# Example 3: Time-Invariant Predictors of Practice Effects (uses same data as Example 2)

***(complete data, syntax, and output available for SAS, SPSS, and STATA electronically)***

In this example we will examine time-invariant predictors of individual differences in intercepts, linear slopes, and quadratic slopes representing improvement in RT (in msec) across six practice sessions. We will examine age, abstract reasoning, and education in sequential conditional (predictor) polynomial models.

## SAS Code for Data Manipulation:

```
* Centering level-2 predictor variables for analysis;
DATA work.example23; SET work.example23;
       age80  = age - 80;         * Convenient value;
       reas22 = absreas - 22;     * Near sample mean;
       LABEL age80  = "age80: Age Centered (0=80)"
             reas22 = "reas22: Abstract Reasoning Centered (0=22)";
       * Make education a grouping variable for purpose of demonstration only;
            IF educyrs LE 12                   THEN educgrp=1;
       ELSE IF educyrs GT 12 AND EducYrs LE 16 THEN educgrp=2;
       ELSE IF educyrs GT 16                   THEN educgrp=3;
       ELSE IF educyrs = .                     THEN educGrp=.;
       LABEL educgrp = "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)";
* Removing cases with missing predictors;
       IF NMISS(age80, reas22, educgrp)>0 THEN DELETE;
RUN;
```

## SPSS Code for Data Manipulation:

```
* Centering level-2 predictor variables for analysis.
DATASET ACTIVATE example23 WINDOW=FRONT.
COMPUTE age80  = age - 80.
COMPUTE reas22 = absreas - 22.
VARIABLE LABELS
       age80 "age80: Age Centered (0=80)"
       reas22 "reas22: Abstract Reasoning Centered (0=22)".
* Make education a grouping variable for purpose of demonstration only.
IF educyrs LE 12                   educgrp=1.
IF educyrs GT 12 AND educyrs LE 16 educgrp=2.
IF educyrs GT 16                   educgrp=3.
VARIABLE LABELS educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)".
* Removing cases with missing predictors.
SELECT IF (NVALID(age80, reas22, educgrp)=3).
EXECUTE.
```

## STATA Code for Data Manipulation:

```
* centering level-2 predictor variables for analysis
gen age80 = age - 80
gen reas22 = absreas - 22
label variable age80 "age80: Age Centered (0=80 years)"
label variable reas22 "reas22: Abstract Reasoning Centered (0=22)"
* make education a grouping variable for purpose of demonstration only
gen educgrp=.
replace educgrp=1 if (educyrs <= 12)
replace educgrp=2 if (educyrs > 12 & educyrs <= 16)
replace educgrp=3 if (educyrs > 16)
label variable educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)"

 * create new variable to hold number of missing cases
 * then drop cases with incomplete predictors
egen nummiss = rowmiss(age80 reas22 educgrp)
drop if nummiss>0
```

## Model 3b. Random Quadratic Time Baseline (in ML now)

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + U_{2i}$

```
TITLE1 "SAS Model 3b: Random Quadratic Time Baseline in ML";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess / SOLUTION DDFM=Satterthwaite OUTPM=work.TimePred;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR V VCORR TYPE=UN SUBJECT=ID;
      REPEATED session / R TYPE=VC SUBJECT=ID; RUN;
PROC CORR NOSIMPLE DATA=work.TimePred; VAR nm3rt pred; RUN;

TITLE "SPSS Model 3b: Random Quadratic Time Baseline in ML".
MIXED nm3rt BY ID session WITH c1sess
      /METHOD  = ML
      /PRINT   = SOLUTION TESTCOV G R
      /FIXED   = c1sess c1sess*c1sess
      /RANDOM  = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predtime).
CORRELATIONS nm3rt predtime.
```

The OUTPM in SAS, /SAVE in SPSS, and predict in STATA calculate outcomes predicted by the fixed effects. We can then correlate the predicted and actual outcomes to get total $R^2$ (actual variance explained).

```
* STATA Model 3b: Random Quadratic Time Baseline in ML
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess,   || id: c1sess c1sess2,  ///
      variance mle covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store Baseline,  // save LL for LRT
      predict predtime           // save fixed-effect predicted outcomes
corr nm3rt predtime              // get total r to make r2
```

### SAS Output:

Estimated G Matrix

| Row | Effect | ID: Participant ID | Col1 | Col2 | Col3 |
|---|---|---|---|---|---|
| 1 | Intercept | 101 | 273306 | -35262 | 3845.38 |
| 2 | c1sess | 101 | -35262 | 25438 | -3837.76 |
| 3 | c1sess*c1sess | 101 | 3845.38 | -3837.76 | 622.81 |

Estimated G Correlation Matrix

| Row | Effect | ID: Participant ID | Col1 | Col2 | Col3 |
|---|---|---|---|---|---|
| 1 | Intercept | 101 | 1.0000 | -0.4229 | 0.2947 |
| 2 | c1sess | 101 | -0.4229 | 1.0000 | **-0.9642** |
| 3 | c1sess*c1sess | 101 | 0.2947 | **-0.9642** | 1.0000 |

Note how correlated the linear and quadratic random slopes are…

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr Z | |
|---|---|---|---|---|---|---|
| UN(1,1) | ID | 273306 | 40828 | 6.69 | <.0001 | Random Intercept variance |
| UN(2,1) | ID | -35262 | 11765 | -3.00 | 0.0027 | Intercept-Linear slope covariance |
| UN(2,2) | ID | 25438 | 5781.19 | 4.40 | <.0001 | Random Linear slope variance |
| UN(3,1) | ID | 3845.38 | 1920.35 | 2.00 | 0.0452 | Intercept-Quadratic slope covariance |
| UN(3,2) | ID | -3837.76 | 968.79 | -3.96 | <.0001 | Linear-Quadratic slope covariance |
| UN(3,3) | ID | 622.81 | 169.99 | 3.66 | 0.0001 | Random Quadratic slope variance |
| session | ID | 20298 | 1649.11 | 12.31 | <.0001 | Residual variance |

Information Criteria

| Neg2LogLike | **Parms** | AIC | AICC | HQIC | BIC | CAIC |
|---|---|---|---|---|---|---|
| 8321.8 | **10** | 8341.8 | 8342.1 | 8352.4 | 8367.9 | 8377.9 |

In ML, the #parms is ALL parms (both sides of model).

