Example 3b: Kinds of Analysis of Variance (ANOVA) Models
(complete syntax, data, and output available for SAS, STATA, and R electronically)

This example comes from Hoffman (2015) chapter 3 (example 3b). We will be examining an outcome of response time (RT) in milliseconds across six practice sessions to a measure of processing speed (as measured by the number match 3 test) in a sample of 101 older adults. Of interest right now is the pattern of variance and covariance in RT across sessions. For an example results section, please see the end of chapter 3. In SAS, the highest category is the reference (as in chapter 3), so the solutions for fixed effects will differ in SAS relative to STATA or R (in which the first category is the default reference). Later in Example 6 we will examine how individual differences in change in RT can be described by alternative growth curve models.

STATA Syntax for Data Import:

```
// Define working directory for files
   cd "C:\Dropbox\24_PSQF6271\PSQF6271_Example3b"

// Import chapter 3b six-occasion long-format data from excel
   clear // clear memory in case a dataset is already open
   import excel "Example3b.xlsx", firstrow case(preserve) clear

// Sort by person and session
   sort PersonID session
```

R Syntax for Data Import and Manipulation (after loading packages *readxl*, *TeachingDemos*, *psych*, *nlme*, and *multcomp*):

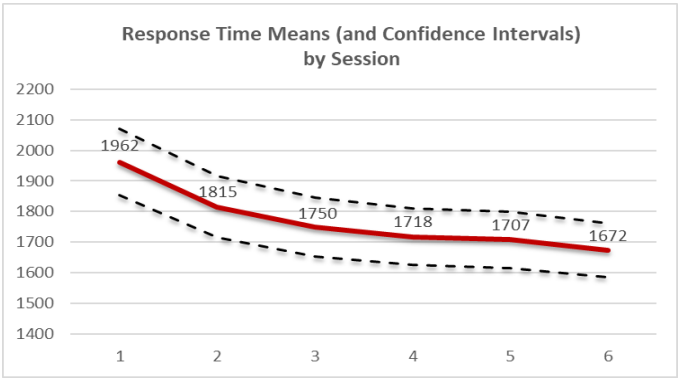
```
# Define variables for working directory and data name
setwd("C:/Dropbox/24_PSQF6271/PSQF6271_Example3b/")

# Import chapter 3b six-occasion long-format data from excel
Example3b = read_excel(path="Example3b.xlsx")
# Convert to data frame as data frame without labels to use for analysis
Example3b = as.data.frame(Example3b)

# Sort by person and occasion (needed for correct R matrix)
Example3b = Example3b[order(Example3b$PersonID, Example3b$session), ]
```

STATA Syntax and Output for Descriptive Statistics using `tabulate`

```
display "Chapter 3b Example: Means by session for RT outcome"
tabulate session, summarize(rt)
```

Note the different means and standard deviations across sessions…

session	Summary of rt: Mean	Std. Dev.	Freq.
1	1961.8934	549.53193	101
2	1815.1724	509.06784	101
3	1750.0346	483.08178	101
4	1717.7965	466.41575	101
5	1707.1757	460.54053	101
6	1672.1360	443.54613	101
Total	1770.7014	494.08763	606



R Syntax for Descriptive Statistics using `describeBy` from `psych` package:

```
print("Chapter 3b Example: Means by session for RT outcome")
describeBy(x=Example3b$rt, digits=2, group=Example3b$session)
```

All models we will examine will have the same "saturated" model for the means:

$$\widehat{RT}_{ti} = \beta_0 + \beta_1(S2_{ti}) + \beta_2(S3_{ti}) + \beta_3(S4_{ti}) + \beta_4(S5_{ti}) + \beta_5(S6_{ti})$$

But they will differ in their model for the variance, corresponding to the three kinds of ANOVA models.

The saturated means model predicts the 6 means from 1 intercept (for session 1) + 5 mean differences (1 per binary session).

