Time-Varying (TV) Predictors in Longitudinal Models of Within-Person Change

- Topics:
  - Review of TV predictors of WP fluctuation
  - Multivariate relations of change – 3 example uses:
    - Distinguish BP and WP sources of variance and their relations (including factor of curves models for intercepts and change)
    - Examine auto-regressive and cross-lagged WP effects
    - Longitudinal mediation of "change"

3 Kinds of Fixed Slopes for TV Predictors

- **Is there a Level-1 Within-Person (WP) slope?**
  - When you have a higher x_{ti} predictor value than usual (*at this occasion*), do you also have a higher (or lower) y_{ti} outcome value than usual (*at same or later occasion*)?
  - If so, the **level-1 within-person *part* of the TV predictor** will reduce the level-1 residual variance (σ_e^2) of the TV outcome

- **Is there a Level-2 Between-Person (BP) slope?**
  - Do people with higher x_{ti} predictor values than other people (*on average over time*) also have higher (or lower) y_{ti} outcomes than other people (*on average over time*)?
  - If so, the **level-2 between-person *part* of the TV predictor** will reduce level-2 random intercept variance ($\tau_{U_0}^2$) of the TV outcome

- **Is there a Level-2 Contextual slope: Do the L2 BP and L1 WP slopes differ?**
  - After controlling for the actual value of TV predictor at that occasion, is there still **an incremental contribution** from the **level-2 between-person *part* of the TV predictor** (i.e., does one's general tendency matter beyond current TVP value)?
  - Equivalently, the **Level-2 Contextual slope = L2 BP slope − L1 WP slope**, so the Level-2 Contextual slope directly tests **if a smushed slope is ok (pry not!)**

3 Options to Prevent Smushed Slopes

- Within Univariate MLM framework (predict only one column):

  1. **Person-mean-centering**: manually carve up TV predictor into its level-specific parts using observed variables (1 predictor per level)
     - More generally, this is "**variable-centering**" because you are **subtracting a variable** (e.g., the cluster/group/person mean or person baseline value)
     - Will always yield **Level-1 Within slopes** and **Level-2 Between slopes**!

  2. **Grand-mean-centering**: do NOT carve up TV predictor into its level-specific parts, but add level-2 mean to distinguish level-specific slopes
     - More generally, this is "**constant-centering**" because you are **subtracting a constant** but still keeping all levels of variance in level-1 TV predictor
     - Choice of constant is irrelevant (changes where 0 is, not what variance it has)
     - Will yield **Level-1 Within slopes** and **Level-2 Contextual slopes (with L2 mean)**

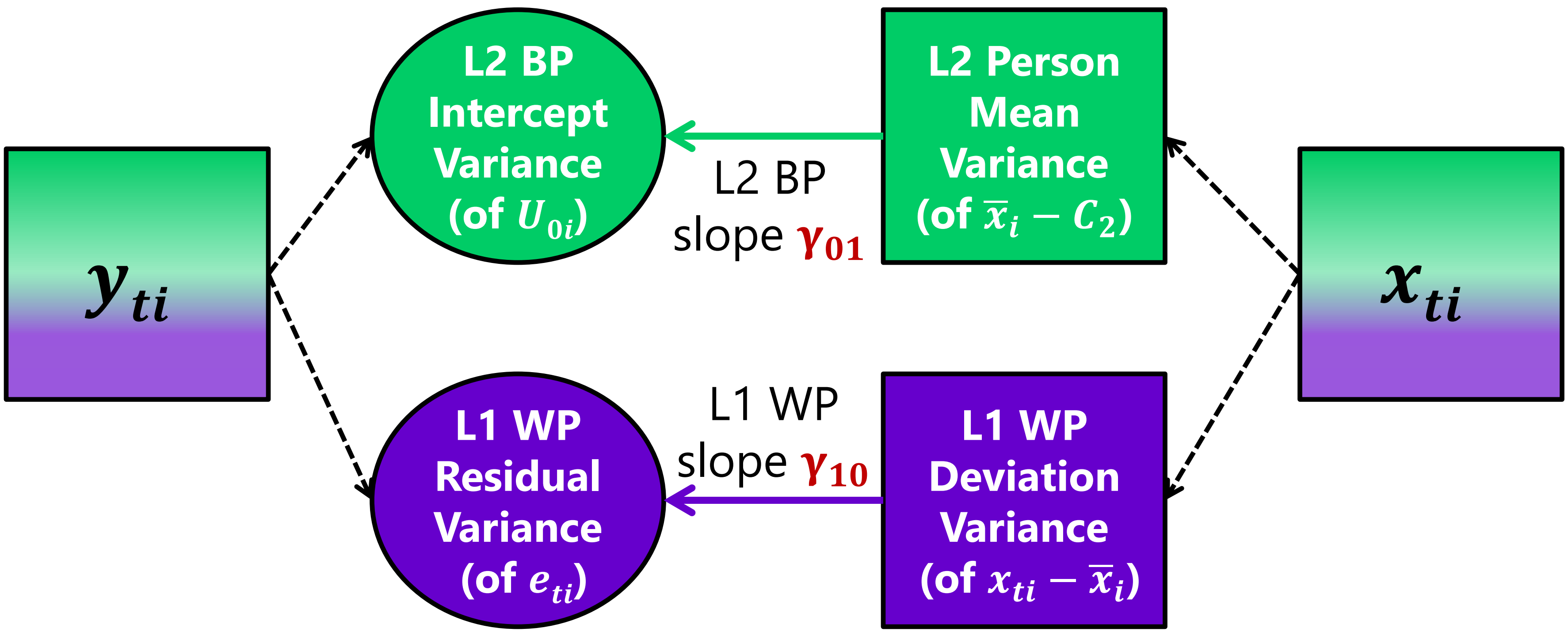
- Within Multivariate MLM framework (via M-SEM or SEM):

  3. **Latent-centering**: Treat the TV predictor as another outcome → let the model carve it up into **level-specific latent variables**
     - Best in theory, but the type of Level-2 slope provided (**Between** or **Contextual**) depends on type of model syntax (and the estimator in Mplus)! (Hoffman, 2019)

Univariate MLM: Variable-Centering*

Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

Manual partitioning of level-1 x_{ti} predictor into level-specific **observed variables**

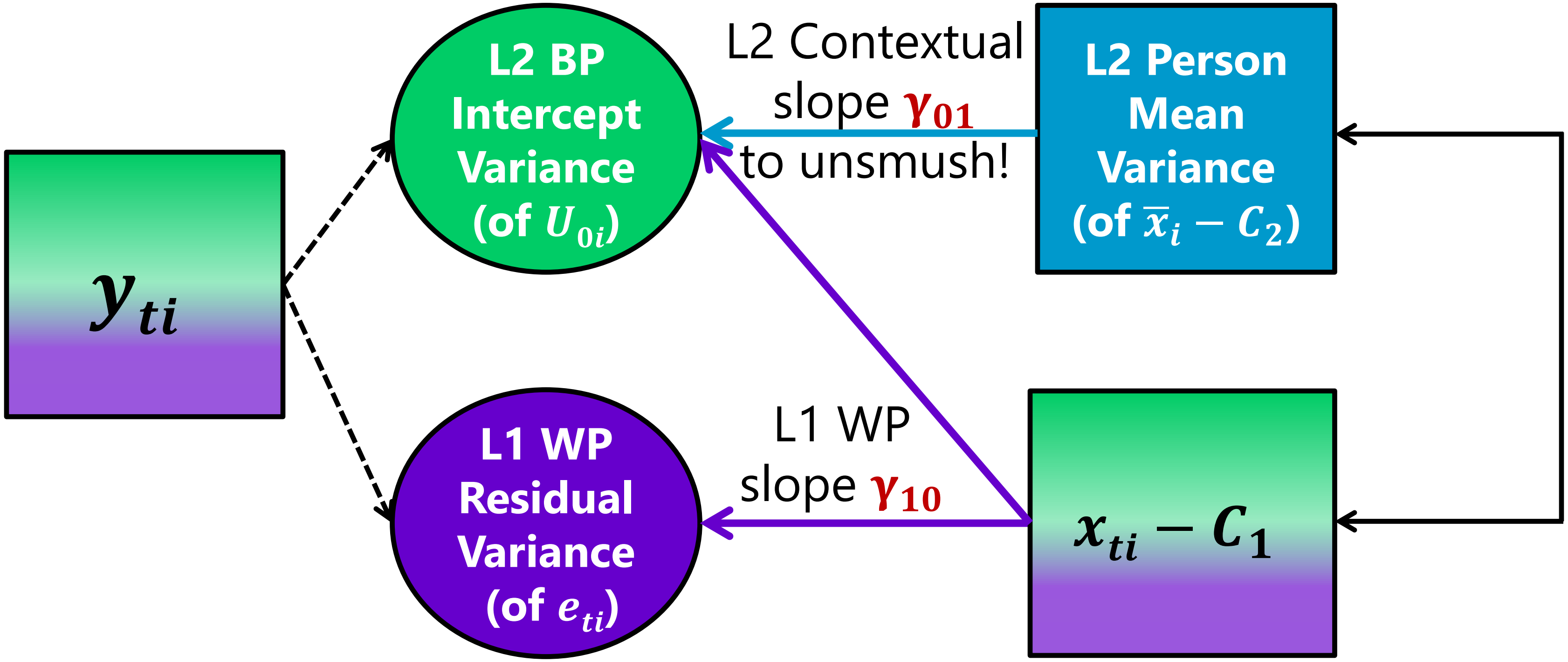


* Known as "person-mean-centering" more generally directly analogous to cluster/group-mean-centering in multilevel models for clustered data)

Univariate: Constant-Centering WITH Level-2 Predictor → OK now!

Model-based partitioning of y_{ti} outcome into level-specific **latent variables**

Level-1 x_{ti} is still NOT partitioned, but person mean $\bar{x}_i - C_2$ is added to allow an **incremental L2 effect**



L2 BP slope = L1 WP slope + Level-2 Contextual slope

Because original x_{ti} still has L2 BP variance, it still carries *some* of the L2 BP effect…

Preventing Smushed (BP=WP) Slopes

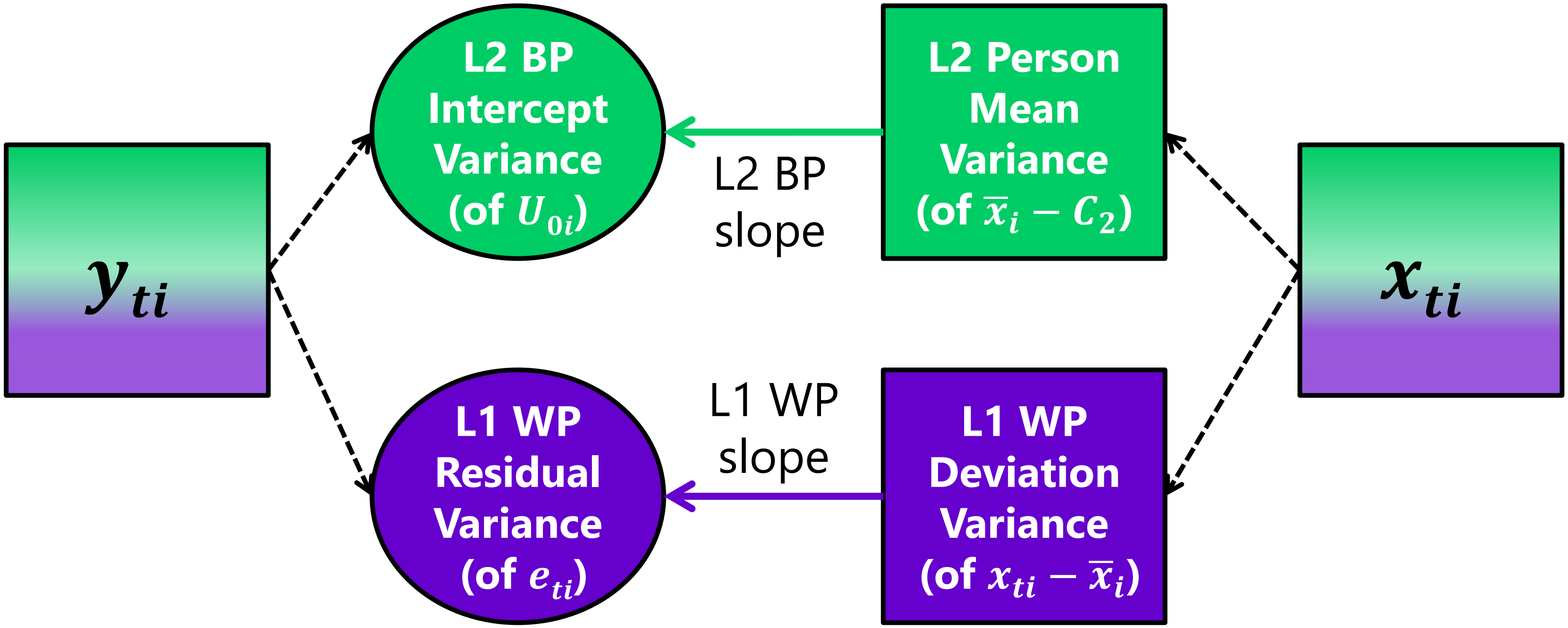
- **Fixed side: 2 strategies to prevent smushed slopes**
  - If using variable-centered (P-MC) L1 TVP ($\mathbf{WPx_{ti}}$), it can only have a **L1 WP slope**, and its L2 $\mathbf{PMx_i}$ can only have a **L2 BP slope** (so no problem)
  - If using constant-C L1 TVP ($\mathbf{TVx_{ti}}$), its L1 slope will be smushed (BP=WP) if you don't add its L2 $\mathbf{PMx_i}$ to allow a **L2 contextual slope = BP − WP**

- **Random side: Only 1 strategy is likely possible!**
  (see Rights & Sterba, MBR 2023, for details)
  - If using variable-centered (P-MC) L1 TVP ($\mathbf{WPx_{ti}}$), its L2 random slope variance **only** captures L2 BP differences in its L1 WP slope (so no problem)
    - Creates a pattern of quadratic heterogeneity of variance **across** $\mathbf{WPx_{ti}}$ **ONLY**
  - If using constant-C L1 TVP ($\mathbf{TVx_{ti}}$), its L2 random slope variance **also** creates **intercept heterogeneity of variance** (beyond BP diffs in L1 WP slope)
    - Enforces **SAME** pattern of quadratic heterogeneity of variance across **L1** $\mathbf{WPx_{ti}}$ and **L2** $\mathbf{PMx_i}$
  - If using $\mathbf{TVx_{ti}}$, you need a "contextual" random slope to allow a different pattern of variance heterogeneity across $\mathbf{PMx_i}$ than $\mathbf{WPx_{ti}}$ (for BP − WP)
    - Requires a L2 BP random "slope ?" variance for **L2** $\mathbf{PMx_i}$ – good luck estimating it!