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1945.85 | 53.5825 | 101 | 36.32 | <.0001 |
| c1sess | -120.90 | 19.9481 | 101 | -6.06 | <.0001 |
| c1sess*c1sess | 13.8656 | 3.3985 | 101 | 4.08 | <.0001 |

Pearson Correlation Coefficients, N = 606
Prob > |r| under H0: Rho=0

| | nm3rt | Pred |
|---|---|---|
| nm3rt | 1.00000 | 0.19167 |
| nm3rt: Number-Match 3 RT in ms | | <.0001 |

R =.1917, so $R^2$ for time = .0367 The model for the means (fixed linear and quadratic session effects so far) accounted for ~4% of the variance in RT.

## Model 4a. Age as Predictor of Intercept, Linear, and Quadratic Time Slopes

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$

```
TITLE1 "SAS Model 4a: Age as Predictor of Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
       CLASS ID session;
       MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                             / SOLUTION DDFM=Satterthwaite OUTPM=work.AgePred;
       RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
       REPEATED session / TYPE=VC SUBJECT=ID;
       * Requesting additional effects for age;
       ESTIMATE "Age Effect at Session 1" age80 1 c1sess*age80 0 c1sess*c1sess*age80 0;
       ESTIMATE "Age Effect at Session 2" age80 1 c1sess*age80 1 c1sess*c1sess*age80 1;
       ESTIMATE "Age Effect at Session 3" age80 1 c1sess*age80 2 c1sess*c1sess*age80 4;
       ESTIMATE "Age Effect at Session 4" age80 1 c1sess*age80 3 c1sess*c1sess*age80 9;
       ESTIMATE "Age Effect at Session 5" age80 1 c1sess*age80 4 c1sess*c1sess*age80 16;
       ESTIMATE "Age Effect at Session 6" age80 1 c1sess*age80 5 c1sess*c1sess*age80 25;
RUN; PROC CORR NOSIMPLE DATA=work.AgePred; VAR nm3rt pred; RUN;

TITLE "SPSS Model 4a: Age as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session WITH c1sess age80
       /METHOD = ML
       /PRINT  = SOLUTION TESTCOV G R
       /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
```

```
       /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
       /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
       /SAVE = FIXPRED (predage)
       /TEST = "Age Effect at Session 1" age80 1 c1sess*age80 0 c1sess*c1sess*age80 0
       /TEST = "Age Effect at Session 2" age80 1 c1sess*age80 1 c1sess*c1sess*age80 1
       /TEST = "Age Effect at Session 3" age80 1 c1sess*age80 2 c1sess*c1sess*age80 4
       /TEST = "Age Effect at Session 4" age80 1 c1sess*age80 3 c1sess*c1sess*age80 9
       /TEST = "Age Effect at Session 5" age80 1 c1sess*age80 4 c1sess*c1sess*age80 16
       /TEST = "Age Effect at Session 6" age80 1 c1sess*age80 5 c1sess*c1sess*age80 25.
CORRELATIONS nm3rt predage.
```

```
* STATA Model 4a: Age as Predictor of Intercept, Linear, and Quadratic
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                      ///
       c.age80 c.age80#c.c1sess c.age80#c.c1sess#c.c1sess,   ///
       || id: c1sess c1sess2,                                 ///
       variance ml covariance(un) residuals(independent,t(session)),
       estat ic, n(101),
       estat recovariance, level(id),
       estimates store age,              // save LL for LRT
       lrtest Age Baseline,              // LRT against non-age baseline
       predict predage                   // save fixed-effect predicted outcomes
       margins, at(c.c1sess=(0(1)5)) dydx(c.age80) vsquish     // age slope per session
       margins, at(c.c1sess=(0(1)5) c.age80=(-5 0 5)) vsquish  // predictions per session
       marginsplot, name(predicted_age, replace)               // plot age predictions
corr nm3rt predage          // get total r to make r2
```

## SAS Output:

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr Z | |
|---|---|---|---|---|---|---|
| UN(1,1) | ID | 242456 | 36490 | 6.64 | <.0001 | → intercept variance reduced by 11.29% |
| UN(2,1) | ID | -29320 | 10863 | -2.70 | 0.0070 | |
| UN(2,2) | ID | 24294 | 5623.77 | 4.32 | <.0001 | → linear slope variance reduced by 4.50% |
| UN(3,1) | ID | 3132.88 | 1792.88 | 1.75 | 0.0806 | |
| UN(3,2) | ID | -3700.54 | 949.39 | -3.90 | <.0001 | |
| UN(3,3) | ID | 606.35 | 167.75 | 3.61 | 0.0002 | → quad slope variance reduced by 2.64% |
| session | ID | 20298 | 1649.11 | 12.31 | <.0001 | → residual variance not reduced |

Information Criteria

| Neg2LogLike | Parms | AIC | AICC | HQIC | BIC | CAIC |
|---|---|---|---|---|---|---|
| 8310.2 | 13 | 8336.2 | 8336.8 | 8350.0 | 8370.2 | 8383.2 |

Is the age model (4a) better than the baseline random quadratic model (3b)?

