Introduction to Multilevel Models for Longitudinal (and other Repeated Measures) Data

- Topics:
  - Features of longitudinal data
  - Features of longitudinal models
  - What can MLM do for you?
  - What to expect in this course

What is this class about?

- "Longitudinal" data
  - Same individual units of analysis measured at different occasions (which can range from milliseconds to decades)

- "Repeated measures" data (*as time permits*)
  - Same individual units of analysis measured via different items, using different stimuli, or under different conditions

- Both of these fall under a more general category of "multivariate" data of varying complexity
  - The link between them is the use of **random effects** to describe covariance of outcomes from the same sampling unit

Data Requirements for Our Models

- A useful outcome variable:
  - Has an interval scale*
    - A one-unit difference means the same thing across all scale points
    - In subscales, each contributing item has an equivalent scale
    - **Other kinds of outcomes will be analyzed using generalized versions of multilevel models instead, but estimation will be more challenging*
  - Has scores with the same meaning over observations
    - Includes meaning of construct
    - Includes how items relate to the scale
    - Implies measurement invariance
- FANCY MODELS STILL CANNOT SAVE BADLY MEASURED VARIABLES OR CONFOUNDED RESEARCH DESIGNS.

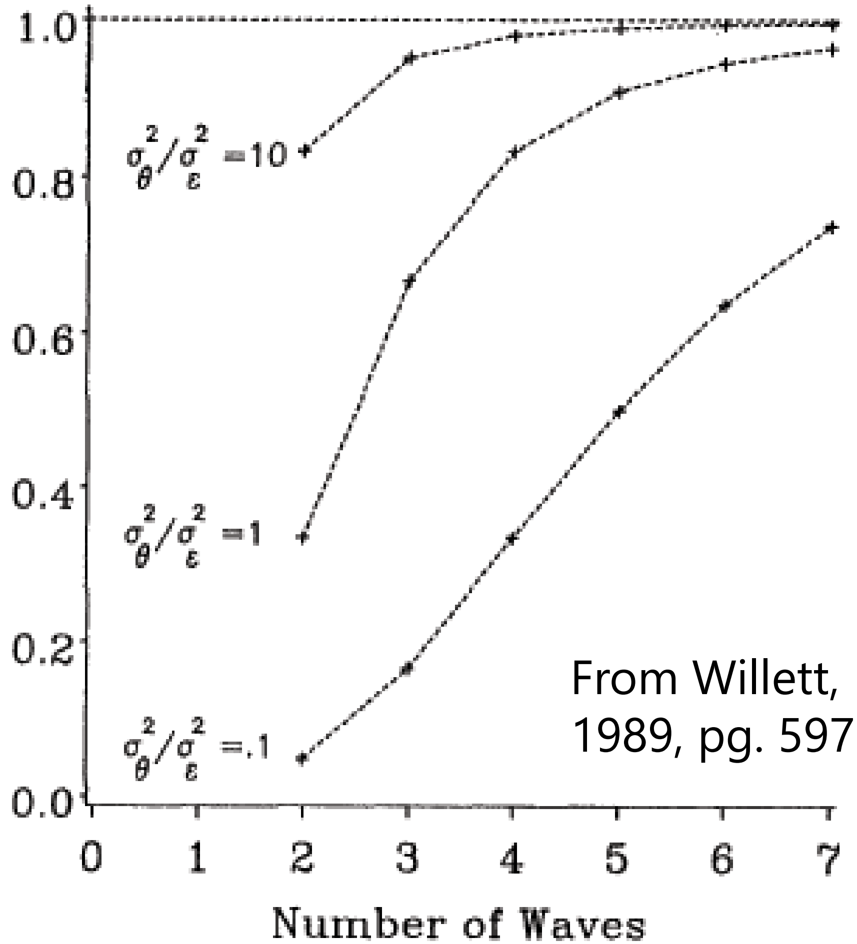
Requirements for Longitudinal Data

- Multiple <u>OCCASIONS</u> from same sampling unit (person)
  - 2 occasions is the minimum, but just 2 can lead to problems:
    - Only 1 kind of change is observable (1 difference)
    - Can't distinguish "real" individual differences in change from error
    - Repeated measures ANOVA is just fine for 2 observations
      - Necessary assumption of "sphericity" is satisfied with only 2 observations even if compound symmetry doesn't hold
