Example 5: Cross-Classified Models for Students Nested within Primary and Secondary Schools
(complete data, syntax, and output available for STATA, R, and SAS electronically)

Cross-classified models (also known as crossed random effects models) are useful in situations in which people belong to more than one type of cluster, but the types of clusters are not nested. To demonstrate, simulated data from Hox (2012) chapter 7 are analyzed below, in which the outcome is 9^{th} grade academic achievement. There are 1000 level-1 children nested within 50 level-2 primary schools AND within 33 level-2 secondary schools, in which **primary and secondary schools are crossed at level 2**. We have predictors for whether the primary and secondary schools are denominational (i.e., religious) and child socio-economic status (SES). The number of children per unique crossing of primary by secondary school ranged from 1–6, which means we have a potential random interaction intercept AND three kinds of contextual effects! Note that these models are different than those in the text, in which contextual effects of child SES were not considered (i.e., it was smushed).

STATA Syntax for Data Import, Manipulation, and Description:

```
// Define global variable for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive
   global filesave "C:\Dropbox\23_PSQF6272\PSQF6272_Example5"

// Open trimmed example excel data file from sheet "grade10" and clear away existing data
   clear // clear memory in case of open data
   import excel "$filesave\Example5_Data.xlsx", firstrow case(preserve) sheet("Hox") clear

// Add labels to original variables
   label variable childID    "childID: Child ID"
   label variable PschoolID  "PschoolID: Primary School ID"
   label variable SschoolID  "SschoolID: Secondary School ID"
   label variable achieve    "achieve: Child Achievement Outcome"
   label variable ses        "ses: Child Socio-Economic Status"
   label variable Pdenom     "Pdenom: Primary School Denomination"
   label variable Sdenom     "Sdenom: Secondary School Denomination"

display "STATA Descriptives for Child Variables"
summarize achieve ses

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     achieve |      1,000      6.3435    .8676812        3.9        9.9
         ses |      1,000       4.098    1.397981          1          6

// Get means per primary school for child variables
   sort PschoolID
   egen PrimN =     count(achieve), by(PschoolID)
   egen PMachieve = mean(achieve),  by(PschoolID)
   egen PMses =     mean(ses),      by(PschoolID)
   label variable PMachieve "PMachieve: Primary Mean Child Achievement"
   label variable PMses     "PMses: Primary Mean Child SES"

display "STATA Descriptives and Correlations for Primary Schools"
preserve  // Save for later use, then compute primary school dataset
collapse  PMachieve Pdenom PMses PrimN, by(PschoolID)
format    PMachieve Pdenom PMses PrimN  %4.2f
summarize PMachieve Pdenom PMses PrimN, format
pwcorr    PMachieve Pdenom PMses PrimN, sig
restore   // Go back to child-level dataset

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
   PMachieve |         50        6.36        0.45       5.28       7.55
      Pdenom |         50        0.60        0.49       0.00       1.00
       PMses |         50        4.10        0.28       3.47       4.73
       PrimN |         50       20.00        4.46      10.00      31.00
```

```
             | PMachi~e   Pdenom    PMses    PrimN
-------------+------------------------------------
   PMachieve |   1.0000 
             |
             |
      Pdenom |   0.2227   1.0000 
             |   0.1200
             |
       PMses |   0.0323  -0.0316   1.0000 
             |   0.8235   0.8276
             |
       PrimN |  -0.1810  -0.2125  -0.0609   1.0000 
             |   0.2085   0.1384   0.6744
```

```
// Get means per secondary school for child variables
   sort SschoolID
   egen SecN =      count(achieve), by(SschoolID)
   egen SMachieve = mean(achieve),  by(SschoolID)
   egen SMses =     mean(ses),      by(SschoolID)
   label variable SMachieve "SMachieve: Secondary Mean Child Achievement"
   label variable SMses     "SMses: Secondary Mean Child SES"

display "STATA Descriptives and Correlations for Secondary Schools"
preserve  // Save for later use, then compute secondary school dataset
collapse  SMachieve Sdenom SMses SecN, by(SschoolID)
format    SMachieve Sdenom SMses SecN  %4.2f
summarize SMachieve Sdenom SMses SecN, format
pwcorr    SMachieve Sdenom SMses SecN, sig
restore   // Go back to child-level dataset
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
   SMachieve |         33        6.32        0.32       5.54       6.91
      Sdenom |         33        0.67        0.48       0.00       1.00
       SMses |         33        4.14        0.34       3.47       5.00
        SecN |         33       30.30       11.66       4.00      48.00
```

```
             | SMachi~e   Sdenom    SMses     SecN
-------------+------------------------------------
   SMachieve |   1.0000 
             |
             |
      Sdenom |   0.2369   1.0000 
             |   0.1843
             |
       SMses |   0.2070   0.2030   1.0000 
             |   0.2477   0.2572
             |
        SecN |   0.1645  -0.0037  -0.2940   1.0000 
             |   0.3603   0.9836   0.0967
```

```
// Get means per unique combination of primary/secondary school for child variables
   sort PschoolID SschoolID
   egen UniqueID =   group(PschoolID SschoolID)  // Create unique ID
   egen UniqueN =    count(achieve), by(PschoolID SschoolID)
   egen PSMachieve = mean(achieve),  by(PschoolID SschoolID)
   egen PSMses =     mean(ses),      by(PschoolID SschoolID)
   label variable PSMachieve "PSMachieve: Unique Primary/Secondary Mean Child Achievement"
   label variable PSMses     "PSMses: Unique Primary/Secondary Mean Child SES"

display "STATA Descriptives and Correlations for Unique Primary/Secondary Combination"
preserve  // Save for later use, then compute secondary school dataset
collapse  PSMachieve PSMses UniqueN, by(PschoolID SschoolID)
format    PSMachieve PSMses UniqueN  %4.2f
```

```
tabulate  UniqueN
summarize PSMachieve PSMses UniqueN, format
pwcorr    PSMachieve PSMses UniqueN, sig
restore   // Go back to child-level dataset
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
  PSMachieve |        652        6.36        0.81       4.00       9.10
      PSMses |        652        4.08        1.27       1.00       6.00
     UniqueN |        652        1.53        0.77       1.00       6.00
```

```
             | PSMach~e   PSMses  UniqueN
-------------+---------------------------
  PSMachieve |   1.0000 
             |
             |
      PSMses |   0.1906   1.0000 
             |   0.0000
             |
     UniqueN |  -0.0496   0.0282   1.0000 
             |   0.2057   0.4724
```