Model 1: Between-Person (e-only) ANOVA *(equal variance with no covariance over sessions)*

STATA Syntax and Output using `mixed`:

STATA wouldn't let me use Satterthwaite DDF ☹

```
display "Eq 3.10: Between-Person (e-only) ANOVA"
mixed rt i.session, || PersonID: , noconstant reml nolog baselevel ///
        residuals(independent,t(session)) dfmethod(residual) dftable(pvalue)
estimates store FitBP       // Save -2LL for LRT
```

```
                                                F(5,   600.00)    =       4.73
Log restricted-likelihood = -4577.7163          Prob > F          =     0.0003
------------------------------------------------------------------------------
          rt |      Coef.   Std. Err.           DF       t    P>|t|  fixed effect
-------------+----------------------------------------------------
     session |
           1 |          0  (base)
           2 |   -146.721   68.47894         600.0    -2.14   0.033  beta1
           3 |  -211.8587   68.47894         600.0    -3.09   0.002  beta2
           4 |  -244.0969   68.47894         600.0    -3.56   0.000  beta3
           5 |  -254.7176   68.47894         600.0    -3.72   0.000  beta4
           6 |  -289.7574   68.47894         600.0    -4.23   0.000  beta5
       _cons |   1961.893   48.42193         600.0    40.52   0.000  beta0
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.      [95% Conf. Interval]
-----------------------------+------------------------------------------------
               var(Residual) |     236813   13604.55      211594.9    265036.5
------------------------------------------------------------------------------
```

```
display "-2LL = " e(ll)*-2  // Print -2LL for model
-2LL = 9155.4325
```

Note these all have the same SE!

```
margins  i.session          // Means per session
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]  b=beta
-------------+----------------------------------------------------------------
     session |
           1 |   1961.893   48.42193    40.52   0.000     1866.988    2056.799  b0
           2 |   1815.172   48.42193    37.49   0.000     1720.267    1910.078  b0+b1
           3 |   1750.035   48.42193    36.14   0.000     1655.129     1844.94  b0+b2
           4 |   1717.796   48.42193    35.48   0.000     1622.891    1812.702  b0+b3
           5 |   1707.176   48.42193    35.26   0.000     1612.271    1802.081  b0+b4
           6 |   1672.136   48.42193    34.53   0.000     1577.231    1767.041  b0+b5
------------------------------------------------------------------------------
```

```
margins  i.session, pwcompare(pveffects) df(600) // Mean differences
------------------------------------------------------
             |            Delta-method    Unadjusted
             |   Contrast   Std. Err.      t    P>|t|
-------------+----------------------------------------
     session |                                        b=beta
     2 vs 1  |   -146.721   68.47894    -2.14   0.033  (b0+b1) - (b0) = b1
     3 vs 1  |  -211.8587   68.47894    -3.09   0.002  (b0+b2) - (b0) = b2
     4 vs 1  |  -244.0969   68.47894    -3.56   0.000  (b0+b3) - (b0) = b3
     5 vs 1  |  -254.7176   68.47894    -3.72   0.000  (b0+b4) - (b0) = b4
     6 vs 1  |  -289.7574   68.47894    -4.23   0.000  (b0+b5) - (b0) = b5
     3 vs 2  |  -65.13774   68.47894    -0.95   0.342  (b0+b2) - (b0+b1) = b2 - b1
     4 vs 2  |  -97.37592   68.47894    -1.42   0.156  (b0+b3) - (b0+b1) = b3 - b1
     5 vs 2  |  -107.9967   68.47894    -1.58   0.115  (b0+b4) - (b0+b1) = b4 - b1
```

Note these all have the same SE!

```
     6 vs 2  |  -143.0364   68.47894    -2.09   0.037   (b0+b5) - (b0+b1) = b5 - b1
     4 vs 3  |  -32.23819   68.47894    -0.47   0.638   (b0+b3) - (b0+b2) = b3 - b2
     5 vs 3  |  -42.85892   68.47894    -0.63   0.532   (b0+b4) - (b0+b2) = b4 - b2
     6 vs 3  |  -77.89864   68.47894    -1.14   0.256   (b0+b5) - (b0+b2) = b5 - b2
     5 vs 4  |  -10.62073   68.47894    -0.16   0.877   (b0+b4) - (b0+b3) = b4 - b3
     6 vs 4  |  -45.66045   68.47894    -0.67   0.505   (b0+b5) - (b0+b3) = b5 - b3
     6 vs 5  |  -35.03972   68.47894    -0.51   0.609   (b0+b5) - (b0+b4) = b5 - b4
-------------------------------------------------------
```

```
contrast i.session, small   // Omnibus F-test for mean diffs (given for model above)
Margins      : asbalanced
-------------------------------------------------------------
             |         df        ddf           F        P>F
-------------+-----------------------------------------------
rt           |
     session |          5     600.00        4.73     0.0003
-------------------------------------------------------------
```

