

Example 5a: Multivariate General Linear Models for Family (Triadic) Data
Part 1 using Univariate Software: SAS MIXED, STATA MIXED, and R GLS (from NLME)
Part 2 using Path Analysis Software: Mplus, STATA SEM, and R LAVAAN
(complete syntax and output available for SAS, STATA, and R electronically)

These data were collected as part of a study of family dynamics conducted at Penn State University. The sample for this example includes 140 families with data from three family members: a mother, a father, and an adult child. The example outcome is a scale mean (range from 1–4) of attitudes about gender roles in marriage, in which higher scores indicate more conservative attitudes. The example predictors are the gender of the adult child (0=girl, 1=boy) and the years of education of each family member (centered such that 0=12 years). In all models, we will use an unstructured **R** matrix (in which the residual variances and covariances are estimated separately for each type of family member).

We will predict all three family members’ outcomes simultaneously two ways. In Part 1 we will estimate multivariate general linear models within univariate software (i.e., with an identity link and conditional multivariate normal distributions) using residual maximum likelihood (REML), and we will (try to) test fixed effects using Satterthwaite denominator degrees of freedom. Note that STATA and R both provide incorrect AIC and BIC values using REML (they count all parameters instead of variance parameters only), so those values are not referred to below. In Part 2, we will estimate the same models using path analysis, whose software restrictions mean we must switch to maximum likelihood and test fixed effects without denominator degrees of freedom.

The marginal outcome distributions of the showed some positive skew (with an observed floor effect for the adult children), but a conditional normal distribution appears to be a reasonable choice among the readily-available options for multivariate models. This is evidenced in the final model by predicted outcomes that stayed within the outcome bounds without the use a link function to do so, and plausible homogeneity of variance across predicted outcomes. In Part 2, we will also invoke robust standard errors that protect against deviations from residual multivariate normality.

Part 1 will require “reshaping” (i.e., stacking) our original data stored in wide (multivariate) format, in which one row holds all variables per family, with per-person versions in separate columns...

	FamilyID: Family ID Number	KidBoy: Kid's Gender (0=girl, 1=boy)	KidEd12: Kid's Years of Education (0=12)	MomEd12: Mother's Years of Education (0=12)	DadEd12: Father's Years of Education (0=12)	KidMarital: Kid's Marital Gender Attitudes Mean (1-4)	MomMarital: Mom's Marital Gender Attitudes Mean (1-4)	DadMarital: Dad's Marital Gender Attitudes Mean (1-4)
1	3996	1	2	2	2	1	1.8333333333	1
2	4425	1	3	0	0	1	1.3333333333	2.5

...into this new format called stacked (long, univariate), with one row per person per family:

	FamilyID: Family ID Number	KidBoy: Kid's Gender (0=girl, 1=boy)	KidEd12: Kid's Years of Education (0=12)	MomEd12: Mother's Years of Education	DadEd12: Father's Years of Education (0=12)	DV: 1K,2M,3D	kid: Is Adult Child (0=no, 1=yes)	mom: Is Mother (0=no, 1=yes)	dad: Is Father (0=no, 1=yes)	marital: Marital Gender Attitudes Mean (1-4)
1	3996	1	2	2	2	1.Kid	1	0	0	1
2	3996	1	2	2	2	2.Mom	0	1	0	1.83333333
3	3996	1	2	2	2	3.Dad	0	0	1	1
4	4425	1	3	0	0	1.Kid	1	0	0	1
5	4425	1	3	0	0	2.Mom	0	1	0	1.33333333
6	4425	1	3	0	0	3.Dad	0	0	1	2.5

Part 2 will use the original wide format for path analysis instead.

SAS Syntax for Importing and Stacking Wide Data into Long (to get one row per person per family):

```

* Location for original files for these models - change this path;
* \\Client\ precedes path in Virtual Desktop outside H drive;
%LET filesave=C:\Dropbox\22_PSQF6270\PSQF6270_Example5a;
LIBNAME filesave "&filesave.";

* Import Example 5a multivariate data into work library and stack it;
DATA work.Example5a; SET filesave.PSQF6270_Example5aWide; * Adding dummy codes while stacking;
  DV="1.Kid"; kid=1; mom=0; dad=0; marital=KidMarital; OUTPUT;
  DV="2.Mom"; kid=0; mom=1; dad=0; marital=MomMarital; OUTPUT;
  DV="3.Dad"; kid=0; mom=0; dad=1; marital=DadMarital; OUTPUT;
  LABEL DV=      "DV: 1K,2M,3D"
         kid=     "kid: Is Adult Child (0=no, 1=yes)"
         mom=     "mom: Is Mother (0=no, 1=yes)"
         dad=     "dad: Is Father (0=no, 1=yes)"
         marital= "marital: Marital Gender Attitudes Mean (1-4)";
  DROP KidMarital MomMarital DadMarital; * Remove original outcomes;
RUN; * Remove missing predictors or row-specific (will happen anyway);
DATA work.Example5a; SET work.Example5a;
  IF NMISS(KidBoy,KidEd12,MomEd12,DadEd12,marital)>0 THEN DELETE;
RUN;

```

STATA Syntax for Importing and Stacking Wide Data into Long (to get one row per person per family):

```

// Defining global variable for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive;
global filesave "C:\Dropbox\22_PSQF6270\PSQF6270_Example5a"
// Import Example 5a wide Stata data
use "$filesave\PSQF6270_Example5aWide.dta", clear
// Rename variables with numeric suffix to use with reshape (old) (new)
rename (kidmarital mommarital dadmarital) (marital1 marital2 marital3)
// Stack data: list multivariate variables first, i(higher index) j(repeated)
reshape long marital, i(familyid) j(DV)
// Create value labels and apply to dv
label define dvlabel 1 "1.Kid" 2 "2.Mom" 3 "3.Dad"
label values DV dvlabel
// Create per-outcome dummy codes
gen kid=0
gen mom=0
gen dad=0
recode kid (0=1) if DV==1
recode mom (0=1) if DV==2
recode dad (0=1) if DV==3
// Label new variables
label variable DV      "DV: 1K,2M,3D"
label variable kid     "kid: Is Adult Child (0=no, 1=yes)"
label variable mom     "mom: Is Mother (0=no, 1=yes)"
label variable dad     "dad: Is Father (0=no, 1=yes)"
label variable marital "marital: Marital Gender Attitudes Mean (1-4)"
// Remove missing predictors or row-specific outcome (will happen anyway)
egen nummiss = rowmiss(kidboy kided12 momed12 daded12 marital)
drop if nummiss>0

```

R Syntax for Importing and Stacking Wide Data into Long (to get one row per person per family):

```

# Define variables for working directory and data name
filesave = "C:\\Dropbox\\22_PSQF6270\\PSQF6270_Example5a\\"
filename = "PSQF6270_Example5aWide.sas7bdat"
setwd(dir=filesave)

# Import Example 5a SAS data
Example5a_wide = read_sas(data_file=paste0(filesave,filename))
# Convert to data frame without labels to use for analysis
Example5a_wide = as.data.frame(Example5a_wide)

