Example 7b: Time-Invariant Predictors in Models of Change

The models for this example use the same response time data as in Hoffman (2015) chapter 6 but will include three new level-2 predictors. Specifically, in a sample of 101 older adults we will be examining baseline age, abstract reasoning, and education group as time-invariant predictors of change in response time (RT) in milliseconds over six practice sessions to a measure of processing speed (as measured by the number match 3 test). This example will first show models for change using piecewise linear slopes, followed by models with linear and quadratic time slopes. Note that because the same diagonal R matrix is used in all example models, no SAS REPEATED (or STATA RESIDUAL) option is used in any of these models (but could be included if other level-1 R matrix structures were desired).

SAS Syntax for Data Import, Manipulation, and Description:

```
* Defining global variable for file location to be replaced in code below;
%LET filesave=C:\Dropbox\21_PSQF7375_Longitudinal\PSQF7375_Longitudinal_Example7b;
* Location for SAS files for these models (uses macro variable filesave);
LIBNAME filesave "&filesave.";

* Bringing data into work library and recoding/centering variables;
* Centering time for polynomial models;
* Creating squared term for use in PROC MEANS only;
DATA work.Example7b; SET filesave.SAS_Example7b;
* Center time at session 1 for polynomial models (also make quadratic version);
  time1=session-1;
  LABEL time1="time1: Session (0=1)";
* Create two slopes for piecewise models;
* (intercept = session 1, breakpoint = session 2);
       IF session EQ 1 THEN DO; slope12=0; slope26=0; END;
  ELSE IF session GE 2 THEN DO; slope12=1; slope26=session-2; END;
  LABEL slope12="slope12: 1-2 Early Practice Slope"
        slope26="slope26: 2-6 Later Practice Slope";
* Center level-2 predictors (based on descriptives below);
  age80=baseage-80;
  reas22=absreas-22;
  LABEL age80= "age80: Age Centered (0=80)"
        reas22="reas22: Abstract Reasoning Centered (0=22)";
* Make education a grouping variable FOR DEMO PURPOSES ONLY;
       IF educyrs=. THEN educgrp=.;
  ELSE IF educyrs LE 12                    THEN educgrp=1;
  ELSE IF educyrs GT 12 AND educyrs LE 16 THEN educgrp=2;
  ELSE IF educyrs GT 16                    THEN educgrp=3;
  LABEL educgrp="educgrp: Education Group (1=HS, 2=BA, 3=GRAD)";
* Remove cases missing any predictors or outcomes used here;
  IF NMISS(age80, reas22, educgrp, session, nm3rt)>0 THEN DELETE;
RUN;

TITLE1 "Descriptive Statistics for Level-2 Quantitative Predictors";
PROC MEANS DATA=work.Example7b; VAR baseage absreas; RUN;
```

STATA Syntax for Data Import, Manipulation, and Description:

```
// Define global variable for file location to be replaced in code below
   global filesave "C:\Dropbox\21_PSQF7375_Longitudinal\PSQF7375_Longitudinal_Example7b"

// Import example 7b stacked data
   use "$filesave\STATA_Example7b.dta", clear

// Center time at session 1 for polynomial time models (also need to make quadratic version)
   gen time1=session-1
   gen time1sq=time1*time1
   label variable time1   "time1: Linear Session (0=1)"
   label variable time1sq "time1sq: Quadratic Session (0=1)"

// Create two slopes for piecewise models
// (intercept = session 1, breakpoint = session 2)
   gen slope12 = session
   recode slope12 (1=0) if session==1
   recode slope12 (2=1) if session==2
```

```
   recode slope12 (3=1) if session==3
   recode slope12 (4=1) if session==4
   recode slope12 (5=1) if session==5
   recode slope12 (6=1) if session==6
   gen slope26 = session
   recode slope26 (1=0) if session==1
   recode slope26 (2=0) if session==2
   recode slope26 (3=1) if session==3
   recode slope26 (4=2) if session==4
   recode slope26 (5=3) if session==5
   recode slope26 (6=4) if session==6
   label variable slope12 "slope12: Early Practice Slope (Session 1-2)"
   label variable slope26 "slope26: Later Practice Slope (Session 2-6)"

// Center level-2 predictors (based on descriptives below)
   gen age80=baseage-80
   gen reas22=absreas-22
   label variable age80  "age80: Age Centered (0=80 years)"
   label variable reas22 "reas22: Abstract Reasoning Centered (0=22)"
// Make education a grouping variable FOR DEMO PURPOSES ONLY
   gen educgrp=.
   replace educgrp=1 if (educyrs <= 12)
   replace educgrp=2 if (educyrs > 12 & educyrs <= 16)
   replace educgrp=3 if (educyrs > 16)
   label variable educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)"
// Create new variable to hold number of missing cases
// Then drop cases with incomplete predictors
   egen nummiss = rowmiss(age80 reas22 educgrp session nm3rt)
   drop if nummiss>0

display "Descriptive Statistics for Level-2 Quantitative Predictors"
summarize baseage absreas
```

R Syntax for Data Import, Manipulation, and Description:

```
# Define variables for working directory and data name
filesave = "C:\\Dropbox/21_PSQF7375_Longitudinal/PSQF7375_Longitudinal_Example7b/"
filename = "SAS_Example7b.sas7bdat"
setwd(dir=filesave)

# Import Example 7b stacked data with labels
Example7b = read_sas(data_file=paste0(filesave,filename))
# Convert to data frame as data frame without labels to use for analysis
Example7b = as.data.frame(Example7b)
# Sort data by PersonID (needed for correct RCOV matrix)
Example7b = sort_asc(Example7b,ID,session)

# Center time at session 1 for polynomial time models
Example7b$time1=Example7b$session-1
# Create two slopes for piecewise models
# (intercept = session 1, breakpoint = session 2)
Example7b$slope12=Example7b$session
Example7b$slope12[which(Example7b$session==1)]=0
Example7b$slope12[which(Example7b$session==2)]=1
Example7b$slope12[which(Example7b$session==3)]=1
Example7b$slope12[which(Example7b$session==4)]=1
Example7b$slope12[which(Example7b$session==5)]=1
Example7b$slope12[which(Example7b$session==6)]=1
Example7b$slope26=Example7b$session
Example7b$slope26[which(Example7b$session==1)]=0
Example7b$slope26[which(Example7b$session==2)]=0
Example7b$slope26[which(Example7b$session==3)]=1
Example7b$slope26[which(Example7b$session==4)]=2
Example7b$slope26[which(Example7b$session==5)]=3
Example7b$slope26[which(Example7b$session==6)]=4

# Center level-2 predictors (based on descriptives below)
Example7b$age80=Example7b$baseage-80  # age80: Age Centered (0=80)
Example7b$reas22=Example7b$absreas-22 # reas22: Abstract Reasoning Centered (0=22)
# Make education a grouping variable FOR DEMO PURPOSES ONLY
Example7b$educgrp = cut(Example7b$educyrs, c(0,12,16,100), labels=c(1:3), right=TRUE)
# Make new variable for educgrp with reference=3 to match other programs
Example7b$educgrp3=relevel(factor(Example7b$educgrp), ref=3)
# Drop cases with missing data
Example7b = Example7b[complete.cases(Example7b[ , 1:6]),]

print("Descriptive Statistics for Level-2 Predictors")
describe(x=Example7b$baseage); describe(x=Example7b$absreas); summary(Example7b$educgrp)
```

1a. Baseline Unconditional Random Two-Piece Time Slopes Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2: **Intercept:** $\beta_{0i} = \gamma_{00} + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + U_{2i}$

Fixed-Effect-Predicted Outcome: $\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(\text{Slope12}_{ti}) + \gamma_{20}(\text{Slope26}_{ti})$

```
TITLE1 "SAS 1a: Random Piecewise Time Unconditional Model";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID session;
MODEL nm3rt = slope12 slope26
            / SOLUTION DDFM=Satterthwaite OUTPM=PredPUnc; * Save yhat;
RANDOM INTERCEPT slope12 slope26 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovPUnc; * Save covparms for comparison;
RUN;
TITLE1 "Correlation of predicted and actual RT";
PROC CORR DATA=PredPUnc OUTP=CorrPUnc; VAR pred; WITH nm3rt; RUN;
```

```
display "STATA 1a: Random Piecewise Time Unconditional Model"
mixed nm3rt c.slope12 c.slope26,                                          ///
           || id: slope12 slope26, variance reml covariance(unstructured) ///
           dfmethod(satterthwaite) dftable(pvalue)
predict predPUnc   // Save yhat
estat ic, n(101)   // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
corr predPUnc nm3rt    // Get total r to make R2
     display r(rho)^2  // Print total R2 relative to empty model
```

```
print("R 1a: Random Piecewise Time Unconditional Model")
PUnc = lmer(data=Example7b, REML=TRUE,
            formula=nm3rt~1+slope12+slope26 +(1+slope12+slope26|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(PUnc, ddf="Satterthwaite"); llikAIC(PUnc, chkREML=FALSE)
print("Save predicted values, show correlation of predicted and actual RT")
Example7b$PredPUnc = predict(PUnc, re.form=NA)
rPUnc = cor.test(Example7b$PredPUnc, Example7b$nm3rt, method="pearson")
rPUnc; print("Total R2"); rPUnc$estimate^2
```

SAS Output:

Estimated G Correlation Matrix

Row	Effect	ID	Col1	Col2	Col3
1	Intercept	101	1.0000	-0.4025	-0.3902
2	slope12	101	-0.4025	1.0000	-0.1293
3	slope26	101	-0.3902	-0.1293	1.0000

These are the correlations among the level-2 random effects. For these models I am not printing the R, G, V, or VCORR matrices (but the parameters in the model for the variance are given below).

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	ID	284312	42731	6.65	<.0001	Level-2 random intercept variance of U_{0i}
UN(2,1)	ID	-54270	18230	-2.98	0.0029	Level-2 random intercept-slope12 covariance
UN(2,2)	ID	63954	13244	4.83	<.0001	Level-2 random slope12 change variance of U_{1i}
UN(3,1)	ID	-10644	3791.26	-2.81	0.0050	Level-2 random intercept-slope26 covariance
UN(3,2)	ID	-1672.30	2097.03	-0.80	0.4252	Level-2 random slope12-slope26 covariance
UN(3,3)	ID	2617.28	636.48	4.11	<.0001	Level-2 random slope26 change variance of U_{2i}
Residual	ID	17673	1435.84	12.31	<.0001	Level-1 residual variance of e_{ti}

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8275.4	**7**	8289.4	8289.6	8296.8	8307.7	8314.7

```
                Solution for Fixed Effects
                      Standard
Effect       Estimate       Error      DF    t Value    Pr > |t|  g = gamma fixed effect
Intercept     1961.89     54.6805     100      35.88      <.0001  g00: Predicted RT when time=0 (session 1)
slope12       -163.64     30.2188     100      -5.42      <.0001  g10: Change/session btw sessions 1 and 2
slope26      -32.8932      6.5888     100      -4.99      <.0001  g20: Change/session btw sessions 2 and 6

Pearson Correlation Coefficients, N = 606
        Prob > |r| under H0: Rho=0
                          Pred
nm3rt                  0.19338
Number Match 3 RT       <.0001
```

r = .19338 → **TOTAL R^2 = .0374**
~ 4% of RT variance is accounted for by 2 piecewise linear effects of session

---

1b. Piecewise Model with Age Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2: **Intercept:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(\text{Slope12}_{ti}) + \gamma_{20}(\text{Slope26}_{ti}) + \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{21}(\text{Slope26}_{ti})(\text{Age}_i - 80)$$

Simple Slopes of Interactions:

$\text{Slope12} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80)$

$\text{Slope26} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

$\text{Age} = \gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

```
TITLE1 "SAS 1b: Add Age Predicting Intercept, Slope12, and Slope26";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID session;
MODEL nm3rt = slope12 slope26 age80 slope12*age80 slope26*age80
              / SOLUTION DDFM=Satterthwaite OUTPM=PredPAge; * Save yhat;
RANDOM INTERCEPT slope12 slope26 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovPAge; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"  age80 1, slope12*age80 1, slope26*age80 1;
* Simple slope12 and slope26 for age 74, 80, 86 (about -1SD, M, +1 SD of age80);
ESTIMATE "Slope12: Age 74"  slope12 1 slope12*age80 -6;
ESTIMATE "Slope12: Age 80"  slope12 1 slope12*age80  0;
ESTIMATE "Slope12: Age 86"  slope12 1 slope12*age80  6;
ESTIMATE "Slope26: Age 74"  slope26 1 slope26*age80 -6;
ESTIMATE "Slope26: Age 80"  slope26 1 slope26*age80  0;
ESTIMATE "Slope26: Age 86"  slope26 1 slope26*age80  6;
* Simple age slope at each session (S);
ESTIMATE "Age Slope: S1"  age80 1 slope12*age80 0 slope26*age80 0;
ESTIMATE "Age Slope: S2"  age80 1 slope12*age80 1 slope26*age80 0;
ESTIMATE "Age Slope: S3"  age80 1 slope12*age80 1 slope26*age80 1;
ESTIMATE "Age Slope: S4"  age80 1 slope12*age80 1 slope26*age80 2;
ESTIMATE "Age Slope: S5"  age80 1 slope12*age80 1 slope26*age80 3;
ESTIMATE "Age Slope: S6"  age80 1 slope12*age80 1 slope26*age80 4;
RUN;
TITLE1 "Total R2 change for time relative to unconditional model";
%TotalR2(DV=nm3rt, PredFewer=PredPUnc, PredMore=PredPAge);
TITLE1 "PseudoR2 for time relative to unconditional model";
%PseudoR2(NCov=7, CovFewer=CovPUnc, CovMore=CovPAge);


display "STATA 1b: Add Age Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80, ///
            || id: slope12 slope26, variance reml covariance(unstructured)   ///
            dfmethod(satterthwaite) dftable(pvalue)
predict predPAge   // Save yhat
estat ic, n(101)   // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.slope12#c.age80=0)(c.slope26#c.age80=0), small
```

```
// Simple slope12 and slope26 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)
lincom c.slope12*1 + c.slope12#c.age80*-6, small  // Slope12: Age 74
lincom c.slope12*1 + c.slope12#c.age80*0 , small  // Slope12: Age 80
lincom c.slope12*1 + c.slope12#c.age80*6 , small  // Slope12: Age 86
margins, at(c.age80=(-6(6)6) c.slope26=0) dydx(c.slope12) df(99) // Same simple slope12
lincom c.slope26*1 + c.slope26#c.age80*-6, small  // Slope26: Age 74
lincom c.slope26*1 + c.slope26#c.age80*0 , small  // Slope26: Age 80
lincom c.slope26*1 + c.slope26#c.age80*6 , small  // Slope26: Age 86
margins, at(c.age80=(-6(6)6) c.slope12=1) dydx(c.slope26) df(99) // Same simple slope26
// Simple age slope at each session (S)
lincom c.age80*1 + c.slope12#c.age80*0 + c.slope26#c.age80*0, small  // Age Slope: S1
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*0, small  // Age Slope: S2
margins, at(c.slope12=(0(1)1) c.slope26=0) dydx(c.age80) df(99) // Same age slope for S1-S2
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*1, small  // Age Slope: S3
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*2, small  // Age Slope: S4
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*3, small  // Age Slope: S5
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*4, small  // Age Slope: S6
margins, at(c.slope12=1 c.slope26=(1(1)4)) dydx(c.age80) df(99) // Same age slope for S3-S6
// Get adjusted means per session and age (start(by)end)
margins, at(c.slope12=(0(1)1) c.slope26=0 c.age80=(-6 0 6))  // Sessions 1-2
     marginsplot       // Plot adjusted means
margins, at(c.slope12=1 c.slope26=(1(1)4) c.age80=(-6 0 6))  // Sessions 3-6
     marginsplot       // Plot adjusted means
corr predPAge nm3rt    // Get total r to make R2
     display r(rho)^2  // Print total R2 relative to empty model
```

```
print("R 1b: Add Age Predicting Intercept, Slope12, and Slope26")
PAge = lmer(data=Example7b, REML=TRUE,
            formula=nm3rt~1+slope12+slope26+age80 +slope12:age80 +slope26:age80
                         +(1+slope12+slope26|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(PAge, ddf="Satterthwaite"); llikAIC(PAge, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(PAge, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0),c(0,0,0,0,1,0),c(0,0,0,0,0,1)))
print("Simple slope12 and slope26 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)")
print("Slope12: Age 74"); contest1D(PAge, ddf="Satterthwaite", L=c(0,1,0,0,-6,0))
print("Slope12: Age 80"); contest1D(PAge, ddf="Satterthwaite", L=c(0,1,0,0, 0,0))
print("Slope12: Age 86"); contest1D(PAge, ddf="Satterthwaite", L=c(0,1,0,0, 6,0))
print("Slope26: Age 74"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,1,0,0,-6))
print("Slope26: Age 80"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,1,0,0, 0))
print("Slope26: Age 86"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,1,0,0, 6))
print("Simple age slope at each session (S)")
print("Age Slope: S1"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,0,0))
print("Age Slope: S2"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,0))
print("Age Slope: S3"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,1))
print("Age Slope: S4"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,2))
print("Age Slope: S5"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,3))
print("Age Slope: S6"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,4))
print("Save yhat and correlation of yhat with y")
Example7b$PredPAge = predict(PAge, re.form=NA)
rPAge = cor.test(Example7b$PredPAge, Example7b$nm3rt, method="pearson")
print("Total R2"); rPAge$estimate^2
print("Total R2 change for age relative to unconditional model")
rPAge$estimate^2-rPUnc$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	254286	38681	6.57	<.0001
UN(2,1)	ID	-46576	17140	-2.72	0.0066
UN(2,2)	ID	62742	13139	4.78	<.0001
UN(3,1)	ID	-9251.94	3583.89	-2.58	0.0098
UN(3,2)	ID	-2106.57	2100.78	-1.00	0.3160
UN(3,3)	ID	2593.60	636.25	4.08	<.0001
Residual	ID	17673	1435.84	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8251.0	7	8265.0	8265.2	8272.4	8283.3	8290.3

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	g = gamma fixed effect
Intercept	1966.86	51.9106	99	37.89	<.0001	g00
slope12	-164.91	30.0311	99	-5.49	<.0001	g10
slope26	-33.1182	6.5734	99	-5.04	<.0001	g20
age80	29.7804	8.5822	99	3.47	0.0008	g01
slope12*age80	-7.5810	4.9650	99	-1.53	0.1300	g11
slope26*age80	-1.3499	1.0868	99	-1.24	0.2171	g21

Interpret the fixed intercept:

Interpret the fixed effect of slope12:

Interpret the fixed effect of slope26:

Interpret the fixed effect of age80:

Interpret the effect of slope12*age80:

Interpret the effect of slope26*age80:

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Slope12: Age 74	-119.42	41.7131	99	-2.86	0.0051
Slope12: Age 80	-164.91	30.0311	99	-5.49	<.0001
Slope12: Age 86	-210.39	42.8789	99	-4.91	<.0001
Slope26: Age 74	-25.0187	9.1305	99	-2.74	0.0073
Slope26: Age 80	-33.1182	6.5734	99	-5.04	<.0001
Slope26: Age 86	-41.2177	9.3857	99	-4.39	<.0001
Age Slope: S1	29.7804	8.5822	99	3.47	0.0008
Age Slope: S2	22.1993	7.9689	99	2.79	0.0064
Age Slope: S3	20.8494	7.5245	99	2.77	0.0067
Age Slope: S4	19.4995	7.2176	99	2.70	0.0081
Age Slope: S5	18.1496	7.0663	99	2.57	0.0117
Age Slope: S6	16.7997	7.0805	99	2.37	0.0196

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	99	4.08	0.0089

Total R2 change for time relative to unconditional model
Total R2 (% Reduction) for PredPUnc vs. PredPAge

This multivariate Wald F-test provides the significance for the change in total R^2 relative to the unconditional model.

Name	Pred Corr	TotalR2	Total R2Diff
PredPUnc	0.19338	0.03740	.
PredPAge	0.32795	0.10755	0.070156

PseudoR2 for time relative to unconditional model
PsuedoR2 (% Reduction) for CovPUnc vs. CovPAge

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	Pseudo R2
CovPUnc	UN(1,1)	ID	284312	42731	6.65	<.0001	.
CovPUnc	UN(2,2)	ID	63954	13244	4.83	<.0001	.
CovPUnc	UN(3,3)	ID	2617.28	636.48	4.11	<.0001	.
CovPUnc	Residual	ID	17673	1435.84	12.31	<.0001	.
CovPAge	UN(1,1)	ID	254286	38681	6.57	<.0001	0.10561
CovPAge	UN(2,2)	ID	62742	13139	4.78	<.0001	0.01895
CovPAge	UN(3,3)	ID	2593.60	636.25	4.08	<.0001	0.00905
CovPAge	Residual	ID	17673	1435.84	12.31	<.0001	0.00000

Which variance component should have been reduced by each new fixed effect of age?

1c. Piecewise Model with Age and Reasoning Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reas}_i - 22) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + U_{2i}$

Simple Slopes of Interactions:

$\text{Slope12} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22)$

$\text{Slope26} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22)$

$\text{Age} = \gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

$\text{Reas} = \gamma_{02} + \gamma_{12}(\text{Slope12}_{ti}) + \gamma_{22}(\text{Slope26}_{ti})$

Fixed-Effect-Predicted Outcome:

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(\text{Slope12}_{ti}) + \gamma_{20}(\text{Slope26}_{ti}) + \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{21}(\text{Slope26}_{ti})(\text{Age}_i - 80) + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(\text{Slope12}_{ti})(\text{Reas}_i - 22) + \gamma_{22}(\text{Slope26}_{ti})(\text{Reas}_i - 22)$$