R Syntax and Output using `gls` from `nlme` Package:

```
print("Eq 3.10: Between-Person (e-only) ANOVA")
BP = gls(data=Example3b, method="REML", model=rt~1+factor(session),
         correlation=NULL) # VC R matrix (no correlation)
print("Show results with -2LL and total leftover variance")
-2*logLik(BP); summary(BP); summary(BP)$sigma^2
```

```
'log Lik.' 9155.4325 (df=7) → -2LL for model
```

```
       AIC      BIC    logLik
  9169.4325 9200.211 -4577.7163
```

Note that the intercept has a different SE (← U+e), but the differences all have the same SE (← e only)

```
Coefficients:
                      Value Std.Error   t-value p-value  fixed effect
(Intercept)      1961.89337 48.421925 40.516633  0.0000  beta0
factor(session)2 -146.72098 68.478944 -2.142571  0.0325  beta1
factor(session)3 -211.85872 68.478944 -3.093779  0.0021  beta2
factor(session)4 -244.09691 68.478944 -3.564554  0.0004  beta3
factor(session)5 -254.71764 68.478944 -3.719649  0.0002  beta4
factor(session)6 -289.75736 68.478944 -4.231335  0.0000  beta5

[1] 236812.97 → total leftover variation
```

```
print("Session means, pairwise mean differences, and omnibus F-test")
emmeans(ref_grid(BP), pairwise~session, adjust="none"); anova(BP)
```

```
$emmeans
 session emmean   SE  df lower.CL upper.CL  b=beta
       1   1962 48.4 600     1867     2057  b0
       2   1815 48.4 600     1720     1910  b0+b1
       3   1750 48.4 600     1655     1845  b0+b2
       4   1718 48.4 600     1623     1813  b0+b3
       5   1707 48.4 600     1612     1802  b0+b4
       6   1672 48.4 600     1577     1767  b0+b5
```

Note these all have the same SE!

```
Degrees-of-freedom method: df.error
$contrasts
 contrast            estimate   SE  df t.ratio p.value  b=beta
 session1 - session2    146.7 68.5 600   2.143  0.0325  (b0) - (b0+b1) = -b1
 session1 - session3    211.9 68.5 600   3.094  0.0021  (b0) - (b0+b2) = -b2
 session1 - session4    244.1 68.5 600   3.565  0.0004  (b0) - (b0+b3) = -b3
 session1 - session5    254.7 68.5 600   3.720  0.0002  (b0) - (b0+b4) = -b4
 session1 - session6    289.8 68.5 600   4.231  <.0001  (b0) - (b0+b5) = -b5
 session2 - session3     65.1 68.5 600   0.951  0.3419  (b0+b1) - (b0+b2) = b1 - b2
 session2 - session4     97.4 68.5 600   1.422  0.1556  (b0+b1) - (b0+b3) = b1 - b3
 session2 - session5    108.0 68.5 600   1.577  0.1153  (b0+b1) - (b0+b4) = b1 - b4
 session2 - session6    143.0 68.5 600   2.089  0.0371  (b0+b1) - (b0+b5) = b1 - b5
 session3 - session4     32.2 68.5 600   0.471  0.6380  (b0+b2) - (b0+b3) = b2 - b3
 session3 - session5     42.9 68.5 600   0.626  0.5316  (b0+b2) - (b0+b4) = b2 - b4
 session3 - session6     77.9 68.5 600   1.138  0.2558  (b0+b2) - (b0+b5) = b2 - b5
 session4 - session5     10.6 68.5 600   0.155  0.8768  (b0+b3) - (b0+b4) = b3 - b4
 session4 - session6     45.7 68.5 600   0.667  0.5052  (b0+b3) - (b0+b3) = b3 - b5
 session5 - session6     35.0 68.5 600   0.512  0.6091  (b0+b4) - (b0+b5) = b4 - b5
```

```
Degrees-of-freedom method: df.error
Denom. DF: 600
                numDF   F-value p-value
(Intercept)         1 8023.3885  <.0001
factor(session)     5    4.7349  0.0003
```

