

# Introduction to this Course: Multilevel Models for Longitudinal Data

- Topics:
  - What to expect this semester
  - Features of longitudinal data
  - Features of longitudinal models
  - What can MLM do for you?

# What To Expect This Semester

- You will expand your knowledge of **longitudinal models** → analysis of *repeated* observations from same sampling *unit*
  - “Units” can be anything: persons, schools, countries, animals...
  - “Repeated” can span any length of time (milliseconds to days to years)
  - “Repeated” can also include trials: across items, conditions, etc.
- This will NOT require anxiety-provoking behaviors like:
  - Calculating things by hand—computers are always better, and more advanced statistical methods cannot be implemented by hand anyway
  - Deriving formulas or results—it’s ok to trust the people who specialize in these areas to have gotten it right and use their work (for now, at least)
  - Memorizing formulas—it’s ok to trust the computer programmers who have implemented various statistical techniques (less so in R, though)
- It WILL require learning and implementing **new language and logic**
  - **Language:** words, notation (symbols and equations), program syntax
  - **Logic:** decision guidelines for matching data and questions to models

# What to Expect This Semester

- I believe that **everyone is capable** and **can significantly benefit\*\*** from learning more types of quantitative methods (*especially* multilevel models)!
- **Philosophy:** Focus on accessibility + mastery learning
- **Materials:** Unit = lecture + example(s); 7 units planned
  - **Lecture** slides present concepts—the (wordy) **what** and the **why**
  - **Example** documents: reinforce the concepts and demonstrate the **how using software**—SAS, STATA, or R (your choice)
  - All available at the [course website](#) (hosted outside of ICON)

\*\* **Benefits** include but are not limited to: Better research, more authorship opportunities, and actual money

# What To Expect This Semester

- I will NOT:
  - Use infrequent high-stakes testing to assess your level of learning
  - Assume you know any models beyond the GLM to begin with!
    - But I will make connections to other modeling frameworks (MLM=SEM)
- I WILL:
  - Use **formative assessments (FA)** to help you figure out what you need to review (7 planned; 14 points for **completing them at all**)
  - Require **online homework (HW) assignments** that give you real-data practice (5 planned; 86 points for **completing them accurately**)
    - Complete demo “HW 0” for 2 points extra credit
    - All canned data this semester, but questions about your own analyses are welcome—and appreciated!—during class or office hours
  - Link research designs, data, questions, and models explicitly
    - If we don’t cover the exact combination you need in class, just ask me—odds are good I have an example from elsewhere that I can send you!

# More About the Course Requirements

- **Everything** is take-home, open-note, and untimed
- Late\* work will be accepted (–2 for HW; –1 for FA)
  - *\*Extensions granted if requested at least 2 weeks in advance*
  - HW due dates **may be pushed later** (to ensure approximately 1 week after covering the material before it's due), but never sooner
- **Formative assessments:** Big-picture questions to provide a structured review (will go over answers at the next class)
  - Also likely to include case studies and open-ended review prompts
- **Homework assignments:** Practice doing data analysis
  - Based directly on examples given (no googling required)
  - You will each have a unique dataset (made with a common story)
  - **Computation** sections: Instant feedback, infinite attempts
  - **Results** (interpretation) sections: Delayed feedback, single attempt (but repetition of concepts and vocabulary across the semester)

# Our Other Responsibilities

- My job (besides providing materials and assignments):
  - **Answer questions** via email, in individual meetings, or in group-based zoom office hours—you can each work on homework during office hours and get (near) immediate assistance (and then keep working)
- Your job (in descending order of timely importance):
  - **Ask questions**—preferably in class, but any time is better than none
  - **Frequently review** the class material, focusing on mastering the vocabulary (words and symbols), logic, and procedural skills
  - Don't wait until the last minute to start homework, and don't be afraid to **ask for help if you get stuck** on one thing for more than 15 minutes
    - Please email me a screenshot of your code+error so I can respond easily
  - **Do the readings** for a broader perspective and additional examples (best after lecture; readings are for the whole unit, not just that day)
  - **Practice** using the software to implement the techniques you are learning **on data you care about**—this will help you so much more!

