Example 7b: Time-Invariant Predictors in Models of Change
(complete syntax, data, and output available for STATA, R, and STATA electronically)

The models for this example use the same response time data as in Hoffman (2015) chapter 6 but will include three new level-2 predictors. Specifically, in a sample of 101 older adults we will be examining baseline age, abstract reasoning, and education group as time-invariant predictors of change in response time (RT) in milliseconds over six practice sessions to a measure of processing speed (as measured by the number match 3 test). This example will first show models for change using piecewise linear slopes, followed by models with linear and quadratic time slopes. Note that the same diagonal R matrix is used in all example models (as is the only possible choice using R lmer).

STATA Syntax for Data Import, Manipulation, and Description:

```
// Define global variable for file location to be replaced in code below
cd "C:\Dropbox\24_PSQF6271\PSQF6271_Example7b"

// Import Example 6b six-occasion long-format data from excel
clear // clear memory in case a dataset is already open
import excel "Example7b_Data.xlsx", firstrow case(preserve) sheet("Example7b") clear

// Center time at session 1 for polynomial time models (also need to make quadratic version)
gen time=session-1
gen timesq=time*time
label variable time   "time: Linear Session (0=1)"
label variable timesq "timesq: Quadratic Session (0=1)"

// Create two slopes for piecewise models
// (intercept = session 1, breakpoint = session 2)
gen slope12 = session
recode slope12 (1=0) if session==1
recode slope12 (2=1) if session==2
recode slope12 (3=1) if session==3
recode slope12 (4=1) if session==4
recode slope12 (5=1) if session==5
recode slope12 (6=1) if session==6
gen slope26 = session
recode slope26 (1=0) if session==1
recode slope26 (2=0) if session==2
recode slope26 (3=1) if session==3
recode slope26 (4=2) if session==4
recode slope26 (5=3) if session==5
recode slope26 (6=4) if session==6
label variable slope12 "slope12: Early Practice Slope (Session 1-2)"
label variable slope26 "slope26: Later Practice Slope (Session 2-6)"

// Center level-2 predictors (based on descriptives below)
gen age80=baseage-80
gen reas22=absreas-22
label variable age80  "age80: Age Centered (0=80 years)"
label variable reas22 "reas22: Abstract Reasoning Centered (0=22)"

// Make education a grouping variable FOR DEMO PURPOSES ONLY
gen educgrp=.
replace educgrp=1 if (educyrs <= 12)
replace educgrp=2 if (educyrs > 12 & educyrs <= 16)
replace educgrp=3 if (educyrs > 16)
label variable educgrp "educgrp: Education Group (1=HS, 2=BA, 3=GRAD)"

// Create new variable to hold number of missing cases
// Then drop cases with incomplete predictors
egen nummiss = rowmiss(age80 reas22 educgrp session nm3rt)
drop if nummiss>0
```

```
display "STATA: Get Variance of Time Predictors for Slope Reliability"
summarize slope12, detail
global S12Var = r(Var) //  variance =  0.1391185
summarize slope26, detail
global S26Var = r(Var) //  variance =  2.225895
summarize time, detail
global LinVar = r(Var) //  variance =  2.921488
summarize timesq, detail
global QuaVar = r(Var) //  variance = 79.2697

// Save number of occasions for use later
global Ntimes = 6

display "Descriptive Statistics for Level-2 Predictors"
summarize baseage absreas

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
     baseage |        606    79.83332    6.051306   66.30801   95.30459
     absreas |        606    22.62376    4.683401         11         32

tabulate educgrp

    educgrp |      Freq.     Percent        Cum.
------------+-----------------------------------
       1 HS |        126       20.79       20.79
       2 BA |        336       55.45       76.24
     3 GRAD |        144       23.76      100.00
------------+-----------------------------------
      Total |        606      100.00
```

R Syntax for Data Import, Manipulation, and Description (after loading 2 custom functions and packages *readxl*, *TeachingDemos*, *psych*, *emmeans*, and *lmerTest*):

```
# Set working directory (to import and export files to)
setwd("C:/Dropbox/24_PSQF6271/PSQF6271_Example7b")

# Import Example 7b six-occasion long-format data from excel -- path = file name
Example7b = read_excel(path="Example7b_Data.xlsx", sheet="Example7b")
# Convert to data frame to use for analysis
Example7b = as.data.frame(Example7b)

# Sort by person and occasion (needed for correct V matrix)
Example7b = Example7b[order(Example7b$ID, Example7b$session), ]

# Center time at session 1 for polynomial time models
Example7b$time1=Example7b$session-1

# Create two slopes for piecewise models
# (intercept = session 1, breakpoint = session 2)
Example7b$slope12=Example7b$session
Example7b$slope12[which(Example7b$session==1)]=0
Example7b$slope12[which(Example7b$session==2)]=1
Example7b$slope12[which(Example7b$session==3)]=1
Example7b$slope12[which(Example7b$session==4)]=1
Example7b$slope12[which(Example7b$session==5)]=1
Example7b$slope12[which(Example7b$session==6)]=1
Example7b$slope26=Example7b$session
Example7b$slope26[which(Example7b$session==1)]=0
Example7b$slope26[which(Example7b$session==2)]=0
Example7b$slope26[which(Example7b$session==3)]=1
Example7b$slope26[which(Example7b$session==4)]=2
Example7b$slope26[which(Example7b$session==5)]=3
Example7b$slope26[which(Example7b$session==6)]=4
```

```
# Center level-2 predictors (based on descriptives below)
Example7b$age80=Example7b$baseage-80  # age80: Age Centered (0=80)
Example7b$reas22=Example7b$absreas-22 # reas22: Abstract Reasoning Centered (0=22)
# Make education a grouping variable FOR DEMO PURPOSES ONLY
Example7b$educgrp = cut(Example7b$educyrs, c(0,12,16,100), labels=c(1:3), right=TRUE)

# Filter to only cases complete on all variables to be used below
Example7b = Example7b[complete.cases(Example7b
                        [ , c("age80","reas22","educgrp","session","nm3rt")]), ]

print("R: Get Variance of Time Predictors for Slope Reliabilities")
S12Var = var(Example7b$slope12); S12Var # variance =  0.1391185
S26Var = var(Example7b$slope26); S26Var # variance =  2.225895
LinVar = var(Example7b$time);    LinVar # variance =  2.921488
QuaVar = var(Example7b$time^2);  QuaVar # variance = 79.2697

# Save number of occasions for use later
Ntimes = 6

print("Descriptive Statistics for Level-2 Predictors")
describe(x=Example7b[ , c("baseage","absreas")])
table(x=Example7b$educgrp,useNA="ifany")
prop.table(table(x=Example7b$educgrp,useNA="ifany"))
```

1a. Baseline Unconditional Random Two-Piece Time Slopes Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Slope12}_{ti}\right) + \beta_{2i}\left(\text{Slope26}_{ti}\right) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + U_{2i}$

Fixed-Effect-Predicted Outcome: $\hat{y}_{ti} = \gamma_{00} + \gamma_{10}\left(\text{Slope12}_{ti}\right) + \gamma_{20}\left(\text{Slope26}_{ti}\right)$

```
display "STATA 1a: Random Piecewise Time Unconditional Model"
mixed nm3rt c.slope12 c.slope26,                                    ///
      || ID: slope12 slope26, reml nolog difficult covariance(unstructured)  ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix PUnc = r(table)   // Save results for computations below
```

```
------------------------------------------------------------------
       nm3rt |      Coef.   Std. Err.          DF       t    P>|t|
-------------+----------------------------------------------------
     slope12 |   -163.644   30.21884        100.0   -5.42    0.000
     slope26 |  -32.89317   6.588755        100.0   -4.99    0.000
       _cons |   1961.893    54.6805        100.0   35.88    0.000
------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ID: Unstructured             |
               var(slope12) |    63954.2    13244.2      42618.17    95971.74
               var(slope26) |   2617.279   636.4813      1624.992    4215.499
                 var(_cons) |   284312.6   42731.62      211768.9    381706.9
        cov(slope12,slope26) |  -1672.296   2097.085     -5782.507    2437.916
          cov(slope12,_cons) |  -54269.95   18230.63     -90001.32   -18538.58
          cov(slope26,_cons) |   -10643.8   3791.317     -18074.64   -3212.951
-----------------------------+------------------------------------------------
               var(Residual) |   17673.02   1435.834      15071.46    20723.64
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 913.11                Prob > chi2 = 0.0000
```

```
display "-2LL = " e(ll)*-2                    // Print -2LL for model
-2LL = 8275.3743

estat recovariance, relevel(ID) correlation   // GCORR matrix

             |   slope12    slope26      _cons
-------------+---------------------------------
     slope12 |         1
     slope26 | -.1292568          1
       _cons | -.4024638  -.3901876          1
```

These negative correlations of the intercept with each slope indicates that slower people (highest RT at session 1) improved more (more negative rates of change).

```
// Build total-R2
predict predPUnc              // Save yhat
corr predPUnc nm3rt           // Get total-r to make R2

             | predPUnc    nm3rt
-------------+------------------
    predPUnc |   1.0000
       nm3rt |   0.1934   1.0000

global R2PUnc = r(rho)^2      // Save total-R2 for comparison
display "Total-R2 = " $R2PUnc   // Print total-R2 relative to empty model
Total-R2 = .03739598

// Save variances for pseudo-R2
matrix list PUnc
```

PUnc[9,10]

	nm3rt: slope12	nm3rt: slope26	nm3rt: _cons	lns1_1_1: _cons	lns1_1_2: _cons	lns1_1_3: _cons	atr1_1_1_2: _cons	atr1_1_1_3: _cons	atr1_1_2_3: _cons	lnsig_e: _cons
b	-163.64399	-32.893166	1961.8934	5.5329612	3.9349453	6.2789148	-.12998398	-.42658548	-.41202135	4.8898971
se	30.218843	6.5887549	54.680496	.1035444	.12159218	.075149	.15503365	.12213021	.14128048	.04062221
t	-5.4152962	-4.992319	35.879217	53.435641	32.361829	83.552873	-.83842431	-3.4928743	-2.9163359	120.37496
pvalue	4.221e-07	2.529e-06	6.426e-59	0	9.46e-230	0	.40179244	.00047785	.00354169	0
ll	-223.59731	-45.965068	1853.4088	5.330018	3.696629	6.1316255	-.43384436	-.66595629	-.688926	4.8102791
ul	-103.69066	-19.821264	2070.3779	5.7359045	4.1732616	6.4262042	.17387639	-.18721466	-.13511669	4.9695152
df	100	100	100	.	.	.	.	.	.	.
crit	1.9839715	1.9839715	1.9839715	1.959964	1.959964	1.959964	1.959964	1.959964	1.959964	1.959964
eform	0	0	0	0	0	0	0	0	0	0

```
global PUncIntVar = exp(PUnc[1,6])^2    // Save as L2 random intercept variance
global PUncS12Var = exp(PUnc[1,4])^2    // Save as L2 random slope12 variance
global PUncS26Var = exp(PUnc[1,5])^2    // Save as L2 random slope26 variance
global PUncResVar = exp(PUnc[1,10])^2   // Save as L1 residual variance
//display $PUncS26Var    // Check to make sure it worked

display "STATA Intercept Reliability = ICC2"
display $PUncIntVar/($PUncIntVar+($PUncResVar/$Ntimes))
.98974615

display "STATA Slope12 Reliability"
display $PUncS12Var/($PUncS12Var+($PUncResVar/($Ntimes*$S12Var)))
.75128141

display "STATA Slope26 Reliability"
display $PUncS26Var/($PUncS26Var+($PUncResVar/($Ntimes*$S26Var)))
.66418827


print("R 1a: Random Piecewise Time Unconditional Model")
PUnc = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
            formula=nm3rt~1+slope12+slope26 +(1+slope12+slope26|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(PUnc, chkREML=FALSE); summary(PUnc, ddf="Satterthwaite")

$AICtab
      AIC       BIC    logLik  deviance  df.resid
 8295.374  8339.443 -4137.687  8275.374   596.000
```

```
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 ID       (Intercept) 284311   533.21
          slope12      63954   252.89   -0.40
          slope26       2617    51.16   -0.39 -0.13
 Residual              17673   132.94

Fixed effects:
            Estimate Std. Error       df t value    Pr(>|t|)
(Intercept) 1961.893     54.680  100.000  35.879     < 2e-16
slope12     -163.644     30.219  100.000  -5.415 0.000000422
slope26      -32.893      6.589  100.000  -4.992 0.000002529

print("Compute squared correlation of predicted and actual RT as total R2")
Example7b$PredPUnc = predict(PUnc, re.form=NA)
rPUnc = cor.test(Example7b$PredPUnc, Example7b$nm3rt, method="pearson"); rPUnc

data:  Example7b$PredPUnc and Example7b$nm3rt
t = 4.844, df = 604, p-value = 0.000001619
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.1155127 0.2688858
sample estimates:
      cor
0.1933804

print("Total R2"); rPUnc$estimate^2
0.03739598

# Total R2 for time relative to empty model using custom function
TotalR2(data=Example7b, dvName="nm3rt", model1=PUnc, name1="Piecewise Time",
        model2=NULL, name2=NULL)
0.03739598

# Compute intercept and slope reliabilities
as.data.frame(VarCorr(PUnc))  # Print variance components to see order

       grp        var1    var2        vcov       sdcor
1       ID (Intercept)    <NA> 284311.486 533.2086701
2       ID     slope12    <NA>  63954.180 252.8916367
3       ID     slope26    <NA>   2617.280  51.1593622
4       ID (Intercept) slope12 -54269.854  -0.4024639
5       ID (Intercept) slope26 -10643.761  -0.3901870
6       ID     slope12 slope26  -1672.293  -0.1292566
7 Residual        <NA>    <NA>  17673.032 132.9399568

PUncIntVar = as.data.frame(VarCorr(PUnc))[1,4]
PUncS12Var = as.data.frame(VarCorr(PUnc))[2,4]
PUncS26Var = as.data.frame(VarCorr(PUnc))[3,4]
PUncResVar = as.data.frame(VarCorr(PUnc))[7,4]

print("R Intercept Reliability = ICC2")
PUncIntVar/(PUncIntVar+(PUncResVar/Ntimes))
0.9897461

print("R Slope12 Reliability")
PUncS12Var/(PUncS12Var+(PUncResVar/(Ntimes*S12Var)))
0.7512812

print("R Slope26 Reliability")
PUncS26Var/(PUncS26Var+(PUncResVar/(Ntimes*S26Var)))
0.6641882
```

1b. Piecewise Model with Age Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Slope12}_{ti}\right) + \beta_{2i}\left(\text{Slope26}_{ti}\right) + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}\left(\text{Slope12}_{ti}\right) + \gamma_{20}\left(\text{Slope26}_{ti}\right)$$
$$+ \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}\left(\text{Slope12}_{ti}\right)\left(\text{Age}_i - 80\right) + \gamma_{21}\left(\text{Slope26}_{ti}\right)\left(\text{Age}_i - 80\right)$$

Simple Slopes of Interactions:

Slope12 = $\gamma_{10} + \gamma_{11}(\text{Age}_i - 80)$

Slope26 = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

Age = $\gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

```
display "STATA 1b: Add Age Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80,  ///
      || ID: slope12 slope26, reml nolog difficult covariance(unstructured)   ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix PAge = r(table)   // Save results for computations below
display "-2LL = " e(ll)*-2                   // Print -2LL for model
estat recovariance, relevel(ID) correlation  // GCORR matrix

// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.slope12#c.age80=0)(c.slope26#c.age80=0), small
```

Simple Slopes of Interactions:

Slope12 = $\gamma_{10} + \gamma_{11}(\text{Age}_i - 80)$

Slope26 = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

```
// Simple slope12 and slope26 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)
margins, at(c.age80=(-6(6)6) c.slope26=0) dydx(c.slope12) df(99)  // As given below
lincom c.slope12*1 + c.slope12#c.age80*-6, small  // Slope12: Age 74
lincom c.slope12*1 + c.slope12#c.age80*0 , small  // Slope12: Age 80
lincom c.slope12*1 + c.slope12#c.age80*6 , small  // Slope12: Age 86

margins, at(c.age80=(-6(6)6) c.slope12=1) dydx(c.slope26) df(99)  // As given below
lincom c.slope26*1 + c.slope26#c.age80*-6, small  // Slope26: Age 74
lincom c.slope26*1 + c.slope26#c.age80*0 , small  // Slope26: Age 80
lincom c.slope26*1 + c.slope26#c.age80*6 , small  // Slope26: Age 86
```

Age Slope = $\gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

```
// Simple age slope at each session (S)
margins, at(c.slope12=(0(1)1) c.slope26=0) dydx(c.age80) df(99)  // As given below
lincom c.age80*1 + c.slope12#c.age80*0 + c.slope26#c.age80*0, small  // Age Slope: S1
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*0, small  // Age Slope: S2
margins, at(c.slope12=1 c.slope26=(1(1)4)) dydx(c.age80) df(99)  // As given below
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*1, small  // Age Slope: S3
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*2, small  // Age Slope: S4
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*3, small  // Age Slope: S5
lincom c.age80*1 + c.slope12#c.age80*1 + c.slope26#c.age80*4, small  // Age Slope: S6

// Get adjusted means per session and age (start(by)end)
margins, at(c.slope12=(0(1)1) c.slope26=0 c.age80=(-6 0 6))  // Sessions 1-2
margins, at(c.slope12=1 c.slope26=(1(1)4) c.age80=(-6 0 6))  // Sessions 3-6
(not shown)

// Build total-R2
predict predPAge                // Save yhat
quietly corr predPAge nm3rt     // Get total-r to make R2
global R2PAge = r(rho)^2        // Save total-R2 for comparison
display "Total-R2 = " $R2PAge   // Print total-R2 relative to empty model
display "Change in Total-R2 = "  $R2PAge - $R2PUnc
```

```
// Save variances and compute pseudo-R2
matrix list PAge
global PAgeIntVar = exp(PAge[1,9])^2    // Save as L2 random intercept variance
global PAgeS12Var = exp(PAge[1,7])^2    // Save as L2 random slope12 variance
global PAgeS26Var = exp(PAge[1,8])^2    // Save as L2 random slope26 variance
global PAgeResVar = exp(PAge[1,13])^2   // Save as L1 residual variance
display "Pseudo-R2 for Intercept = " 1-($PAgeIntVar/$PUncIntVar)
display "Pseudo-R2 for Slope12 = "   1-($PAgeS12Var/$PUncS12Var)
display "Pseudo-R2 for Slope26 = "   1-($PAgeS26Var/$PUncS26Var)
display "Pseudo-R2 for Residual = "  1-($PAgeResVar/$PUncResVar)
```

```
print("R 1b: Add Age Predicting Intercept, Slope12, and Slope26")
PAge = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
            formula=nm3rt~1+slope12+slope26+age80 +slope12:age80 +slope26:age80
                    +(1+slope12+slope26|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(PAge, chkREML=FALSE); summary(PAge, ddf="Satterthwaite")
```

```
$AICtab
      AIC       BIC    logLik  deviance  df.resid
 8277.036  8334.325 -4125.518  8251.036   593.000

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 ID       (Intercept) 254285   504.27
          slope12      62742   250.48   -0.37
          slope26       2594    50.93   -0.36 -0.17
 Residual              17673   132.94

Fixed effects:
              Estimate Std. Error       df t value    Pr(>|t|)
(Intercept)   1966.857     51.910   99.001  37.889     < 2e-16
slope12       -164.908     30.031   99.000  -5.491 0.000000309
slope26        -33.118      6.573   99.000  -5.038 0.000002118
age80           29.780      8.582   99.001   3.470    0.000772
slope12:age80   -7.581      4.965   99.000  -1.527    0.129973
slope26:age80   -1.350      1.087   99.000  -1.242    0.217121
```

Interpret the fixed intercept:

Interpret the fixed effect of slope12:

Interpret the fixed effect of slope26:

Interpret the fixed effect of age80:

Interpret the effect of slope12*age80:

Interpret the effect of slope26*age80:

```
print("DF=3 Wald Test for all Age Slopes")
contestMD(PAge, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0),c(0,0,0,0,1,0),c(0,0,0,0,0,1)))
```

```
    Sum Sq  Mean Sq NumDF    DenDF  F value      Pr(>F)
1 216170.2 72056.73     3 99.00025 4.077208 0.008926335
```

This multivariate Wald F-test provides the significance for the change in total R^2 relative to the unconditional model.