Yes, $-2\Delta LL=11.6$ on df=3, *p*=.009

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1950.69 | 50.6713 | 101 | 38.50 | <.0001 |
| c1sess | -121.83 | 19.6695 | 101 | -6.19 | <.0001 |
| c1sess*c1sess | 13.9774 | 3.3757 | 101 | 4.14 | <.0001 |
| age80 | 29.0495 | 8.3774 | 101 | 3.47 | 0.0008 |
| c1sess*age80 | -5.5946 | 3.2519 | 101 | -1.72 | 0.0884 |
| c1sess*c1sess*age80 | 0.6709 | 0.5581 | 101 | 1.20 | 0.2321 |

Estimates

| Label | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Age Effect at Session 1 | 29.0495 | 8.3774 | 101 | 3.47 | 0.0008 |
| Age Effect at Session 2 | 24.1258 | 7.6097 | 101 | 3.17 | 0.0020 |
| Age Effect at Session 3 | 20.5439 | 7.4593 | 101 | 2.75 | 0.0070 |
| Age Effect at Session 4 | 18.3038 | 7.3302 | 101 | 2.50 | 0.0141 |
| Age Effect at Session 5 | 17.4056 | 7.0715 | 101 | 2.46 | 0.0155 |
| Age Effect at Session 6 | 17.8492 | 7.0545 | 101 | 2.53 | 0.0129 |

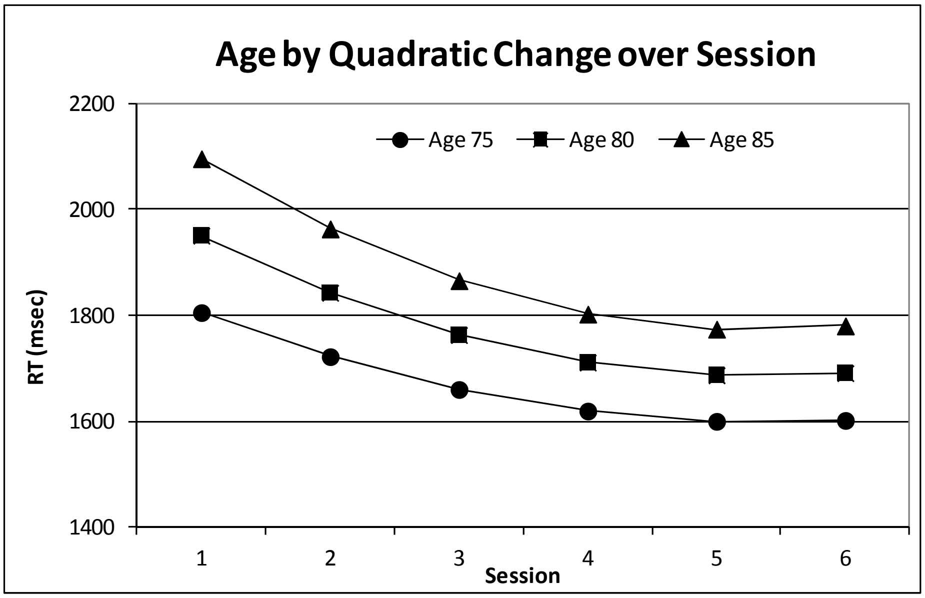
These are the simple slopes for the effect of age per session.

```
        Pearson Correlation Coefficients, N = 606
                Prob > |r| under H0: Rho=0
                                          nm3rt          Pred
nm3rt                                   1.00000       0.32688
nm3rt: Number-Match 3 RT in ms                         <.0001
```

R =.32689, so $R^2$ for time+age = .1069

The fixed effects of time before accounted for ~3.7% of the variance in RT, so there is a net increase of ~7% due to age.

The pattern of the interaction is shown by the simple effects of age at each session, graphed below.




## Model 5a. +Abstract Reasoning as Predictor of Intercept, Linear, and Quadratic Time Slopes

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(Session_{ti} - 1\right) + \beta_{2i}\left(Session_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(Age_i - 80\right) + \gamma_{02}\left(Reason_i - 22\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(Age_i - 80\right) + \gamma_{12}\left(Reason_i - 22\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(Age_i - 80\right) + \gamma_{22}\left(Reason_i - 22\right) + U_{2i}$

```
TITLE1 "SAS Model 5a: +Reasoning as Predictor of Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
       CLASS ID session;
       MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                     reas22 c1sess*reas22 c1sess*c1sess*reas22
                           / SOLUTION DDFM=Satterthwaite OUTPM=work.ReasPred;
       RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
       REPEATED session / TYPE=VC SUBJECT=ID;
RUN; PROC CORR NOSIMPLE DATA=work.ReasPred; VAR nm3rt pred; RUN;
TITLE "SPSS Model 5a: +Reasoning as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session WITH c1sess age80 reas22
       /METHOD = ML
       /PRINT  = SOLUTION TESTCOV G R
       /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                 reas22 c1sess*reas22 c1sess*c1sess*reas22
       /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
       /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
       /SAVE = FIXPRED (predreas).
CORRELATIONS nm3rt predreas
```

```
* STATA Model 5a: +Reasoning as Predictor of Intercept, Linear, and Quadratic
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                      ///
       c.age80  c.age80#c.c1sess c.age80#c.c1sess#c.c1sess    ///
       c.reas22 c.reas22#c.c1sess c.reas22#c.c1sess#c.c1sess, ///
       || id: c1sess c1sess2,                                 ///
       variance ml covariance(un) residuals(independent,t(session)),
       estat ic, n(101),
       estat recovariance, level(id),
       estimates store Reas,          // save LL for LRT
       lrtest Reas Age,               // LRT against age baseline
       predict predreas               // save fixed-effect predicted outcomes
corr nm3rt predreas                   // get total r to make r2
```

## SAS Output:

```
                 Covariance Parameter Estimates
                                  Standard         Z
Cov Parm     Subject     Estimate      Error     Value        Pr Z
UN(1,1)      ID            228049      34464      6.62      <.0001 → intercept variance reduced by 5.94%
UN(2,1)      ID            -31230      10649     -2.93      0.0034
UN(2,2)      ID             24041    5588.98      4.30      <.0001 → linear slope variance reduced by 1.04%
UN(3,1)      ID           3748.22    1746.63      2.15      0.0319
UN(3,2)      ID          -3618.98     937.05     -3.86      0.0001
UN(3,3)      ID            580.07     164.19      3.53      0.0002 → quad slope variance reduced by 4.33%
session      ID             20298    1649.11     12.31      <.0001 → residual variance not reduced

                           Information Criteria
Neg2LogLike      Parms        AIC        AICC        HQIC         BIC        CAIC
     8297.7         16     8329.7      8330.7      8346.7      8371.6      8387.6
```

Is the reasoning model (5a) better than the age model (4a)?