R Matrix from SAS (would not print in STATA or R for an e-only model)

Estimated R Matrix for PersonID 101

Row	Col1	Col2	Col3	Col4	Col5	Col6
1	236813					
2		236813				
3			236813			
4				236813		
5					236813	
6						236813

The **R matrix** shows the model-implied pattern of total variance and covariance over time. A BP (e-only) model implies constant variance over time with no covariance between occasions ("independent" in STATA or "diagonal" elsewhere). But this is a testable hypothesis!

Model 2: Univariate Repeated Measures (U+e) ANOVA *(WP equal variance; equal covariance)*

STATA Syntax and Output using `mixed`:

```
display "Eq 3.10: Univariate Repeated Measures (U+e) ANOVA"
mixed rt i.session, || PersonID: , noconstant reml nolog baselevel ///
        residuals(exchangeable,t(session)) dfmethod(satterthwaite) dftable(pvalue)
```

```
                                              F(5,   500.00)    =     32.85
Log restricted-likelihood = -4176.6776        Prob > F          =    0.0000
------------------------------------------------------------------------------
          rt |      Coef.   Std. Err.           DF       t    P>|t|  fixed effect
-------------+----------------------------------------------------------------
     session |
           1 |          0  (base)
           2 |   -146.721   25.99932         500.0    -5.64   0.000  beta1
           3 |  -211.8587   25.99932         500.0    -8.15   0.000  beta2
           4 |  -244.0969   25.99932         500.0    -9.39   0.000  beta3
           5 |  -254.7176   25.99932         500.0    -9.80   0.000  beta4
           6 |  -289.7574   25.99932         500.0   -11.14   0.000  beta5
       _cons |   1961.893   48.42193         128.7    40.52   0.000  beta0
------------------------------------------------------------------------------
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.      [95% Conf. Interval]
-----------------------------+------------------------------------------------
PersonID:            (empty) |
-----------------------------+------------------------------------------------
Residual: Exchangeable       |
                     var(e) |     236813    29522.3      185477.2    302357.4  Total var
                     cov(e) |   202676.8   29469.63      144917.4    260436.2  Var(U0)
------------------------------------------------------------------------------
LR test vs. linear model: chi2(1) = 802.08                Prob > chi2 = 0.0000
```