```
TITLE1 "SAS 1c: Keep Age, Add Reasoning Predicting Intercept, Slope12, and Slope26";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
CLASS ID session;
MODEL nm3rt = slope12 slope26 age80 slope12*age80 slope26*age80
              reas22 slope12*reas22 slope26*reas22
              / SOLUTION DDFM=Satterthwaite OUTPM=PredPReas; * Save yhat;
RANDOM INTERCEPT slope12 slope26 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovPReas; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"         age80 1,  slope12*age80 1,  slope26*age80 1;
CONTRAST "DF=3 Wald Test for all Reasoning Slopes"  reas22 1, slope12*reas22 1, slope26*reas22 1;
* Simple slope12 and slope26 for reasoning 17, 22, 27 (about -1SD, M, +1 SD of reas22);
ESTIMATE "Slope12: Reasoning 17"  slope12 1 slope12*reas22 -5;
ESTIMATE "Slope12: Reasoning 22"  slope12 1 slope12*reas22  0;
ESTIMATE "Slope12: Reasoning 27"  slope12 1 slope12*reas22  5;
ESTIMATE "Slope26: Reasoning 17"  slope26 1 slope26*reas22 -5;
ESTIMATE "Slope26: Reasoning 22"  slope26 1 slope26*reas22  0;
ESTIMATE "Slope26: Reasoning 27"  slope26 1 slope26*reas22  5;
* Simple reasoning slope at each session (S);
ESTIMATE "Reasoning Slope: S1"  reas22 1 slope12*reas22 0 slope26*reas22 0;
ESTIMATE "Reasoning Slope: S2"  reas22 1 slope12*reas22 1 slope26*reas22 0;
ESTIMATE "Reasoning Slope: S3"  reas22 1 slope12*reas22 1 slope26*reas22 1;
ESTIMATE "Reasoning Slope: S4"  reas22 1 slope12*reas22 1 slope26*reas22 2;
ESTIMATE "Reasoning Slope: S5"  reas22 1 slope12*reas22 1 slope26*reas22 3;
ESTIMATE "Reasoning Slope: S6"  reas22 1 slope12*reas22 1 slope26*reas22 4;
RUN;
TITLE1 "Total R2 change for reasoning relative to age-only model";
%TotalR2(DV=nm3rt, PredFewer=PredPAge, PredMore=PredPReas);
TITLE1 "PseudoR2 for reasoning relative to age-only model";
%PseudoR2(NCov=7, CovFewer=CovPAge, CovMore=CovPReas);
```

```
display "STATA 1c: Keep Age, Add Reasoning Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80 ///
           c.reas22 c.slope12#c.reas22 c.slope26#c.reas22,                  ///
           || id: slope12 slope26, variance reml covariance(unstructured)  ///
           dfmethod(satterthwaite) dftable(pvalue)
predict predPReas  // Save yhat
estat ic, n(101)   // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.slope12#c.age80=0)(c.slope26#c.age80=0), small
// DF=3 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.slope12#c.reas22=0)(c.slope26#c.reas22=0), small
// Simple slope12 and slope26 for reasoning 17, 22, 27 (about -1SD, M, +1 SD of reas22)
lincom c.slope12*1 + c.slope12#c.reas22*-5, small  // Slope12: Reasoning 17
lincom c.slope12*1 + c.slope12#c.reas22*0 , small  // Slope12: Reasoning 22
lincom c.slope12*1 + c.slope12#c.reas22*5 , small  // Slope12: Reasoning 27
margins, at(c.age80=0 c.reas22=(-5(5)5) c.slope26=0) dydx(c.slope12) df(98) // Same simple slope12
lincom c.slope26*1 + c.slope26#c.reas22*-5, small  // Slope12: Reasoning 17
lincom c.slope26*1 + c.slope12#c.reas22*0 , small  // Slope12: Reasoning 22
lincom c.slope26*1 + c.slope26#c.reas22*5 , small  // Slope12: Reasoning 27
margins, at(c.age80=0 c.reas22=(-5(5)5) c.slope12=1) dydx(c.slope26) df(98) // Same simple slope26
// Simple reasoning slope at each session (S)
lincom c.reas22*1 + c.slope12#c.reas22*0 + c.slope26#c.reas22*0, small  // Reasoning Slope: S1
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*0, small  // Reasoning Slope: S2
margins, at(c.age80=0 c.slope12=(0(1)1) c.slope26=0) dydx(c.reas22) df(98) // Same reas slope for S1-S2
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*1, small  // Reasoning Slope: S3
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*2, small  // Reasoning Slope: S4
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*3, small  // Reasoning Slope: S5
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*4, small  // Reasoning Slope: S6
margins, at(c.age80=0 c.slope12=1 c.slope26=(1(1)4)) dydx(c.reas22) df(98) // Same reas slope for S3-S6
// Get adjusted means per session and reasoning (start(by)end), hold age80-0
margins, at(c.age80=0 c.slope12=(0(1)1) c.slope26=0 c.reas22=(-5 0 5))  // Sessions 1-2
     marginsplot       // Plot adjusted means
margins, at(c.age80=0 c.slope12=1 c.slope26=(1(1)4) c.reas22=(-5 0 5))  // Sessions 3-6
     marginsplot       // Plot adjusted means
corr predPReas nm3rt   // Get total r to make R2
     display r(rho)^2  // Print total R2 relative to empty model
```

```
print("R 1c: Keep Age, Add Reasoning Predicting Intercept, Slope12, and Slope26")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
PReas = lmer(data=Example7b, REML=TRUE,
            formula=nm3rt~1+slope12+slope26+age80+reas22 +slope12:age80 +slope26:age80
                          +slope12:reas22 +slope26:reas22 +(1+slope12+slope26|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(PReas, ddf="Satterthwaite"); llikAIC(PReas, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(PReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,1,0,0,0,0,0),c(0,0,0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,1,0,0)))
print("DF=3 Wald Test for all Reasoning Slopes")
contestMD(PReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,0,1,0,0,0,0),c(0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,1)))
print("Simple slope12 and slope26 for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22)")
print("Slope12: Reas 17"); contest1D(PReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,-5,0))
print("Slope12: Reas 22"); contest1D(PReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,0, 0,0))
print("Slope12: Reas 27"); contest1D(PReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,0, 5,0))
print("Slope26: Reas 17"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0,0,-5))
print("Slope26: Reas 22"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0,0, 0))
print("Slope26: Reas 27"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0,0, 5))
print("Simple age slope at each session (S)")
print("Reas Slope: S1"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0,0,0))
print("Reas Slope: S2"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,0))
print("Reas Slope: S3"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,1))
print("Reas Slope: S4"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,2))
print("Reas Slope: S5"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,3))
print("Reas Slope: S6"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,4))
print("Save yhat and correlation of yhat with y")
Example7b$PredPReas = predict(PReas, re.form=NA)
rPReas = cor.test(Example7b$PredPReas, Example7b$nm3rt, method="pearson")
print("Total R2"); rPReas$estimate^2
print("Total R2 change for reasoning relative to age-only model")
rPReas$estimate^2-rPAge$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	242192	37151	6.52	<.0001
UN(2,1)	ID	-49817	17064	-2.92	0.0035
UN(2,2)	ID	63222	13272	4.76	<.0001
UN(3,1)	ID	-7510.98	3414.18	-2.20	0.0278
UN(3,2)	ID	-1845.11	2068.67	-0.89	0.3724
UN(3,3)	ID	2411.55	614.00	3.93	<.0001
Residual	ID	17673	1435.84	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8226.5	7	8240.5	8240.7	8247.9	8258.8	8265.8

Solution for Fixed Effects

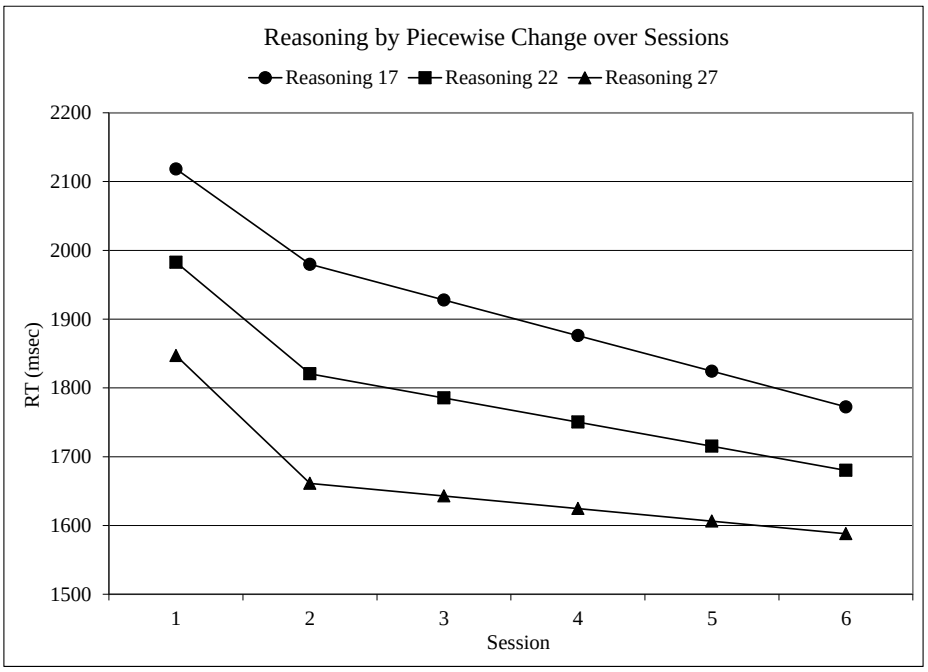
Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept	1982.64	51.1793	98	38.74	<.0001	g00
slope12	-162.16	30.3688	98	-5.34	<.0001	g10
slope26	-35.0669	6.4901	98	-5.40	<.0001	g20
age80	23.0041	8.8639	98	2.60	0.0109	g01
slope12*age80	-8.7589	5.2597	98	-1.67	0.0990	g11
slope26*age80	-0.5135	1.1240	98	-0.46	0.6488	g21
reas22	-27.1200	11.4528	98	-2.37	0.0198	g02
slope12*reas22	-4.7141	6.7959	98	-0.69	0.4895	g12
slope26*reas22	3.3476	1.4523	98	2.30	0.0233	g22

Which fixed effects are conditional on age?

Which fixed effects are conditional on reasoning?

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Slope12: Reasoning 17	-138.59	48.4325	98	-2.86	0.0052
Slope12: Reasoning 22	-162.16	30.3688	98	-5.34	<.0001
Slope12: Reasoning 27	-185.73	42.5208	98	-4.37	<.0001
Slope26: Reasoning 17	-51.8049	10.3504	98	-5.01	<.0001
Slope26: Reasoning 22	-35.0669	6.4901	98	-5.40	<.0001
Slope26: Reasoning 27	-18.3288	9.0870	98	-2.02	0.0464
Reasoning Slope: S1	-27.1200	11.4528	98	-2.37	0.0198
Reasoning Slope: S2	-31.8340	10.4508	98	-3.05	0.0030
Reasoning Slope: S3	-28.4864	9.9154	98	-2.87	0.0050
Reasoning Slope: S4	-25.1388	9.5724	98	-2.63	0.0100
Reasoning Slope: S5	-21.7912	9.4427	98	-2.31	0.0231
Reasoning Slope: S6	-18.4436	9.5350	98	-1.93	0.0560



Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	98	2.37	0.0756
DF=3 Wald Test for all Reasoning Slopes	3	98	3.50	0.0183

Total R2 change for reasoning relative to age-only model
Total R2 (% Reduction) for PredPAge vs. PredPReas

Name	Pred Corr	TotalR2	Total R2Diff
PredPAge	0.32795	0.10755	.
PredPReas	0.40163	0.16131	0.053755

The second multivariate Wald F-test provides the significance for the change in total R^2 relative to the age-only model.

PseudoR2 for reasoning relative to age-only model
PsuedoR2 (% Reduction) for CovPAge vs. CovPReas

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovPAge	UN(1,1)	ID	254286	38681	6.57	<.0001	.
CovPAge	UN(2,2)	ID	62742	13139	4.78	<.0001	.
CovPAge	UN(3,3)	ID	2593.60	636.25	4.08	<.0001	.
CovPAge	Residual	ID	17673	1435.84	12.31	<.0001	.
CovPReas	UN(1,1)	ID	242192	37151	6.52	<.0001	0.047560
CovPReas	UN(2,2)	ID	63222	13272	4.76	<.0001	-0.007643
CovPReas	UN(3,3)	ID	2411.55	614.00	3.93	<.0001	0.070193
CovPReas	Residual	ID	17673	1435.84	12.31	<.0001	-0.000000

Which variance component should have been reduced by each new fixed effect of reasoning?