60% of the unique combinations have only one child, whose variance will then go to any "unique" level-2 variable

```
UniqueN |      Freq.     Percent        Cum.
--------+-----------------------------------
   1.00 |        394       60.43       60.43
   2.00 |        185       28.37       88.80
   3.00 |         60        9.20       98.01
   4.00 |         10        1.53       99.54
   5.00 |          2        0.31       99.85
   6.00 |          1        0.15      100.00
--------+-----------------------------------
  Total |        652      100.00
```

```
// Constant-centered predictors
   gen ses4    = ses - 4
   gen PMses4  = PMses - 4
   gen SMses4  = SMses - 4
   gen PSMses4 = PSMses – 4

// Cluster-mean-centered level-1 child predictors
   gen WPses  = ses - PMses   // Within primary only
   gen WSses  = ses - SMses   // Within secondary only
   gen WPSses = ses - PSMses  // Within unique combination
   label variable WPses  "WPses: Within-Primary Centered Child SES"
   label variable WSses  "WSses: Within-Secondary Centered Child SES"
   label variable WPSses "WPSses: Within-Primary/Secondary Centered Child SES"

display "STATA Descriptives and Correlations in Child-Level Data"
summarize WPSses, detail
pwcorr PMses SMses PSMses WPses WSses WPSses, sig
```

```
             |    PMses    SMses   PSMses    WPses    WSses   WPSses
-------------+------------------------------------------------------
       PMses |   1.0000 
             |
             |
       SMses |   0.0154   1.0000 
             |   0.6261
             |
      PSMses |   0.2387   0.2238   1.0000 
             |   0.0000   0.0000
             |
       WPses |   0.0000   0.1849   0.7921   1.0000 
             |   1.0000   0.0000   0.0000
             |
       WSses |   0.1972  -0.0000   0.7960   0.9629   1.0000 
             |   0.0000   1.0000   0.0000   0.0000
             |
      WPSses |  -0.0000  -0.0000   0.0000   0.5785   0.5771   1.0000 
             |   1.0000   1.0000   1.0000   0.0000   0.0000
```

As shown, only the within-primary AND within-secondary centered SES variable (whose variance = 0.62871 for slope reliability) is completely uncorrelated with the primary school AND secondary SES school means.

R Syntax for Data Import, Manipulation, and Description (after loading packages *readxl*, *TeachingDemos*, *Hmisc*, *psych*, *lme4*, *lmerTest*, and *performance*):

```
# Define variables for working directory and data name -- CHANGE THESE
filesave = "C:\\Dropbox/23_PSQF6272/PSQF6272_Example5/"
filename = "Example5_Data.xlsx"
setwd(dir=filesave)
# Load Jonathan's custom R functions from folder within working directory
functions = paste0("R functions/",dir("R functions/"))
temp = lapply(X=functions, FUN=source)
#  Import trimmed example excel data file from sheet "Hox"
Example5 = read_excel(paste0(filesave,filename), sheet="Hox")
# Convert to data frame to use in analysis
Example5 = as.data.frame(Example5)

print("R Descriptives for Child Variables")
describe(x=Example5[ , c("achieve","ses")])

# Get means per primary school for child variables using Jonathan's function
Example5 = addUnitMeans(data=Example5, unitVariable="PschoolID",
                        meanVariables=c("achieve","ses"), newNames=c("PMachieve","PMses"))

print("R Descriptives and Correlations for Primary Schools")
Primary = unique(Example5[,c("PschoolID","PMachieve","PMses","NperPschoolID")])
describe(x=Primary[ , c("PMachieve","PMses","NperPschoolID")])
rcorr(x=as.matrix(Primary[ , c("PMachieve","PMses","NperPschoolID")]), type="pearson")

# Get means per secondary school for child variables using Jonathan's function
Example5 = addUnitMeans(data=Example5, unitVariable="SschoolID",
                        meanVariables=c("achieve","ses"), newNames=c("SMachieve","SMses"))

print("R Descriptives and Correlations for Secondary Schools")
Secondary = unique(Example5[,c("SschoolID","SMachieve","SMses","NperSschoolID")])
describe(x=Secondary[ , c("SMachieve","SMses","NperSschoolID")])
rcorr(x=as.matrix(Secondary[ , c("SMachieve","SMses","NperSschoolID")]), type="pearson")

# Create unique ID variable for primary/secondary school combination
uniqueIDs = unique(Example5[c("PschoolID", "SschoolID")])
uniqueIDs$UniqueID = 100000 + 1:nrow(uniqueIDs)
# Merge unique ID back into child-level data
Example5 = merge(x=Example5, y=uniqueIDs, by=c("PschoolID","SschoolID"))

# Get means per unique combination of primary/secondary school for child variables using
Jonathan's function
Example5 = addUnitMeans(data=Example5, unitVariable=c("UniqueID"),
                       meanVariables=c("achieve","ses"), newNames=c("PSMachieve","PSMses"))

print("R Descriptives and Correlations for Unique Primary/Secondary Combination")
Unique = unique(Example5[,c("UniqueID","PSMachieve","PSMses","NperUniqueID")])
table(x=Unique$NperUniqueID, useNA="ifany")
prop.table(table(x=Unique$NperUniqueID, useNA="ifany"))
describe(x=Unique[ , c("PSMachieve","PSMses","NperUniqueID")])
rcorr(x=as.matrix(Unique[ , c("PSMachieve","PSMses","NperUniqueID")]), type="pearson")

# Constant-centered predictors
Example5$ses4    = Example5$ses - 4
Example5$PMses4  = Example5$PMses - 4
Example5$SMses4  = Example5$SMses - 4
Example5$PSMses4 = Example5$PSMses - 4

# Cluster-mean-centered level-1 child predictors
Example5$WPses  = Example5$ses - Example5$PMses
Example5$WSses  = Example5$ses - Example5$SMses
Example5$WPSses = Example5$ses - Example5$PSMses

print("R Descriptives and Correlations in Child-Level Data")
var(Example5$WPSses)
rcorr(x=as.matrix(Example5[ , c("PMses","SMses","PSMses","WPses","WSses","WPSses")]),
      type="pearson")
```

Model 1a: Empty Means, Primary Random Intercept Only

$Achieve_{cps} = \gamma_{000} + U_{0p0} + e_{cps}$

For crossed models, this composite equation can be easier to understand!