The **`chi(1)`** test above is a likelihood ratio test (LRT) comparing this model to the single-level model (without a random intercept, as **`linear model`**) using a chi-square (χ^2) distribution with DF=1 for Var(U0).

```
display "-2LL = " e(ll)*-2  // Print -2LL for model
-2LL = 8353.3551

estat wcorrelation, covariance  // R matrix

Covariances for PersonID = 101: Total leftover variance and covariance
         obs |      1       2       3       4       5       6
-------------+------------------------------------------------
           1 |  2.4e+05
           2 |  2.0e+05   2.4e+05
           3 |  2.0e+05   2.0e+05   2.4e+05
           4 |  2.0e+05   2.0e+05   2.0e+05   2.4e+05
           5 |  2.0e+05   2.0e+05   2.0e+05   2.0e+05   2.4e+05
           6 |  2.0e+05   2.0e+05   2.0e+05   2.0e+05   2.0e+05   2.4e+05
```

```
estat wcorrelation              // RCORR matrix
Correlations: Total leftover correlations

         obs |      1       2       3       4       5       6
-------------+------------------------------------------------
           1 |  1.000
           2 |  0.856   1.000
           3 |  0.856   0.856   1.000
           4 |  0.856   0.856   0.856   1.000
           5 |  0.856   0.856   0.856   0.856   1.000
           6 |  0.856   0.856   0.856   0.856   0.856   1.000

margins  i.session              // Means per session
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]  b=beta
-------------+----------------------------------------------------------------
     session |
           1 |   1961.893   48.42193    40.52   0.000     1866.988    2056.799  b0
           2 |   1815.172   48.42193    37.49   0.000     1720.267    1910.078  b0+b1
           3 |   1750.035   48.42193    36.14   0.000     1655.129     1844.94  b0+b2
           4 |   1717.796   48.42193    35.48   0.000     1622.891    1812.702  b0+b3
           5 |   1707.176   48.42193    35.26   0.000      1612.27    1802.081  b0+b4
           6 |   1672.136   48.42193    34.53   0.000     1577.231    1767.041  b0+b5
------------------------------------------------------------------------------

margins  i.session, pwcompare(pveffects) df(500) // Mean differences
-----------------------------------------------------
             |            Delta-method    Unadjusted
             |   Contrast   Std. Err.      t    P>|t|
-------------+---------------------------------------
     session |                                        b=beta
     2 vs 1  |   -146.721   25.99932    -5.64   0.000  (b0+b1) - (b0) = b1
     3 vs 1  |  -211.8587   25.99932    -8.15   0.000  (b0+b2) - (b0) = b2
     4 vs 1  |  -244.0969   25.99932    -9.39   0.000  (b0+b3) - (b0) = b3
     5 vs 1  |  -254.7176   25.99932    -9.80   0.000  (b0+b4) - (b0) = b4
     6 vs 1  |  -289.7574   25.99932   -11.14   0.000  (b0+b5) - (b0) = b5
     3 vs 2  |  -65.13774   25.99932    -2.51   0.013  (b0+b2) - (b0+b1) = b2 - b1
     4 vs 2  |  -97.37592   25.99932    -3.75   0.000  (b0+b3) - (b0+b1) = b3 - b1
     5 vs 2  |  -107.9967   25.99932    -4.15   0.000  (b0+b4) - (b0+b1) = b4 - b1
     6 vs 2  |  -143.0364   25.99932    -5.50   0.000  (b0+b5) - (b0+b1) = b5 - b1
     4 vs 3  |  -32.23819   25.99932    -1.24   0.216  (b0+b3) - (b0+b2) = b3 - b2
     5 vs 3  |  -42.85892   25.99932    -1.65   0.100  (b0+b4) - (b0+b2) = b4 - b2
     6 vs 3  |  -77.89864   25.99932    -3.00   0.003  (b0+b5) - (b0+b2) = b5 - b2
     5 vs 4  |  -10.62073   25.99932    -0.41   0.683  (b0+b4) - (b0+b3) = b4 - b3
     6 vs 4  |  -45.66045   25.99932    -1.76   0.080  (b0+b5) - (b0+b3) = b5 - b3
     6 vs 5  |  -35.03972   25.99932    -1.35   0.178  (b0+b5) - (b0+b4) = b5 - b4
-----------------------------------------------------

contrast i.session, small       // Omnibus F-test for mean diffs (given for model above)
Margins      : asbalanced
------------------------------------------------------------
             |         df        ddf           F        P>F
-------------+----------------------------------------------
rt           |
     session |          5     500.00       32.85     0.0000
------------------------------------------------------------

estimates store FitUWP          // Save -2LL for LRT
lrtest FitUWP FitBP             // LRT model comparison (more, fewer), given above

Likelihood-ratio test                                 LR chi2(1)  =     802.08
(Assumption: FitBP nested in FitUWP)                  Prob > chi2 =     0.0000
```

Because Model 2 fits better than Model 1, Model 2's F-test is more accurate. But is it accurate enough?