1d. Piecewise Model with Age, Reasoning, and Education Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{03}(\text{HighvsLowEd}_i) + \gamma_{04}(\text{HighvsMedEd}_i) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{HighvsLowEd}_i) + \gamma_{14}(\text{HighvsMedEd}_i) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + \gamma_{23}(\text{HighvsLowEd}_i) + \gamma_{24}(\text{HighvsMedEd}_i) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$\begin{aligned}\hat{y}_{ti} = {} & \gamma_{00} + \gamma_{10}(\text{Slope12}_{ti}) + \gamma_{20}(\text{Slope26}_{ti}) \\ & + \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{21}(\text{Slope26}_{ti})(\text{Age}_i - 80) \\ & + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(\text{Slope12}_{ti})(\text{Reas}_i - 22) + \gamma_{22}(\text{Slope26}_{ti})(\text{Reas}_i - 22) \\ & + \gamma_{03}(\text{HighvsLowEd}_i) + \gamma_{13}(\text{Slope12}_{ti})(\text{HighvsLowEd}_i) + \gamma_{23}(\text{Slope26}_{ti})(\text{HighvsLowEd}_i) \\ & + \gamma_{04}(\text{HighvsMedEd}_i) + \gamma_{14}(\text{Slope12}_{ti})(\text{HighvsMedEd}_i) + \gamma_{24}(\text{Slope26}_{ti})(\text{HighvsMedEd}_i)\end{aligned}$$

Simple Slopes of Interactions:

Slope12 = $\gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{HighvsLowEd}_i) + \gamma_{14}(\text{HighvsMedEd}_i)$

Slope26 = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + \gamma_{23}(\text{HighvsLowEd}_i) + \gamma_{24}(\text{HighvsMedEd}_i)$

Age = $\gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

Reasoning = $\gamma_{02} + \gamma_{12}(\text{Slope12}_{ti}) + \gamma_{22}(\text{Slope26}_{ti})$

High vs Low Ed = $\gamma_{03} + \gamma_{13}(\text{Slope12}_{ti}) + \gamma_{23}(\text{Slope26}_{ti})$

High vs Med Ed = $\gamma_{04} + \gamma_{14}(\text{Slope12}_{ti}) + \gamma_{24}(\text{Slope26}_{ti})$

Med vs Low Ed = $\gamma_{04} + \gamma_{14}(\text{Slope12}_{ti}) + \gamma_{24}(\text{Slope26}_{ti}) - \gamma_{03} - \gamma_{13}(\text{Slope12}_{ti}) - \gamma_{23}(\text{Slope26}_{ti})$

```
TITLE1 "SAS 1d: Keep Age & Reasoning, Add Education Group Predicting Intercept, Slope12, and Slope26";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID educgrp session; * On CLASS = educgrp is categorical (contrasts made automatically);
MODEL nm3rt = slope12 slope26 age80 slope12*age80 slope26*age80
              reas22 slope12*reas22 slope26*reas22 educgrp slope12*educgrp slope26*educgrp
              / SOLUTION DDFM=Satterthwaite OUTPM=PredPEduc; * Save yhat;
RANDOM INTERCEPT slope12 slope26 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovPEduc; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"          age80 1,  slope12*age80 1,  slope26*age80 1;
CONTRAST "DF=3 Wald Test for all Reasoning Slopes"  reas22 1, slope12*reas22 1, slope26*reas22 1;
CONTRAST "DF=6 Wald Test for all Education Slopes"  educgrp -1 1 0, educgrp -1 0 1,
        educgrp*slope12 -1 1 0, educgrp*slope12 -1 0 1, educgrp*slope26 -1 1 0, educgrp*slope26 -1 0 1;
* LSMEANS gives adjusted means and diffs per group only for education simple main effect;
LSMEANS educgrp / AT (slope12 slope26 age80 reas22) = (0 0 0 0) DIFF=ALL; * At beginning;
LSMEANS educgrp / AT (slope12 slope26 age80 reas22) = (1 4 0 0) DIFF=ALL; * At end;
* ESTIMATE statements can also give specific group differences;
ESTIMATE "1Low vs 3High Educ:  Intercept"  educgrp -1  0  1;
ESTIMATE "2Med vs 3High Educ:  Intercept"  educgrp  0 -1  1;
ESTIMATE "1Low vs 2Med  Educ:  Intercept"  educgrp -1  1  0;
ESTIMATE "1Low vs 3High Educ:  Slope12"    slope12*educgrp -1  0  1;
ESTIMATE "2Med vs 3High Educ:  Slope12"    slope12*educgrp  0 -1  1;
ESTIMATE "1Low vs 2Med  Educ:  Slope12"    slope12*educgrp -1  1  0;
ESTIMATE "1Low vs 3High Educ:  Slope26"    slope26*educgrp -1  0  1;
ESTIMATE "2Med vs 3High Educ:  Slope26"    slope26*educgrp  0 -1  1;
ESTIMATE "1Low vs 2Med  Educ:  Slope26"    slope26*educgrp -1  1  0;
RUN;
TITLE1 "Total R2 change for education relative to model with age and reasoning";
%TotalR2(DV=nm3rt, PredFewer=PredPReas, PredMore=PredPEduc);
TITLE1 "PseudoR2 for education relative to model with age and reasoning";
%PseudoR2(NCov=7, CovFewer=CovPReas, CovMore=CovPEduc);
```

```
display "STATA 1d: Keep Age & Reas, Add Education Group Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80          ///
            c.reas22 c.slope12#c.reas22 c.slope26#c.reas22                          ///
            ib(last).educgrp c.slope12#ib(last).educgrp c.slope26#ib(last).educgrp, ///
            || id: slope12 slope26, variance reml covariance(un)                    ///
            dfmethod(satterthwaite) dftable(pvalue)
predict predPEduc  // Save yhat
estat ic, n(101)   // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.slope12#c.age80=0)(c.slope26#c.age80=0), small
// DF=3 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.slope12#c.reas22=0)(c.slope26#c.reas22=0), small
// DF=2 Wald test for education on intercept, slope12, slope26, and DF=6 joint test
contrast i.educgrp c.slope12#i.educgrp c.slope26#i.educgrp, small overall
// Estimating adjusted means and mean diffs per group at first and last session
margins ib(last).educgrp, at(c.slope12=0 c.slope26=0 c.age80=0 c.reas22=0)
margins ib(last).educgrp, at(c.slope12=0 c.slope26=0 c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)
margins ib(last).educgrp, at(c.slope12=1 c.slope26=4 c.age80=0 c.reas22=0)
margins ib(last).educgrp, at(c.slope12=1 c.slope26=4 c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)
// Contrasts between groups on intercept, linear, and quadratic slopes
test 1.educgrp=3.educgrp, small                          // 1Low vs 3High: Intercept
test 2.educgrp=3.educgrp, small                          // 2Med vs 2High: Intercept
test 1.educgrp=2.educgrp, small                          // 1Low vs 2Med:  Intercept
test 1.educgrp#c.slope12=3.educgrp#c.slope12, small   // 1Low vs 3High: Slope12
test 2.educgrp#c.slope12=3.educgrp#c.slope12, small   // 2Med vs 3High: Slope12
test 1.educgrp#c.slope12=2.educgrp#c.slope12, small   // 1Low vs 2Med:  Slope12
test 1.educgrp#c.slope26=3.educgrp#c.slope26, small   // 1Low vs 3High: Slope26
test 2.educgrp#c.slope26=3.educgrp#c.slope26, small   // 2Med vs 3High: Slope26
test 1.educgrp#c.slope26=2.educgrp#c.slope26, small   // 1Low vs 2Med:  Slope26
// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.age80=0 c.reas22=0 c.slope12=(0(1)1) c.slope26=0 educgrp=(1 2 3))
     marginsplot        // Plot adjusted means
margins, at(c.age80=0 c.reas22=0 c.slope12=1 c.slope26=(1(1)4) educgrp=(1 2 3))
     marginsplot        // Plot adjusted means
corr predPEduc nm3rt    // Get total r to make R2
     display r(rho)^2   // Print total R2 relative to empty model


print("R 1d: Keep Age & Reasoning, Add Education Group Predicting Intercept, Slope12, and Slope26")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
PEduc = lmer(data=Example7b, REML=TRUE,
             formula=nm3rt~1+slope12+slope26+age80+reas22+factor(educgrp3) +slope12:age80
                           +slope26:age80 +slope12:reas22 +slope26:reas22 +slope12:factor(educgrp3)
                           +slope26:factor(educgrp3) +(1+slope12+slope26|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(PEduc, ddf="Satterthwaite"); llikAIC(PEduc, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(PEduc, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0,0,0,0,0,0,0,0,0,0),
                 c(0,0,0,0,0,0,0,1,0,0,0,0,0,0,0),c(0,0,0,0,0,0,0,0,1,0,0,0,0,0,0)))
print("DF=3 Wald Test for all Reasoning Slopes")
contestMD(PEduc, ddf="Satterthwaite", L=rbind(c(0,0,0,0,1,0,0,0,0,0,0,0,0,0,0),
               c(0,0,0,0,0,0,0,0,1,0,0,0,0,0,0),c(0,0,0,0,0,0,0,0,0,0,1,0,0,0,0)))
print("DF=2 Wald Test for Each Education Effect")
anova(PEduc)
print("DF=6 Wald Test for all Education Slopes")
contestMD(PEduc, ddf="Satterthwaite", L=rbind(
                 c(0,0,0,0,0,1,0,0,0,0,0,0,0,0,0),c(0,0,0,0,0,0,1,0,0,0,0,0,0,0,0),
                 c(0,0,0,0,0,0,0,0,0,0,1,0,0,0,0),c(0,0,0,0,0,0,0,0,0,0,0,1,0,0,0),
                 c(0,0,0,0,0,0,0,0,0,0,0,0,1,0,0),c(0,0,0,0,0,0,0,0,0,0,0,0,0,1,0),
                 c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,1)))
print("Adjusted means and diffs per group only for education simple main effect")
print("Education diffs at session 1")
Ps1mean = ref_grid(PEduc, at=list(slope12=0,slope26=0,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Ps1mean, pairwise~educgrp3, lmer.df="satterthwaite", adjust="none")
print("Education diffs at session 6")
Ps6mean = ref_grid(PEduc, at=list(slope12=1,slope26=4,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Ps6mean, pairwise~educgrp3, lmer.df="satterthwaite", adjust="none")
print("Specific education group differences on intercept, slope12 and slope16")
print("1Low vs 3High Educ: Intercept"); contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,-1,0,0,0,0,0,0,0,0,0))
print("2Med vs 3High Educ: Intercept"); contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,-1,0,0,0,0,0,0,0,0))
print("1Low vs 2Med  Educ: Intercept"); contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,-1,1,0,0,0,0,0,0,0,0))
print("1Low vs 3High Educ: Slope12");   contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0))
print("2Med vs 3High Educ: Slope12");   contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0))
print("1Low vs 2Med  Educ: Slope12");   contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,-1,1,0,0))
print("1Low vs 3High Educ: Slope26");   contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0))
print("2Med vs 3High Educ: Slope26");   contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1))
print("1Low vs 2Med  Educ: Slope26");   contest1D(PEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,-1,1))
```

```
print("Save yhat and correlation of yhat with y")
Example7b$PredPEduc = predict(PEduc, re.form=NA)
rPEduc = cor.test(Example7b$PredPEduc, Example7b$nm3rt, method="pearson")
print("Total R2"); rPEduc$estimate^2
print("Total R2 change relative to age and reasoning main effects model")
rPEduc$estimate^2-rPReas$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	246920	38218	6.46	<.0001
UN(2,1)	ID	-52254	17491	-2.99	0.0028
UN(2,2)	ID	63495	13444	4.72	<.0001
UN(3,1)	ID	-7543.48	3493.64	-2.16	0.0308
UN(3,2)	ID	-1820.21	2099.75	-0.87	0.3860
UN(3,3)	ID	2446.05	624.87	3.91	<.0001
Residual	ID	17673	1435.84	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8164.2	7	8178.2	8178.4	8185.6	8196.5	8203.5

Solution for Fixed Effects

Effect	Education Group (1=HS,2=BA,3=GRAD)	Estimate	Standard Error	DF	t Value	Pr > |t|	g = gamma fixed effect
Intercept		1978.15	105.83	96	18.69	<.0001	g00
slope12		-153.14	62.3250	96	-2.46	0.0158	g10
slope26		-24.6403	13.3543	96	-1.85	0.0681	g20
age80		22.9367	8.9490	96	2.56	0.0119	g01
slope12*age80		-8.9054	5.2704	96	-1.69	0.0943	g11
slope26*age80		-0.5289	1.1293	96	-0.47	0.6406	g21
reas22		-28.5673	11.9710	96	-2.39	0.0190	g02
slope12*reas22		-7.0891	7.0501	96	-1.01	0.3172	g12
slope26*reas22		3.4883	1.5106	96	2.31	0.0231	g22
educgrp	1	-41.9718	157.35	96	-0.27	0.7902	g03
educgrp	2	25.4470	125.54	96	0.20	0.8398	g04
educgrp	3	0	.	.	.	.	
slope12*educgrp	1	-85.9455	92.6714	96	-0.93	0.3560	g13
slope12*educgrp	2	18.5834	73.9371	96	0.25	0.8021	g14
slope12*educgrp	3	0	.	.	.	.	
slope26*educgrp	1	-6.3237	19.8566	96	-0.32	0.7508	g23
slope26*educgrp	2	-16.5965	15.8424	96	-1.05	0.2975	g24
slope26*educgrp	3	0	.	.	.	.	

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
slope12	1	96	28.16	<.0001
slope26	1	96	20.73	<.0001
age80	1	96	6.57	0.0119
slope12*age80	1	96	2.86	0.0943
slope26*age80	1	96	0.22	0.6406
reas22	1	96	5.69	0.0190
slope12*reas22	1	96	1.01	0.3172
slope26*reas22	1	96	5.33	0.0231
educgrp	**2**	**96**	**0.12**	**0.8831**
slope12*educgrp	**2**	**96**	**0.85**	**0.4289**
slope26*educgrp	**2**	**96**	**0.60**	**0.5516**

I normally omit this box if the CLASS statement is not used for predictors (because it is redundant). But here the last three entries give us the omnibus (df=2) tests for whether there are any education group differences on the intercept, slope12, or slope26 time slopes.

Estimates

Label		Estimate	Standard Error	DF	t Value	Pr > |t|
1Low vs 3High Educ:	Intercept	41.9718	157.35	96	0.27	0.7902
2Med vs 3High Educ:	Intercept	-25.4470	125.54	96	-0.20	0.8398
1Low vs 2Med Educ:	Intercept	67.4187	136.36	96	0.49	0.6221
1Low vs 3High Educ:	Slope12	85.9455	92.6714	96	0.93	0.3560
2Med vs 3High Educ:	Slope12	-18.5834	73.9371	96	-0.25	0.8021
1Low vs 2Med Educ:	Slope12	104.53	80.3066	96	1.30	0.1962
1Low vs 3High Educ:	Slope26	6.3237	19.8566	96	0.32	0.7508
2Med vs 3High Educ:	Slope26	16.5965	15.8424	96	1.05	0.2975
1Low vs 2Med Educ:	Slope26	-10.2728	17.2072	96	-0.60	0.5519

Least Squares Means

Effect	Education Group (1=HS, 2=BA,3=GRAD)	slope12	slope26	age80	reas22	Estimate	Standard Error	DF	t Value	Pr > |t|
educgrp	1	0.00	0.00	0.00	0.00	1936.18	114.13	96	16.97	<.0001
educgrp	2	0.00	0.00	0.00	0.00	2003.60	70.3593	96	28.48	<.0001
educgrp	3	0.00	0.00	0.00	0.00	1978.15	105.83	96	18.69	<.0001
educgrp	1	1.00	4.00	0.00	0.00	1573.24	94.3228	96	16.68	<.0001
educgrp	2	1.00	4.00	0.00	0.00	1704.10	58.1509	96	29.30	<.0001
educgrp	3	1.00	4.00	0.00	0.00	1726.45	87.4643	96	19.74	<.0001

Differences of Least Squares Means

Effect	(1=HS, 2=BA, 3=GRAD)	(1=HS, 2=BA, 3=GRAD)	slope12	slope26	age80	reas22	Estimate	Standard Error	DF	t Value	Pr > |t|
educgrp	1	2	0.00	0.00	0.00	0.00	-67.4187	136.36	96	-0.49	0.6221
educgrp	1	3	0.00	0.00	0.00	0.00	-41.9718	157.35	96	-0.27	0.7902
educgrp	2	3	0.00	0.00	0.00	0.00	25.4470	125.54	96	0.20	0.8398
educgrp	1	2	1.00	4.00	0.00	0.00	-130.86	112.70	96	-1.16	0.2485
educgrp	1	3	1.00	4.00	0.00	0.00	-153.21	130.05	96	-1.18	0.2417
educgrp	2	3	1.00	4.00	0.00	0.00	-22.3558	103.76	96	-0.22	0.8299

In LSMEANS, you must specify a value at which to hold each quantitative predictor.

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	96	2.32	0.0802
DF=3 Wald Test for all Reasoning Slopes	3	96	3.97	0.0103
DF=6 Wald Test for all Education Slopes	6	96	0.73	0.6264

Total R2 change for education relative to model with age and reasoning

Total R2 (% Reduction) for PredPReas vs. PredPEduc

Name	Pred Corr	TotalR2	Total R2Diff
PredPReas	0.40163	0.16131	.
PredPEduc	0.41669	0.17363	0.012322

The third multivariate Wald F-test provides the significance for the change in total R^2 relative to the age and reasoning model.

PseudoR2 for education relative to model with age and reasoning

PsuedoR2 (% Reduction) for CovPReas vs. CovPEduc

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovPReas	UN(1,1)	ID	242192	37151	6.52	<.0001	.
CovPReas	UN(2,2)	ID	63222	13272	4.76	<.0001	.
CovPReas	UN(3,3)	ID	2411.55	614.00	3.93	<.0001	.
CovPReas	Residual	ID	17673	1435.84	12.31	<.0001	.
CovPEduc	UN(1,1)	ID	246920	38218	6.46	<.0001	-0.019521
CovPEduc	UN(2,2)	ID	63495	13444	4.72	<.0001	-0.004322
CovPEduc	UN(3,3)	ID	2446.05	624.87	3.91	<.0001	-0.014309
CovPEduc	Residual	ID	17673	1435.84	12.31	<.0001	0.000000

Given that education group has no significant effects, we can drop it entirely before moving on to examine potential interactions among the time-invariant predictors of baseline age and reasoning.

1e. Piecewise Model with Age*Reasoning Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reason}_i - 22) + \gamma_{03}(\text{Age}_i - 80)(\text{Reason}_i - 22) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reason}_i - 22) + \gamma_{13}(\text{Age}_i - 80)(\text{Reason}_i - 22) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reason}_i - 22) + \gamma_{23}(\text{Age}_i - 80)(\text{Reason}_i - 22) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$\begin{aligned}\hat{y}_{ti} = {} & \gamma_{00} + \gamma_{10}(\text{Slope12}_{ti}) + \gamma_{20}(\text{Slope26}_{ti}) \\ & + \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{21}(\text{Slope26}_{ti})(\text{Age}_i - 80) \\ & + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(\text{Slope12}_{ti})(\text{Reas}_i - 22) + \gamma_{22}(\text{Slope26}_{ti})(\text{Reas}_i - 22) \\ & + \gamma_{03}(\text{Age}_i - 80)(\text{Reason}_i - 22) + \gamma_{13}(\text{Slope12}_{ti})(\text{Age}_i - 80)(\text{Reason}_i - 22) + \gamma_{23}(\text{Slope26}_{ti})(\text{Age}_i - 80)(\text{Reason}_i - 22)\end{aligned}$$

Simple Slopes of Interactions:

Slope12 = $\gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{Age}_i - 80)(\text{Reas}_i - 22)$

Slope26 = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + \gamma_{23}(\text{Age}_i - 80)(\text{Reas}_i - 22)$

Age = $\gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti}) + \gamma_{03}(\text{Reas}_i - 22) + \gamma_{13}(\text{Slope12}_{ti})(\text{Reas}_i - 22) + \gamma_{23}(\text{Slope26}_{ti})(\text{Reas}_i - 22)$

Reasoning = $\gamma_{02} + \gamma_{12}(\text{Slope12}_{ti}) + \gamma_{22}(\text{Slope26}_{ti}) + \gamma_{03}(\text{Age}_i - 80) + \gamma_{13}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{23}(\text{Slope26}_{ti})(\text{Age}_i - 80)$

Age*Reasoning = $\gamma_{03} + \gamma_{13}(\text{Slope12}_{ti}) + \gamma_{23}(\text{Slope26}_{ti})$

Age*Slope12 = $\gamma_{11} + \gamma_{13}(\text{Reas}_i - 22)$

Age*Slope26 = $\gamma_{21} + \gamma_{23}(\text{Reas}_i - 22)$

Reasoning*Slope12 = $\gamma_{12} + \gamma_{13}(\text{Age}_i - 80)$

Reasoning*Slope26 = $\gamma_{22} + \gamma_{23}(\text{Age}_i - 80)$