This model 1a predicts 9th grade academic achievement for child c who previously went to primary school p and currently goes to secondary school s. The inclusion of only a random intercept for primary school creates an expected correlation only among children who went to the same primary school (so far)

```
display "STATA Model 1a: Empty Means, Primary Random Intercept Only"
mixed achieve  , || PschoolID: ,    reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2       // Print -2LL for model
estat icc                        // Intraclass correlation
estimates store FitEmpty1        // Save for LRT

print("R Model 1a: Empty Means, Primary Random Intercept")
Model1a = lmer(data=Example5, REML=TRUE, formula=achieve~1+(1|PschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model1a, chkREML=FALSE); summary(Model1a, ddf="Satterthwaite")
print("Show intraclass correlation and its LRT")
icc(Model1a); ranova(Model1a)
```

```
       AIC         BIC      logLik   deviance    df.resid
 2393.7365   2408.4597 -1193.8682  2387.7365    997.0000

Random effects:
 Groups    Name        Variance Std.Dev.
 PschoolID (Intercept) 0.17555  0.41899
 Residual              0.57708  0.75966
Number of obs: 1000, groups:  PschoolID, 50

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  6.359093   0.064141 49.241213  99.142 < 2.2e-16

# Intraclass Correlation Coefficient
    Adjusted ICC: 0.233
  Unadjusted ICC: 0.233

ANOVA-like table for random-effects: Single term deletions
                npar   logLik     AIC     LRT Df Pr(>Chisq)
<none>             3 -1193.87 2393.74
(1 | PschoolID)    2 -1279.18 2562.37 170.632  1 < 2.22e-16
```

r = .233 of children from the same primary school

Model 1b: Empty Means, Primary by Secondary Crossed Random Intercepts

$Achieve_{cps} = \gamma_{000} + U_{0p0} + U_{00s} + e_{cps}$

In model 1b, the secondary school random intercept U_{00s} introduces a separate correlation of children from the same secondary school (beyond the primary school random intercept U_{0p0}). In the STATA code below, the `_all: R.` should be used for whichever crossing has fewer possible units to speed up estimation. See further details in Example 8 of the STATA MIXED manual: https://www.stata.com/manuals/me.pdf or these slides from Don Hedeker: https://prevention.nih.gov/sites/default/files/2022-11/MtG-HedekerSlides-FINAL-508.pdf

```
display "STATA Model 1b: Empty Means, Primary by Secondary Crossed Random Intercepts"
mixed achieve  , || _all: R.SschoolID || PschoolID: ,  ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2       // Print -2LL for model
estimates store FitEmpty2        // Save for LRT
lrtest FitEmpty2 FitEmpty1       // LRT for secondary intercept variance
```

```
-------------------------------------------------------------------
     achieve | Coefficient  Std. err.          df       t    P>|t|
-------------+-----------------------------------------------------
       _cons |   6.341978   .0808221         67.8    78.47   0.000
-------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
_all: Identity               |
             var(R.SschoolID) |   .0787319    .025134      .0421131     .147192
-----------------------------+------------------------------------------------
PschoolID: Identity          |
                  var(_cons) |   .1746989   .0407036      .1106537    .2758127
-----------------------------+------------------------------------------------
               var(Residual) |   .5051556   .0235792      .4609921      .55355
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 246.10                Prob > chi2 = 0.0000
-2LL = 2312.2688

Likelihood-ratio test: Assumption: FitEmpty1 nested within FitEmpty2
 LR chi2(1) =   75.47
Prob > chi2 = 0.0000
```

```
print("R Model 1b: Empty Means, Primary by Secondary Crossed Random Intercepts")
Model1b = lmer(data=Example5, REML=TRUE, formula=achieve~1+(1|PschoolID)+(1|SschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model1b, chkREML=FALSE); summary(Model1b, ddf="Satterthwaite")
print("Show LRT for each random intercept"); ranova(Model1b)
```

```
$AICtab
       AIC         BIC      logLik    deviance    df.resid
 2320.2688   2339.8998 -1156.1344   2312.2688    996.0000

Random effects:
 Groups    Name        Variance Std.Dev.
 PschoolID (Intercept) 0.174699 0.41797
 SschoolID (Intercept) 0.078732 0.28059
 Residual              0.505156 0.71074
Number of obs: 1000, groups:  PschoolID, 50; SschoolID, 33

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  6.341978   0.080822 66.878773  78.468 < 2.2e-16

ANOVA-like table for random-effects: Single term deletions
                npar   logLik     AIC      LRT Df Pr(>Chisq)
<none>             4 -1156.13 2320.27
(1 | PschoolID)    3 -1250.29 2506.58 188.3088  1 < 2.22e-16
(1 | SschoolID)    3 -1193.87 2393.74  75.4677  1 < 2.22e-16
```

All sources of variance are orthogonal, so we can sum them to compute the proportion of variance due to each sampling dimension, as shown below.

