Example 6d: Modeling Change over Time Using Log Time to Approximate Exponential Trends *(complete data, syntax, and output available for SAS electronically)*

These data for these example models come from Hoffman (2015) chapter 6. We will be examining change in response time (RT) in milliseconds over six practice sessions (balanced time) to a measure of processing speed in a sample of 101 older adults. Previously in Example6c we used SAS NLMIXED to estimate a truly nonlinear model with an exponential curve, in which REML is not available. Below we examine a strategy to mimic the same shape that uses linear and quadratic effects of natural-log-transformed time instead, such that we can use REML in MIXED. At the end we will compare the fit of the “best” version of each family of models examined (polynomial, piecewise, and exponential).

SAS Syntax for Data Import and Manipulation:

```
* Define global variable for file location to be replaced in code below;
%LET filesave=C:\Dropbox\21_PSQF7375_Longitudinal\PSQF7375_Longitudinal_Example6;
* Location for SAS files for these models (uses macro variable filesave);
LIBNAME filesave "&filesave.";

DATA work.Example6; SET filesave.SAS_Chapter6;
* Log-transform time as new predictor;
  logtime=LOG(session); LABEL logtime="logtime: Log-Transformed Time (0=1)";
* Create session dummy codes for testing means model absolute fit in REML;
  IF session=1 THEN s1=1; ELSE s1=0;
  IF session=2 THEN s2=1; ELSE s2=0;
  IF session=3 THEN s3=1; ELSE s3=0;
  IF session=4 THEN s4=1; ELSE s4=0;
RUN;
```

STATA Syntax for Data Import and Manipulation:

```
// Define global variable for file location to be replaced in code below
   global filesave "C:\Dropbox\21_PSQF7375_Longitudinal\PSQF7375_Longitudinal_Example6"
// Import chapter 6 stacked data and create time predictor variables
   use "$filesave\STATA_Chapter6.dta", clear

// Log-transform time as new predictor
   gen logtime=log(session)
   gen logtimesq=log(session)*log(session)
   label variable logtime   "logtime: Natural Log Session"
   label variable logtimesq "logtimesq: Quadratic Natural Log Session"

// Create session dummy codes for testing means model absolute fit in REML
   gen s1=0
   gen s2=0
   gen s3=0
   gen s4=0
   replace s1=1 if session==1
   replace s2=1 if session==2
   replace s3=1 if session==3
   replace s4=1 if session==4
```

R Syntax for Data Import and Manipulation:

```
# Define variables for working directory and data name
filesave = "C:\\Dropbox/21_PSQF7375_Longitudinal/PSQF7375_Longitudinal_Example6/"
filename = "SAS_Chapter6.sas7bdat"
setwd(dir=filesave)

# Import chapter 6 stacked data with labels
Example6 = read_sas(data_file=paste0(filesave,filename))
# Convert to data frame as data frame without labels to use for analysis
Example6 = as.data.frame(Example6)
# Sort data by PersonID (needed for correct RCOV matrix)
Example6 = sort_asc(Example6,PersonID,wave)

# Log-transform time as new predictor
Example6$logtime=log(Example6$session)
```

```
# Create session dummy codes for testing means model absolute fit in REML
Example6$s1=0
Example6$s2=0
Example6$s3=0
Example6$s4=0
Example6$s1[which(Example6$session==1)]=1
Example6$s2[which(Example6$session==2)]=1
Example6$s3[which(Example6$session==3)]=1
Example6$s4[which(Example6$session==4)]=1
```

Note—a TYPC=VC R matrix will be used in all of the models that follow (not shown to save space).

Model 7a. Fixed Linear Log Time, Random Intercept Model

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{LogTime}_{ti}\right) + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear LogTime: } \beta_{1i} = \gamma_{10}$$

```
TITLE1 "SAS: 7a: Fixed Linear Log Time, Random Intercept Model";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT / G V VCORR TYPE=UN SUBJECT=PersonID;
     REPEATED session / R TYPE=VC SUBJECT=PersonID;
     ODS OUTPUT CovParms=CovFixLinLog InfoCrit=FitFixLinLog; * Save for pseudo-R2 and LRT;
     * Get conditional mean per occasion from value of time predictor;
     ESTIMATE "Intercept at Session=1"    intercept 1 logtime 0;
     ESTIMATE "Intercept at Session=2"    intercept 1 logtime 0.6931;
     ESTIMATE "Intercept at Session=3"    intercept 1 logtime 1.0986;
     ESTIMATE "Intercept at Session=4"    intercept 1 logtime 1.3863;
     ESTIMATE "Intercept at Session=5"    intercept 1 logtime 1.6094;
     ESTIMATE "Intercept at Session=6"    intercept 1 logtime 1.7918;
RUN;
TITLE1 "Calculate pseudo R2 -- variance accounted for by fixed linear log time";
%PseudoR2(NCov=2, CovFewer=CovEmpty, CovMore=CovFixLinLog); TITLE1;
```

```
display "STATA: 7a: Fixed Linear Log Time, Random Intercept Model"
mixed rt c.logtime, || personid: , variance reml covariance(unstructured) ///
           residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      estat ic, n(101)               // AIC and BIC
      estat wcorrelation, covariance // V matrix
      estat wcorrelation             // VCORR matrix
      // Get conditional mean per occasion from value of time predictor
      lincom _cons*1 + c.logtime*0          // Intercept at Session=1
      lincom _cons*1 + c.logtime*0.6931     // Intercept at Session=2
      lincom _cons*1 + c.logtime*1.0986     // Intercept at Session=3
      lincom _cons*1 + c.logtime*1.3863     // Intercept at Session=4
      lincom _cons*1 + c.logtime*1.6094     // Intercept at Session=5
      lincom _cons*1 + c.logtime*1.7918     // Intercept at Session=6
      estimates store FitFixLinLog  // Save for LRT
```

```
print("R 7a: Fixed Linear Log Time, Random Intercept Model")
FixLinLog = lmer(data=Example6, REML=TRUE, formula=rt~1+logtime+(1|PersonID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(FixLinLog, ddf="Satterthwaite"); llikAIC(FixLinLog, chkREML=FALSE)
print("Get conditional mean per occasion from value of time predictor")
print("Intercept at Session=1"); contest1D(FixLinLog, ddf="Satterthwaite", L=c(1,0))
print("Intercept at Session=2"); contest1D(FixLinLog, ddf="Satterthwaite", L=c(1,0.6931))
print("Intercept at Session=3"); contest1D(FixLinLog, ddf="Satterthwaite", L=c(1,1.0986))
print("Intercept at Session=4"); contest1D(FixLinLog, ddf="Satterthwaite", L=c(1,1.3863))
print("Intercept at Session=5"); contest1D(FixLinLog, ddf="Satterthwaite", L=c(1,1.6094))
print("Intercept at Session=6"); contest1D(FixLinLog, ddf="Satterthwaite", L=c(1,1.7918))
```

SAS Output:

```
                         Estimated V Matrix for PersonID 101
 Row         Col1        Col2        Col3        Col4        Col5        Col6
   1       236852      202669      202669      202669      202669      202669
   2       202669      236852      202669      202669      202669      202669
   3       202669      202669      236852      202669      202669      202669
   4       202669      202669      202669      236852      202669      202669
   5       202669      202669      202669      202669      236852      202669
   6       202669      202669      202669      202669      202669      236852
```

