Two-Level Models for Cross-Classified Data

- Topics:
  - Cross-sectional cross-classification (time-invariant groups)
  - Longitudinal cross-classification (time-varying groups)
  - No smushing allowed in cross-classified models, either

More Complex Multilevel Designs

- Multilevel models are specified based on the relevant dimensions by which observations differ each other, and how the units are organized

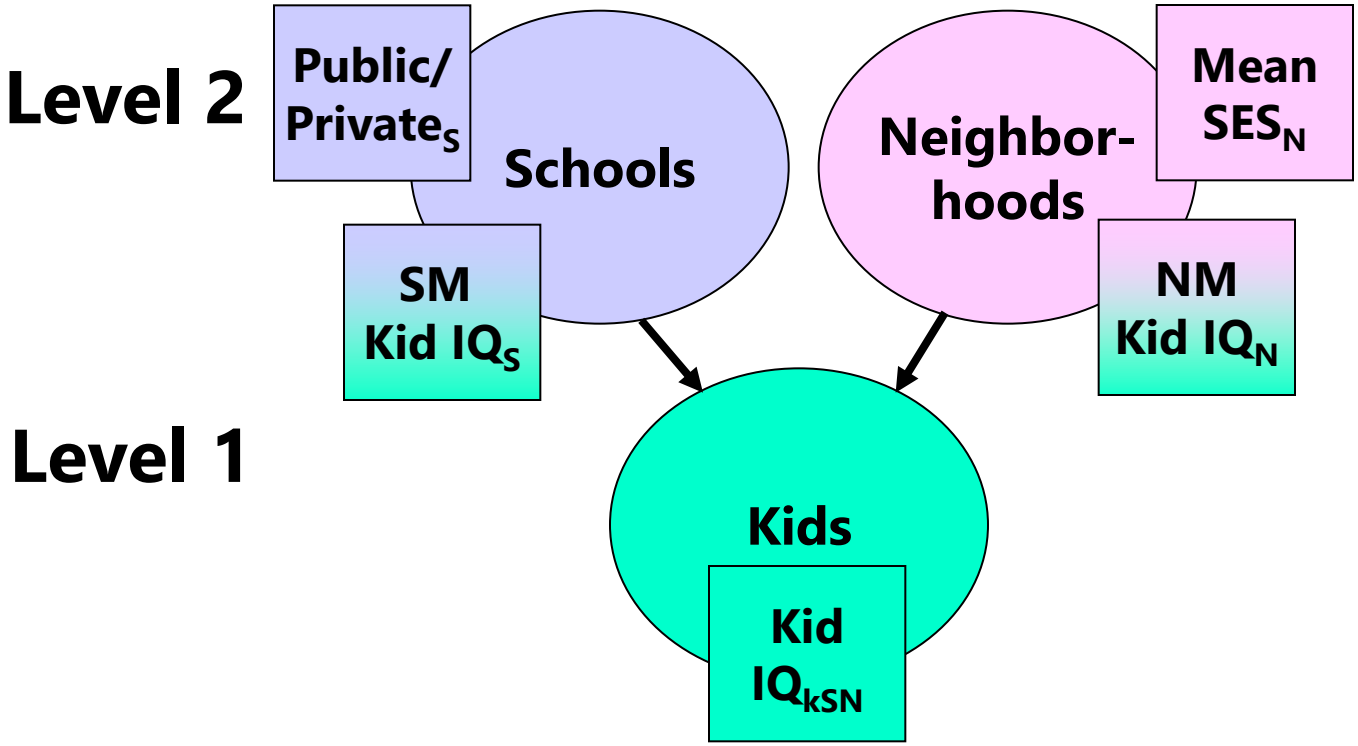
- Two-level models have at least two piles of variance, in which level-1 units are nested within level-2 units:
  - Longitudinal Data: Occasions nested within Persons
  - Clustered Data: Students nested within Teachers

- Three-level models have at least three piles of variance, in which level-2 units are nested within level-3 units (stay tuned):
  - Longitudinal Data: Occasions nested within Persons within Families
  - Clustered Data: Students nested within Teachers within Schools

- In other designs, multiple sources of systematic variation may be present, but the sampling may be crossed instead…
  - Models with crossed random effects are known as "cross-classified" models in the context of clustered data
  - Here are a few examples on when this might happen…

Kids, Schools, and Neighborhoods

- Kids are nested within schools AND within neighborhoods
- Not all kids from same neighborhood live in same school, so schools and neighborhoods are crossed at level 2
- Can include predictors for each source of variation



Kids, Schools, and Neighborhoods

$y_{ksn} = \gamma_{000}$ → *fixed intercept (all x's = 0)*

$+ \gamma_{010}(\text{Private}_s) + \gamma_{020}(\text{SMIQ}_s)$ → *school effects*

$+ \gamma_{001}(\text{SES}_n) + \gamma_{002}(\text{NMIQ}_n)$ → *neighborhood effects*