Empty Model Proportions of Variance (from SAS)

PschoolID	SschoolID	Residual	Total	PropL2Primary	PropL2Second	PropResid
0.1747	0.07873	0.5052	0.75857	0.23028	0.10378	0.66594

Of the total variation of 0.75857 (from summing all three orthogonal variances):

0.1747 / 0.75857 = **.230** reflects mean achievement differences between **primary** schools

0.0787 / 0.75857 = **.104** reflects mean achievement differences between **secondary** schools

0.5052 / 0.75857 = **.666** reflects **remaining** achievement differences between children within schools

How do these proportions of variance translate into ICCs for different types of children?

```
print("Show saved variances from model 1b")
as.data.frame(VarCorr(Model1b))
        grp        var1 var2        vcov     sdcor
1 PschoolID (Intercept) <NA> 0.174698688 0.41796972
2 SschoolID (Intercept) <NA> 0.078731963 0.28059216
3  Residual        <NA> <NA> 0.505155614 0.71074300

# Compute total variance from model 1b saved variances
total1b = as.data.frame(VarCorr(Model1b))[1,4] +
          as.data.frame(VarCorr(Model1b))[2,4] +
          as.data.frame(VarCorr(Model1b))[3,4]

print("ICC for Children in Same Primary School but Different Secondary Schools")
as.data.frame(VarCorr(Model1b))[1,4] / total1b
[1] 0.23029509

print("ICC for Children in Same Secondary School but Different Primary Schools")
as.data.frame(VarCorr(Model1b))[2,4] / total1b
[1] 0.10378775

print("ICC for Children in Same Primary School and Same Secondary School")
(as.data.frame(VarCorr(Model1b))[1,4] +
 as.data.frame(VarCorr(Model1b))[2,4]) / total1b
[1] 0.33408284
```

Here is an easier (but less transparent way) to compute ICCs using the `performance` R package:

```
print("Show ICC for each school type"); icc(Model1b, by_group=TRUE)
Group     |   ICC
-----------------
PschoolID | 0.230
SschoolID | 0.104

print("Show ICC for same primary and secondary"); icc(Model1b)
  Unadjusted ICC: 0.334
```

95% random effect confidence interval for the intercept across each type of school:
Fixed effect ± 1.96*SQRT(random variance)

Primary: 6.342 ± 1.96*SQRT(0.1747) = 5.523 to 7.161
→ 95% of primary schools are predicted to have school mean achievement from 5.523 to 7.161

Secondary: 6.342 ± 1.96*SQRT(0.07873) = 5.792 to 6.892
→ 95% of secondary schools are predicted to have school mean achievement from 5.792 to 6.892

Model 1c: Empty Means, Primary by Secondary AND Unique Crossed Random Intercepts

$$Achieve_{cps} = \gamma_{000} + U_{0p0} + U_{00s} + U_{0ps} + e_{cps}$$

Given 1–6 children per unique combination of primary and secondary school, we can test whether there is an extra correlation among children who have both schools in common—a random interaction intercept U_{0ps}!

```
display "STATA Model 1c: Empty Means, Primary by Secondary and Unique Crossed Intercepts"
mixed achieve  , || _all: R.SschoolID || PschoolID: || UniqueID: ,  ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2      // Print -2LL for model
estimates store FitEmpty3       // Save for LRT
lrtest FitEmpty3 FitEmpty2      // LRT for unique intercept variance
```

```
print("R Model 1c: Empty Means, Primary by Secondary and Unique Crossed Random Intercepts")
Model1c = lmer(data=Example5, REML=TRUE,
formula=achieve~1+(1|PschoolID)+(1|SschoolID)+(1|UniqueID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model1c, chkREML=FALSE); summary(Model1c, ddf="Satterthwaite")
print("Show LRT for each random intercept"); ranova(Model1c)

      AIC        BIC     logLik   deviance   df.resid
 2321.2836  2345.8224 -1155.6418  2311.2836   995.0000

Random effects:
 Groups    Name        Variance Std.Dev.
 UniqueID  (Intercept) 0.026993 0.16430
 PschoolID (Intercept) 0.173769 0.41686
 SschoolID (Intercept) 0.077787 0.27890
 Residual              0.480183 0.69295
Number of obs: 1000, groups:  UniqueID, 652; PschoolID, 50; SschoolID, 33

Fixed effects:
             Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)  6.343576   0.080679 66.344823  78.627 < 2.2e-16

ANOVA-like table for random-effects: Single term deletions
                npar  logLik     AIC      LRT Df Pr(>Chisq)
<none>             5 -1155.64 2321.28
(1 | PschoolID)    4 -1232.26 2472.52 153.2332  1 < 2.22e-16
(1 | SschoolID)    4 -1188.12 2384.24  64.9612  1 7.6388e-16
(1 | UniqueID)     4 -1156.13 2320.27   0.9852  1    0.32092
```

The LRT indicates we do not need the random interaction variance, so we will remove it.

```
print("Show saved variances from model 1c")
as.data.frame(VarCorr(Model1c))
        grp        var1 var2        vcov      sdcor
1  UniqueID (Intercept) <NA> 0.026993200 0.16429607
2 PschoolID (Intercept) <NA> 0.173769173 0.41685630
3 SschoolID (Intercept) <NA> 0.077786917 0.27890306
4  Residual        <NA> <NA> 0.480183039 0.69295241

# Compute total variance from model 1c saved variances
total1b = as.data.frame(VarCorr(Model1b))[1,4] +
          as.data.frame(VarCorr(Model1b))[2,4] +
          as.data.frame(VarCorr(Model1b))[3,4] +
          as.data.frame(VarCorr(Model1b))[4,4]

print("ICC for Children in Same Primary School but Different Secondary Schools")
as.data.frame(VarCorr(Model1b))[2,4] / total1c
[1] 0.22902566

print("ICC for Children in Same Secondary School but Different Primary Schools")
as.data.frame(VarCorr(Model1b))[3,4] / total1c
[1] 0.10252221

print("ICC for Children in Same Primary School and Same Secondary School")
(as.data.frame(VarCorr(Model1b))[1,4] +
 as.data.frame(VarCorr(Model1b))[2,4] +
 as.data.frame(VarCorr(Model1b))[3,4]) / total1c
[1] 0.36712458
```