```

```

# Stack into long format (one row per outcome per family)
Example5a = reshape(Example5a_wide, direction="long", idvar="FamilyID",
                    varying=c("KidMarital","MomMarital","DadMarital"),
                    v.names="marital", timevar="DVnum", times=c(1,2,3))
# Work-around to add value labels for categorical variables DV
# Make a concatenated list of labels in order of values to be labeled
DVLabels = c("1.Kid", "2.Mom", "3.Dad")
# Create new text-format string variable with the labels instead of values
Example5a$DV = DVLabels[Example5a$DVnum]
# Create per-person dummy codes
Example5a$kid=0
Example5a$mom=0
Example5a$dad=0
Example5a$kid[which(Example5a$DVnum==1)]=1
Example5a$mom[which(Example5a$DVnum==2)]=1
Example5a$dad[which(Example5a$DVnum==3)]=1
# Make new variable for DV with reference=Dad to match other programs
Example5a$DV3=relevel(factor(Example5a$DV), ref=3)

# Remove missing predictors or row-specific outcome (will happen anyway)
Example5a = Example5a[complete.cases(Example5a[ ,
                    c("KidBoy", "KidEd12", "MomEd12", "DadEd12", "marital")]), ]

```

Part 1: Multivariate General Linear Models via Univariate Software

Model 0: Empty Means, Unstructured Variance Model for Marital Conservative Gender Attitudes

Two Ways for General Intercept Version: $\widehat{Marital}_{fi} = \beta_{00} + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi})$

```

TITLE1 "SAS Empty Means, Unstructured Variance Models for Marital Attitudes";
TITLE2 "SAS Model 0a: General Intercept (Dad=Ref DV) using 2 Dummy Codes";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 METHOD=REML;
* CLASS is for ID variables and program-categorical predictors;
  CLASS FamilyID DV; * Fixed intercept will be for dad (as omitted);
  MODEL marital = kid mom / SOLUTION DDFM=Satterthwaite;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID;
  CONTRAST "DF=2 Intercept Diff" kid 1, mom 1;
  ESTIMATE "Kid Intercept (Dad+diff)" intercept 1 kid 1 mom 0;
  ESTIMATE "Mom Intercept (Dad+diff)" intercept 1 kid 0 mom 1;
  ESTIMATE "Kid vs. Mom: Intercept Diff" kid -1 mom 1;
RUN; TITLE2;

TITLE2 "SAS Model 0b: General Intercept (Dad=Ref DV) using Program-Categorical DV";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 METHOD=REML;
* CLASS is for ID variables and program-categorical predictors;
  CLASS FamilyID DV; * Fixed intercept is for last DV (dad here);
  MODEL marital = DV / SOLUTION DDFM=Satterthwaite;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID;
* F-test given by default when program-categorical DV is used;
  CONTRAST "DF=2 Intercept Diff" DV -1 0 1, DV -1 1 0;
* LSMEANS gives all means and mean diffs (given by ESTIMATES below);
  LSMEANS DV / DIFF=ALL;
  ESTIMATE "Kid Intercept (Dad+diff)" intercept 1 DV 1 0 0;
  ESTIMATE "Mom Intercept (Dad+diff)" intercept 1 DV 0 1 0;
  ESTIMATE "Kid vs. Mom: Intercept Diff" DV -1 1 0;
RUN; TITLE2;

display "STATA Empty Means, Unstructured Variance Models for Marital Attitudes"
display "STATA Model 0a: General Intercept (Dad=Ref DV) using 2 Dummy Codes"
mixed marital c.kid c.mom, /// Fixed intercept will be for dad (as omitted)
|| familyid: , noconstant /// This NOCONSTANT removes family random intercept
variance reml residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
dfmethod(satterthwaite) dftable(pvalue) // Use Satterthwaite denominator DF
display "-2LL=" e(l1)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix
test (c.kid=0)(c.mom=0), small // DF=2 Intercept Diff (small = use denominator DF)

```

```

lincom _cons*1 + c.kid*1, small // Kid Intercept (Dad + diff)
lincom _cons*1 + c.mom*1, small // Mom Intercept (Dad + diff)
lincom c.kid*-1 + c.mom*1, small // Kid vs. Mom: Intercept Diff

display "STATA Model 0b: General Intercept (Dad=Ref DV) using Program-Categorical DV"
mixed marital ib(last).DV, // Fixed intercept is for last DV (dad here)
|| familyid: , noconstant // This NOCONSTANT removes family random intercept
variance reml residuals(unstructured,t(DV)) // Unstructured R matrix by DV
dfmethod(satterthwaite) dftable(pvalue) // Use Satterthwaite denominator DF
display "-2LL=" e(l1)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix
margins DV // Means per person (as given by LINCOM below)
margins DV, pwcompare(pveffects) df(139) // Pairwise differences (using denominator DF)
test (1.DV=2.DV) (1.DV=3.DV), small // DF=2 Intercept Diff (small = use denominator DF)
// These LINCOMs would be given by margins, but here's how to get them this way
lincom _cons*1 + 1.DV*1, small // Kid Intercept (Dad + diff)
lincom _cons*1 + 2.DV*1, small // Mom Intercept (Dad + diff)
lincom 1.DV*-1 + 2.DV*1, small // Kid vs. Mom: Intercept Diff

print("R Empty Means, Unstructured Variance Models for Marital Attitudes")
print("R Model 0a: General Intercept (Dad=Ref DV) using 2 Dummy Codes")
Model0a = gls(data=Example5a, method="REML",
model=marital~1+kid+mom, # Fixed intercept will be for dad (as omitted)
correlation=corSymm(form=~DVnum|FamilyID), # Unstructured correlations
weights=varIdent(form=~1|DV)) # Separate variance by DV
print("Print -2LL and Results") # Btw, AIC and BIC are incorrect (match STATA)
-2*logLik(Model0a); summary(Model0a)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model0a, individual="3996");
corMatrix(Model0a$modelStruct$corStruct)[[3]] # 3=Dimensions of R here

print("DF=2 Intercept Diff -- Get error that it used Chi-Square instead of F")
F0a = glht(model=Model0a, linfct=rbind(c(0,1,0),c(0,0,1)), df=139)
SaveF0a = summary(F0a, test=Ftest()); SaveF0a # Joint F-test
print("Get and show hidden results for F, dfnum, dfden, and p-value")
SaveF0a$test$fstat; SaveF0a$test$df; SaveF0a$df
pf(SaveF0a$test$fstat, df1=SaveF0a$test$df, df2=SaveF0a$df, lower.tail=FALSE)

print("Missing Intercepts and Difference -- Had to give it correct Denominator DF")
summary(glht(model=Model0a, df=139, linfct=rbind(
"Kid Intercept (Dad+Diff)" = c(1,1,0), # in order of fixed effects
"Mom Intercept (Dad+Diff)" = c(1,0,1),
"Kid vs. Mom: Intercept Diff" = c(0,-1,1))), test=adjusted("none"))

print("R Model 0b: General Intercept (Dad=Ref DV) using Program-Categorical DV")
Model0b = gls(data=Example5a, method="REML",
model=marital~1+factor(DV3), # Fixed intercept will be for DV=3
correlation=corSymm(form=~DVnum|FamilyID),
weights=varIdent(form=~1|DV))
print("Print -2LL and Results") # Btw, AIC and BIC are incorrect (match STATA)
-2*logLik(Model0b); summary(Model0b)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model0b, individual="3996");
corMatrix(Model0b$modelStruct$corStruct)[[3]] # 3=Dimensions of R here
print("DV means, pairwise mean differences, and omnibus F-test")
print("Error message says appx-satterthwaite used instead -- close but not quite")
emmeans(ref_grid(Model0b), pairwise~DV3, adjust="none"); joint_tests(Model0b)

```

SAS Output—Variance parameters, fit statistics, and contrasts are the same from all Model 0 variants:

Iteration History				
Iteration	Evaluations	-2 Res Log Like	Criterion	
0	1	729.02493914		
1	2	706.95470511	0.00000000	

For your homework using SAS, get your -2LL value from this table to get two digits after the decimal.