  - More data is better (with diminishing returns)
    - More occasions → better description of the form of change
    - More units (persons) → better estimates of amount of individual differences in change; better prediction of those individual differences
    - More items/stimuli/groups → more power to show effects of differences between items/stimuli/groups

Power in Longitudinal Data

- More occasions are better!
  - Can examine more complex growth functions
  - Can get more reliable individual growth parameters
- More units (people) are better!
  - Can get more reliable measures of individual differences

Reliability of Slopes by Signal to Noise Ratio and # Occasions



From Willett, 1989, pg. 597

Levels of Analysis in Longitudinal Data

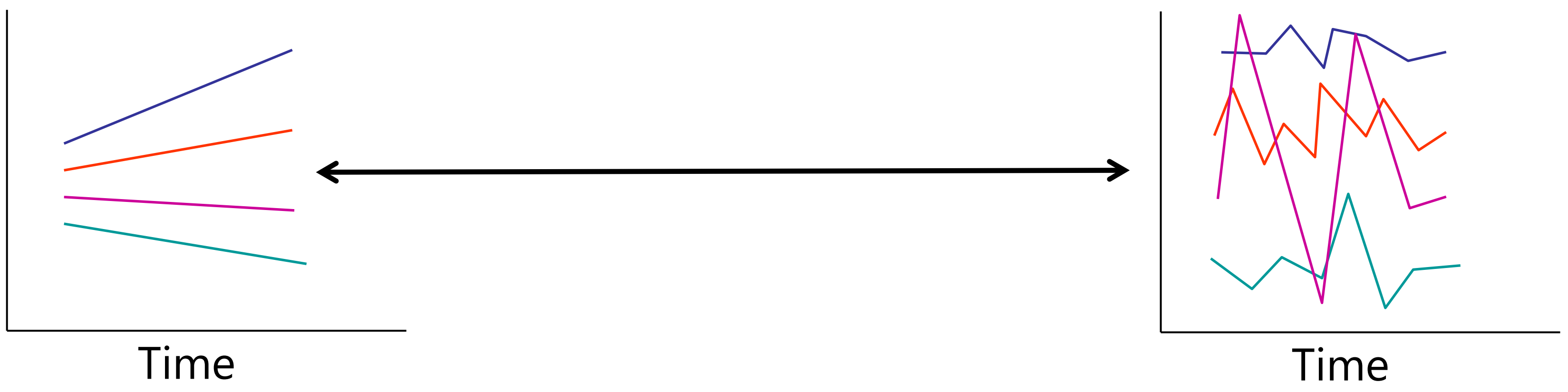
- Between-Person (BP) Variation:
  - **Level-2** – "**INTER**-individual Differences" – Time-Invariant
  - All longitudinal studies begin as cross-sectional studies
- Within-Person (WP) Variation:
  - **Level-1** – "**INTRA**-individual Differences" – Time-Varying
  - Only longitudinal studies can provide this extra information
- Longitudinal studies allow examination of both types of relationships simultaneously (and their interactions)
  - Any variable measured over time usually has both BP and WP variation
  - BP = more/less than other people; WP = more/less than one's average
- I use "person" here, but level-2 units can be anything that is measured repeatedly (like animals, schools, countries…)

A Longitudinal Data Continuum

- **Within-Person Change**: Systematic change
  - Magnitude or direction of change can be different across individuals
  - “Growth curve models” → Time is meaningfully sampled

- **Within-Person Fluctuation**: No systematic change
  - Outcome just varies/fluctuates over time (e.g., emotion, stress)
  - Time is just a way to get lots of data per individual

Pure WP Change   Pure WP Fluctuation



What is a Multilevel Model (MLM)?