Yes, $-2\Delta LL = 12.5$ on df=3, $p = .0059$, so $\Delta R^2$ is significant

```
                        Solution for Fixed Effects
                                   Standard
Effect                   Estimate      Error      DF    t Value    Pr > |t|
Intercept                 1966.47    49.6658     101      39.59      <.0001
c1sess                    -119.74    19.7742     101      -6.06      <.0001
c1sess*c1sess             13.3036     3.3656     101       3.95      0.0001
age80                     22.2782     8.6018     101       2.59      0.0110
c1sess*age80              -6.4921     3.4247     101      -1.90      0.0609
c1sess*c1sess*age80        0.9601     0.5829     101       1.65      0.1026
reas22                   -27.1004    11.1141     101      -2.44      0.0165
c1sess*reas22             -3.5917     4.4250     101      -0.81      0.4189
c1sess*c1sess*reas22       1.1575     0.7531     101       1.54      0.1274

        Pearson Correlation Coefficients, N = 606
                Prob > |r| under H0: Rho=0
                                         nm3rt          Pred
nm3rt                                  1.00000       0.40108
nm3rt: Number-Match 3 RT in ms                        <.0001
```

$R = .4011$, so $R^2$ for time+age+reas = .1609

The fixed effects of time and age before accounted for ~10.7% of the variance in RT, so there is a net increase of ~5.4% due to reasoning.

## Model 5b. Abstract Reasoning on Intercept and Linear Time Slope Only

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$

```
TITLE1 "SAS Model 5b: Reasoning on Intercept and Linear Time Slope Only";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session;
      MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                    reas22 c1sess*reas22
                  / SOLUTION DDFM=Satterthwaite OUTPM=work.ReasPred2;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
      REPEATED session / TYPE=VC SUBJECT=ID;
      * Requesting additional effects for reasoning instead;
      ESTIMATE "Reasoning Effect at Session 1" reas22 1 c1sess*reas22 0;
      ESTIMATE "Reasoning Effect at Session 2" reas22 1 c1sess*reas22 1;
      ESTIMATE "Reasoning Effect at Session 3" reas22 1 c1sess*reas22 2;
      ESTIMATE "Reasoning Effect at Session 4" reas22 1 c1sess*reas22 3;
      ESTIMATE "Reasoning Effect at Session 5" reas22 1 c1sess*reas22 4;
      ESTIMATE "Reasoning Effect at Session 6" reas22 1 c1sess*reas22 5;
RUN; PROC CORR NOSIMPLE DATA=work.ReasPred2; VAR nm3rt pred; RUN;

TITLE "SPSS Model 5b: +Reasoning as Predictor of Intercept, Linear Time Slope Only".
MIXED nm3rt BY ID session WITH c1sess age80 reas22
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                reas22 c1sess*reas22
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (predreas2)
      /TEST = "Reasoning Effect at Session 1" reas22 1 c1sess*reas22 0
      /TEST = "Reasoning Effect at Session 2" reas22 1 c1sess*reas22 1
      /TEST = "Reasoning Effect at Session 3" reas22 1 c1sess*reas22 2
      /TEST = "Reasoning Effect at Session 4" reas22 1 c1sess*reas22 3
      /TEST = "Reasoning Effect at Session 5" reas22 1 c1sess*reas22 4
      /TEST = "Reasoning Effect at Session 6" reas22 1 c1sess*reas22 5.
CORRELATIONS nm3rt predreas2.

* STATA Model 5b: +Reasoning as Predictor of Intercept, Linear Time Slope Only
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                          ///
      c.age80  c.age80#c.c1sess c.age80#c.c1sess#c.c1sess    ///
      c.reas22 c.reas22#c.c1sess, || id: c1sess c1sess2,     ///
      variance ml covariance(un) residuals(independent,t(session)),
      estat ic, n(101),
      estat recovariance, level(id),
      estimates store Reas2,           // save LL for LRT
      lrtest Reas2 Age,                // LRT against age baseline
      margins, at(c.c1sess=(0(1)5)) dydx(c.reas22) vsquish     // reas slope per session
      margins, at(c.c1sess=(0(1)5) c.reas22=(-5 0 5)) vsquish  // predictions per session
      marginsplot, name(predicted_reas, replace)               // plot reas predictions
      predict predreas2                // save fixed-effect predicted outcomes
corr nm3rt predreas2                   // get total r to make r2
```

## SAS Output:

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr Z | |
|---|---|---|---|---|---|---|
| UN(1,1) | ID | 228688 | 34635 | 6.60 | <.0001 | → intercept variance reduced by 5.68% |
| UN(2,1) | ID | -31959 | 10877 | -2.94 | 0.0033 | |
| UN(2,2) | ID | 24872 | 5711.58 | 4.35 | <.0001 | → linear slope variance reduced by -2.38% |
| UN(3,1) | ID | 3877.66 | 1786.86 | 2.17 | 0.0300 | |
| UN(3,2) | ID | -3766.27 | 957.50 | -3.93 | <.0001 | |
| UN(3,3) | ID | 606.16 | 167.70 | 3.61 | 0.0002 | → quad slope variance reduced by 0.03% |
| session | ID | 20298 | 1649.11 | 12.31 | <.0001 | → residual variance not reduced |

Information Criteria

| Neg2LogLike | Parms | AIC | AICC | HQIC | BIC | CAIC |
|---|---|---|---|---|---|---|
| 8300.1 | 15 | 8330.1 | 8330.9 | 8345.9 | 8369.3 | 8384.3 |

Is the revised reasoning model (5b) still better than the age model (4a)?