Estimated R Matrix for FAMILYID 3996				Estimated R Correlation Matrix for FAMILYID 3996			
Row	Col1	Col2	Col3	Row	Col1	Col2	Col3
1	0.3312	0.04133	0.08240	1	1.0000	0.1264	0.2533
2	0.04133	0.3230	0.09371	2	0.1264	1.0000	0.2917
3	0.08240	0.09371	0.3196	3	0.2533	0.2917	1.0000

Covariance Parameter Estimates					
Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
UN(1,1)	FamilyID	0.3312	0.03973	8.34	<.0001
UN(2,1)	FamilyID	0.04133	0.02796	1.48	0.1394
UN(2,2)	FamilyID	0.3230	0.03875	8.34	<.0001
UN(3,1)	FamilyID	0.08240	0.02847	2.89	0.0038
UN(3,2)	FamilyID	0.09371	0.02839	3.30	0.0010
UN(3,3)	FamilyID	0.3196	0.03834	8.34	<.0001

Information Criteria						
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
707.0	6	719.0	719.2	726.1	736.6	742.6

Contrasts				
Label	Num DF	Den DF	F Value	Pr > F
DF=2 Diff in Intercept?	2	139	16.19	<.0001

Partial STATA Output for variance parameters, fit statistics, and contrasts for all Model 0 variants:

```
Mixed-effects REML regression
Group variable: familyid
Number of obs = 420
Number of groups = 140
Obs per group:
    min = 3
    avg = 3.0
    max = 3
DF method: Satterthwaite
DF:
    min = 139.00
    avg = 139.00
    max = 139.00
Log restricted-likelihood = -353.47735
F(2, 139.00) = 16.19 → Multiv Wald test given
Prob > F = 0.0000
```

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
familyid: (empty)	(No random effect variances in this model)			
Residual: Unstructured				
var(e1)	.3311924	.0397272	.2618044	.4189707
var(e2)	.3230136	.0387461	.2553391	.4086242
var(e3)	.3195886	.0383353	.2526318	.4042916
cov(e1,e2)	.041334	.027963	-.0134724	.0961405
cov(e1,e3)	.0824049	.0284663	.026612	.1381978
cov(e2,e3)	.0937145	.0283876	.0380758	.1493531

R and RCORR from estat wcorrelation

SDs for R also printed (not shown here)

Covariances for familyid = 3996:				Correlations:			
DV	1	2	3	DV	1	2	3
1	0.331			1	1.000		
2	0.041	0.323		2	0.126	1.000	
3	0.082	0.094	0.320	3	0.253	0.292	1.000

Partial R Output for variance parameters, fit statistics, and contrasts for all Model 0 variants:

```
[1] "Print -2LL and Results"
> -2 * logLik(Model0a)
'log Lik.' 706.95471 (df=9)
```

-2LL requested separately

```
Generalized least squares fit by REML
Model: marital ~ 1 + kid + mom
Data: Example5a
      AIC      BIC    logLik
724.95471 761.25248 -353.47735
```

LL instead, and incorrect version of AIC and BIC (counting all parameters instead of just variance model parameters, as in STATA too)

Correlation Structure: General

```
 1      2
2 0.126
3 0.253 0.292
```

Inside of RCORR (given in full below)

Variance function:

Structure: Different standard deviations per stratum

Formula: ~1 | DV

Parameter estimates:

```
 1.Kid  2.Mom  3.Dad
1.0000000 0.98757870 0.98232045
```

Weird multiplication factors to compute SD relative to first DV → ignore this

Standardized residuals:

```
      Min      Q1      Med      Q3      Max
-1.690995001 -0.806539334  0.077916333  0.643753422  3.402105330
```

Residual standard error: 0.57549394

Degrees of freedom: 420 total; 417 residual

Naïve denominator DF given

[1] "Show R and RCORR matrices for first family in the data"

```
> getVarCov(Model0a, individual = "3996")
```

Marginal variance covariance matrix

```
      [,1]      [,2]      [,3]
[1,] 0.331190 0.041336 0.082407
[2,] 0.041336 0.323020 0.093715
[3,] 0.082407 0.093715 0.319590
```

Actual R matrix!

Standard Deviations: 0.57549 0.56835 0.56532

```
> corMatrix(Model0a$modelStruct$corStruct)[[3]]
```

```
      [,1]      [,2]      [,3]
[1,] 1.00000000 0.12637845 0.25329512
[2,] 0.12637845 1.00000000 0.29167759
[3,] 0.25329512 0.29167759 1.00000000
```

Actual RCORR matrix!

[1] "DF=2 Intercept Diff -- Get error that it used Chi-Square instead of F"

```
> F0a = glht(model = Model0a, linfct = rbind(c(0, 1, 0), c(0, 0, 1)), df = 139)
```

```
> SaveF0a = summary(F0a, test = Ftest()); SaveF0a
```

General Linear Hypotheses

Linear Hypotheses:

```
Estimate
1 == 0 -0.32643
2 == 0 -0.05619
```

Global Test:

```
Chisq DF Pr(>Chisq)
1 32.376 2 0.00000009324
```

R told me it wouldn't compute the F test... except it secretly did! So I just asked for it

[1] "Get and show hidden results for F, dfnum, dfden, and p-value"

```
> SaveF0a$test$fstat
```

```
      [,1]
[1,] 16.18809
```

```
> SaveF0a$test$df
```

```
[1] 2
```

```
> SaveF0a$df
```

```
[1] 139
```

```
> pf(SaveF0a$test$fstat, df1 = SaveF0a$test$df, df2 = SaveF0a$df,
+ lower.tail = FALSE)
```

```
      [,1]
[1,] 0.00000047859907
```

Model-Estimated Fixed Effects from General Intercept Version using Dummy Codes:

$$\text{Solution for Fixed Effects } \widehat{Marital}_{fi} = \beta_{00} + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi})$$

Effect	Estimate	Standard Error	DF	t Value	Pr > t	
Intercept	1.9560	0.04778	139	40.94	<.0001	Dad intercept B00
kid	-0.3264	0.05892	139	-5.54	<.0001	Kid intercept diff B01
mom	-0.05619	0.05702	139	-0.99	0.3261	Mom intercept diff B02

Model-Estimated Fixed Effects from General Intercept Version using Categorical DV:

Solution for Fixed Effects $\widehat{Marital}_{fi} = \beta_{00} + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi})$

Effect	DV:		Standard				
	1K,2M,3D	Estimate	Error	DF	t Value	Pr > t	
Intercept		1.9560	0.04778	139	40.94	<.0001	Dad intercept B00
DV	1.Kid	-0.3264	0.05892	139	-5.54	<.0001	Kid intercept diff B01
DV	2.Mom	-0.05619	0.05702	139	-0.99	0.3261	Mom intercept diff B02
DV	3.Dad	0

Requested Linear Combination Estimates from General Intercept Version either way:

Estimates $\widehat{Marital}_{fi} = \beta_{00} + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi})$

Label	Estimate		Standard		DF	t Value	Pr > t	
	Estimate	Error	Error	Standard				
Kid Intercept (Dad+diff)	1.6295	0.04864	0.04864	0.04864	139	33.50	<.0001	B00 + B01
Mom Intercept (Dad+diff)	1.8998	0.04803	0.04803	0.04803	139	39.55	<.0001	B00 + B02
Kid vs. Mom: Intercept Diff	0.2702	0.06389	0.06389	0.06389	139	4.23	<.0001	B02 - B01

Output from Program-Categorical SAS LSMEANS (Model 0b)—EMMEANS in R, MARGINS in STATA