Fixed-Effect-Predicted Outcome:

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}\left(\text{Slope12}_{ti}\right) + \gamma_{20}\left(\text{Slope26}_{ti}\right)$$
$$+ \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}\left(\text{Slope12}_{ti}\right)\left(\text{Age}_i - 80\right) + \gamma_{21}\left(\text{Slope26}_{ti}\right)\left(\text{Age}_i - 80\right)$$

Simple Slopes of Interactions:

Slope12 = $\gamma_{10} + \gamma_{11}(\text{Age}_i - 80)$

Slope26 = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

```
print("Simple slope12 and slope26 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)")
print("Slope12: Age 74"); contest1D(PAge, ddf="Satterthwaite", L=c(0,1,0,0,-6, 0))
print("Slope12: Age 80"); contest1D(PAge, ddf="Satterthwaite", L=c(0,1,0,0, 0, 0))
print("Slope12: Age 86"); contest1D(PAge, ddf="Satterthwaite", L=c(0,1,0,0, 6, 0))
print("Slope26: Age 74"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,1,0, 0,-6))
print("Slope26: Age 80"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,1,0, 0, 0))
print("Slope26: Age 86"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,1,0, 0, 6))
```

Estimates (from SAS, for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Slope12: Age 74	-119.42	41.7131	99	-2.86	0.0051
Slope12: Age 80	-164.91	30.0311	99	-5.49	<.0001
Slope12: Age 86	-210.39	42.8789	99	-4.91	<.0001
Slope26: Age 74	-25.0187	9.1305	99	-2.74	0.0073
Slope26: Age 80	-33.1182	6.5734	99	-5.04	<.0001
Slope26: Age 86	-41.2177	9.3857	99	-4.39	<.0001

Simple Slopes of Interactions:

Age Slope $= \gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

```
print("Simple age slope at each session (S)")
print("Age Slope: S1"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,0,0))
print("Age Slope: S2"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,0))
print("Age Slope: S3"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,1))
print("Age Slope: S4"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,2))
print("Age Slope: S5"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,3))
print("Age Slope: S6"); contest1D(PAge, ddf="Satterthwaite", L=c(0,0,0,1,1,4))
```

Estimates (from SAS, for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Age Slope: S1	29.7804	8.5822	99	3.47	0.0008
Age Slope: S2	22.1993	7.9689	99	2.79	0.0064
Age Slope: S3	20.8494	7.5245	99	2.77	0.0067
Age Slope: S4	19.4995	7.2176	99	2.70	0.0081
Age Slope: S5	18.1496	7.0663	99	2.57	0.0117
Age Slope: S6	16.7997	7.0805	99	2.37	0.0196

```
# Total R2 for time and time+age relative to empty model using custom function
TotalR2(data=Example7b, dvName="nm3rt", model1=PUnc, name1="Piecewise Time",
        model2=PAge, name2="Age")

   totalR2.1 totalR2.2   changeR2
1 0.03739598  0.107552 0.07015599
```

```
# Pseudo-R2 for age relative to unconditional model using custom function
PseudoR2(data=Example7b, baseModel=PUnc, model1=PAge, name1="Age")

Pseudo-R2 for Age
         term      base     model1 pseudoR2.model1
1 (Intercept) 284311.49 254284.559          0.1056
2     slope12  63954.18  62742.156          0.0190
3     slope26   2617.28   2593.603          0.0090
7    Residual  17673.03  17673.058         -0.0000
```

Which variance component should have been reduced by each new fixed effect of age?

1c. Piecewise Model with Age and Reasoning Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Slope12}_{ti}\right) + \beta_{2i}\left(\text{Slope26}_{ti}\right) + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reas}_i - 22\right) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reas}_i - 22\right) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{22}\left(\text{Reas}_i - 22\right) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$
\begin{aligned}
\hat{y}_{ti} = {} & \gamma_{00} + \gamma_{10}\left(\text{Slope12}_{ti}\right) + \gamma_{20}\left(\text{Slope26}_{ti}\right) \\
& + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}\left(\text{Slope12}_{ti}\right)\left(\text{Age}_i - 80\right) + \gamma_{21}\left(\text{Slope26}_{ti}\right)\left(\text{Age}_i - 80\right) \\
& + \gamma_{02}\left(\text{Reas}_i - 22\right) + \gamma_{12}\left(\text{Slope12}_{ti}\right)\left(\text{Reas}_i - 22\right) + \gamma_{22}\left(\text{Slope26}_{ti}\right)\left(\text{Reas}_i - 22\right)
\end{aligned}
$$

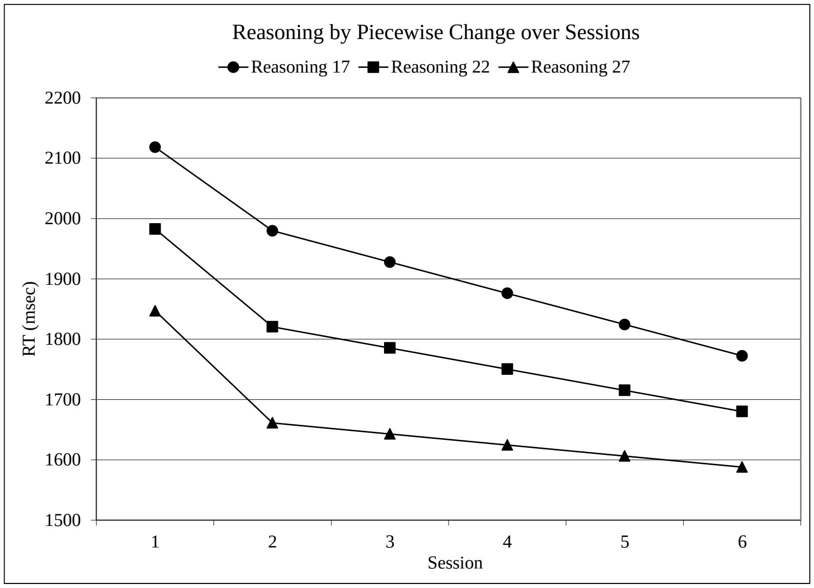
```
display "STATA 1c: Keep Age, Add Reasoning Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80   ///
            c.reas22 c.slope12#c.reas22 c.slope26#c.reas22,                   ///
      || ID: slope12 slope26, reml nolog difficult covariance(unstructured)   ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix PReas = r(table)    // Save results for computations below
```

```
------------------------------------------------------------------------
           nm3rt |      Coef.   Std. Err.           DF       t    P>|t|
-----------------+------------------------------------------------------
         slope12 |  -162.1635   30.36884          98.0   -5.34    0.000
         slope26 |  -35.06686   6.490066          98.0   -5.40    0.000
           age80 |    23.0041   8.863877          98.0    2.60    0.011
 c.slope12#c.age80 | -8.758887  5.259654          98.0   -1.67    0.099
 c.slope26#c.age80 | -.5134742  1.124031          98.0   -0.46    0.649
          reas22 |  -27.11998    11.4528          98.0   -2.37    0.020
c.slope12#c.reas22 | -4.714051  6.795868          98.0   -0.69    0.490
c.slope26#c.reas22 |  3.347607  1.452332          98.0    2.30    0.023
           _cons |   1982.644   51.17934          98.0   38.74    0.000
------------------------------------------------------------------------
```

Interpret the fixed effect of reas22:

Interpret the effect of slope12*reas22:

Interpret the effect of slope26*reas22:



See excel workbook online for how this plot was made

```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ID: Unstructured             |
                var(slope12) |   63221.91   13271.61      41896.96    95400.95
                var(slope26) |    2411.55   614.0023      1464.107    3972.097
                  var(_cons) |   242192.2   37151.54      179303.4    327138.7
        cov(slope12,slope26) |  -1845.106   2068.729     -5899.741    2209.529
          cov(slope12,_cons) |   -49816.7   17063.72     -83260.97   -16372.42
          cov(slope26,_cons) |  -7510.989   3414.251      -14202.8   -819.1804
-----------------------------+------------------------------------------------
               var(Residual) |   17673.01   1435.833      15071.46    20723.64
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 849.71                Prob > chi2 = 0.0000
```

```
display "-2LL = " e(ll)*-2                   // Print -2LL for model
-2LL = 8226.4671

estat recovariance, relevel(ID) correlation  // GCORR matrix
             |   slope12    slope26      _cons
-------------+---------------------------------
     slope12 |         1
     slope26 | -.1494305          1
       _cons | -.4025882  -.3107913          1

// DF=3 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.slope12#c.reas22=0)(c.slope26#c.reas22=0), small

       F(  3, 98.00) =    3.50
            Prob > F =    0.0183
```

Simple Slopes of Interactions:

$\text{Slope12} = \gamma_{10} + \gamma_{12}(\text{Reas}_i - 22)$

$\text{Slope26} = \gamma_{20} + \gamma_{22}(\text{Reas}_i - 22)$

```
// Simple slope12 and slope26 for reasoning 17, 22, 27 (about -1SD, M, +1 SD of reas22)
margins, at(c.age80=0 c.reas22=(-5(5)5) c.slope26=0) dydx(c.slope12) df(98) // As given below
lincom c.slope12*1 + c.slope12#c.reas22*-5, small  // Slope12: Reasoning 17
lincom c.slope12*1 + c.slope12#c.reas22*0 , small  // Slope12: Reasoning 22
lincom c.slope12*1 + c.slope12#c.reas22*5 , small  // Slope12: Reasoning 27

------------------------------------------------------------------------------
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
slope12   _at |
          1  |  -138.5932   48.43248    -2.86   0.005    -234.7059   -42.48058
          2  |  -162.1635   30.36884    -5.34   0.000    -222.4295   -101.8975
          3  |  -185.7338   42.52081    -4.37   0.000    -270.1149   -101.3526
------------------------------------------------------------------------------

margins, at(c.age80=0 c.reas22=(-5(5)5) c.slope12=1) dydx(c.slope26) df(98) // As given below
lincom c.slope26*1 + c.slope26#c.reas22*-5, small  // Slope26: Reasoning 17
lincom c.slope26*1 + c.slope12#c.reas22*0 , small  // Slope26: Reasoning 22
lincom c.slope26*1 + c.slope26#c.reas22*5 , small  // Slope26: Reasoning 27

------------------------------------------------------------------------------
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
slope26   _at |
          1  |   -51.8049   10.35041    -5.01   0.000    -72.34495   -31.26484
          2  |  -35.06686   6.490066    -5.40   0.000    -47.94619   -22.18754
          3  |  -18.32883    9.08704    -2.02   0.046    -36.36177   -.2958963
------------------------------------------------------------------------------
```

Reasoning Slope $= \gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

```
// Simple reasoning slope at each session (S)
margins, at(c.age80=0 c.slope12=(0(1)1) c.slope26=0) dydx(c.reas22) df(98) // As given below
lincom c.reas22*1 + c.slope12#c.reas22*0 + c.slope26#c.reas22*0, small  // Reasoning Slope: S1
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*0, small  // Reasoning Slope: S2

------------------------------------------------------------------------------
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
reas22   _at |
          1  |  -27.11998    11.4528    -2.37   0.020    -49.84768    -4.39228
          2  |  -31.83403   10.45082    -3.05   0.003    -52.57335   -11.09471
------------------------------------------------------------------------------

margins, at(c.age80=0 c.slope12=1 c.slope26=(1(1)4)) dydx(c.reas22) df(98) // As given below
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*1, small  // Reasoning Slope: S3
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*2, small  // Reasoning Slope: S4
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*3, small  // Reasoning Slope: S5
lincom c.reas22*1 + c.slope12#c.reas22*1 + c.slope26#c.reas22*4, small  // Reasoning Slope: S6

------------------------------------------------------------------------------
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
reas22   _at |
          1  |  -28.48642   9.915436    -2.87   0.005    -48.16328   -8.809564
          2  |  -25.13882   9.572385    -2.63   0.010     -44.1349    -6.14273
          3  |  -21.79121   9.442658    -2.31   0.023    -40.52986   -3.052564
          4  |   -18.4436   9.534964    -1.93   0.056    -37.36543    .4782225
------------------------------------------------------------------------------

// Get adjusted means per session and reasoning (start(by)end), hold age80-0
margins, at(c.age80=0 c.slope12=(0(1)1) c.slope26=0 c.reas22=(-5 0 5))  // Sessions 1-2
margins, at(c.age80=0 c.slope12=1 c.slope26=(1(1)4) c.reas22=(-5 0 5))  // Sessions 3-6

// Build total-R2
predict predPReas                // Save yhat
quietly corr predPReas nm3rt     // Get total-r to make R2
global R2PReas = r(rho)^2        // Save total-R2 for comparison
display "Total-R2 = " $R2PReas   // Print total-R2 relative to empty model
Total-R2 = .16130712
display "Change in Total-R2 = "  $R2PReas - $R2PAge
Change in Total-R2 = .05375515

// Save variances and compute pseudo-R2 and change in pseudo-R2
matrix list PReas
global PReasIntVar = exp(PReas[1,12])^2   // Save as L2 random intercept variance
global PReasS12Var = exp(PReas[1,10])^2   // Save as L2 random slope12 variance
global PReasS26Var = exp(PReas[1,11])^2   // Save as L2 random slope26 variance
global PReasResVar = exp(PReas[1,16])^2   // Save as L1 residual variance

display "Pseudo-R2 for Intercept = " 1-($PReasIntVar/$PUncIntVar)
Pseudo-R2 for Intercept = .14814817

display "Pseudo-R2 for Slope12 = "   1-($PReasS12Var/$PUncS12Var)
Pseudo-R2 for Slope12 = .01145021

display "Pseudo-R2 for Slope26 = "   1-($PReasS26Var/$PUncS26Var)
Pseudo-R2 for Slope26 = .07860409

display "Pseudo-R2 for Residual = "  1-($PReasResVar/$PUncResVar)
Pseudo-R2 for Residual = 1.323e-07

display "Change in Pseudo-R2 for Intercept = " (1-($PReasIntVar/$PUncIntVar)) ///
                                             - (1-($PAgeIntVar/$PUncIntVar))
Change in Pseudo-R2 for Intercept = .04253677
```

```
display "Change in Pseudo-R2 for Slope12 = "   (1-($PReasS12Var/$PUncS12Var)) ///
                                             - (1-($PAgeS12Var/$PUncS12Var))
Change in Pseudo-R2 for Slope12 = -.00749827

display "Change in Pseudo-R2 for Slope26 = "   (1-($PReasS26Var/$PUncS26Var)) ///
                                             - (1-($PAgeS26Var/$PUncS26Var))
Change in Pseudo-R2 for Slope26 = .06955813
display "Change in Pseudo-R2 for Residual = "  (1-($PReasResVar/$PUncResVar)) ///
                                             - (1-($PAgeResVar/$PUncResVar))
Change in Pseudo-R2 for Residual = 5.528e-08

print("R 1c: Keep Age, Add Reasoning Predicting Intercept, Slope12, and Slope26")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
PReas = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
             formula=nm3rt~1+slope12+slope26+age80+reas22 +slope12:age80 +slope26:age80
                     +slope12:reas22 +slope26:reas22 +(1+slope12+slope26|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(PReas, chkREML=FALSE); summary(PReas, ddf="Satterthwaite")

print("DF=3 Wald Test for all Reasoning Slopes")
contestMD(PReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,0,1,0,0,0,0),c(0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,1)))

print("Simple slope12 and slope26 for reas 17, 22, 27 (about -1SD, M, +1 SD of reas22)")
print("Slope12: Reas 17"); contest1D(PReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,-5, 0))
print("Slope12: Reas 22"); contest1D(PReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 0, 0))
print("Slope12: Reas 27"); contest1D(PReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 5, 0))
print("Slope26: Reas 17"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0, 0,-5))
print("Slope26: Reas 22"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0, 0, 0))
print("Slope26: Reas 27"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0, 0, 5))

print("Simple reasoning slope at each session (S)")
print("Reas Slope: S1"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0,0))
print("Reas Slope: S2"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,0))
print("Reas Slope: S3"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,1))
print("Reas Slope: S4"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,2))
print("Reas Slope: S5"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,3))
print("Reas Slope: S6"); contest1D(PReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1,4))

# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=PAge, name1="Age",
        model2=PReas, name2="Reasoning")

  totalR2.1 totalR2.2   changeR2
1  0.107552 0.1613071 0.05375515

PseudoR2(data=Example7b, baseModel=PUnc, model1=PAge, name1="Age",
         model2=PReas, name2="Reasoning")

Pseudo-R2 and Change in Pseudo-R2 for Age vs Reasoning
         term      base     model1     model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 284311.49 254284.559 242192.031          0.1056          0.1481          0.0425
2     slope12  63954.18  62742.156  63221.876          0.0190          0.0115         -0.0075
3     slope26   2617.28   2593.603   2411.556          0.0090          0.0786          0.0696
7    Residual  17673.03  17673.058  17673.023         -0.0000          0.0000          0.0000
```