UniqueID random intercept variance now contributes extra to the same-school ICC

```
print("Show ICC for each school type"); icc(Model1c, by_group=TRUE)
Group     |   ICC
-----------------
UniqueID  | 0.036
PschoolID | 0.229
SschoolID | 0.103

print("Show ICC for same primary and secondary"); icc(Model1c)
  Unadjusted ICC: 0.367 → was 0.334 before random interaction (LRT → not different)
```

Child SES: Empty Means, Primary by Secondary AND Unique Crossed Random Intercepts

$$SES_{cps} = \gamma_{000} + U_{0p0} + U_{00s} + U_{0ps} + e_{cps}$$

```
display "STATA Empty Means, Three-Way Crossed Model for SES Predictor"
mixed ses  , || _all: R.SschoolID || PschoolID: || UniqueID: ,  ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog

print("R Empty Means, Three-Way Crossed Model for SES Predictor")
EmptySES = lmer(data=Example5, REML=TRUE,
                formula=ses~1+(1|PschoolID)+(1|SschoolID)+(1|UniqueID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(EmptySES, chkREML=FALSE); summary(EmptySES, ddf="Satterthwaite")
print("Show intraclass correlation by random type and its LRT")
icc(EmptySES, by_group=TRUE); icc(EmptySES); ranova(EmptySES)
```

```
Random effects:
 Groups    Name        Variance          Std.Dev.
 UniqueID  (Intercept) 0.03207762714616 0.179102281
 PschoolID (Intercept) 0.00000000000000 0.000000000
 SschoolID (Intercept) 0.00000000022738 0.000015079
 Residual              1.92235299693585 1.386489451

Fixed effects:
              Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)   4.097514   0.044536 469.460512  92.004 < 2.2e-16

optimizer (nloptwrap) convergence code: 0 (OK)
boundary (singular) fit: see help('isSingular')
```

ICC values are "NA" indicating no detectable school variance in SES…

```
ANOVA-like table for random-effects: Single term deletions
                npar  logLik     AIC       LRT Df Pr(>Chisq)
<none>             5 -1755.62 3521.24
(1 | PschoolID)    4 -1755.62 3519.24 0.0000000  1    1.00000
(1 | SschoolID)    4 -1755.62 3519.24 0.0000000  1    1.00000
(1 | UniqueID)     4 -1755.67 3519.33 0.0964561  1    0.75612
```

Even though SES does not have significant school variance, we will still examine its contextual effects to demonstrate proper specification of fixed effects of level-1 predictors in cross-classified models. This is also warranted by the significant correlation between unique primary/secondary achievement and SES.

Model 2: Model 1b + Primary*Secondary School Denomination (0= not religious, 1= religious)

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}(Pdenom_p) + \gamma_{001}(Sdenom_s) + \gamma_{011}(Pdenom_p)(Sdenom_s) + U_{0p0} + U_{00s} + e_{cps}$$

```
display "STATA Model 2: Model 1b + Primary*Secondary School Denomination"
mixed achieve c.Pdenom c.Sdenom c.Pdenom#c.Sdenom ,  ///
      || _all: R.SschoolID || PschoolID: ,           ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2       // Print -2LL for model


print("R Model 2: Model 1b + Primary*Secondary School Denomination")
Model2 = lmer(data=Example5, REML=TRUE,
              formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom+(1|PschoolID)+(1|SschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model2, chkREML=FALSE); summary(Model2, ddf="Satterthwaite")
```

```
       AIC        BIC     logLik   deviance   df.resid
 2326.5332  2360.8875 -1156.2666  2312.5332   993.0000
```

```
Random effects:
 Groups    Name        Variance Std.Dev.
 PschoolID (Intercept) 0.172047 0.41479
 SschoolID (Intercept) 0.072549 0.26935
 Residual              0.504291 0.71013
Fixed effects:
                Estimate Std. Error         df t value Pr(>|t|)
(Intercept)     6.167016   0.137467  78.411238 44.8617   <2e-16
Pdenom          0.073058   0.144450  75.522971  0.5058   0.6145
Sdenom          0.100602   0.126084  45.008101  0.7979   0.4291
Pdenom:Sdenom   0.160770   0.100562 949.193305  1.5987   0.1102
```

Pseudo-R2 Relative to CovEmpty2 (from SAS)

Name	CovParm	Subject	Estimate	StdErr	PseudoR2
CovEmpty2	UN(1,1)	PschoolID	0.1747	0.04070	.
CovEmpty2	UN(1,1)	SschoolID	0.07873	0.02513	.
CovEmpty2	Residual		0.5052	0.02358	.
CovPxSdenom	UN(1,1)	PschoolID	0.1720	0.04059	0.015208
CovPxSdenom	UN(1,1)	SschoolID	0.07255	0.02407	0.078470
CovPxSdenom	Residual		0.5043	0.02356	0.001712

Which fixed slope should have caused the reduction in each pile of variance?

Model 3a: Add Level-1 Child SES (centered at 4)

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}(Pdenom_p) + \gamma_{001}(Sdenom_s) + \gamma_{011}(Pdenom_p)(Sdenom_s) + \gamma_{100}(SES_{cps} - 4) + U_{0p0} + U_{00s} + e_{cps}$$

```
display "STATA Model 3a: Add Child SES"
mixed achieve c.Pdenom c.Sdenom c.Pdenom#c.Sdenom c.ses4, ///
      || _all: R.SschoolID || PschoolID: ,              ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2      // Print -2LL for model
```

```
print("R Model 3a: Add Child SES")
Model3a = lmer(data=Example5, REML=TRUE, formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom
                +ses4+(1|PschoolID)+(1|SschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model3a, chkREML=FALSE); summary(Model3a, ddf="Satterthwaite")
```

```
      AIC       BIC     logLik  deviance   df.resid
2292.8588 2332.1208 -1138.4294 2276.8588   992.0000

Random effects:
 Groups    Name        Variance Std.Dev.
 PschoolID (Intercept) 0.172560 0.41540
 SschoolID (Intercept) 0.067836 0.26045
 Residual              0.483284 0.69519

Fixed effects:
                Estimate Std. Error         df t value         Pr(>|t|)
(Intercept)     6.170877   0.135422  79.188519 45.5679        < 2.2e-16
Pdenom          0.051524   0.143693  74.532897  0.3586          0.72093
Sdenom          0.072529   0.122555  45.283271  0.5918          0.55692
ses4            0.106591   0.016259 939.145507  6.5557 0.00000000009125
Pdenom:Sdenom   0.198812   0.098621 947.929758  2.0159          0.04409
```

What are we assuming in estimating this level-1 child SES fixed slope by itself?