Least Squares Means

Effect	DV:		Standard				
	1K,2M,3D	Estimate	Error	DF	t Value	Pr > t	
DV	1.Kid	1.6295	0.04864	139	33.50	<.0001	Kid intercept B00 + B01
DV	2.Mom	1.8998	0.04803	139	39.55	<.0001	Mom intercept B00 + B02
DV	3.Dad	1.9560	0.04778	139	40.94	<.0001	Dad intercept B00

Differences of Least Squares Means

Effect	DV:		DV:		Standard					
	1K,2M,3D	1K,2M,3D	1K,2M,3D	1K,2M,3D	Estimate	Error	DF	t Value		Pr > t
DV	1.Kid	2.Mom			-0.2702	0.06389	139	-4.23	<.0001	B01 - B02
DV	1.Kid	3.Dad			-0.3264	0.05892	139	-5.54	<.0001	B01
DV	2.Mom	3.Dad			-0.05619	0.05702	139	-0.99	0.3261	B02

Model 0, continued: Empty Means, Unstructured Variance Model for Marital Conservative Gender Attitudes

Two Ways for DV-Specific Intercept Version: $\widehat{Marital}_{fi} = \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi})$

```
TITLE2 "SAS Model 0c: DV-Specific Intercepts using All 3 Dummy Codes";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
* CLASS is for ID variables and program-categorical predictors;
  CLASS FamilyID DV; * NOINT removes general fixed intercept;
  MODEL marital = kid mom dad / NOINT SOLUTION DDFM=Satterthwaite;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID; * UN = unstructured R;
  CONTRAST "DF=2 Intercept Diff" kid -1 mom 1, kid -1 dad 1;
  ESTIMATE "Kid vs. Mom: Intercept Diff" kid -1 mom 1;
  ESTIMATE "Kid vs. Dad: Intercept Diff" kid -1 dad 1;
  ESTIMATE "Mom vs. Dad: Intercept Diff" mom -1 dad 1;
RUN; TITLE2;
```

```
TITLE2 "SAS Model 0d: DV-Specific Intercepts using Program-Categorical DV";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
* CLASS is for ID variables and program-categorical predictors;
  CLASS FamilyID DV; * NOINT removes general fixed intercept;
  MODEL marital = DV / NOINT SOLUTION DDFM=Satterthwaite;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID;
* F-test given by default is INCORRECT without a general intercept;
  CONTRAST "DF=2 Intercept Diff" DV -1 0 1, DV -1 1 0;
* LSMEANS gives all means and mean diffs (given by ESTIMATEs below);
  LSMEANS DV / DIFF=ALL;
  ESTIMATE "Kid vs. Mom: Intercept Diff" DV -1 1 0;
  ESTIMATE "Kid vs. Dad: Intercept Diff" DV -1 0 1;
  ESTIMATE "Mom vs. Dad: Intercept Diff" DV 0 -1 1;
RUN; TITLE2; TITLE1;
```

```

display "STATA Model 0c: DV-Specific Intercepts using All 3 Dummy Codes"
mixed marital c.kid c.mom c.dad, noconstant /// This NOCONSTANT removes general fixed intercept
  || familyid: , noconstant /// This NOCONSTANT removes family random intercept
  variance reml residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
  dfmethod(satterthwaite) dftable(pvalue) // Use Satterthwaite denominator DF
display "-2LL=" e(11)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix
test (c.kid=c.mom)(c.kid=c.dad), small // DF=2 Intercept Diff (small = use denominator DF)
lincom c.kid*-1 + c.mom*1, small // Kid vs. Mom: Intercept Diff
lincom c.kid*-1 + c.dad*1, small // Kid vs. Dad: Intercept Diff
lincom c.mom*-1 + c.dad*1, small // Mom vs. Dad: Intercept Diff

display "STATA Model 0d: DV-Specific Intercepts using Program-Categorical DV"
display "ibn. = use all factor levels (no base as reference)"
mixed marital ibn.DV, noconstant /// This NOCONSTANT removes general fixed intercept
  || familyid: , noconstant /// This NOCONSTANT removes family random intercept
  variance reml residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
  dfmethod(satterthwaite) dftable(pvalue) // Use Satterthwaite denominator DF
display "-2LL=" e(11)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix
test (1.DV=2.DV)(1.DV=3.DV), small // DF=2 Intercept Diff (small = use denominator DF)
lincom 1.DV*-1 + 2.DV*1, small // Kid vs. Mom: Intercept Diff
lincom 1.DV*-1 + 3.DV*1, small // Kid vs. Dad: Intercept Diff
lincom 2.DV*-1 + 3.DV*1, small // Mom vs. Dad: Intercept Diff

print("R Model 0c: DV-Specific Intercepts using All 3 Dummy Codes")
Model0c = gls(data=Example5a, method="REML",
  model=marital~0+kid+mom+dad, # 0 removes fixed intercept
  correlation=corSymm(form=~DVnum|FamilyID), # Unstructured correlations
  weights=varIdent(form=~1|DV)) # Separate variance by DV
print("Print -2LL and Results") # Btw, AIC and BIC are incorrect (match STATA)
-2*logLik(Model0c); summary(Model0c)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model0c, individual="3996");
corMatrix(Model0c$modelStruct$corStruct)[[3]] # 3=Dimensions of R here

print("DF=2 Intercept Diff -- Get error that it used Chi-Square instead of F")
F0c = glht(model=Model0c, linfct=rbind(c(-1,1,0),c(0,-1,1)), df=139)
SaveF0c = summary(F0c, test=Ftest()); SaveF0a # Joint F-test
print("Get and show hidden results for F, dfnum, dfden, and p-value")
SaveF0c$test$fstat; SaveF0c$test$ddf; SaveF0c$df
pf(SaveF0c$test$fstat,df1=SaveF0c$test$ddf,df2=SaveF0c$df,lower.tail=FALSE)

print("Pairwise Intercept Diffs -- Had to give it correct Denominator DF")
summary(glht(model=Model0c, df=139, linfct=rbind(
  "Kid vs. Mom: Intercept Diff" = c(-1,1,0), # in order of fixed effects
  "Kid vs. Dad: Intercept Diff" = c(-1,0,1),
  "Mom vs. Dad: Intercept Diff" = c(0,-1,1))), test=adjusted("none"))

print("R Model 0d: DV-Specific Intercepts using Program-Categorical DV")
Model0d = gls(data=Example5a, method="REML",
  model=marital~0+factor(DV), # 0 removes fixed intercept
  correlation=corSymm(form=~DVnum|FamilyID),
  weights=varIdent(form=~1|DV))
print("Print -2LL and Results") # Btw, AIC and BIC are incorrect (match STATA)
-2*logLik(Model0d); summary(Model0d)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model0d, individual="3996");
corMatrix(Model0d$modelStruct$corStruct)[[3]] # 3=Dimensions of R here
print("DV means, pairwise mean differences, and omnibus F-test")
print("Error message says appx-satterthwaite used instead -- close but not quite")
emmeans(ref_grid(Model0d), pairwise~DV, adjust="none"); joint_tests(Model0d)

```


Model-Estimated Fixed Effects from DV-Specific Intercept Version using Dummy Codes:

$$\text{Solution for Fixed Effects } \widehat{\text{Marital}}_{fi} = \beta_{00}(\text{Dad}_{fi}) + \beta_{01}(\text{Kid}_{fi}) + \beta_{02}(\text{Mom}_{fi})$$

Effect	Estimate	Standard Error	DF	t Value	Pr > t	
kid	1.6295	0.04864	139	33.50	<.0001	Kid intercept B01
mom	1.8998	0.04803	139	39.55	<.0001	Mom intercept B02
dad	1.9560	0.04778	139	40.94	<.0001	Dad intercept B00