1d. Piecewise Model with Age, Reasoning, and Education Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Slope12}_{ti}) + \beta_{2i}(\text{Slope26}_{ti}) + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reas}_i - 22) + \gamma_{03}(\text{LowvsMedEd}_i) + \gamma_{04}(\text{LowvsHighEd}_i) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{LowvsMedEd}_i) + \gamma_{14}(\text{LowvsHighEd}_i) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + \gamma_{23}(\text{LowvsMedEd}_i) + \gamma_{24}(\text{LowvsHighEd}_i) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$\begin{aligned}
\hat{y}_{ti} = &\gamma_{00} + \gamma_{10}(\text{Slope12}_{ti}) + \gamma_{20}(\text{Slope26}_{ti}) \\
&+ \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{21}(\text{Slope26}_{ti})(\text{Age}_i - 80) \\
&+ \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(\text{Slope12}_{ti})(\text{Reas}_i - 22) + \gamma_{22}(\text{Slope26}_{ti})(\text{Reas}_i - 22) \\
&+ \gamma_{03}(\text{LowvsMedEd}_i) + \gamma_{13}(\text{Slope12}_{ti})(\text{LowvsMedEd}_i) + \gamma_{23}(\text{Slope26}_{ti})(\text{LowvsMedEd}_i) \\
&+ \gamma_{04}(\text{LowvsHighEd}_i) + \gamma_{14}(\text{Slope12}_{ti})(\text{LowvsHighEd}_i) + \gamma_{24}(\text{Slope26}_{ti})(\text{LowvsHighEd}_i)
\end{aligned}$$

```
display "STATA 1d: Keep Age & Reas, Add Education Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80    ///
            c.reas22 c.slope12#c.reas22 c.slope26#c.reas22                      ///
            i.educgrp c.slope12#i.educgrp c.slope26#i.educgrp,                  ///
      || ID: slope12 slope26, reml nolog difficult covariance(un) baselevels    ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix PEduc = r(table)   // Save results for computations below
display "-2LL = " e(ll)*-2                  // Print -2LL for model
estat recovariance, relevel(ID) correlation  // GCORR matrix

// DF=2 Wald test for education on intercept, slope12, slope26, and DF=6 joint test
contrast i.educgrp c.slope12#i.educgrp c.slope26#i.educgrp, small overall
```

	df	ddf	F	P>F
nm3rt				
educgrp	2	96.00	0.12	0.8831
educgrp#c.slope12	2	96.00	0.85	0.4289
educgrp#c.slope26	2	96.00	0.60	0.5516
Overall	6	96.00	0.73	0.6264

```
// Estimating adjusted means and mean diffs per group at first and last session
margins i.educgrp, at(c.slope12=0 c.slope26=0 c.age80=0 c.reas22=0)
margins i.educgrp, at(c.slope12=0 c.slope26=0 c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)
margins i.educgrp, at(c.slope12=1 c.slope26=4 c.age80=0 c.reas22=0)
margins i.educgrp, at(c.slope12=1 c.slope26=4 c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)

// Contrasts between groups on intercept, linear, and quadratic slopes
test 1.educgrp=3.educgrp, small                          // 1Low vs 3High: Intercept
test 2.educgrp=3.educgrp, small                          // 2Med vs 2High: Intercept
test 1.educgrp=2.educgrp, small                          // 1Low vs 2Med:  Intercept
test 1.educgrp#c.slope12=3.educgrp#c.slope12, small    // 1Low vs 3High: Slope12
test 2.educgrp#c.slope12=3.educgrp#c.slope12, small    // 2Med vs 3High: Slope12
test 1.educgrp#c.slope12=2.educgrp#c.slope12, small    // 1Low vs 2Med:  Slope12
test 1.educgrp#c.slope26=3.educgrp#c.slope26, small    // 1Low vs 3High: Slope26
test 2.educgrp#c.slope26=3.educgrp#c.slope26, small    // 2Med vs 3High: Slope26
test 1.educgrp#c.slope26=2.educgrp#c.slope26, small    // 1Low vs 2Med:  Slope26

// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.age80=0 c.reas22=0 c.slope12=(0(1)1) c.slope26=0 educgrp=(1 2 3))
margins, at(c.age80=0 c.reas22=0 c.slope12=1 c.slope26=(1(1)4) educgrp=(1 2 3))
```

```
// Build total-R2
predict predPEduc  // Save yhat
quietly corr predPEduc nm3rt     // Get total-r to make R2
global R2PEduc = r(rho)^2        // Save total-R2 for comparison
display "Total-R2 = " $R2PEduc   // Print total-R2 relative to empty model
display "Change in Total-R2 = "  $R2PEduc - $R2PReas

// Save variances and compute pseudo-R2 and change in pseudo-R2
matrix list PEduc
global PEducIntVar = exp(PEduc[1,21])^2   // Save as L2 random intercept variance
global PEducS12Var = exp(PEduc[1,19])^2   // Save as L2 random slope12 variance
global PEducS26Var = exp(PEduc[1,20])^2   // Save as L2 random slope26 variance
global PEducResVar = exp(PEduc[1,25])^2   // Save as L1 residual variance

display "Pseudo-R2 for Intercept = " 1-($PEducIntVar/$PUncIntVar)
display "Pseudo-R2 for Slope12 = "   1-($PEducS12Var/$PUncS12Var)
display "Pseudo-R2 for Slope26 = "   1-($PEducS26Var/$PUncS26Var)
display "Pseudo-R2 for Residual = "  1-($PEducResVar/$PUncResVar)

display "Change in Pseudo-R2 for Intercept = " (1-($PEducIntVar/$PUncIntVar)) ///
                                             - (1-($PReasIntVar/$PUncIntVar))
display "Change in Pseudo-R2 for Slope12 = "   (1-($PEducS12Var/$PUncS12Var)) ///
                                             - (1-($PReasS12Var/$PUncS12Var))
display "Change in Pseudo-R2 for Slope26 = "   (1-($PEducS26Var/$PUncS26Var)) ///
                                             - (1-($PReasS26Var/$PUncS26Var))
display "Change in Pseudo-R2 for Residual = "  (1-($PEducResVar/$PUncResVar)) ///
                                             - (1-($PReasResVar/$PUncResVar))

print("R 1d: Keep Age & Reasoning, Add Education Predicting Intercept, Slope12, and Slope26")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
PEduc = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
             formula=nm3rt~1+slope12+slope26+age80+reas22+factor(educgrp) +slope12:age80
                    +slope26:age80 +slope12:reas22 +slope26:reas22 +slope12:factor(educgrp)
                    +slope26:factor(educgrp) +(1+slope12+slope26|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(PEduc, chkREML=FALSE); summary(PEduc, ddf="Satterthwaite")
```

```
      AIC       BIC    logLik  deviance  df.resid
 8208.209  8305.161 -4082.105  8164.209   584.000

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 ID       (Intercept) 246919   496.91
          slope12      63495   251.98   -0.42
          slope26       2446    49.46   -0.31 -0.15
 Residual              17673   132.94
Number of obs: 606, groups:  ID, 101

Fixed effects:
                          Estimate Std. Error       df t value Pr(>|t|)
(Intercept)              1936.1816   114.1253  96.0003  16.965  < 2e-16
slope12                  -239.0857    67.2122  96.0000  -3.557 0.000584
slope26                   -30.9640    14.4015  95.9996  -2.150 0.034063
age80                      22.9367     8.9490  96.0002   2.563 0.011929
reas22                    -28.5673    11.9710  96.0002  -2.386 0.018976
factor(educgrp)2           67.4187   136.3593  96.0003   0.494 0.622139
factor(educgrp)3           41.9718   157.3546  96.0003   0.267 0.790246
slope12:age80              -8.9054     5.2704  96.0000  -1.690 0.094329
slope26:age80              -0.5289     1.1293  95.9996  -0.468 0.640616
slope12:reas22             -7.0891     7.0501  96.0000  -1.006 0.317167
slope26:reas22              3.4883     1.5106  95.9996   2.309 0.023079
slope12:factor(educgrp)2  104.5289    80.3066  96.0000   1.302 0.196161
slope12:factor(educgrp)3   85.9455    92.6715  96.0000   0.927 0.356034
slope26:factor(educgrp)2  -10.2728    17.2072  95.9996  -0.597 0.551908
slope26:factor(educgrp)3    6.3237    19.8566  95.9996   0.318 0.750821
```

```
print("DF=2 Wald Test for Each Education Effect")
anova(PEduc)

Type III Analysis of Variance Table with Satterthwaite's method
                       Sum Sq Mean Sq NumDF DenDF F value       Pr(>F)
slope12                497750  497750     1    96 28.1644 0.0000007161
slope26                366394  366394     1    96 20.7318 0.0000154997
age80                  116097  116097     1    96  6.5692      0.01193
reas22                 100645  100645     1    96  5.6948      0.01898
factor(educgrp)          4399    2200     2    96  0.1245      0.88312
slope12:age80           50459   50459     1    96  2.8551      0.09433
slope26:age80            3876    3876     1    96  0.2193      0.64062
slope12:reas22          17869   17869     1    96  1.0111      0.31717
slope26:reas22          94236   94236     1    96  5.3322      0.02308
slope12:factor(educgrp) 30189   15094     2    96  0.8541      0.42888
slope26:factor(educgrp) 21161   10581     2    96  0.5987      0.55157

print("DF=6 Wald Test for all Education Slopes")
contestMD(PEduc, ddf="Satterthwaite", L=rbind(
                 c(0,0,0,0,0,1,0,0,0,0,0,0,0,0,0),c(0,0,0,0,0,0,1,0,0,0,0,0,0,0,0),
                 c(0,0,0,0,0,0,0,0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,0,0,0,0,0,1,0,0),
                 c(0,0,0,0,0,0,0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,1)))

    Sum Sq  Mean Sq NumDF    DenDF   F value    Pr(>F)
1 77427.52 12904.59     6 96.00007 0.7301851 0.6264171

print("Adjusted means and diffs per group only for education simple main effect")
print("Education diffs at session 1")
Ps1mean = ref_grid(PEduc, at=list(slope12=0,slope26=0,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Ps1mean, pairwise~educgrp, lmer.df="satterthwaite", adjust="none")

$emmeans
educgrp emmean    SE df lower.CL upper.CL
 1        1936 114.1 96     1710     2163
 2        2004  70.4 96     1864     2143
 3        1978 105.8 96     1768     2188

$contrasts
 contrast            estimate  SE df t.ratio p.value
 educgrp1 - educgrp2    -67.4 136 96  -0.494  0.6221
 educgrp1 - educgrp3    -42.0 157 96  -0.267  0.7902
 educgrp2 - educgrp3     25.4 126 96   0.203  0.8398

print("Education diffs at session 6")
Ps6mean = ref_grid(PEduc, at=list(slope12=1,slope26=4,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Ps6mean, pairwise~educgrp, lmer.df="satterthwaite", adjust="none")

$emmeans
 educgrp emmean   SE df lower.CL upper.CL
 1        1573 94.3 96     1386     1760
 2        1704 58.2 96     1589     1820
 3        1726 87.5 96     1553     1900

$contrasts
 contrast            estimate  SE df t.ratio p.value
 educgrp1 - educgrp2   -130.9 113 96  -1.161  0.2485
 educgrp1 - educgrp3   -153.2 130 96  -1.178  0.2417
 educgrp2 - educgrp3    -22.4 104 96  -0.215  0.8299
```

Simple Slopes of Interactions:

$$\text{Slope12} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{LowvsMedEd}_i) + \gamma_{14}(\text{HighvsMedEd}_i)$$

$$\text{Slope26} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + \gamma_{23}(\text{LowvsMedEd}_i) + \gamma_{24}(\text{HighvsMedEd}_i)$$

Age Slope = $\gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti})$

Reasoning Slope = $\gamma_{02} + \gamma_{12}(\text{Slope12}_{ti}) + \gamma_{22}(\text{Slope26}_{ti})$

Low vs Med Ed Slope = $\gamma_{03} + \gamma_{13}(\text{Slope12}_{ti}) + \gamma_{23}(\text{Slope26}_{ti})$

Low vs High Ed Slope = $\gamma_{04} + \gamma_{14}(\text{Slope12}_{ti}) + \gamma_{24}(\text{Slope26}_{ti})$

Med vs High Ed Slope = $\gamma_{04} + \gamma_{14}(\text{Slope12}_{ti}) + \gamma_{24}(\text{Slope26}_{ti}) - \gamma_{03} - \gamma_{13}(\text{Slope12}_{ti}) - \gamma_{23}(\text{Slope26}_{ti})$

```
print("Specific education group differences on intercept, slope12 and slope16")
print("1Low vs 3High Educ: Intercept"); contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,-1,0,0,0,0,0,0,0,0,0))
print("2Med vs 3High Educ: Intercept"); contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,-1,0,0,0,0,0,0,0,0))
print("1Low vs 2Med  Educ: Intercept"); contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,-1,1,0,0,0,0,0,0,0,0))
print("1Low vs 3High Educ: Slope12");   contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0))
print("2Med vs 3High Educ: Slope12");   contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0))
print("1Low vs 2Med  Educ: Slope12");   contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,-1,1,0,0))
print("1Low vs 3High Educ: Slope26");   contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0))
print("2Med vs 3High Educ: Slope26");   contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1))
print("1Low vs 2Med  Educ: Slope26");   contest1D(PEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,-1,1))
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
1Low vs 3High Educ: Intercept	41.9718	157.35	96	0.27	0.7902
2Med vs 3High Educ: Intercept	-25.4470	125.54	96	-0.20	0.8398
1Low vs 2Med Educ: Intercept	67.4187	136.36	96	0.49	0.6221
1Low vs 3High Educ: Slope12	85.9455	92.6714	96	0.93	0.3560
2Med vs 3High Educ: Slope12	-18.5834	73.9371	96	-0.25	0.8021
1Low vs 2Med Educ: Slope12	104.53	80.3066	96	1.30	0.1962
1Low vs 3High Educ: Slope26	6.3237	19.8566	96	0.32	0.7508
2Med vs 3High Educ: Slope26	16.5965	15.8424	96	1.05	0.2975
1Low vs 2Med Educ: Slope26	-10.2728	17.2072	96	-0.60	0.5519

```
# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=PReas, name1="Reasoning",
        model2=PEduc, name2="Education")

  totalR2.1 totalR2.2   changeR2
1 0.1613071 0.1736287 0.01232161

PseudoR2(data=Example7b, baseModel=PUnc, model1=PReas, name1="Reasoning",
         model2=PEduc, name2="Education")

Pseudo-R2 and Change in Pseudo-R2 for Reasoning vs Education
         term      base     model1     model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 284311.49 242192.031 246919.170          0.1481          0.1315         -0.0166
2     slope12  63954.18  63221.876  63495.158          0.0115          0.0072         -0.0043
3     slope26   2617.28   2411.556   2446.061          0.0786          0.0654         -0.0132
7    Residual  17673.03  17673.023  17673.035          0.0000         -0.0000         -0.0000
```

Given that education group has no significant effects, we can drop it entirely before moving on to examine potential interactions among the time-invariant predictors of baseline age and reasoning.

1e. Piecewise Model with Age*Reasoning Predicting Intercept, Slope12, and Slope26

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Slope12}_{ti}\right) + \beta_{2i}\left(\text{Slope26}_{ti}\right) + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reason}_i - 22\right) + \gamma_{03}\left(\text{Age}_i - 80\right)\left(\text{Reason}_i - 22\right) + U_{0i}$

Slope12: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reason}_i - 22\right) + \gamma_{13}\left(\text{Age}_i - 80\right)\left(\text{Reason}_i - 22\right) + U_{1i}$

Slope26: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{22}\left(\text{Reason}_i - 22\right) + \gamma_{23}\left(\text{Age}_i - 80\right)\left(\text{Reason}_i - 22\right) + U_{2i}$

Fixed-Effect-Predicted Outcome:

$$
\begin{aligned}
\hat{y}_{ti} &= \gamma_{00} + \gamma_{10}\left(\text{Slope12}_{ti}\right) + \gamma_{20}\left(\text{Slope26}_{ti}\right) \\
&+ \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}\left(\text{Slope12}_{ti}\right)\left(\text{Age}_i - 80\right) + \gamma_{21}\left(\text{Slope26}_{ti}\right)\left(\text{Age}_i - 80\right) \\
&+ \gamma_{02}\left(\text{Reas}_i - 22\right) + \gamma_{12}\left(\text{Slope12}_{ti}\right)\left(\text{Reas}_i - 22\right) + \gamma_{22}\left(\text{Slope26}_{ti}\right)\left(\text{Reas}_i - 22\right) \\
&+ \gamma_{03}\left(\text{Age}_i - 80\right)\left(\text{Reason}_i - 22\right) + \gamma_{13}\left(\text{Slope12}_{ti}\right)\left(\text{Age}_i - 80\right)\left(\text{Reason}_i - 22\right) + \gamma_{23}\left(\text{Slope26}_{ti}\right)\left(\text{Age}_i - 80\right)\left(\text{Reason}_i - 22\right)
\end{aligned}
$$

```
display "STATA 1e: Drop Education, Add Age*Reasoning Predicting Intercept, Slope12, and Slope26"
mixed nm3rt c.slope12 c.slope26 c.age80 c.slope12#c.age80 c.slope26#c.age80          ///
            c.reas22 c.slope12#c.reas22 c.slope26#c.reas22                           ///
            c.age80#c.reas22 c.slope12#c.age80#c.reas22 c.slope26#c.age80#c.reas22,  ///
      || ID: slope12 slope26, reml nolog difficult covariance(unstructured)          ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix PAgeReas = r(table)   // Save results for computations below
```

Which fixed effects are now conditional on age?

Which fixed effects are now conditional on reasoning?

