in change; better prediction of those individual differences
        - More items/stimuli →more power to show effects of differences between items/stimuli/conditions

Power in Longitudinal Data

- **More occasions are better!**
  - ➤ Can examine more complex growth functions
  - ➤ Can get more reliable individual growth parameters

- **More people are better!**
  - ➤ Can get more reliable measures of individual differences

Reliability of Slopes by Signal to Noise Ratio and # Occasions

$\sigma_\theta^2/\sigma_\varepsilon^2 = 10$

$\sigma_\theta^2/\sigma_\varepsilon^2 = 1$

$\sigma_\theta^2/\sigma_\varepsilon^2 = .1$

From Willett, 1989, pg. 597

Number of Waves

Levels of Analysis in Longitudinal Data

- ## Between-Person (BP) Variation:
  - ➢ **Level-2** – "**INTER**-individual Differences" – Time-Invariant
  - ➢ All longitudinal studies begin as cross-sectional studies

- ## Within-Person (WP) Variation:
  - ➢ **Level-1** – "**INTRA**-individual Differences" – Time-Varying
  - ➢ Only longitudinal studies can provide this extra information

- ## Longitudinal studies allow examination of both types of relationships simultaneously (and their interactions)
  - ➢ Any variable measured over time usually has both BP and WP variation
  - ➢ BP = more/less than other people; WP = more/less than one's average

- ## Later, Between-Item Variation will also show up as another Level-2 source of variation in other repeated measures data

A Longitudinal Data Continuum

- **Within-Person Change**: Systematic change
  - ➢ Magnitude or direction of change can be different across individuals
  - ➢ "Growth curve models" → Time is meaningfully sampled

- **Within-Person Fluctuation**: No systematic change
  - ➢ Outcome just varies/fluctuates over time (e.g., emotion, stress)
  - ➢ Time is just a way to get lots of data per individual

Pure WP Change
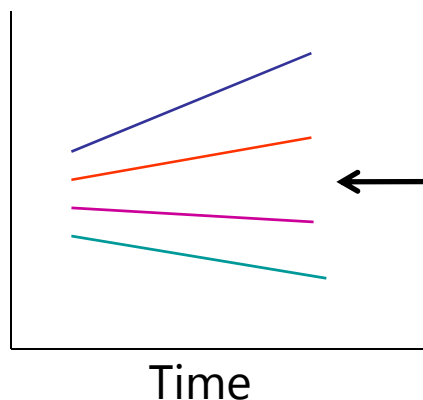
Pure WP Fluctuation

Time

Time
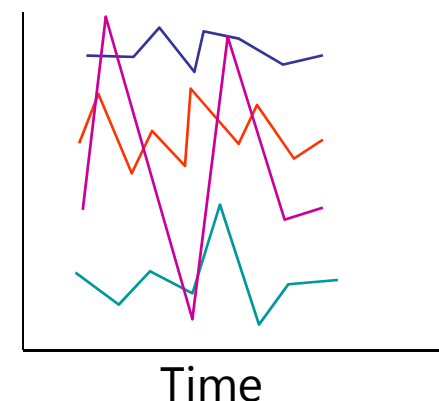
What is a Multilevel Model (MLM)?

- <u>Same as other terms you have heard of:</u>
  - **General Linear Mixed Model** (if you are from statistics)
    - *Mixed* = Fixed and Random effects
  - **Random Coefficients Model** (also if you are from statistics)
    - Random coefficients = Random effects
  - **Hierarchical Linear Model** (if you are from education)
    - Not the same as hierarchical regression

- <u>Special cases of MLM:</u>
  - Random Effects ANOVA or Repeated Measures ANOVA
  - (Latent) Growth Curve Model (where "Latent" → SEM)
  - Within-Person Variation Model (e.g., for daily diary data)
  - Clustered/Nested Observations Model (e.g., for kids in schools)
  - Cross-Classified Models (e.g., "value-added" models)

The Two Sides of Any Model

- ## **Model for the Means:**

  - ➤ *Aka* **Fixed Effects**, Structural Part of Model

  - ➤ What you are used to **caring about for testing hypotheses**

  - ➤ How the expected outcome for a given observation varies as a function of values on predictor variables