Pseudo-R2 Relative to CovEmpty2 (from SAS)
Change in Pseudo-R2 for CovPxSdenom vs. CovSES1

Name	CovParm	Subject	Estimate	StdErr	PseudoR2	PseudoR2Change
CovEmpty2	UN(1,1)	PschoolID	0.1747	0.04070	.	.
CovEmpty2	UN(1,1)	SschoolID	0.07873	0.02513	.	.
CovEmpty2	Residual		0.5052	0.02358	.	.
CovPxSdenom	UN(1,1)	PschoolID	0.1720	0.04059	0.01521	.
CovPxSdenom	UN(1,1)	SschoolID	0.07255	0.02407	0.07847	.
CovPxSdenom	Residual		0.5043	0.02356	0.00171	.
CovSES1	UN(1,1)	PschoolID	0.1726	0.04045	0.01215	-0.003058
CovSES1	UN(1,1)	SschoolID	0.06784	0.02266	0.13832	0.059847
CovSES1	Residual		0.4833	0.02259	0.04330	0.041592

Model 3b: Add Contextual SES Slopes (each centered at 4)

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}\left(Pdenom_p\right) + \gamma_{001}\left(Sdenom_s\right) + \gamma_{011}\left(Pdenom_p\right)\left(Sdenom_s\right)$$
$$+ \gamma_{100}\left(SES_{cps} - 4\right) + \gamma_{020}\left(\overline{SES}_p - 4\right) + \gamma_{002}\left(\overline{SES}_s - 4\right) + \gamma_{022}\left(\overline{SES}_{ps} - 4\right)$$
$$+ U_{0p0} + U_{00s} + e_{cps}$$

```
display "STATA Model 3b: Add Child SES Contextual Effects"
mixed achieve c.Pdenom c.Sdenom c.Pdenom#c.Sdenom  ///
              c.ses4 c.PMses4 c.SMses4 c.PSMses4,  ///
      || _all: R.SschoolID || PschoolID: ,         ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2      // Print -2LL for model
estimates store FitFix          // Save for LRT

print("R Model 3b: Add Child SES Contextual Effects")
Model3b = lmer(data=Example5, REML=TRUE, formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom
                +ses4+PMses4+SMses4+PSMses4+(1|PschoolID)+(1|SschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model3b, chkREML=FALSE); summary(Model3b, ddf="Satterthwaite")
```

```
       AIC         BIC      logLik    deviance    df.resid
 2304.8990   2358.8843 -1141.4495   2282.8990    989.0000

Random effects:
 Groups    Name        Variance Std.Dev.
 PschoolID (Intercept) 0.177091 0.42082
 SschoolID (Intercept) 0.066498 0.25787
 Residual              0.483690 0.69548
```

Previous SES slope:
ses4 0.106591

```
Fixed effects:
               Estimate Std. Error         df t value Pr(>|t|)
(Intercept)    6.166405   0.138285  77.484221 44.5920  < 2e-16
Pdenom         0.049632   0.145087  72.327641  0.3421  0.73328
Sdenom         0.054282   0.122940  42.604036  0.4415  0.66106
ses4           0.089837   0.027751 914.494237  3.2373  0.00125
PMses4        -0.036125   0.230406  48.395383 -0.1568  0.87606
SMses4         0.179069   0.175583  38.457301  1.0199  0.31417
PSMses4        0.023378   0.034383 915.093916  0.6799  0.49673
Pdenom:Sdenom  0.200339   0.098777 945.776817  2.0282  0.04282
```

What do the new SES effects represent?

```
optimizer (nloptwrap) convergence code: 0 (OK)
Model failed to converge with max|grad| = 0.00241881 (tol = 0.002, component 1)
```

R said the model had estimation problems, whereas SAS and STATA said it was fine, so…?

Pseudo-R2 Relative to CovEmpty2 (from SAS)
Change in Pseudo-R2 for CovSES1 vs. CovSES2

Name	CovParm	Subject	Estimate	StdErr	PseudoR2	PseudoR2Change
CovEmpty2	UN(1,1)	PschoolID	0.1747	0.04070	.	.
CovEmpty2	UN(1,1)	SschoolID	0.07873	0.02513	.	.
CovEmpty2	Residual		0.5052	0.02358	.	.
CovPxSdenom	UN(1,1)	PschoolID	0.1720	0.04059	0.01521	.
CovPxSdenom	UN(1,1)	SschoolID	0.07255	0.02407	0.07847	.
CovPxSdenom	Residual		0.5043	0.02356	0.00171	.
CovSES2	UN(1,1)	PschoolID	0.1771	0.04182	-0.01367	-0.028876
CovSES2	UN(1,1)	SschoolID	0.06646	0.02274	0.15581	0.077343
CovSES2	Residual		0.4837	0.02262	0.04248	0.040769