```
--------------------------------------------------------------------------------
                       nm3rt |      Coef.   Std. Err.          DF       t    P>|t|
-----------------------------+--------------------------------------------------
                     slope12 |  -151.5164   31.78285         97.0   -4.77   0.000
                     slope26 |  -34.17831   6.829407         97.0   -5.00   0.000
                       age80 |   22.75975   8.911254         97.0    2.55   0.012
           c.slope12#c.age80 |  -8.436569   5.260677         97.0   -1.60   0.112
           c.slope26#c.age80 |  -.4865751   1.130399         97.0   -0.43   0.668
                      reas22 |  -28.04481   11.64376         97.0   -2.41   0.018
          c.slope12#c.reas22 |  -3.494116   6.873786         97.0   -0.51   0.612
          c.slope26#c.reas22 |   3.449416   1.477019         97.0    2.34   0.022
            c.age80#c.reas22 |  -.9317339   1.857941         97.0   -0.50   0.617
c.slope12#c.age80#c.reas22 |   1.229037   1.096819         97.0    1.12   0.265
c.slope26#c.age80#c.reas22 |   .1025695   .2356813         97.0    0.44   0.664
                       _cons |   1974.573   53.83813         97.0   36.68   0.000
--------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ID: Unstructured             |
               var(slope12) |   62983.86   13304.13      41632.19    95286.05
               var(slope26) |   2446.404   621.8551      1486.483    4026.209
                 var(_cons) |   244192.3   37629.28      180536.3    330293.1
        cov(slope12,slope26) |  -1999.155   2088.736     -6093.001    2094.692
          cov(slope12,_cons) |   -49616.9   17178.99     -83287.11   -15946.69
          cov(slope26,_cons) |  -7513.682   3458.039     -14291.31     -736.05
-----------------------------+------------------------------------------------
              var(Residual) |   17673.01   1435.833      15071.46    20723.64
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 849.81                Prob > chi2 = 0.0000
```

```
display "-2LL = " e(ll)*-2                  // Print -2LL for model
-2LL = 8220.9291
```

```
estat recovariance, relevel(ID) correlation  // GCORR matrix

             |   slope12    slope26      _cons
-------------+---------------------------------
     slope12 |         1
     slope26 | -.1610526          1
       _cons |  -.400082  -.3074134          1

// DF=3 Wald test for all Age*Reasoning Slopes
test (c.age80#c.reas22=0)(c.slope12#c.age80#c.reas22=0)(c.slope26#c.age80#c.reas22=0), small

       F(  3, 97.00) =    0.66
            Prob > F =    0.5791
```

Simple Slopes of Interactions (each of these is for a model-implied slope of that predictor):

Slope12 = $\gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{13}(\text{Age}_i - 80)(\text{Reas}_i - 22)$

Slope26 = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{22}(\text{Reas}_i - 22) + \gamma_{23}(\text{Age}_i - 80)(\text{Reas}_i - 22)$

Age = $\gamma_{01} + \gamma_{11}(\text{Slope12}_{ti}) + \gamma_{21}(\text{Slope26}_{ti}) + \gamma_{03}(\text{Reas}_i - 22) + \gamma_{13}(\text{Slope12}_{ti})(\text{Reas}_i - 22) + \gamma_{23}(\text{Slope26}_{ti})(\text{Reas}_i - 22)$

Reasoning = $\gamma_{02} + \gamma_{12}(\text{Slope12}_{ti}) + \gamma_{22}(\text{Slope26}_{ti}) + \gamma_{03}(\text{Age}_i - 80) + \gamma_{13}(\text{Slope12}_{ti})(\text{Age}_i - 80) + \gamma_{23}(\text{Slope26}_{ti})(\text{Age}_i - 80)$

Age*Reasoning = $\gamma_{03} + \gamma_{13}(\text{Slope12}_{ti}) + \gamma_{23}(\text{Slope26}_{ti})$

Age*Slope12 = $\gamma_{11} + \gamma_{13}(\text{Reas}_i - 22)$

Age*Slope26 = $\gamma_{21} + \gamma_{23}(\text{Reas}_i - 22)$

Reasoning*Slope12 = $\gamma_{12} + \gamma_{13}(\text{Age}_i - 80)$

Reasoning*Slope26 = $\gamma_{22} + \gamma_{23}(\text{Age}_i - 80)$

```
// Age simple slopes (for about -1SD, M, +1SD of reas22) to decompose interactions
lincom c.age80*1 + c.age80#c.reas22*-5, small  // Age on Intercept: Reas 17
lincom c.age80*1 + c.age80#c.reas22*0 , small  // Age on Intercept: Reas 22
lincom c.age80*1 + c.age80#c.reas22*5 , small  // Age on Intercept: Reas 27
lincom c.slope12#c.age80*1 + c.slope12#c.age80#c.reas22*-5, small  // Age on Slope12: Reas 17
lincom c.slope12#c.age80*1 + c.slope12#c.age80#c.reas22*0 , small  // Age on Slope12: Reas 22
lincom c.slope12#c.age80*1 + c.slope12#c.age80#c.reas22*5 , small  // Age on Slope12: Reas 27
lincom c.slope26#c.age80*1 + c.slope26#c.age80#c.reas22*-5, small  // Age on Slope26: Reas 17
lincom c.slope26#c.age80*1 + c.slope26#c.age80#c.reas22*0 , small  // Age on Slope26: Reas 22
lincom c.slope26#c.age80*1 + c.slope26#c.age80#c.reas22*5 , small  // Age on Slope26: Reas 27

// Reasoning simple slopes (for about -1SD, M, +1SD of age80) to decompose interactions
lincom c.reas22*1 + c.age80#c.reas22*-6, small  // Reasoning on Intercept: Age 74
lincom c.reas22*1 + c.age80#c.reas22*0 , small  // Reasoning on Intercept: Age 80
lincom c.reas22*1 + c.age80#c.reas22*6 , small  // Reasoning on Intercept: Age 86
lincom c.slope12#c.reas22*1 + c.slope12#c.age80#c.reas22*-6, small  // Reas on Slope12: Age 74
lincom c.slope12#c.reas22*1 + c.slope12#c.age80#c.reas22*0 , small  // Reas on Slope12: Age 80
lincom c.slope12#c.reas22*1 + c.slope12#c.age80#c.reas22*6 , small  // Reas on Slope12: Age 86
lincom c.slope26#c.reas22*1 + c.slope26#c.age80#c.reas22*-6, small  // Reas on Slope26: Age 74
lincom c.slope26#c.reas22*1 + c.slope26#c.age80#c.reas22*0 , small  // Reas on Slope26: Age 80
lincom c.slope26#c.reas22*1 + c.slope26#c.age80#c.reas22*6 , small  // Reas on Slope26: Age 86
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Age on Intercept: Reas 17	27.4184	12.5162	97	2.19	0.0309
Age on Intercept: Reas 22	22.7598	8.9112	97	2.55	0.0122
Age on Intercept: Reas 27	18.1011	13.2197	97	1.37	0.1741
Age on Slope12: Reas 17	-14.5818	7.3888	97	-1.97	0.0513
Age on Slope12: Reas 22	-8.4366	5.2607	97	-1.60	0.1120
Age on Slope12: Reas 27	-2.2914	7.8042	97	-0.29	0.7697
Age on Slope26: Reas 17	-0.9994	1.5877	97	-0.63	0.5305
Age on Slope26: Reas 22	-0.4866	1.1304	97	-0.43	0.6678
Age on Slope26: Reas 27	0.02627	1.6769	97	0.02	0.9875

```
Reasoning on Intercept: Age 74    -22.4544      14.7895       97       -1.52       0.1322
Reasoning on Intercept: Age 80    -28.0448      11.6437       97       -2.41       0.0179
Reasoning on Intercept: Age 86    -33.6352      17.3483       97       -1.94       0.0554
Reasoning on Slope12:   Age 74    -10.8683       8.7309       97       -1.24       0.2162
Reasoning on Slope12:   Age 80     -3.4941       6.8738       97       -0.51       0.6124
Reasoning on Slope12:   Age 86      3.8801      10.2414       97        0.38       0.7056
Reasoning on Slope26:   Age 74      2.8340       1.8761       97        1.51       0.1341
Reasoning on Slope26:   Age 80      3.4494       1.4770       97        2.34       0.0216
Reasoning on Slope26:   Age 86      4.0648       2.2006       97        1.85       0.0678
```

```
// Build total-R2
predict predPAgeReas                   // Save yhat
quietly corr predPAgeReas nm3rt        // Get total-r to make R2
global R2PAgeReas = r(rho)^2           // Save total-R2 for comparison
display "Total-R2 = " $R2PAgeReas      // Print total-R2 relative to empty model
Total-R2 = .16245537

display "Change in Total-R2 = "  $R2PAgeReas - $R2PReas
Change in Total-R2 = .00114825

// Save variances and compute pseudo-R2 and change in pseudo-R2
matrix list PAgeReas
global PAgeReasIntVar = exp(PAgeReas[1,15])^2   // Save as L2 random intercept variance
global PAgeReasS12Var = exp(PAgeReas[1,13])^2   // Save as L2 random slope12 variance
global PAgeReasS26Var = exp(PAgeReas[1,14])^2   // Save as L2 random slope26 variance
global PAgeReasResVar = exp(PAgeReas[1,19])^2   // Save as L1 residual variance
display "Pseudo-R2 for Intercept = " 1-($PAgeReasIntVar/$PUncIntVar)
display "Pseudo-R2 for Slope12 = "   1-($PAgeReasS12Var/$PUncS12Var)
display "Pseudo-R2 for Slope26 = "   1-($PAgeReasS26Var/$PUncS26Var)
display "Pseudo-R2 for Residual = "  1-($PAgeReasResVar/$PUncResVar)
display "Change in Pseudo-R2 for Intercept = " (1-($PAgeReasIntVar/$PUncIntVar)) ///
                                             - (1-($PReasIntVar/$PUncIntVar))
display "Change in Pseudo-R2 for Slope12 = "   (1-($PAgeReasS12Var/$PUncS12Var)) ///
                                             - (1-($PReasS12Var/$PUncS12Var))
display "Change in Pseudo-R2 for Slope26 = "   (1-($PAgeReasS26Var/$PUncS26Var)) ///
                                             - (1-($PReasS26Var/$PUncS26Var))
display "Change in Pseudo-R2 for Residual = "  (1-($PAgeReasResVar/$PUncResVar)) ///
                                             - (1-($PReasResVar/$PUncResVar))
(shown below)
```

```
print("R 1e: Drop Education, Add Age*Reasoning Predicting Intercept, Slope12, and Slope26")
PAgeReas = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
                formula=nm3rt~1+slope12+slope26+age80+reas22 +slope12:age80 +slope26:age80
                        +slope12:reas22 +slope26:reas22 +age80:reas22 +slope12:age80:reas22
                        +slope26:age80:reas22 +(1+slope12+slope26|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(PAgeReas, chkREML=FALSE); summary(PAgeReas, ddf="Satterthwaite")

print("DF=3 Wald Test for all Age*Reasoning Slopes")
contestMD(PAgeReas, ddf="Satterthwaite", L=rbind(c(0,0,0,0,0,0,0,0,0,1,0,0),
                    c(0,0,0,0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,0,0,0,1)))

print("Age simple slopes (for about -1SD, M, +1SD of reas22) to decompose interactions")
print("Age on Intercept: Reas 17"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,1,0,0,0,0,0,-5,0,0))
print("Age on Intercept: Reas 22"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,1,0,0,0,0,0, 0,0,0))
print("Age on Intercept: Reas 27"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,1,0,0,0,0,0, 5,0,0))
print("Age on Slope12: Reas 17");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,1,0,0,0, 0,-5,0))
print("Age on Slope12: Reas 22");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,1,0,0,0, 0, 0,0))
print("Age on Slope12: Reas 27");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,1,0,0,0, 0, 5,0))
print("Age on Slope26: Reas 17");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,1,0,0, 0, 0,-5))
print("Age on Slope26: Reas 22");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,1,0,0, 0, 0, 0))
print("Age on Slope26: Reas 27");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,1,0,0, 0, 0, 5))

print("Reasoning simple slopes (for about -1SD, M, +1SD of age80) to decompose interactions")
print("Reas on Intercept: Age 74"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0,0,-6,0,0))
print("Reas on Intercept: Age 80"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0,0, 0,0,0))
print("Reas on Intercept: Age 86"); contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0,0, 6,0,0))
```

```
print("Reas on Slope12: Age 74");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,0, 0,-6,0))
print("Reas on Slope12: Age 80");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,0, 0, 0,0))
print("Reas on Slope12: Age 86");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,0, 0, 6,0))
print("Reas on Slope26: Age 74");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,1, 0, 0,-6))
print("Reas on Slope26: Age 80");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,1, 0, 0, 0))
print("Reas on Slope26: Age 86");   contest1D(PAgeReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,0,1, 0, 0, 6))

# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=PReas, name1="Reasoning",
        model2=PAgeReas, name2="Age*Reas")

  totalR2.1 totalR2.2    changeR2
1 0.1613071 0.1624554 0.001148258

PseudoR2(data=Example7b, baseModel=PUnc, model1=PReas, name1="Reasoning",
         model2=PAgeReas, name2="Age*Reas")

From R for better organization:
         term      base     model1    model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 284311.49 242192.031 244192.07          0.1481          0.1411         -0.0070
2     slope12  63954.18  63221.876  62983.60          0.0115          0.0152          0.0037
3     slope26   2617.28   2411.556   2446.39          0.0786          0.0653         -0.0133
7    Residual  17673.03  17673.023  17673.05          0.0000         -0.0000         -0.0000
```

Based on the nonsignificance of the higher-order interactions, I'd say we're done with this model. Age and reasoning as main effects in predicting the intercept, slope12, and slope26 seems to be the best piecewise slopes model...

2a. Baseline Unconditional Random Quadratic Time Model

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2: Intercept: $\beta_{0i} = \gamma_{00} + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + U_{2i}$

Simple Slopes of Interactions (T = Session_{ti} – 1):

Linear Time = $\gamma_{10} + \gamma_{20}(2T)$

Fixed-Effect-Predicted Outcome: $\hat{y}_{ti} = \gamma_{00} + \gamma_{10}\left(\text{Session}_{ti} - 1\right) + \gamma_{20}\left(\text{Session}_{ti} - 1\right)^2$

```
display "STATA 2a: Random Quadratic Time Unconditional Model"
mixed nm3rt c.time c.time#c.time,                                      ///
      || ID: time timesq, reml nolog difficult covariance(unstructured)  ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix QUnc = r(table)    // Save results for computations below

--------------------------------------------------------------------
       nm3rt |      Coef.   Std. Err.           DF       t    P>|t|
-------------+------------------------------------------------------
        time |  -120.8999   20.04752         100.0   -6.03    0.000
c.time#c.time |  13.86561    3.41541         100.0    4.06    0.000
       _cons |    1945.85   53.84993         100.0   36.13    0.000
--------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ID: Unstructured             |
                   var(time) |   25839.79    5864.68      16561.42    40316.27
                 var(timesq) |   634.4658    172.375      372.5197    1080.605
                  var(_cons) |   276207.8   41445.49      205831.3    370646.8
            cov(time,timesq) |   -3903.29   982.6245     -5829.199   -1977.382
             cov(time,_cons) |  -35734.05   11947.78     -59151.26   -12316.83
           cov(timesq,_cons) |   3901.973   1950.274      79.50683     7724.44
-----------------------------+------------------------------------------------
               var(Residual) |   20298.19   1649.118      17310.19    23801.96
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 890.51                Prob > chi2 = 0.0000
```

```
display "-2LL = " e(ll)*-2                   // Print -2LL for model
-2LL = 8302.7457
estat recovariance, relevel(ID) correlation  // GCORR matrix

             |      time     timesq      _cons
-------------+---------------------------------
        time |         1
      timesq | -.9640116          1
       _cons | -.4229799   .2947557          1
```

These are the correlations among the random effects. Note the strong correlation among linear (at time 0) and quadratic change.

```
// Build total-R2
predict predQUnc                // Save yhat
quietly corr predQUnc nm3rt     // Get total-r to make R2
global R2QUnc = r(rho)^2        // Save total-R2 for comparison
display "Total-R2 = " $R2QUnc   // Print total-R2 relative to empty model
Total-R2 = .03673722

// Save variances for pseudo-R2
matrix list QUnc
```