Yes, -2ΔLL = 10.1 on df=2, *p*=.006 (so only 2.4 of the previous -2ΔLL was due to reason*quad)

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1969.80 | 49.6821 | 101 | 39.65 | <.0001 |
| c1sess | -123.54 | 19.8277 | 101 | -6.23 | <.0001 |
| c1sess*c1sess | 13.9774 | 3.3754 | 101 | 4.14 | <.0001 |
| age80 | 20.8470 | 8.5613 | 103 | 2.44 | 0.0166 |
| c1sess*age80 | -4.8610 | 3.2905 | 102 | -1.48 | 0.1427 |
| c1sess*c1sess*age80 | 0.6709 | 0.5580 | 101 | 1.20 | 0.2321 |
| reas22 | -32.8284 | 10.4706 | 101 | -3.14 | 0.0022 |
| c1sess*reas22 | 2.9363 | 1.2412 | 101 | 2.37 | 0.0199 |

Estimates

| Label | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Reasoning Effect at Session 1 | -32.8284 | 10.4706 | 101 | -3.14 | 0.0022 |
| Reasoning Effect at Session 2 | -29.8921 | 9.9615 | 101 | -3.00 | 0.0034 |
| Reasoning Effect at Session 3 | -26.9558 | 9.5870 | 101 | -2.81 | 0.0059 |
| Reasoning Effect at Session 4 | -24.0195 | 9.3632 | 101 | -2.57 | 0.0118 |
| Reasoning Effect at Session 5 | -21.0831 | 9.3012 | 101 | -2.27 | 0.0255 |
| Reasoning Effect at Session 6 | -18.1468 | 9.4040 | 101 | -1.93 | 0.0564 |

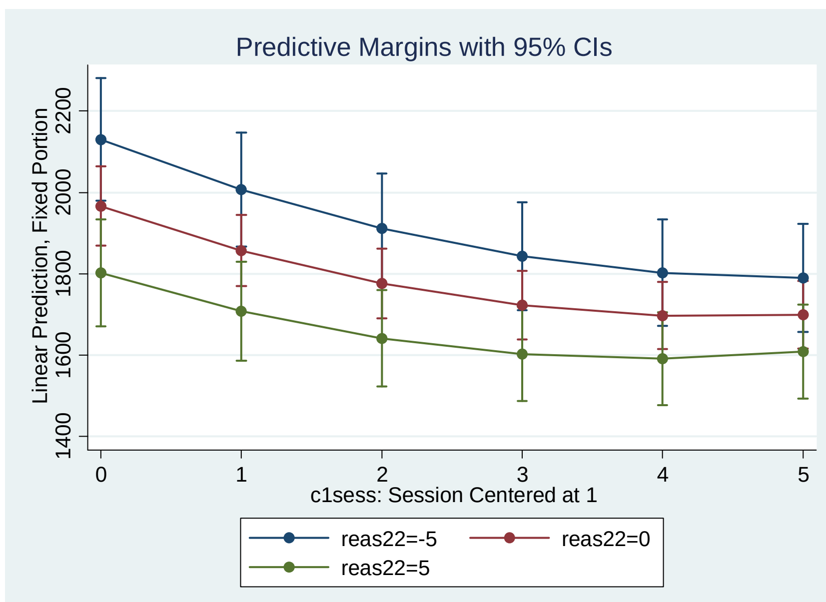
These are the simple slopes for the effect of reasoning per session.

Pearson Correlation Coefficients, N = 606
Prob > |r| under H0: Rho=0

| | nm3rt | Pred |
|---|---|---|
| nm3rt | 1.00000 | 0.40008 |
| nm3rt | | <.0001 |

R =.4001, so $R^2$ for time+age+reas = .1601

So ~0.1% of the variance accounted for previously was due to reason*quad




This plot was produced by the marginsplot in STATA.

## Model 6a. +Education Group on Intercept, Linear, and Quadratic Time Slopes

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(Session_{ti} - 1\right) + \beta_{2i}\left(Session_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(Age_i - 80\right) + \gamma_{02}\left(Reason_i - 22\right) + \gamma_{03}\left(Highvs.LowEd_i\right) + \gamma_{04}\left(Highvs.MedEd_i\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(Age_i - 80\right) + \gamma_{12}\left(Reason_i - 22\right) + \gamma_{13}\left(Highvs.LowEd_i\right) + \gamma_{14}\left(Highvs.MedEd_i\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(Age_i - 80\right) + \gamma_{23}\left(Highvs.LowEd_i\right) + \gamma_{24}\left(Highvs.MedEd_i\right) + U_{2i}$

### Additional model-implied group differences:

Medium vs. Low education intercept = $(\gamma_{00} + \gamma_{04}) - (\gamma_{00} + \gamma_{03}) = \gamma_{04} - \gamma_{03}$

Medium vs. Low education linear session = $(\gamma_{10} + \gamma_{14}) - (\gamma_{10} + \gamma_{13}) = \gamma_{14} - \gamma_{13}$