The predicted **V** matrix still has a compound symmetry pattern because we have not yet added to the model for the variance (still only a random intercept variance in **G**).

```
                   Estimated V Correlation Matrix for PersonID 101
 Row         Col1        Col2        Col3        Col4        Col5        Col6
   1       1.0000      0.8557      0.8557      0.8557      0.8557      0.8557
   2       0.8557      1.0000      0.8557      0.8557      0.8557      0.8557
   3       0.8557      0.8557      1.0000      0.8557      0.8557      0.8557
   4       0.8557      0.8557      0.8557      1.0000      0.8557      0.8557
   5       0.8557      0.8557      0.8557      0.8557      1.0000      0.8557
   6       0.8557      0.8557      0.8557      0.8557      0.8557      1.0000
```

This random intercept only model for the variance (CS) translates into equal correlation over time as shown in the VCORR matrix (the ICC).

```
                      Covariance Parameter Estimates
                                          Standard         Z
Cov Parm     Subject     Estimate         Error       Value      Pr > Z
UN(1,1)      PersonID      202669         29470        6.88      <.0001   Level-2 random intercept variance of U0i
session      PersonID       34183       2153.35       15.87      <.0001   Level-1 residual variance of eti
```

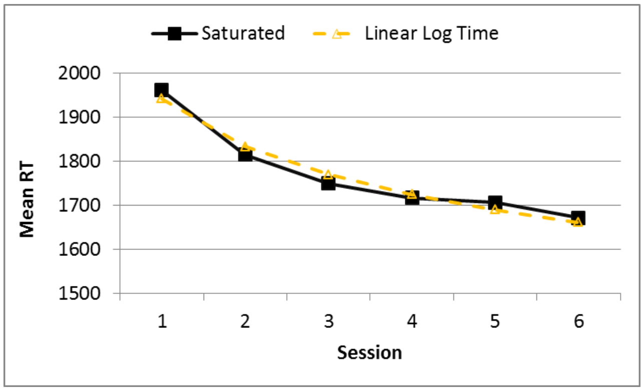
```
  Null Model Likelihood Ratio Test
    DF     Chi-Square      Pr > ChiSq
     1         805.80          <.0001
```

This LRT is for the conditional ICC > 0.

```
                                  Information Criteria
Neg2LogLike      Parms         AIC         AICC        HQIC         BIC        CAIC
     8391.3          2      8395.3       8395.3      8397.4      8400.5      8402.5
```

Only variance model parameters are “counted” as parms using REML.

```
                         Solution for Fixed Effects
                               Standard
Effect         Estimate           Error       DF    t Value     Pr > |t|
Intercept       1942.55         47.4172      119      40.97       <.0001   gamma00
logtime         -156.72         12.4160      504     -12.62       <.0001   gamma10
                                         Estimates
                                             Standard
Label                          Estimate         Error      DF     t Value     Pr > |t| gamma00(1)+gamma10(logtime)
Intercept at Session=1          1942.55       47.4172     119       40.97       <.0001
Intercept at Session=2          1833.93       45.6960     102       40.13       <.0001
Intercept at Session=3          1770.38       45.4206     100       38.98       <.0001
Intercept at Session=4          1725.29       45.5629     101       37.87       <.0001
Intercept at Session=5          1690.33       45.8648     104       36.85       <.0001
Intercept at Session=6          1661.74       46.2336     107       35.94       <.0001
```



The fixed linear effect of log time mimics an exponential curve (and appears to fit pretty well to the saturated means).

As shown below, the fixed linear slope for log time accounted for 24% of the level-1 residual variance.

```
Calculate pseudo R2 -- variance accounted for by fixed linear log time
PsuedoR2 (% Reduction) for CovEmpty vs. CovFixLinLog
Name            CovParm     Subject      Estimate       StdErr      ZValue      ProbZ     PseudoR2
CovEmpty        UN(1,1)     PersonID       200883        29471        6.82      <.0001        .
CovEmpty        session     PersonID        44900      2825.63       15.89      <.0001        .
CovFixLinLog    UN(1,1)     PersonID       202669        29470        6.88      <.0001     -0.00889
CovFixLinLog    session     PersonID        34183      2153.35       15.87      <.0001      0.23868
```

Model 7b. Random Linear Log Time

$$\text{Level 1:}\quad y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{LogTime}_{ti}\right) + e_{ti}$$

$$\text{Level 2:}\quad \text{Intercept:}\qquad \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear LogTime:}\qquad \beta_{1i} = \gamma_{10} + U_{1i}$$

```
TITLE1 "SAS: 7b: Random Linear Log Time Model";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT logtime / G GCORR V VCORR TYPE=UN SUBJECT=PersonID;
     REPEATED session / R TYPE=VC SUBJECT=PersonID;
     ODS OUTPUT CovParms=CovRandLinLog  InfoCrit=FitRandLinLog; * Save for pseudo-R2 and LRT;
RUN;
TITLE1 "Calculate LRT -- does random linear log time slope improve model fit?";
%FitTest(FitFewer=FitFixLinLog, FitMore=FitRandLinLog); TITLE1;
```

```
display "STATA: 7b: Random Linear Log Time Model"
mixed rt c.logtime,  || personid: logtime, variance reml covariance(unstructured) ///
           residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      estat ic, n(101)                                  // AIC and BIC
      estat recovariance, relevel(personid)             // G matrix
      estat recovariance, relevel(personid) correlation // GCORR matrix
      estat wcorrelation, covariance                    // V matrix
      estat wcorrelation                                // VCORR matrix
      estimates store FitRandLinLog      // Save for LRT
      lrtest FitRandLinLog FitFixLinLog  // Does random linear log time slope improve model fit?
```

```
print("R 7b: Random Linear Log Time Model")
RandLinLog = lmer(data=Example6, REML=TRUE, formula=rt~1+logtime+(1+logtime|PersonID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(RandLinLog, ddf="Satterthwaite"); llikAIC(RandLinLog, chkREML=FALSE)
print("Does random linear time slope improve model fit?")
ranova(RandLinLog, reduce.term=TRUE) # Remove random slope and covariance
```

SAS Output:

```
                  Estimated G Matrix
Row    Effect       PersonID           Col1        Col2
  1     Intercept    101              274252      -44541
  2     logtime      101              -44541       23101
            Estimated G Correlation Matrix
Row    Effect       PersonID           Col1        Col2
  1     Intercept    101              1.0000     -0.5596
  2     logtime      101             -0.5596      1.0000
```

This **level-2 G matrix** (always **unstructured**) still contains a random intercept variance (1,1), and we have added a random linear log time slope variance (2,2), as well as a covariance between the random intercept and random linear log time slope for the same person (2,1). The intercept–slope correlation is shown in GCORR (2,1).

```
                   Estimated V Matrix for PersonID 101
 Row        Col1        Col2        Col3        Col4        Col5        Col6
   1      298372      243378      225318      212505      202566      194445
   2      243378      247724      212036      203829      197463      192262
   3      225318      212036      228387      198754      194478      190984
   4      212505      203829      198754      219274      192360      190078
   5      202566      197463      194478      192360      214838      189375
   6      194445      192262      190984      190078      189375      212922
```

The marginal **V** matrix now predicts that **variance changes quadratically over time** (as a function of logtime^2).

```
               Estimated V Correlation Matrix for PersonID 101
 Row         Col1         Col2         Col3         Col4         Col5         Col6
   1       1.0000       0.8952       0.8631       0.8308       0.8001       0.7714
   2       0.8952       1.0000       0.8914       0.8746       0.8559       0.8371
   3       0.8631       0.8914       1.0000       0.8882       0.8780       0.8661
   4       0.8308       0.8746       0.8882       1.0000       0.8863       0.8797
   5       0.8001       0.8559       0.8780       0.8863       1.0000       0.8854
   6       0.7714       0.8371       0.8661       0.8797       0.8854       1.0000
```