# More About Your Experience in this Class

- **Attendance:** Strongly recommended but not required
  - **You choose** (for any reason): In-person or zoom
  - **Masks** are \*STRONGLY encouraged\* for in-person attendees
  - **Please do not attend in-person if you were exposed to Covid!** (or monkey pox, or whatever new pandemic is next)
  - You won't miss out: I will post **YouTube-hosted recordings** (audio + screenshare only) for each class at the [course website](#)
  - **Ask questions aloud or in the zoom chat window (+DM)** (even if you are attending class in person)
- **Changes** will be sent via email by 9 am on class days
  - I will change to zoom-only if I am exposed to Covid!
  - I will change to zoom-only for dangerous weather
  - Nothing is more important than our health and safety...

# Class-Sponsored Statistical Software

- To help address the needs of different Iowa degree programs, I will show examples using **SAS, STATA, and R** software
  - **SAS** = “Statistical Analysis System” (likely for the last time ☹)
  - **STATA** (aka, Stata) = “Software for Statistics and Data Science”
  - **R** = free implementation of what was initially the “S” language
- **Why not SPSS?** Because it doesn’t have as much room to grow (and thus it isn’t used in any other EMS advanced classes)
  - As in SPSS, drop-down windows can also generate syntax in STATA and in SAS “enterprise” (which I don’t use, and you won’t need to)
  - SPSS could be used for some—but not all—of our content
- **My story:** After SPSS, I became a heavy-duty **SAS enthusiast** who:
  - Picked up enough STATA initially to teach workshops using it, and I am learning it better now that I teach it in my classes
  - Is (begrudgingly) learning enough (base) R to add it to my classes, so it’s possible you learned how to do the same things differently but correctly
  - So if you have **STATA or R tips**, please share them with me!



# Which Program: SAS, STATA, or R?

- **I am assuming you know how to use at least one of these!**
  - Each is available (with VPN) in the free [U Iowa Virtual Desktop](#)
  - More programs = more “technical skills” for your CV; easier collaboration with colleagues (who only know one program)
  - For intro handouts and videos, please see materials [posted 2/7 here](#)
- **To consider** when choosing which program to focus on:
  - Future use: R can be freely installed on your own machine; SAS has a free web-based [SAS OnDemand](#); STATA install = \$\$\$
  - **STATA** is popular in fields that use **large, weighted survey data** (e.g., sociology, political science, public health, EPLS at Iowa)
  - **R** will be used exclusively in classes by Drs. Aloe, LeBeau, or Templin, and it has become increasingly mainstream, **but**:
    - R packages are only as good as their authors (so little quality control)
    - Syntax and capabilities are idiosyncratic to the packages (grrrrrr)
    - I have found incorrect or impossible results, even for “good” packages

# Class-Sponsored Statistical Software

- **SAS** will be the primary output emphasized, as its MIXED (and GLIMMIX) packages are the most comprehensive and stable
- [PilesOfVariance.com](https://PilesOfVariance.com) currently has examples of every model in my textbook using SAS, SPSS, STATA, and Mplus (and some using R)
  - Some textbook examples will be used in class, along with unique examples
  - **STATA MIXED and R (various packages):** can used for homework and I will provide syntax and output for all unique class examples
  - **SPSS MIXED:** syntax is very similar to that of SAS MIXED, and it has largely the same functionality (so you can use it for course homework), but I will not be adding it to any unique course examples
  - **Mplus:** syntax is very different, and it does not have REML estimation, and thus you cannot use it for homework in this class
    - But stay tuned for an “advanced longitudinal (SEM)” as PSQF 7375 in Spring 2023 that will use Mplus (along with the other packages when possible)

# This Semester: Longitudinal Multilevel Models

- **Background:** how this relates to what you already know
  - Concepts in longitudinal modeling (and why use MLMs)
  - Review of concepts to carry forward from single-level linear models
  - Within-person analysis via Repeated Measures ANOVA
- **“Unconditional” longitudinal models:** describing an outcome’s mean, variance, and covariance over time
  - Due to within-person fluctuation via alternative covariance structures
  - Due to within-person change via fixed and random effects of time (polynomial, piecewise, and exponential models)
  - This lengthy component can be skipped for other kinds of multilevel data
- **“Conditional” longitudinal models:** adding predictors
  - Time-invariant (level-2) cross-sectional predictors
  - Time-varying (level-1) predictors within multivariate MLMs (“M-SEM”) will be addressed in the Spring 2023 PSQF 7375 advanced class instead