```
TITLE1 "SAS 1e: Drop Education, Add Age*Reasoning Predicting Intercept, Slope12, and Slope26";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
CLASS ID session;
MODEL nm3rt = slope12 slope26 age80 slope12*age80 slope26*age80
              reas22 slope12*reas22 slope26*reas22
              age80*reas22 slope12*age80*reas22 slope26*age80*reas22
              / SOLUTION DDFM=Satterthwaite OUTPM=PredPAgeReas; * Save yhat;
RANDOM INTERCEPT slope12 slope26 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovPAgeReas; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age*Reasoning Slopes"
         age80*reas22 1, slope12*age80*reas22 1, slope26*age80*reas22 1;
* Age simple slopes (for about -1SD, M, +1SD of reas22) to decompose interactions;
ESTIMATE "Age on Intercept: Reas 17"  age80 1 age80*reas22 -5;
ESTIMATE "Age on Intercept: Reas 22"  age80 1 age80*reas22  0;
ESTIMATE "Age on Intercept: Reas 27"  age80 1 age80*reas22  5;
ESTIMATE "Age on Slope12:   Reas 17"  slope12*age80 1 slope12*age80*reas22 -5;
ESTIMATE "Age on Slope12:   Reas 22"  slope12*age80 1 slope12*age80*reas22  0;
ESTIMATE "Age on Slope12:   Reas 27"  slope12*age80 1 slope12*age80*reas22  5;
ESTIMATE "Age on Slope26:   Reas 17"  slope26*age80 1 slope26*age80*reas22 -5;
ESTIMATE "Age on Slope26:   Reas 22"  slope26*age80 1 slope26*age80*reas22  0;
ESTIMATE "Age on Slope26:   Reas 27"  slope26*age80 1 slope26*age80*reas22  5;
* Reasoning simple slopes (for about -1SD, M, +1SD of age80) to decompose interactions;
ESTIMATE "Reasoning on Intercept: Age 74"  reas22 1 age80*reas22 -6;
ESTIMATE "Reasoning on Intercept: Age 80"  reas22 1 age80*reas22  0;
ESTIMATE "Reasoning on Intercept: Age 86"  reas22 1 age80*reas22  6;
ESTIMATE "Reasoning on Slope12:   Age 74"  slope12*reas22 1 slope12*age80*reas22 -6;
ESTIMATE "Reasoning on Slope12:   Age 80"  slope12*reas22 1 slope12*age80*reas22  0;
ESTIMATE "Reasoning on Slope12:   Age 86"  slope12*reas22 1 slope12*age80*reas22  6;
ESTIMATE "Reasoning on Slope26:   Age 74"  slope26*reas22 1 slope26*age80*reas22 -6;
ESTIMATE "Reasoning on Slope26:   Age 80"  slope26*reas22 1 slope26*age80*reas22  0;
ESTIMATE "Reasoning on Slope26:   Age 86"  slope26*reas22 1 slope26*age80*reas22  6;
RUN;
```

```
TITLE1 "Total R2 change for age*reas relative to age and reasoning main effects model";
%TotalR2(DV=nm3rt, PredFewer=PredPReas, PredMore=PredPAgeReas);
TITLE1 "PseudoR2 for age*reas relative to age and reasoning main effects model";
%PseudoR2(NCov=7, CovFewer=CovPReas, CovMore=CovPAgeReas);
TITLE1;


display "STATA 1e: Drop Education, Add Age*Reasoning Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80          ///
            c.reas22 c.slope12#c.reas22 c.slope26#c.reas22                          ///
            c.age80#c.reas22 c.slope12#c.age80#c.reas22 c.slope26#c.age80#c.reas22,  ///
            || id: slope12 slope26, variance reml covariance(unstructured)          ///
            dfmethod(satterthwaite) dftable(pvalue)
predict predPAgeReas   // Save yhat
estat ic, n(101)       // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age*Reasoning Slopes
test (c.age80#c.reas22=0)(c.slope12#c.age80#c.reas22=0)(c.slope26#c.age80#c.reas22=0), small
// Age simple slopes (for about -1SD, M, +1SD of reas22) to decompose interactions
lincom c.age80*1 + c.age80#c.reas22*-5, small  // Age on Intercept: Reas 17
lincom c.age80*1 + c.age80#c.reas22*0 , small  // Age on Intercept: Reas 22
lincom c.age80*1 + c.age80#c.reas22*5 , small  // Age on Intercept: Reas 27
lincom c.slope12#c.age80*1 + c.slope12#c.age80#c.reas22*-5, small  // Age on Slope12: Reas 17
lincom c.slope12#c.age80*1 + c.slope12#c.age80#c.reas22*0 , small  // Age on Slope12: Reas 22
lincom c.slope12#c.age80*1 + c.slope12#c.age80#c.reas22*5 , small  // Age on Slope12: Reas 27
lincom c.slope26#c.age80*1 + c.slope26#c.age80#c.reas22*-5, small  // Age on Slope26: Reas 17
lincom c.slope26#c.age80*1 + c.slope26#c.age80#c.reas22*0 , small  // Age on Slope26: Reas 22
lincom c.slope26#c.age80*1 + c.slope26#c.age80#c.reas22*5 , small  // Age on Slope26: Reas 27
// Reasoning simple slopes (for about -1SD, M, +1SD of age80) to decompose interactions
lincom c.reas22*1 + c.age80#c.reas22*-6, small  // Reasoning on Intercept: Age 74
lincom c.reas22*1 + c.age80#c.reas22*0 , small  // Reasoning on Intercept: Age 80
lincom c.reas22*1 + c.age80#c.reas22*6 , small  // Reasoning on Intercept: Age 86
lincom c.slope12#c.reas22*1 + c.slope12#c.age80#c.reas22*-6, small  // Reasoning on Slope12: Age 74
lincom c.slope12#c.reas22*1 + c.slope12#c.age80#c.reas22*0 , small  // Reasoning on Slope12: Age 80
lincom c.slope12#c.reas22*1 + c.slope12#c.age80#c.reas22*6 , small  // Reasoning on Slope12: Age 86
lincom c.slope26#c.reas22*1 + c.slope26#c.age80#c.reas22*-6, small  // Reasoning on Slope26: Age 74
lincom c.slope26#c.reas22*1 + c.slope26#c.age80#c.reas22*0 , small  // Reasoning on Slope26: Age 80
lincom c.slope26#c.reas22*1 + c.slope26#c.age80#c.reas22*6 , small  // Reasoning on Slope26: Age 86


print("R 1e: Drop Education, Add Age*Reasoning Predicting Intercept, Slope12, and Slope26")
PAgeReas = lmer(data=Example7b, REML=TRUE,
            formula=nm3rt~1+slope12+slope26+age80+reas22 +slope12:age80 +slope26:age80
            +slope12:reas22 +slope26:reas22 +age80:reas22 +slope12:age80:reas22
            +slope26:age80:reas22 +(1+slope12+slope26|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(PAgeReas, ddf="Satterthwaite"); llikAIC(PAgeReas, chkREML=FALSE)
print("DF=3 Wald Test for all Age*Reasoning Slopes")
contestMD(PAgeReas, ddf="Satterthwaite", L=rbind(c(0,0,0,0,0,0,0,0,0,0,1,0,0),
                  c(0,0,0,0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,0,0,0,1)))
print("Age simple slopes (for about -1SD, M, +1SD of reas22) to decompose interactions")
print("Age on Intercept: Reas 17"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,1,0,0,0,0,0,-5,0,0))
print("Age on Intercept: Reas 22"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,1,0,0,0,0,0, 0,0,0))
print("Age on Intercept: Reas 27"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,1,0,0,0,0,0, 5,0,0))
print("Age on Slope12: Reas 17");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,1,0,0,0, 0,-5,0))
print("Age on Slope12: Reas 22");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,1,0,0,0, 0, 0,0))
print("Age on Slope12: Reas 27");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,1,0,0,0, 0, 5,0))
print("Age on Slope26: Reas 17");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,0,0, 0, 0,-5))
print("Age on Slope26: Reas 22");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,0,0, 0, 0, 0))
print("Age on Slope26: Reas 27");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,0,0, 0, 0, 5))
print("Reasoning simple slopes (for about -1SD, M, +1SD of age80) to decompose interactions")
print("Reas on Intercept: Age 74"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,1,0,0,0,0,-6,0,0))
print("Reas on Intercept: Age 80"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,1,0,0,0,0, 0,0,0))
print("Reas on Intercept: Age 86"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,1,0,0,0,0, 6,0,0))
print("Reas on Slope12: Age 74");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,1,0, 0,-6,0))
print("Reas on Slope12: Age 80");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,1,0, 0, 0,0))
print("Reas on Slope12: Age 86");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,1,0, 0, 6,0))
print("Reas on Slope26: Age 74");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,1, 0, 0,-6))
print("Reas on Slope26: Age 80");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,1, 0, 0, 0))
print("Reas on Slope26: Age 86");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,1, 0, 0, 6))
print("Save yhat and correlation of yhat with y")
Example7b$PredPAgeReas = predict(PAgeReas, re.form=NA)
rPAgeReas = cor.test(Example7b$PredPAgeReas, Example7b$nm3rt, method="pearson")
print("Total R2"); rPAgeReas$estimate^2
print("Total R2 change relative to age and reasoning main effects model")
rPAgeReas$estimate^2-rPReas$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	244192	37629	6.49	<.0001
UN(2,1)	ID	-49617	17179	-2.89	0.0039
UN(2,2)	ID	62984	13304	4.73	<.0001
UN(3,1)	ID	-7513.67	3457.96	-2.17	0.0298
UN(3,2)	ID	-1999.16	2088.67	-0.96	0.3385
UN(3,3)	ID	2446.40	621.86	3.93	<.0001
Residual	ID	17673	1435.84	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8220.9	7	8234.9	8235.1	8242.3	8253.2	8260.2

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept	1974.57	53.8381	97	36.68	<.0001	g00
slope12	-151.52	31.7828	97	-4.77	<.0001	g10
slope26	-34.1783	6.8294	97	-5.00	<.0001	g20
age80	22.7598	8.9112	97	2.55	0.0122	g01
slope12*age80	-8.4366	5.2607	97	-1.60	0.1120	g11
slope26*age80	-0.4866	1.1304	97	-0.43	0.6678	g20
reas22	-28.0448	11.6437	97	-2.41	0.0179	g02
slope12*reas22	-3.4941	6.8738	97	-0.51	0.6124	g12
slope26*reas22	3.4494	1.4770	97	2.34	0.0216	g22
age80*reas22	-0.9317	1.8579	97	-0.50	0.6172	g03
slope12*age80*reas22	1.2290	1.0968	97	1.12	0.2652	g13
slope26*age80*reas22	0.1026	0.2357	97	0.44	0.6644	g23

Which fixed effects are now conditional on age?

Which fixed effects are now conditional on reasoning?

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Age on Intercept: Reas 17	27.4184	12.5162	97	2.19	0.0309
Age on Intercept: Reas 22	22.7598	8.9112	97	2.55	0.0122
Age on Intercept: Reas 27	18.1011	13.2197	97	1.37	0.1741
Age on Slope12: Reas 17	-14.5818	7.3888	97	-1.97	0.0513
Age on Slope12: Reas 22	-8.4366	5.2607	97	-1.60	0.1120
Age on Slope12: Reas 27	-2.2914	7.8042	97	-0.29	0.7697
Age on Slope26: Reas 17	-0.9994	1.5877	97	-0.63	0.5305
Age on Slope26: Reas 22	-0.4866	1.1304	97	-0.43	0.6678
Age on Slope26: Reas 27	0.02627	1.6769	97	0.02	0.9875
Reasoning on Intercept: Age 74	-22.4544	14.7895	97	-1.52	0.1322
Reasoning on Intercept: Age 80	-28.0448	11.6437	97	-2.41	0.0179
Reasoning on Intercept: Age 86	-33.6352	17.3483	97	-1.94	0.0554
Reasoning on Slope12: Age 74	-10.8683	8.7309	97	-1.24	0.2162
Reasoning on Slope12: Age 80	-3.4941	6.8738	97	-0.51	0.6124
Reasoning on Slope12: Age 86	3.8801	10.2414	97	0.38	0.7056
Reasoning on Slope26: Age 74	2.8340	1.8761	97	1.51	0.1341
Reasoning on Slope26: Age 80	3.4494	1.4770	97	2.34	0.0216
Reasoning on Slope26: Age 86	4.0648	2.2006	97	1.85	0.0678

```
                                Contrasts
                                                    Num     Den
Label                                                DF      DF    F Value    Pr > F
DF=3 Wald Test for all Age*Reasoning Slopes           3      97       0.66    0.5791
```

Total R2 change for age*reasoning relative to age and reasoning main effects model
Total R2 (% Reduction) for PredPReas vs. PredPAgeReas

This multivariate Wald F-test provides the significance for the change in total R^2 relative to the age and reasoning main effects model.

```
                  Pred                      Total
    Name          Corr      TotalR2        R2Diff
PredPReas       0.40163     0.16131     .
PredPAgeReas    0.40306     0.16246     .001148258
```

PseudoR2 for age*reasoning relative to age and reasoning main effects model
PsuedoR2 (% Reduction) for CovPReas vs. CovPAgeReas

```
   Name         CovParm     Subject     Estimate      StdErr     ZValue      ProbZ      PseudoR2
CovPReas        UN(1,1)       ID          242192       37151       6.52     <.0001        .
CovPReas        UN(2,2)       ID           63222       13272       4.76     <.0001        .
CovPReas        UN(3,3)       ID         2411.55      614.00       3.93     <.0001        .
CovPReas        Residual      ID           17673     1435.84      12.31     <.0001        .
CovPAgeReas     UN(1,1)       ID          244192       37629       6.49     <.0001    -0.008258
CovPAgeReas     UN(2,2)       ID           62984       13304       4.73     <.0001     0.003765
CovPAgeReas     UN(3,3)       ID         2446.40      621.86       3.93     <.0001    -0.014453
CovPAgeReas     Residual      ID           17673     1435.84      12.31     <.0001     0.000000
```

Based on the nonsignificance of the higher-order interactions, I'd say we're done with this model. Age and reasoning as main effects in predicting the intercept, slope12, and slope26 seems to be the best piecewise slopes model...

2a. Baseline Unconditional Random Quadratic Time Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti} - 1) + \beta_{2i}(\text{Session}_{ti} - 1)^2 + e_{ti}$

Level 2: **Intercept:** $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + U_{2i}$

Simple Slopes of Interactions (T = Session$_{ti}$ – 1):

Linear Time = $\gamma_{10} + 2\gamma_{20}(T)$

Fixed-Effect-Predicted Outcome: $\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(\text{Session}_{ti} - 1) + \gamma_{20}(\text{Session}_{ti} - 1)^2$

```
TITLE1 "SAS 2a: Random Quadratic Time Unconditional Model";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID session;
MODEL nm3rt = time1 time1*time1
              / SOLUTION DDFM=Satterthwaite OUTPM=PredQUnc; * Save yhat;
RANDOM INTERCEPT time1 time1*time1 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovQUnc; * Save covparms for comparison;
RUN;
TITLE1 "Correlation of predicted and actual RT";
PROC CORR DATA=PredQUnc OUTP=CorrQUnc;
     VAR pred; WITH nm3rt; RUN;


display "STATA 2a: Random Quadratic Time Unconditional Model"
mixed nm3rt c.time1 c.time1#c.time1,                                    ///
            || id: time1 time1sq, variance reml covariance(unstructured) ///
            dfmethod(satterthwaite) dftable(pvalue)
predict predQUnc    // Save yhat
estat ic, n(101)    // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
corr predQUnc nm3rt    // Get total r to make R2
     display r(rho)^2  // Print total R2 relative to empty model
```

```
print("R 2a: Random Quadratic Time Unconditional Model")
QUnc = lmer(data=Example7b, REML=TRUE,
            formula=nm3rt~1+time1+I(time1^2) +(1+time1+I(time1^2)|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(QUnc, ddf="Satterthwaite"); llikAIC(QUnc, chkREML=FALSE)
print("Save predicted values, show correlation of predicted and actual RT")
Example7b$PredQUnc = predict(QUnc, re.form=NA)
rQUnc = cor.test(Example7b$PredQUnc, Example7b$nm3rt, method="pearson")
rQUnc; print("Total R2"); rQUnc$estimate^2
```

SAS Output:

These are the correlations among the random effects. Note the strong correlation among linear (at time 0) and quadratic change. For these models I am not printing the R, G, V, or VCORR matrices (but the parameters in the model for the variance are given below).