The marginal **VCORR** matrix now predicts that the covariance changes over time as well (in a time-dependent pattern).

```
                      Covariance Parameter Estimates
                                          Standard         Z
Cov Parm     Subject      Estimate       Error       Value       Pr Z
UN(1,1)      PersonID       274252       41240        6.65       <.0001   Level-2 random intercept variance of U0i
UN(2,1)      PersonID       -44541       11493       -3.88       0.0001   Level-2 random intercept-linear covariance
UN(2,2)      PersonID        23101     4882.26        4.73       <.0001   Level-2 random linear slope variance of U1i
session      PersonID        24120     1697.11       14.21       <.0001   Level-1 residual variance of eti

  Null Model Likelihood Ratio Test
    DF     Chi-Square       Pr > ChiSq
     3         873.64           <.0001
```

This LRT tells us whether we need the full 2x2 **G** matrix (so it is not helpful).

```
                                Information Criteria
Neg2LogLike      Parms        AIC        AICC        HQIC         BIC        CAIC
     8323.4          4     8331.4      8331.5      8335.7      8341.9      8345.9

                         Solution for Fixed Effects
                               Standard
Effect          Estimate         Error        DF    t Value     Pr > |t|
Intercept        1942.55       53.7211       100      36.16       <.0001
logtime          -156.72       18.3711       100      -8.53       <.0001
```

```
Calculate LRT -- does random linear log time slope improve model fit?
Likelihood Ratio Test for FitFixLinLog vs. FitRandLinLog
                 Neg2Log
    Name           Like       Parms         AIC        BIC     DevDiff     DFdiff       Pvalue
FitFixLinLog      8391.3        2        8395.3     8400.5        .           .              .
FitRandLinLog     8323.4        4        8331.4     8341.9     67.8448        2       1.8874E-15
```

This LRT tells us that the random log time slope improves model fit.

<u>95% Random Effects Confidence Intervals that describe the *predicted* range of *individual* random effects:</u>

$$\text{Random Effect 95\% CI} = \text{fixed effect} \pm \left(1.96*\sqrt{\text{Random Variance}}\right)$$

$$\text{Intercept 95\% CI} = \gamma_{00} \pm \left(1.96*\sqrt{\tau^2_{U_0}}\right) \rightarrow 1{,}942.55 \pm \left(1.96*\sqrt{274{,}252}\right) = 916 \text{ to } 2{,}969$$

$$\text{Linear LogTime Slope 95\% CI} = \gamma_{10} \pm \left(1.96*\sqrt{\tau^2_{U_1}}\right) \rightarrow -156.72 \pm \left(1.96*\sqrt{23{,}101}\right) = -455 \text{ to } 141$$

The absolute fit of the linear log time model for the means can be tested by mimicking a saturated means model using the *same random linear log time slope* (i.e., holding the model for the variance constant):

```
TITLE1 "SAS: Test Absolute Fit of Linear Log Time Means (Using Random Linear Log Time Variance Model)";
TITLE2 "Add 4 session dummy codes to saturate the means model";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime s1 s2 s3 s4 / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT logtime / G GCORR V VCORR TYPE=UN SUBJECT=PersonID;
     REPEATED session / R TYPE=VC SUBJECT=PersonID;
     CONTRAST "Does fixed linear log time reproduce saturated means?" s1 1, s2 1, s3 1, s4 1 / CHISQ;
RUN; TITLE1; TITLE2;
```

```
display "STATA: Test Absolute Fit of Linear Log Time Means (Using Random Linear Log Time Variance Model)"
display "Add 3 session dummy codes to saturate the means model"
mixed rt c.logtime c.s1 c.s2 c.s3 c.s4, ///
           || personid: logtime, variance reml covariance(unstructured) ///
           residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      test (c.s1=0)(c.s2=0)(c.s3=0)(c.s4=0), small  // Does fixed linear log time reproduce sat means?
```

```
print("R: Test Absolute Fit of Linear Log Time Means (Using Random Linear Log Time Variance Model)")
print("Add 3 session dummy codes to second piece to saturate the means model")
LinLogMean = lmer(data=Example6, REML=TRUE, formula=rt~1+logtime+s1+s2+s3+s4+(1+logtime|PersonID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(LinLogMean, ddf="Satterthwaite"); llikAIC(LinLogMean, chkREML=FALSE)
print("Does fixed two-piece slope reproduce saturated means?")
contestMD(LinLogMean, ddf="Satterthwaite",
          L=rbind(c(0,0,1,0,0,0),c(0,0,0,1,0,0),c(0,0,0,0,1,0) ,c(0,0,0,0,0,1)))
```

SAS Output (relevant tables only):

```
                  Solution for Fixed Effects
                        Standard
Effect       Estimate      Error      DF    t Value    Pr > |t|
Intercept     2016.49     210.04     449       9.60      <.0001
logtime       -192.19     120.43     418      -1.60      0.1113
s1           -54.5944     204.05     400      -0.27      0.7892
s2           -68.1019     121.83     400      -0.56      0.5765
s3           -55.3148    74.3533     400      -0.74      0.4573
s4           -32.2644    42.0226     400      -0.77      0.4431

                                                   Contrasts
                                                    Num    Den
Label                                                DF     DF   Chi-Square   F Value   Pr > ChiSq    Pr > F
Does fixed linear log time reproduce saturated means?  4    400         6.69      1.67       0.1531    0.1554
```

Because there are now **6 fixed effects for the 6 means**, this model is equivalent to **saturated means** (even if the linear log time fixed slope is largely uninterpretable). In an SEM context, this model would be specified by letting four of the observed occasions' intercepts be estimated (as discrepancies).

The multivariate Wald test using CONTRAST indicates that the 4 extra session contrasts did not improve model fit (which is good news here).

The absolute fit of the random linear log time model for the variance can be tested against a UN variance model using the *same fixed linear log time slope* (i.e., holding the model for the means constant):