Univariate MLM: Variable-Centering

Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

Manual partitioning of level-1 x_{ti} predictor into level-specific **observed variables**

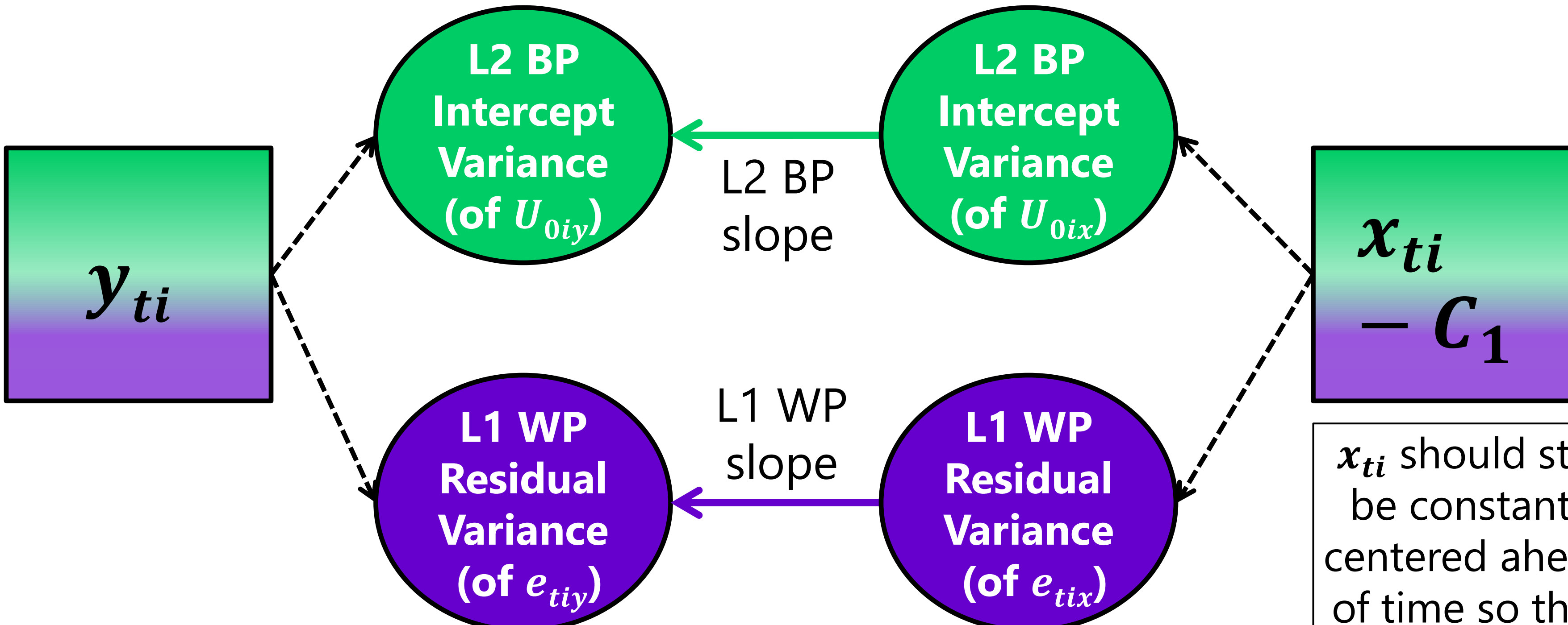


Why not let the model estimate variance components for x_{ti}, too? We can do so using multivariate MLM (via SEM or M-SEM).

Multivariate MLM: Latent-Centering

Model-based partitioning of level-1 y_{ti} outcome into level-specific **latent variables**

Model-based partitioning of level-1 x_{ti} predictor (= outcome now) into level-specific **latent variables**



x_{ti} should still be constant-centered ahead of time so that 0 is meaningful

Univariate MLM software can be tricked into multivariate MLM if relationships of x_{ti} and y_{ti} at each level are phrased as covariances, but not if you want directed slopes (or moderators thereof)

Time-Varying Predictors that Change **Need** Multivariate MLMs (via SEM or M-SEM)

- Univariate MLMs for time-varying predictors can still be reasonable if a time-varying predictor has only **fixed effect(s) of time**
  - Adding fixed time slopes → other "unique" effects controlling for time

- But if a TV predictor has ***individual differences* in change**, univariate MLM cannot fully separate its BP and WP variance
  - There are then **at least two "kinds" of BP variance** to be concerned with: in **intercept and change** (and possibly more kinds for nonlinear change)

If people change differently over time, then BP rank orders change over time, too (Hoffman, 2015, ch. 9)

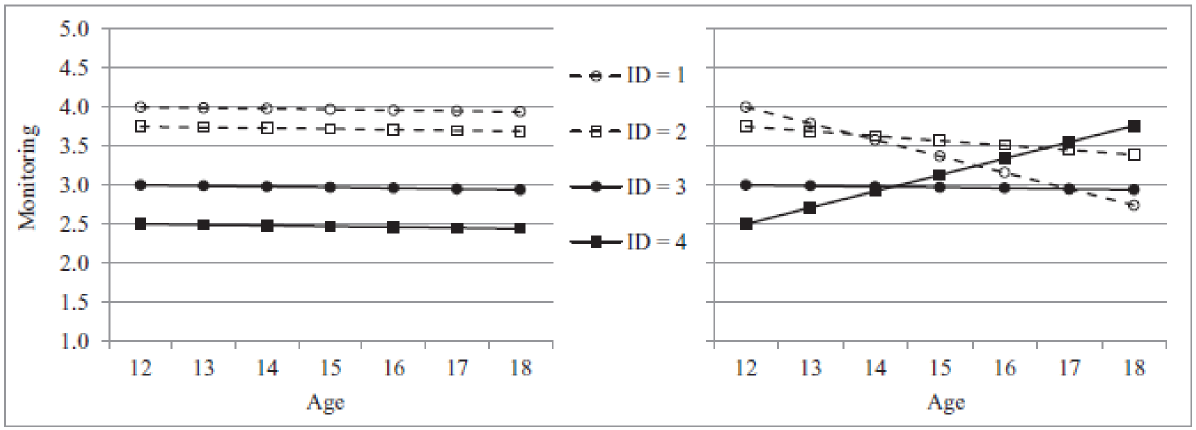
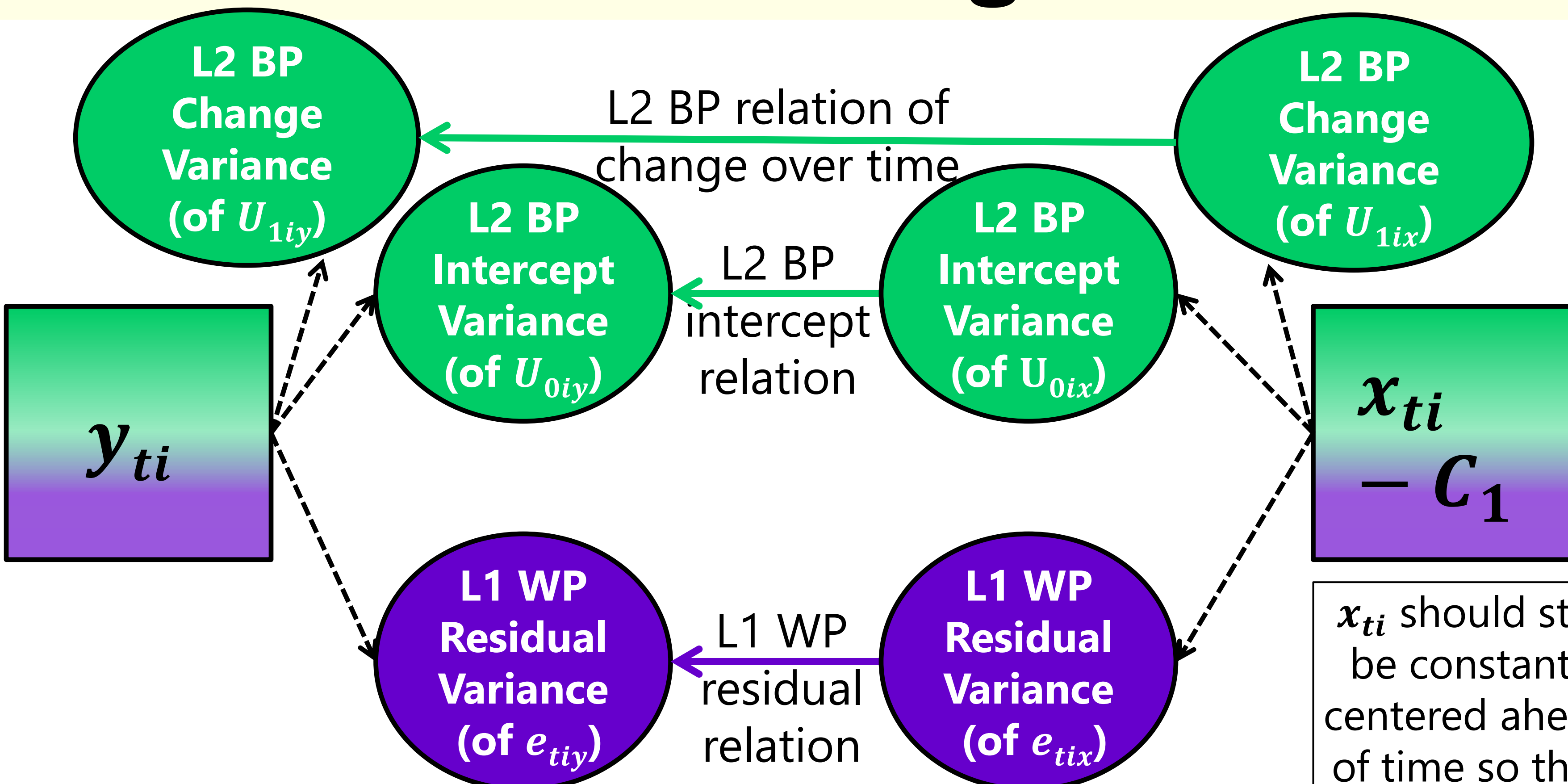


Figure 9.1 Individual trajectories for monitoring across age given a fixed slope (left panel) or a random slope (right panel).

Multivariate Modeling of Time-Varying Predictors that ***Change*** over Time



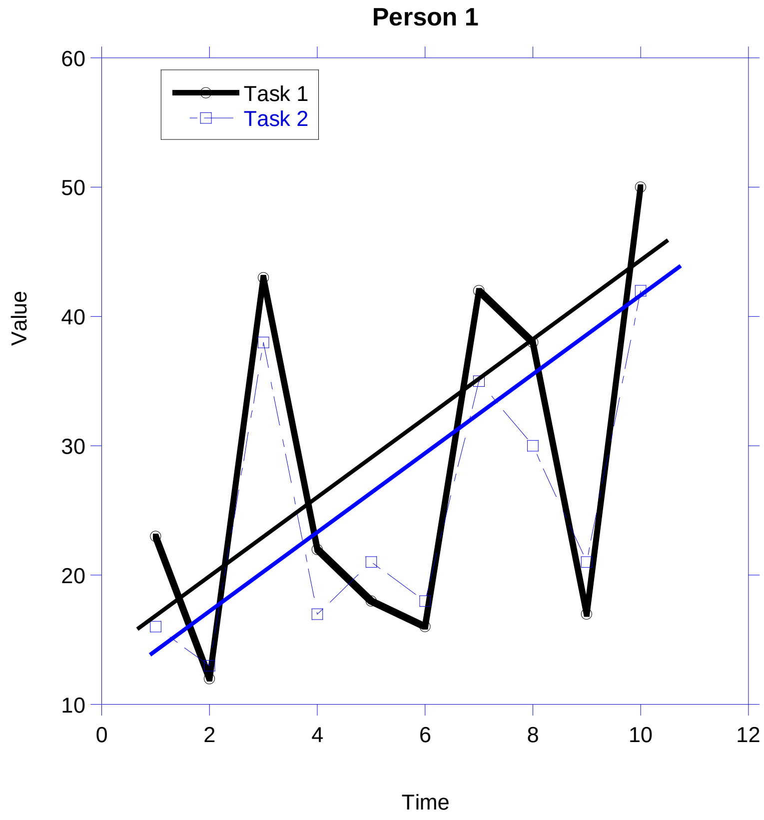
x_{ti} should still be constant-centered ahead of time so that 0 is meaningful