Estimated G Correlation Matrix

Row	Effect	ID	Col1	Col2	Col3
1	Intercept	101	1.0000	-0.4230	0.2948
2	time1	101	**-0.4230**	1.0000	-0.9640
3	time1*time1	101	**0.2948**	**-0.9640**	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z	
UN(1,1)	ID	276206	41442	6.66	<.0001	Level-2 random intercept variance of U_{0i}
UN(2,1)	ID	-35734	11941	-2.99	0.0028	Level-2 random intercept-linear covariance
UN(2,2)	ID	25840	5864.41	4.41	<.0001	Level-2 random linear time variance of U_{1i}
UN(3,1)	ID	3901.96	1949.06	2.00	0.0453	Level-2 random intercept-quadratic covariance
UN(3,2)	ID	-3903.32	982.61	-3.97	<.0001	Level-2 random linear-quadratic covariance
UN(3,3)	ID	634.47	172.37	3.68	0.0001	Level-2 random quadratic time variance of U_{2i}
Residual	ID	20298	1649.11	12.31	<.0001	Level-1 residual variance of e_{ti}

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8302.7	7	8316.7	8316.9	8324.2	8335.1	8342.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept	1945.85	53.8497	100	36.13	<.0001	g00: Predicted RT when time=0 (session 1)
time1	-120.90	20.0476	100	-6.03	<.0001	g10: RT change/session at session=1
time1*time1	13.8656	3.4154	100	4.06	<.0001	g20: Half rate change in linear slope/session

Pearson Correlation Coefficients, N = 606
Prob > |r| under H0: Rho=0

	Pred
nm3rt	0.19167
Number Match 3 RT	<.0001

r = .19167 → **TOTAL R^2 = .0367**
~ 4% of RT variance is accounted for by linear and quadratic effects of session

---

2b. Quadratic Model with Age Predicting Intercept, Linear Time, and Quadratic Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = Session$_{ti}$ – 1):

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2 + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}(T)\left(\text{Age}_i - 80\right) + \gamma_{21}(T)^2\left(\text{Age}_i - 80\right)$$

Simple Slopes of Interactions (T = Session$_{ti}$ – 1):

Linear Time = $\gamma_{10} + 2\gamma_{20}(T) + \gamma_{11}(\text{Age}_i - 80) + 2\gamma_{21}(T)(\text{Age}_i - 80)$

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

Age = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

```
TITLE1 "SAS 2b: Add Age Predicting Intercept, Linear Time, and Quadratic Time";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID session;
MODEL nm3rt = time1 time1*time1 age80 time1*age80 time1*time1*age80
            / SOLUTION DDFM=Satterthwaite OUTPM=PredQAge; * Save yhat;
RANDOM INTERCEPT time1 time1*time1 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovQAge; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"  age80 1, age80*time1 1, age80*time1*time1 1;
* Simple linear time slope: sessions 1, 3, 5 for age 74, 80, 86 (about -1SD, M, +1 SD of age80);
* Use 2*time for both quadratic terms;
ESTIMATE "Linear Time: S1, Age 74"  time1 1 time1*time1 0 time1*age80 -6 time1*time1*age80   0;
ESTIMATE "Linear Time: S3, Age 74"  time1 1 time1*time1 4 time1*age80 -6 time1*time1*age80 -24;
ESTIMATE "Linear Time: S5, Age 74"  time1 1 time1*time1 8 time1*age80 -6 time1*time1*age80 -48;
ESTIMATE "Linear Time: S1, Age 80"  time1 1 time1*time1 0 time1*age80  0 time1*time1*age80   0;
ESTIMATE "Linear Time: S3, Age 80"  time1 1 time1*time1 4 time1*age80  0 time1*time1*age80   0;
ESTIMATE "Linear Time: S5, Age 80"  time1 1 time1*time1 8 time1*age80  0 time1*time1*age80   0;
ESTIMATE "Linear Time: S1, Age 86"  time1 1 time1*time1 0 time1*age80  6 time1*time1*age80   0;
ESTIMATE "Linear Time: S3, Age 86"  time1 1 time1*time1 4 time1*age80  6 time1*time1*age80  24;
ESTIMATE "Linear Time: S5, Age 86"  time1 1 time1*time1 8 time1*age80  6 time1*time1*age80  48;
* Simple quadratic time slope for age 74, 80, 86 (about -1SD, M, +1 SD of age80);
ESTIMATE "Quadratic Time: Age 74"   time1*time1 1 time1*time1*age80 -6;
ESTIMATE "Quadratic Time: Age 80"   time1*time1 1 time1*time1*age80  0;
ESTIMATE "Quadratic Time: Age 86"   time1*time1 1 time1*time1*age80  6;
* Simple age slope at each session (S): use time and time^2;
ESTIMATE "Age Slope: S1"  age80 1 time1*age80 0 time1*time1*age80 0;
ESTIMATE "Age Slope: S2"  age80 1 time1*age80 1 time1*time1*age80 1;
ESTIMATE "Age Slope: S3"  age80 1 time1*age80 2 time1*time1*age80 4;
ESTIMATE "Age Slope: S4"  age80 1 time1*age80 3 time1*time1*age80 9;
ESTIMATE "Age Slope: S5"  age80 1 time1*age80 4 time1*time1*age80 16;
ESTIMATE "Age Slope: S6"  age80 1 time1*age80 5 time1*time1*age80 25;
* Simple age*linear time interaction slope at each session (S): use 2*time;
ESTIMATE "Age*Linear Time: S1"  time1*age80 1 time1*time1*age80 0;
ESTIMATE "Age*Linear Time: S2"  time1*age80 1 time1*time1*age80 2;
ESTIMATE "Age*Linear Time: S3"  time1*age80 1 time1*time1*age80 4;
ESTIMATE "Age*Linear Time: S4"  time1*age80 1 time1*time1*age80 6;
ESTIMATE "Age*Linear Time: S5"  time1*age80 1 time1*time1*age80 8;
ESTIMATE "Age*Linear Time: S6"  time1*age80 1 time1*time1*age80 10;
RUN;
TITLE1 "Total R2 change for age relative to unconditional model";
%TotalR2(DV=nm3rt, PredFewer=PredQUnc, PredMore=PredQAge);
TITLE1 "PseudoR2 for age relative to unconditional model";
%PseudoR2(NCov=7, CovFewer=CovQUnc, CovMore=CovQAge);


display "STATA 2b: Add Age Predicting Intercept, Linear Time, and Quadratic Time"
mixed nm3rt c.time1 c.time1#c.time1 c.age80 c.time1#c.age80 c.time1#c.time1#c.age80, ///
           || id: time1 time1sq, variance reml covariance(un)                        ///
           dfmethod(satterthwaite) dftable(pvalue)
predict predQage     // Save yhat
estat ic, n(101)     // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.time1#c.age80=0)(c.time1#c.time1#c.age80=0), small
// Simple linear time slope at session 1, 3, 5 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)
// Use 2*time for both quadratic terms
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.age80*-6 + c.time1#c.time1#c.age80*0  , small // S1, Age 74
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.age80*-6 + c.time1#c.time1#c.age80*-24, small // S3, Age 74
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.age80*-6 + c.time1#c.time1#c.age80*-48, small // S5, Age 74
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.age80*0  + c.time1#c.time1#c.age80*0  , small // S1, Age 80
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.age80*0  + c.time1#c.time1#c.age80*0  , small // S3, Age 80
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.age80*0  + c.time1#c.time1#c.age80*0  , small // S5, Age 80
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.age80*6  + c.time1#c.time1#c.age80*0  , small // S1, Age 86
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.age80*6  + c.time1#c.time1#c.age80*24 , small // S3, Age 86
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.age80*6  + c.time1#c.time1#c.age80*48 , small // S5, Age 86
margins, at(c.age80=(-6(6)6) c.time1=(0(2)4)) dydx(c.time1) df(99) // Same simple linear time slopes
// Simple quadratic time slope for age 74, 80, 86 (about -1SD, M, +1 SD of age80)
lincom c.time1#c.time1*1 + c.time1#c.time1#c.age80*-6, small  // Age 74
lincom c.time1#c.time1*1 + c.time1#c.time1#c.age80*0 , small  // Age 80
lincom c.time1#c.time1*1 + c.time1#c.time1#c.age80*6 , small  // Age 86
```

```
// Simple age slope at each session (S): use time and time^2
lincom c.age80*1 + c.time1#c.age80*0 + c.time1#c.time1#c.age80*0 , small  // S1
lincom c.age80*1 + c.time1#c.age80*1 + c.time1#c.time1#c.age80*1 , small  // S2
lincom c.age80*1 + c.time1#c.age80*2 + c.time1#c.time1#c.age80*4 , small  // S3
lincom c.age80*1 + c.time1#c.age80*3 + c.time1#c.time1#c.age80*9 , small  // S4
lincom c.age80*1 + c.time1#c.age80*4 + c.time1#c.time1#c.age80*16, small  // S5
lincom c.age80*1 + c.time1#c.age80*5 + c.time1#c.time1#c.age80*25, small  // S6
margins, at(c.time1=(0(1)5)) dydx(c.age80) df(99) // Same simple age slope per session
// Simple age*linear time interaction slope at each session (S): use 2*time
lincom c.time1#c.age80*1 + c.time1#c.time1#c.age80*0 , small  // S1
lincom c.time1#c.age80*1 + c.time1#c.time1#c.age80*2 , small  // S2
lincom c.time1#c.age80*1 + c.time1#c.time1#c.age80*4 , small  // S3
lincom c.time1#c.age80*1 + c.time1#c.time1#c.age80*6 , small  // S4
lincom c.time1#c.age80*1 + c.time1#c.time1#c.age80*8 , small  // S5
lincom c.time1#c.age80*1 + c.time1#c.time1#c.age80*10, small  // S6
// Get adjusted means per session and age (start(by)end)
margins, at(c.time1=(0(1)5) c.age80=(-6 0 6))
     marginsplot        // Plot adjusted means
corr predQage nm3rt     // Get total r to make R2
     display r(rho)^2   // Print total R2 relative to empty model

print("R 2b: Add Age Predicting Intercept, Linear Time, and Quadratic Time")
QAge = lmer(data=Example7b, REML=TRUE,
            formula=nm3rt~1+time1+I(time1^2)+age80 +time1:age80 +I(time1^2):age80
            +(1+time1+I(time1^2)|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(QAge, ddf="Satterthwaite"); llikAIC(QAge, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(QAge, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0),c(0,0,0,0,1,0),c(0,0,0,0,0,1)))
print("Simple linear time slope: sessions 1, 3, 5 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)")
print("Use 2*time for both quadratic terms")
print("Linear Time: S1, Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,0,0,-6,  0))
print("Linear Time: S3, Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,4,0,-6,-24))
print("Linear Time: S5, Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,8,0,-6,-48))
print("Linear Time: S1, Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,0,0, 0,  0))
print("Linear Time: S3, Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,4,0, 0,  0))
print("Linear Time: S5, Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,8,0, 0,  0))
print("Linear Time: S1, Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,0,0, 6,  0))
print("Linear Time: S3, Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,4,0, 6, 24))
print("Linear Time: S5, Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,8,0, 6, 48))
print("Simple quadratic time slope for age 74, 80, 86 (about -1SD, M, +1 SD of age80)")
print("Quadratic Time: Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,1,0,0,-6))
print("Quadratic Time: Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,1,0,0, 0))
print("Quadratic Time: Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,1,0,0, 6))
print("Simple age slope at each session (S): use time and time^2")
print("Age Slope: S1"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,0, 0))
print("Age Slope: S2"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,1, 1))
print("Age Slope: S3"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,2, 4))
print("Age Slope: S4"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,3, 9))
print("Age Slope: S5"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,4,16))
print("Age Slope: S6"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,5,25))
print("Simple age*linear time interaction slope at each session (S): use 2*time")
print("Age*Linear Time: S1"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 0))
print("Age*Linear Time: S2"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 2))
print("Age*Linear Time: S3"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 4))
print("Age*Linear Time: S4"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 6))
print("Age*Linear Time: S5"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 8))
print("Age*Linear Time: S6"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1,10))
print("Save yhat and correlation of yhat with y")
Example7b$PredQAge = predict(QAge, re.form=NA)
rQAge = cor.test(Example7b$PredQAge, Example7b$nm3rt, method="pearson")
print("Total R2"); rQAge$estimate^2
print("Total R2 change for age relative to unconditional model")
rQAge$estimate^2-rQUnc$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	247691	37599	6.59	<.0001
UN(2,1)	ID	-30154	11191	-2.69	0.0070
UN(2,2)	ID	25083	5787.37	4.33	<.0001
UN(3,1)	ID	3232.78	1847.12	1.75	0.0801
UN(3,2)	ID	-3830.21	976.76	-3.92	<.0001
UN(3,3)	ID	629.58	172.51	3.65	0.0001
Residual	ID	20298	1649.11	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8283.2	7	8297.2	8297.3	8304.6	8315.5	8322.5

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept	1950.69	51.1806	99	38.11	<.0001	g00
time1	-121.83	19.8672	99	-6.13	<.0001	g10
time1*time1	13.9774	3.4096	99	4.10	<.0001	g20
age80	29.0495	8.4616	99	3.43	0.0009	g01
time1*age80	-5.5946	3.2846	99	-1.70	0.0916	g11
time1*time1*age80	0.6709	0.5637	99	1.19	0.2368	g21

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Linear Time: S1, Age 74	-88.2647	27.5955	99	-3.20	0.0019
Linear Time: S3, Age 74	-48.4568	10.9025	99	-4.44	<.0001
Linear Time: S5, Age 74	-8.6489	13.9267	99	-0.62	0.5360
Linear Time: S1, Age 80	-121.83	19.8672	99	-6.13	<.0001
Linear Time: S3, Age 80	-65.9227	7.8492	99	-8.40	<.0001
Linear Time: S5, Age 80	-10.0129	10.0264	99	-1.00	0.3204
Linear Time: S1, Age 86	-155.40	28.3668	99	-5.48	<.0001
Linear Time: S3, Age 86	-83.3886	11.2072	99	-7.44	<.0001
Linear Time: S5, Age 86	-11.3769	14.3159	99	-0.79	0.4287
Quadratic Time: Age 74	9.9520	4.7360	99	2.10	0.0381
Quadratic Time: Age 80	13.9774	3.4096	99	4.10	<.0001
Quadratic Time: Age 86	18.0029	4.8683	99	3.70	0.0004
Age Slope: S1	29.0495	8.4616	99	3.43	0.0009
Age Slope: S2	24.1258	7.6862	99	3.14	0.0022
Age Slope: S3	20.5439	7.5343	99	2.73	0.0076
Age Slope: S4	18.3038	7.4038	99	2.47	0.0151
Age Slope: S5	17.4056	7.1425	99	2.44	0.0166
Age Slope: S6	17.8492	7.1254	99	2.51	0.0139
Age*Linear Time: S1	-5.5946	3.2846	99	-1.70	0.0916
Age*Linear Time: S2	-4.2528	2.2283	99	-1.91	0.0592
Age*Linear Time: S3	-2.9110	1.2977	99	-2.24	0.0271
Age*Linear Time: S4	-1.5692	0.9720	99	-1.61	0.1096
Age*Linear Time: S5	-0.2273	1.6576	99	-0.14	0.8912
Age*Linear Time: S6	1.1145	2.6632	99	0.42	0.6765

Interpret the fixed intercept:

Interpret the fixed effect of linear time:

Interpret the fixed effect of quadratic time:

Interpret the effect of age80:

Interpret the effect of linear*age80:

Interpret the effect of quadratic*age80:

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	99	4.00	0.0098

Total R2 change for age relative to unconditional model
Total R2 (% Reduction) for PredQUnc vs. PredQAge

Name	Pred Corr	TotalR2	Total R2Diff
PredQUnc	0.19167	0.03674	.
PredQAge	0.32688	0.10685	0.070114

This multivariate Wald F-test provides the significance for the change in total R^2 relative to the unconditional model.

PseudoR2 for age relative to unconditional model
PsuedoR2 (% Reduction) for CovQUnc vs. CovQAge

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	Pseudo R2
CovQUnc	UN(1,1)	ID	276206	41442	6.66	<.0001	.
CovQUnc	UN(2,2)	ID	25840	5864.41	4.41	<.0001	.
CovQUnc	UN(3,3)	ID	634.47	172.37	3.68	0.0001	.
CovQUnc	Residual	ID	20298	1649.11	12.31	<.0001	.
CovQAge	UN(1,1)	ID	247691	37599	6.59	<.0001	0.10324
CovQAge	UN(2,2)	ID	25083	5787.37	4.33	<.0001	0.02931
CovQAge	UN(3,3)	ID	629.58	172.51	3.65	0.0001	0.00770
CovQAge	Residual	ID	20298	1649.11	12.31	<.0001	0.00000

Which variance component should have been reduced by each new fixed effect of age?

2c. Quadratic Model with Age and Reasoning Predicting Intercept, Linear Time, and Quadratic Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reas}_i - 22\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reas}_i - 22\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{22}\left(\text{Reas}_i - 22\right) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = Session$_{ti}$ −1):

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2 + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}(T)\left(\text{Age}_i - 80\right) + \gamma_{21}(T)^2\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reas}_i - 22\right) + \gamma_{12}(T)\left(\text{Reas}_i - 22\right) + \gamma_{22}(T)^2\left(\text{Reas}_i - 22\right)$$

Simple Slopes of Interactions (T = Session$_{ti}$ −1):

Linear Time = $\gamma_{10} + 2\gamma_{20}(T) + \gamma_{11}(\text{Age}_i - 80) + 2\gamma_{21}(T)(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + 2\gamma_{22}(T)(\text{Reas}_i - 22)$

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{21}(\text{Reas}_i - 22)$

Age = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

Reasoning = $\gamma_{02} + \gamma_{12}(T) + \gamma_{22}(T)^2$

```
TITLE1 "SAS 2c: Keep Age, Add Reasoning Predicting Intercept, Linear Time, and Quadratic Time";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID session;
MODEL nm3rt = time1 time1*time1 age80 time1*age80 time1*time1*age80
              reas22 time1*reas22 time1*time1*reas22
              / SOLUTION DDFM=Satterthwaite OUTPM=PredQReas; * Save yhat;
RANDOM INTERCEPT time1 time1*time1 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovQReas; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"         age80 1,  age80*time1 1,  age80*time1*time1 1;
CONTRAST "DF=3 Wald Test for all Reasoning Slopes"  reas22 1, reas22*time1 1, reas22*time1*time1 1;

