

## ANOVA Longitudinal Models for the Practice Effects Data: via GLM

### Model 1. Saturated Means Model for Session, E-only Variances Model (BP)

Means Model: 6 means (1 intercept + 5 mean differences)  
 Variances Model: NO correlation, EQUAL variance across time

#### SPSS GLM:

```
TITLE "SPSS Saturated Means, E-Only Variances Model via GLM".
SUBTITLE "BP ANOVA".
UNIANOVA nm3rt BY session
  /EMMEANS = TABLES(session)
  /DESIGN = session.
```

#### SAS GLM:

```
TITLE1 "SAS: Saturated Means, E-Only Variances Model via GLM";
TITLE2 "BP ANOVA";
PROC GLM DATA=stacked;
  CLASS session;
  MODEL nm3rt = session / SOLUTION;
  LSMEANS session / STDERR; RUN; QUIT;
```

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
<b>Model</b>	<b>5</b>	<b>5606381.8</b>	<b>1121276.4</b>	<b>4.73</b>	<b>0.0003</b>
<b>Error</b>	<b>600</b>	<b>142087780.8</b>	<b>236813.0</b>		
Corrected Total	605	147694162.6			

R-Square	Coeff Var	Root MSE	nm3rt Mean
0.037959	27.48257	486.6343	1770.701

Parameter	Estimate	Standard Error	t Value	Pr >  t
Intercept	1672.136012 B	48.42192532	34.53	<.0001
Session 1	289.757355 B	68.47894350	4.23	<.0001
Session 2	143.036372 B	68.47894350	2.09	0.0371
Session 3	77.898636 B	68.47894350	1.14	0.2558
Session 4	45.660447 B	68.47894350	0.67	0.5052
Session 5	35.039719 B	68.47894350	0.51	0.6091
Session 6	0.000000 B	.	.	.

NOTE: The X'X matrix has been found to be singular, and a generalized inverse was used to solve the normal equations.

Terms whose estimates are followed by the letter 'B' are not uniquely estimable.

#### Least Squares Means

Session	nm3rt LSMEAN	Standard Error	Pr >  t
<b>1</b>	<b>1961.89337</b>	<b>48.42193</b>	<b>&lt;.0001</b>
<b>2</b>	<b>1815.17238</b>	<b>48.42193</b>	<b>&lt;.0001</b>
<b>3</b>	<b>1750.03465</b>	<b>48.42193</b>	<b>&lt;.0001</b>
<b>4</b>	<b>1717.79646</b>	<b>48.42193</b>	<b>&lt;.0001</b>
<b>5</b>	<b>1707.17573</b>	<b>48.42193</b>	<b>&lt;.0001</b>
<b>6</b>	<b>1672.13601</b>	<b>48.42193</b>	<b>&lt;.0001</b>

**Model 2. Saturated Means Model for Session,  $U_0 + e$  (CS) Variances Model (WP)**

**Model 3. Saturated Means Model for Session, Saturated (Multivariate) Variances Model (WP)**

**Means Model: 6 means (1 intercept + 5 mean differences)**

**Variances Model: EQUAL correlation, EQUAL variance across time → CS → Univ. RM ANOVA**

**Variances Model: Completely UNEQUAL correlation and variance across time → UN → Multiv. RM ANOVA**

**SPSS GLM:**

```

DATASET ACTIVATE chapmultiv WINDOW=FRONT.
TITLE "SPSS Saturated Means, CS and Unstructured Variances Models via GLM".
SUBTITLE "RM ANOVA".
GLM nm3rt1 nm3rt2 nm3rt3 nm3rt4 nm3rt5 nm3rt6
  /WSFACTOR = session 6
  /EMMEANS = TABLES(session).
    
```

**SAS GLM:**

```

TITLE "SAS Saturated Means, CS and Unstructured Variances Models via GLM";
TITLE2 "RM ANOVA";
PROC GLM DATA=multiv;
  MODEL nm3rt1-nm3rt6 = / SOLUTION NOUNI;
  REPEATED session / PRINT;
RUN; QUIT;
    
```