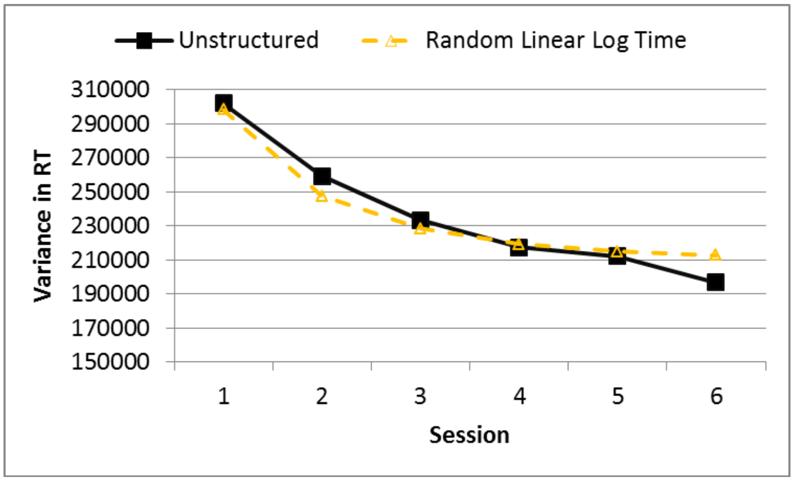
```
TITLE1 "SAS: Test Absolute Fit of Linear Log Time Variance (Using Fixed Linear Log Time Means Model)";
TITLE2 "Change to Unstructured R matrix as variance model answer key";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime / SOLUTION DDFM=Satterthwaite;
     REPEATED session / R RCORR TYPE=UN SUBJECT=PersonID;
     ODS OUTPUT InfoCrit=FitFixLinLogUN; * Save for LRT;
RUN; TITLE2;
TITLE1 "Calculate LRT -- does random linear log time reproduce UN variance model?";
 %FitTest(FitFewer=FitRandLinLog, FitMore=FitFixLinLogUN); TITLE1;
```

```
display "STATA: Test Absolute Fit of Linear Log Time Variance (Using Fixed Linear Log Time Means Model)"
display "Change to Unstructured R matrix as variance model answer key"
mixed rt c.logtime, || personid: , noconstant variance reml ///
           residuals(unstructured,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      estimates store FitFixLinLogUN      // Save for LRT
      lrtest FitFixLinLogUN FitRandLinLog // Does random linear log time reproduce UN variance model?
```

```
print("R: Test Absolute Fit of Linear Log Time Variance Model (Using Fixed Linear Log Time Means Model)")
print("Change to Unstructured R matrix as variance model answer key in GLS")
LinLogUN = gls(data=Example6, method="REML", model=rt~1+logtime,
                correlation=corSymm(form=~as.numeric(session)|PersonID),
                weights=varIdent(form=~1|session))
print("Have to re-run random quadratic log time model using LME to get LRT")
RandLinLoglme = lme(data=Example6, method="REML", rt~1+logtime, random=~1+logtime|PersonID)
anova(LinLogUN,RandLinLoglme) # anova does LRT using LME versions
```

SAS Output (relevant table only):

```
Calculate LRT -- does random linear log time reproduce UN variance model?
Likelihood Ratio Test for FitRandLinLog vs. FitRandUN
                  Neg2Log
    Name            Like       Parms        AIC         BIC     DevDiff    DFdiff         Pvalue
FitRandLinLog     8323.4         4       8331.4      8341.9        .           .        .
FitRandUN         8265.4        21       8307.4      8362.3     58.0097       17       .000002230
```



The random linear slope of log time approximates most of the observed variances from the unstructured model, but not well enough according to the LRT (and the same is likely true of the covariances, not shown here).

Model 7c. Fixed Quadratic, Random Linear Log Time

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{LogTime}_{ti}\right) + \beta_{2i}\left(\text{LogTime}_{ti}\right)^2 + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear LogTime: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Quadratic LogTime: } \beta_{2i} = \gamma_{20}$$

Predicted intercept at any occasion:

$$= \gamma_{00} + \gamma_{10}(\text{LogTime}_{ti}) + \gamma_{20}(\text{LogTime}_{ti})^2$$

Instantaneous linear time slope at any occasion:

$$= \gamma_{10} + 2\gamma_{20}(\text{LogTime}_{ti})$$

```
TITLE1 "SAS: 7c: Fixed Quadratic, Random Linear Log Time Model";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime logtime*logtime / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT logtime / G GCORR V VCORR TYPE=UN SUBJECT=PersonID;
     REPEATED session / R TYPE=VC SUBJECT=PersonID;
     ODS OUTPUT CovParms=CovFixQuadLog InfoCrit=FitFixQuadLog; * Save for pseudo-R2 and LRT;
     * Get conditional mean per occasion from values of time predictors;
     ESTIMATE "Intercept at Session=1"    intercept 1 logtime 0      logtime*logtime 0;
     ESTIMATE "Intercept at Session=2"    intercept 1 logtime 0.6931 logtime*logtime 0.4805;
     ESTIMATE "Intercept at Session=3"    intercept 1 logtime 1.0986 logtime*logtime 1.2069;
     ESTIMATE "Intercept at Session=4"    intercept 1 logtime 1.3863 logtime*logtime 1.9218;
     ESTIMATE "Intercept at Session=5"    intercept 1 logtime 1.6094 logtime*logtime 2.5903;
     ESTIMATE "Intercept at Session=6"    intercept 1 logtime 1.7918 logtime*logtime 3.2104;
     * Get instantaneous linear slope per occasion from 2*value of time predictor;
     ESTIMATE "Linear Slope at Session=1" logtime 1 logtime*logtime 0;
     ESTIMATE "Linear Slope at Session=2" logtime 1 logtime*logtime 1.3863;
     ESTIMATE "Linear Slope at Session=3" logtime 1 logtime*logtime 2.1972;
     ESTIMATE "Linear Slope at Session=4" logtime 1 logtime*logtime 2.7726;
     ESTIMATE "Linear Slope at Session=5" logtime 1 logtime*logtime 3.2189;
     ESTIMATE "Linear Slope at Session=6" logtime 1 logtime*logtime 3.5835;
RUN;
TITLE1 "Calculate pseudo R2 -- variance accounted for by fixed quadratic log time";
%PseudoR2(NCov=4, CovFewer=CovRandLinLog, CovMore=CovFixQuadLog); TITLE1;
```

Because twice the quadratic slope is how the linear slope changes per unit time, the value for log time used in estimating the linear slope per session gets multiplied by 2.

```
display "STATA: 7c: Fixed Quadratic, Random Linear Log Time Model"
mixed rt c.logtime c.logtime#c.logtime, || personid: logtime, variance reml covariance(unstructured) ///
         residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
     estat ic, n(101)                                      // AIC and BIC
     estat recovariance, relevel(personid)                 // G matrix
     estat recovariance, relevel(personid) correlation // GCORR matrix
     estat wcorrelation, covariance                        // V matrix
     estat wcorrelation                                    // VCORR matrix
     * Get conditional mean per occasion from values of time predictors
     lincom _cons*1 + c.logtime*0      + c.logtime#c.logtime*0        // Intercept at Session=1
     lincom _cons*1 + c.logtime*0.6931 + c.logtime#c.logtime*0.4805   // Intercept at Session=2
     lincom _cons*1 + c.logtime*1.0986 + c.logtime#c.logtime*1.2069   // Intercept at Session=3
     lincom _cons*1 + c.logtime*1.3863 + c.logtime#c.logtime*1.9218   // Intercept at Session=4
     lincom _cons*1 + c.logtime*1.6094 + c.logtime#c.logtime*2.5903   // Intercept at Session=5
     lincom _cons*1 + c.logtime*1.7918 + c.logtime#c.logtime*3.2104   // Intercept at Session=6
     // Get instantaneous linear slope per occasion from 2*value of time predictor
     lincom c.logtime*1 + c.logtime#c.logtime*0     , small    // Linear Slope at Session=1
     lincom c.logtime*1 + c.logtime#c.logtime*1.3863, small    // Linear Slope at Session=2
     lincom c.logtime*1 + c.logtime#c.logtime*2.1972, small    // Linear Slope at Session=3
     lincom c.logtime*1 + c.logtime#c.logtime*2.7726, small    // Linear Slope at Session=4
     lincom c.logtime*1 + c.logtime#c.logtime*3.2189, small    // Linear Slope at Session=5
     lincom c.logtime*1 + c.logtime#c.logtime*3.5835, small    // Linear Slope at Session=6
     estimates store FitFixQuadLog   // Save for LRT
```

```
print("R 7c: Fixed Quadratic, Random Linear Log Time Model")
FixQuadLog = lmer(data=Example6, REML=TRUE, formula=rt~1+logtime+I(logtime^2)+(1+logtime|PersonID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(FixQuadLog, ddf="Satterthwaite"); llikAIC(FixQuadLog, chkREML=FALSE)
print("Get conditional mean per occasion from values of time predictors")
print("Intercept at Session=1"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(1,0,0))
print("Intercept at Session=2"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(1,0.6931,0.4805))
print("Intercept at Session=3"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(1,1.0986,1.2069))
print("Intercept at Session=4"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(1,1.3863,1.9218))
print("Intercept at Session=5"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(1,1.6094,2.5903))
print("Intercept at Session=6"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(1,1.7918,3.2104))
print("Get instantaneous linear slope per occasion from 2*value of time predictor")
print("Linear Slope at Session=1 Time=0"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(0,1,0))
print("Linear Slope at Session=2 Time=1"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(0,1,1.3863))
print("Linear Slope at Session=3 Time=2"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(0,1,2.1972))
print("Linear Slope at Session=4 Time=3"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(0,1,2.7726))
print("Linear Slope at Session=5 Time=4"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(0,1,3.2189))
print("Linear Slope at Session=6 Time=5"); contest1D(FixQuadLog, ddf="Satterthwaite", L=c(0,1,3.5835))
```

