ANOVA Longitudinal Models for the Example 3b Practice Effects Data via MIXED Syntax

This example comes from Hoffman (2015) chapter 3 (example 3b). We will be examining response time (RT) in milliseconds across six practice sessions to a measure of processing speed (as measured by the number match 3 test) in a sample of 101 older adults. Of interest right now is the pattern of variance and covariance in RT across sessions. Later we will examine the extent to which individual differences in change in RT can be described by polynomial models, piecewise slopes models, and exponential models.

All models we will examine will have the same model for the means, saturated by session:

$$RT_{ti} = \beta_0 + \beta_1(T1_{ti}) + \beta_2(T2_{ti}) + \beta_3(T3_{ti}) + \beta_4(T4_{ti}) + \beta_5(T5_{ti})$$

But they will differ in their model for the variance, corresponding to the three kinds of ANOVAs.

The saturated means model predicts 6 means from 1 intercept (for session 6) + 5 mean differences (1 for each of the 5 other sessions).

Model 1: Saturated Session Means, E-only Variance (BP → no covariance over sessions)

Variance Model: NO covariance or correlation; EQUAL variances across sessions

```
TITLE1 'Eq 3b.10: Between-Person Independent ANOVA';
PROC MIXED DATA=work.Chapter3b COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = session / SOLUTION CHISQ DDFM=Satterthwaite;
     REPEATED session / R RCORR TYPE=VC SUBJECT=PersonID;
     ODS OUTPUT InfoCrit=FitBPANOVA;
     LSMEANS session / DIFF=ALL;
RUN; TITLE1;
```

The ODS OUTPUT saves the Information Criteria table to a SAS dataset called "FitBPANOVA" for later use in my %FitTest macro.

```
           Dimensions
Covariance Parameters              1   still just e in model for variances
Columns in X                       7   should be 6 but it counts the unidentified one
Columns in Z                       0   still no U's yet
Subjects                         101
Max Obs per Subject                6
```

```
                    Estimated R Matrix for ID 101
 Row        Col1        Col2        Col3        Col4        Col5        Col6
   1      236813
   2                  236813
   3                              236813
   4                                          236813
   5                                                      236813
   6                                                                  236813
```

The **R matrix** is the **unstandardized** matrix of the error variances and covariances for each session. So far no covariance is allowed across time, with equal variance across time.

```
               Covariance Parameter Estimates
                                 Standard         Z
Cov Parm    Subject    Estimate      Error     Value        Pr Z
Session     ID           236813      13672     17.32      <.0001   E variance after accounting for means
```

```
          Fit Statistics
-2 Res Log Likelihood          9155.4
AIC (smaller is better)        9157.4
AICC (smaller is better)       9157.4
BIC (smaller is better)        9160.0
```

```
  Null Model Likelihood Ratio Test
    DF    Chi-Square      Pr > ChiSq
     0          0.00          1.0000
```

No other parameters in the model for the variance besides residual variance to test yet…

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
9155.4	1	9157.4	9157.4	9158.5	9160.0	9161.0

In REML, #parms is the number of variance model parms specifically (1 here).

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		1672.14	48.4219	600	34.53	<.0001
Session	1	289.76	68.4789	600	4.23	<.0001
Session	2	143.04	68.4789	600	2.09	0.0371
Session	3	77.8986	68.4789	600	1.14	0.2558
Session	4	45.6604	68.4789	600	0.67	0.5052
Session	5	35.0397	68.4789	600	0.51	0.6091
Session	6	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
session	5	600	23.67	4.73	0.0003	0.0003

This multivariate Wald test tells us there is a significant "omnibus" effect of time.

Least Squares Means

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Session	1	1961.89	48.4219	600	40.52	<.0001
Session	2	1815.17	48.4219	600	37.49	<.0001
Session	3	1750.03	48.4219	600	36.14	<.0001
Session	4	1717.80	48.4219	600	35.48	<.0001
Session	5	1707.18	48.4219	600	35.26	<.0001
Session	6	1672.14	48.4219	600	34.53	<.0001

Model 2: Saturated Session Means, $U_{0i} + e_{ti}$ (CS) Variance
(WP → constant covariance across sessions)

Variance Model: EQUAL covariance/correlation, EQUAL variances across sessions

```
TITLE1 'Eq 3b.10: Univariate Repeated Measures ANOVA';
PROC MIXED DATA=work.Chapter3b COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = session / SOLUTION CHISQ DDFM=Satterthwaite;
     REPEATED session / R RCORR TYPE=CS SUBJECT=PersonID;
     ODS OUTPUT InfoCrit=FitUnivANOVA;
     LSMEANS session / DIFF=ALL;
RUN; TITLE1;
```

The ODS OUTPUT saves the Information Criteria table to a SAS dataset called "FitUnivANOVA" for later use in my %FitTest macro.