Univariate MLM software can be tricked into multivariate MLM if relationships of x_{ti} and y_{ti} at each level are phrased as covariances, but not if you want directed slopes (or moderators thereof)

Multivariate Relations of Models of Change

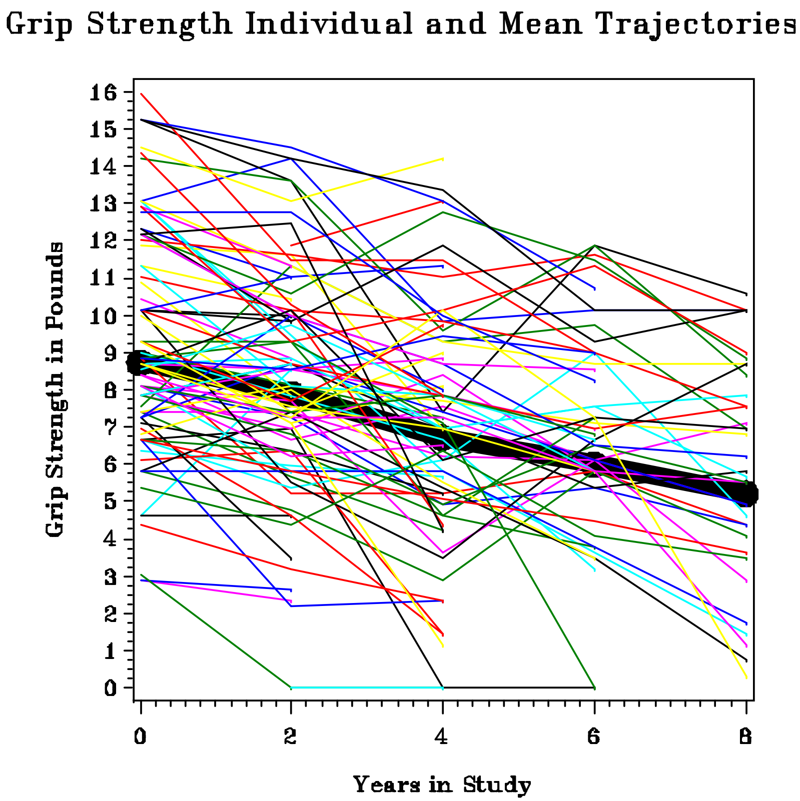
- Relations among **random effects for individual differences**
  - **Intercepts**: Are the predicted means (at time = 0) of x_{ti} and y_{ti} related?
  - **Time Slopes**: Are the predicted rates of change of x_{ti} and y_{ti} related?
  - These are **Between-Person** relations → relative to other people

- Relations among **residuals for within-person variation**: If I am higher than my predicted trajectory on x_{ti}, am I also likely higher than predicted on y_{ti} at…
  - Same occasion (*concurrent* relation)?
  - Next occasion (*lagged* relation)?
    - Btw, fitting same lagged relation across time only makes sense for equal-interval balanced longitudinal data

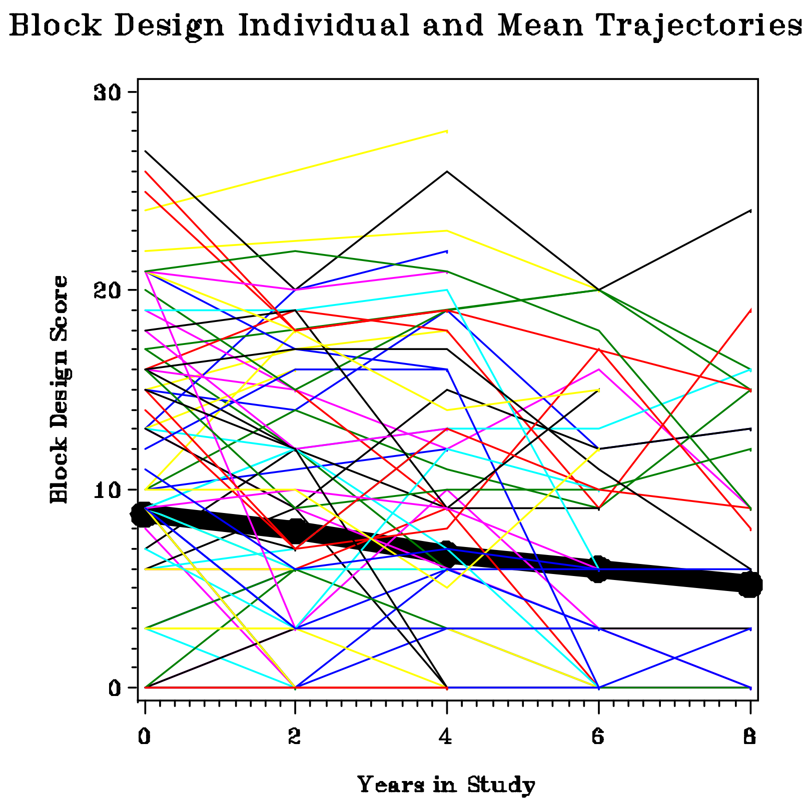


Individual Relations of Functional and Cognitive Change in Old Age

Functional Change

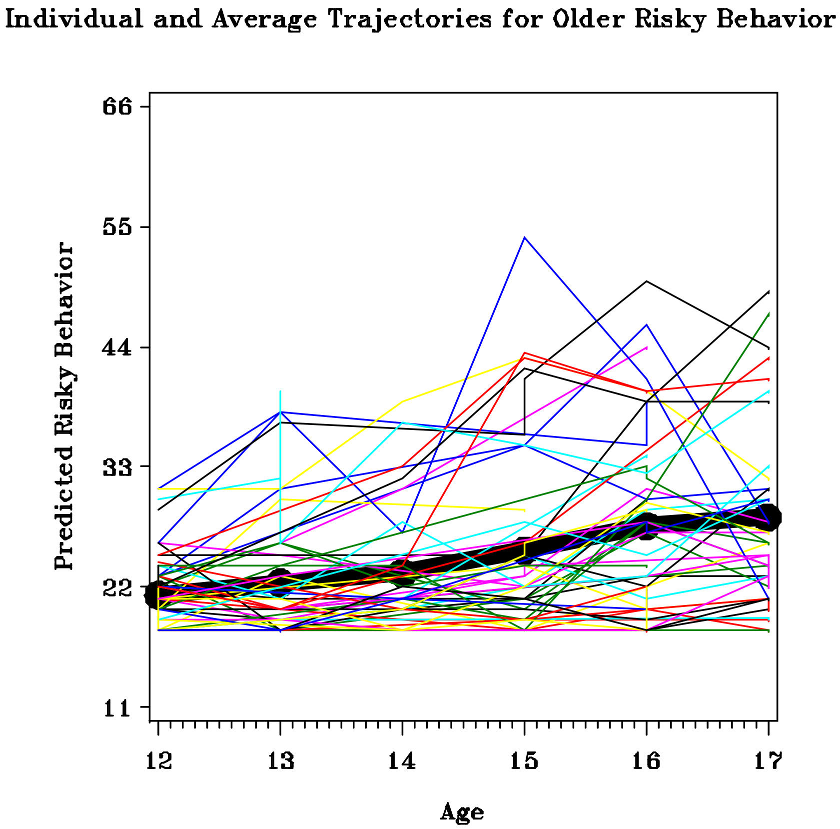


Cognitive Change

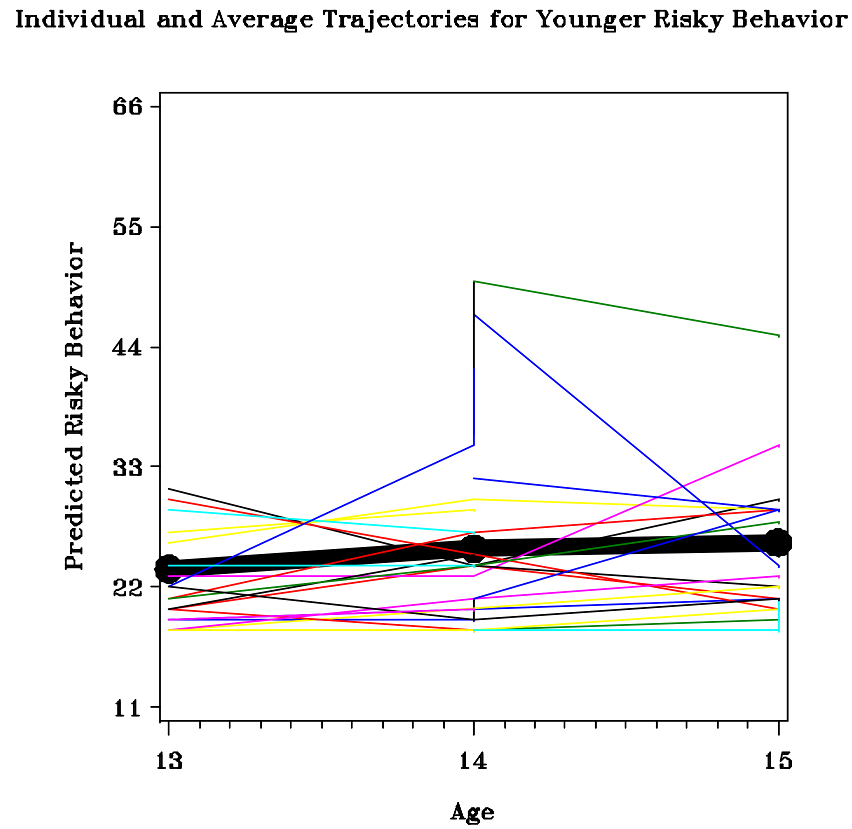


Individual Relations of Change in Risky Behavior Across Siblings

Older Siblings

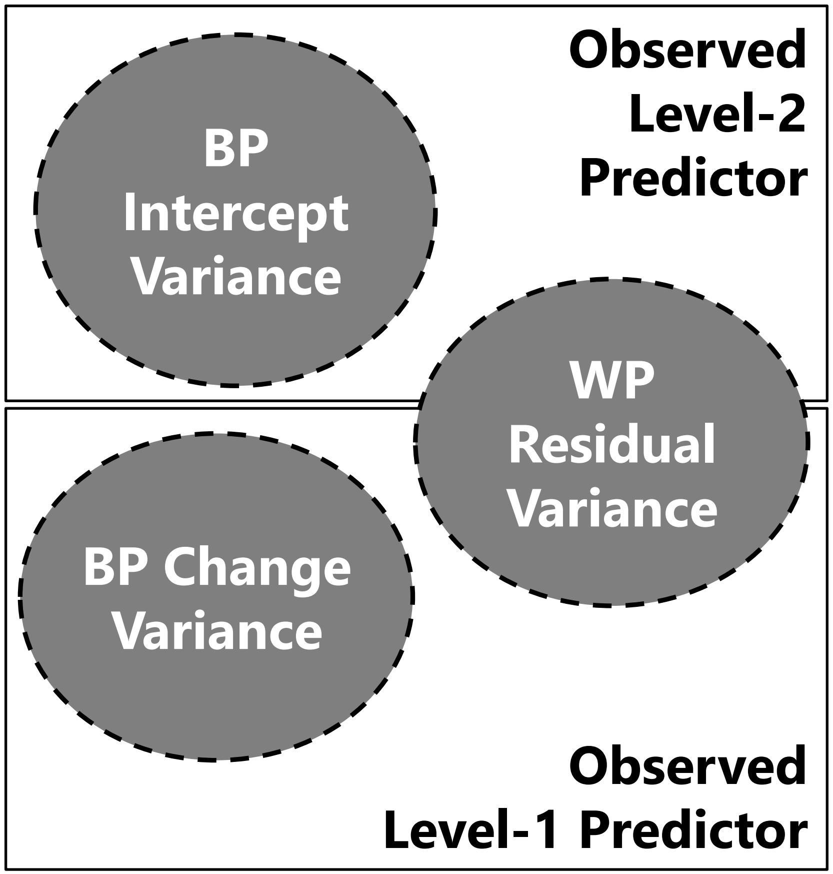


Younger Siblings



Distinguishing Longitudinal Relations

- If a TV predictor has both individual differences in change (U_{1i}) and residual deviations from change (e_{ti}), they should each have their own relationship(s) to y_{ti} (Hoffman 2015, Figure 9.3)
- Otherwise, they are **smushed into the level-1 WP relation**
  - If the TV predictor's **WP residual** still contains the TV predictor's unmodeled BP change variance, **the level-1 WP relation will be smushed with the missing L2 BP change relation!** (bottom panel)
  - Different than more well-known result of observed vs. latent person mean (top panel) due to True $\tau^2_{U_0}$= observed $\tau^2_{U_0} - \frac{\sigma^2_e}{L1n}$



Time Slope Smushing (Hoffman 2015, ch. 9)