Model 4a: Add Random WPS-Centered Child SES across Primary Schools

$$\begin{aligned} Achieve_{cps} &= \gamma_{000} + \gamma_{010}\left(Pdenom_p\right) + \gamma_{001}\left(Sdenom_s\right) + \gamma_{011}\left(Pdenom_p\right)\left(Sdenom_s\right) \\ &+ \gamma_{100}\left(SES_{cps} - 4\right) + \gamma_{020}\left(\overline{SES}_p - 4\right) + \gamma_{002}\left(\overline{SES}_s - 4\right) + \gamma_{022}\left(\overline{SES}_{ps} - 4\right) \\ &+ U_{0p0} + U_{2p0}\left(SES_{cps} - \overline{SES}_{ps}\right) + U_{00s} + e_{cps} \end{aligned}$$

```
display "STATA Model 4a: Add Random WPS-Centered Child SES across Primary Schools"
mixed achieve c.Pdenom c.Sdenom c.Pdenom#c.Sdenom ///
              c.ses4 c.PMses4 c.SMses4 c.PSMses4, ///
      || _all: R.SschoolID || PschoolID: WPSses, cov(un) ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2       // Print -2LL for model
estat recovariance, relevel(PschoolID) correlation  // Random effect correlations
estimates store FitRandP         // Save for LRT
lrtest FitRandP FitFix           // LRT for random slope over primary?

print("R Model 4a: Add Random WPS-Centered Child SES across Primary Schools")
Model4a = lmer(data=Example5, REML=TRUE, formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom
               +ses4+PMses4+SMses4+PSMses4+(1+WPSses|PschoolID)+(1|SschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model4a, chkREML=FALSE); summary(Model4a, ddf="Satterthwaite")
print("LRT for random slope"); ranova(Model4a)
```

```
      AIC        BIC     logLik   deviance   df.resid
 2308.7927  2372.5935 -1141.3963  2282.7927   987.0000

Random effects:
 Groups    Name        Variance    Std.Dev.  Corr
 PschoolID (Intercept) 0.177109195 0.4208434
           WPSses      0.000095715 0.0097834 1.000
 SschoolID (Intercept) 0.066420623 0.2577220
 Residual              0.483636425 0.6954397
```

Note that a correlation for the new random slope was estimated only with the primary school random intercept, not with the secondary school random intercept.

Btw, slope reliability = .002!

```
Fixed effects:
                Estimate Std. Error        df t value  Pr(>|t|)
(Intercept)     6.165546   0.138209 77.499766 44.6105 < 2.2e-16
Pdenom          0.051368   0.144961 72.344770  0.3544  0.724101
```

```
Sdenom            0.054003   0.122892  42.626423  0.4394  0.662566
ses4              0.090360   0.027785 824.144510  3.2521  0.001192
PMses4           -0.036419   0.230191  48.395037 -0.1582  0.874947
SMses4            0.178770   0.175511  38.469166  1.0186  0.314772
PSMses4           0.022876   0.034409 877.591073  0.6648  0.506328
Pdenom:Sdenom     0.200605   0.098770 945.766909  2.0310  0.042532

ANOVA-like table for random-effects: Single term deletions
                                   npar  logLik    AIC     LRT Df Pr(>Chisq)
<none>                               13 -1141.40 2308.79
WPSses in (1 + WPSses | PschoolID)   11 -1141.45 2304.90  0.1063  2    0.94822
(1 | SschoolID)                      12 -1170.26 2364.52 57.7235  1 3.0168e-14
```

95% random effect confidence interval for student SES slope across primary schools:
Fixed effect ± 1.96*SQRT(random variance)
Primary Student SES Slope: 0.090 ± 1.96*SQRT(0.000095715) = 0.071 to 0.110 (so not much variation)

What kind of fixed effects would have explained the **WPSses** random slope variance over primary schools?

Model 4b: Model 3b + Random WPS-Centered Child SES across Secondary Schools

$$
\begin{aligned}
Achieve_{cps} = {} & \gamma_{000} + \gamma_{010}\left(Pdenom_p\right) + \gamma_{001}\left(Sdenom_s\right) + \gamma_{011}\left(Pdenom_p\right)\left(Sdenom_s\right) \\
& + \gamma_{100}\left(SES_{cps} - 4\right) + \gamma_{020}\left(\overline{SES}_p - 4\right) + \gamma_{002}\left(\overline{SES}_s - 4\right) + \gamma_{022}\left(\overline{SES}_{ps} - 4\right) \\
& + U_{0p0} + U_{20s}\left(SES_{cps} - \overline{SES}_{ps}\right) + U_{00s} + e_{cps}
\end{aligned}
$$

Note that I had to switch the assignment of the random model parts (from **R.SschoolID** to **R.PschoolID**) in STATA to estimate the secondary school random slope and its covariance with the secondary random intercept. I have not been successful in getting any crossed model with random slopes for each level-2 dimension to work, as the **_all: R** option does not allow random slopes. Without it, STATA assumes that the second set of random effects are nested in the first set (i.e., a three-level model, not a two-level crossed model).

```
display "STATA Model 4b: Model 3b + Random WPS-Centered Child SES across Secondary Schools"
mixed achieve c.Pdenom c.Sdenom c.Pdenom#c.Sdenom  ///
              c.ses4 c.PMses4 c.SMses4 c.PSMses4,  ///
      || _all: R.PschoolID || SschoolID: WPSses, cov(un) ///
      reml dfmethod(satterthwaite) dftable(pvalue) nolog
display "-2LL = " e(ll)*-2       // Print -2LL for model
estat recovariance, relevel(PschoolID) correlation  // Random effect correlations
estimates store FitRandS         // Save for LRT
lrtest FitRandS FitFix           // LRT for random slope over secondary?

print("R Model 4b: Model 3b + Random WPS-Centered Child SES across Secondary Schools")
Model4b = lmer(data=Example5, REML=TRUE, formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom
               +ses4+PMses4+SMses4+PSMses4+(1|PschoolID)+(1+WPSses|SschoolID))
print("Show results using Satterthwaite DDF including -2LL as deviance")
llikAIC(Model4a, chkREML=FALSE); summary(Model4b, ddf="Satterthwaite")
print("LRT for random slope"); ranova(Model4b)
```

```
      AIC        BIC     logLik   deviance   df.resid
 2308.7927  2372.5935 -1141.3963 2282.7927   987.0000

Random effects:
 Groups    Name        Variance   Std.Dev. Corr
 PschoolID (Intercept) 0.17693994 0.420642
 SschoolID (Intercept) 0.06651145 0.257898
           WPSses      0.00013417 0.011583 1.000
 Residual              0.48361733 0.695426
```

Note that a correlation for the new random slope was estimated only with the secondary school random intercept, not with the primary school random intercept.