Dimensions

Covariance Parameters	2 → 2 variance model parameters
Columns in X	7
Columns in Z	0
Subjects	101
Max Obs per Subject	6

Estimated R Matrix for ID 101 → COMBINED $U_{0i} + e_{ti}$ VARIANCE AFTER DIFFERENT MEANS

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**236813**	202677	202677	202677	202677	202677
2	202677	**236813**	202677	202677	202677	202677
3	202677	202677	**236813**	202677	202677	202677
4	202677	202677	202677	**236813**	202677	202677
5	202677	202677	202677	202677	**236813**	202677
6	202677	202677	202677	202677	202677	**236813**

Estimated R Correlation Matrix for PersonID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8559	0.8559	0.8559	0.8559	0.8559
2	0.8559	1.0000	0.8559	0.8559	0.8559	0.8559
3	0.8559	0.8559	1.0000	0.8559	0.8559	0.8559
4	0.8559	0.8559	0.8559	1.0000	0.8559	0.8559
5	0.8559	0.8559	0.8559	0.8559	1.0000	0.8559
6	0.8559	0.8559	0.8559	0.8559	0.8559	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
CS	**ID**	**202677**	**29470**	**6.88**	**<.0001**	**Var(U_{0i}) after accounting for means**
Residual		**34136**	**2158.96**	**15.81**	**<.0001**	**Var(e_{ti}) after accounting for means**

Fit Statistics

-2 Res Log Likelihood	8353.4
AIC (Smaller is Better)	8357.4
AICC (Smaller is Better)	8357.4
BIC (Smaller is Better)	8362.6

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	**802.08**	<.0001

Now 1 other parameter in the model for the variance besides residual variance

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8353.4	2	8357.4	8357.4	8359.5	8362.6	8364.6

In REML, #parms is the number of variance model parms specifically (2 here).

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		**1672.14**	**48.4219**	**129**	**34.53**	**<.0001**
Session	**1**	**289.76**	**25.9993**	**500**	**11.14**	**<.0001**
Session	**2**	**143.04**	**25.9993**	**500**	**5.50**	**<.0001**
Session	**3**	**77.8986**	**25.9993**	**500**	**3.00**	**0.0029**
Session	**4**	**45.6604**	**25.9993**	**500**	**1.76**	**0.0797**
Session	**5**	**35.0397**	**25.9993**	**500**	**1.35**	**0.1784**
Session	**6**	**0**	**.**	**.**	**.**	**.**

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
session	5	500	164.24	32.85	<.0001	<.0001

This multivariate Wald test tells us there is a significant "omnibus" effect of time.

Least Squares Means

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Session	1	1961.89	48.4219	129	40.52	<.0001
Session	2	1815.17	48.4219	129	37.49	<.0001
Session	3	1750.03	48.4219	129	36.14	<.0001
Session	4	1717.80	48.4219	129	35.48	<.0001
Session	5	1707.18	48.4219	129	35.26	<.0001
Session	6	1672.14	48.4219	129	34.53	<.0001

Model 3: Saturated Session Means, Saturated (Multivariate) Variance (WP → all possible covariances/correlations, variances across sessions)

Variance Model: Completely UNEQUAL covariance, correlation, and variance across time

```
TITLE1 'Eq 3b.10: Multivariate Repeated Measures ANOVA';
PROC MIXED DATA=work.Chapter3b COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = session / SOLUTION CHISQ DDFM=Satterthwaite;
     REPEATED session / R RCORR TYPE=UN SUBJECT=PersonID;
     ODS OUTPUT InfoCrit=FitMultivANOVA;
     LSMEANS session / DIFF=ALL;
RUN; TITLE1;
```

Dimensions		
Covariance Parameters	21	→ number of total variance model parameters
Columns in X	7	
Columns in Z	0	
Subjects	101	
Max Obs per Subject	6	