* Simple linear time slope: session 1, 3, 5 for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22);
* Use 2*time for both quadratic terms, hold age80=0;
ESTIMATE "Linear Time: S1, Reas 17"  time1 1 time1*time1 0 time1*reas22 -5 time1*time1*reas22   0;
ESTIMATE "Linear Time: S3, Reas 17"  time1 1 time1*time1 4 time1*reas22 -5 time1*time1*reas22 -20;
ESTIMATE "Linear Time: S5, Reas 17"  time1 1 time1*time1 8 time1*reas22 -5 time1*time1*reas22 -40;
ESTIMATE "Linear Time: S1, Reas 22"  time1 1 time1*time1 0 time1*reas22  0 time1*time1*reas22   0;
ESTIMATE "Linear Time: S3, Reas 22"  time1 1 time1*time1 4 time1*reas22  0 time1*time1*reas22   0;
ESTIMATE "Linear Time: S5, Reas 22"  time1 1 time1*time1 8 time1*reas22  0 time1*time1*reas22   0;
ESTIMATE "Linear Time: S1, Reas 27"  time1 1 time1*time1 0 time1*reas22  5 time1*time1*reas22   0;
ESTIMATE "Linear Time: S3, Reas 27"  time1 1 time1*time1 4 time1*reas22  5 time1*time1*reas22  20;
ESTIMATE "Linear Time: S5, Reas 27"  time1 1 time1*time1 8 time1*reas22  5 time1*time1*reas22  40;
* Simple quadratic time slope for reasoning 17, 22, 27 (about -1SD, M, +1 SD of reas22), hold age80=0;
ESTIMATE "Quadratic Time: Reas 17"  time1*time1 1 time1*time1*reas22 -5;
ESTIMATE "Quadratic Time: Reas 22"  time1*time1 1 time1*time1*reas22  0;
ESTIMATE "Quadratic Time: Reas 27"  time1*time1 1 time1*time1*reas22  5;
* Simple reasoning slope at each session (S): use time and time^2;
ESTIMATE "Reasoning Slope: S1"  reas22 1 time1*reas22 0 time1*time1*reas22 0;
ESTIMATE "Reasoning Slope: S2"  reas22 1 time1*reas22 1 time1*time1*reas22 1;
ESTIMATE "Reasoning Slope: S3"  reas22 1 time1*reas22 2 time1*time1*reas22 4;
ESTIMATE "Reasoning Slope: S4"  reas22 1 time1*reas22 3 time1*time1*reas22 9;
ESTIMATE "Reasoning Slope: S5"  reas22 1 time1*reas22 4 time1*time1*reas22 16;
ESTIMATE "Reasoning Slope: S6"  reas22 1 time1*reas22 5 time1*time1*reas22 25;
* Simple reasoning*linear time interaction slope at each session (S): use 2*time;
ESTIMATE "Reasoning*Linear Time: S1"  time1*reas22 1 time1*time1*reas22 0;
ESTIMATE "Reasoning*Linear Time: S2"  time1*reas22 1 time1*time1*reas22 2;
ESTIMATE "Reasoning*Linear Time: S3"  time1*reas22 1 time1*time1*reas22 4;
ESTIMATE "Reasoning*Linear Time: S4"  time1*reas22 1 time1*time1*reas22 6;
ESTIMATE "Reasoning*Linear Time: S5"  time1*reas22 1 time1*time1*reas22 8;
ESTIMATE "Reasoning*Linear Time: S6"  time1*reas22 1 time1*time1*reas22 10;
RUN;
TITLE1 "Total R2 change for full reasoning relative to age-only model";
%TotalR2(DV=nm3rt, PredFewer=PredQAge, PredMore=PredQReas);
TITLE1 "PseudoR2 for full reasoning relative to age-only model";
%PseudoR2(NCov=7, CovFewer=CovQAge, CovMore=CovQReas);


display "STATA 2c: Keep Age, Add Reasoning Predicting Intercept, Linear Time, and Quadratic Time"
mixed nm3rt c.time1 c.time1#c.time1 c.age80 c.time1#c.age80 c.time1#c.time1#c.age80 ///
            c.reas22 c.time1#c.reas22 c.time1#c.time1#c.reas22,                      ///
            || id: time1 time1sq, variance reml covariance(un)                       ///
            dfmethod(satterthwaite) dftable(pvalue)
predict predQReas    // Save yhat
estat ic, n(101)     // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.time1#c.age80=0)(c.time1#c.time1#c.age80=0), small
// DF=3 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.time1#c.reas22=0)(c.time1#c.time1#c.reas22=0), small
// Simple linear time slope at session 1, 3, 5 for reasoning 17, 22, 27 (about -1SD, M, +1 SD of age80)
// Use 2*time for both quadratic terms, hold age80=0
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.reas22*-5 + c.time1#c.time1#c.reas22*0  , small // S1, Reas 17
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.reas22*-5 + c.time1#c.time1#c.reas22*-20, small // S3, Reas 17
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.reas22*-5 + c.time1#c.time1#c.reas22*-40, small // S5, Reas 17
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.reas22*0  + c.time1#c.time1#c.reas22*0  , small // S1, Reas 22
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.reas22*0  + c.time1#c.time1#c.reas22*0  , small // S3, Reas 22
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.reas22*0  + c.time1#c.time1#c.reas22*0  , small // S5, Reas 22
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.reas22*5  + c.time1#c.time1#c.reas22*0  , small // S1, Reas 27
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.reas22*5  + c.time1#c.time1#c.reas22*20 , small // S3, Reas 27
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.reas22*5  + c.time1#c.time1#c.reas22*40 , small // S5, Reas 27
margins, at(c.age80=0 c.reas22=(-5(5)5) c.time1=(0(2)4)) dydx(c.time1) df(98) // Same linear slopes
```

```
// Simple quadratic time slope for reasoning 17, 22, 27 (about -1SD, M, +1 SD of reas22), hold age80=0
lincom c.time1#c.time1*1 + c.time1#c.time1#c.reas22*-5, small  // Reas 17
lincom c.time1#c.time1*1 + c.time1#c.time1#c.reas22*0 , small  // Reas 22
lincom c.time1#c.time1*1 + c.time1#c.time1#c.reas22*5 , small  // Reas 27
// Simple reasoning slope at each session (S): use time and time^2
lincom c.reas22*1 + c.time1#c.reas22*0 + c.time1#c.time1#c.reas22*0 , small  // S1
lincom c.reas22*1 + c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*1 , small  // S2
lincom c.reas22*1 + c.time1#c.reas22*2 + c.time1#c.time1#c.reas22*4 , small  // S3
lincom c.reas22*1 + c.time1#c.reas22*3 + c.time1#c.time1#c.reas22*9 , small  // S4
lincom c.reas22*1 + c.time1#c.reas22*4 + c.time1#c.time1#c.reas22*16, small  // S5
lincom c.reas22*1 + c.time1#c.reas22*5 + c.time1#c.time1#c.reas22*25, small  // S6
margins, at(c.age80=0 c.time1=(0(1)5)) dydx(c.reas22) df(98) // Same simple age slope per session
// Simple reasoning*linear time interaction slope at each session (S): use 2*time
lincom c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*0 , small  // S1
lincom c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*2 , small  // S2
lincom c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*4 , small  // S3
lincom c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*6 , small  // S4
lincom c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*8 , small  // S5
lincom c.time1#c.reas22*1 + c.time1#c.time1#c.reas22*10, small  // S6
// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.age80=0 c.time1=(0(1)5) c.reas22=(-6 0 6))
     marginsplot        // Plot adjusted means
corr predQReas nm3rt    // Get total r to make R2
     display r(rho)^2   // Print total R2 relative to empty model
```

```
print("R 2c: Keep Age, Add Reasoning Predicting Intercept, time1, and I(time1^2)")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
QReas = lmer(data=Example7b, REML=TRUE,
             formula=nm3rt~1+time1+I(time1^2)+age80+reas22 +time1:age80 +I(time1^2):age80
             +time1:reas22 +I(time1^2):reas22 +(1+time1+I(time1^2)|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(QReas, ddf="Satterthwaite"); llikAIC(QReas, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(QReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,1,0,0,0,0,0),c(0,0,0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,1,0,0)))
print("DF=3 Wald Test for all Reasoning Slopes")
contestMD(QReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,0,1,0,0,0,0),c(0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,1)))
print("Simple linear time slope: sessions 1, 3, 5 for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22)")
print("Use 2*time for both quadratic terms, hold age80=0")
print("Linear Time: S1, Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,-5,  0))
print("Linear Time: S3, Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0,-5,-20))
print("Linear Time: S5, Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0,-5,-40))
print("Linear Time: S1, Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 0,  0))
print("Linear Time: S3, Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 0,  0))
print("Linear Time: S5, Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 0,  0))
print("Linear Time: S1, Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 5,  0))
print("Linear Time: S3, Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 5, 20))
print("Linear Time: S5, Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 5, 40))
print("Simple quadratic time slope for for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22), hold age80=0")
print("Quadratic Time: Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0,-5))
print("Quadratic Time: Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0, 0))
print("Quadratic Time: Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0, 5))
print("Simple reasoning slope at each session (S): use time and time^2")
print("Reasoning Slope: S1"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0, 0))
print("Reasoning Slope: S2"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1, 1))
print("Reasoning Slope: S3"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,2, 4))
print("Reasoning Slope: S4"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,3, 9))
print("Reasoning Slope: S5"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,4,16))
print("Reasoning Slope: S6"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,5,25))
print("Simple reasoning*linear time interaction slope at each session (S): use 2*time")
print("Reasoning*Linear Time: S1"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 0))
print("Reasoning*Linear Time: S2"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 2))
print("Reasoning*Linear Time: S3"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 4))
print("Reasoning*Linear Time: S4"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 6))
print("Reasoning*Linear Time: S5"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 8))
print("Reasoning*Linear Time: S6"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,10))
print("Save yhat and correlation of yhat with y")
Example7b$PredQReas = predict(QReas, re.form=NA)
rQReas = cor.test(Example7b$PredQReas, Example7b$nm3rt, method="pearson")
print("Total R2"); rQReas$estimate^2
print("Total R2 change for reasoning relative to age-only model")
rQReas$estimate^2-rQAge$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	235541	36056	6.53	<.0001
UN(2,1)	ID	-32552	11138	-2.92	0.0035
UN(2,2)	ID	25228	5835.93	4.32	<.0001
UN(3,1)	ID	3918.44	1826.88	2.14	0.0320
UN(3,2)	ID	-3812.99	978.05	-3.90	<.0001
UN(3,3)	ID	614.47	171.25	3.59	0.0002
Residual	ID	20298	1649.11	12.31	<.0001

Information Criteria

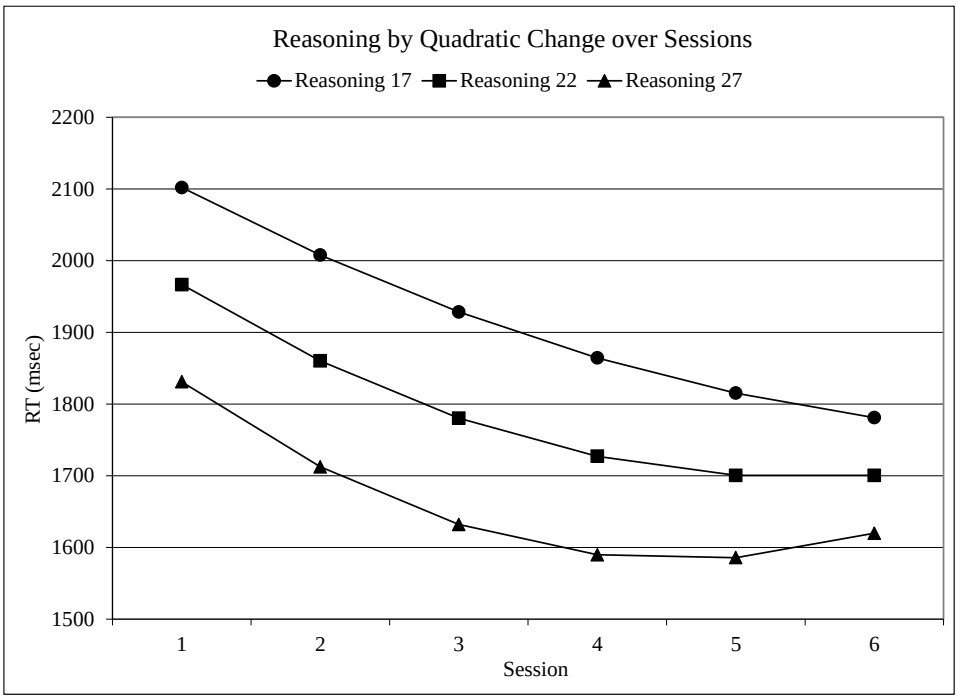
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8261.0	7	8275.0	8275.2	8282.4	8293.3	8300.3

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept	1966.47	50.4203	98	39.00	<.0001	g00
time1	-119.74	20.0746	98	-5.96	<.0001	g10
time1*time1	13.3036	3.4167	98	3.89	0.0002	g20
age80	22.2782	8.7324	98	2.55	0.0123	g01
time1*age80	-6.4921	3.4768	98	-1.87	0.0649	g11
time1*time1*age80	0.9601	0.5917	98	1.62	0.1079	g21
reas22	-27.1004	11.2829	98	-2.40	0.0182	g02
time1*reas22	-3.5917	4.4922	98	-0.80	0.4259	g12
time1*time1*reas22	1.1575	0.7646	98	1.51	0.1333	g22

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Linear Time: S1, Reas 17	-101.78	32.0151	98	-3.18	0.0020
Linear Time: S3, Reas 17	-71.7192	12.6677	98	-5.66	<.0001
Linear Time: S5, Reas 17	-41.6554	15.6876	98	-2.66	0.0092
Linear Time: S1, Reas 22	-119.74	20.0746	98	-5.96	<.0001
Linear Time: S3, Reas 22	-66.5272	7.9431	98	-8.38	<.0001
Linear Time: S5, Reas 22	-13.3127	9.8367	98	-1.35	0.1791
Linear Time: S1, Reas 27	-137.70	28.1073	98	-4.90	<.0001
Linear Time: S3, Reas 27	-61.3351	11.1215	98	-5.52	<.0001
Linear Time: S5, Reas 27	15.0301	13.7728	98	1.09	0.2778
Quadratic Time: Reas 17	7.5159	5.4490	98	1.38	0.1709
Quadratic Time: Reas 22	13.3036	3.4167	98	3.89	0.0002
Quadratic Time: Reas 27	19.0913	4.7839	98	3.99	0.0001
Reasoning Slope: S1	-27.1004	11.2829	98	-2.40	0.0182
Reasoning Slope: S2	-29.5346	10.1156	98	-2.92	0.0043
Reasoning Slope: S3	-29.6537	9.8944	98	-3.00	0.0035
Reasoning Slope: S4	-27.4578	9.7730	98	-2.81	0.0060
Reasoning Slope: S5	-22.9468	9.5224	98	-2.41	0.0178
Reasoning Slope: S6	-16.1207	9.6403	98	-1.67	0.0977
Reasoning*Linear Time: S1	-3.5917	4.4922	98	-0.80	0.4259
Reasoning*Linear Time: S2	-1.2767	3.0547	98	-0.42	0.6769
Reasoning*Linear Time: S3	1.0384	1.7775	98	0.58	0.5604
Reasoning*Linear Time: S4	3.3535	1.2900	98	2.60	0.0108
Reasoning*Linear Time: S5	5.6686	2.2012	98	2.58	0.0115
Reasoning*Linear Time: S6	7.9836	3.5642	98	2.24	0.0274



Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	98	2.40	0.0727
DF=3 Wald Test for all Reasoning Slopes	3	98	4.29	0.0068

Total R2 change for full reasoning relative to age-only model
Total R2 (% Reduction) for PredQAge vs. PredQReas

The second multivariate Wald F-test provides the significance for the change in total R^2 relative to the age-only model.

Name	Pred Corr	TotalR2	Total R2Diff
PredQAge	0.32688	0.10685	.
PredQReas	0.40108	0.16086	0.054011

PseudoR2 for full reasoning relative to age-only model
PsuedoR2 (% Reduction) for CovQAge vs. CovQReas

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovQAge	UN(1,1)	ID	247691	37599	6.59	<.0001	.
CovQAge	UN(2,2)	ID	25083	5787.37	4.33	<.0001	.
CovQAge	UN(3,3)	ID	629.58	172.51	3.65	0.0001	.
CovQAge	Residual	ID	20298	1649.11	12.31	<.0001	.
CovQReas	UN(1,1)	ID	235541	36056	6.53	<.0001	0.049052
CovQReas	UN(2,2)	ID	25228	5835.93	4.32	<.0001	-0.005808
CovQReas	UN(3,3)	ID	614.47	171.25	3.59	0.0002	0.024008
CovQReas	Residual	ID	20298	1649.11	12.31	<.0001	-0.000000

From these results *it appears* we could remove both the interaction of reasoning with both the linear and quadratic time slopes, but keep in mind how correlated those terms are… let's see what happens if we just remove just the reasoning*quadratic time interaction for now.