QUnc[9,10]

	nm3rt: time	nm3rt: c.time# c.time	nm3rt: _cons	lns1_1_1: _cons	lns1_1_2: _cons	lns1_1_3: _cons	atr1_1_1_2: _cons	atr1_1_1_3: _cons	atr1_1_2_3: _cons	lnsig_e: _cons
b	-120.89992	13.865613	1945.8498	5.0798354	3.2263917	6.2644543	-1.9997732	-.45131573	.30376654	4.9591434
se	20.047524	3.4154096	53.849926	.11348157	.13584264	.07502594	.18927919	.12818523	.13982446	.04062229
t	-6.0306661	4.059722	36.134679	44.76353	23.75095	83.497178	-10.565203	-3.5208091	2.1724849	122.07938
pvalue	2.761e-08	.00009768	3.324e-59	0	1.07e-124	0	4.320e-26	.00043023	.02981911	0
ll	-160.67364	7.0895381	1839.0131	4.8574157	2.960145	6.1174062	-2.3707536	-.70255416	.02971562	4.8795252
ul	-81.126206	20.641689	2052.6865	5.3022552	3.4926384	6.4115024	-1.6287928	-.20007729	.57781745	5.0387617
df	100	100	100	.	.	.	.	.	.	.
crit	1.9839715	1.9839715	1.9839715	1.959964	1.959964	1.959964	1.959964	1.959964	1.959964	1.959964
eform	0	0	0	0	0	0	0	0	0	0

```
global QUncIntVar = exp(QUnc[1,6])^2    // Save as L2 random intercept variance
global QUncLinVar = exp(QUnc[1,4])^2    // Save as L2 random linear time variance
global QUncQuaVar = exp(QUnc[1,5])^2    // Save as L2 random quadratic time variance
global QUncResVar = exp(QUnc[1,10])^2   // Save as L1 residual variance
//display $QUncQuaVar    // Check to make sure it worked

display "STATA Intercept Reliability = ICC2"
display $QUncIntVar/($QUncIntVar+($QUncResVar/$Ntimes))
.98790006

display "STATA Linear Time Reliability"
display $QUncLinVar/($QUncLinVar+($QUncResVar/($Ntimes*$LinVar)))
.95710822

display "STATA Quadratic Time Reliability"
display $QUncQuaVar/($QUncQuaVar+($QUncResVar/($Ntimes*$QuaVar)))
.93697422


print("R 2a: Random Quadratic Time Unconditional Model")
QUnc = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
            formula=nm3rt~1+time+I(time^2) +(1+time+I(time^2)|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(QUnc, chkREML=FALSE); summary(QUnc, ddf="Satterthwaite")

$AICtab
      AIC       BIC    logLik  deviance  df.resid
 8322.746  8366.814 -4151.373  8302.746   596.000

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 ID       (Intercept) 276206.2 525.55
          time         25839.6 160.75   -0.42
          I(time^2)      634.5  25.19    0.29 -0.96
 Residual              20298.2 142.47
```

Note the strong correlation among the random effects for linear (at time 0) and quadratic change.

```
Fixed effects:
              Estimate Std. Error       df t value     Pr(>|t|)
(Intercept) 1945.850     53.850  100.000  36.135      < 2e-16
time         -120.900     20.047  100.001  -6.031 0.0000000276
I(time^2)      13.866      3.415  100.001   4.060 0.0000976788

# Effect size using custom functions
TotalR2(data=Example7b, dvName="nm3rt", model1=QUnc, name1="Quadratic Time")
[1] 0.03673722

# Compute intercept and slope reliabilities
as.data.frame(VarCorr(QUnc))  # Print variance components to see order
QUncIntVar = as.data.frame(VarCorr(QUnc))[1,4]
QUncLinVar = as.data.frame(VarCorr(QUnc))[2,4]
QUncQuaVar = as.data.frame(VarCorr(QUnc))[3,4]
QUncResVar = as.data.frame(VarCorr(QUnc))[7,4]

print("R Intercept Reliability = ICC2")
QUncIntVar/(QUncIntVar+(QUncResVar/Ntimes))
[1] 0.9879

print("R Linear Time Reliability")
QUncLinVar/(QUncLinVar+(QUncResVar/(Ntimes*LinVar)))
[1] 0.9571079

print("R Quadratic Time Reliability")
QUncQuaVar/(QUncQuaVar+(QUncResVar/(Ntimes*QuaVar)))
[1] 0.9369736
```

2b. Quadratic Model with Age Predicting Intercept, Linear Time, and Quadratic Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = $\text{Session}_{ti} - 1$):

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}\left(T\right) + \gamma_{20}\left(T\right)^2$$
$$+ \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}\left(T\right)\left(\text{Age}_i - 80\right) + \gamma_{21}\left(T\right)^2\left(\text{Age}_i - 80\right)$$

Simple Slopes of Interactions (T = Session_{ti} – 1):

Linear = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(\text{Age}_i - 80) + \gamma_{21}(2T)(\text{Age}_i - 80)$

Quadratic = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

Age = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

Age*Time = $\gamma_{11} + \gamma_{21}(2T)$

```
display "STATA 2b: Add Age Predicting Intercept, Linear Time, and Quadratic Time"
mixed nm3rt c.time c.time#c.time c.age80 c.time#c.age80 c.time#c.time#c.age80,  ///
      || ID: time timesq, variance reml difficult covariance(un)                 ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix QAge = r(table)   // Save results for computations below

----------------------------------------------------------------------------
                  nm3rt |      Coef.   Std. Err.           DF        t    P>|t|
------------------------+---------------------------------------------------
                   time |  -121.8325   19.86717          99.0    -6.13    0.000
          c.time#c.time |   13.97744   3.409614          99.0     4.10    0.000
                  age80 |   29.04954   8.461584          99.0     3.43    0.001
         c.time#c.age80 |  -5.594634   3.284586          99.0    -1.70    0.092
  c.time#c.time#c.age80 |   .6709122   .5637023          99.0     1.19    0.237
                  _cons |   1950.692   51.18081          99.0    38.11    0.000
----------------------------------------------------------------------------
```

Interpret the fixed intercept:

Interpret the fixed effect of linear time:

Interpret the fixed effect of quadratic time:

Interpret the effect of age80:

Interpret the effect of linear*age80:

Interpret the effect of quadratic*age80:

```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ID: Unstructured             |
                   var(time) |   25082.47   5787.732      15957.21    39426.06
                 var(timesq) |   629.5796   172.5156      367.9653    1077.195
                  var(_cons) |   247692.6   37603.92      183944.2    333533.7
           cov(time,timesq) |   -3830.19   976.7921     -5744.668   -1915.713
            cov(time,_cons) |  -30154.34   11200.07     -52106.08   -8202.611
          cov(timesq,_cons) |   3232.798   1848.713     -390.6124    6856.208
-----------------------------+------------------------------------------------
               var(Residual) |   20298.18   1649.117      17310.18    23801.95
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 855.04                Prob > chi2 = 0.0000

display "-2LL = " e(ll)*-2                   // Print -2LL for model
-2LL = 8283.1557

estat recovariance, relevel(ID) correlation  // GCORR matrix
             |      time     timesq      _cons
-------------+---------------------------------
        time |         1
      timesq |  -.963851          1
       _cons | -.3825677   .2588789          1

// DF=3 Wald test for all Age Slopes
test (c.age80=0)(c.time#c.age80=0)(c.time#c.time#c.age80=0), small

       F(  3, 99.00) =    4.00
            Prob > F =    0.0098
```

Simple Slopes of Interactions (T = $Session_{ti}$ – 1):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(Age_i - 80) + \gamma_{21}(2T)(Age_i - 80)$

```
// Simple linear time slope at session 1, 3, 5 for age 74, 80, 86 (about -1SD, M, +1 SD of age80)
margins, at(c.age80=(-6(6)6) c.time=(0(2)4)) dydx(c.time) df(99) // As given below
// Use 2*time for both quadratic terms
lincom c.time*1 + c.time#c.time*0 + c.time#c.age80*-6 + c.time#c.time#c.age80*0   , small // S1, Age 74
lincom c.time*1 + c.time#c.time*4 + c.time#c.age80*-6 + c.time#c.time#c.age80*-24, small // S3, Age 74
lincom c.time*1 + c.time#c.time*8 + c.time#c.age80*-6 + c.time#c.time#c.age80*-48, small // S5, Age 74
lincom c.time*1 + c.time#c.time*0 + c.time#c.age80*0  + c.time#c.time#c.age80*0   , small // S1, Age 80
lincom c.time*1 + c.time#c.time*4 + c.time#c.age80*0  + c.time#c.time#c.age80*0   , small // S3, Age 80
lincom c.time*1 + c.time#c.time*8 + c.time#c.age80*0  + c.time#c.time#c.age80*0   , small // S5, Age 80
lincom c.time*1 + c.time#c.time*0 + c.time#c.age80*6  + c.time#c.time#c.age80*0   , small // S1, Age 86
lincom c.time*1 + c.time#c.time*4 + c.time#c.age80*6  + c.time#c.time#c.age80*24  , small // S3, Age 86
lincom c.time*1 + c.time#c.time*8 + c.time#c.age80*6  + c.time#c.time#c.age80*48  , small // S5, Age 86
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Linear Time: S1, Age 74	-88.2647	27.5955	99	-3.20	0.0019
Linear Time: S3, Age 74	-48.4568	10.9025	99	-4.44	<.0001
Linear Time: S5, Age 74	-8.6489	13.9267	99	-0.62	0.5360
Linear Time: S1, Age 80	-121.83	19.8672	99	-6.13	<.0001
Linear Time: S3, Age 80	-65.9227	7.8492	99	-8.40	<.0001
Linear Time: S5, Age 80	-10.0129	10.0264	99	-1.00	0.3204
Linear Time: S1, Age 86	-155.40	28.3668	99	-5.48	<.0001
Linear Time: S3, Age 86	-83.3886	11.2072	99	-7.44	<.0001
Linear Time: S5, Age 86	-11.3769	14.3159	99	-0.79	0.4287

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80)$

```
// Simple quadratic time slope for age 74, 80, 86 (about -1SD, M, +1 SD of age80)
lincom c.time#c.time*1 + c.time#c.time#c.age80*-6, small  // Age 74
lincom c.time#c.time*1 + c.time#c.time#c.age80*0 , small  // Age 80
lincom c.time#c.time*1 + c.time#c.time#c.age80*6 , small  // Age 86
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Quadratic Time: Age 74	9.9520	4.7360	99	2.10	0.0381
Quadratic Time: Age 80	13.9774	3.4096	99	4.10	<.0001
Quadratic Time: Age 86	18.0029	4.8683	99	3.70	0.0004

Age Slope = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

```
// Simple age slope at each session (S): use time and time^2
margins, at(c.time=(0(1)5)) dydx(c.age80) df(99) // Same simple age slope per session
lincom c.age80*1 + c.time#c.age80*0 + c.time#c.time#c.age80*0 , small  // S1
lincom c.age80*1 + c.time#c.age80*1 + c.time#c.time#c.age80*1 , small  // S2
lincom c.age80*1 + c.time#c.age80*2 + c.time#c.time#c.age80*4 , small  // S3
lincom c.age80*1 + c.time#c.age80*3 + c.time#c.time#c.age80*9 , small  // S4
lincom c.age80*1 + c.time#c.age80*4 + c.time#c.time#c.age80*16, small  // S5
lincom c.age80*1 + c.time#c.age80*5 + c.time#c.time#c.age80*25, small  // S6
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Age Slope: S1	29.0495	8.4616	99	3.43	0.0009
Age Slope: S2	24.1258	7.6862	99	3.14	0.0022
Age Slope: S3	20.5439	7.5343	99	2.73	0.0076
Age Slope: S4	18.3038	7.4038	99	2.47	0.0151
Age Slope: S5	17.4056	7.1425	99	2.44	0.0166
Age Slope: S6	17.8492	7.1254	99	2.51	0.0139

Age*Time Slope = $\gamma_{11} + \gamma_{21}(2T)$

```
// Simple age*linear time interaction slope at each session (S): use 2*time
lincom c.time#c.age80*1 + c.time#c.time#c.age80*0 , small  // S1
lincom c.time#c.age80*1 + c.time#c.time#c.age80*2 , small  // S2
lincom c.time#c.age80*1 + c.time#c.time#c.age80*4 , small  // S3
lincom c.time#c.age80*1 + c.time#c.time#c.age80*6 , small  // S4
lincom c.time#c.age80*1 + c.time#c.time#c.age80*8 , small  // S5
lincom c.time#c.age80*1 + c.time#c.time#c.age80*10, small  // S6
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Age*Linear Time: S1	-5.5946	3.2846	99	-1.70	0.0916
Age*Linear Time: S2	-4.2528	2.2283	99	-1.91	0.0592
Age*Linear Time: S3	-2.9110	1.2977	99	-2.24	0.0271

```
Age*Linear Time: S4          -1.5692      0.9720      99      -1.61      0.1096
Age*Linear Time: S5          -0.2273      1.6576      99      -0.14      0.8912
Age*Linear Time: S6           1.1145      2.6632      99       0.42      0.6765
```

```
// Get adjusted means per session and age (start(by)end)
margins, at(c.time=(0(1)5) c.age80=(-6 0 6))
marginsplot    // Plot adjusted means (not shown)

// Build total-R2
predict predQAge                // Save yhat
quietly corr predQAge nm3rt     // Get total-r to make R2
global R2QAge = r(rho)^2        // Save total-R2 for comparison
display "Total-R2 = " $R2QAge   // Print total-R2 relative to empty model
Total-R2 = .1068511

display "Change in Total-R2 = "  $R2QAge - $R2QUnc
Change in Total-R2 = .07011388

// Save variances and compute pseudo-R2
matrix list QAge
global QAgeIntVar = exp(QAge[1,9])^2    // Save as L2 random intercept variance
global QAgeLinVar = exp(QAge[1,7])^2    // Save as L2 random linear time variance
global QAgeQuaVar = exp(QAge[1,8])^2    // Save as L2 random quadratic time variance
global QAgeResVar = exp(QAge[1,13])^2   // Save as L1 residual variance
display "Pseudo-R2 for Intercept = "        1-($QAgeIntVar/$QUncIntVar)
display "Pseudo-R2 for Linear Time = "      1-($QAgeLinVar/$QUncLinVar)
display "Pseudo-R2 for Quadratic Time = "   1-($QAgeQuaVar/$QUncQuaVar)
display "Pseudo-R2 for Residual = "         1-($QAgeResVar/$QUncResVar)
(given below instead)
```

```
print("R 2b: Add Age Predicting Intercept, Linear Time, and Quadratic Time")
QAge = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
            formula=nm3rt~1+time+I(time^2)+age80 +time:age80 +I(time^2):age80
                    +(1+time+(time^2)|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(QAge, chkREML=FALSE); summary(QAge, ddf="Satterthwaite")

print("DF=3 Wald Test for all Age Slopes")
contestMD(QAge, ddf="Satterthwaite", L=rbind(c(0,0,0,1,0,0),c(0,0,0,0,1,0),c(0,0,0,0,0,1)))
```

Simple Slopes of Interactions ($T = Session_{ti} - 1$):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(Age_i - 80) + \gamma_{21}(2T)(Age_i - 80)$

```
print("Simple linear time slope: sessions 1, 3, 5 for age 74, 80, 86 (-1SD, M, +1 SD of age80)")
print("Use 2*time for both quadratic terms")
print("Linear Time: S1, Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,0,0,-6,  0))
print("Linear Time: S3, Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,4,0,-6,-24))
print("Linear Time: S5, Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,8,0,-6,-48))
print("Linear Time: S1, Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,0,0, 0,  0))
print("Linear Time: S3, Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,4,0, 0,  0))
print("Linear Time: S5, Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,8,0, 0,  0))
print("Linear Time: S1, Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,0,0, 6,  0))
print("Linear Time: S3, Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,4,0, 6, 24))
print("Linear Time: S5, Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,1,8,0, 6, 48))
```

Quadratic Time = $\gamma_{20} + \gamma_{21}(Age_i - 80)$

```
print("Simple quadratic time slope for age 74, 80, 86 (about -1SD, M, +1 SD of age80)")
print("Quadratic Time: Age 74"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,1,0,0,-6))
print("Quadratic Time: Age 80"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,1,0,0, 0))
print("Quadratic Time: Age 86"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,1,0,0, 6))
```

Age Slope = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

```
print("Simple age slope at each session (S): use time and time^2")
print("Age Slope: S1"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,0, 0))
```

```
print("Age Slope: S2"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,1, 1))
print("Age Slope: S3"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,2, 4))
print("Age Slope: S4"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,3, 9))
print("Age Slope: S5"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,4,16))
print("Age Slope: S6"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,1,5,25))
```

Age*Time Slope = $\gamma_{11} + \gamma_{21}(2T)$

```
print("Simple age*linear time interaction slope at each session (S): use 2*time")
print("Age*Linear Time: S1"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 0))
print("Age*Linear Time: S2"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 2))
print("Age*Linear Time: S3"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 4))
print("Age*Linear Time: S4"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 6))
print("Age*Linear Time: S5"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1, 8))
print("Age*Linear Time: S6"); contest1D(QAge, ddf="Satterthwaite", L=c(0,0,0,0,1,10))

# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=QUnc, name1="Quadratic Time",
        model2=QAge, name2="Age")
PseudoR2(data=Example7b, baseModel=QUnc, model1=QAge, name1="Age")
```

```
Pseudo-R2 for Age
         term        base      model1 pseudoR2.model1
1 (Intercept) 276206.2261 247691.1000          0.1032
2        time  25839.6306  25082.6896          0.0293
3   I(time^2)    634.4598    629.5866          0.0077
7    Residual  20298.2117  20298.1640          0.0000
```

2c. Quadratic Model with Age and Reasoning Predicting Intercept, Linear Time, and Quadratic Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reas}_i - 22\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reas}_i - 22\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{22}\left(\text{Reas}_i - 22\right) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = $\text{Session}_{ti} - 1$):

$$\begin{aligned}\hat{y}_{ti} &= \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2 \\ &+ \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}(T)\left(\text{Age}_i - 80\right) + \gamma_{21}(T)^2\left(\text{Age}_i - 80\right) \\ &+ \gamma_{02}\left(\text{Reas}_i - 22\right) + \gamma_{12}(T)\left(\text{Reas}_i - 22\right) + \gamma_{22}(T)^2\left(\text{Reas}_i - 22\right)\end{aligned}$$

```
display "STATA 2c: Keep Age, Add Reasoning Predicting Intercept, Linear Time, and Quadratic Time"
mixed nm3rt c.time c.time#c.time c.age80 c.time#c.age80 c.time#c.time#c.age80   ///
            c.reas22 c.time#c.reas22 c.time#c.time#c.reas22,                     ///
      || ID: time timesq, variance reml difficult covariance(un)                  ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix QReas = r(table)   // Save results for computations below
display "-2LL = " e(ll)*-2                  // Print -2LL for model
estat recovariance, relevel(ID) correlation  // GCORR matrix

// DF=3 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.time#c.reas22=0)(c.time#c.time#c.reas22=0), small
```