Medium vs. Low education quadratic session = $(\gamma_{20} + \gamma_{24}) - (\gamma_{20} + \gamma_{23}) = \gamma_{24} - \gamma_{23}$

```
TITLE1 "SAS Model 6a: +Education Group on Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.example23 NOCLPRINT NOITPRINT COVTEST NAMELEN=100 IC METHOD=ML;
      CLASS ID session educgrp;
      MODEL nm3rt = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                    reas22 c1sess*reas22 educgrp c1sess*educgrp c1sess*c1sess*educgrp
                          / SOLUTION DDFM=Satterthwaite OUTPM=work.EducPred;
      RANDOM INTERCEPT c1sess c1sess*c1sess / G GCORR TYPE=UN SUBJECT=ID;
      REPEATED session / TYPE=VC SUBJECT=ID;
      * Estimating group means at first and last sessions
      LSMEANS educgrp  / AT (c1sess) = (0) DIFF=ALL;
      LSMEANS educgrp  / AT (c1sess) = (5) DIFF=ALL;
      * Contrasts between groups on intercept, linear, and quadratic slopes
      ESTIMATE "L vs. H Educ for Intercept Main Effect"  educgrp -1  0  1 ;
      ESTIMATE "M vs. H Educ for Intercept Main Effect"  educgrp  0 -1  1 ;
      ESTIMATE "L vs. M Educ for Intercept Main Effect"  educgrp -1  1  0 ;
      ESTIMATE "L vs. H Educ for Linear Session"  c1sess*educgrp -1  0  1 ;
      ESTIMATE "M vs. H Educ for Linear Session"  c1sess*educgrp  0 -1  1 ;
      ESTIMATE "L vs. M Educ for Linear Session"  c1sess*educgrp -1  1  0 ;
      ESTIMATE "L vs. H Educ for Quadratic Session"  c1sess*c1sess*educgrp -1  0  1 ;
      ESTIMATE "M vs. H Educ for Quadratic Session"  c1sess*c1sess*educgrp  0 -1  1 ;
      ESTIMATE "L vs. M Educ for Quadratic Session"  c1sess*c1sess*educgrp -1  1  0 ;
RUN; PROC CORR NOSIMPLE DATA=work.EducPred; VAR nm3rt pred; RUN;
```

Think of the −1 as the "0" and the "1" as the "1" in a dummy code.

The table of model fixed effects will have the highest group as the 0 instead, though.

```
TITLE "SPSS Model 6a: +Education as Predictor of Intercept, Linear, and Quadratic".
MIXED nm3rt BY ID session educgrp WITH c1sess age80 reas22
      /METHOD = ML
      /PRINT  = SOLUTION TESTCOV G R
      /FIXED  = c1sess c1sess*c1sess age80 c1sess*age80 c1sess*c1sess*age80
                reas22 c1sess*reas22 educgrp c1sess*educgrp c1sess*c1sess*educgrp
      /RANDOM = INTERCEPT c1sess c1sess*c1sess | SUBJECT(ID) COVTYPE(UN)
      /REPEATED = session | SUBJECT(ID) COVTYPE(ID)
      /SAVE = FIXPRED (prededuc)
      /EMMEANS = TABLES(educgrp) WITH (c1sess=0) COMPARE(educgrp)
      /EMMEANS = TABLES(educgrp) WITH (c1sess=5) COMPARE(educgrp)
      /TEST = "L vs. H Educ for for Main Effect" educgrp -1  0  1
      /TEST = "M vs. H Educ for for Main Effect" educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Main Effect" educgrp -1  1  0
      /TEST = "L vs. H Educ for for Linear Session" c1sess*educgrp -1  0  1
      /TEST = "M vs. H Educ for for Linear Session" c1sess*educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Linear Session" c1sess*educgrp -1  1  0
      /TEST = "L vs. H Educ for for Quadratic Session" c1sess*c1sess*educgrp -1  0  1
      /TEST = "M vs. H Educ for for Quadratic Session" c1sess*c1sess*educgrp  0 -1  1
      /TEST = "L vs. M Educ for for Quadratic Session" c1sess*c1sess*educgrp -1  1  0.
CORRELATIONS nm3rt prededuc.
```

```
* STATA Model 6a: +Education Group on Intercept, Linear, and Quadratic
xtmixed nm3rt c.c1sess c.c1sess#c.c1sess                               ///
       c.age80  c.age80#c.c1sess c.age80#c.c1sess#c.c1sess            ///
       c.reas22 c.reas22#c.c1sess                                     ///
       ib(last).educgrp ib(last).educgrp#c.c1sess                     ///
       ib(last).educgrp#c.c1sess#c.c1sess, || id: c1sess c1sess2,     ///
       variance ml covariance(un) residuals(independent,t(session)),
       estat ic, n(101),
       estat recovariance, level(id),
       estimates store Educ,
       lrtest Educ Reas2,
 * Estimating group means at first and last sessions
       margins ib(last).educgrp,  at(c.c1sess=(0 5))
* Contrasts between groups on intercept, linear, and quadratic slopes
       test 1.educgrp=3.educgrp                        // Low vs. High: Intercept
       test 2.educgrp=3.educgrp                        // Med vs. High: Intercept
       test 1.educgrp=2.educgrp                        // Low vs. Med:  Intercept
       test 1.educgrp#c.c1sess=3.educgrp#c.c1sess            // Low vs. High: Linear
       test 2.educgrp#c.c1sess=3.educgrp#c.c1sess            // Med vs. High: Linear
       test 1.educgrp#c.c1sess=2.educgrp#c.c1sess            // Low vs. Med:  Linear
       test 1.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess  // Low vs. High: Quad
       test 2.educgrp#c.c1sess#c.c1sess=3.educgrp#c.c1sess#c.c1sess  // Med vs. High: Quad
       test 1.educgrp#c.c1sess#c.c1sess=2.educgrp#c.c1sess#c.c1sess  // Low vs. Med:  Quad
       contrast educgrp,                                     // omnibus group diff on intercept
       contrast educgrp#c.c1sess,                            // omnibus group diff on linear
       contrast educgrp#c.c1sess#c.c1sess,                   // omnibus group diff on quadratic
       margins, at(c.c1sess=(0(1)5) educgrp=(1 2 3)) vsquish  // predictions per session
       marginsplot, name(predicted_educ, replace)             // plot educ predictions
       predict prededuc                        // save fixed-effect predicted outcomes
corr nm3rt prededuc                            // get total r to make r2
```