# Data Requirements for Our Models

- A useful outcome variable:
  - Has an interval scale\*
    - A one-unit difference means the same thing across all scale points
    - In subscales, each contributing item has an equivalent scale
    - *\*Other kinds of outcomes can be analyzed using generalized multilevel models instead, to be addressed in the Spring 2023 advanced class*
  - Has scores\* with the same meaning over observations
    - Includes meaning of construct and/or how items relate to the construct
    - Implies measurement invariance over time
    - *\*This class will address observed outcomes only; change over time in latent variables will be addressed in the Spring 2023 advanced class*
- FANCY MODELS STILL CANNOT SAVE BADLY MEASURED VARIABLES OR CONFOUNDED RESEARCH DESIGNS.

# Requirements for Longitudinal Data

- Multiple **occasions** from same sampling unit (=person)
  - 2 occasions is the minimum, but just 2 can lead to problems:
    - Only 1 kind of change is observable (1 linear difference)
    - Can't distinguish "real" individual differences in change from error
    - Repeated measures ANOVA is just fine for 2 (complete) observations
      - Necessary assumption of "sphericity" is satisfied with only 2 observations even if compound symmetry doesn't hold
  - More data is better (with diminishing returns)
    - More occasions → better description of the form of change; more power to show effects of time-varying predictors (measured repeatedly)
    - More units (persons) → better estimates of the amount of individual differences in change; better prediction of those individual differences
    - More items/stimuli/groups → more power to show effects of differences between items/stimuli/groups (i.e., for each dimension of sampling)

# Power in Longitudinal Data

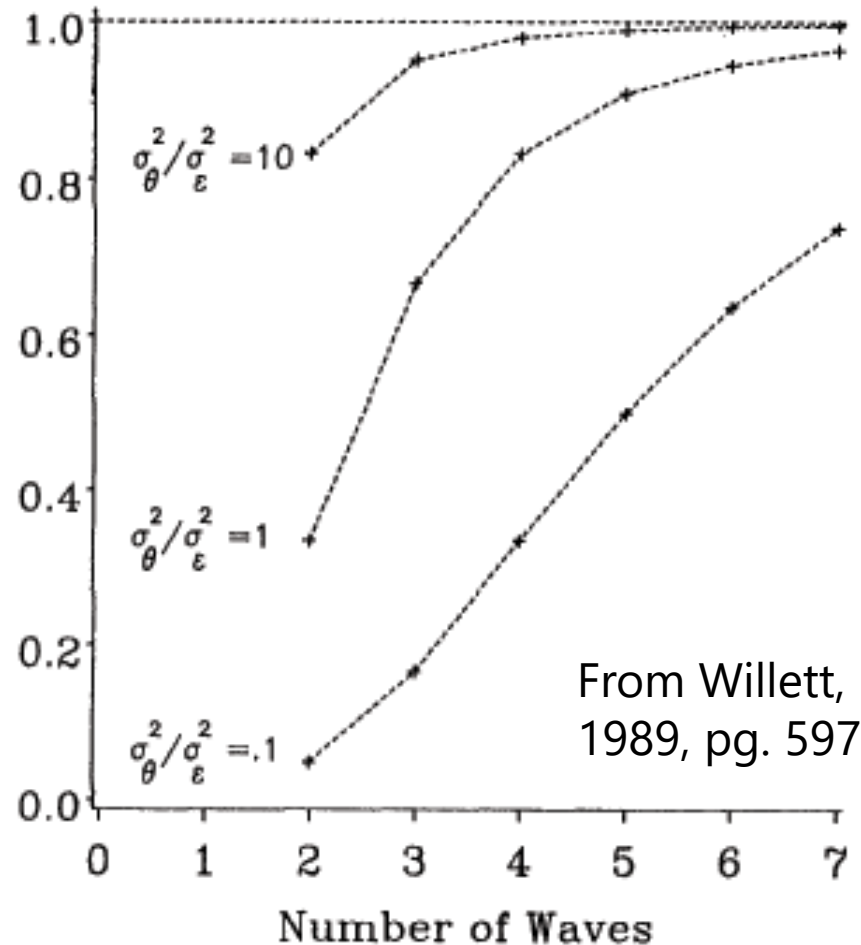
- **More occasions are better!**

- Can examine more complex growth functions
- Can get more reliable individual growth parameters