R Syntax and Output using `gls` from `nlme` Package:

```
print("Eq 3.10: Univariate Repeated Measures (U+e) ANOVA")
UWP = gls(data=Example3b, method="REML", model=rt~1+factor(session),
          correlation=corCompSymm(form=~1|PersonID) )
print("Show results with -2LL")
-2*logLik(UWP); summary(UWP)
```

```
'log Lik.' 8353.3551 (df=8) → -2LL for model

        AIC       BIC     logLik
  8369.3551 8404.5305 -4176.6776

Correlation Structure: Compound symmetry
       Rho
0.85585159

Coefficients:
                     Value Std.Error    t-value p-value  fixed effect
(Intercept)      1961.89337 48.421924  40.516634       0  beta0
factor(session)2 -146.72098 25.999318  -5.643263       0  beta1
factor(session)3 -211.85872 25.999318  -8.148626       0  beta2
factor(session)4 -244.09691 25.999318  -9.388589       0  beta3
factor(session)5 -254.71764 25.999318  -9.797089       0  beta4
factor(session)6 -289.75736 25.999318 -11.144806       0  beta5
```

```
print("Show R and RCORR matrices for first person in the data")
getVarCov(UWP, individual="101"); corMatrix(UWP$modelStruct$corStruct)[[5]]
```

```
Marginal variance covariance matrix → Total leftover variance and covariance (and correlations below)
       [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] 236810 202680 202680 202680 202680 202680
[2,] 202680 236810 202680 202680 202680 202680
[3,] 202680 202680 236810 202680 202680 202680
[4,] 202680 202680 202680 236810 202680 202680
[5,] 202680 202680 202680 202680 236810 202680
[6,] 202680 202680 202680 202680 202680 236810

           [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
[1,] 1.00000000 0.85585159 0.85585159 0.85585159 0.85585159 0.85585159
[2,] 0.85585159 1.00000000 0.85585159 0.85585159 0.85585159 0.85585159
[3,] 0.85585159 0.85585159 1.00000000 0.85585159 0.85585159 0.85585159
[4,] 0.85585159 0.85585159 0.85585159 1.00000000 0.85585159 0.85585159
[5,] 0.85585159 0.85585159 0.85585159 0.85585159 1.00000000 0.85585159
[6,] 0.85585159 0.85585159 0.85585159 0.85585159 0.85585159 1.00000000
```

```
print("Session means, pairwise mean differences, and omnibus F-test using correct DDF")
emmeans(ref_grid(UWP), pairwise~session6, adjust="none"); anova(UWP)
```

```
$emmeans
 session emmean   SE  df lower.CL upper.CL  b=beta
       1   1962 48.4 129     1866     2058  b0
       2   1815 48.4 129     1719     1911  b0+b1
       3   1750 48.4 129     1654     1846  b0+b2
       4   1718 48.4 129     1622     1814  b0+b3
       5   1707 48.4 129     1611     1803  b0+b4
       6   1672 48.4 129     1576     1768  b0+b5

Degrees-of-freedom method: satterthwaite
$contrasts
 contrast            estimate SE  df t.ratio p.value  b=beta
 session1 - session2    146.7 26 500   5.643  <.0001  (b0) - (b0+b1) = -b1
 session1 - session3    211.9 26 500   8.149  <.0001  (b0) - (b0+b2) = -b2
 session1 - session4    244.1 26 500   9.389  <.0001  (b0) - (b0+b3) = -b3
 session1 - session5    254.7 26 500   9.797  <.0001  (b0) - (b0+b4) = -b4
 session1 - session6    289.8 26 500  11.145  <.0001  (b0) - (b0+b5) = -b5
 session2 - session3     65.1 26 500   2.505  0.0125  (b0+b1) - (b0+b2) = b1 - b2
 session2 - session4     97.4 26 500   3.745  0.0002  (b0+b1) - (b0+b3) = b1 - b3
 session2 - session5    108.0 26 500   4.154  <.0001  (b0+b1) - (b0+b4) = b1 - b4
 session2 - session6    143.0 26 500   5.502  <.0001  (b0+b1) - (b0+b5) = b1 - b5
 session3 - session4     32.2 26 500   1.240  0.2156  (b0+b2) - (b0+b3) = b2 - b3
 session3 - session5     42.9 26 500   1.648  0.0999  (b0+b2) - (b0+b4) = b2 - b4
 session3 - session6     77.9 26 500   2.996  0.0029  (b0+b2) - (b0+b5) = b2 - b5
 session4 - session5     10.6 26 500   0.409  0.6831  (b0+b3) - (b0+b4) = b3 - b4
 session4 - session6     45.7 26 500   1.756  0.0797  (b0+b3) - (b0+b3) = b3 - b5
 session5 - session6     35.0 26 500   1.348  0.1784  (b0+b4) - (b0+b5) = b4 - b5

Degrees-of-freedom method: satterthwaite Denom. DF: 600
                numDF    F-value p-value
(Intercept)         1 1519.79482  <.0001
factor(session)     5   32.84712  <.0001
```

```
print("Show likelihood ratio test comparing model fit: BP vs. Univ WP")
anova(UWP,BP)  # Order of "more", "fewer"
```

```
     Model df       AIC       BIC     logLik   Test   L.Ratio p-value
UWP      1  8 8369.3551 8404.5305 -4176.6776
BP       2  7 9169.4325 9200.2110 -4577.7163 1 vs 2 802.07743  <.0001
```

Because Model 2 fits better than Model 1, Model 2's F-test is more accurate. But is it accurate enough?