Variables	Sphericity Tests			
	DF	Mauchly's Criterion	Chi-Square	Pr > ChiSq
Transformed Variates	14	0.1004957	225.39847	<.0001
Orthogonal Components	14	0.3154422	113.18581	<.0001

**MANOVA Test Criteria and Exact F Statistics for the Hypothesis of no session Effect**

H = Type III SSCP Matrix for session  
 E = Error SSCP Matrix

S=1 M=1.5 N=47

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.54465555	16.05	5	96	<.0001
Pillai's Trace	0.45534445	16.05	5	96	<.0001
Hottelling-Lawley Trace	0.83602279	16.05	5	96	<.0001
Roy's Greatest Root	0.83602279	16.05	5	96	<.0001

**Repeated Measures Analysis of Variance**

**Univariate Tests of Hypotheses for Within Subject Effects**

Source	DF	Type III SS	Mean Square	F Value	Pr > F	Adj Pr > F	
						G - G	H - F
session	5	5606381.77	1121276.35	32.85	<.0001	<.0001	<.0001
Error(session)	500	17068104.97	34136.21				
Greenhouse-Geisser Epsilon		0.6550					
Huynh-Feldt Epsilon		0.6798					

## ANOVA Longitudinal Models for the Practice Effects Data: via MIXED Syntax

### SAS Syntax:

**Model 1. Saturated Means Model for Session, E-only Variances Model (BP – no covariance over time)**

```
TITLE1 "SAS: Saturated Means, E-Only Variances Model via MIXED";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  LSMEANS session / ; RUN;
```

**Model 2. Saturated Means Model for Session,  $U_0 + e_{ti}$  (CS) Variances Model (WP – constant covariance)**

```
TITLE1 "SAS: Saturated Means, CS Variances Model";
TITLE2 "Univariate RM ANOVA";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
  REPEATED session / R RCORR TYPE=CS SUBJECT=ID;
  LSMEANS session / ; RUN;
```

**Model 3. Saturated Means Model for Session, Saturated (Multivariate) Variances Model (WP – any covariance)**

```
TITLE1 "SAS: Saturated Means, Unstructured Variances Model";
TITLE2 "Multivariate RM ANOVA";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
  REPEATED session / R RCORR TYPE=UN SUBJECT=ID;
  LSMEANS session / ; RUN;
```

### SPSS Syntax:

**Model 1. Saturated Means Model for Session, E-only Variances Model (BP – no covariance over time)**

```
TITLE "SPSS Saturated Means, E-Only Variances Model via MIXED".
MIXED nm3rt BY ID session
  /METHOD = REML
  /FIXED = session
  /PRINT = SOLUTION TESTCOV R
  /REPEATED = session | COVTYPE(ID) SUBJECT(ID)
  /EMMEANS = TABLES(session).
```

**Model 2. Saturated Means Model for Session,  $U_0 + e_{ti}$  (CS) Variances Model (WP – constant covariance)**

```
TITLE "SPSS Saturated Means, CS Variances Model".
SUBTITLE "Univariate RM ANOVA".
MIXED nm3rt BY ID session
  /METHOD = REML
  /FIXED = session
  /PRINT = SOLUTION TESTCOV R
  /REPEATED = session | COVTYPE(CS) SUBJECT(ID)
  /EMMEANS = TABLES(session).
```

**Model 3. Saturated Means Model for Session, Saturated (Multivariate) Variances Model (WP – any covariance)**

```
TITLE "SPSS Saturated Means, Unstructured Variances Model".
SUBTITLE "Multivariate RM ANOVA".
MIXED nm3rt BY ID session
  /METHOD = REML
  /FIXED = session
  /PRINT = SOLUTION TESTCOV R
  /REPEATED = session | COVTYPE(UN) SUBJECT(ID)
  /EMMEANS = TABLES(session).
```

**Model 1. Saturated Means Model for Session, E-Only Variances Model (BP – no covariance over time)**

**Means Model: 6 means (1 intercept + 5 mean differences)**

**Variances Model: NO correlation, EQUAL variance across time**

```
TITLE1 "SAS: Saturated Means, E-Only Variances Model via MIXED";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
  REPEATED session / R TYPE=VC SUBJECT=ID;
  LSMEANS session / ; RUN;
```