- **More units (people) are better!**

- Can get more reliable estimates and prediction of individual differences

Reliability of Slopes (y-axis) by Signal-to-Noise Ratio and # Occasions



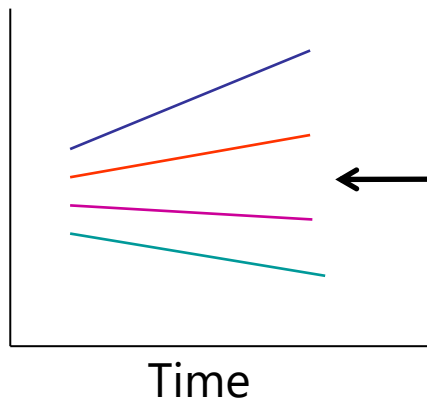
# Levels of Analysis in Longitudinal Data

- **Between-Person\* (BP) Variation:**
  - Macro – **Level-2** – “**INTER**-individual Differences” – Time-Invariant
  - All longitudinal studies that begin as cross-sectional studies have this
- **Within-Person\* (WP) Variation:**
  - Micro – **Level-1** – “**INTRA**-individual Differences” – Time-Varying
  - Only longitudinal studies can provide this extra type of information!
- Longitudinal studies allow examination of **both types** of relationships simultaneously (and their interactions)
  - **Any** variable measured over time usually has both BP and WP variation
  - BP = more/less than other people; WP = more/less than usual
- *\*I will use **person**, but “between” units can be anything that is measured repeatedly (like animals, schools, countries...)*

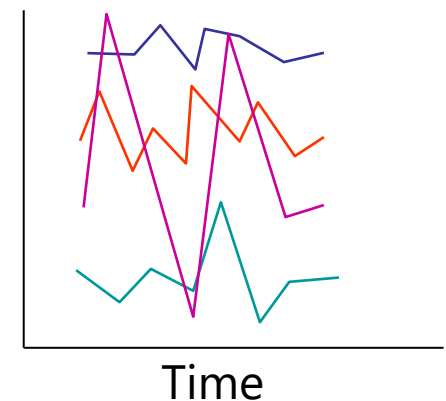
# A Longitudinal Data Continuum

- **Within-Person (WP) Change:** Expect systematic effect(s) of time
  - e.g., "(Latent) Growth Curve Models" → **Time is meaningfully sampled**
  - If magnitude or direction of change differs across individuals, then the outcome's variance and covariance will change over time, too!
- **Within-Person (WP) Fluctuation:** Few *expected* effects of time
  - Outcome just varies/fluctuates over time (e.g., emotion, mood, stress)
  - **Time is just a way to get lots of data per person** (e.g., EMA studies)
  - Lends itself to questions about effects of relative changes and inconsistency

Pure WP Change



Pure WP Fluctuation





# Sources of “Time” in Longitudinal Data

- What aspects of “**time**” are relevant?
  - **WP change**: e.g., time in study, age, grade, time to/from event
  - **WP fluctuation**: e.g., time of day, day of week, day in study
- Does time vary **within persons (WP)** AND **between persons (BP)**?
  - If people differ in time at the study beginning (e.g., accelerated designs), we will need to **differentiate BP time effects from WP time effects**
  - If there is more than one kind of WP “time” (e.g., occasions within days), we will need to **differentiate distinct sources of WP time effects**
- Is time *balanced* or *unbalanced*?
  - **Balanced** = **shared** measurement schedule (not necessarily equal interval)
    - Although some people may miss some occasions, making their data “incomplete”
  - **Unbalanced** = people have **different** possible time values
    - By definition, the possible outcomes are at least partially “incomplete” across persons
    - This may be a consequence of using a time metric that also varies between persons