Btw, slope reliability = .004!

```
Fixed effects:
               Estimate Std. Error         df t value  Pr(>|t|)
(Intercept)     6.164304   0.138176  77.593983 44.6120 < 2.2e-16
Pdenom          0.049769   0.145040  72.329355  0.3431  0.732490
Sdenom          0.059055   0.122717  42.762567  0.4812  0.632808
ses4            0.089433   0.027833 613.654686  3.2132  0.001381
PMses4         -0.035359   0.230319  48.386651 -0.1535  0.878625
SMses4          0.171820   0.175345  38.575153  0.9799  0.333245
PSMses4         0.023738   0.034447 762.649047  0.6891  0.490961
Pdenom:Sdenom   0.200014   0.098768 945.739665  2.0251  0.043139

ANOVA-like table for random-effects: Single term deletions
                                  npar   logLik     AIC      LRT Df Pr(>Chisq)
<none>                              13 -1141.38 2308.77
(1 | PschoolID)                     12 -1237.88 2499.77 193.0031  1    < 2e-16
WPSses in (1 + WPSses | SschoolID)  11 -1141.45 2304.90   0.1315  2    0.93636
```

What kind of fixed effects would have explained the **`WPSses`** random slope variance over secondary schools?

95% random effect confidence interval for student SES slope across secondary schools:
Fixed effect ± 1.96*SQRT(random variance)
Secondary Student SES Slope: 0.089 ± 1.96*SQRT(0.00013417) = 0.067 to 0.112

Sample Results Section [indicates notes about what to change]

The extent to which 9th grade academic achievement could be predicted from school denomination and child socio-economic status (SES) was examined in a series of multilevel models with crossed random effects (i.e., for child cross-classification). Specifically, the 1,000 students at level 1 were modeled as nested within their 50 primary schools at level 2, as well as within their 33 secondary schools at level 2, such that primary and secondary schools were crossed sampling dimensions at level 2. Residual maximum likelihood (REML) within SAS MIXED [or STATA MIXED or R `lmer`] was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with Wald tests using Satterthwaite denominator degrees of freedom, whereas random effects were evaluated via likelihood ratio tests (i.e., $-2\Delta LL$ with degrees of freedom equal to the number of new random effects variances and covariances). Alpha was chosen as .05. Effect size was evaluated via pseduo-R^2 values for the proportion reduction in each variance component.

We first examined the extent of dependency due to mean differences by including a random intercept variance for each type of school. Relative to a model assuming independent children (i.e., with only a single model residual), adding a random intercept variance across primary schools significantly improved model fit, $-2\Delta LL(1) = 170.63$, $p < .001$. Adding another random intercept variance across secondary schools also significantly improved model fit, $-2\Delta LL(1) = 75.47$, $p < .001$, providing empirical support for the need to model the cross-classification of students within primary and secondary schools. Given 1–6 children within each unique combination of primary and secondary schools, we also examined the need for a random primary by secondary interaction. It was removed given that it did not significantly improve model fit, $-2\Delta LL(1) = 0.99$, $p = .321$, indicating no extra correlation of children from the same unique combination. Of the total variation in child achievement, 23.0% reflected mean differences between primary schools, 10.4% reflected mean differences between secondary schools, and 66.6% reflected reamining between-children differences after controlling for primary and secondary school additive effects. A 95% random effects confidence interval was calculated for each source of intercept variation as the fixed intercept ± 1.96* SQRT(random intercept variance), which revealed that 95% of the primary schools were predicted to have intercepts for school mean achievement between 5.52 and 7.16, whereas 95% of the secondary schools were predicted to have intercepts for school mean achievement between 5.79 and 6.89.

We then added the effects for the denomination status (0 = not religious, 1 = religious) for the primary school and for the secondary school, as well as for their interaction. Both indicated nonsignificantly greater achievement outcomes for denominational schools with no significant interaction. Primary school denomination captured 1.52% of the primary school random intercept variance, secondary school denomination captured 7.85% of the secondary school random intercept variance, and their interaction captured 0.17% of the level-1 residual variance. However, all three denomination predictors were retained in the model as control variables.

We then considered the effects of child SES (centered at 4). Its fixed slope was significantly positive, such that child achievement was expected to be larger by 0.107 per unit SES. However, the incusion of a single fixed slope for child SES assumes no contextual effects of any kind. To test this assumption, and to ensure proper interpretation of the child-level SES fixed effect as the within-school effect, we added three level-2 contextual SES effects (each centered at 4): primary school mean SES, secondary school mean SES, and the unique combination of primary by secondary school mean SES.

The level-1 SES effect—now representing the pure within-school effect—was significantly positive and indicated that child achievement was expected to be larger by 0.107 per unit greater SES than the mean of the child's primary and secondary school combination. The following level-2 contextual effects are each interpreted as the incremental contribution of the school after controlling for child SES. The level-2 contextual SES effect for primary schools indicated that primary school achievement was nonsignificantly lower by 0.036 per unit higher primary mean SES. Likewise, the level-2 contextual SES effect for secondary schools indicated that secondary school achievement was nonsignificantly higher by 0.179 per unit higher secondary mean SES. Finally, the level-2 contextual SES effect for the unique combination of primary and secondary schools indicated that child achievement was expected to be nonsignificantly higher by be 0.023 per unit higher school combination mean SES. The SES effects in total accounteed for none of the primary school random intercept variance, 7.73% of the secondary school random intercept variance, and 4.08% of the level-1 residual variance.

Lastly, we considered the potential for random slopes for the child SES effect (using a within-unique-combination centered predictor to avoid conflated random slopes). The SES slope variation resulted in non-positive-definite matrices of random effect variances and covariances. Within-school child SES slope variation was nonsignificant across primary schools, $-2\Delta LL(\sim 2) = 0.106$, $p = .948$, as well as across secondary schools, $-2\Delta LL(\sim 2) = 0.132$, $p = .936$, indicating that the size of the relative child SES advantage did not differ significantly across each type of school.