SAS Output:

```
                   Estimated G Matrix

 Row    Effect       PersonID         Col1         Col2

   1    Intercept         101       274453       -44682
   2    logtime           101       -44682        23229


             Estimated G Correlation Matrix

 Row    Effect       PersonID         Col1         Col2

   1    Intercept         101       1.0000      -0.5596
   2    logtime           101      -0.5596       1.0000
```

The model for the variance has not changed, so it has the same structure as before (**G** is 2x2).

```
                     Estimated V Matrix for PersonID 101

 Row        Col1        Col2        Col3        Col4        Col5        Col6

   1      298292      243482      225365      212511      202540      194394
   2      243482      247510      212083      203861      197483      192273
   3      225365      212083      228152      198801      194525      191032
   4      212511      203861      198801      219050      192426      190151
   5      202540      197483      194525      192426      214637      189468
   6      194394      192273      191032      190151      189468      212749
```

```
               Estimated V Correlation Matrix for PersonID 101
 Row        Col1        Col2        Col3        Col4        Col5        Col6
   1      1.0000      0.8961      0.8639      0.8314      0.8005      0.7717
   2      0.8961      1.0000      0.8925      0.8755      0.8568      0.8379
   3      0.8639      0.8925      1.0000      0.8893      0.8790      0.8671
   4      0.8314      0.8755      0.8893      1.0000      0.8874      0.8808
   5      0.8005      0.8568      0.8790      0.8874      1.0000      0.8866
   6      0.7717      0.8379      0.8671      0.8808      0.8866      1.0000

                  Covariance Parameter Estimates
                                  Standard        Z
Cov Parm    Subject     Estimate       Error    Value      Pr Z
UN(1,1)     PersonID      274453       41239     6.66    <.0001  Level-2 random intercept variance of U0i
UN(2,1)     PersonID      -44682       11493    -3.89    0.0001  Level-2 random intercept-linear covariance
UN(2,2)     PersonID       23229     4880.99     4.76    <.0001  Level-2 random linear time slope variance of U1i
session     PersonID       23839     1679.38    14.20    <.0001  Level-1 residual variance of eti

  Null Model Likelihood Ratio Test
    DF    Chi-Square      Pr > ChiSq
     3        876.51          <.0001
```

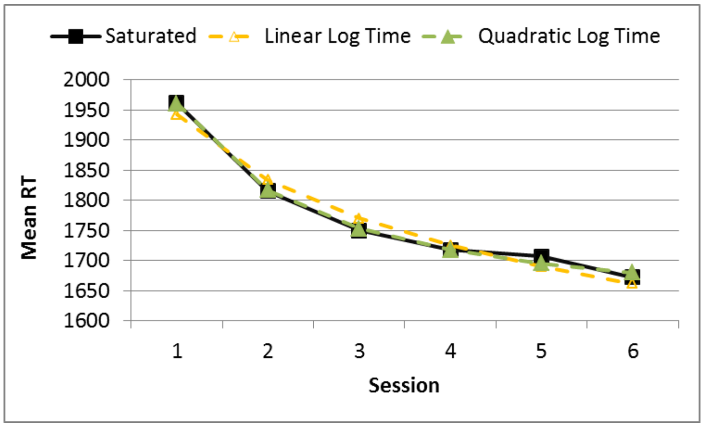
This LRT tells us whether we need the full 2x2 **G** matrix (so it is not helpful).

```
                            Information Criteria
Neg2LogLike    Parms        AIC       AICC       HQIC        BIC       CAIC
     8309.9        4     8317.9     8318.0     8322.1     8328.4     8332.4
```

Is the fixed quadratic log time slope significant? How do we know?

```
                      Solution for Fixed Effects
                                 Standard
Effect              Estimate        Error      DF    t Value    Pr > |t|
Intercept            1960.96      54.2653     104      36.14      <.0001  gamma00
logtime              -240.61      39.4593     502      -6.10      <.0001  gamma10
logtime*logtime      46.9146      19.5288     403       2.40      0.0167  gamma20

                                    Estimates
                                        Standard
Label                        Estimate       Error      DF    t Value    Pr > |t|  g = gamma, logt = logtime
Intercept at Session=1        1960.96     54.2653     104      36.14      <.0001  g00(1)+ g10(logt)+ g20(logt^2)
Intercept at Session=2        1816.74     48.1936     105      37.70      <.0001
Intercept at Session=3        1753.25     45.9683     105      38.14      <.0001
Intercept at Session=4        1717.57     44.6261     101      38.49      <.0001
Intercept at Session=5        1695.25     44.2821     100      38.28      <.0001
Intercept at Session=6        1680.45     44.9706     106      37.37      <.0001
Linear Slope at Session=1     -240.61     39.4593     502      -6.10      <.0001  g10(1) + 2*g20(logt)
Linear Slope at Session=2     -175.57     19.9776     139      -8.79      <.0001
Linear Slope at Session=3     -137.53     20.0322     140      -6.87      <.0001
Linear Slope at Session=4     -110.53     26.5904     338      -4.16      <.0001
Linear Slope at Session=5    -89.5967     33.4381     472      -2.68      0.0076
Linear Slope at Session=6    -72.4917     39.5812     502      -1.83      0.0676
```



The quadratic effect of log time appears to make the predicted line a little more bendy, such that it fits the saturated means slightly (but significantly) better.

```
Calculate pseudo R2 -- variance accounted for by fixed quadratic log time
PsuedoR2 (% Reduction) for CovRandLinLog vs. CovFixQuadLog
    Name         CovParm    Subject     Estimate      StdErr     ZValue     ProbZ     PseudoR2
CovRandLinLog    UN(1,1)    PersonID      274252       41240       6.65    <.0001          .
CovRandLinLog    UN(2,2)    PersonID       23101     4882.26       4.73    <.0001          .
CovRandLinLog    session    PersonID       24120     1697.11      14.21    <.0001          .
CovFixQuadLog    UN(1,1)    PersonID      274453       41239       6.66    <.0001    -0.000733
CovFixQuadLog    UN(2,2)    PersonID       23229     4880.99       4.76    <.0001    -0.005551
CovFixQuadLog    session    PersonID       23839     1679.38      14.20    <.0001     0.011672
```