Model-Estimated Fixed Effects from DV-Specific Intercept Version using Categorical DV:

$$\text{Solution for Fixed Effects } \widehat{\text{Marital}}_{fi} = \beta_{00}(\text{Dad}_{fi}) + \beta_{01}(\text{Kid}_{fi}) + \beta_{02}(\text{Mom}_{fi})$$

Effect	DV:	Estimate	Standard Error	DF	t Value	Pr > t	
DV	1.Kid	1.6295	0.04864	139	33.50	<.0001	Kid intercept B01
DV	2.Mom	1.8998	0.04803	139	39.55	<.0001	Mom intercept B02
DV	3.Dad	1.9560	0.04778	139	40.94	<.0001	Dad intercept B00

Requested Linear Combination Estimates from DV-Specific Intercept Version either way:

$$\text{Estimates } \widehat{\text{Marital}}_{fi} = \beta_{00}(\text{Dad}_{fi}) + \beta_{01}(\text{Kid}_{fi}) + \beta_{02}(\text{Mom}_{fi})$$

Label	Estimate	Standard Error	DF	t Value	Pr > t	
Kid vs. Mom: Intercept Diff	0.2702	0.06389	139	4.23	<.0001	B02 - B01
Kid vs. Dad: Intercept Diff	0.3264	0.05892	139	5.54	<.0001	B00 - B01
Mom vs. Dad: Intercept Diff	0.05619	0.05702	139	0.99	0.3261	B00 - B02

Output from Program-Categorical SAS LSMEANS (Model 0d)—EMMEANS in R, MARGINS in STATA

Least Squares Means							
Effect	DV:	Estimate	Standard Error	DF	t Value	Pr > t	
DV	1.Kid	1.6295	0.04864	139	33.50	<.0001	Kid intercept B01
DV	2.Mom	1.8998	0.04803	139	39.55	<.0001	Mom intercept B02
DV	3.Dad	1.9560	0.04778	139	40.94	<.0001	Dad intercept B00

Differences of Least Squares Means								
Effect	DV:	DV:	Estimate	Standard Error	DF	t Value	Pr > t	
DV	1.Kid	2.Mom	-0.2702	0.06389	139	-4.23	<.0001	B01 - B02
DV	1.Kid	3.Dad	-0.3264	0.05892	139	-5.54	<.0001	B01 - B00
DV	2.Mom	3.Dad	-0.05619	0.05702	139	-0.99	0.3261	B02 - B00

To avoid confusion, we will proceed using Model 0c: DV-specific intercepts implemented via three dummy codes. This approach also aligns most directly with path model variants of these models (Part 2).

Model 1: DV-Specific Intercepts adding Kid's Gender as Predictor for Each Attitude

$$\widehat{\text{Marital}}_{fi} = \beta_{00}(\text{Dad}_{fi}) + \beta_{01}(\text{Kid}_{fi}) + \beta_{02}(\text{Mom}_{fi}) + \beta_{10}(\text{Dad}_{fi})(\text{KidBoy}_f) + \beta_{11}(\text{Kid}_{fi})(\text{KidBoy}_f) + \beta_{12}(\text{Mom}_{fi})(\text{KidBoy}_f)$$

```
TITLE "SAS Model 1: DV-Specific Intercepts -- Add Kid Gender";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
  CLASS FamilyID DV; * CLASS is for ID variables;
  MODEL marital = kid mom dad kid*KidBoy mom*KidBoy dad*KidBoy
    / NOINT SOLUTION DDFM=Satterthwaite;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID;
  CONTRAST "DF=2 Diff in KidBoy Slope?" kid*KidBoy -1 Mom*KidBoy 1, kid*KidBoy -1 Dad*KidBoy 1;
  ESTIMATE "Kid vs. Mom: KidBoy Slope Diff" kid*KidBoy -1 Mom*KidBoy 1;
  ESTIMATE "Kid vs. Dad: KidBoy Slope Diff" kid*KidBoy -1 Dad*KidBoy 1;
  ESTIMATE "Mom vs. Dad: KidBoy Slope Diff" Mom*KidBoy -1 Dad*KidBoy 1;
  ESTIMATE "Parent KidBoy Slope" Mom*KidBoy 1 Dad*KidBoy 1 / DIVISOR=2;
  ESTIMATE "Kid vs. Parents: KidBoy Diff" Kid*KidBoy -2 Mom*KidBoy 1 Dad*KidBoy 1 / DIVISOR=2;
RUN; TITLE;
```

```

display "STATA Model 1: DV-Specific Intercepts -- Add Kid Gender"
mixed marital c.kid c.mom c.dad c.kid#c.kidboy c.mom#c.kidboy c.dad#c.kidboy, noconstant ///
  || familyid: , noconstant      /// This NOCONSTANT removes family random intercept
  variance reml residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
  dfmethod(satterthwaite) dftable(pvalue)    // Use Satterthwaite denominator DF
display "-2LL=" e(ll)*-2                // Print -2LL for model
estat wcorrelation, covariance          // R matrix
estat wcorrelation                      // RCORR matrix
// DF=2 Diff in Kidboy Slope
test (c.kid#c.kidboy=c.mom#c.kidboy) (c.kid#c.kidboy=c.dad#c.kidboy), small
lincom c.kid#c.kidboy*-1 + c.mom#c.kidboy*1, small // Kid vs. Mom: Kidboy Slope Diff
lincom c.kid#c.kidboy*-1 + c.dad#c.kidboy*1, small // Kid vs. Dad: Kidboy Slope Diff
lincom c.mom#c.kidboy*-1 + c.dad#c.kidboy*1, small // Mom vs. Dad: Kidboy Slope Diff
lincom 0.5*(c.mom#c.kidboy*1 + c.dad#c.kidboy*1), small // Parent: Kidboy Slope
// Mom vs. Dad: Kidboy Slope Diff
lincom 0.5*(c.kid#c.kidboy*-2 + c.mom#c.kidboy*1 + c.dad#c.kidboy*1), small

print("R Model 1: DV-Specific Intercepts -- Add Kid Gender")
Modell = gls(data=Example5a, method="REML",
  model=marital~0+kid+mom+dad+kid:KidBoy+mom:KidBoy+dad:KidBoy,
  correlation=corSymm(form=~DVnum|FamilyID), # Unstructured correlations
  weights=varIdent(form=~1|DV)) # Separate variance by DV
print("Print -2LL and Results")
-2*logLik(Modell); summary(Modell)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Modell, individual="3996");
corMatrix(Modell$modelStruct$corStruct)[[3]] # 3=Dimensions of R here

print("DF=2 Diff in KidBoy Effect -- Get error that it used Chi-Square instead of F")
F1 = glht(model=Modell, linfct=rbind(c(0,0,0,-1,1,0),c(0,0,0,-1,0,1)), df=138)
SaveF1 = summary(F1, test=Ftest()); SaveF0a # Joint F-test
print("Get and show hidden results for F, dfnum, dfden, and p-value")
SaveF1$test$stat; SaveF1$test$df; SaveF1$df
pf(SaveF1$test$stat,df1=SaveF1$test$df,df2=SaveF1$df,lower.tail=FALSE)

print("KidBoy Slope Diffs -- Had to give it correct Denominator DF")
summary(glht(model=Modell, df=138, linfct=rbind(
  "Kid vs. Mom: KidBoy Slope Diff" = c(0,0,0,-1,1,0), # in order of fixed effects
  "Kid vs. Dad: KidBoy Slope Diff" = c(0,0,0,-1,0,1),
  "Mom vs. Dad: KidBoy Slope Diff" = c(0,0,0,0,-1,1),
  "Parent KidBoy Effect" = c(0,0,0,0,1/2,1/2),
  "Kids vs. Parent KidBoy Effect Diff" = c(0,0,0,-1,1/2,1/2))), test=adjusted("none"))