Dimensions  
 Covariance Parameters 1 still just e in model for variances  
 Columns in X 7 should be 6 but it counts the unidentified one  
 Columns in Z 0 still no U's yet

Estimated R Matrix for ID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	236813					
2		236813				
3			236813			
4				236813		
5					236813	
6						236813

The **R matrix** is the **unstandardized** matrix of the error variances and covariances for each session. So far no covariance is allowed across time, with equal variance across time.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr >  Z
Session	ID	236813	13672	17.32	<.0001

E variance after accounting for means

Fit Statistics

-2 Res Log Likelihood	9155.4
AIC (smaller is better)	9157.4
AICC (smaller is better)	9157.4
BIC (smaller is better)	9160.0

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr >  t
Intercept		1672.14	48.4219	600	34.53	<.0001
Session	1	289.76	68.4789	600	4.23	<.0001
Session	2	143.04	68.4789	600	2.09	0.0371
Session	3	77.8986	68.4789	600	1.14	0.2558
Session	4	45.6604	68.4789	600	0.67	0.5052
Session	5	35.0397	68.4789	600	0.51	0.6091
Session	6	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Session	5	600	4.73	0.0003

The test of the fixed effects ( $p < .05$ ) tells us that there is a significant effect of time overall.  
 This test matches the results from BP ANOVA.

Least Squares Means

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr >  t
Session	1	1961.89	48.4219	600	40.52	<.0001
Session	2	1815.17	48.4219	600	37.49	<.0001
Session	3	1750.03	48.4219	600	36.14	<.0001
Session	4	1717.80	48.4219	600	35.48	<.0001
Session	5	1707.18	48.4219	600	35.26	<.0001
Session	6	1672.14	48.4219	600	34.53	<.0001

**Model 2. Saturated Means Model for Session,  $U_0 + e_{ti}$  (CS) Variances Model (WP – constant covariance)**

Means Model: 6 means (1 intercept + 5 mean differences)

Variances Model: EQUAL correlation, EQUAL variance across time

```
TITLE1 "SAS: Saturated Means, CS Variances Model";
TITLE2 "Univariate RM ANOVA";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
  REPEATED session / R RCORR TYPE=CS SUBJECT=ID;
  LSMEANS session / ; RUN;
```

Dimensions  
 Covariance Parameters 2  
 Columns in X 7  
 Columns in Z 0

Estimated R Matrix for ID 101 → COMBINED  $U_{0i} + e_{ti}$  VARIANCE AFTER DIFFERENT MEANS

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	<b>236813</b>	202677	202677	202677	202677	202677
2	202677	<b>236813</b>	202677	202677	202677	202677
3	202677	202677	<b>236813</b>	202677	202677	202677
4	202677	202677	202677	<b>236813</b>	202677	202677
5	202677	202677	202677	202677	<b>236813</b>	202677
6	202677	202677	202677	202677	202677	<b>236813</b>

Estimated R Correlation = .8559

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr >  Z	
<b>CS</b>	<b>ID</b>	<b>202677</b>	<b>29470</b>	<b>6.88</b>	<b>&lt;.0001</b>	Var( $U_{0i}$ ) after accounting for means
<b>Residual</b>		<b>34136</b>	<b>2158.96</b>	<b>15.81</b>	<b>&lt;.0001</b>	Var( $e_{ti}$ ) after accounting for means

Fit Statistics

-2 Res Log Likelihood	8353.4
AIC (smaller is better)	8357.4
AICC (smaller is better)	8357.4
BIC (smaller is better)	8362.6

**Does saturated means, CS variances model fit better than the saturated means, no correlation variances model?**

**Model 1 vs. Model 2**  
 9155.4 vs. 8353.4 diff for df=1 > 2.7, so yes

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr >  t
Intercept		1672.14	48.4219	129	34.53	<.0001
Session	1	289.76	25.9993	500	11.14	<.0001
Session	2	143.04	25.9993	500	5.50	<.0001
Session	3	77.8986	25.9993	500	3.00	0.0029
Session	4	45.6604	25.9993	500	1.76	0.0797
Session	5	35.0397	25.9993	500	1.35	0.1784
Session	6	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
<b>Session</b>	<b>5</b>	<b>500</b>	<b>32.85</b>	<b>&lt;.0001</b>

The test of the fixed effects ( $p < .05$ ) tells us that there is a significant effect of time overall.