# The Two Sides of \*Any\* Model

- Model for the Means:

- Aka **Fixed Effects**, Structural Part of Model
- What you are used to **caring about for testing hypotheses**
- How the expected outcome for a given observation varies as a weighted function of its values of the predictor variables
  - Fixed slopes are **estimated constants** that multiply predictors

- Model for the Variance (how many “piles”):

- Aka **Random Effects and Residuals**, Stochastic Part of Model
- What you \*were\* used to **making assumptions about** instead
- How **residuals are distributed and related** across sampling dimensions (persons, occasions) → these relationships are known as “**dependency**” and **this is the primary way that longitudinal models differ from “regular” (GLM) regression models**

# A Statistician's World View

- Outcome type: General (normal) vs. Generalized (not normal)
- Dimensions of sampling: One (so one variance term per outcome) vs. **Multiple** (so multiple variance terms per outcome) → **OUR WORLD**
- **General Linear Models**: conditionally normal outcome distribution, **fixed effects** (identity link; only one dimension of sampling) Note: OLS is only for GLM
- **Generalized Linear Models**: **any conditional outcome distribution**, **fixed effects** through **link functions**, no random effects (one dimension)
- **General Linear Mixed Models**: conditionally normal outcome distribution, **fixed and random effects** (identity link, but multiple sampling dimensions)
- **Generalized Linear Mixed Models**: **any conditional outcome distribution**, **fixed and random effects** through **link functions** (multiple dimensions)
  - Same concepts as for this course, but with more complexity in estimation
- "Linear" means fixed effects predict the *link-transformed* conditional mean of DV in a linear combination of (slope\*predictor) + (slope\*predictor)...

# Multilevel Model (MLM) Word Salad

- MLM is the same as other terms you have heard of:
  - **Linear Mixed-Effects Model** (fixed + random effects, of which intercepts and slopes are specific kinds of effects)
  - **Random Coefficients Model** (because coefficients also = effects)
  - **Hierarchical Linear Model** (not same as hierarchical regression)
- Special cases of MLM:
  - Random Effects ANOVA or Repeated Measures ANOVA
  - (Latent) Growth Curve Model (where “Latent” implies SEM software)
    - Btw, most MLMs can be equivalently estimated as single-level SEMs
  - Within-Person Fluctuation Model (e.g., for EMA or daily diary data)
    - See also “dynamic” SEM or multilevel SEM (even without measurement models!)
  - Clustered/Nested Observations Model (e.g., for kids in schools)
    - If followed over time in same group, is “clustered longitudinal model”
  - Cross-Classified Models (e.g., teacher “value-added” models)
  - Psychometric Models (e.g., factor analysis, item response theory, SEM)

# Options for Longitudinal Models

- Although models and software are logically separate, longitudinal data can be analyzed via multiple analytic frameworks (which are then usually tied to software):
  - **“Multilevel/Mixed-Effects/Hierarchical Linear Models”**
    - Dependency over time, persons, groups, etc. is modeled via random effects (multivariate through “levels” using stacked/long data)
    - Builds on GLM, generalizes easier to additional levels of analysis
  - **“Structural Equation Models”**
    - Dependency over time *only* is modeled via latent variables (single-level analysis using multivariate/wide data)
    - Generalizes easier to analysis of latent constructs, multivariate, mediation
  - Because random effects and latent variables are the same thing, many longitudinal models can be specified/estimated either way
    - And now “Multilevel Structural Equation Models” can do it all (maybe)...

# What can MLM do for you?

## 1. **Model dependency across observations**

- Longitudinal, clustered, and/or cross-classified data? No problem!
- Tailor your model of sources of correlation to your data

## 2. **Include categorical or continuous predictors at any level**

- Time-varying, person-level, group-level predictors for each variance
- Explore reasons for dependency, don't just control for dependency

## 3. **Does not require same data structure for each person**

- Unbalanced or missing data? No problem!