```

Partial SAS Output for Model 1: DV-Specific Intercepts adding Kid’s Gender as Predictor for Each Attitude

$$\widehat{Marital}_{fi} = \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{10}(Dad_{fi})(KidBoy_f) + \beta_{11}(Kid_{fi})(KidBoy_f) + \beta_{12}(Mom_{fi})(KidBoy_f)$$

Estimated R Matrix for FAMILYID 3996				Estimated R Correlation Matrix for FAMILYID 3996			
Row	Col1	Col2	Col3	Row	Col1	Col2	Col3
1	0.3136	0.03725	0.07733	1	1.0000	0.1168	0.2440
2	0.03725	0.3244	0.09315	2	0.1168	1.0000	0.2890
3	0.07733	0.09315	0.3203	3	0.2440	0.2890	1.0000

Information Criteria						
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
707.0	6	719.0	719.2	726.2	736.7	742.7

Contrasts					
Label	Num DF	Den DF	F Value	Pr > F	
DF=2 Diff in KidBoy Effect?	2	138	1.90	0.1529	

Solution for Fixed Effects						
Effect	Estimate	Standard Error	DF	t Value	Pr > t	
kid	1.4950	0.06554	138	22.81	<.0001	Kid intercept B01
mom	1.8703	0.06666	138	28.06	<.0001	Mom intercept B02
dad	1.9178	0.06624	138	28.95	<.0001	Dad intercept B00
kid*KidBoy	0.2811	0.09474	138	2.97	0.0035	girl vs boy for Kid B11
mom*KidBoy	0.06152	0.09636	138	0.64	0.5242	girl vs boy for Mom B12
dad*KidBoy	0.07970	0.09575	138	0.83	0.4066	girl vs boy for Dad B10

Estimates						
Label	Estimate	Standard Error	DF	t Value	Pr > t	
Kid vs. Mom: KidBoy Slope Diff	-0.2196	0.1270	138	-1.73	0.0860	B12 - B11
Kid vs. Dad: KidBoy Slope Diff	-0.2014	0.1171	138	-1.72	0.0877	B10 - B11
Mom vs. Dad: KidBoy Slope Diff	0.01818	0.1145	138	0.16	0.8741	B10 - B12
Parent KidBoy Slope	0.07061	0.07711	138	0.92	0.3614	0.5*(B10+B12)
Kid vs. Parents: KidBoy Slope Diff	-0.2105	0.1079	138	-1.95	0.0531	0.5*(B10+B12)-B11

It looks like we need to control for the effect of kid gender only for the kid (which makes sense, since we don't know about the gender of their siblings). Next, we'll test the effects of each person's education on their own attitude, followed by the incremental effect of dad's education on kid and mom attitudes after controlling for own education.

Model 2: DV-Specific Intercepts adding Own Education as Predictor of Own Attitude

$$\widehat{Marital}_{fi} = \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{11}(Kid_{fi})(KidBoy_f) \\ + \beta_{20}(Dad_{fi})(DadEd_f - 12) + \beta_{31}(Kid_{fi})(KidEd_f - 12) + \beta_{42}(Mom_{fi})(MomEd_f - 12)$$

```
TITLE "SAS Model 2: DV-Specific Intercepts -- KidBoy on Kid Only, Add Own Educ";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 IC METHOD=REML;
  CLASS FamilyID DV; * CLASS is for ID variables;
  MODEL marital = kid mom dad kid*KidBoy kid*KidEd12 mom*MomEd12 dad*DadEd12
    / NOINT SOLUTION DDFM=Satterthwaite;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID;
RUN; TITLE;

display "STATA Model 2: DV-Specific Intercepts -- KidBoy on Kid Only, Add Own Education"
mixed marital c.kid c.mom c.dad c.kid#c.kidboy ///
  c.kid#c.kided12 c.mom#c.momed12 c.dad#c.daded12, noconstant ///
  || familyid: , noconstant /// This NOCONSTANT removes family random intercept
  variance reml residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
  dfmethod(satterthwaite) dftable(pvalue) // Use Satterthwaite denominator DF
display "-2LL=" e(11)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix

print("R Model 2: DV-Specific Intercepts -- KidBoy on Kid Only, Add Own Educ")
Model2 = gls(data=Example5a, method="REML",
  model=marital~0+kid+mom+dad +kid:KidBoy +kid:KidEd12+mom:MomEd12+dad:DadEd12,
  correlation=corSymm(form=~DVnum|FamilyID), # Unstructured correlations
  weights=varIdent(form=~1|DV) # Separate variance by DV
print("Print -2LL and Results")
-2*logLik(Model2); summary(Model2)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model2, individual="3996");
corMatrix(Model2$modelStruct$corStruct) [[3]] # 3=Dimensions of R here
```

Partial SAS Output for Model 2:

$$\widehat{Marital}_{fi} = \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{11}(Kid_{fi})(KidBoy_f) \\ + \beta_{20}(Dad_{fi})(DadEd_f - 12) + \beta_{31}(Kid_{fi})(KidEd_f - 12) + \beta_{42}(Mom_{fi})(MomEd_f - 12)$$

Estimated R Matrix for FAMILYID 3996				Estimated R Correlation Matrix for FAMILYID 3996			
Row	Col1	Col2	Col3	Row	Col1	Col2	Col3
1	0.3156	0.03837	0.07669	1	1.0000	0.1207	0.2501
2	0.03837	0.3205	0.08441	2	0.1207	1.0000	0.2732
3	0.07669	0.08441	0.2979	3	0.2501	0.2732	1.0000

Information Criteria						
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
708.6	6	720.6	720.8	727.8	738.3	744.3

Solution for Fixed Effects						
Effect	Estimate	Standard Error	DF	t Value	Pr > t	
kid	1.5117	0.09814	141	15.40	<.0001	Kid intercept B01
mom	1.9359	0.05976	142	32.39	<.0001	Mom intercept B02
dad	2.0700	0.05663	145	36.55	<.0001	Dad intercept B00
kid*KidBoy	0.2641	0.09204	137	2.87	0.0048	girl vs boy for Kid B11
kid*KidEd12	-0.00280	0.02344	138	-0.12	0.9052	Kid Ed for kid B31
mom*MomEd12	-0.01725	0.01711	142	-1.01	0.3150	Mom Ed for mom B42
dad*DadEd12	-0.05447	0.01570	143	-3.47	0.0007	Dad Ed for dad B20