2d. Quadratic Model Removing Reasoning Predicting Quadratic Time Slope

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti} - 1) + \beta_{2i}(\text{Session}_{ti} - 1)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reason}_i - 22) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reason}_i - 22) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = Session$_{ti}$ −1):

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2 + \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(T)(\text{Age}_i - 80) + \gamma_{21}(T)^2(\text{Age}_i - 80) + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(T)(\text{Reas}_i - 22)$$

Simple Slopes of Interactions (T = Session$_{ti}$ −1):

Linear Time = $\gamma_{10} + 2\gamma_{20}(T) + \gamma_{11}(\text{Age}_i - 80) + 2\gamma_{21}(T)(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22)$

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

Age = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

Reasoning = $\gamma_{02} + \gamma_{12}(T)$

```
TITLE1 "SAS 2d: Remove Reasoning Predicting Quadratic Time";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID session;
MODEL nm3rt = time1 time1*time1 age80 time1*age80 time1*time1*age80
              reas22 time1*reas22  / SOLUTION DDFM=Satterthwaite OUTPM=PredLReas; * Save yhat;
RANDOM INTERCEPT time1 time1*time1 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovLReas; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"    age80 1,  age80*time1 1, age80*time1*time1 1;
CONTRAST "DF=2 Wald Test for all Reasoning Slopes"  reas22 1, reas22*time1 1;
* Simple linear time slope: session 1, 3, 5 for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22);
* Use 2*time for quadratic term, hold age80=0;
ESTIMATE "Linear Time: S1, Reas 17"  time1 1 time1*time1 0 time1*reas22 -5;
ESTIMATE "Linear Time: S3, Reas 17"  time1 1 time1*time1 4 time1*reas22 -5;
ESTIMATE "Linear Time: S5, Reas 17"  time1 1 time1*time1 8 time1*reas22 -5;
ESTIMATE "Linear Time: S1, Reas 22"  time1 1 time1*time1 0 time1*reas22  0;
ESTIMATE "Linear Time: S3, Reas 22"  time1 1 time1*time1 4 time1*reas22  0;
ESTIMATE "Linear Time: S5, Reas 22"  time1 1 time1*time1 8 time1*reas22  0;
ESTIMATE "Linear Time: S1, Reas 27"  time1 1 time1*time1 0 time1*reas22  5;
ESTIMATE "Linear Time: S3, Reas 27"  time1 1 time1*time1 4 time1*reas22  5;
ESTIMATE "Linear Time: S5, Reas 27"  time1 1 time1*time1 8 time1*reas22  5;
* Simple reasoning slope at each session (S): use time only;
ESTIMATE "Reasoning Slope: S1"  reas22 1 time1*reas22 0;
ESTIMATE "Reasoning Slope: S2"  reas22 1 time1*reas22 1;
ESTIMATE "Reasoning Slope: S3"  reas22 1 time1*reas22 2;
ESTIMATE "Reasoning Slope: S4"  reas22 1 time1*reas22 3;
ESTIMATE "Reasoning Slope: S5"  reas22 1 time1*reas22 4;
ESTIMATE "Reasoning Slope: S6"  reas22 1 time1*reas22 5;
RUN;
TITLE1 "Total R2 change for reduced reasoning relative to age-only model";
%TotalR2(DV=nm3rt, PredFewer=PredQAge, PredMore=PredLReas);
TITLE1 "PseudoR2 for reduced reasoning relative to age-only model";
%PseudoR2(NCov=7, CovFewer=CovQAge, CovMore=CovLReas);


display "STATA 2d: Remove Reasoning Predicting Quadratic Time"
mixed nm3rt c.time1 c.time1#c.time1 c.age80 c.time1#c.age80 c.time1#c.time1#c.age80 ///
            c.reas22 c.time1#c.reas22,                                              ///
            || id: time1 time1sq, variance reml covariance(un)                      ///
            dfmethod(satterthwaite) dftable(pvalue)
predict predLReas    // Save yhat
estat ic, n(101)     // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
```

```
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.time1#c.age80=0)(c.time1#c.time1#c.age80=0), small
// DF=2 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.time1#c.reas22=0), small
// Simple linear time slope at session 1, 3, 5 for reasoning 17, 22, 27 (about -1SD, M, +1 SD of age80)
// Use 2*time for quadratic term, hold age80=0
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.reas22*-5, small // S1, Reas 17
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.reas22*-5, small // S3, Reas 17
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.reas22*-5, small // S5, Reas 17
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.reas22*0 , small // S1, Reas 22
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.reas22*0 , small // S3, Reas 22
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.reas22*0 , small // S5, Reas 22
lincom c.time1*1 + c.time1#c.time1*0 + c.time1#c.reas22*5 , small // S1, Reas 27
lincom c.time1*1 + c.time1#c.time1*4 + c.time1#c.reas22*5 , small // S3, Reas 27
lincom c.time1*1 + c.time1#c.time1*8 + c.time1#c.reas22*5 , small // S5, Reas 27
margins, at(c.age80=0 c.reas22=(-5(5)5) c.time1=(0(2)4)) dydx(c.time1) df(98) // Same linear slopes
// Simple reasoning slope at each session (S): use time only
lincom c.reas22*1 + c.time1#c.reas22*0, small  // S1
lincom c.reas22*1 + c.time1#c.reas22*1, small  // S2
lincom c.reas22*1 + c.time1#c.reas22*2, small  // S3
lincom c.reas22*1 + c.time1#c.reas22*3, small  // S4
lincom c.reas22*1 + c.time1#c.reas22*4, small  // S5
lincom c.reas22*1 + c.time1#c.reas22*5, small  // S6
margins, at(c.age80=0 c.time1=(0(1)5)) dydx(c.reas22) df(98)  // Same simple reas slopes per session
// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.time1=(0(1)5) c.reas22=(-5 0 5)) vsquish
     marginsplot        // Plot adjusted means
corr predLReas nm3rt    // Get total r to make R2
     display r(rho)^2   // Print total R2 relative to empty model
```

```
print("R 2d: Remove Reasoning Predicting Quadratic Time")
LReas = lmer(data=Example7b, REML=TRUE,
             formula=nm3rt~1+time1+I(time1^2)+age80+reas22 +time1:age80 +I(time1^2):age80
             +time1:reas22 +(1+time1+I(time1^2)|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(LReas, ddf="Satterthwaite"); llikAIC(LReas, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(LReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,1,0,0,0,0),c(0,0,0,0,0,1,0,0),c(0,0,0,0,0,0,1,0)))
print("DF=2 Wald Test for all Reasoning Slopes")
contestMD(LReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,1)))
print("Simple linear time slope: sessions 1, 3, 5 for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22)")
print("Use 2*time for quadratic term, hold age80=0")
print("Linear Time: S1, Reas 17"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,-5))
print("Linear Time: S3, Reas 17"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0,-5))
print("Linear Time: S5, Reas 17"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0,-5))
print("Linear Time: S1, Reas 22"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 0))
print("Linear Time: S3, Reas 22"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 0))
print("Linear Time: S5, Reas 22"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 0))
print("Linear Time: S1, Reas 27"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 5))
print("Linear Time: S3, Reas 27"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 5))
print("Linear Time: S5, Reas 27"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 5))
print("Simple reasoning slope at each session (S): use time and time^2")
print("Reasoning Slope: S1"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0))
print("Reasoning Slope: S2"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1))
print("Reasoning Slope: S3"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,2))
print("Reasoning Slope: S4"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,3))
print("Reasoning Slope: S5"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,4))
print("Reasoning Slope: S6"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,5))
print("Save yhat and correlation of yhat with y")
Example7b$PredLReas = predict(LReas, re.form=NA)
rLReas = cor.test(Example7b$PredLReas, Example7b$nm3rt, method="pearson")
print("Total R2"); rLReas$estimate^2
print("Total R2 change for reasoning relative to age-only model")
rLReas$estimate^2-rQAge$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	235909	36153	6.53	<.0001
UN(2,1)	ID	-32972	11262	-2.93	0.0034

UN(2,2)	ID	25707	5883.65	4.37	<.0001
UN(3,1)	ID	3993.04	1848.58	2.16	0.0308
UN(3,2)	ID	-3897.93	985.52	-3.96	<.0001
UN(3,3)	ID	629.52	172.50	3.65	0.0001
Residual	ID	20298	1649.11	12.31	<.0001

Information Criteria

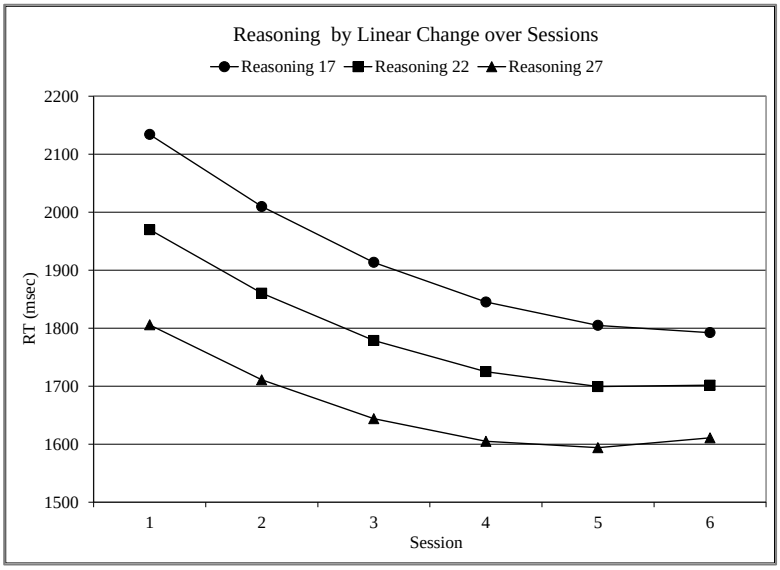
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8264.6	7	8278.6	8278.8	8286.0	8296.9	8303.9

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept	1969.80	50.4084	98.1	39.08	<.0001	g00
time1	-123.54	20.0358	98.9	-6.17	<.0001	g10
time1*time1	13.9774	3.4095	99	4.10	<.0001	g20
age80	20.8470	8.6868	99.7	2.40	0.0183	g01
time1*age80	-4.8610	3.3252	100	-1.46	0.1469	g11
time1*time1*age80	0.6709	0.5637	99	1.19	0.2368	g21
reas22	-32.8281	10.6297	98	-3.09	0.0026	g02
time1*reas22	2.9362	1.2602	98	2.33	0.0219	g12 → **Different result!**

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Linear Time: S1, Reas 17	-138.22	21.2222	120	-6.51	<.0001
Linear Time: S3, Reas 17	-82.3130	10.5786	130	-7.78	<.0001
Linear Time: S5, Reas 17	-26.4032	12.0595	141	-2.19	0.0302
Linear Time: S1, Reas 22	-123.54	20.0359	98.9	-6.17	<.0001
Linear Time: S3, Reas 22	-67.6319	7.9348	98.5	-8.52	<.0001
Linear Time: S5, Reas 22	-11.7221	9.8227	98.6	-1.19	0.2356
Linear Time: S1, Reas 27	-108.86	20.7821	112	-5.24	<.0001
Linear Time: S3, Reas 27	-52.9508	9.6653	126	-5.48	<.0001
Linear Time: S5, Reas 27	2.9589	11.2669	130	0.26	0.7933
Reasoning Slope: S1	-32.8281	10.6298	98	-3.09	0.0026
Reasoning Slope: S2	-29.8919	10.1129	98	-2.96	0.0039
Reasoning Slope: S3	-26.9557	9.7327	98	-2.77	0.0067
Reasoning Slope: S4	-24.0195	9.5055	98	-2.53	0.0131
Reasoning Slope: S5	-21.0833	9.4425	98	-2.23	0.0278
Reasoning Slope: S6	-18.1471	9.5469	98	-1.90	0.0603



Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	100	1.99	0.1200
DF=2 Wald Test for all Reasoning Slopes	2	98	5.29	0.0066

Total R2 change for reduced reasoning relative to age-only model
Total R2 (% Reduction) for PredQAge vs. PredLReas

Name	Pred Corr	TotalR2	Total R2Diff
PredQAge	0.32688	0.10685	.
PredLReas	0.40008	0.16006	0.053213

The second multivariate Wald F-test provides the significance for the change in total R^2 relative to the age-only model.

PseudoR2 for reduced reasoning relative to age-only model
PsuedoR2 (% Reduction) for CovQAge vs. CovLReas

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovQAge	UN(1,1)	ID	247691	37599	6.59	<.0001	.
CovQAge	UN(2,2)	ID	25083	5787.37	4.33	<.0001	.
CovQAge	UN(3,3)	ID	629.58	172.51	3.65	0.0001	.
CovQAge	Residual	ID	20298	1649.11	12.31	<.0001	.
CovLReas	UN(1,1)	ID	235909	36153	6.53	<.0001	0.047565
CovLReas	UN(2,2)	ID	25707	5883.65	4.37	<.0001	-0.024908
CovLReas	UN(3,3)	ID	629.52	172.50	3.65	0.0001	0.000095
CovLReas	Residual	ID	20298	1649.11	12.31	<.0001	-0.000000

2e. Quadratic Model adding Education Group Predicting Intercept, Linear Time, and Quadratic Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti} - 1) + \beta_{2i}(\text{Session}_{ti} - 1)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{03}(\text{HighvsLowEd}_i) + \gamma_{04}(\text{HighvsMedEd}_i) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{HighvsLowEd}_i) + \gamma_{14}(\text{HighvsMedEd}_i) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{23}(\text{HighvsLowEd}_i) + \gamma_{24}(\text{HighvsMedEd}_i) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = $\text{Session}_{ti} - 1$):

$$\begin{aligned}\hat{y}_{ti} = {} & \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2 \\ & + \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(T)(\text{Age}_i - 80) + \gamma_{21}(T)^2(\text{Age}_i - 80) \\ & + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(T)(\text{Reas}_i - 22) + \gamma_{22}(T)^2(\text{Reas}_i - 22) \\ & + \gamma_{03}(\text{HighvsLowEd}_i) + \gamma_{13}(T)(\text{HighvsLowEd}_i) + \gamma_{23}(T)^2(\text{HighvsLowEd}_i) \\ & + \gamma_{04}(\text{HighvsMedEd}_i) + \gamma_{14}(T)(\text{HighvsMedEd}_i) + \gamma_{24}(T)^2(\text{HighvsMedEd}_i)\end{aligned}$$

Simple Slopes of Interactions (T = $\text{Session}_{ti} - 1$):

Linear Time = $\gamma_{10} + 2\gamma_{20}(T) + \gamma_{11}(\text{Age}_i - 80) + 2\gamma_{21}(T)(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22)$
$+ \gamma_{13}(\text{HighvsLowEd}_i) + 2\gamma_{23}(T)(\text{HighvsLowEd}_i) + \gamma_{14}(\text{HighvsMedEd}_i) + 2\gamma_{24}(T)(\text{HighvsMedEd}_i)$

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{23}(\text{HighvsLowEd}_i) + \gamma_{24}(\text{HighvsMedEd}_i)$

Age = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

Reasoning = $\gamma_{02} + \gamma_{12}(T)$

```
TITLE1 "SAS 2e: Keep Age & Reas, Add Education Group Predicting Intercept, Linear, and Quadratic";
PROC MIXED DATA=work.Example7b NOCLPRINT COVTEST IC NAMELEN=100 METHOD=REML;
CLASS ID educgrp session;
MODEL nm3rt = time1 time1*time1 age80 time1*age80 time1*time1*age80
              reas22 time1*reas22 educgrp time1*educgrp time1*time1*educgrp
              / SOLUTION DDFM=Satterthwaite  OUTPM=PredQEduc; * Save yhat;
RANDOM INTERCEPT time1 time1*time1 / GCORR TYPE=UN SUBJECT=ID;
ODS OUTPUT CovParms=CovQEduc; * Save covparms for comparison;
CONTRAST "DF=3 Wald Test for all Age Slopes"         age80 1,  age80*time1 1, age80*time1*time1 1;
CONTRAST "DF=2 Wald Test for all Reasoning Slopes"  reas22 1, reas22*time1 1;
CONTRAST "DF=6 Wald Test for all Education Slopes"  educgrp -1 1 0, educgrp -1 0 1,
          educgrp*time1 -1 1 0, educgrp*time1 -1 0 1,
          educgrp*time1*time1 -1 1 0, educgrp*time1*time1 -1 0 1;
* LSMEANS gives adjusted means and diffs per group at first and last session;
LSMEANS educgrp / AT (time1 age80 reas22) = (0 0 0) DIFF=ALL; * At beginning;
LSMEANS educgrp / AT (time1 age80 reas22) = (5 0 0) DIFF=ALL; * At end;
* ESTIMATE statements can also give specific group differences;
ESTIMATE "1Low vs 3High Educ:  Intercept "      educgrp -1  0  1;
ESTIMATE "2Med vs 3High Educ:  Intercept"       educgrp  0 -1  1;
ESTIMATE "1Low vs 2Med  Educ:  Intercept"       educgrp -1  1  0 ;
ESTIMATE "1Low vs 3High Educ:  Linear Time"     time1*educgrp -1  0  1;
ESTIMATE "2Med vs 3High Educ:  Linear Time"     time1*educgrp  0 -1  1;
ESTIMATE "1Low vs 2Med  Educ:  Linear Time"     time1*educgrp -1  1  0;
ESTIMATE "1Low vs 3High Educ:  Quadratic Time"  time1*time1*educgrp -1  0  1;
ESTIMATE "2Med vs 3High Educ:  Quadratic Time"  time1*time1*educgrp  0 -1  1;
ESTIMATE "1Low vs 2Med  Educ:  Quadratic Time"  time1*time1*educgrp -1  1  0;
RUN;
TITLE1 "Total R2 change for education relative to model with reasoning*linear only";
%TotalR2(DV=nm3rt, PredFewer=PredLReas, PredMore=PredQEduc);
TITLE1 "PseudoR2 change for education relative to model with reasoning*linear only";
%PseudoR2(NCov=7, CovFewer=CovLReas, CovMore=CovQEduc);
TITLE1;


display "STATA 2e: Keep Age & Reas, Add Education Group Predicting Intercept, Linear, and Quadratic"
mixed nm3rt c.time1 c.time1#c.time1 c.age80 c.time1#c.age80 c.time1#c.time1#c.age80     ///
           c.reas22 c.time1#c.reas22                                                    ///
           ib(last).educgrp c.time1#ib(last).educgrp c.time1#c.time1#ib(last).educgrp, ///
           || id: time1 time1sq, variance reml covariance(un)                          ///
           dfmethod(satterthwaite) dftable(pvalue)
predict predQEduc    // Save yhat
estat ic, n(101)     // AIC and BIC
estat recovariance, relevel(id) correlation  // GCORR matrix
// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.time1#c.age80=0)(c.time1#c.time1#c.age80=0), small
// DF=2 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.time1#c.reas22=0), small
// DF=2 Wald test for education on intercept, linear, quadratic, and DF=6 joint test
contrast i.educgrp c.time1#i.educgrp c.time1#c.time1#i.educgrp, small overall
// Estimating adjusted means and mean diffs per group at first and last session
margins ib(last).educgrp, at(c.time1=(0 5) c.age80=0 c.reas22=0)
margins ib(last).educgrp, at(c.time1=(0) c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)
margins ib(last).educgrp, at(c.time1=(5) c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)
// Contrasts between groups on intercept, linear, and quadratic slopes
test 1.educgrp=3.educgrp, small                                      // 1Low vs 3High: Intercept
test 2.educgrp=3.educgrp, small                                      // 2Med vs 2High: Intercept
test 1.educgrp=2.educgrp, small                                      // 1Low vs 2Med:  Intercept
test 1.educgrp#c.time1=3.educgrp#c.time1, small                      // 1Low vs 3High: Linear Time
test 2.educgrp#c.time1=3.educgrp#c.time1, small                      // 2Med vs 3High: Linear Time
test 1.educgrp#c.time1=2.educgrp#c.time1, small                      // 1Low vs 2Med:  Linear Time
test 1.educgrp#c.time1#c.time1=3.educgrp#c.time1#c.time1, small // 1Low vs 3High: Quadratic Time
test 2.educgrp#c.time1#c.time1=3.educgrp#c.time1#c.time1, small // 2Med vs 3High: Quadratic Time
test 1.educgrp#c.time1#c.time1=2.educgrp#c.time1#c.time1, small // 1Low vs 2Med:  Quadratic Time
// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.age80=0 c.reas22=0 c.time1=(0(1)5) educgrp=(1 2 3))
     marginsplot        // Plot adjusted means
corr predQEduc nm3rt    // Get total r to make R2
     display r(rho)^2   // Print total R2 relative to empty model
```