Simple Slopes of Interactions (T = $\text{Session}_{ti} - 1$):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(\text{Age}_i - 80) + \gamma_{21}(2T)(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22) + \gamma_{22}(2T)(\text{Reas}_i - 22)$

```
// Simple linear time slope at session 1, 3, 5 for reasoning 17, 22, 27 (about -1SD, M, +1 SD of age80)
margins, at(c.age80=0 c.reas22=(-5(5)5) c.time=(0(2)4)) dydx(c.time) df(98) // As given below
```

```
// Use 2*time for both quadratic terms, hold age80=0
lincom c.time*1 + c.time#c.time*0 + c.time#c.reas22*-5 + c.time#c.time#c.reas22*0  , small // S1, Reas 17
lincom c.time*1 + c.time#c.time*4 + c.time#c.reas22*-5 + c.time#c.time#c.reas22*-20, small // S3, Reas 17
lincom c.time*1 + c.time#c.time*8 + c.time#c.reas22*-5 + c.time#c.time#c.reas22*-40, small // S5, Reas 17
lincom c.time*1 + c.time#c.time*0 + c.time#c.reas22*0  + c.time#c.time#c.reas22*0  , small // S1, Reas 22
lincom c.time*1 + c.time#c.time*4 + c.time#c.reas22*0  + c.time#c.time#c.reas22*0  , small // S3, Reas 22
lincom c.time*1 + c.time#c.time*8 + c.time#c.reas22*0  + c.time#c.time#c.reas22*0  , small // S5, Reas 22
lincom c.time*1 + c.time#c.time*0 + c.time#c.reas22*5  + c.time#c.time#c.reas22*0  , small // S1, Reas 27
lincom c.time*1 + c.time#c.time*4 + c.time#c.reas22*5  + c.time#c.time#c.reas22*20 , small // S3, Reas 27
lincom c.time*1 + c.time#c.time*8 + c.time#c.reas22*5  + c.time#c.time#c.reas22*40 , small // S5, Reas 27
```

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{21}(\text{Reas}_i - 22)$

```
// Simple quadratic time slope for reasoning 17, 22, 27 (about -1SD, M, +1 SD of reas22), hold age80=0
lincom c.time#c.time*1 + c.time#c.time#c.reas22*-5, small  // Reas 17
lincom c.time#c.time*1 + c.time#c.time#c.reas22*0 , small  // Reas 22
lincom c.time#c.time*1 + c.time#c.time#c.reas22*5 , small  // Reas 27
```

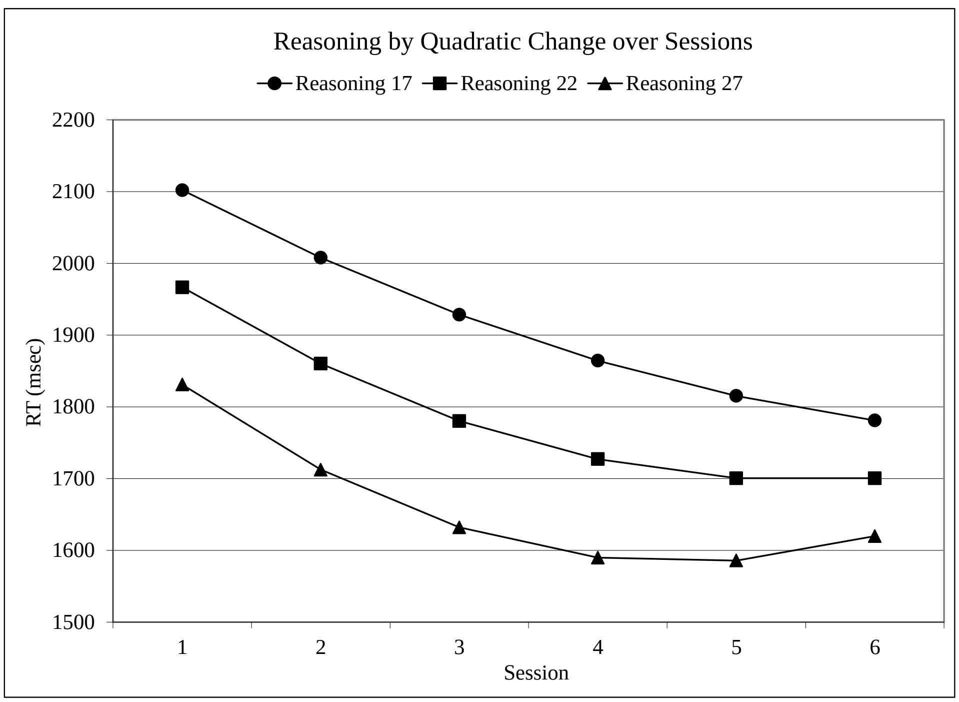
Reasoning Slope = $\gamma_{02} + \gamma_{12}(T) + \gamma_{22}(T)^2$

```
// Simple reasoning slope at each session (S): use time and time^2
margins, at(c.age80=0 c.time=(0(1)5)) dydx(c.reas22) df(98) // As given below
lincom c.reas22*1 + c.time#c.reas22*0 + c.time#c.time#c.reas22*0 , small  // S1
lincom c.reas22*1 + c.time#c.reas22*1 + c.time#c.time#c.reas22*1 , small  // S2
lincom c.reas22*1 + c.time#c.reas22*2 + c.time#c.time#c.reas22*4 , small  // S3
lincom c.reas22*1 + c.time#c.reas22*3 + c.time#c.time#c.reas22*9 , small  // S4
lincom c.reas22*1 + c.time#c.reas22*4 + c.time#c.time#c.reas22*16, small  // S5
lincom c.reas22*1 + c.time#c.reas22*5 + c.time#c.time#c.reas22*25, small  // S6
```

Reasoning*Time Slope = $\gamma_{12} + \gamma_{22}(2T)$

```
// Simple reasoning*linear time interaction slope at each session (S): use 2*time
lincom c.time#c.reas22*1 + c.time#c.time#c.reas22*0 , small  // S1
lincom c.time#c.reas22*1 + c.time#c.time#c.reas22*2 , small  // S2
lincom c.time#c.reas22*1 + c.time#c.time#c.reas22*4 , small  // S3
lincom c.time#c.reas22*1 + c.time#c.time#c.reas22*6 , small  // S4
lincom c.time#c.reas22*1 + c.time#c.time#c.reas22*8 , small  // S5
lincom c.time#c.reas22*1 + c.time#c.time#c.reas22*10, small  // S6

// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.age80=0 c.time=(0(1)5) c.reas22=(-6 0 6))
marginsplot    // Plot adjusted means

// Build total-R2
predict predQReas                 // Save yhat
quietly corr predQReas nm3rt      // Get total-r to make R2
global R2QReas = r(rho)^2         // Save total-R2 for comparison
display "Total-R2 = " $R2QReas    // Print total-R2 relative to empty model
display "Change in Total-R2 = "  $R2QReas - $R2QAge

// Save variances and compute pseudo-R2 and change in pseudo-R2
matrix list QReas
global QReasIntVar = exp(QReas[1,12])^2   // Save as L2 random intercept variance
global QReasLinVar = exp(QReas[1,10])^2   // Save as L2 random linear time variance
global QReasQuaVar = exp(QReas[1,11])^2   // Save as L2 random quadratic time variance
global QReasResVar = exp(QReas[1,16])^2   // Save as L1 residual variance
display "Pseudo-R2 for Intercept = "        1-($QReasIntVar/$QUncIntVar)
display "Pseudo-R2 for Linear Time = "      1-($QReasLinVar/$QUncLinVar)
display "Pseudo-R2 for Quadratic Time = "   1-($QReasQuaVar/$QUncQuaVar)
display "Pseudo-R2 for Residual = "         1-($QReasResVar/$QUncResVar)
display "Change in Pseudo-R2 for Intercept = "       (1-($QReasIntVar/$QUncIntVar)) ///
                                                   - (1-($QAgeIntVar/$QUncIntVar))
display "Change in Pseudo-R2 for Linear Time = "     (1-($QReasLinVar/$QUncLinVar)) ///
                                                   - (1-($QAgeLinVar/$QUncLinVar))
display "Change in Pseudo-R2 for Quadratic Time = "  (1-($QReasQuaVar/$QUncQuaVar)) ///
                                                   - (1-($QAgeQuaVar/$QUncQuaVar))
display "Change in Pseudo-R2 for Residual = "        (1-($QReasResVar/$QUncResVar)) ///
                                                   - (1-($QAgeResVar/$QUncResVar))
```

```
print("R 2c: Keep Age, Add Reasoning Predicting Intercept, time1, and I(time1^2)")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
QReas = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
             formula=nm3rt~1+time+I(time^2)+age80+reas22 +time:age80 +I(time^2):age80
                     +time:reas22 +I(time^2):reas22 +(1+time+I(time^2)|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(QReas, chkREML=FALSE); summary(QReas, ddf="Satterthwaite")

$AICtab
      AIC       BIC    logLik  deviance  df.resid
 8293.015  8363.526 -4130.508  8261.015   590.000

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 ID       (Intercept) 235541.2 485.33
          time         25228.3 158.83   -0.42
          I(time^2)      614.5  24.79    0.33 -0.97
 Residual              20298.2 142.47

Fixed effects:
                  Estimate Std. Error       df t value      Pr(>|t|)
(Intercept)      1966.4674    50.4203  97.9998  39.001       < 2e-16
time             -119.7417    20.0746  97.9996  -5.965 0.0000000389
I(time^2)          13.3036     3.4167  97.9997   3.894       0.00018
age80              22.2782     8.7324  97.9998   2.551       0.01228
reas22            -27.1004    11.2829  97.9998  -2.402       0.01820
time:age80         -6.4921     3.4768  97.9996  -1.867       0.06485
I(time^2):age80     0.9601     0.5917  97.9996   1.623       0.10790
time:reas22        -3.5917     4.4922  97.9996  -0.800       0.42591
I(time^2):reas22    1.1575     0.7646  97.9996   1.514       0.13326
```



See excel workbook online for how this plot was made

```
print("DF=3 Wald Test for all Reasoning Slopes")
contestMD(QReas, ddf="Satterthwaite",
          L=rbind(c(0,0,0,0,1,0,0,0,0),c(0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,1)))

    Sum Sq  Mean Sq NumDF    DenDF  F value      Pr(>F)
1 261471.2 87157.07     3 97.99995 4.293836 0.006844106
```

Simple Slopes of Interactions (T = Session$_{ti}$ −1):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(Age_i - 80) + \gamma_{21}(2T)(Age_i - 80) + \gamma_{12}(Reas_i - 22) + \gamma_{22}(2T)(Reas_i - 22)$

```
print("Simple linear time slope: sessions 1, 3, 5 for reas 17, 22, 27 (-1SD, M, +1 SD of reas22)")
print("Use 2*time for both quadratic terms, hold age80=0")
print("Linear Time: S1, Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,-5,  0))
print("Linear Time: S3, Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0,-5,-20))
print("Linear Time: S5, Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0,-5,-40))
print("Linear Time: S1, Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 0,  0))
print("Linear Time: S3, Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 0,  0))
print("Linear Time: S5, Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 0,  0))
print("Linear Time: S1, Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 5,  0))
print("Linear Time: S3, Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 5, 20))
print("Linear Time: S5, Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 5, 40))
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Linear Time: S1, Reas 17	-101.78	32.0151	98	-3.18	0.0020
Linear Time: S3, Reas 17	-71.7192	12.6677	98	-5.66	<.0001
Linear Time: S5, Reas 17	-41.6554	15.6876	98	-2.66	0.0092
Linear Time: S1, Reas 22	-119.74	20.0746	98	-5.96	<.0001
Linear Time: S3, Reas 22	-66.5272	7.9431	98	-8.38	<.0001
Linear Time: S5, Reas 22	-13.3127	9.8367	98	-1.35	0.1791
Linear Time: S1, Reas 27	-137.70	28.1073	98	-4.90	<.0001
Linear Time: S3, Reas 27	-61.3351	11.1215	98	-5.52	<.0001
Linear Time: S5, Reas 27	15.0301	13.7728	98	1.09	0.2778

Quadratic Time = $\gamma_{20} + \gamma_{21}(Age_i - 80) + \gamma_{21}(Reas_i - 22)$

```
print("Simple quadratic time slope for reas 17, 22, 27 (-1SD, M, +1 SD of reas22), hold age80=0")
print("Quadratic Time: Reas 17"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0,-5))
print("Quadratic Time: Reas 22"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0, 0))
print("Quadratic Time: Reas 27"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,1,0,0,0,0,0, 5))
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Quadratic Time: Reas 17	7.5159	5.4490	98	1.38	0.1709
Quadratic Time: Reas 22	13.3036	3.4167	98	3.89	0.0002
Quadratic Time: Reas 27	19.0913	4.7839	98	3.99	0.0001

Reasoning Slope = $\gamma_{02} + \gamma_{12}(T) + \gamma_{22}(T)^2$

```
print("Simple reasoning slope at each session (S): use time and time^2")
print("Reasoning Slope: S1"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0, 0))
print("Reasoning Slope: S2"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1, 1))
print("Reasoning Slope: S3"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,2, 4))
print("Reasoning Slope: S4"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,3, 9))
print("Reasoning Slope: S5"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,4,16))
print("Reasoning Slope: S6"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,5,25))
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Reasoning Slope: S1	-27.1004	11.2829	98	-2.40	0.0182
Reasoning Slope: S2	-29.5346	10.1156	98	-2.92	0.0043
Reasoning Slope: S3	-29.6537	9.8944	98	-3.00	0.0035
Reasoning Slope: S4	-27.4578	9.7730	98	-2.81	0.0060
Reasoning Slope: S5	-22.9468	9.5224	98	-2.41	0.0178
Reasoning Slope: S6	-16.1207	9.6403	98	-1.67	0.0977

Reasoning*Time Slope = $\gamma_{12} + 2\gamma_{22}(T)$

```
print("Simple reasoning*linear time interaction slope at each session (S): use 2*time")
print("Reasoning*Linear Time: S1"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 0))
print("Reasoning*Linear Time: S2"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 2))
print("Reasoning*Linear Time: S3"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 4))
print("Reasoning*Linear Time: S4"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 6))
print("Reasoning*Linear Time: S5"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1, 8))
print("Reasoning*Linear Time: S6"); contest1D(QReas, ddf="Satterthwaite", L=c(0,0,0,0,0,0,0,1,10))
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Reasoning*Linear Time: S1	-3.5917	4.4922	98	-0.80	0.4259
Reasoning*Linear Time: S2	-1.2767	3.0547	98	-0.42	0.6769
Reasoning*Linear Time: S3	1.0384	1.7775	98	0.58	0.5604
Reasoning*Linear Time: S4	3.3535	1.2900	98	2.60	0.0108
Reasoning*Linear Time: S5	5.6686	2.2012	98	2.58	0.0115
Reasoning*Linear Time: S6	7.9836	3.5642	98	2.24	0.0274

```
# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=QAge, name1="Age",
        model2=QReas, name2="Reasoning*Quad")
```

```
  totalR2.1 totalR2.2   changeR2
1 0.1068511 0.1608621 0.05401102
```

```
PseudoR2(data=Example7b, baseModel=QUnc, model1=QAge, name1="Age",
         model2=QReas, name2="Reasoning*Quad")
```

Pseudo-R2 and Change in Pseudo-R2 for Age vs Reasoning*Quad

	term	base	model1	model2	pseudoR2.model1	pseudoR2.model2	pseudoR2.change
1	(Intercept)	276206.2261	247691.1000	235541.1580	0.1032	0.1472	0.0440
2	time	25839.6306	25082.6896	25228.3489	0.0293	0.0237	-0.0056
3	I(time^2)	634.4598	629.5866	614.4702	0.0077	0.0315	0.0238
7	Residual	20298.2117	20298.1640	20298.1819	0.0000	0.0000	-0.0000

From these results *it appears* we could remove both the interaction of reasoning with both the linear and quadratic time slopes, but keep in mind how correlated those terms are… let's see what happens if we just remove just the reasoning*quadratic time interaction for now.