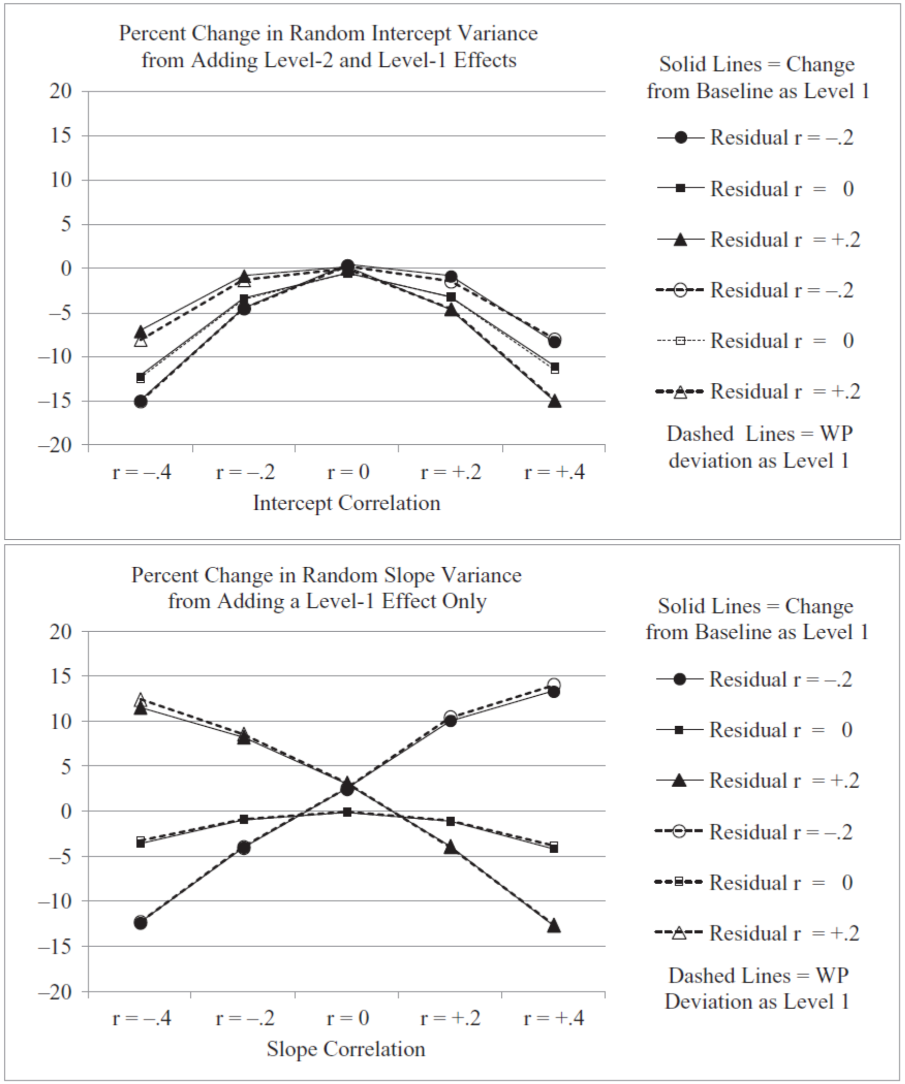


Figure 9.4 Changes in level-2 variance components from fitting univariate models to simulated data with varying intercept, slope, and residual correlation of a time-varying predictor and outcome.

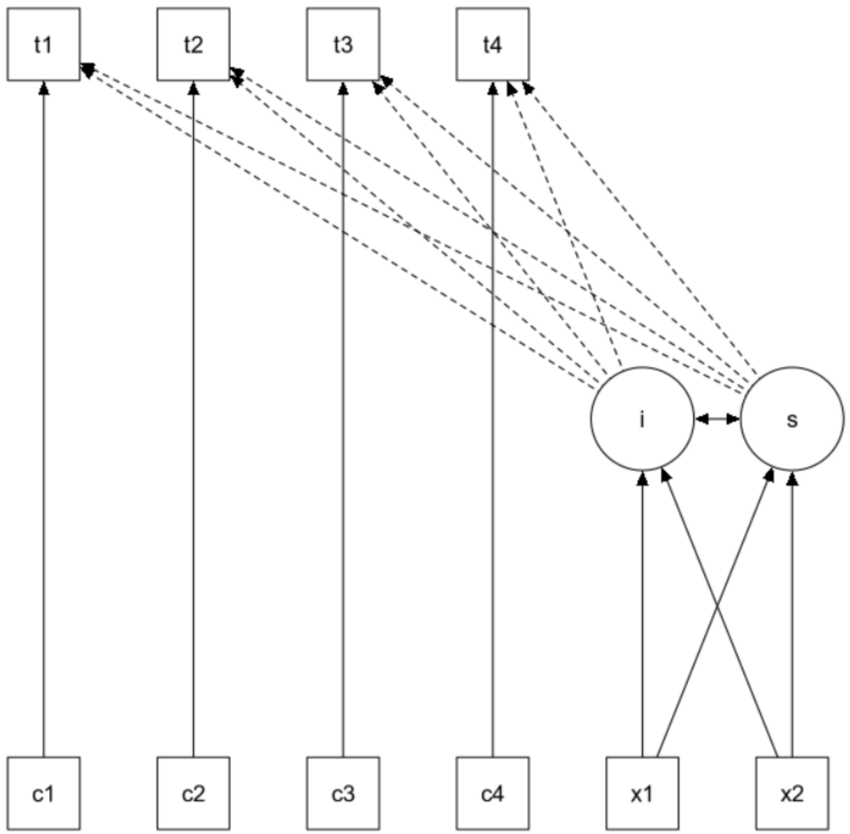
- What if we use **Person-MC** (or baseline-centering) in univariate MLM for a TV predictor whose BP intercepts, BP change, and WP residuals are correlated with those of the TV outcome?
  - **Top:** Pseudo-R^2 for BP L2 intercept relation is biased in direction of WP residual L1 slope because observed person mean still has a little WP residual variance in it
  - **Bottom:** Amount of BP change variance explained is biased in direction of WP residual L1 slope
  - **Conclusion:** Change variance in TV predictor needs to be its own variable, which can only be done correctly in a **multivariate MLM**!

What *Not* to Do with Longitudinal Data: Wide-Data SEM Edition!

- Mis-specified wide-data-format path models (with observed variables only) and SEMs (adding latent variables) for longitudinal data are still too common
  - Models often examine auto-regressive effects, cross-lagged effects, and observed variable mediation effects, which involve different variables each measured on two or more occasions
  - Following slides give common exemplars to watch out for!

- The problem in each is a lack of differentiation of sources (piles) of variance, and thus what their paths (slopes) mean
  - Big picture: If the path model variables have not been de-trended for person mean differences (AND for any individual change over time), then **all paths reflect smushed BP/WP relations to some degree…**
  - … and this problem will not necessarily be reflected by bad model fit, as models can have unrecognized compensatory parameters!

Time-Varying Predictors in Wide-Data SEM: What *Not* to Do… in R lavaan



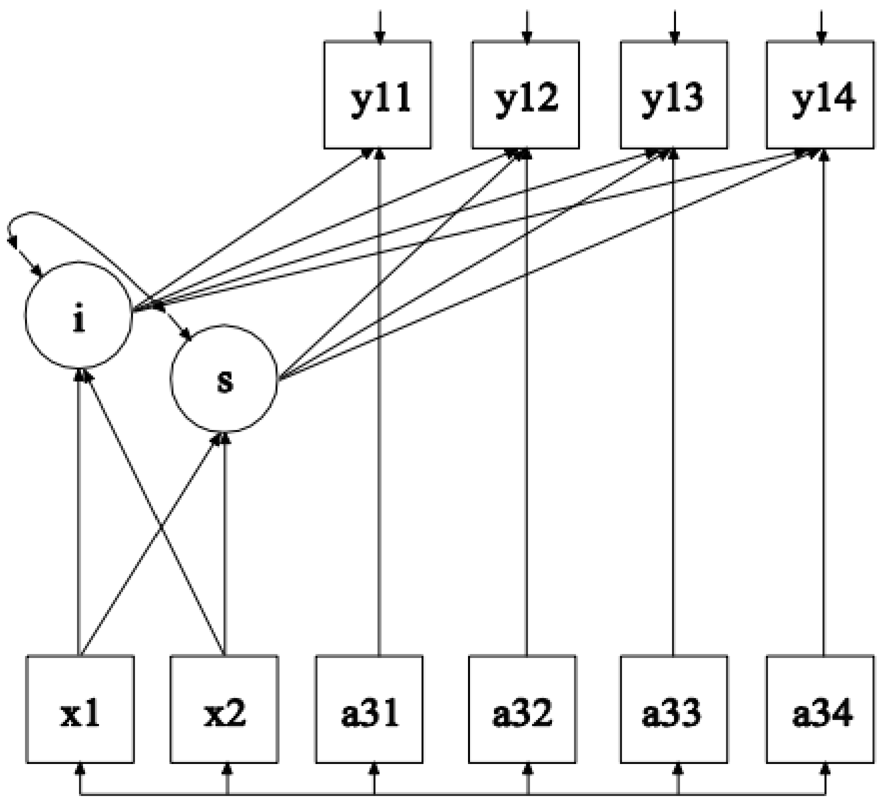

```
# a linear growth model with a time-varying covariate
model <- '
  # intercept and slope with fixed coefficients
    i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
    s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4
  # regressions
    i ~ x1 + x2
    s ~ x1 + x2
  # time-varying covariates
    t1 ~ c1
    t2 ~ c2
    t3 ~ c3
    t4 ~ c4
'
fit <- growth(model, data = Demo.growth)
summary(fit)
```

This diagram is from the (current) lavaan tutorial on growth curves.

Although the t1–t4 outcomes are predicted by latent intercept and time slope factors (separating two kinds of BP variance from WP variance), this is not the case for the c1–c4 TVPs.

Consequently, in the model shown here, the **c→y paths will be smushed**.

Time-Varying Predictors in Wide-Data SEM: What *Not* to Do… in Mplus



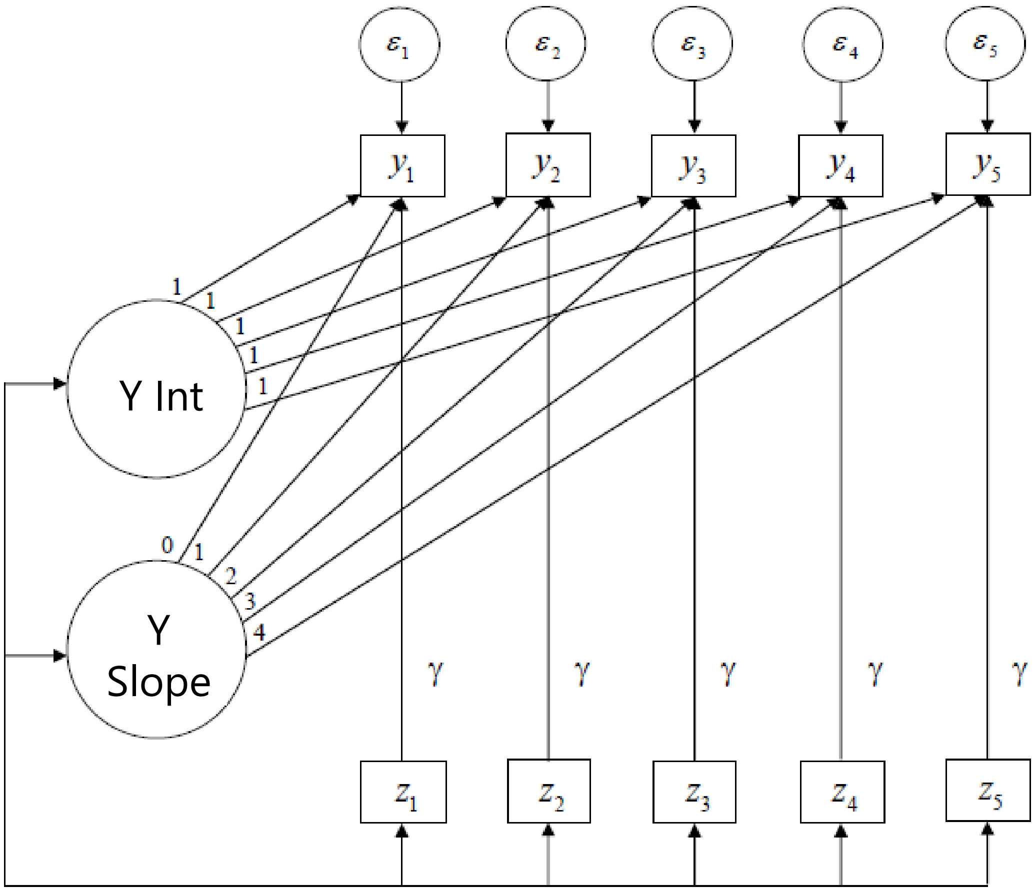

```
TITLE:     this is an example of a linear growth
           model for a continuous outcome with time-
           invariant and time-varying covariates
DATA:      FILE IS ex6.10.dat;
VARIABLE:  NAMES ARE y11-y14 x1 x2 a31-a34;
MODEL:     i s | y11@0 y12@1 y13@2 y14@3;
           i s ON x1 x2;
           y11 ON a31;
           y12 ON a32;
           y13 ON a33;
           y14 ON a34;
```

This diagram is from the (current) Mplus v. 8 Users Guide example 6.10.

Although the y11–y14 outcomes are predicted by latent intercept and time slope factors (separating two kinds of BP variance from WP variance), this is not the case for the a31–a34 TVPs.

Consequently, in the model shown here, the **a→y paths will be smushed**.

Time-Varying Predictors in Wide-Data SEM: What *Should* We *Do*?



This diagram is from Curran et al. (2012). The time-varying predictors z1–z5 boxes have directed effects onto the y1–y5 outcomes at the same time.

If you constrain these paths to be equal (as γ), you get **a smushed effect** (they call it an "aggregate" effect).

IF you add covariances of the z's with the intercept, then γ becomes **the WP effect**. But the BP effect is not provided!