$+ \gamma_{100}(\text{KidIQ}_{ksn})$ → *kid effects*

$+ U_{0s0}$ → *random intercept of school*

$+ U_{00n}$ → *random intercept of neighborhood*

$+ e_{ksn}$ → *residual kid-to-kid variation*

Because cross-classified models have two types of level-2 units, it can be much easier to write a single, composite equation

Time (t), Students (s), and Classes (c)

- Students are nested within Classes at each occasion…
- But if students move into different classes across time…
  - Time at level 1 is nested within Student AND within Classes
  - Student is crossed with Class at level 2
- How to model a time-varying random classroom effect?
  - This is the basis of so-called "value-added models"

- (At least) Two options via fixed or random effects:

  - Acute effect: Effect for class operates only when kids are in the class
    - e.g., Class effect ← teacher bias
    - Once a student is out of the class, class effect is no longer present

  - Transfer effect: Effect for class operates now and in the future…
    - e.g., Class effect ← differential learning
    - Effect stays with the student in the future (i.e., a "layered" value-added model)

Time (t), Students (s), and Classes (c)

- Custom-built intercepts for time-varying effects of classes
  - An intercept is usually a column of 1's, but ours will be 0's and 1's to serve as switches that turn on/off class effects

				Per-Year Class ID (−99 = missing)			Intercepts for Acute Effects			Intercepts for Transfer Effects		
Student ID	Class ID	Grade	Year	Year 0 Class	Year 1 Class	Year 2 Class	Year 0 Intercept	Year 1 Intercept	Year 2 Intercept	Year 0 Effect	Year 1 Effect	Year 2 Effect
101	1	3	0	1	−99	43	1	0	0	1	0	0
101	−99	4	1	1	−99	43	0	0	0	0	0	0
101	43	5	2	1	−99	43	0	0	1	1	0	1
102	3	3	0	3	21	42	1	0	0	1	0	0
102	21	4	1	3	21	42	0	1	0	1	1	0
102	42	5	2	3	21	42	0	0	1	1	1	1

Time (t), Students (s), and Classes (c)

- Hoffman (2015) Equation 11.3: fixed effects model for class as a categorical time-varying predictor:
  - Allows for control of classes only….

$$\begin{aligned}\text{Effort}_{tsc} &= \gamma_{000} + \gamma_{100}\left(\text{Year01}_{tsc}\right) + \gamma_{200}\left(\text{Year12}_{tsc}\right) + \text{U}_{0s0} + \text{e}_{tsc} \\ &+ \gamma^{0}_{001}\left(\text{Class1}_{c}\right)\left(\text{Int0}_{tsc}\right) + \gamma^{0}_{002}\left(\text{Class2}_{c}\right)\left(\text{Int0}_{tsc}\right)\cdots + \gamma^{0}_{00C}\left(\text{ClassC}_{c}\right)\left(\text{Int0}_{tsc}\right) \\ &+ \gamma^{1}_{001}\left(\text{Class1}_{c}\right)\left(\text{Int1}_{tsc}\right) + \gamma^{1}_{002}\left(\text{Class2}_{c}\right)\left(\text{Int1}_{tsc}\right)\cdots + \gamma^{1}_{00C}\left(\text{ClassC}_{c}\right)\left(\text{Int1}_{tsc}\right) \\ &+ \gamma^{2}_{001}\left(\text{Class1}_{c}\right)\left(\text{Int2}_{tsc}\right) + \gamma^{2}_{002}\left(\text{Class2}_{c}\right)\left(\text{Int2}_{tsc}\right)\cdots + \gamma^{2}_{00C}\left(\text{ClassC}_{c}\right)\left(\text{Int2}_{tsc}\right)\end{aligned}$$

- Hoffman (2015) Equation 11.4: class as a random effects crossed with students at level 2:
  - Controls and models class-related variance so it can be predicted

$$\begin{aligned}\text{Effort}_{tsc} &= \gamma_{000} + \gamma_{100}\left(\text{Year01}_{tsc}\right) + \gamma_{200}\left(\text{Year12}_{tsc}\right) + \text{U}_{0s0} + \text{e}_{tsc} \\ &+ \text{U}^{0}_{00c}\left(\text{Int0}_{tsc}\right) + \text{U}^{1}_{00c}\left(\text{Int1}_{tsc}\right) + \text{U}^{2}_{00c}\left(\text{Int2}_{tsc}\right)\end{aligned}$$

More on Cross-Classified Models

- In crossed models, lower-level predictors can have random slopes of over higher levels AND random slopes of the other crossed factor at the same level
  - Example: Kids, Schools, and Neighborhoods (data permitting)
    - Kid effects could vary over schools AND/OR neighborhoods
    - School effects could vary over neighborhoods (both level 2)
    - Neighborhood effects could vary over schools (both level 2)

- Concerns about smushing still apply over both level-2's
  - Separate contextual effects of kid predictors for schools and neighborhoods (e.g., after controlling for how smart you are, it matters incrementally whether you go to a smart school AND if you live in a neighborhood with smart kids)