Model 3: Multivariate Repeated Measures (Unstructured) ANOVA
(WP → all possible unequal covariances/correlations, unequal variances across sessions)

This is the most correct answer because it fully reproduces the original RT variances and covariances (which can differ from their descriptive statistics given any missing data).

STATA Syntax using `mixed`:

```
display "Eq 3.10: Multivariate Repeated Measures (Unstructured) ANOVA"
mixed rt i.session, || PersonID: , noconstant reml nolog baselevel ///
        residuals(unstructured,t(session)) dfmethod(satterthwaite) dftable(pvalue)
display "-2LL = " e(ll)*-2      // Print -2LL for model
estat wcorrelation, covariance  // R matrix
estat wcorrelation              // RCORR matrix
margins  i.session              // Means per session
margins  i.session, pwcompare(pveffects) df(500) // Mean differences
contrast i.session, small       // Omnibus F-test for mean diffs
estimates store FitUWP          // Save -2LL for LRT
lrtest FitMWP FitUWP            // LRT model comparison (more, fewer)
```

R Syntax and Output using `gls` from `nlme` Package:

```
print("Eq 3.10: Multivariate Repeated Measures (Unstructured) ANOVA")
MWP = gls(data=Example3b, method="REML", model=rt~1+factor(session),
          correlation=corSymm(form=~as.numeric(session)|PersonID), # UN correlations
          weights=varIdent(form=~1|session))                       # Het var by session
print("Show results with -2LL")
-2*logLik(MWP); summary(MWP)
```

```
'log Lik.' 8229.7885 (df=27) → -2LL for model

        AIC       BIC     logLik
  8283.7885 8402.5056 -4114.8942

Correlation Structure: General
Correlation:
  1     2     3     4     5
2 0.842
3 0.821 0.936
4 0.790 0.898 0.911
5 0.759 0.862 0.885 0.902
6 0.801 0.856 0.880 0.896 0.920

Variance function:
 Structure: Different standard deviations per stratum
         1          2          3          4          5          6
1.00000000 0.92635959 0.87908417 0.84875089 0.83806122 0.80713831
```

This variance function parameterization is unnecessarily confusing, so you can ignore it given our requests for RCOV and R below. But, what is given is each session's multiplier of the "residual standard error" for the first session that creates a session-specific SD (as shown at the bottom of this page). For example: Session 2 SD = 549.525*0.926 ≈ 509.06