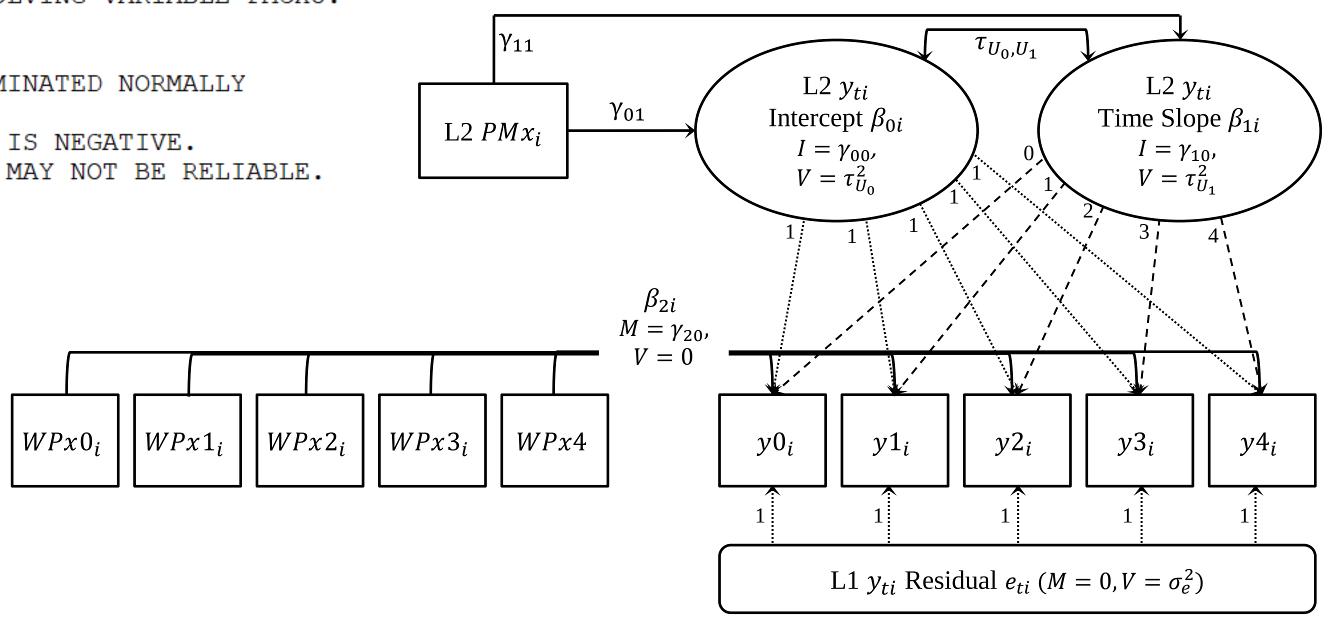
How to Fix it In Wide-Data SEM Using Observed Variables for TVx

- Person-MC for TV predictors *can* provide BP effects in SEM using Mplus, but it will result in an error because PMx_i is redundant (ipsative) with the $WPxt_i$ variables (t because separate columns)
- Looking at this way makes the better alternative more obvious…

```
WARNING:  THE SAMPLE COVARIANCE OF THE INDEPENDENT VARIABLES
IS SINGULAR. PROBLEM INVOLVING VARIABLE PMCX5.


THE MODEL ESTIMATION TERMINATED NORMALLY

THE CHI-SQUARE STATISTIC IS NEGATIVE.
THE LOGLIKELIHOOD VALUES MAY NOT BE RELIABLE.
```



Why Multivariate Change? 3 Example Uses:

1. To fully disaggregate BP and WP sources of variance and their corresponding relations across multiple variables
   - Prevent both intercept and time slope smushing that could happen when using observed-predictor approaches (Person-MC as in univariate MLM)
   - If longitudinal invariance falls apart, to salvage an intended model of "curve of factors" with an alternative "factor of curves" (as in end of Example 4b)

2. To examine auto-regressive and cross-lagged effects of multiple variables in multiple directions
   - e.g., previous x_{ti} → current y_{ti}; previous y_{ti} → current x_{ti}
   - But all BP sources of variance must be distinguished to prevent smushing!

3. Longitudinal mediation of change
   - BP mediation among intercepts and change factors; WP residual mediation

Multivariate Change: First things First

- Univariate change for **each variable separately comes first**!
- For any variable measured repeatedly (regardless of whether it is to become a "predictor" or "outcome"), first build its **unconditional model for change** (like we did last semester)
- Options to do so depends on whether you are in a long-data univariate MLM, long-data M-SEM, or wide-data SEM!

1. Estimate **saturated means** → What kind of **fixed** time slopes are needed (i.e., what is the average trajectory shape)?
   - **In MLM:** Add occasion as a categorical predictor (may need to round time into convenient intervals for unbalanced occasions)
   - **In M-SEM:** Create binary contrast for each (rounded) occasion; estimate their slopes as manual way of treating occasion as categorical
   - **In SEM:** Estimate per-occasion intercept (requiring balanced time)

Univariate change for each variable separately comes first, continued

2. Estimate **unstructured (UN) variances and covariances** → Heterogeneity of variance and correlation over time suggests **level-2 random time slopes are needed**
   - **In MLM:** UN R matrix directly in MIXED or R GLS without random effects
   - **In M-SEM:** Not possible as far without significant trickery as far as I know in standard software (so do it using MLM or SEM instead)
   - **In SEM:** Estimate H1 model as UN R matrix to get per-occasion intercepts, variances, and covariances (requiring balanced time)

- If you do not have balanced time (and it's too far unbalanced to be rounded for descriptive purposes), then skip this step
  - Acknowledge that you do not have an "answer key" with which to assess absolute fit (also true after adding random TV predictor slopes in SEM)

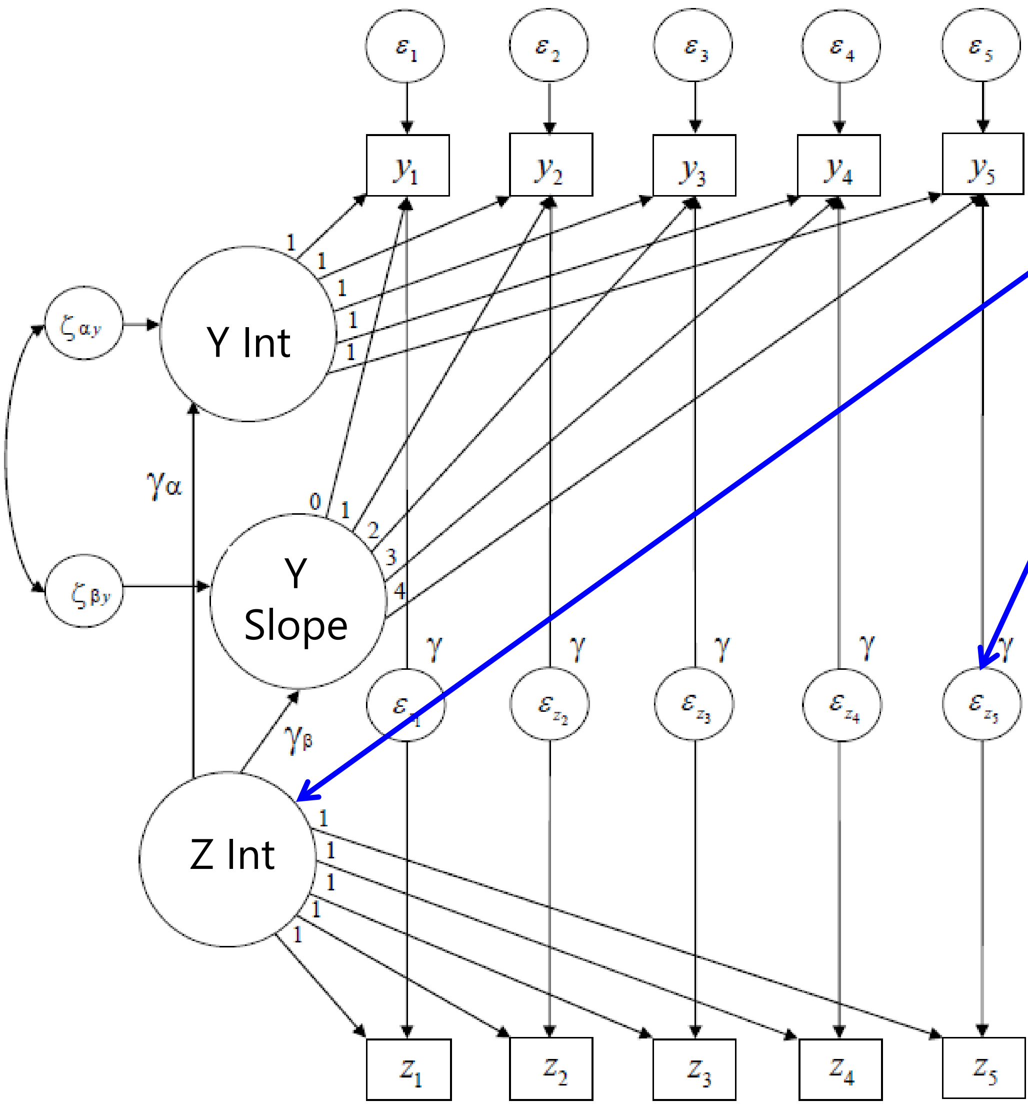
Univariate change for each variable separately comes first, continued

3. Once you have whatever fixed and random time slopes are needed, consider if **level-1 residual variances need to differ by occasion**

- **MLM/M-SEM default = same; SEM default = different!**
  - **In MLM:** For balanced time, compare models with residual variances all the same or all different (no readily available in-between options)
  - **In M-SEM:** Not possible as far without significant trickery as far as I know in standard software (so do it using MLM or SEM instead)
  - **In SEM:** Examine local misfit and modification indices to see if any residual variances really want to be different from each other, and free just those ones (so model can be customized as needed)

4. Also consider **level-1 residual relations** (as covariances or slopes)
   - **In MLM:** Add R matrix covariance pattern (e.g., AR1, Lag-1 Toeplitz)
   - **In M-SEM:** AR relations only possible through slopes, not covariances!
   - **In SEM:** Add residual covariances (or slopes with "structured residuals")

Fix It in Wide-Data SEM (by Curran et al., 2012)

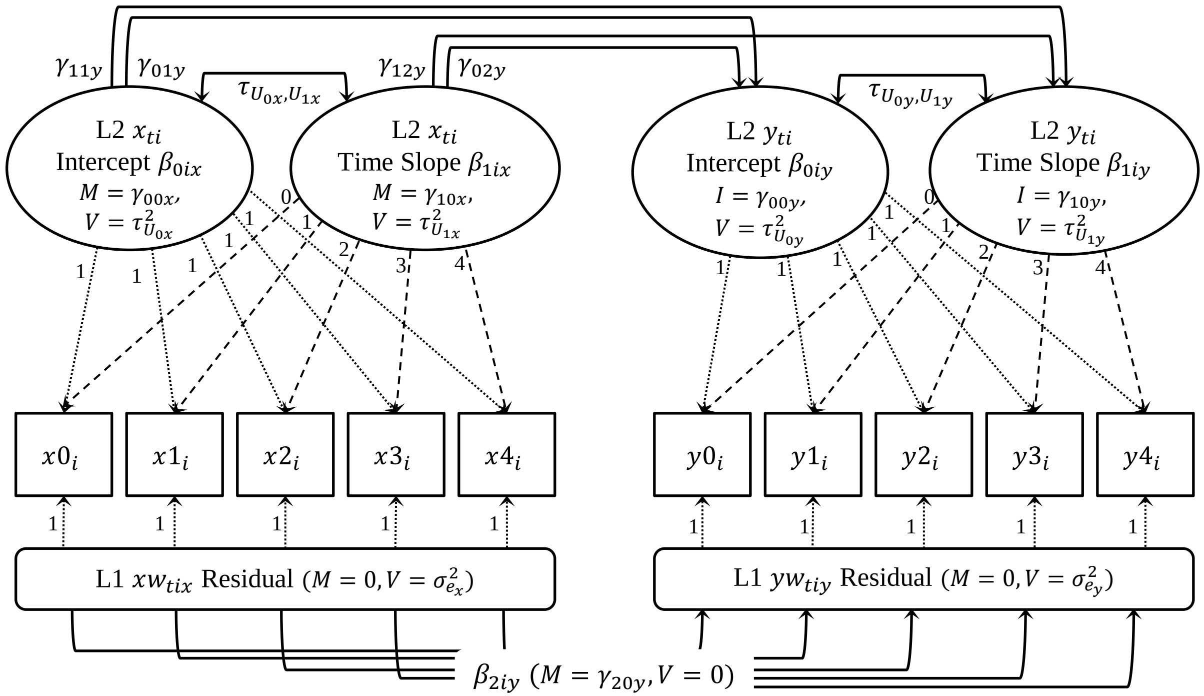


The z1–z5 TV predictor slopes are unsmushed if they have their own BP intercept (and time slope as needed) factor(s), which directly represents their level-2 BP source(s) of variance.

The **Zint → Yint slope γ_α** is a L2 **BP slope** because of the **structured residuals**: the new ε_z latent variables to which the level-1 residual variances of z1–z5 have been moved. The **WP effect** is now given by **γ** from $\varepsilon_{z1\text{-}z5}$ → y1–y5.

If z1–z5 had predicted y1–y5 directly, the **Zint → Yint** slope would be a **L2 contextual** effect instead of a L2 BP effect.

Fix It in Wide-Data SEM with Two Variables with Individual Change



Change factor(s) needs to be in the model for each variable that shows random change → multiple kinds of "BP" relations!