Model 7d. Random Quadratic Log Time

$$\text{Level 1: } y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{LogTime}_{ti}\right) + \beta_{2i}\left(\text{LogTime}_{ti}\right)^2 + e_{ti}$$

$$\text{Level 2: Intercept: } \beta_{0i} = \gamma_{00} + U_{0i}$$

$$\text{Linear LogTime: } \beta_{1i} = \gamma_{10} + U_{1i}$$

$$\text{Quadratic LogTime: } \beta_{2i} = \gamma_{20} + U_{2i}$$

Predicted intercept at any occasion:

$$= \gamma_{00} + \gamma_{10}(\text{LogTime}_{ti}) + \gamma_{20}(\text{LogTime}_{ti})^2$$

Instantaneous linear time slope at any occasion:

$$= \gamma_{10} + 2\gamma_{20}(\text{LogTime}_{ti})$$

```
TITLE1 "SAS: 7d: Random Quadratic Log Time Model";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime logtime*logtime / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT logtime logtime*logtime / G GCORR V VCORR TYPE=UN SUBJECT=PersonID;
     REPEATED session / R TYPE=VC SUBJECT=PersonID;
     ODS OUTPUT InfoCrit=FitRandQuadLog; * Save for LRT;
RUN;
TITLE1 "Calculate LRT -- does random quadratic log time slope improve model fit?";
%FitTest(FitFewer=FitFixQuadLog, FitMore=FitRandQuadLog); TITLE1;
```

```
display "STATA: 7d: Random Quadratic Log Time Model"
mixed rt c.logtime c.logtime#c.logtime, || personid: logtime logtimesq, variance reml
covariance(unstructured) ///
           residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      estat ic, n(101)                                  // AIC and BIC
      estat recovariance, relevel(personid)             // G matrix
      estat recovariance, relevel(personid) correlation // GCORR matrix
      estat wcorrelation, covariance                    // V matrix
      estat wcorrelation                                // VCORR matrix
      estimates store FitRandQuadLog         // Save for LRT
      lrtest FitRandQuadLog FitFixQuadLog    // Does random quadratic log time slope improve model fit?
```

```
print("R 7d: Random Quadratic Log Time Model -- reports convergence problem")
RandQuadLog = lmer(data=Example6, REML=TRUE,
formula=rt~1+logtime+I(logtime^2)+(1+logtime+I(logtime^2)|PersonID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(RandQuadLog, ddf="Satterthwaite"); llikAIC(RandQuadLog, chkREML=FALSE)
print("Does random quadratic time slope improve model fit?")
ranova(RandQuadLog, reduce.term=TRUE) # Remove random slope and covariance
```

SAS Output:

```
                          Estimated G Matrix
 Row     Effect            PersonID        Col1         Col2         Col3
   1     Intercept              101      285072       -80403        15959
   2     logtime                101      -80403       199200       -86830
   3     logtime*logtime        101       15959       -86830        43025

                     Estimated G Correlation Matrix
Row     Effect             PersonID        Col1         Col2         Col3
  1      Intercept              101      1.0000      -0.3374       0.1441
  2      logtime                101     -0.3374       1.0000      -0.9379
  3      logtime*logtime        101      0.1441      -0.9379       1.0000
```

The **level-2 G matrix** (always **unstructured**) still contains variances for the random intercept (1,1) and random linear log time slope (2,2), as well as their covariance (2,1). We have now added a variance for the random quadratic log time slope (3,3) and its covariances with the random intercept (3,1) and random linear log time slope (3,2) for the same person. **GCORR** provides the corresponding correlations among the random effects.

```
                   Estimated V Matrix for PersonID 101
 Row          Col1         Col2         Col3         Col4         Col5         Col6
   1        302304       237009       216002       204280       197007       192244
   2        237009       253981       226105       213856       201669       189971
   3        216002       226105       236995       209779       198899       187976
   4        204280       213856       209779       219785       194472       186263
   5        197007       201669       198899       194472       206863       184764
   6        192244       189971       187976       186263       184764       200661
```

The marginal **V** matrix now predicts the variances to change in a **quartic pattern (logtime^4)**.

```
              Estimated V Correlation Matrix for PersonID 101
 Row          Col1         Col2         Col3         Col4         Col5         Col6
   1        1.0000       0.8553       0.8070       0.7925       0.7878       0.7805
   2        0.8553       1.0000       0.9216       0.9052       0.8798       0.8415
   3        0.8070       0.9216       1.0000       0.9192       0.8983       0.8620
   4        0.7925       0.9052       0.9192       1.0000       0.9120       0.8869
   5        0.7878       0.8798       0.8983       0.9120       1.0000       0.9069
   6        0.7805       0.8415       0.8620       0.8869       0.9069       1.0000
```

The marginal **VCORR** matrix now predicts that the covariance changes over time in a more complex time-dependent pattern than for random linear log time.

```
                Covariance Parameter Estimates
                                       Standard       Z
Cov Parm    Subject      Estimate       Error     Value      Pr Z
UN(1,1)     PersonID       285072       42684      6.68     <.0001   Level-2 random intercept variance of U0i
UN(2,1)     PersonID       -80403       31879     -2.52     0.0117   Level-2 random intercept-linear slope covariance
UN(2,2)     PersonID       199200       42606      4.68     <.0001   Level-2 random linear log time slope variance of U1i
UN(3,1)     PersonID        15959       14893      1.07     0.2839   Level-2 random intercept-quadratic slope covariance
UN(3,2)     PersonID       -86830       20318     -4.27     <.0001   Level-2 random linear-quadratic slope covariance
UN(3,3)     PersonID        43025       10274      4.19     <.0001   Level-2 random quadratic time slope variance of U2i
session     PersonID        17232     1399.97     12.31     <.0001   Level-1 residual variance of eti

  Null Model Likelihood Ratio Test
     DF     Chi-Square      Pr > ChiSq
      6         920.29          <.0001
```

This LRT tells us whether we need the full 3x3 **G** matrix (so it is not helpful).

```
                          Information Criteria
Neg2LogLike      Parms        AIC        AICC        HQIC         BIC        CAIC
     8266.1          7     8280.1      8280.3      8287.5      8298.4      8305.4
```

```
                      Solution for Fixed Effects
                               Standard
Effect             Estimate       Error      DF    t Value    Pr > |t|
Intercept           1960.96     54.6521     100      35.88      <.0001   gamma00
logtime             -240.61     54.1434     100      -4.44      <.0001   gamma10
logtime*logtime     46.9146     26.4888     100       1.77      0.0796   gamma20
```

```
Calculate LRT -- does random quadratic log time slope improve model fit?
Likelihood Ratio Test for FitFixQuadLog vs. FitRandQuadLog
                  Neg2Log
     Name          Like      Parms       AIC        BIC      DevDiff     DFdiff       Pvalue
FitFixQuadLog     8309.9       4       8317.9     8328.4        .           .              .
FitRandQuadLog    8266.1       7       8280.1     8298.4     43.7825        3       1.6786E-9
```

Is the random quadratic time slope significant? How do we know?

<u>95% Random Effect Confidence Intervals that describe the *predicted* range of *individual* random effects:</u>

Random Effect 95% CI = fixed effect ± $\left(1.96*\sqrt{\text{Random Variance}}\right)$

Intercept 95% CI = $\gamma_{00} \pm \left(1.96*\sqrt{\tau^2_{U_0}}\right) \rightarrow 1,960.96 \pm \left(1.96*\sqrt{285,072}\right) = 914$ to 3,007

Linear LogTime Slope 95% CI = $\gamma_{10} \pm \left(1.96*\sqrt{\tau^2_{U_1}}\right) \rightarrow -240.61 \pm \left(1.96*\sqrt{199,200}\right) = -115$ to 634

Quadratic LogTime Slope 95% CI = $\gamma_{20} \pm \left(1.96*\sqrt{\tau^2_{U_2}}\right) \rightarrow 46.91 \pm \left(1.96*\sqrt{43025}\right) = -360$ to 453

Is it a problem that the CIs for the linear and quadratic log time slopes overlap 0? What does this mean?