- Same as other terms you have heard of:
  - **General Linear Mixed-Effects Model** (if you are from statistics)
    - *Mixed Effects* = Fixed and Random effects
  - **Random Coefficients Model** (also if you are from statistics)
    - Random coefficients = Random effects = latent variables/factors
  - **Hierarchical Linear Model** (if you are from education)
    - Not the same as *hierarchical regression*

- Special cases of MLM:
  - Random Effects ANOVA or Repeated Measures ANOVA
  - (Latent) Growth Curve Model (where "Latent" implies SEM)
  - Within-Person Fluctuation Model (e.g., for daily diary data)
  - Clustered/Nested Observations Model (e.g., for kids in schools)
  - Cross-Classified Models (e.g., "value-added" models)
  - Psychometric Models (e.g., factor analysis, item response theory)

The Two Sides of Any Model

- **Model for the Means:**
  - *Aka* **Fixed Effects**, Structural Part of Model
  - What you are used to **caring about for testing hypotheses**
  - How the expected outcome for a given observation varies as a function of values on predictor variables

- **Model for the Variance:**
  - *Aka* **Random Effects and Residuals**, Stochastic Part of Model
  - What you are used to **making assumptions about** instead
  - How residuals are distributed and related across observations (persons, groups, time, etc.) → these relationships are called "dependency" and ***this is the primary way that multilevel models differ from general linear models*** *(e.g., regression)*

Dimensions for Organizing Models

- Outcome type: General (normal) vs. General*ized* (not normal)
- Dimensions of sampling: One (so one variance term per outcome) vs. **Multiple** (so multiple variance terms per outcome) → **OUR WORLD**

- **General Linear Models:** conditionally normal outcome distribution, **fixed effects** (identity link; only one dimension of sampling)
- **General*ized* Linear Models:** **any conditional outcome distribution**, **fixed** effects through **link functions**, no random effects (one dimension)

Note: Least Squares is only for GLM

- **General Linear Mixed Models:** conditionally normal outcome distribution, **fixed and random effects** (identity link, but multiple sampling dimensions)
- **General*ized* Linear Mixed Models:** **any conditional outcome distribution**, **fixed and random effects** through **link functions** (multiple dimensions)
  - Same concepts as for this course, but with more complexity in estimation

- "Linear" means fixed effects predict the *link-transformed* conditional mean of DV in a linear combination of (effect*predictor) + (effect*predictor)…

Options for Longitudinal Models

- Although models and software are logically separate, longitudinal data can be analyzed via multiple analytic frameworks (which are then usually bound to software):
  - "Multilevel/Mixed-Effects Models"
    - Dependency over time, persons, groups, etc. is modeled via random effects (multivariate through "levels" using stacked/long data)
    - Builds on GLM, generalizes easier to additional levels of analysis
  - "Structural Equation Models"
    - Dependency over time *only* is modeled via latent variables (single-level analysis using multivariate/wide data)
    - Generalizes easier to analysis of latent constructs, multivariate, mediation
  - Because random effects and latent variables are the same thing, many longitudinal models can be specified/estimated either way
    - And now "Multilevel Structural Equation Models" can do it all (maybe)…

What can MLM do for you?

1. **Model dependency across observations**
   - Longitudinal, clustered, and/or cross-classified data? No problem!
   - Tailor your model of sources of correlation to your data

2. **Include categorical or continuous predictors at any level**
   - Time-varying, person-level, group-level predictors for each variance
   - Explore reasons for dependency, don't just control for dependency

3. **Does not require same data structure for each person**
   - Unbalanced or missing data? No problem!