## SAS Output:

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr Z | |
|---|---|---|---|---|---|---|
| UN(1,1) | ID | 228585 | 34693 | 6.59 | <.0001 | → intercept variance reduced by 0.05% |
| UN(2,1) | ID | -33273 | 10909 | -3.05 | 0.0023 | |
| UN(2,2) | ID | 24129 | 5614.80 | 4.30 | <.0001 | → linear slope variance reduced by 2.99% |
| UN(3,1) | ID | 4125.93 | 1788.44 | 2.31 | 0.0211 | |
| UN(3,2) | ID | -3633.38 | 939.20 | -3.87 | 0.0001 | |
| UN(3,3) | ID | 581.50 | 164.31 | 3.54 | 0.0002 | → quad slope variance reduced by 4.07% |
| session | ID | 20298 | 1649.11 | 12.31 | <.0001 | → residual variance not reduced |

Information Criteria

| Neg2LogLike | Parms | AIC | AICC | HQIC | BIC | CAIC |
|---|---|---|---|---|---|---|
| 8295.4 | 21 | 8337.4 | 8338.9 | 8359.6 | 8392.3 | 8413.3 |

Is the education model (6a) better than the revised reasoning model (5b)?

No, −2ΔLL = 4.7 on df=6, $p$ = .583

Solution for Fixed Effects

| Effect | Education Group (1=HS, 2=BA,3=GRAD) | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | | 1961.89 | 101.79 | 100 | 19.27 | <.0001 |
| c1sess | | -106.50 | 40.2761 | 101 | -2.64 | 0.0095 |
| c1sess*c1sess | | 12.4797 | 6.8474 | 101 | 1.82 | 0.0713 |
| age80 | | 20.2893 | 8.5600 | 102 | 2.37 | 0.0196 |
| c1sess*age80 | | -4.5758 | 3.2667 | 102 | -1.40 | 0.1643 |
| c1sess*c1sess*age80 | | 0.6177 | 0.5533 | 101 | 1.12 | 0.2669 |
| reas22 | | -36.6228 | 10.7638 | 101 | -3.40 | 0.0010 |
| c1sess*reas22 | | 2.9788 | 1.2799 | 101 | 2.33 | 0.0219 |
| educgrp | 1 | -51.3811 | 151.06 | 101 | -0.34 | 0.7345 |
| educgrp | 2 | 37.6427 | 120.87 | 100 | 0.31 | 0.7561 |
| educgrp | 3 | 0 | . | . | . | . |

| | | | | | | |
|---|---|---|---|---|---|---|
| c1sess*educgrp | 1 | -70.2445 | 59.0672 | 101 | -1.19 | 0.2371 |
| c1sess*educgrp | 2 | -4.3577 | 48.1238 | 100 | -0.09 | 0.9280 |
| c1sess*educgrp | 3 | 0 | . | . | . | . |
| c1sess*c1sess*educgrp | 1 | 11.0653 | 10.0300 | 101 | 1.10 | 0.2726 |
| c1sess*c1sess*educgrp | 2 | -1.4641 | 8.1865 | 101 | -0.18 | 0.8584 |
| c1sess*c1sess*educgrp | 3 | 0 | . | . | . | . |

Type 3 Tests of Fixed Effects (truncated)

| Effect | Num DF | Den DF | F Value | Pr > F |
|---|---|---|---|---|
| educgrp | 2 | 101 | 0.24 | 0.7874 |
| c1sess*educgrp | 2 | 101 | 0.96 | 0.3860 |
| c1sess*c1sess*educgrp | 2 | 101 | 1.09 | 0.3395 |

Estimates

| Label | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| L vs. H Educ for Intercept Main Effect | 51.3811 | 151.06 | 101 | 0.34 | 0.7345 |
| M vs. H Educ for Intercept Main Effect | -37.6427 | 120.87 | 100 | -0.31 | 0.7561 |
| **L vs. M Educ for Intercept Main Effect** | **89.0238** | **130.74** | **102** | **0.68** | **0.4975** |
| L vs. H Educ for Linear Session | 70.2445 | 59.0672 | 101 | 1.19 | 0.2371 |
| M vs. H Educ for Linear Session | 4.3577 | 48.1238 | 100 | 0.09 | 0.9280 |
| **L vs. M Educ for Linear Session** | **65.8868** | **50.7047** | **101** | **1.30** | **0.1967** |
| L vs. H Educ for Quadratic Session | -11.0653 | 10.0300 | 101 | -1.10 | 0.2726 |
| M vs. H Educ for Quadratic Session | 1.4641 | 8.1865 | 101 | 0.18 | 0.8584 |
| **L vs. M Educ for Quadratic Session** | **-12.5294** | **8.6028** | **101** | **-1.46** | **0.1484** |

Least Squares Means

| Effect | Education Group (1=HS, 2=BA,3=GRAD) | c1sess | age80 | reas22 | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|---|---|---|
| **educgrp** | **1** | **0.00** | **-0.17** | **0.62** | **1884.28** | **110.93** | **101** | **16.99** | **<.0001** |
| **educgrp** | **2** | **0.00** | **-0.17** | **0.62** | **1973.30** | **66.5665** | **100** | **29.64** | **<.0001** |
| **educgrp** | **3** | **0.00** | **-0.17** | **0.62** | **1935.66** | **101.24** | **100** | **19.12** | **<.0001** |
| **educgrp** | **1** | **5.00** | **-0.17** | **0.62** | **1599.71** | **94.5405** | **101** | **16.92** | **<.0001** |
| **educgrp** | **2** | **5.00** | **-0.17** | **0.62** | **1704.94** | **56.6069** | **101** | **30.12** | **<.0001** |
| **educgrp** | **3** | **5.00** | **-0.17** | **0.62** | **1725.69** | **86.0606** | **101** | **20.05** | **<.0001** |