The absolute fit of the quadratic log time model for the means can be tested by mimicking a saturated means model using the *same random quadratic log time slopes* (i.e., holding the variance model constant):

```
TITLE1 "SAS: Test Absolute Fit of Quadratic Log Time Means (Using Random Quadratic Log Time Variance)";
TITLE2 "Add 3 session dummy codes to saturate the means model";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime logtime*logtime s1 s2 s3 / SOLUTION DDFM=Satterthwaite;
     RANDOM INTERCEPT logtime logtime*logtime / G GCORR V VCORR TYPE=UN SUBJECT=PersonID;
     REPEATED session / R TYPE=VC SUBJECT=PersonID;
     CONTRAST "Does fixed quadratic log time reproduce saturated means?" s1 1, s2 1, s3 1 / CHISQ;
RUN; TITLE1; TITLE2;
```

```
display "STATA: Test Absolute Fit of Quadratic Log Time Means (Using Random Quadratic Log Time Variance)"
display "Add 3 session dummy codes to saturate the means model"
mixed rt c.logtime c.logtime#c.logtime c.s1 c.s2 c.s3, ///
           || personid: logtime logtimesq, variance reml covariance(unstructured) ///
           residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      test (c.s1=0)(c.s2=0)(c.s3=0), small  // Does fixed quadratic log time reproduce saturated means?
```

```
print("R: Test Absolute Fit of Quadratic Log Time Means (Using Random Quadratic Log Time Variance)")
print("Add 3 session dummy codes to second piece to saturate the means model")
QuadLogMean = lmer(data=Example6, REML=TRUE,
                   formula=rt~1+logtime+I(logtime^2)+s1+s2+s3+(1+logtime+I(logtime^2)|PersonID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
summary(QuadLogMean, ddf="Satterthwaite"); llikAIC(QuadLogMean, chkREML=FALSE)
print("Does fixed two-piece slope reproduce saturated means?")
contestMD(QuadLogMean, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0),c(0,0,0,0,1,0),c(0,0,0,0,0,1)))
```

SAS Output (relevant tables only):

Because there are now **6 fixed effects for the 6 means**, this model is equivalent to **saturated means** (even if the linear and quadratic fixed slopes are largely uninterpretable). In an SEM context, this model would be specified by letting three of the observed occasions' intercepts be estimated (as discrepancies).

The multivariate Wald test using CONTRAST indicates that the 3 extra session contrasts did not improve model fit (which is good news here).

```
                         Solution for Fixed Effects
                                  Standard
Effect             Estimate          Error      DF    t Value    Pr > |t|
Intercept            988.14         986.36     302       1.00      0.3172
logtime             1020.69        1254.21     301       0.81      0.4164
logtime*logtime     -356.60         395.51     303      -0.90      0.3680
s1                   973.75         985.01     300       0.99      0.3237
s2                   290.87         306.89     300       0.95      0.3440
s3                  70.9505        87.1698     300       0.81      0.4163

                                                                 Contrasts
                                                                   Num    Den
Label                                                               DF     DF    Chi-Square   F Value    Pr > ChiSq    Pr > F
Does fixed quadratic log time reproduce saturated means?             3    300          1.31      0.44        0.7257    0.7259
```

The absolute fit of the random quadratic log time model for the variance can be tested against a UN variance model using the *same fixed quadratic log time slopes* (i.e., holding the model for the means constant):

```
TITLE1 "SAS: Test Absolute Fit of Quadratic Log Time Variance (Using Fixed Quadratic Log Time Means)";
TITLE2 "Change to Unstructured R matrix as variance model answer key";
PROC MIXED DATA=work.Example6 COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
     CLASS PersonID session;
     MODEL rt = logtime logtime*logtime / SOLUTION DDFM=Satterthwaite;
     REPEATED session / R RCORR TYPE=UN SUBJECT=PersonID;
     ODS OUTPUT InfoCrit=FitFixQuadLogUN; * Save for LRT;
RUN; TITLE2;
TITLE1 "Calculate LRT -- does random quadratic log time reproduce UN variance model?";
%FitTest(FitFewer=FitRandQuadLog, FitMore=FitFixQuadLogUN);

display "STATA: Test Absolute Fit of Quadratic Log Time Variance (Using Fixed Quadratic Log Time Means)"
display "Change to Unstructured R matrix as variance model answer key"
mixed rt c.logtime c.logtime#c.logtime, || personid: , noconstant variance reml ///
           residuals(unstructured,t(session)) dfmethod(satterthwaite) dftable(pvalue)
      estimates store FitFixQuadLogUN       // Save for LRT
      lrtest FitFixQuadLogUN FitRandQuadLog // Does random quadratic log time reproduce UN variance?
print("R: Test Absolute Fit of Quadratic Log Time Variance (Using Fixed Quadratic Log Time Means)")
print("Change to Unstructured R matrix as variance model answer key in GLS")
QuadLogUN = gls(data=Example6, method="REML", model=rt~1+logtime+I(logtime^2),
                correlation=corSymm(form=~as.numeric(session)|PersonID),
                weights=varIdent(form=~1|session))
print("Have to re-run random quadratic log time model using LME to get LRT")
RandQuadLoglme = lme(data=Example6, method="REML", rt~1+logtime+I(logtime^2),
                     random=~1+logtime+I(logtime^2)|PersonID)
anova(QuadLogUN,RandQuadLoglme) # anova does LRT using LME versions
```

SAS Output (relevant table only):

```
Calculate LRT -- does random quadratic log time reproduce UN variance model?
Likelihood Ratio Test for FitRandQuadLog vs. FitRandUN
                  Neg2Log
Name               Like      Parms       AIC        BIC     DevDiff    DFdiff     Pvalue
FitRandQuadLog    8266.1        7     8280.1     8298.4        .          .          .
FitRandUN         8254.1       21     8296.1     8351.1     11.9846      14      0.60754
```