WP relation among "structured residuals" shown above → paths go from X residuals to Y residuals so that level-2 intercept–change relations stay BP instead of contextual

Multivariate Change: Use Case 1

- Estimate all univariate change models together for relations among BP intercepts, BP changes, and WP residuals
  - If a TV predictor has **random change variance**, it **must become a time-predicted outcome** in a multivariate model (via SEM or M-SEM)
  - For TV predictors with **fixed change** only, you *could* use Person-MC or constant-C + PMx in univ MLM (b/c individual differences are constant)
    - With a "large enough" L2 sample size, latent centering should yield more accurate L2 slopes (because the expected WP variance is then removed from latent X intercept)

- At L2: relations can be covariances or fixed slopes, but anything that predicts a variable's change should* predict its intercept too!
  - X Int → Y Intercept = X Int main effect
  - X Int → Y Change = X Int*time interaction (on change in Y)
  - X Change → Y Intercept = X Change main effect
  - X Change → Y Change = X Change*time interaction (on change in Y)
  - **But are these between-person or contextual relations???**

Multivariate MLM Chaos (Hoffman, 2019)

Table 1. Summary of Modeling Choices and Level 2 Results

Level 1 source of variance and type of effects	Mplus syntax for the Level 1 effect	Resulting Level 2 effect
Univariate MLM with ML or Bayesian estimation		
Variable-centered observed variable		
Fixed effects only	Level 1 direct	Between[a]
Fixed effects only	Level 1 placeholder	Between[a]
Fixed and random effects	Level 1 placeholder	Between[a]
Constant-centered observed variable		
Fixed effects only	Level 1 direct	Contextual[a]
Fixed effects only	Level 1 placeholder	Contextual[a]
Fixed and random effects	Level 1 placeholder	Contextual[a]
Multivariate MLM using Mplus multilevel structural equation modeling with ML estimation		
Within-level latent variable		
Fixed effects only	Level 1 direct	Between[b]
Uncentered observed variable		
Fixed effects only	Level 1 placeholder	Contextual[b]
Fixed and random effects	Level 1 placeholder	Contextual[b]
Multivariate MLM using Mplus 8.1+ multilevel structural equation modeling with Bayesian estimation		
Within-level latent variable		
Fixed effects only	Level 1 direct	Between[b]
Fixed effects only	Level 1 placeholder	Between[b]
Fixed and random effects	Level 1 placeholder	Between[b]
Multivariate MLM using general structural equation modeling with ML or Bayesian estimation		
Latent residual of observed variable		
Fixed effects only	Residual direct	Contextual[b]
Fixed effects only	Structured residual	Between[b]
Fixed and random effects	Residual direct through placeholder	Contextual[b]
Fixed and random effects	Structured residual through placeholder	NA

Note: MLM = multilevel model; ML = maximum likelihood; NA = not available.
[a]These Level 2 effects are fixed effects for observed Level 2 mean predictors (included in all univariate models).
[b]These Level 2 effects are effects for latent Level 2 intercept predictors (included in all multivariate models).

Under %WITHIN% in M-SEM syntax:

- *Level-1 "direct" slope:* **`Y ON X;`**
  - Can only be used for fixed L1 slopes
- *Level-1 "placeholder" slope:* **`L1slope | Y ON X;`**
  - Needed to add random L1 slopes and/or cross-level interactions across level-2 units in %BETWEEN% model

Troubleshooting Tips: Are My Level-2 Slopes Between or Contextual?

- Start with a simplified multivariate MLM in which each Y pile of variance is predicted by only one X pile of variance at a time
  - Goal: Recover bivariate relations without contamination by how slopes change when they are "unique" effects controlling for other predictors

- Concern is relevant when same variables have **slopes at both levels**
  - e.g., model for X → BPx Intercept, BPx Change, WPx Residual
  - e.g., model for Y → BPy Intercept, BPy Change, WPy Residual
  - If there is a WPx → WPy slope in the L1 model, then fixed slopes for at least some of the BPx → BPy intercept/change relations could be **L2 contextual slopes instead of L2 BP slopes** (based on previous table)

- **How to check?** Compare L2 slope results from two models:
  - A) WPx → WPy **fixed slope** (no random slope variance or cross-level ints)
  - B) WPx ←→ WPy **covariance** (no random slope variance or cross-level ints)
  - If any L2 slopes changed notably, they must be L2 contextual (because they are **controlled for the L1 slope only in A,** whereas BP slopes don't control)

Curve of Factors vs. Factor of Curves

- In Example 4b we looked at a "**curve of factors**" model:
  - **Lower-order factors →** Latent factor measurement per occasion
  - **Higher-order factors →** Change over time in latent factor
  - Answers the question, do I have fixed and/or random change over time in my *single* latent variable, assuming all outcomes change the same way?
  - Requires at least partial longitudinal invariance to ensure that the per-occasion factor represents the same latent construct over time!

- If invariance falls apart, one alternative is a "**factor of curves**" model based in the idea of multivariate change instead (also in 4b)
  - **Lower-order factors →** Change over time in *each* observed outcome
  - **Higher-order factors →** Common factors for intercept and change
  - Answers the question, are the patterns of correlation among my lower-order intercept and change factors consistent with a "common" higher-order intercept factor and a "common" higher-order change factor?
  - Does NOT assume all observed outcomes change the exact same way!

Curve of Factors vs. Factor of Curves

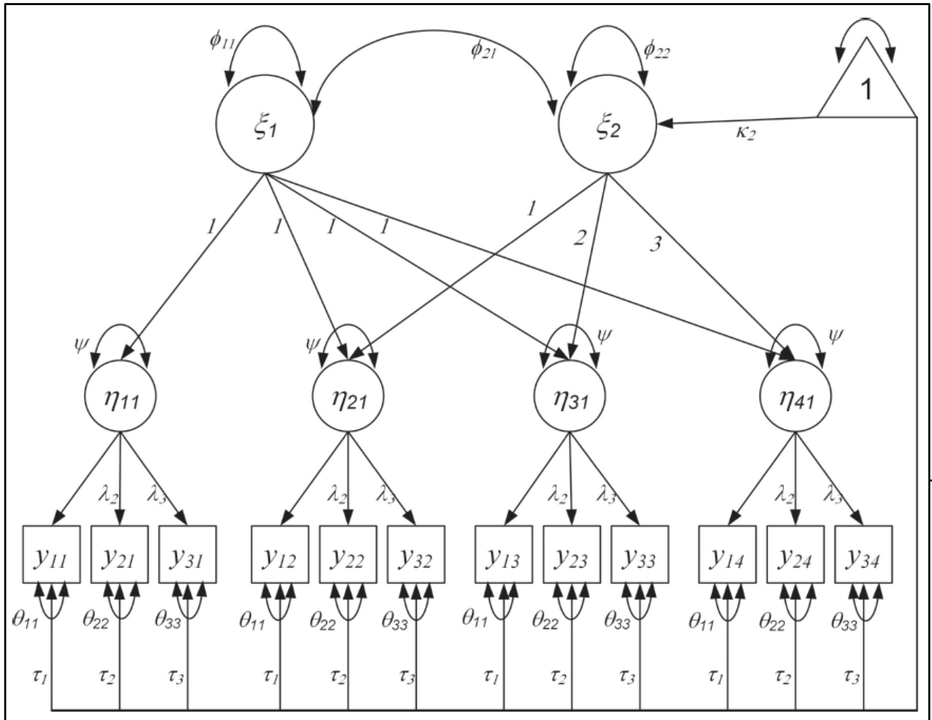


FIGURE 14.3. Path diagram of a second-order growth model.

Left: Curve of Factors
([Grimm et al., 2016](#))

- Lower-order factors → Latent factor per occasion
- Higher-order factors → Change over time in latent factor

Right: Factor of Curves
([Isordia et al., 2017](#))

- Lower-order factors → Change over time per outcome
- Higher-order factors → Common intercept and change

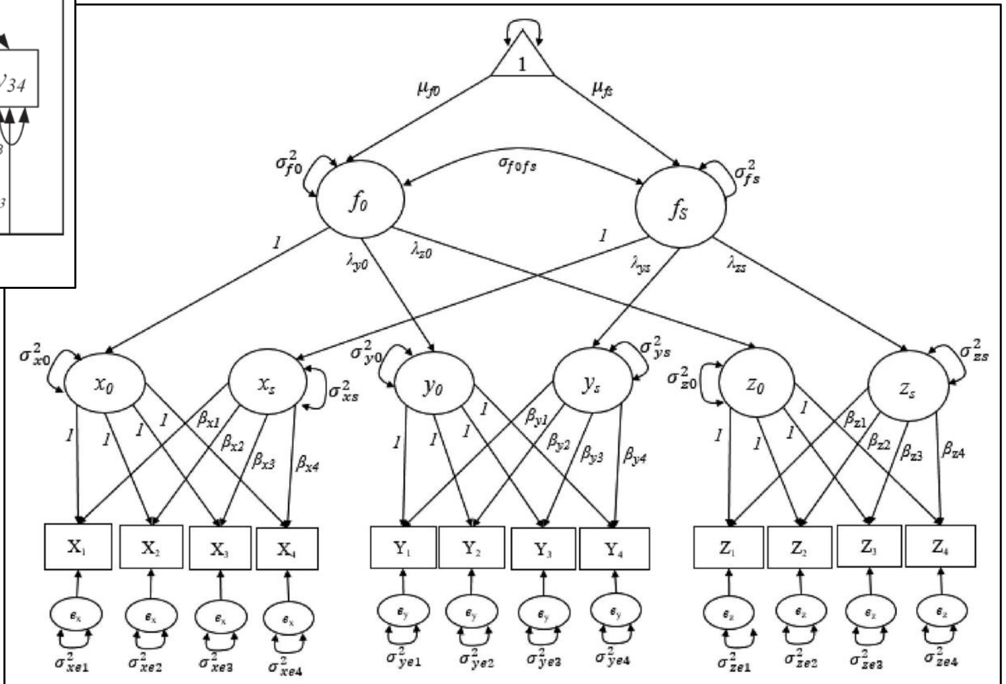


Figure 2. A path diagram of a factor of curves (FOCUS) model with three constructs across four measurement occasions.

Use Case 2: Cross-Lagged Relations

- All the within-person (WP) relations described so far have been concurrent—between x_{ti} and y_{ti} at the same occasion
- Lagged WP relations can be examined in univariate MLM, but:
  - Rows with unpredicted y_{ti} at prior occasions will be dropped by default
  - Relations go in one direction only: observed WPx_{ti} → latent y_{ti} (as e_{tiy})

- To examine "cross-lagged" reciprocal relations between x_{ti} and y_{ti} at different occasions, the model needs to somehow have access to all the occasions at once!
  - Although one can create lagged observed WPx_{ti} variables, there are no comparable **observed** WPy_{ti} variables to lag
  - Thus, cross-lagged relations are easier to examine in wide-data SEM (or Mplus M-SEM using "dynamic" SEM lagging features)

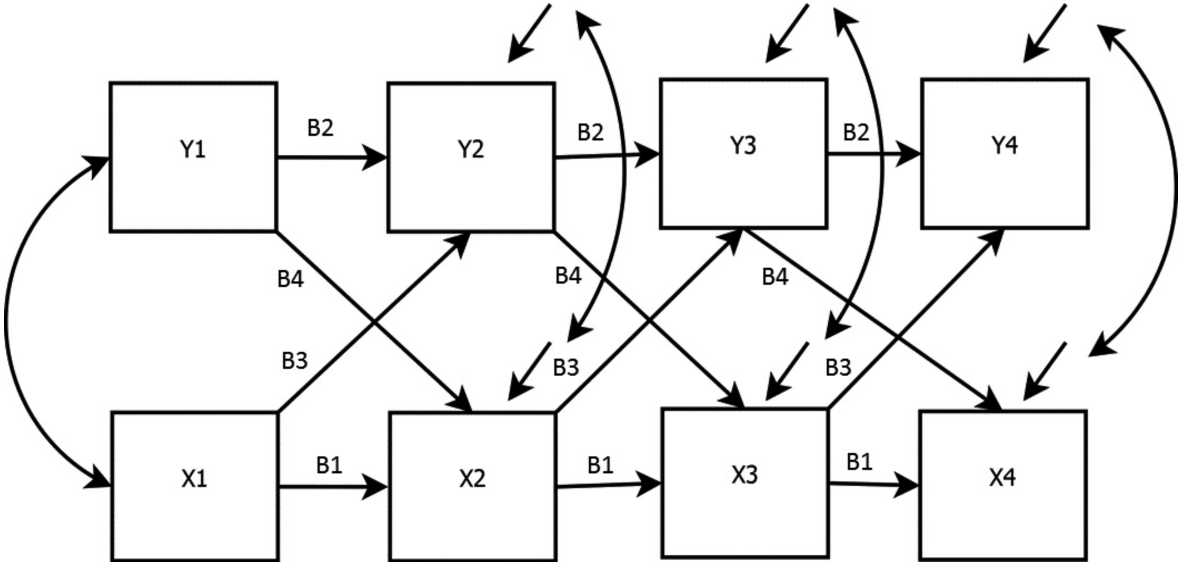
- However, the same issues of using centering to avoid smushed effects are still relevant (even though it's not as obvious)…!
  - Just having "longitudinal" paths (e.g., T1 → T2) is not enough!

What *Not* to Do with Longitudinal Data

- Mis-specified path models (involving observed variables only) for longitudinal data are still far too common
  - Using different variables each measured on three or more occasions, these models often examine auto-regressive effects (within same variable over time), cross-lagged effects (between different variables over time), and observed variable mediation effects
  - Next slides give some common exemplars to watch out for! Not shown are "accumulating" versions of models that are even harder to interpret (see Usami et al., 2019 or Clark et al., 2021 for elaboration on those)

- The problem in each is a lack of differentiation of sources (piles) of variance, and thus what their paths (slopes) mean
  - Big picture: If the path model variables have not been de-trended for person mean differences (AND for any individual change over time), then **all paths reflect smushed BP/WP relations to some degree…**
  - … and this problem will not necessarily be reflected by bad model fit!