2d. Quadratic Model Removing Reasoning Predicting Quadratic Time Slope

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\text{Session}_{ti} - 1) + \beta_{2i}(\text{Session}_{ti} - 1)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}(\text{Age}_i - 80) + \gamma_{02}(\text{Reason}_i - 22) + U_{0i}$

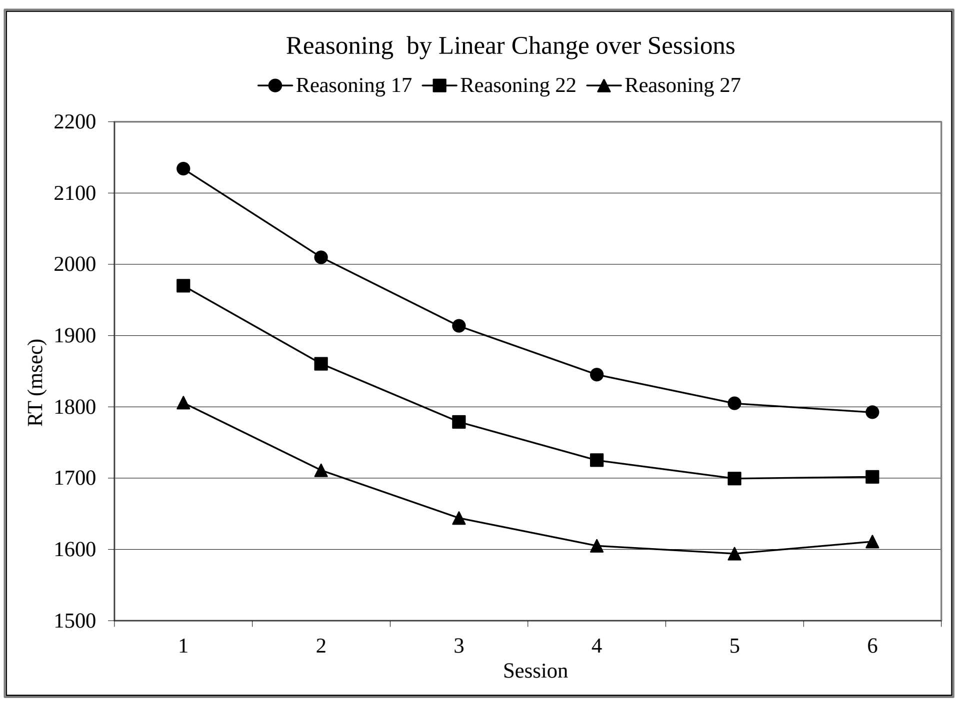
Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}(\text{Age}_i - 80) + \gamma_{12}(\text{Reason}_i - 22) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = Session$_{ti}$ − 1):

$$\hat{y}_{ti} = \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2$$
$$+ \gamma_{01}(\text{Age}_i - 80) + \gamma_{11}(T)(\text{Age}_i - 80) + \gamma_{21}(T)^2(\text{Age}_i - 80)$$
$$+ \gamma_{02}(\text{Reas}_i - 22) + \gamma_{12}(T)(\text{Reas}_i - 22)$$

```
display "STATA 2d: Remove Reasoning Predicting Quadratic Time"
mixed nm3rt c.time c.time#c.time c.age80 c.time#c.age80 c.time#c.time#c.age80  ///
            c.reas22 c.time#c.reas22,                                            ///
      || ID: time timesq, reml nolog difficult covariance(un)                    ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix LReas = r(table)    // Save results for computations below

----------------------------------------------------------------------------
               nm3rt |      Coef.   Std. Err.           DF       t    P>|t|
---------------------+------------------------------------------------------
                time |  -123.5416    20.0362          99.1   -6.17   0.000
       c.time#c.time |   13.97744   3.409614          99.0    4.10   0.000
               age80 |   20.84705   8.686875          99.9    2.40   0.018
      c.time#c.age80 |  -4.860994   3.325251         100.5   -1.46   0.147
c.time#c.time#c.age80 |   .6709122   .5637022          99.0    1.19   0.237
              reas22 |  -32.82806   10.62976          98.0   -3.09   0.003
      c.time#c.reas22 |   2.936178   1.260211          98.0    2.33   0.022 -> different result!
               _cons |   1969.802   50.40858          98.3   39.08   0.000
----------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ID: Unstructured             |
                   var(time) |   25708.82   5884.389       16415.5    40263.38
                 var(timesq) |   629.5787   172.5153      367.9648    1077.194
                  var(_cons) |     235911   36157.97      174696.8    318574.8
            cov(time,timesq) |  -3898.199   985.6229     -5829.984   -1966.413
             cov(time,_cons) |   -32973.4   11270.06     -55062.31   -10884.49
           cov(timesq,_cons) |   3993.212   1849.988      367.3024    7619.121
-----------------------------+------------------------------------------------
               var(Residual) |    20298.2   1649.117       17310.2    23801.97
------------------------------------------------------------------------------
LR test vs. linear model: chi2(6) = 828.33                Prob > chi2 = 0.0000
```



See excel workbook online for how this plot was made

```
display "-2LL = " e(ll)*-2                    // Print -2LL for model
-2LL = 8264.6002
```

```
estat recovariance, relevel(ID) correlation  // GCORR matrix

             |      time     timesq      _cons
-------------+---------------------------------
        time |         1
      timesq | -.9689422          1
       _cons | -.4233977   .3276599          1

// DF=2 Wald test for all Reasoning Slopes
test (c.reas22=0)(c.time#c.reas22=0), small

       F(  2, 98.00) =    5.29
            Prob > F =    0.0066
```

Simple Slopes of Interactions (T = Session$_{ti}$ −1):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(Age_i - 80) + \gamma_{21}(2T)(Age_i - 80) + \gamma_{12}(Reas_i - 22)$

```
// Simple linear time slope at session 1, 3, 5 for reas 17, 22, 27 (-1SD, M, +1 SD of age80)
margins, at(c.age80=0 c.reas22=(-5(5)5) c.time=(0(2)4)) dydx(c.time) df(98) // As given below
// Use 2*time for quadratic term, hold age80=0
lincom c.time*1 + c.time#c.time*0 + c.time#c.reas22*-5, small // S1, Reas 17
lincom c.time*1 + c.time#c.time*4 + c.time#c.reas22*-5, small // S3, Reas 17
lincom c.time*1 + c.time#c.time*8 + c.time#c.reas22*-5, small // S5, Reas 17
lincom c.time*1 + c.time#c.time*0 + c.time#c.reas22*0 , small // S1, Reas 22
lincom c.time*1 + c.time#c.time*4 + c.time#c.reas22*0 , small // S3, Reas 22
lincom c.time*1 + c.time#c.time*8 + c.time#c.reas22*0 , small // S5, Reas 22
lincom c.time*1 + c.time#c.time*0 + c.time#c.reas22*5 , small // S1, Reas 27
lincom c.time*1 + c.time#c.time*4 + c.time#c.reas22*5 , small // S3, Reas 27
lincom c.time*1 + c.time#c.time*8 + c.time#c.reas22*5 , small // S5, Reas 27
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Linear Time: S1, Reas 17	-138.22	21.2222	120	-6.51	<.0001
Linear Time: S3, Reas 17	-82.3130	10.5786	130	-7.78	<.0001
Linear Time: S5, Reas 17	-26.4032	12.0595	141	-2.19	0.0302
Linear Time: S1, Reas 22	-123.54	20.0359	98.9	-6.17	<.0001
Linear Time: S3, Reas 22	-67.6319	7.9348	98.5	-8.52	<.0001
Linear Time: S5, Reas 22	-11.7221	9.8227	98.6	-1.19	0.2356
Linear Time: S1, Reas 27	-108.86	20.7821	112	-5.24	<.0001
Linear Time: S3, Reas 27	-52.9508	9.6653	126	-5.48	<.0001
Linear Time: S5, Reas 27	2.9589	11.2669	130	0.26	0.7933

Reasoning Slope = $\gamma_{02} + \gamma_{12}(T)$

```
// Simple reasoning slope at each session (S): use time only
margins, at(c.age80=0 c.time=(0(1)5)) dydx(c.reas22) df(98)  // As given below
lincom c.reas22*1 + c.time#c.reas22*0, small  // S1
lincom c.reas22*1 + c.time#c.reas22*1, small  // S2
lincom c.reas22*1 + c.time#c.reas22*2, small  // S3
lincom c.reas22*1 + c.time#c.reas22*3, small  // S4
lincom c.reas22*1 + c.time#c.reas22*4, small  // S5
lincom c.reas22*1 + c.time#c.reas22*5, small  // S6
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
Reasoning Slope: S1	-32.8281	10.6298	98	-3.09	0.0026
Reasoning Slope: S2	-29.8919	10.1129	98	-2.96	0.0039
Reasoning Slope: S3	-26.9557	9.7327	98	-2.77	0.0067
Reasoning Slope: S4	-24.0195	9.5055	98	-2.53	0.0131
Reasoning Slope: S5	-21.0833	9.4425	98	-2.23	0.0278
Reasoning Slope: S6	-18.1471	9.5469	98	-1.90	0.0603

```
// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.time=(0(1)5) c.reas22=(-5 0 5)) vsquish
marginsplot   // Plot adjusted means (not shown)

// Build total-R2
predict predLReas                 // Save yhat
quietly corr predLReas nm3rt      // Get total-r to make R2
global R2LReas = r(rho)^2         // Save total-R2 for comparison
display "Total-R2 = " $R2LReas    // Print total-R2 relative to empty model
display "Change in Total-R2 = "  $R2LReas - $R2QAge

// Save variances and compute pseudo-R2 and change in pseudo-R2
matrix list LReas
global LReasIntVar = exp(LReas[1,11])^2   // Save as L2 random intercept variance
global LReasLinVar = exp(LReas[1,9])^2    // Save as L2 random linear time variance
global LReasQuaVar = exp(LReas[1,10])^2   // Save as L2 random quadratic time variance
global LReasResVar = exp(LReas[1,15])^2   // Save as L1 residual variance
display "Pseudo-R2 for Intercept = "        1-($LReasIntVar/$QUncIntVar)
display "Pseudo-R2 for Linear Time = "      1-($LReasLinVar/$QUncLinVar)
display "Pseudo-R2 for Quadratic Time = "   1-($LReasQuaVar/$QUncQuaVar)
display "Pseudo-R2 for Residual = "         1-($LReasResVar/$QUncResVar)
display "Change in Pseudo-R2 for Intercept = "       (1-($LReasIntVar/$QUncIntVar)) ///
                                                   - (1-($QAgeIntVar/$QUncIntVar))
display "Change in Pseudo-R2 for Linear Time = "     (1-($LReasLinVar/$QUncLinVar)) ///
                                                   - (1-($QAgeLinVar/$QUncLinVar))
display "Change in Pseudo-R2 for Quadratic Time = "  (1-($LReasQuaVar/$QUncQuaVar)) ///
                                                   - (1-($QAgeQuaVar/$QUncQuaVar))
display "Change in Pseudo-R2 for Residual = "        (1-($LReasResVar/$QUncResVar)) ///
                                                   - (1-($QAgeResVar/$QUncResVar))
```

```
print("R 2d: Remove Reasoning Predicting Quadratic Time")
LReas = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
             formula=nm3rt~1+time+I(time^2)+age80+reas22 +time:age80 +I(time^2):age80
                    +time:reas22 +(1+time+I(time^2)|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(LReas, chkREML=FALSE); summary(LReas, ddf="Satterthwaite")

print("DF=2 Wald Test for all Reasoning Slopes")
contestMD(LReas, ddf="Satterthwaite", L=rbind(c(0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,1)))
```

Simple Slopes of Interactions (T = Session_{ti} −1):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(Age_i - 80) + \gamma_{21}(2T)(Age_i - 80) + \gamma_{12}(Reas_i - 22)$

```
print("Simple linear slope: sessions 1, 3, 5 for reas 17, 22, 27 (-1SD, M, +1 SD of reas22)")
print("Use 2*time for quadratic term, hold age80=0")
print("Linear Time: S1, Reas 17"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0,-5))
print("Linear Time: S3, Reas 17"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0,-5))
print("Linear Time: S5, Reas 17"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0,-5))
print("Linear Time: S1, Reas 22"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 0))
print("Linear Time: S3, Reas 22"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 0))
print("Linear Time: S5, Reas 22"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 0))
print("Linear Time: S1, Reas 27"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,0,0,0,0,0, 5))
print("Linear Time: S3, Reas 27"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,4,0,0,0,0, 5))
print("Linear Time: S5, Reas 27"); contest1D(LReas, ddf="Satterthwaite", L=c(0,1,8,0,0,0,0, 5))
```

Reasoning Slope = $\gamma_{02} + \gamma_{12}(T)$

```
print("Simple reasoning slope at each session (S): use time and time^2")
print("Reasoning Slope: S1"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,0))
print("Reasoning Slope: S2"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,1))
print("Reasoning Slope: S3"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,2))
print("Reasoning Slope: S4"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,3))
print("Reasoning Slope: S5"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,4))
print("Reasoning Slope: S6"); contest1D(LReas, ddf="Satterthwaite", L=c(0,0,0,0,1,0,0,5))
```

```
# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=QAge, name1="Age",
        model2=LReas, name2="Reasoning*Lin")

  totalR2.1 totalR2.2   changeR2
1 0.1068511 0.1600643 0.05321322

PseudoR2(data=Example7b, baseModel=QUnc, model1=QAge, name1="Age",
        model2=LReas, name2="Reasoning*Lin")

Pseudo-R2 and Change in Pseudo-R2 for Age vs Reasoning*Lin
         term        base      model1      model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 276206.2261 247691.1000 235910.2146          0.1032          0.1459          0.0427
2        time  25839.6306  25082.6896  25709.0953          0.0293          0.0051         -0.0242
3   I(time^2)    634.4598    629.5866    629.5864          0.0077          0.0077          0.0000
7    Residual  20298.2117  20298.1640  20298.1835          0.0000          0.0000         -0.0000
```

2e. Quadratic Model adding Education Group Predicting Intercept, Linear Time, and Quadratic Time

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}\left(\text{Session}_{ti} - 1\right) + \beta_{2i}\left(\text{Session}_{ti} - 1\right)^2 + e_{ti}$

Level 2:

Intercept: $\beta_{0i} = \gamma_{00} + \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{02}\left(\text{Reas}_i - 22\right) + \gamma_{03}\left(\text{LowvsMedEd}_i\right) + \gamma_{04}\left(\text{LowvsHighEd}_i\right) + U_{0i}$

Linear: $\beta_{1i} = \gamma_{10} + \gamma_{11}\left(\text{Age}_i - 80\right) + \gamma_{12}\left(\text{Reas}_i - 22\right) + \gamma_{13}\left(\text{LowvsMedEd}_i\right) + \gamma_{14}\left(\text{LowvsHighEd}_i\right) + U_{1i}$

Quadratic: $\beta_{2i} = \gamma_{20} + \gamma_{21}\left(\text{Age}_i - 80\right) + \gamma_{23}\left(\text{LowvsMedEd}_i\right) + \gamma_{24}\left(\text{LowvsHighEd}_i\right) + U_{2i}$

Fixed-Effect-Predicted Outcome (T = $\text{Session}_{ti} - 1$):

$$\begin{aligned}
\hat{y}_{ti} &= \gamma_{00} + \gamma_{10}(T) + \gamma_{20}(T)^2 \\
&+ \gamma_{01}\left(\text{Age}_i - 80\right) + \gamma_{11}(T)\left(\text{Age}_i - 80\right) + \gamma_{21}(T)^2\left(\text{Age}_i - 80\right) \\
&+ \gamma_{02}\left(\text{Reas}_i - 22\right) + \gamma_{12}(T)\left(\text{Reas}_i - 22\right) + \gamma_{22}(T)^2\left(\text{Reas}_i - 22\right) \\
&+ \gamma_{03}\left(\text{LowvsMedEd}_i\right) + \gamma_{13}(T)\left(\text{LowvsMedEd}_i\right) + \gamma_{23}(T)^2\left(\text{LowvsMedEd}_i\right) \\
&+ \gamma_{04}\left(\text{LowvsHighEd}_i\right) + \gamma_{14}(T)\left(\text{LowvsHighEd}_i\right) + \gamma_{24}(T)^2\left(\text{LowvsHighEd}_i\right)
\end{aligned}$$

Simple Slopes of Interactions (T = $\text{Session}_{ti} - 1$):

Linear Time = $\gamma_{10} + \gamma_{20}(2T) + \gamma_{11}(\text{Age}_i - 80) + \gamma_{21}(2T)(\text{Age}_i - 80) + \gamma_{12}(\text{Reas}_i - 22)$
$+ \gamma_{13}(\text{LowvsMedEd}_i) + \gamma_{23}(2T)(\text{LowvsMedEd}_i) + \gamma_{14}(\text{LowvsHighEd}_i) + \gamma_{24}(2T)(\text{LowvsHighEd}_i)$

Quadratic Time = $\gamma_{20} + \gamma_{21}(\text{Age}_i - 80) + \gamma_{23}(\text{LowvsMedEd}_i) + \gamma_{24}(\text{LowvsHighEd}_i)$