This test matches the results from Univ. RM ANOVA.

Least Squares Means

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr >  t
Session	1	1961.89	48.4219	129	40.52	<.0001
Session	2	1815.17	48.4219	129	37.49	<.0001
Session	3	1750.03	48.4219	129	36.14	<.0001
Session	4	1717.80	48.4219	129	35.48	<.0001
Session	5	1707.18	48.4219	129	35.26	<.0001
Session	6	1672.14	48.4219	129	34.53	<.0001

**Model 3. Saturated Means Model for Session, Saturated (Multivariate) Variances Model (WP – any covariance)**

**Means Model: 6 means (1 intercept + 5 mean differences)**  
**Variances Model: Completely UNEQUAL correlation and variance across time**

```
TITLE1 "SAS: Saturated Means, Unstructured Variances Model";
TITLE2 "Multivariate RM ANOVA";
PROC MIXED DATA=&datafile. NOCLPRINT NOITPRINT COVTEST METHOD=REML;
  CLASS ID session;
  MODEL nm3rt = session / SOLUTION DDFM=Satterthwaite;
  REPEATED session / R RCORR TYPE=UN SUBJECT=ID;
  LSMEANS session / ; RUN;
```

Dimensions  
 Covariance Parameters 21  
 Columns in X 7  
 Columns in Z 0

**Estimated R Matrix for ID 101 → TOTAL COVARIANCE MATRIX AFTER DIFFERENT MEANS**

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	<b>301985</b>	235659	217994	202607	192154	195360
2	235659	<b>259150</b>	230217	213232	202092	193268
3	217994	230217	<b>233368</b>	205209	196919	188604
4	202607	213232	205209	<b>217544</b>	193676	185321
5	192154	202092	196919	193676	<b>212098</b>	187840
6	195360	193268	188604	185321	187840	<b>196733</b>

**Estimated R Correlation Matrix for ID 101 → TOTAL CORRELATION MATRIX AFTER DIFFERENT MEANS**

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8424	0.8212	0.7905	0.7593	0.8015
2	0.8424	1.0000	0.9361	0.8981	0.8620	0.8559
3	0.8212	0.9361	1.0000	0.9108	0.8851	0.8802
4	0.7905	0.8981	0.9108	1.0000	0.9016	0.8958
5	0.7593	0.8620	0.8851	0.9016	1.0000	0.9196
6	0.8015	0.8559	0.8802	0.8958	0.9196	1.0000

Fit Statistics  
 -2 Res Log Likelihood 8229.8  
 AIC (smaller is better) 8271.8  
 AICC (smaller is better) 8273.4  
 BIC (smaller is better) 8326.7

**Does saturated means, unstructured variances model fit better than the saturated means, CS variances model?**

**Model 2 vs. Model 3**  
 8353.4 vs. 8229.8 diff for df=19 > 30.1, so yes

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr >  t
Intercept		1672.14	44.1345	100	37.89	<.0001
Session	1	289.76	32.7000	100	8.86	<.0001
Session	2	143.04	26.2031	100	5.46	<.0001
Session	3	77.8986	22.8842	100	3.40	0.0010
Session	4	45.6604	20.7853	100	2.20	0.0303
Session	5	35.0397	18.1168	100	1.93	0.0559
Session	6	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
<b>Session</b>	<b>5</b>	<b>100</b>	<b>16.72</b>	<b>&lt;.0001</b>

Least Squares Means

The test of the fixed effects (p < .05) tells us that there is a significant effect of time overall.

This test matches the results from Multiv. RM ANOVA.

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr >  t
Session	1	1961.89	54.6805	100	35.88	<.0001
Session	2	1815.17	50.6541	100	35.83	<.0001
Session	3	1750.03	48.0684	100	36.41	<.0001
Session	4	1717.80	46.4101	100	37.01	<.0001
Session	5	1707.18	45.8255	100	37.25	<.0001
Session	6	1672.14	44.1345	100	37.89	<.0001