A Model that Needs to Go Away*

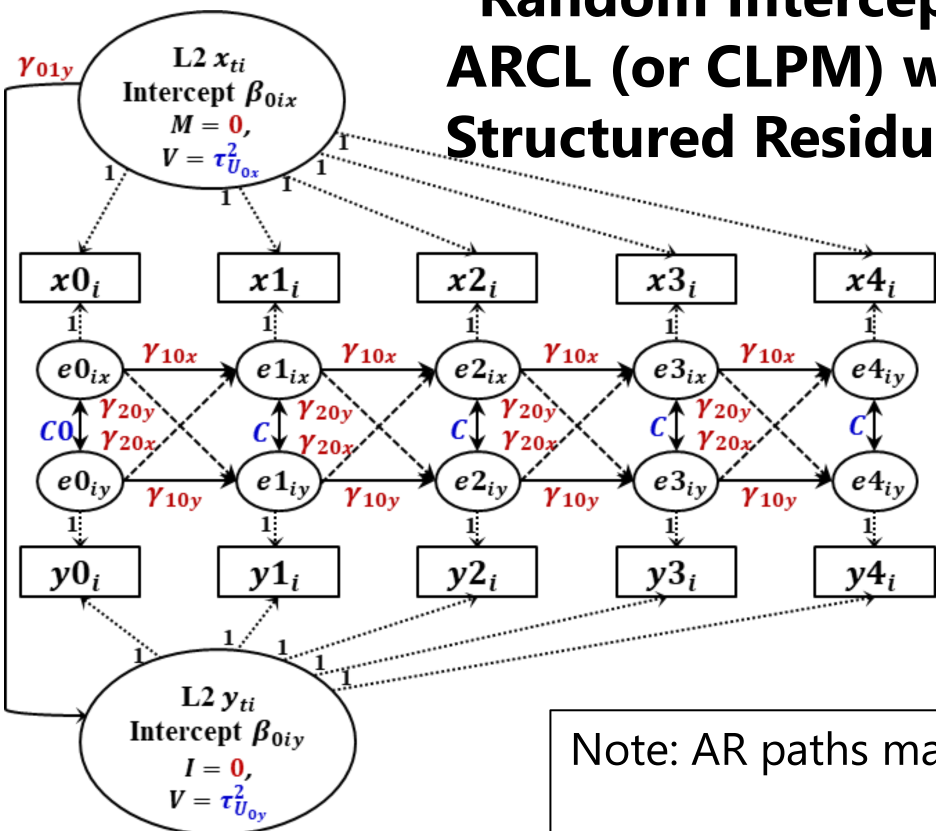


Autoregressive cross-legged panel model

* Emphasis mine, picture from [Berry & Willoughby (2017)]

- **Logic:** By including **auto-regressive paths** (B1 and B2) to "control" for previous occasions, the **cross-lagged paths** (B3 and B4) then represent effects of "change" on each variable in predicting the other (so they are "longitudinal" predictions of time *t*-1 predicting time *t*)
- **Reality:** By allowing only one path (usually constrained equal over time), it reflects smushed effects across sources of variance—BP intercept, BP time slope(s), WP residual; AR paths do NOT adequately control for BP differences because they assume an AR(1) pattern over time

Remedies for *Intercept* Smushing



"Random Intercept" ARCL (or CLPM) with Structured Residuals

Many authors have also pointed out the need to distinguish constant BP effects from WP effects via:

$$x_{tix} = \gamma_{t0x} + \gamma_{10x}(x_{t-1i}) + \gamma_{20x}(y_{t-1i}) + \boxed{U_{0ix}} + e_{tix}$$

$$y_{tiy} = \gamma_{t0y} + \gamma_{10y}(y_{t-1i}) + \gamma_{20y}(x_{t-1i}) + \boxed{U_{0iy}} + e_{tiy}$$

Btw, equal AR and CL paths over time only make sense for equal-interval balanced occasions

Note: AR paths may not be needed given random intercepts!

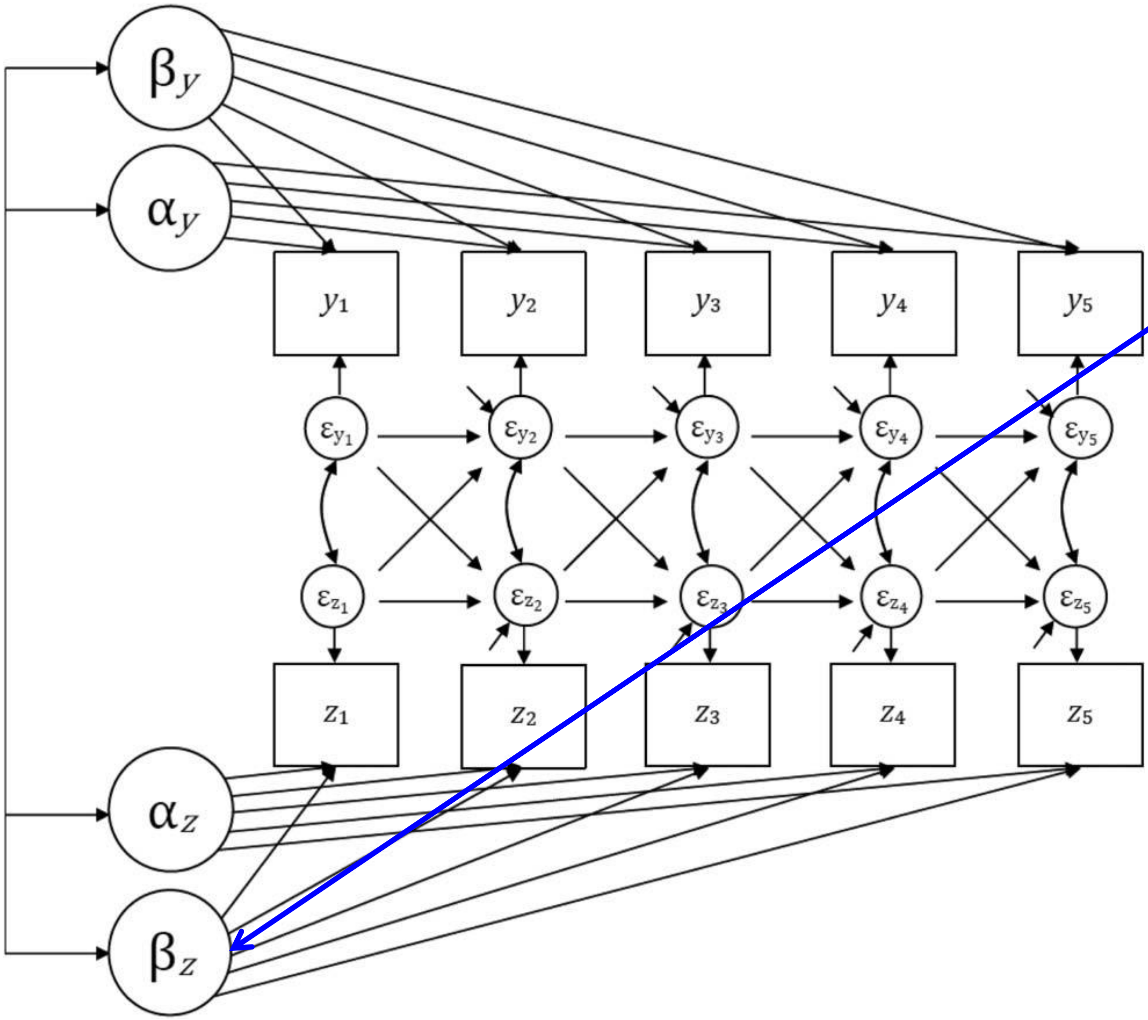
Given the interest in cross-lagged "which came first" level-1 WP residual paths, the level-2 random intercept relationship is usually specified as a covariance instead of a slope—and whether a slope would capture the between or contextual effects differs by software, estimator, and model specification…

What about Change over Time?

- The RI-CLPM is appropriate for longitudinal data that show fluctuation—but not individual change—over time
  - Whether each variable's AR1 paths are still needed after controlling for its random intercept factor is then an empirical question (and they could become covariances instead in single-level SEM)
  - Analysts can decide whether to specify concurrent or lagged paths in one variable predicting another, or covariances (whatever makes sense)

- For outcomes that contain individual differences in change, how to properly specify unsmushed effects of "time-varying predictors" (TVPs) is *still* not well-understood…
  - Big picture: TVPs will usually carry at least one source of BP variance (random intercept for mean differences), possibly more (random time slopes for individual change; random scale factor for volatility)
  - Each source of level-2 variance can have its own set of relations; otherwise, you can have time-smushed WP effects beyond intercept-smushed effects
  - So let's see how the standard SEM latent growth curve model would need to adapt to address this… (I bet you can guess!)

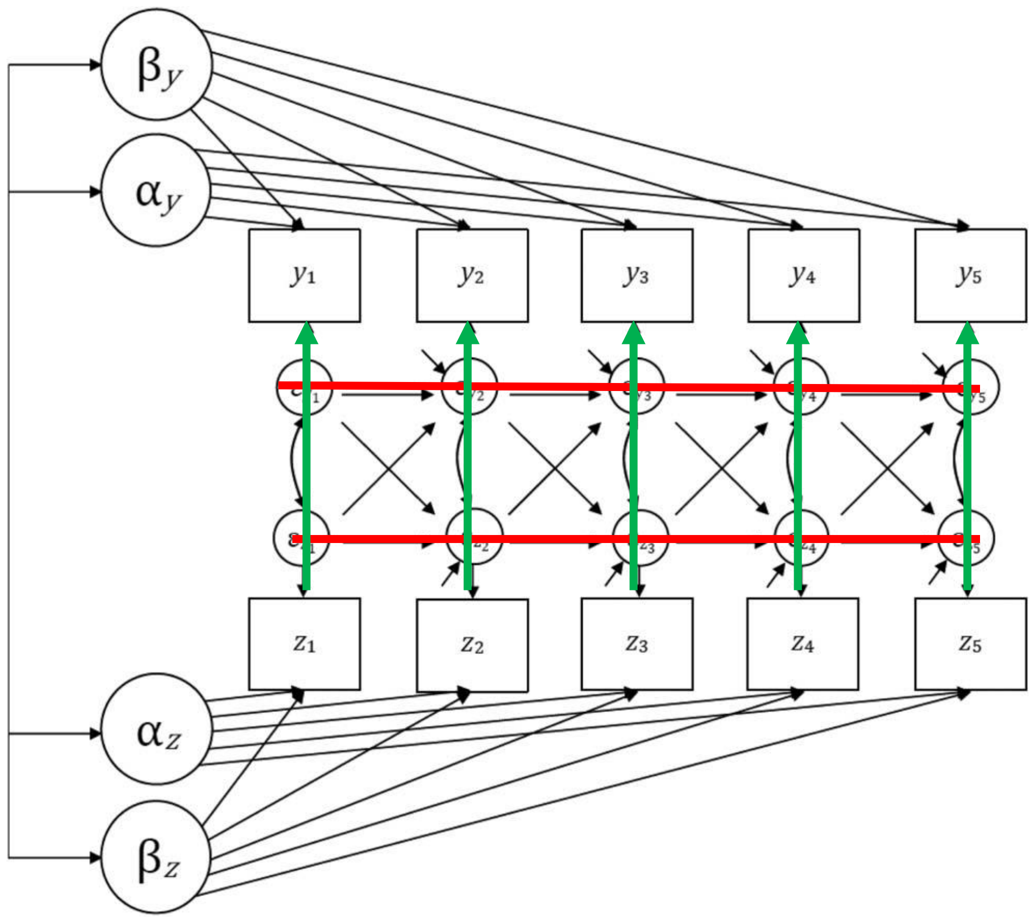
Change + ARCL Model (by [Curran et al., 2014])



If z1–z5 has individual differences in change over time instead of just fluctuation, **just add a random time slope factor for z1–z5**—you'd have the multivariate change model we began with, but including structured residuals.

When **using level-1 structured residuals,** all paths among the intercept and slope factors will represent their **total level-2 BP effects**. But structured residuals then don't allow random slopes (or other modifications), at least in ML in Mplus…

How To Fix It Without Structured Residuals



See [Hoffman & Hall (2024)](#) for more on these kinds of CPLMs with random effects

IF you predict the y1–y5 residuals directly from z1–z5 (without structured residuals), **that effect is still the level-1 WP effect.**

The problem is that some of the paths among the intercept and slope factors become **BP contextual effects** instead. These include paths for intercept → intercept (and slope → slope), but not for intercept → slope (or slope → intercept).

In either version, you can still get the missing L2 effect (BP or BP contextual) by requesting a linear combination (e.g., in Mplus MODEL CONSTRAINT).

Lagged Effects in Long-Data M-SEM?