```
print("R 2e: Keep Age & Reasoning, Add Education Group Predicting Intercept, Linear, and Quadratic")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
QEduc = lmer(data=Example7b, REML=TRUE,
             formula=nm3rt~1+time1+I(time1^2)+age80+reas22+factor(educgrp3)
             +time1:age80 +I(time1^2):age80 +time1:reas22 +time1:factor(educgrp3)
             +I(time1^2):factor(educgrp3) +(1+time1+I(time1^2)|ID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(QEduc, ddf="Satterthwaite"); llikAIC(QEduc, chkREML=FALSE)
print("DF=3 Wald Test for all Age Slopes")
contestMD(QEduc, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0,0,0,0,0,0,0,0,0),
                  c(0,0,0,0,0,0,0,1,0,0,0,0,0,0),c(0,0,0,0,0,0,0,0,1,0,0,0,0,0)))
print("DF=3 Wald Test for all Reasoning Slopes")
contestMD(QEduc, ddf="Satterthwaite", L=rbind(c(0,0,0,0,1,0,0,0,0,0,0,0,0,0),
                                              c(0,0,0,0,0,0,0,0,0,1,0,0,0,0)))
print("DF=2 Wald Test for Each Education Effect")
anova(QEduc)
print("DF=6 Wald Test for all Education Slopes")
contestMD(QEduc, ddf="Satterthwaite", L=rbind(
  c(0,0,0,0,0,1,0,0,0,0,0,0,0,0),c(0,0,0,0,0,0,1,0,0,0,0,0,0,0),
  c(0,0,0,0,0,0,0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,0,0,0,0,1,0,0),
  c(0,0,0,0,0,0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,0,0,0,0,0,1)))
print("Adjusted means and diffs per group only for education simple main effect")
print("Education diffs at session 1")
Qs1mean = ref_grid(QEduc, at=list(time1=0,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Qs1mean, pairwise~educgrp3, lmer.df="satterthwaite", adjust="none")
print("Education diffs at session 6")
Qs6mean = ref_grid(QEduc, at=list(time1=5,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Qs6mean, pairwise~educgrp3, lmer.df="satterthwaite", adjust="none")
print("Specific education group differences on intercept, time1 and slope16")
print("1Low vs 3High Educ: Intercept"); contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,-1,0,0,0,0,0,0,0,0))
print("2Med vs 3High Educ: Intercept"); contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,-1,0,0,0,0,0,0,0))
print("1Low vs 2Med  Educ: Intercept"); contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,-1,1,0,0,0,0,0,0,0))
print("1Low vs 3High Educ: Linear");    contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,-1,0,0))
print("2Med vs 3High Educ: Linear");    contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,0))
print("1Low vs 2Med  Educ: Linear");    contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,-1,1,0))
print("1Low vs 3High Educ: Quadratic"); contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,0))
print("2Med vs 3High Educ: Quadratic"); contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,-1))
print("1Low vs 2Med  Educ: Quadratic"); contest1D(QEduc, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,1))
print("Save yhat and correlation of yhat with y")
Example7b$PredQEduc = predict(QEduc, re.form=NA)
rQEduc = cor.test(Example7b$PredQEduc, Example7b$nm3rt, method="pearson")
print("Total R2"); rQEduc$estimate^2
print("Total R2 change relative to age and reasoning main effects model")
rQEduc$estimate^2-rLReas$estimate^2
```

SAS Output:

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	ID	241027	37339	6.46	<.0001
UN(2,1)	ID	-35271	11645	-3.03	0.0025
UN(2,2)	ID	25772	5956.96	4.33	<.0001
UN(3,1)	ID	4371.57	1907.59	2.29	0.0219
UN(3,2)	ID	-3896.53	995.30	-3.91	<.0001
UN(3,3)	ID	628.15	173.93	3.61	0.0002
Residual	ID	20298	1649.11	12.31	<.0001

Information Criteria

Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
8211.4	7	8225.4	8225.6	8232.8	8243.7	8250.7

Solution for Fixed Effects

Effect	Education Group (1=HS,2=BA,3=GRAD)	Estimate	Standard Error	DF	t Value	Pr > |t|	**g = gamma fixed effect**
Intercept		1961.89	104.34	95.7	18.80	<.0001	g00
time1		-106.50	41.1184	96.7	-2.59	0.0111	g10
time1*time1		12.4797	6.9879	97	1.79	0.0772	g20
age80		20.2894	8.7750	97.5	2.31	0.0229	g01
time1*age80		-4.5759	3.3351	98	-1.37	0.1732	g11
time1*time1*age80		0.6177	0.5646	97	1.09	0.2767	g21
reas22		-36.6221	11.0407	96	-3.32	0.0013	g02
time1*reas22		2.9786	1.3130	96.1	2.27	0.0255	g12

educgrp	1	-51.3792	154.85	96.3	-0.33	0.7408	g03
educgrp	2	37.6426	123.90	95.4	0.30	0.7619	g04
educgrp	3	0	.	.	.	.	
time1*educgrp	1	-70.2451	60.3032	97.1	-1.16	0.2469	g13
time1*educgrp	2	-4.3577	49.1299	96.5	-0.09	0.9295	g14
time1*educgrp	3	0	.	.	.	.	
time1*time1*educgrp	1	11.0653	10.2358	97	1.08	0.2824	g23
time1*time1*educgrp	2	-1.4641	8.3545	97	-0.18	0.8612	g24
time1*time1*educgrp	3	0	.	.	.	.	

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
time1	1	96.5	35.77	<.0001
time1*time1	1	97	17.62	<.0001
age80	1	97.5	5.35	0.0229
time1*age80	1	98	1.88	0.1732
time1*time1*age80	1	97	1.20	0.2767
reas22	1	96	11.00	0.0013
time1*reas22	1	96.1	5.15	0.0255
educgrp	**2**	**96.1**	**0.23**	**0.7965**
time1*educgrp	**2**	**97**	**0.92**	**0.4012**
time1*time1*educgrp	**2**	**97**	**1.05**	**0.3545**

I normally skip this box if the CLASS statement is not used for predictors (it is redundant). But here the last three entries give us the omnibus (df=2) tests for whether there are any education group differences on the intercept, linear, or quadratic time slopes.

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
1Low vs 3High Educ: Intercept	51.3789	154.86	96.3	0.33	0.7408
2Med vs 3High Educ: Intercept	-37.6426	123.90	95.4	-0.30	0.7619
1Low vs 2Med Educ: Intercept	89.0215	134.03	96.7	0.66	0.5081
1Low vs 3High Educ: Linear Time	70.2452	60.3044	97.1	1.16	0.2469
2Med vs 3High Educ: Linear Time	4.3577	49.1308	96.5	0.09	0.9295
1Low vs 2Med Educ: Linear Time	65.8875	51.7672	97.4	1.27	0.2061
1Low vs 3High Educ: Quadratic Time	-11.0653	10.2361	97	-1.08	0.2824
2Med vs 3High Educ: Quadratic Time	1.4641	8.3547	97	0.18	0.8613
1Low vs 2Med Educ: Quadratic Time	-12.5294	8.7795	97	-1.43	0.1568

Least Squares Means

Effect	Education Group (1=HS, 2=BA,3=GRAD)	time1	Age80	Reas22	Estimate	Standard Error	DF	t Value	Pr > |t|
educgrp	1	0.00	0.00	0.00	1910.51	112.41	96.1	17.00	<.0001
educgrp	2	0.00	0.00	0.00	1999.53	69.2521	96.3	28.87	<.0001
educgrp	3	0.00	0.00	0.00	1961.89	104.34	95.7	18.80	<.0001
educgrp	1	5.00	0.00	0.00	1615.41	95.7340	96	16.87	<.0001
educgrp	2	5.00	0.00	0.00	1720.63	59.0119	96.1	29.16	<.0001
educgrp	3	5.00	0.00	0.00	1741.38	88.7908	95.9	19.61	<.0001

In LSMEANS, you must specify a value at which to hold each quantitative predictor.

Differences of Least Squares Means

Effect	(1=HS, 2=BA, 3=GRAD)	(1=HS, 2=BA, 3=GRAD)	time1	age80	reas22	Estimate	Standard Error	DF	t Value	Pr > |t|
educgrp	1	2	0.00	0.00	0.00	-89.0215	134.03	96.7	-0.66	0.5081
educgrp	1	3	0.00	0.00	0.00	-51.3789	154.86	96.3	-0.33	0.7408
educgrp	2	3	0.00	0.00	0.00	37.6426	123.90	95.4	0.30	0.7619
educgrp	1	2	5.00	0.00	0.00	-105.22	114.33	96.2	-0.92	0.3597
educgrp	1	3	5.00	0.00	0.00	-125.97	131.97	96.1	-0.95	0.3422
educgrp	2	3	5.00	0.00	0.00	-20.7486	105.36	95.9	-0.20	0.8443

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
DF=3 Wald Test for all Age Slopes	3	98.1	1.83	0.1469
DF=2 Wald Test for all Reasoning Slopes	2	96	5.85	0.0040
DF=6 Wald Test for all Education Slopes	6	96.4	0.76	0.5994

Total R2 change for education relative to model with reasoning*linear only
Total R2 (% Reduction) for PredLReas vs. PredQEduc

The third multivariate Wald F-test provides the significance for the change in total R^2 relative to the age and reasoning model.

Name	Pred Corr	TotalR2	Total R2Diff
PredLReas	0.40008	0.16006	.
PredQEduc	0.41510	0.17231	0.012242

PseudoR2 change for education relative to model with reasoning*linear only
PsuedoR2 (% Reduction) for CovLReas vs. CovQEduc

Name	CovParm	Subject	Estimate	StdErr	ZValue	ProbZ	PseudoR2
CovLReas	UN(1,1)	ID	235909	36153	6.53	<.0001	.
CovLReas	UN(2,2)	ID	25707	5883.65	4.37	<.0001	.
CovLReas	UN(3,3)	ID	629.52	172.50	3.65	0.0001	.
CovLReas	Resdiual	ID	20298	1649.11	12.31	<.0001	.
CovQEduc	UN(1,1)	ID	241027	37339	6.46	<.0001	-0.021693
CovQEduc	UN(2,2)	ID	25772	5956.96	4.33	<.0001	-0.002519
CovQEduc	UN(3,3)	ID	628.15	173.93	3.61	0.0002	0.002185
CovQEduc	Resdiual	ID	20298	1649.11	12.31	<.0001	-0.000000

Based on the lack of significance of the effect of education, I'd say we're done with this model (I had previously tried age*reasoning, and none of those higher-order effects were significant).

The age*quadratic interaction could probably be removed, but I choose to leave it in as a control.

Simple Processing Speed: Example Conditional Models of Change Results

The extent to which individual differences in response time (RT) in milliseconds over six sessions for a simple processing speed test (number match three) could be predicted from baseline age, abstract reasoning, and education group was examined in a series of multilevel models (i.e., general linear mixed models) in which the six practice sessions were modeled as nested within each participant. Residual maximum likelihood (REML) was used in estimating and reporting all model parameters; denominator degrees of freedom were estimated using the Satterthwaite method. The significance of new fixed effects were evaluated with univariate and multivariate Wald tests. Session (i.e., the index of time) was centered at the first occasion, age was centered at 80 years, abstract reasoning was centered at 22 (near the mean of the scale), and graduate-level education was the reference group for education level (with separate contrasts for high school or less and for bachelor's level education). Effect size for the fixed effects was evaluated via pseduo-R^2 values for the proportion reduction in each variance component, as well as with total-R^2, the squared correlation between the actual outcome values and the outcomes predicted by the model fixed effects.

Piecewise Time Models

The best-fitting unconditional growth model specified linear decline from sessions 1–2 and a second, shallower rate of linear decline from sessions 2–6, along with significant individual differences in the intercept and in each piecewise linear slope. In the unconditional piecewise slopes model, the two fixed slopes for linear change across sessions accounted for approximately 4% of the total variation in RT. Next, age was added as a predictor of the intercept and each piecewise linear slope. Although the three effects of age together resulted in a significant omnibus effect, $F(3, 99) = 4.08$, $p < .01$, only the fixed effect of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher (slower) by 29.78 ($p < .001$). In terms of pseudo-R^2, age accounted for 10.56% of the level-2 random intercept variance, 1.90% of the level-2 random variance in linear change from sessions 1–2, and 0.91% of the level-2 random variance in linear change from sessions 2–6. As expected given that baseline age is a time-invariant predictor, the level-1 residual variance was not reduced. The cumulative total-R^2 from piecewise session slopes and age was $R^2 = .11$, approximately a 7% increase due to age.

Although the interactions of age with the linear piecewise slopes were not significant, they were retained in the model to fully control for age effects on change across sessions before examining the other predictors.

Abstract reasoning was then added as a predictor of the intercept and each piecewise linear slope. The three effects of abstract reasoning together resulted in a significant omnibus effect, $F(3, 98) = 3.50$, $p = .02$. The significant fixed effects of abstract reasoning on the intercept and second slope indicated that for every additional unit of reasoning above 22, RT at the first session was predicted to be significantly lower (faster) by 27.10 ($p < .001$) and to increase by an additional 3.35 ms after session 2.The nonsignificant effect of reasoning on the first slope was retained to facilitate interpretation of the separate effects of reasoning on each aspect of change. Relative to the age-only model, reasoning accounted for 4.76% of the level-2 random intercept variance, none of the level-2 random first slope variance, and 0.70% of the level-2 second slope variance. The cumulative total-R^2 from piecewise session slopes, age, and reasoning was 16%, approximately a 5% increase due to reasoning.

Education group (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept and each linear slope. These six effects of education did not significantly improve model fit, $F(6, 96) = 0.73$, $p = .63$. No omnibus main effects of education level on the intercept, linear, or quadratic slopes were significant, and no pairwise comparisons were significant as well. Relative to the age and reasoning model, education accounted for no measurable variance in the level-2 random intercept or either level-2 random linear slope. The cumulative R^2 from piecewise session slopes, age, reasoning, and education was total-$R^2 = .17$, approximately a 1% increase due to education. Finally, we examined the interactive effects of age and reasoning in predicting the intercept and each linear slope, although none was significant. (From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)

Quadratic Time Models

The best-fitting unconditional growth model specified quadratic decline across the six sessions (i.e., a decelerating negative function) with significant individual differences in the intercept, linear, and quadratic time effects. In the unconditional growth model, the fixed effects for linear and quadratic change accounted for approximately 4% of the total variation in RT.

Next, age was added as a predictor of the intercept, linear slope, and quadratic slope. Although the three effects of age together resulted in a significant omnibus effect, $F(3, 99) = 4.00$, $p < .01$, only the fixed effect of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher (slower) by 29.05 ($p < .001$). In terms of pseudo-R^2, age accounted for 10.32% of the level-2 random intercept variance, 2.93% of the level-2 random linear slope variance, and 0.77% of the level-2 random quadratic slope variance. As expected given that baseline age is a time-invariant predictor, the level-1 residual variance was not reduced. The cumulative total-R^2 from linear and quadratic slopes for session and age was 11%, approximately a 7% increase due to age. Although the interactions of age with the linear and quadratic slopes were not significant, they were retained in the model to fully control for age effects on change across sessions before examining the effects of other predictors.

Abstract reasoning was then added as a predictor of the intercept, linear slope, and quadratic slope. As with the effects of age, although the three effects of abstract reasoning together resulted in a significant omnibus effect, $F(3, 98) = 4.29$, $p < .01$, only the fixed effect of abstract reasoning on the intercept was significant, indicating that for every additional unit of reasoning above 22, RT at the first session was predicted to be significantly lower (faster) by 27.10 ($p < .001$).The nonsignificant effect of reasoning on the quadratic slope was then removed, revealing a significant effect of reasoning on both the intercept and linear slope, $F(2, 98) = 5.29$, $p < .01$, such that for every unit higher reasoning above 22, RT at the first session was expected to be lower by 32.83 and the linear rate of improvement in RT (as evaluated at the first session given the quadratic slope) was expected to be less negative by 2.94 (i.e., faster initial RT with less improvement in persons with greater reasoning). Relative to the age-only model, reasoning accounted for 4.76% of the level-2 random intercept variance but had no measurable reduction of the level-2 random linear and quadratic slope variances. The cumulative total-R^2 from linear and quadratic slopes for session, age, and reasoning was 16%, approximately a 5% increase due to reasoning.

Education group (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept, linear slope, and quadratic slope. These six effects of education did not significantly improve model fit, $F(6, 96) = 0.76$, $p = .60$. No omnibus main effects of education level on the intercept, linear, or quadratic slopes were significant, and no pairwise comparisons were significant as well. Relative to the age and reasoning model, education accounted for no measurable random intercept or random linear slope variance, and 0.22% of the random quadratic slope variance. The cumulative total-R^2 from linear and quadratic slopes for session, age, reasoning, and education was 17%, approximately a 1% increase due to education. Finally, we examined the interactive effects of age and reasoning in predicting the intercept and each linear slope, although none was significant. (From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)