## 4. **You already know how (or you will soon)!**

- Use **SAS Mixed**, STATA Mixed, R, SPSS Mixed, Mplus, HLM, MlwiN...
- What's an intercept? What's a slope? What's a pile of variance?

# 1. Model Dependency

- Outcomes from the same sampling unit (i.e., person) will have one or more sources of **dependency** → **correlated residuals**
- **“Levels” for dependency** = “levels of random effects”
  - More generally, a **“level”** refers to a **dimension of sampling** that has **unexplained** outcome variability represented by 1+ random effects
  - Sampling dimensions can be **nested** in each other
    - e.g., time within person, time within group, trial within person
  - If you can't figure out the direction of your nesting structure, odds are good you have a **crossed sampling design** instead
    - e.g., persons crossed with items, raters crossed with targets
  - To have a “level”, there must be random outcome variation due to sampling that **remains** after including the model's fixed effects
    - e.g., treatment vs. control does not create another level of “group” (but it would if you had multiple treatment and multiple control groups)

# Longitudinal dependency comes from...

- **Mean differences** across sampling units (e.g., persons)
  - Creates constant dependency (residual correlation) over time
  - Will be represented by a **random intercept** in our models (or could be addressed by fixed main effects of person ID)
- **Slope differences** in effects of time-varying predictors
  - Individual differences in change over time, stress reactivity
  - Creates non-constant dependency, the size of which depends varies over time based on the value of the time-varying predictor
  - Will be represented by **random slopes** in our models (or could be addressed by fixed interaction effects of person ID)
- Non-constant within-person correlation for other unknown reasons (time-dependent autocorrelation)
  - Can add other patterns of correlation as needed for this (what I will call "alternative covariance structure" models)



# Why should we care about dependency?

- In other words, what happens if we have the wrong model for the variance (i.e., assume independent residuals instead)?
- **Validity of the tests of the predictors** depends on having the “most right” model for the variance
  - Estimates of slopes for level-specific predictors (i.e., only level 2 or level 1) will usually be ok (because they are from the model for the means)
  - Estimates of slopes for most time-varying predictors will NOT be ok!
  - **Standard errors** (and thus **p-values**) of slopes can be **incorrect**
- The sources of variation that are present in your outcome will dictate **what kinds of predictors** will be useful
  - Between-Person variation needs time-invariant predictors
  - Within-Person (time) variation needs time-varying predictors
  - Between-whatever variation needs Between-whatever predictors...
  - Bottom line: If it don't vary, GAME OVER for predicting it

## 2. Include categorical or continuous predictors at any level of analysis

- “ANOVA” test differences among discrete categories measured once (Between) or repeatedly measured (RM)
- “Regression” tests slopes for quantitative predictors measured once on outcomes measured once per person
- What if a predictor is measured repeatedly but can't be characterized by discrete “conditions”?
  - ANOVA or Regression won't work → you need MLM
- Some things don't change over time → time-invariant
- Some things do change over time → time-varying
- Some things are measured at higher levels
- Interactions are possible at same level or across levels

### 3. Does not require same data structure per person (by accident or by design)

RM ANOVA: uses **multivariate** (wide) data structure:

ID	Sex	T1	T2	T3	T4
100	0	5	6	8	12
101	1	4	7	.	11

People missing any data are excluded (data from ID 101 are not included at all)

MLM: uses **stacked** (long) data structure:

Only rows missing data are excluded

100 uses 4 cases

101 uses 3 cases

ID	Sex	Time	Y
100	0	1	5
100	0	2	6
100	0	3	8
100	0	4	12
-----			
101	1	1	4
101	1	2	7
101	1	3	.
101	1	4	11

Time can also be **unbalanced** across people such that each person can have their own measurement schedule: Time = "0.9" "1.4" "3.5" "4.2"...

# 4. You already know how (or you will soon)!

- If you can do GLMs, you can do MLMs  
(and if you can do generalized linear models,  
you can do generalized multilevel models, too)
- Let's review: How do you interpret an estimate for...
  - the intercept?
  - the slope of a quantitative predictor?
  - the slope of a categorical predictor?
  - a variance component ("pile of variance")?