Age = $\gamma_{01} + \gamma_{11}(T) + \gamma_{21}(T)^2$

Reasoning = $\gamma_{02} + \gamma_{12}(T)$

```
display "STATA 2e: Keep Age & Reas, Add Education Predicting Intercept, Linear, and Quadratic"
mixed nm3rt c.time c.time#c.time c.age80 c.time#c.age80 c.time#c.time#c.age80            ///
         c.reas22 c.time#c.reas22 i.educgrp c.time#i.educgrp c.time#c.time#i.educgrp,   ///
      || ID: time timesq, variance reml difficult covariance(un)                        ///
      residuals(independent,t(session)) dfmethod(satterthwaite) dftable(pvalue)
matrix QEduc = r(table)   // Save results for computations below
display "-2LL = " e(ll)*-2                    // Print -2LL for model
estat recovariance, relevel(ID) correlation  // GCORR matrix

// DF=2 Wald test for education on intercept, linear, quadratic, and DF=6 joint test
contrast i.educgrp c.time#i.educgrp c.time#c.time#i.educgrp, small overall
```

```
------------------------------------------------------------------
                     |         df        ddf           F        P>F
---------------------+--------------------------------------------
nm3rt                |
             educgrp |          2      96.71        0.23     0.7965
      educgrp#c.time |          2      97.36        0.92     0.4013
educgrp#c.time#c.time |         2      97.00        1.05     0.3546
             Overall |          6      97.05        0.76     0.5995
------------------------------------------------------------------
```

```
// Estimating adjusted means and mean diffs per group at first and last session
margins i.educgrp, at(c.time=(0 5) c.age80=0 c.reas22=0)
margins i.educgrp, at(c.time=(0) c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)
margins i.educgrp, at(c.time=(5) c.age80=0 c.reas22=0) pwcompare(pveffects) df(96)

// Contrasts between groups on intercept, linear, and quadratic slopes
test 1.educgrp=3.educgrp, small                              // 1Low vs 3High: Intercept
test 2.educgrp=3.educgrp, small                              // 2Med vs 2High: Intercept
test 1.educgrp=2.educgrp, small                              // 1Low vs 2Med:  Intercept
test 1.educgrp#c.time=3.educgrp#c.time, small                    // 1Low vs 3High: Linear Time
test 2.educgrp#c.time=3.educgrp#c.time, small                    // 2Med vs 3High: Linear Time
test 1.educgrp#c.time=2.educgrp#c.time, small                    // 1Low vs 2Med:  Linear Time
test 1.educgrp#c.time#c.time=3.educgrp#c.time#c.time, small // 1Low vs 3High: Quadratic Time
test 2.educgrp#c.time#c.time=3.educgrp#c.time#c.time, small // 2Med vs 3High: Quadratic Time
test 1.educgrp#c.time#c.time=2.educgrp#c.time#c.time, small // 1Low vs 2Med:  Quadratic Time

// Get adjusted means per session and reasoning (start(by)end), hold age80=0
margins, at(c.age80=0 c.reas22=0 c.time=(0(1)5) educgrp=(1 2 3))
marginsplot   // Plot adjusted means

// Build total-R2
predict predQEduc                // Save yhat
quietly corr predQEduc nm3rt     // Get total-r to make R2
global R2QEduc = r(rho)^2        // Save total-R2 for comparison
display "Total-R2 = " $R2QEduc   // Print total-R2 relative to empty model
display "Change in Total-R2 = "  $R2QEduc - $R2LReas

// Save variances and compute pseudo-R2 and change in pseudo-R2
matrix list QEduc
global QEducIntVar = exp(QEduc[1,20])^2   // Save as L2 random intercept variance
global QEducLinVar = exp(QEduc[1,18])^2    // Save as L2 random linear time variance
global QEducQuaVar = exp(QEduc[1,19])^2   // Save as L2 random quadratic time variance
global QEducResVar = exp(QEduc[1,24])^2   // Save as L1 residual variance
display "Pseudo-R2 for Intercept = "        1-($QEducIntVar/$QUncIntVar)
display "Pseudo-R2 for Linear Time = "      1-($QEducLinVar/$QUncLinVar)
display "Pseudo-R2 for Quadratic Time = "   1-($QEducQuaVar/$QUncQuaVar)
display "Pseudo-R2 for Residual = "         1-($QEducResVar/$QUncResVar)
display "Change in Pseudo-R2 for Intercept = "        (1-($QEducIntVar/$QUncIntVar)) ///
                                                    - (1-($LReasIntVar/$QUncIntVar))
display "Change in Pseudo-R2 for Linear Time = "      (1-($QEducLinVar/$QUncLinVar)) ///
                                                    - (1-($LReasLinVar/$QUncLinVar))
display "Change in Pseudo-R2 for Quadratic Time = "   (1-($QEducQuaVar/$QUncQuaVar)) ///
                                                    - (1-($LReasQuaVar/$QUncQuaVar))
display "Change in Pseudo-R2 for Residual = "         (1-($QEducResVar/$QUncResVar)) ///
                                                    - (1-($LReasResVar/$QUncResVar))
```

```
print("R 2e: Keep Age & Reasoning, Add Education Predicting Intercept, Linear, and Quadratic")
print("LMER re-orders all main effects to be first, so I wrote them in that order")
QEduc = lmer(data=Example7b, REML=TRUE, control=lmerControl(optimizer="Nelder_Mead"),
             formula=nm3rt~1+time+I(time^2)+age80+reas22+factor(educgrp)
                     +time:age80 +I(time^2):age80 +time:reas22 +time:factor(educgrp)
                     +I(time^2):factor(educgrp) +(1+time+I(time^2)|ID))
print("Show results with -2LL using Satterthwaite DDF")
llikAIC(QEduc, chkREML=FALSE); summary(QEduc, ddf="Satterthwaite");
```

```
$AICtab
      AIC       BIC    logLik  deviance  df.resid
 8253.362  8345.906 -4105.681  8211.362   585.000

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 ID       (Intercept) 241036.8 490.95
          time         25780.2 160.56   -0.45
          I(time^2)      628.4  25.07    0.36 -0.97
 Residual              20298.2 142.47
Number of obs: 606, groups:  ID, 101

Fixed effects:
                           Estimate Std. Error       df t value Pr(>|t|)
(Intercept)               1910.5091   112.4113  96.0512  16.996  < 2e-16
time                      -176.7446    44.0290  96.8687  -4.014 0.000118
I(time^2)                   23.5449     7.4780  97.0000   3.149 0.002181
age80                       20.2896     8.7752  97.4881   2.312 0.022872
reas22                     -36.6212    11.0409  96.0001  -3.317 0.001286
factor(educgrp)2            89.0193   134.0275  96.7464   0.664 0.508151
factor(educgrp)3            51.3767   154.8557  96.3425   0.332 0.740782
time:age80                  -4.5760     3.3354  97.9924  -1.372 0.173219
I(time^2):age80              0.6177     0.5647  97.0000   1.094 0.276752
time:reas22                  2.9783     1.3132  95.9999   2.268 0.025569
time:factor(educgrp)2       65.8882    51.7714  97.3791   1.273 0.206163
time:factor(educgrp)3       70.2459    60.3092  97.0758   1.165 0.246970
I(time^2):factor(educgrp)2 -12.5294     8.7804  97.0000  -1.427 0.156799
I(time^2):factor(educgrp)3 -11.0653    10.2371  97.0000  -1.081 0.282424
```

```
print("DF=2 Wald Test for Each Education Effect")
anova(QEduc)
```

```
Type III Analysis of Variance Table with Satterthwaite's method
                          Sum Sq Mean Sq NumDF  DenDF F value        Pr(>F)
time                      725823  725823     1 96.454 35.7580 0.00000003773
I(time^2)                 357641  357641     1 97.000 17.6194 0.00005983704
age80                     108517  108517     1 97.488  5.3461      0.022872
reas22                    223312  223312     1 96.000 11.0016      0.001286
factor(educgrp)             9257    4628     2 96.106  0.2280      0.796536
time:age80                 38205   38205     1 97.992  1.8822      0.173219
I(time^2):age80            24285   24285     1 97.000  1.1964      0.276752
time:reas22               104413  104413     1 96.000  5.1440      0.025569
time:factor(educgrp)       37420   18710     2 96.932  0.9217      0.401282
I(time^2):factor(educgrp)  42547   21273     2 97.000  1.0480      0.354560
```

```
print("DF=6 Wald Test for all Education Slopes")
contestMD(QEduc, ddf="Satterthwaite", L=rbind(
  c(0,0,0,0,0,1,0,0,0,0,0,0,0,0),c(0,0,0,0,0,0,1,0,0,0,0,0,0,0),
  c(0,0,0,0,0,0,0,0,0,0,1,0,0,0),c(0,0,0,0,0,0,0,0,0,0,0,1,0,0),
  c(0,0,0,0,0,0,0,0,0,0,0,0,1,0),c(0,0,0,0,0,0,0,0,0,0,0,0,0,1)))
```

```
    Sum Sq  Mean Sq NumDF    DenDF   F value    Pr(>F)
1 93120.35 15520.06     6 96.32969 0.7646034 0.5995175
```

```
print("Adjusted means and diffs per group only for education simple main effect")
print("Education diffs at session 1")
Qs1mean = ref_grid(QEduc, at=list(time1=0,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Qs1mean, pairwise~educgrp, lmer.df="satterthwaite", adjust="none")
```

```
$emmeans
 educgrp emmean    SE   df lower.CL upper.CL
 1         1911 112.4 96.0     1687     2134
 2         2000  69.3 96.3     1862     2137
 3         1962 104.3 95.7     1755     2169
```

```
$contrasts
 contrast            estimate  SE   df t.ratio p.value
 educgrp1 - educgrp2    -89.0 134 96.8  -0.664  0.5082
 educgrp1 - educgrp3    -51.4 155 96.3  -0.332  0.7408
 educgrp2 - educgrp3     37.6 124 95.4   0.304  0.7619
```

```
print("Education diffs at session 6")
Qs6mean = ref_grid(QEduc, at=list(time1=5,age80=0,reas22=0), disable.pbkrtest=TRUE)
emmeans(Qs6mean, pairwise~educgrp, lmer.df="satterthwaite", adjust="none")
```

```
$emmeans
 educgrp emmean   SE   df lower.CL upper.CL
 1         1615 95.7 96.0     1425     1805
 2         1721 59.0 96.1     1603     1838
 3         1741 88.8 95.9     1565     1918

$contrasts
 contrast            estimate  SE   df t.ratio p.value
 educgrp1 - educgrp2   -105.2 114 96.2  -0.920  0.3597
 educgrp1 - educgrp3   -126.0 132 96.1  -0.955  0.3422
 educgrp2 - educgrp3    -20.7 105 95.8  -0.197  0.8443
```

```
print("Specific education group differences on intercept, time1 and slope16")
print("1Low vs 3High Educ: Intercept"); contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,-1,0,0,0,0,0,0,0,0))
print("2Med vs 3High Educ: Intercept"); contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,-1,0,0,0,0,0,0,0))
print("1Low vs 2Med  Educ: Intercept"); contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,-1,1,0,0,0,0,0,0,0))
print("1Low vs 3High Educ: Linear");    contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,-1,0,0,0))
print("2Med vs 3High Educ: Linear");    contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,-1,0,0))
print("1Low vs 2Med  Educ: Linear");    contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,-1,1,0,0))
print("1Low vs 3High Educ: Quadratic"); contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,0))
print("2Med vs 3High Educ: Quadratic"); contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,0,-1))
print("1Low vs 2Med  Educ: Quadratic"); contest1D(QEduc, ddf="Satterthwaite",
      L=c(0,0,0,0,0,0,0,0,0,0,0,0,-1,1))
```

Estimates (from SAS for better organization)

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
1Low vs 3High Educ: Intercept	51.3789	154.86	96.3	0.33	0.7408
2Med vs 3High Educ: Intercept	-37.6426	123.90	95.4	-0.30	0.7619
1Low vs 2Med Educ: Intercept	89.0215	134.03	96.7	0.66	0.5081
1Low vs 3High Educ: Linear Time	70.2452	60.3044	97.1	1.16	0.2469
2Med vs 3High Educ: Linear Time	4.3577	49.1308	96.5	0.09	0.9295
1Low vs 2Med Educ: Linear Time	65.8875	51.7672	97.4	1.27	0.2061
1Low vs 3High Educ: Quadratic Time	-11.0653	10.2361	97	-1.08	0.2824
2Med vs 3High Educ: Quadratic Time	1.4641	8.3547	97	0.18	0.8613
1Low vs 2Med Educ: Quadratic Time	-12.5294	8.7795	97	-1.43	0.1568

```
# Effect sizes using custom functions
TotalR2(data=Example7b,  dvName="nm3rt", model1=LReas, name1="Reasoning*Linear",
        model2=QEduc, name2="Education")
```

```
  totalR2.1 totalR2.2   changeR2
1 0.1600643 0.1723062 0.01224183
```

```
PseudoR2(data=Example7b, baseModel=QUnc, model1=LReas, name1="Reasoning*Linear",
        model2=QEduc, name2="Education")
```

```
Pseudo-R2 and Change in Pseudo-R2 for Reasoning*Linear vs Education
         term        base      model1      model2 pseudoR2.model1 pseudoR2.model2 pseudoR2.change
1 (Intercept) 276206.2261 235910.2146 241036.7665          0.1459          0.1273         -0.0186
2        time  25839.6306  25709.0953  25780.2415          0.0051          0.0023         -0.0028
3   I(time^2)    634.4598    629.5864    628.4481          0.0077          0.0095          0.0018
7    Residual  20298.2117  20298.1835  20298.1821          0.0000          0.0000          0.0000
```

Based on the lack of significance of the effect of education, I'd say we're done with this model (I had previously tried age*reasoning, and none of those higher-order effects were significant).

The age*quadratic interaction could probably be removed, but I choose to leave it in as a control.

Simple Processing Speed: Example Conditional Models of Change Results

The extent to which individual differences in response time (RT) in milliseconds over six sessions for a simple processing speed test (number match three) could be predicted from baseline age, abstract reasoning, and education group was examined in a series of multilevel models (i.e., general linear mixed models) in which the six practice sessions at level 1 were modeled as nested within participants at level 2. Residual maximum likelihood (REML) was used to estimate all model parameters; denominator degrees of freedom were estimated using the Satterthwaite method. The significance of new fixed effects was evaluated with univariate and multivariate Wald tests. Session (i.e., the index of time) was centered at the first occasion, age was centered at 80 years, abstract reasoning was centered at 22 (near the sample mean of that predictor), and high-school-level education was the reference group for education level (with two contrasts for bachelor-level education and graduate-level education). Effect size for the fixed effects was evaluated via pseduo-R^2 values for the proportion reduction in each variance component, as well as with total-R^2, the squared correlation between the actual outcome and the outcome predicted by the model fixed effects.

Piecewise Time Models

The best-fitting unconditional growth model specified linear decline from sessions 1–2 and a second, shallower rate of linear decline from sessions 2–6, along with significant individual differences in the intercept and in each piecewise linear slope. In the unconditional piecewise slopes model, the two fixed slopes for linear change across sessions accounted for 3.74% of the total variance in RT. Intercept reliability (i.e., ICC2) was .9897, and reliability for the two random piecewise slopes was .75 for sessions 1–2 and .66 for sessions 2–6.

Next, age was added as a predictor of the intercept and each piecewise linear slope. Although the three slopes of age together resulted in a significant omnibus effect, $F(3, 99) = 4.08$, $p < .01$, only the fixed slope of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher (slower) by 29.78 ($p < .001$). In terms of pseudo-R^2, age accounted for 10.56% of the level-2 random intercept variance, 1.90% of the level-2 random variance in linear change from sessions 1–2, and 0.90% of the level-2 random variance in linear change from sessions 2–6. As expected given that baseline age is a time-invariant predictor, the level-1 residual variance was not reduced. The piecewise session slopes and age together accounted for 10.76% of the variance in RT, a 7.02% increase due to the slopes of age. Although the interactions of age with the linear piecewise slopes were not significant, they were retained in the model to fully control for any age effects on change across sessions before examining the effects of other predictors.

Abstract reasoning was then added as a predictor of the intercept and each piecewise linear slope. The three slopes of abstract reasoning together resulted in a significant omnibus effect, $F(3, 98) = 3.50$, $p = .018$. The significant fixed effects of abstract reasoning on the intercept and second slope indicated that for every additional unit of reasoning above 22, RT at the first session was predicted to be significantly lower (faster) by 27.10 ($p < .001$) and to increase by an additional 3.35 ms after session 2.The nonsignificant effect of reasoning on the first slope was retained to facilitate interpretation of the separate effects of reasoning on each aspect of change. Relative to the age-only model, reasoning accounted for an additional 4.25% of the level-2 random intercept variance, none of the level-2 random first slope variance, and 6.96% of the level-2 second slope variance. The piecewise session slopes, age, and reasoning together accounted for 16.13% of the variance in RT, a 5.38% increase due to reasoning.

Education group (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept and each linear slope. These six slopes of education did not result in a significant omnibus effect, $F(6, 96) = 0.73$, $p = .626$. No omnibus main effects of education level on the intercept, linear, or quadratic slopes were significant, and no pairwise comparisons were significant as well. Relative to the age and reasoning model, education accounted for no measurable variance in the level-2 random intercept or either level-2 random linear slope. The piecewise session slopes, age, reasoning, and education accounted for 17.36%

of the variance in RT, a 1.23% increase due to education. Finally, we examined the interactive effects of age and reasoning in predicting the intercept and each linear slope, although none was significant. (From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)

Quadratic Time Models

The best-fitting unconditional growth model specified quadratic decline across the six sessions (i.e., a decelerating negative function) with significant individual differences in the intercept, linear, and quadratic time effects. In the unconditional growth model, the fixed effects for linear and quadratic change accounted for 3.67% of the total variance in RT. Intercept reliability (i.e., ICC2) was .9879, and reliability for the random linear and quadratic time slopes was .96 and .94, respectively.

Next, age was added as a predictor of the intercept, linear slope, and quadratic slope. Although the three slopes of age together resulted in a significant omnibus effect, $F(3, 99) = 4.00$, $p < .01$, only the fixed effect of age on the intercept was significant, indicating that for every additional year of age above 80, RT at the first session was predicted to be significantly higher (slower) by 29.05 ($p < .001$). In terms of pseudo-R^2, age accounted for 10.32% of the level-2 random intercept variance, 2.93% of the level-2 random linear slope variance, and 0.77% of the level-2 random quadratic slope variance. As expected given that baseline age is a time-invariant predictor, the level-1 residual variance was not reduced. The linear and quadratic slopes for session and age accounted for 10.69% of the variance in RT, a 7.01% increase due to age. Although the interactions of age with the linear and quadratic time slopes were not significant, they were retained in the model to fully control for any age effects on change across sessions before examining the effects of other predictors.

Abstract reasoning was then added as a predictor of the intercept, linear slope, and quadratic slope. As with the effects of age, the three slopes of abstract reasoning together resulted in a significant omnibus effect, $F(3, 98) = 4.29$, $p < .01$, but only the fixed effect of abstract reasoning on the intercept was significant, indicating that for every additional unit of reasoning above 22, RT at the first session was predicted to be significantly lower (faster) by 27.10 ($p < .001$). The nonsignificant effect of reasoning on the quadratic slope was then removed, revealing a significant effect of reasoning on both the intercept and linear slope, $F(2, 98) = 5.29$, $p < .01$, such that for every unit higher reasoning above 22, RT at the first session was expected to be lower by 32.83 and the linear rate of improvement in RT (as evaluated at the first session given the quadratic slope) was expected to be less negative by 2.94 (i.e., faster initial RT with less improvement in persons with greater reasoning). Relative to the age-only model, reasoning accounted for an additional 4.27% of the level-2 random intercept variance but had no measurable reduction of the level-2 random linear and quadratic slope variances. The linear and quadratic slopes for session, age, and reasoning accounted for 16.00% of the variance in RT, a 5.32% increase due to reasoning.

Education group (high school or less, bachelor's level, or graduate level) was then added as a predictor of the intercept, linear slope, and quadratic slope. These six slopes of education did not result in a significant omnibus effect, $F(6, 96) = 0.76$, $p = .600$. No omnibus main effects of education group on the intercept, linear, or quadratic slopes were significant, and no pairwise comparisons were significant as well. Relative to the age and reasoning model, education accounted for no measurable random intercept or random linear slope variance, and an additional 0.18% of the random quadratic slope variance. The linear and quadratic slopes for session, age, reasoning, and education accounted for 17.23% of the variance in RT, a 1.22% increase due to education. Finally, we examined the interactive effects of age and reasoning in predicting the intercept and each linear slope, although none was significant. (From here one might remove nonsignificant model effects and/or add other effects as needed to fully answer all research questions…)