Estimated R Matrix for ID 101 → TOTAL COVARIANCE MATRIX AFTER DIFFERENT MEANS

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	**301985**	235659	217994	202607	192154	195360
2	235659	**259150**	230217	213232	202092	193268
3	217994	230217	**233368**	205209	196919	188604
4	202607	213232	205209	**217544**	193676	185321
5	192154	202092	196919	193676	**212098**	187840
6	195360	193268	188604	185321	187840	**196733**

Estimated R Correlation Matrix for ID 101 → TOTAL CORRELATION MATRIX AFTER DIFFERENT MEANS

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	1.0000	0.8424	0.8212	0.7905	0.7593	0.8015
2	0.8424	1.0000	0.9361	0.8981	0.8620	0.8559
3	0.8212	0.9361	1.0000	0.9108	0.8851	0.8802
4	0.7905	0.8981	0.9108	1.0000	0.9016	0.8958
5	0.7593	0.8620	0.8851	0.9016	1.0000	0.9196
6	0.8015	0.8559	0.8802	0.8958	0.9196	1.0000

Fit Statistics	
-2 Res Log Likelihood	8229.8
AIC (smaller is better)	8271.8
AICC (smaller is better)	8273.4
BIC (smaller is better)	8326.7

Null Model Likelihood Ratio Test		
DF	Chi-Square	Pr > ChiSq
20	925.64	<.0001

Now 20 other parameter in the model for the variance besides residual variance

Information Criteria						
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8229.8	21	8271.8	8273.4	8294.0	8326.7	8347.7

In REML, #parms is the number of variance model parms specifically (21 here).

Solution for Fixed Effects

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		1672.14	44.1345	100	37.89	<.0001
Session	1	289.76	32.7000	100	8.86	<.0001
Session	2	143.04	26.2031	100	5.46	<.0001
Session	3	77.8986	22.8842	100	3.40	0.0010
Session	4	45.6604	20.7853	100	2.20	0.0303
Session	5	35.0397	18.1168	100	1.93	0.0559
Session	6	0	.	.	.	.

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
session	5	100	83.60	**16.72**	<.0001	<.0001

This multivariate Wald test tells us there is a significant "omnibus" effect of time.

Least Squares Means

Effect	Session #	Estimate	Standard Error	DF	t Value	Pr > |t|
Session	1	1961.89	54.6805	100	35.88	<.0001
Session	2	1815.17	50.6541	100	35.83	<.0001
Session	3	1750.03	48.0684	100	36.41	<.0001
Session	4	1717.80	46.4101	100	37.01	<.0001
Session	5	1707.18	45.8255	100	37.25	<.0001
Session	6	1672.14	44.1345	100	37.89	<.0001

We now have three different answers as to the significance of our model fixed effects, so which is the "right" one? To find out, we can compare the fit of the **model for the variance**.

BP ANOVA: Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
session	5	600	23.67	**4.73**	0.0003	0.0003

WP Univariate ANOVA: Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
session	5	500	164.24	**32.85**	<.0001	<.0001

BP vs. WP Univariate ANOVA:

Does the saturated means, CS variance model fit better than the saturated means, e-only model?

```
%FitTest(FitFewer=FitBPANOVA, FitMore=FitUnivANOVA);
```

Likelihood Ratio Test for FitBPANOVA vs. FitUnivANOVA

Name	Neg2Log Like	Parms	AIC	BIC	DevDiff	DFdiff	Pvalue
FitBPANOVA	9155.4	1	9157.4	9160.0	.	.	.
FitUnivANOVA	8353.4	2	8357.4	8362.6	**802.077**	1	0

WP Univariate ANOVA vs. WP Multivariate ANOVA:

Does the saturated means, UN variance model fit better than the saturated means, CS variance model?

```
%FitTest(FitFewer=FitUnivANOVA, FitMore=FitMultivANOVA);
```

Likelihood Ratio Test for FitUnivANOVA vs. FitMultivANOVA

Name	Neg2Log Like	Parms	AIC	BIC	DevDiff	DFdiff	Pvalue
FitUnivANOVA	8353.4	2	8357.4	8362.6	.	.	.
FitMultivANOVA	8229.8	21	8271.8	8326.7	**123.567**	19	0