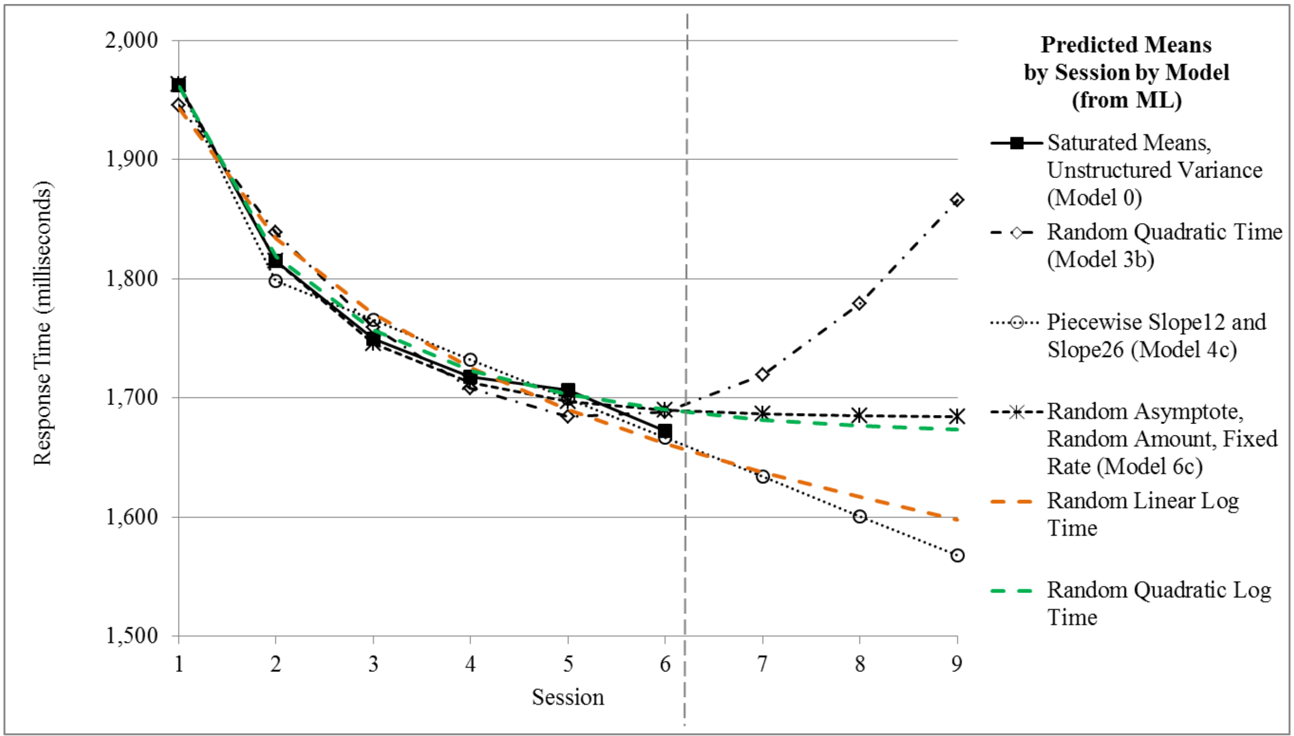
What does this nonsignificant LRT result indicate about our random quadratic log time model?

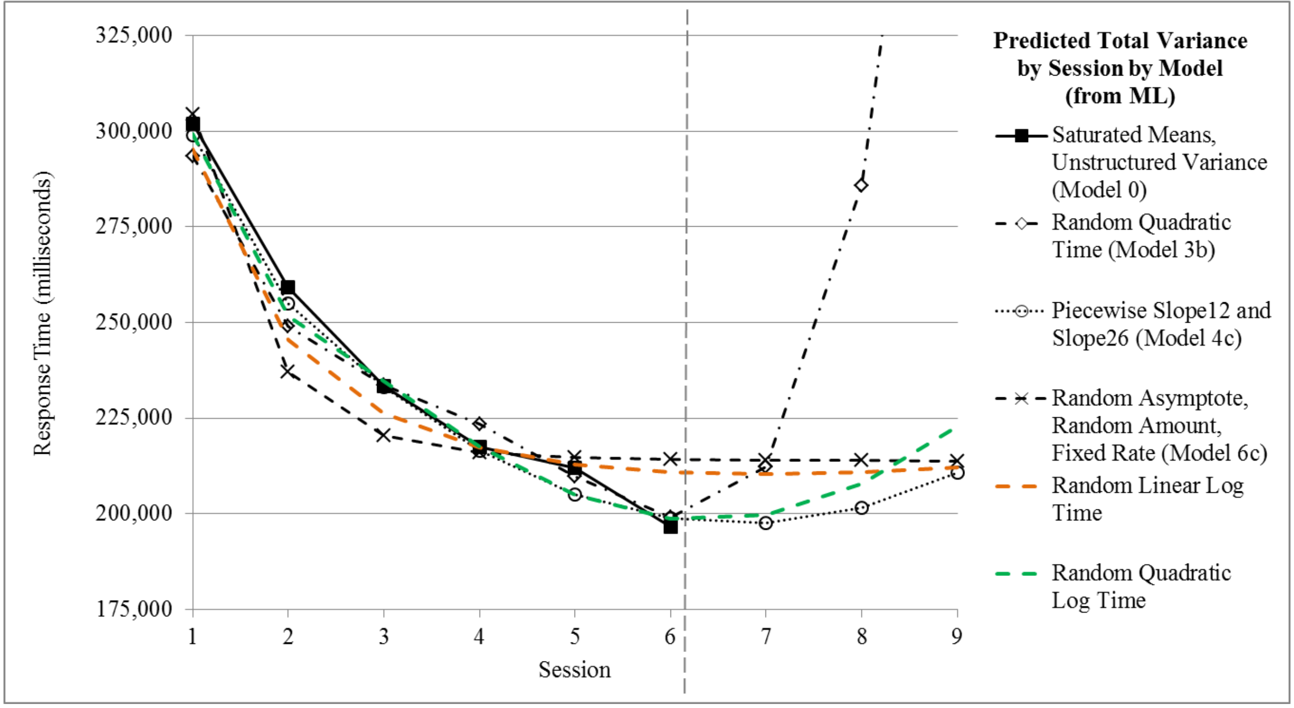
At last, the grand finale comparison: Tests of absolute fit of each side of the model for the "best" version of each model as presented in Examples 6a–6d, as well as tests of absolute fit of both sides of the model combined (see example syntax and output online for the latter)....

Summary of Tests of Absolute Model Fit For Each Side of the Model		Means Side Test			Variance Side Test		
		Chi-Square	Test DF	p-value	Chi-Square	Test DF	p-value
3b	Polynomial: Random Quadratic Time	9.07	3	0.030	35.76	14	0.001
4c	**Piecewise: Random Slope12, Random Slope26**	4.74	3	**0.195**	15.77	14	**0.327**
6c	Negative Exponential: Random Asymptote, Random Amount, Fixed Rate	0.87	3	**0.404**	46.50	17	0.000
7c	**Random Linear Log Time**	**6.69**	**4**	**0.155**	58.01	17	0.000
7d	**Random Quadratic Log Time**	**1.31**	**3**	**0.726**	**11.98**	**14**	**0.608**

	Model	Total # Parameters	ML -2LL	ML AIC	ML BIC
7c	Random Linear Log Time	6	8340.49	8352.5	8368.2
6c	Negative Exponential: Random Asymptote, Random Amount, Fixed Rate	7	8327.30	8341.3	8359.6
3b	Polynomial: Random Quadratic Time	10	8321.77	8341.8	8367.9
4c	Piecewise: Random Slope12, Random Slope26	10	8298.95	8318.9	8345.1
7d	**Random Quadratic Log Time**	10	8291.51	**8311.5**	**8337.7**
0	Least Parsimonious Baseline: Saturated Means, Unstructured Variance	27	**8278.09**	8332.1	8402.7
0	Absolute Fit worse than Saturated Means, Unstructured Variance Model?	df	Chi-Square	p-value	
7c	Random Linear Log Time	21	62.41	.000	
6c	Negative Exponential: Random Asymptote, Random Amount, Fixed Rate	20	49.21	.000	
3b	Polynomial: Random Quadratic Time	17	43.68	.000	
4c	**Piecewise: Random Slope12, Random Slope26**	**17**	**20.86**	**.233**	
7d	**Random Quadratic Log Time**	**17**	**13.42**	**.708**	

Predicted means/variances all in ML for comparability





I think random quadratic log time wins—and that's going in chapter 6 when I revise my book!