Output given later (but that you need to understand the variance function):

```
Standard Deviations: 549.52 509.06 483.08 466.41 460.54 443.54
Residual standard error: 549.52468
```

```
Coefficients:
                      Value Std.Error   t-value p-value  fixed effect
(Intercept)      1961.89337 54.679749 35.879707       0  beta0
factor(session)2 -146.72098 29.820968 -4.920061       0  beta1
factor(session)3 -211.85872 31.366023 -6.754402       0  beta2
factor(session)4 -244.09691 33.642973 -7.255509       0  beta3
factor(session)5 -254.71764 35.845925 -7.105902       0  beta4
factor(session)6 -289.75736 32.700282 -8.861005       0  beta5
```

```
print("Show R and RCORR matrices for first person in the data")
getVarCov(MWP, individual="101"); corMatrix(MWP$modelStruct$corStruct)[[5]]
```

```
Marginal variance covariance matrix → Leftover variance and covariance (and correlations below)
       [,1]   [,2]   [,3]   [,4]   [,5]   [,6]
[1,] 301980 235650 217990 202600 192150 195350
[2,] 235650 259140 230210 213220 202080 193260
[3,] 217990 230210 233360 205200 196910 188600
[4,] 202600 213220 205200 217540 193670 185320
[5,] 192150 202080 196910 193670 212090 187840
[6,] 195350 193260 188600 185320 187840 196730

           [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
[1,] 1.00000000 0.84238790 0.82115911 0.79046587 0.75924409 0.80149155
[2,] 0.84238790 1.00000000 0.93613757 0.89805284 0.86199181 0.85594057
[3,] 0.82115911 0.93613757 1.00000000 0.91075553 0.88510835 0.88021939
[4,] 0.79046587 0.89805284 0.91075553 1.00000000 0.90163881 0.89579863
[5,] 0.75924409 0.86199181 0.88510835 0.90163881 1.00000000 0.91956267
[6,] 0.80149155 0.85594057 0.88021939 0.89579863 0.91956267 1.00000000
```

```
print("Session means, pairwise mean differences, and omnibus F-test")
emmeans(ref_grid(MWP), pairwise~session, adjust="none"); anova(MWP)
```

```
Analytical Satterthwaite method not available; using appx-satterthwaite
$emmeans
 session emmean   SE    df lower.CL upper.CL  b=beta
       1   1962 54.7  98.2     1853     2070  b0
       2   1815 50.7 102.0     1715     1916  b0+b1
       3   1750 48.1 101.6     1655     1845  b0+b2
       4   1718 46.4 101.4     1626     1810  b0+b3
       5   1707 45.8 101.3     1616     1798  b0+b4
       6   1672 44.1 100.8     1585     1760  b0+b5

Degrees-of-freedom method: appx-satterthwaite
$contrasts
contrast            estimate   SE    df t.ratio p.value   b=beta
 session1 - session2    146.7 29.8  99.5   4.920  <.0001  (b0) - (b0+b1) = -b1
 session1 - session3    211.9 31.4 100.1   6.754  <.0001  (b0) - (b0+b2) = -b2
 session1 - session4    244.1 33.6  99.9   7.256  <.0001  (b0) - (b0+b3) = -b3
 session1 - session5    254.7 35.8 100.3   7.106  <.0001  (b0) - (b0+b4) = -b4
 session1 - session6    289.8 32.7  99.6   8.861  <.0001  (b0) - (b0+b5) = -b5
 session2 - session3     65.1 17.8 100.2   3.655  0.0004  (b0+b1) - (b0+b2) = b1 - b2
 session2 - session4     97.4 22.3  99.2   4.367  <.0001  (b0+b1) - (b0+b3) = b1 - b3
 session2 - session5    108.0 25.8 100.4   4.191  0.0001  (b0+b1) - (b0+b4) = b1 - b4
 session2 - session6    143.0 26.2  98.7   5.459  <.0001  (b0+b1) - (b0+b5) = b1 - b5
 session3 - session4     32.2 20.0  98.2   1.610  0.1106  (b0+b2) - (b0+b3) = b2 - b3
 session3 - session5     42.9 22.6  99.6   1.896  0.0609  (b0+b2) - (b0+b4) = b2 - b4
 session3 - session6     77.9 22.9  97.3   3.404  0.0010  (b0+b2) - (b0+b5) = b2 - b5
 session4 - session5     10.6 20.5  98.7   0.519  0.6049  (b0+b3) - (b0+b4) = b3 - b4
 session4 - session6     45.7 20.8  95.9   2.197  0.0304  (b0+b3) - (b0+b3) = b3 - b5
 session5 - session6     35.0 18.1  95.9   1.934  0.0560  (b0+b4) - (b0+b5) = b4 - b5

Degrees-of-freedom method: appx-satterthwaite
Denom. DF: 600
                numDF    F-value p-value
(Intercept)         1 1472.07423  <.0001
factor(session)     5   16.72028  <.0001
```

```
print("Show likelihood ratio test comparing model fit: Univ WP vs. Multiv WP")
anova(MWP,UWP)  # Order of "more", "fewer"
```

```
    Model df       AIC       BIC     logLik   Test   L.Ratio p-value
MWP     1 27 8283.7885 8402.5056 -4114.8942
UWP     2  8 8369.3551 8404.5305 -4176.6776 1 vs 2 123.56663  <.0001
```