- ## **Model for the Variances:**

  - ➤ *Aka* **Random Effects and Residuals**, Stochastic Part of Model

  - ➤ What you are used to **making assumptions about** instead

  - ➤ How residuals are distributed and related across observations (persons, groups, time, etc.) → these relationships are called "dependency" and *this is the primary way that MLM differs from the GLM*

Dimensions for Organizing Models

- Outcome type: General (normal) vs. General*ized* (not normal)

- Dimensions of sampling: One (so one variance term per outcome) vs. Multiple (so multiple variance terms per outcome) **→ OUR WORLD**

- **General Linear Models:** conditionally normal outcome distribution, **fixed effects** (identity link; only one dimension of sampling)

  Note: Least Squares is only for GLM

- **General*ized* Linear Models: any conditional outcome distribution**, **fixed** effects through **link functions**, no random effects (one dimension)

- **General Linear Mixed Models:** conditionally normal outcome distribution, **fixed and random effects** (identity link, but multiple sampling dimensions)

- **General*ized* Linear Mixed Models: any conditional outcome distribution**, **fixed and random effects** through **link functions** (multiple dimensions)

- "Linear" means the fixed effects predict the *link-transformed* DV in a linear combination of (effect*predictor) + (effect*predictor)...

Options for Longitudinal Models

- Although models and software are logically separate, longitudinal data can be analyzed via multiple analytic frameworks:
  - "Multilevel/Mixed Models"
    - Dependency over time, persons, groups, etc. is modeled via random effects (multivariate through "levels" using stacked/long data)
    - Builds on GLM, generalizes easier to additional levels of analysis
  - "Structural Equation Models"
    - Dependency over time *only* is modeled via latent variables (single-level analysis using multivariate/wide data)
    - Generalizes easier to broader analysis of latent constructs, mediation

  - Because random effects and latent variables are the same thing, many longitudinal models can be specified/estimated either way
    - And now "Multilevel Structural Equation Models" can do it all…

What can MLM do for you?

1. **Model dependency across observations**
   - Longitudinal, clustered, and/or cross-classified data? No problem!
   - Tailor your model of sources of correlation to your data

2. **Include categorical or continuous predictors at any level**
   - Time-varying, person-level, group-level predictors for each variance
   - Explore reasons for dependency, don't just control for dependency

3. **Does not require same data structure for each person**
   - Unbalanced or missing data? No problem!

4. **You already know how (or you will soon)!**
   - Use SPSS Mixed, **SAS Mixed**, Stata, Mplus, R, HLM, MlwiN…
   - What's an intercept? What's a slope? What's a pile of variance?

1. Model Dependency

- Sources of dependency depend on the sources of variation created by your sampling design: residuals for outcomes from the same unit are likely to be related, which violates the GLM "independence" assumption

- **"Levels" for dependency** = "levels of random effects"
  - Sampling dimensions can be **nested**
    - e.g., time within person, person within group, school within district
  - If you can't figure out the direction of your nesting structure, odds are good you have a **crossed sampling design** instead
    - e.g., persons crossed with items, raters crossed with targets
  - To have a "level", there must be random outcome variation due to sampling that remains after including the model's fixed effects
    - e.g., treatment vs. control does not create another level of "group"

Longitudinal dependency comes from…

- Mean differences across sampling units (e.g., persons)
  - Creates *constant* dependency over time
  - Will be represented by a random intercept in our models

- Individual differences in effects of predictors
  - e.g., individual differences in growth, stress reactivity
  - Creates non-constant dependency, the size of which depends on the value of the predictor at each occasion
  - Will be represented by random slopes in our models

- Non-constant within-person correlation for unknown reasons (time-dependent autocorrelation)
  - Can add other patterns of correlation as needed for this

Why care about dependency?

- In other words, what happens if we have the wrong model for the variances (assume independence instead)?