4. **You already know how (or you will soon)!**
   - Use SPSS Mixed, **SAS Mixed**, Stata, Mplus, R, HLM, MlwiN…
   - What's an intercept? What's a slope? What's a pile of variance?

1. Model Dependency

- Sources of dependency depend on the sources of **variation** created by your sampling design: residuals for outcomes from the same unit are likely to be related, which violates the GLM "independence" assumption

- **"Levels" for dependency** ="levels of random effects"
  - Sampling dimensions can be **nested**
    - e.g., time within person, time within group, trial within person
  - If you can't figure out the direction of your nesting structure, odds are good you have a **crossed sampling design** instead
    - e.g., persons crossed with items, raters crossed with targets
  - To have a "level", there must be random outcome variation due to sampling that **remains** after including the model's fixed effects
    - e.g., treatment vs. control does not create another level of "group" (but it would if you had multiple treatment and multiple control groups)

Longitudinal dependency comes from…

- Mean differences across sampling units (e.g., persons)
  - Creates *constant* dependency over time
  - Will be represented by a random intercept in our models

- Individual differences in effects of predictors
  - Individual differences in change over time, stress reactivity
  - Creates <u>non-constant</u> dependency, the size of which depends on the value of the predictor at each occasion
  - Will be represented by random slopes in our models

- Non-constant within-person correlation for unknown reasons (time-dependent autocorrelation)
  - Can add other patterns of correlation as needed for this

Why care about dependency?

- In other words, what happens if we have the wrong model for the variance (assume independence instead)?

- **Validity of the tests of the predictors** depends on having the "most right" model for the variance
  - Estimates will usually be ok → come from model for the means
  - Standard errors (and thus *p*-values) can be compromised

- The sources of variation that exist in your outcome will dictate **what kinds of predictors** will be useful
  - Between-Person variation needs Between-Person predictors
  - Within-Person variation needs Within-Person predictors
  - Between-whatever variation needs Between-whatever predictors…

2. Include categorical or continuous predictors at any level of analysis

- "ANOVA" test differences among discrete groups
- "Regression" tests slopes for continuous predictors
- What if a predictor is assessed repeatedly but can't be characterized by discrete "conditions"?
  - ANOVA or Regression won't work → you need MLM

- Some things don't change over time → time-invariant
- Some things do change over time → time-varying
- Some things are measured at higher levels
- Interactions are possible at same level or across levels

3. Does not require same data structure per person (by accident or by design)

RM ANOVA: uses **multivariate** (wide) data structure:

ID	Sex	T1	T2	T3	T4
100	0	5	6	8	12
101	1	4	7	.	11

People missing any data are excluded (data from ID 101 are not included at all)

MLM: uses **stacked** (long) data structure:

Only rows missing data are excluded

100 uses 4 cases
101 uses 3 cases

ID	Sex	Time	Y
100	0	1	5
100	0	2	6
100	0	3	8
100	0	4	12
101	1	1	4
101	1	2	7
101	1	3	.
101	1	4	11

Time can also be **unbalanced** across people such that each person can have his or her own measurement schedule: Time “0.9” “1.4” “3.5” “4.2”…

4. You already know how!

- If you can do GLM, you can do MLM
  (*and if you can do generalized linear models, you can do generalized multilevel models, too*)

- How do you interpret an estimate for…
  - the intercept?
  - the effect of a continuous variable?
  - the effect of a categorical variable?
  - a variance component ("pile of variance")?

This Semester's Topics

- Make Friends with SAS (to translate into SPSS, Stata, R…)
- Review of single-level linear models
- Within-person analysis via ANOVA
- Describing within-person fluctuation via alternative covariance structure models
  - R matrix structures with and without a random intercept
- Describing within-person change via random effects
  - Polynomial, piecewise, and exponential models
- Review of interactions in single-level linear models
- Time-invariant (level-2) predictors
- As time permits, crossed random effects models for other kinds of repeated measures data (subjects by items)
- *Next course: time-varying predictors and multivariate MLMs*