Model 3: DV-Specific Intercepts adding Dad Education as Predictor of Kid and Mom Attitudes

$$\widehat{Marital}_{fi} = \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{11}(Kid_{fi})(KidBoy_f) \\ + \beta_{20}(Dad_{fi})(DadEd_f - 12) + \beta_{31}(Kid_{fi})(KidEd_f - 12) + \beta_{42}(Mom_{fi})(MomEd_f - 12) \\ + \beta_{21}(Kid_{fi})(DadEd_f - 12) + \beta_{22}(Mom_{fi})(DadEd_f - 12)$$

```
TITLE "SAS Model 3: DV-Specific Intercepts -- Add Dad Educ (Control for Own Educ)";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 METHOD=REML;
  CLASS FamilyID DV; * CLASS is for ID variables;
  MODEL marital = kid mom dad kid*KidBoy Kid*KidEd12 Mom*MomEd12 Dad*DadEd12
    Kid*DadEd12 Mom*DadEd12 / NOINT SOLUTION DDFM=Satterthwaite RESIDUAL;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID; * RESIDUAL requests plots of residuals;
  ESTIMATE "Kid vs. Mom: DadEd12 Slope Diff" kid*DadEd12 -1 mom*DadEd12 1;
  ESTIMATE "Kid vs. Dad: DadEd12 Slope Diff" kid*DadEd12 -1 dad*DadEd12 1;
  ESTIMATE "Mom vs. Dad: DadEd12 Slope Diff" mom*DadEd12 -1 dad*DadEd12 1;
RUN; TITLE;

display "STATA Model 3: DV-Specific Intercepts -- Add Dad Educ (Control for Own Educ)"
mixed marital c.kid c.mom c.dad c.kid#c.kidboy c.kid#c.kided12 c.mom#c.momed12 ///
  c.dad#c.daded12 c.kid#c.daded12 c.mom#c.daded12, noconstant ///
  || familyid: , noconstant /// This NOCONSTANT removes family random intercept
  variance reml residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
  dfmethod(satterthwaite) dftable(pvalue) // Use Satterthwaite denominator DF
display "-2LL=" e(11)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix
lincom c.kid#c.daded12*-1 + c.mom#c.daded12*1, small // Kid vs. Mom: DadEd12 Slope Diff
lincom c.kid#c.daded12*-1 + c.dad#c.daded12*1, small // Kid vs. Dad: DadEd12 Slope Diff
lincom c.mom#c.daded12*-1 + c.dad#c.daded12*1, small // Mom vs. Dad: DadEd12 Slope Diff
predict Model3pred, xb // Save yhat from fixed effects
predict Model3res, rstandard // Save "standardized" residuals from fixed effects
hist Model3res // Histogram of residuals (for normality)
graph export "$filesave\STATA Model 3 Residual Histogram.png", replace
twoway (scatter Model3res Model3pred) // Scatterplot by predicted (for constant variance)
graph export "$filesave\STATA Model 3 Residual Scatterplot.png", replace
```

```

print("R Model 3: DV-Specific Intercepts -- Add Dad Educ (Control for Own Educ)")
Model3 = gls(data=Example5a, method="REML",
             model=marital~0+kid+mom+dad+ kid:KidBoy +kid:KidEd12+mom:MomEd12+dad:DadEd12
             +kid:DadEd12+mom:DadEd12,
             correlation=corSymm(form=~DVnum|FamilyID), # Unstructured correlations
             weights=varIdent(form=~1|DV) # Separate variance by DV
print("Print -2LL and Results") # Btw, AIC and BIC are incorrect (match STATA)
-2*logLik(Model3); summary(Model3)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model3, individual="3996");
corMatrix(Model3$modelStruct$corStruct)[[3]] # 3=Dimensions of R here

print("DadEd Slope Diffs -- Had to give it correct Denominator DF")
summary(glht(model=Model3, df=136, linfct=rbind(
  "Kid vs. Mom: DadEd12 Slope Diff" = c(0,0,0,0,0,0,-1,1), # in order of fixed effects
  "Kid vs. Dad: DadEd12 Slope Diff" = c(0,0,0,0,0,0,1,-1,0),
  "Mom vs. Dad: DadEd12 Slope Diff" = c(0,0,0,0,0,0,1,0,-1))), test=adjusted("none"))

print("Save yhat from fixed effects and Pearson residuals")
Example5a$Model3pred = predict(Model3, type="response")
Example5a$Model3res = residuals(Model3, type="pearson")
print ("Histogram of Residuals for normality")
hist(x=Example5a$Model3res, freq=FALSE, ylab="Density",xlab="Model 3 Residuals")
print ("Scatterplot of residuals by predicted for constant variance")
plot(x=Example5a$Model3res, y=Example5a$Model3pred,
     ylab="Residual",xlab="Model 3 Predicted Outcome")

```

Partial SAS Output for Model 3:

$$\begin{aligned}
 \widehat{Marital}_{fi} = & \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{11}(Kid_{fi})(KidBoy_f) \\
 & + \beta_{20}(Dad_{fi})(DadEd_f - 12) + \beta_{31}(Kid_{fi})(KidEd_f - 12) + \beta_{42}(Mom_{fi})(MomEd_f - 12) \\
 & + \beta_{21}(Kid_{fi})(DadEd_f - 12) + \beta_{22}(Mom_{fi})(DadEd_f - 12)
 \end{aligned}$$

Estimated R Matrix for FAMILYID 3996				Estimated R Correlation Matrix for FAMILYID 3996			
Row	Col1	Col2	Col3	Row	Col1	Col2	Col3
1	0.3179	0.03856	0.07720	1	1.0000	0.1204	0.2508
2	0.03856	0.3229	0.08514	2	0.1204	1.0000	0.2744
3	0.07720	0.08514	0.2982	3	0.2508	0.2744	1.0000

Information Criteria						
Neg2LogLike	Parms	AIC	AICC	HQIC	BIC	CAIC
720.8	6	732.8	733.0	739.9	750.4	756.4

Solution for Fixed Effects						
Effect	Estimate	Standard Error	DF	t Value	Pr > t	
kid	1.5123	0.1003	140	15.08	<.0001	Kid intercept B01
mom	1.9373	0.06305	138	30.73	<.0001	Mom intercept B02
dad	2.0707	0.05769	138	35.89	<.0001	Dad intercept B00
kid*KidBoy	0.2639	0.09258	136	2.85	0.0050	girl vs boy for Kid B11
kid*KidEd12	-0.00264	0.02458	136	-0.11	0.9147	Kid Ed for kid B31
mom*MomEd12	-0.01624	0.02068	137	-0.79	0.4338	Mom Ed for mom B42
dad*DadEd12	-0.05484	0.01654	138	-3.32	0.0012	Dad Ed for dad B20
kid*DadEd12	-0.00048	0.01791	138	-0.03	0.9787	Dad Ed for kid B21
mom*DadEd12	-0.00169	0.02069	141	-0.08	0.9349	Dad Ed for mom B22

Estimates						
Label	Estimate	Standard Error	DF	t Value	Pr > t	
Kid vs. Mom: DadEd12 Slope Diff	-0.00121	0.02601	162	-0.05	0.9629	B22 - B21
Kid vs. Dad: DadEd12 Slope Diff	-0.05436	0.02127	143	-2.56	0.0117	B20 - B21
Mom vs. Dad: DadEd12 Slope Diff	-0.05314	0.02335	154	-2.28	0.0242	B20 - B22