- **Validity of the tests of the predictors** depends on having the "most right" model for the variances

  - ➤ Estimates will usually be ok → come from model for the means

  - ➤ Standard errors (and thus p-values) can be compromised

- The sources of variation that exist in your outcome will dictate **what kinds of predictors** will be useful

  - ➤ Between-Person variation needs Between-Person predictors

  - ➤ Within-Person variation needs Within-Person predictors

  - ➤ Between-Item variation needs Between-Item predictors

2. Include categorical or continuous predictors at any level of analysis

- <u>ANOVA</u>: test differences among <u>discrete IV factor levels</u>
  - Between-Groups: Gender, Intervention, Age Groups
  - Within-Subjects (Repeated Measures): Condition, Time
  - Test main effects of continuous covariates (ANCOVA)

- <u>Regression</u>: test whether <u>slopes</u> relating predictors to outcomes are different from 0
  - Persons measured once, differ categorically or continuously on a set of time-invariant (person-level) covariates

- What if a predictor is assessed repeatedly (time-varying predictors) but can't be characterized by 'conditions'?
  - ANOVA or Regression won't work → need MLM

2. Include categorical or continuous predictors at any level of analysis

- Some things <u>don't change</u> over measurements…
  - ➢ Sex, Ethnicity
    - → <u>Time-Invariant</u> Predictor = <u>Person</u> Level
- Some things <u>do change</u> over measurements…
  - ➢ Health Status, Stress Levels, Living Arrangements
    - → <u>Time-Varying</u> Predictor = <u>Time</u> Level
- Some predictors might be measured at <u>higher levels</u>
  - ➢ Family SES, length of marriage, school size
- Interactions between levels may be included, too
  - ➢ Does the effect of <u>health status</u> differ by <u>gender</u> and <u>SES</u>?

Level: Time   Person   Family

3. Does not require same data structure per person (by accident or by design)

RM ANOVA: uses **multivariate** (wide) data structure:

ID	Sex	T1	T2	T3	T4
100	0	5	6	8	12
101	1	4	7	.	11

<u>People</u> missing any data are excluded (data from ID 101 are not included at all)

MLM: uses **stacked** (long) data structure:

Only <u>rows</u> missing data are excluded

100 uses 4 cases
101 uses 3 cases

ID	Sex	Time	Y
100	0	1	5
100	0	2	6
100	0	3	8
100	0	4	12
101	1	1	4
101	1	2	7
101	1	3	.
101	1	4	11

Time can also be **unbalanced** across people such that each person can have his or her own measurement schedule: Time "0.9" "1.4" "3.5" "4.2"...

4. You already know how!

- If you can do GLM, you can do MLM.

- How do you interpret an estimate for...
  - ➢ the intercept?

  - ➢ the effect of a continuous variable?

  - ➢ the effect of a categorical variable?

  - ➢ a variance component ("pile of variance")?

This Semester's Topics

- Review of linear models and interactions
- Within-person analysis via ANOVA
- Describing within-person fluctuation via alternative covariance structure models
  - ➢ R matrix structures with and without a random intercept
- Describing within-person change via random effects
  - ➢ Polynomial, piecewise, and nonlinear models
- Time-invariant (level-2) predictors
- Level-2 predictors in crossed repeated measures designs
- Time-varying (level-1) predictors
- MLM for persons within groups (clustered data)
- Multivariate longitudinal models
- Three-level models

Coursework Structure

- SAS PROC MIXED for everything!
  - Available in 234 Burnett on Mondays, and during office hours from 2-4 Wednesdays and Fridays (in 230 or 227 otherwise)
  - Available for purchase for $25 from the Psych department, see: http://psychology.unl.edu/sas-distribution-information
  - Available remotely for free via Holland Computing Center, see handout on websites for 944 (for SAS) or 948 (for Mplus)

- 8 weekly online homeworks (until spring break)
  - Mix of data analysis, results sections, and conceptual questions

- Readings are not considered optional!
  - 9 chapters from my book written specifically for this class
  - Other articles that I've found to be readable and useful

(Mostly) after Spring Break…

- **Individual projects!**
  - Outline due 2/14 as planning mechanism
  - Unconditional models due 4/4
  - Conditional models due 4/18
  - Individual feedback and opportunity for revision of each part

- 3 main kinds of data structures to choose from:
  - Within-person change (at least three occasions)
  - Within-person fluctuation (at least three occasions)
  - Other repeated measures designs (nested or crossed)

- Outcome needs to be suitable for general models (i.e., residuals should be normally distributed)