Differences of Least Squares Means

| Effect | Education Group (1=HS, 2=BA, 3=GRAD) | Education Group (1=HS, 2=BA, 3=GRAD) | c1sess | age80 | reas22 | Estimate | Standard Error | DF | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|---|---|---|---|
| educgrp | 1 | 2 | 0.00 | -0.17 | 0.62 | -89.0238 | 130.74 | 102 | -0.68 | 0.4975 |
| educgrp | 1 | 3 | 0.00 | -0.17 | 0.62 | -51.3811 | 151.06 | 101 | -0.34 | 0.7345 |
| **educgrp** | **2** | **3** | **0.00** | **-0.17** | **0.62** | **37.6427** | **120.87** | **100** | **0.31** | **0.7561** |
| **educgrp** | **1** | **2** | **5.00** | **-0.17** | **0.62** | **-105.22** | **111.48** | **101** | **-0.94** | **0.3475** |
| **educgrp** | **1** | **3** | **5.00** | **-0.17** | **0.62** | **-125.97** | **128.68** | **101** | **-0.98** | **0.3299** |
| **educgrp** | **2** | **3** | **5.00** | **-0.17** | **0.62** | **-20.7487** | **102.72** | **101** | **-0.20** | **0.8403** |

Pearson Correlation Coefficients, N = 606
Prob > |r| under H0: Rho=0

| | nm3rt | Pred |
|---|---|---|
| nm3rt | 1.00000 | 0.41510 |
| nm3rt: Number-Match 3 RT in ms | | <.0001 |

R =.41510, so $R^2$ for time+age+reas+educ = .172

The fixed effects of time, age, and reasoning before accounted for ~16.0% of the variance in RT, so there is a net increase of 1.2% due to education (which is not significant).

## Simple Processing Speed: Example Conditional Models of Change Results

The extent to which individual differences in response time (RT) over six sessions for a simple processing speed test (number match three) could be predicted from baseline age, abstract reasoning, and education level was examined in a series of multilevel models (i.e., general linear mixed models) in which the six practice sessions were nested within each participant. Given the interest in comparing models differing in fixed effects, maximum likelihood (ML) was used in estimating and reporting all model parameters; denominator degrees of freedom were estimated using the Satterthwaite method. The significance of new fixed effects were evaluated with individual Wald tests (i.e., of estimate / SE) as well as with likelihood ratio tests (i.e., −2ΔLL), with degrees of freedom equal to the number of new fixed effects. Session (i.e., the index of time) was centered at the first occasion, age was centered at 80 years, abstract reasoning was centered at 22 (near the mean of the scale), and graduate-level education was the reference group for education level (with separate contrasts for high school or less and for bachelor's level education).

The best-fitting unconditional growth model specified quadratic decline across the six sessions (i.e., a decelerating negative function) with significant individual differences in the intercept, linear, and quadratic effects. Accordingly, effect size was evaluated via pseduo-$R^2$ values for the proportion reduction in each random effect variance, as well as with total $R^2$, the squared correlation between the actual outcome values and the outcomes predicted by the model fixed effects. In the unconditional growth model, the fixed effects for linear and quadratic change across sessions accounted for approximately 4% of the total variation in RT.

Next, age was added as a predictor of the intercept, linear slope, and quadratic slope. The age model fit significantly better than the unconditional model as indicated by a significant likelihood ratio test, −2ΔLL(3) = 11.6, $p$ = .009; the AIC was lower, although the BIC was not. However, only the fixed effect of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher by 29.05 ($p$ = .0008). In terms of pseudo-$R^2$, age accounted for 11.29% of the random intercept variance, 4.50% of the random linear slope variance, and 2.64% of the random quadratic slope variance. As expected given that baseline age is a time-invariant predictor, the residual variance was not reduced. The total cumulative $R^2$ from session and age was $R^2$ = .11, approximately a 7% increase due to age (which was significant, as indicated by the likelihood ratio test). Although the interactions of age with the linear and quadratic slopes were not significant, they were retained in the model to fully control for age effects before examining the effects of other predictors.

Abstract reasoning was then added as a predictor of the intercept, linear slope, and quadratic slope. The abstract reasoning model fit significantly better than the age model, −2ΔLL(3) = 12.5, $p$ = .006; the AIC was lower, although the BIC was not. However, only the fixed effect of reasoning on the intercept was significant. The nonsignificant effect of reasoning on the quadratic slope was then removed, revealing a significant effect of reasoning on both the intercept and linear slope, such that for every unit higher reasoning above 22, RT at the first session was expected to be lower by 32.82 and the linear rate of improvement in RT (as evaluated at the first session given the quadratic slope) was expected to be less negative by 2.94 (i.e., faster initial RT with less improvement in persons with greater reasoning). These two effects still resulted in a significant improvement in model fit over the age model, −2ΔLL(2) = 10.1, $p$ = .006, with a lower AIC and BIC. Reasoning accounted for 5.58% of the random intercept variance but had no measurable reduction of the random linear and quadratic slope variances. The total cumulative R2 from session, age, and reasoning was $R^2$ = .16, approximately a 5% increase due to reasoning (which was significant, as indicated by the likelihood ratio test).

Finally, education level (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept, linear slope, and quadratic slope. The education model did not fit significantly better than the reasoning model, −2ΔLL(6) = 4.7, $p$ = .583, with a higher AIC and BIC. None of the omnibus main effects of group on the intercept, linear, or quadratic slopes were significant, F's(2,101) < 1.10, $p$'s > .05, and none of the pairwise group comparisons were significant as well. Education accounted for 0.05% of the random intercept variance, 2.99% of the random linear slope variance, and 4.07% of the random quadratic slope variance. The total cumulative $R^2$ from session, age, reasoning, and education was $R^2$ = .17, approximately a 1% increase due to education (which was not significant, as indicated by the likelihood ratio test). (From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)