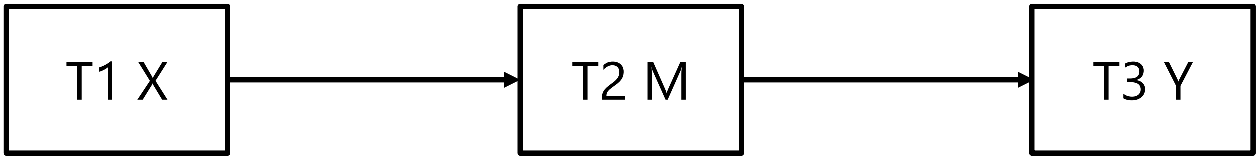
- The original M-SEM versions of AR and CL slopes (labeled as "dynamic" SEM) created smushed effects or inconsistency
  - e.g., in Mplus: `Y ON Y&1;` creates an AR1 slope of previous occasion's original (unpartitioned) Y to current Y → smushed AR1 slope
  - e.g., in Mplus: `Y ON X&1;` creates an CL1 slope of previous occasion's original (unpartitioned) X to current Y → smushed CL1 slope
  - Could only be solved by Person-MC to try to get the lagged slope of the WP part of the observed predictor specifically (even while any concurrent effects used latent centering for WP outcome instead)

- "Residual (dynamic?) SEM" now allows lagged effects using model-partitioned WP residuals as predictors
  - e.g., in Mplus: `Y ON Y^1;` creates an AR1 slope from previous occasion's WP part of latent-centered Y to current Y → like structured residuals
  - e.g., using Mplus: `Y ON X^1;` creates an CL1 slope from previous occasion's WP part of latent-centered X to current Y → like structured residuals

- Btw, these features are only available using Bayes estimation

Use Case 3: "Longitudinal" Mediation

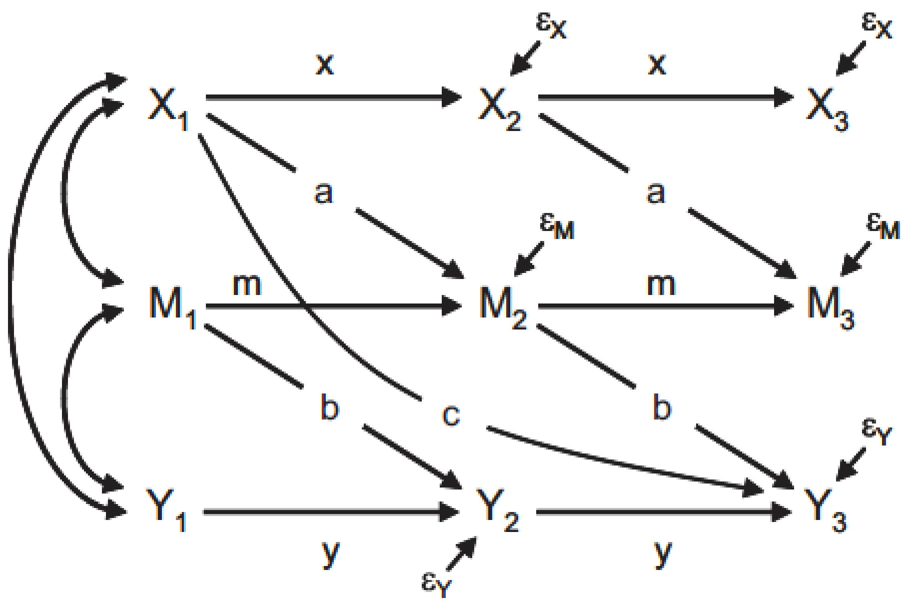
- Because causal effects should take time to happen, below is a commonly estimated "longitudinal" (mediation) model:



- However, if each variable contains individual differences in change over time, then each of these occasion-adjacent slopes reflects some combination of **4 (or more) distinct sources of relation**:
  - (1) WP residual → WP residual is "longitudinal" because it could be estimated using data from only one person! (although slope could differ BP)
  - (2) BP intercept → BP intercept is "cross-sectional" (>1 person needed)
  - (3) BP change → BP change, and (4) BP intercept → BP change
  - 3 and 4 are harder to label: "BP change" is actually BP differences (which are cross-sectional) in WP change (which is longitudinal)

- So why not use all the occasions for each variable to differentiate these kinds of relations, as well as a "multivariate change" model!

Mediation: What Not To Do (Still)



Typical longitudinal mediation model

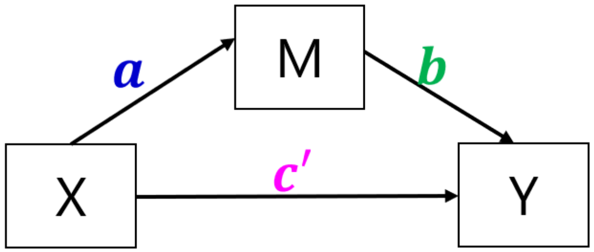
X= predictor, M= mediator, Y= outcome

* My point of view only, picture from Maxwell & Cole (2007)

- **Logic:** Mediation should time to occur, so indirect effects should be specified across different occasions (as before, of "change")
- Agreed, but if these variables haven't been de-trended for ALL sources of BP variance, then **the *b* and *c* paths are smushed**
- And what about **BP mediation**? Capturing BP variances in the same model would allow examination of that, too…
  - BP intercept mediation, BP change mediation, WP residual mediation…

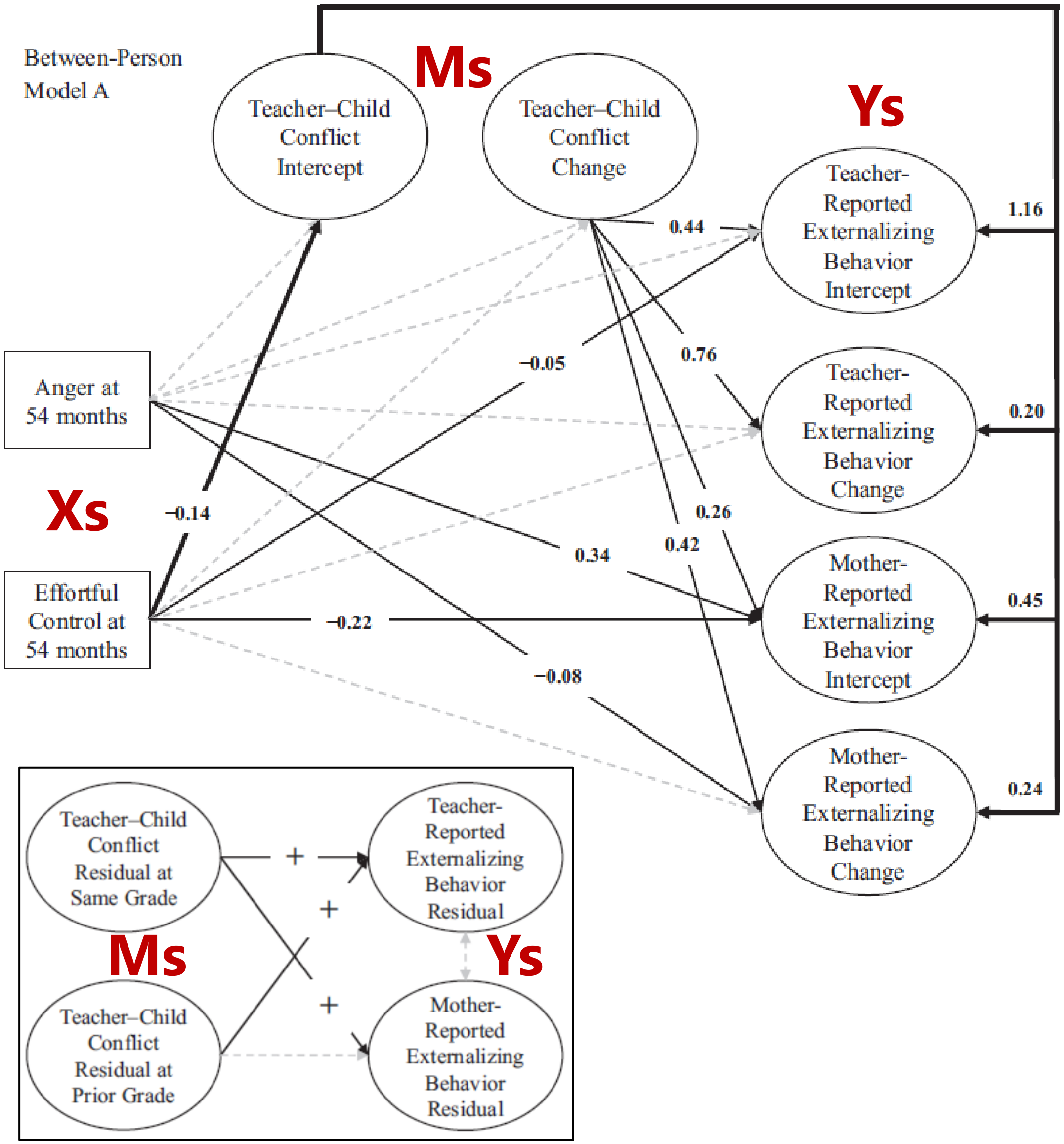
Implications for Multilevel Mediation

- Mediation is only logically possible at two levels for **one combination**, as shown below
  - By mediation, I mean "M is part of the reason why X → Y" theoretically
  - Although indirect effects can always be computed, they may not make sense
  - Below: Is each variable measured at Level 2 or Level 1 (= both L1+L2)
  - Note that Level 2 can include change as part of mediation, too!



X predictor	M mediator	Y outcome	L1 mediation?	L2 mediation?
2	2	2	no	yes
2	2	1	no	yes
2	1	2	no	yes
2	1	1	no	yes
1	2	2	no	yes
1	2	1	no	yes
1	1	2	no	yes
1	**1**	**1**	**yes**	**yes**

Actual Longitudinal Mediation: Example 1



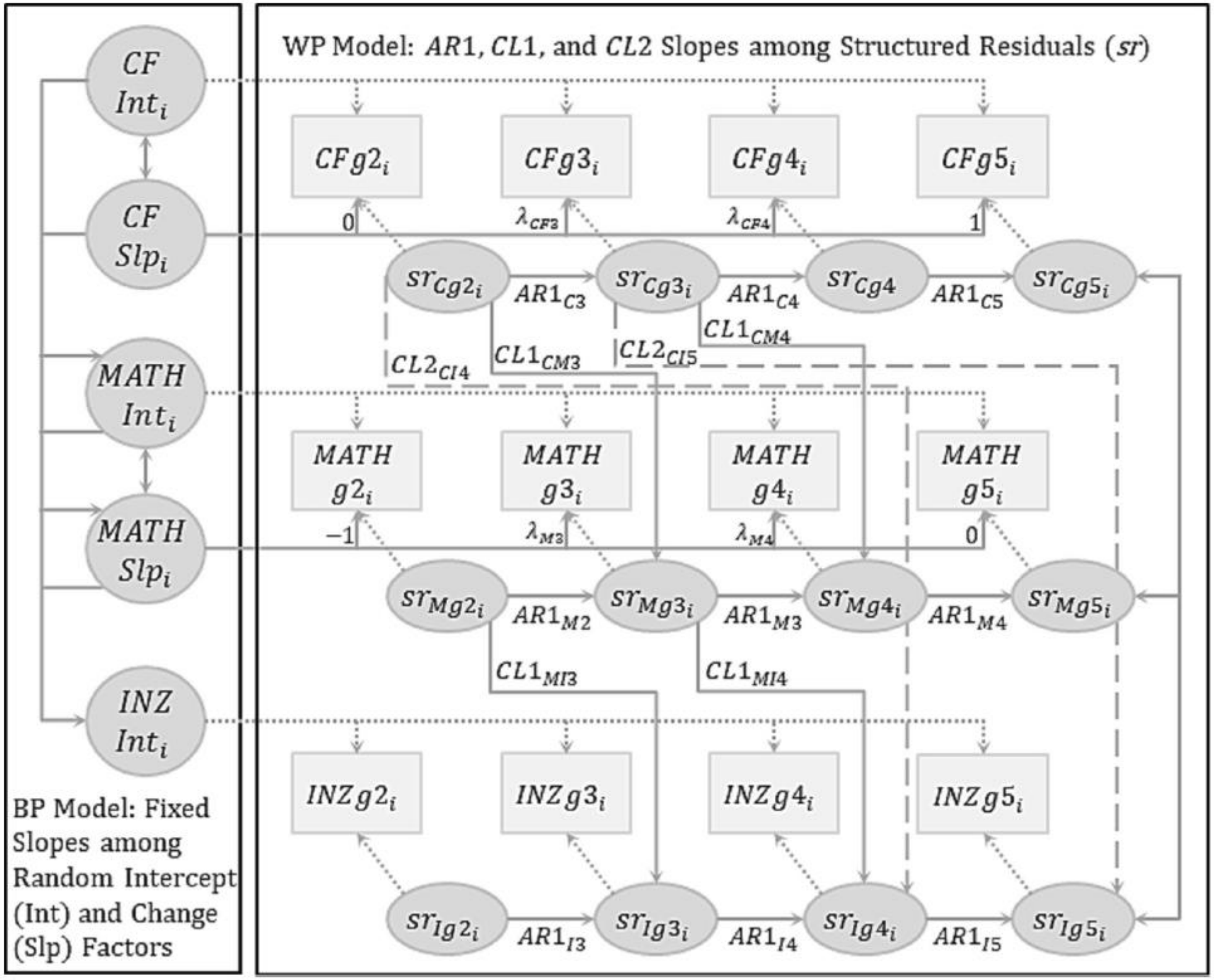
Mediation cannot be meaningfully examined using smushed effects!

Example from [Crockett et al. 2019] using latent basis change in single-level SEM

Top: Between-Person Model (A) of direct and indirect effects among level-2 random intercepts and time slopes of 3 longitudinal variables

Bottom: Within-Person Model (A) of direct effects among level-1 residuals (no indirect effects possible because X = time-invariant)

Actual Longitudinal Mediation: Example 2



[Hoffman & Hall (2024)](#)
Cognitive flexibility → Math → Internalizing in grades 2–5

Internalizing did not show change over time, so it had a random intercept only

Summary

- If a time-varying "predictor" contains individual differences in change, then observed-variable centering strategies (Person-MC, constant-C) will not adequately distinguish BP change variance from WP residual variance in the observed predictors

- The solution is to predict both time-varying "predictors" and "outcomes" as outcomes in a multivariate MLM → multivariate change via single-level SEM (wide data) or M-SEM (long data)

- When examining lagged effects and/or mediation, make sure to properly distinguish and BP sources of variance (and their across-variable relations) FIRST, otherwise those slopes are smushed (only WP model logically can show lagged relations)