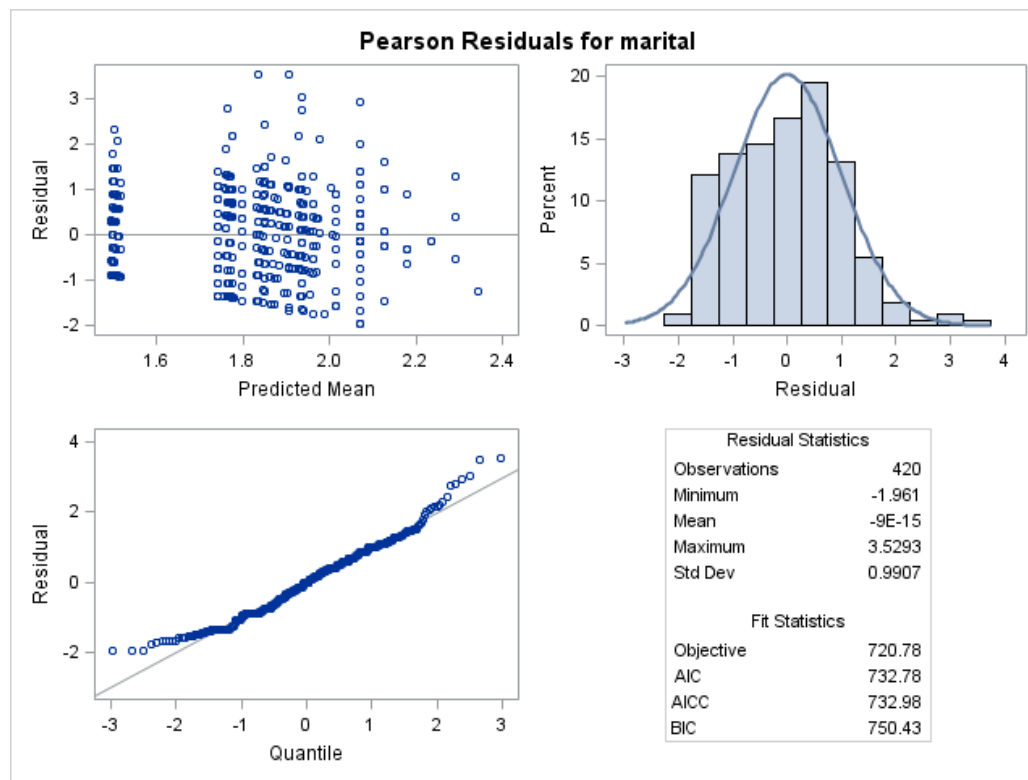
Moral of the story? Multivariate models can be estimated in univariate software to capture the relationships between person-specific predictors and person-specific outcomes (such as in “actor–partner” models for dyadic data as well).

Example results section for Part 1 Models 0–3:

The extent to which gender and education predicted marital attitudes was examined in 140 families, in which responses were collected from adult children, their mothers, and their fathers. Higher outcomes indicated more conservative marital attitudes (i.e., gender-traditional attitudes measured as the mean across items on a scale of 1 to 4). Given that the outcomes were correlated within families, multivariate general linear models (i.e., with conditionally multivariate normal residuals) were used to predict all three outcomes for each family simultaneously. All models were estimated using residual maximum likelihood and Satterthwaite denominator degrees of freedom. All models allowed separate means and residual variances across the three outcomes for the three types of family members, as well as covariances among the residuals from the same family. ESTIMATE statements were used to estimate simple slopes and simple slope differences as linear combinations of the model fixed effects. Prior to adding predictors, an empty means model (i.e., an unconditional model with no predictors) revealed significant differences in marital attitudes across type of family member, $F(2, 139) = 16.19, p < .001$. Although mean attitudes were similar across mothers and fathers (1.90 and 1.96, respectively, $p = .27$), the mean attitudes of children (1.63) were significantly less conservative on average than those of their parents ($p < .001$ for both comparisons).

To begin, we examined the extent to which the gender of the adult child (coded 0=woman, 1=man) who was surveyed was related to the marital attitudes of each type of family member. Although the attitudes of adult male children were significantly more conservative than those of adult female children (diff = 0.28, $p = .004$), there were no significant effects of the gender of the adult child for the marital attitudes of their mothers or fathers. Thus, we retained a predictor for the gender of the adult child only for the adult child’s outcome. We then examined the extent to which the education (centered at 12 years) of each type of family member predicted their own attitudes, which was significant only for the father: for every additional year of father’s education, his own attitudes were expected to be less conservative by 0.05 ($p < .001$). Next, we examined whether father’s education incrementally predicted the marital attitudes of the mother or adult child after controlling for their own education, but neither effect was significant (and the effect of father’s education on his own attitudes was significantly larger).

But how do we know if Model 3 is sufficient?? One aspect concerns the fit of the conditional distribution—in absence of Pearson χ^2/DF , we can examine residual plots, such as shown for SAS below:



These plots suggest some deviation from normality of the residuals, although the assumption of constant variance looks not terribly unreasonable.

Unfortunately, multivariate options for generalized linear models do not include beta-binomial alternatives that might have been useful here (given that the outcomes are bounded by 1 and 4). Also, given that all predicted outcomes stayed in bounds, it appears we don’t need a link function.

Instead, we can see how the results differ using “robust” standard errors...so stay tuned for Part 2!

The other issue whether all relationships among the predictors and outcomes have been captured adequately by the model... for a more efficient way to answer that question, **stay tuned for Part 2 using path analysis!**

Part 2: Multivariate General Linear Models via Path Analysis Software

In Part 2, we begin by estimating Model 3 using path analysis in Mplus, STATA SEM, and R LAVAAN, which each require us to switch to maximum likelihood and test fixed effects without denominator degrees of freedom. For Model 4, we will also invoke “robust” standard errors (that correct for deviations from multivariate non-normality).

SAS Syntax to prepare wide-format data file in .csv format for Mplus:

```
* Export original wide format to Mplus;
DATA work.ForMplus; SET filesave.PSQF6270_Example5aWide;
  * Fixing any missing values;
  ARRAY vars(8) FamilyID KidBoy KidEd12 MomEd12 DadEd12 KidMarital MomMarital DadMarital;
  DO i=1 TO 8; IF vars(i)=. THEN vars(i)=-999; END; DROP i; RUN;
PROC EXPORT DATA=work.ForMplus OUTFILE= "&filesave.\PSQF6270_Example5aWide.csv"
  DBMS=CSV REPLACE; PUTNAMES=NO; RUN;
```

STATA Syntax to prepare wide-format data file in .csv format for Mplus:

```
// Import Example 5a wide STATA data
use "$filesave\PSQF6270_Example5aWide.dta", clear

// Example of how to export a .csv file for use in Mplus
// Replace all missing values with -999 for Mplus
mvencode _all, mv(-999)
// export delimited below: using lists the path and name of the new .csv file
// replace means it will be replaced if a file already exists with that name
// delimiter indicates a comma-delimited file
// nolabel will save actual data (numbers) instead of any value labels included
// novarnames tells it not to write the names to the top of the .csv file
export delimited using "$filesave\PSQF6270_Example5aWide_STATA.csv", ///
  delimiter(",") replace nolabel novarnames
```

R Syntax to prepare wide-format data file in .csv format for Mplus:

```
# Example of how to export a .csv file for use in Mplus
# Copy data, replace all missing values with -999 for Mplus
Example5a_Mplus = Example5a wide
Example5a_Mplus[is.na(Example5a_Mplus)] <- -999
# Write to .csv file without column names
write.table(x=Example5a_Mplus, col.names=FALSE, row.names=FALSE, sep="," ,
  file=paste0(filesave,"PSQF6270_Example5aWide_R.csv"))
```

Model 3: DV-Specific Intercept Version adding Dad Education as Predictor of Kid and Mom Attitudes

$$\widehat{Marital}_{fi} = \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{11}(Kid_{fi})(KidBoy_f) \\ + \beta_{20}(Dad_{fi})(DadEd_f - 12) + \beta_{31}(Kid_{fi})(KidEd_f - 12) + \beta_{42}(Mom_{fi})(MomEd_f - 12) \\ + \beta_{21}(Kid_{fi})(DadEd_f - 12) + \beta_{22}(Mom_{fi})(DadEd_f - 12)$$

STATA Syntax and Output for Previous Model 3 as a Path Model (estimated with ML; regular SEs):

```
// Import Example 5a wide STATA data
use "$filesave\PSQF6270_Example5aWide.dta", clear

* /// means continue the command + comment
* // means comment only
```

See STATA code updated online for how to get H1 saturated model LL and # of estimated parameters:
 display "LL for H1 Model= " e(critvalue_s)
 display "# of parameters= " e(df_m)

```
display "STATA Model 3: Own Education + Dad Education a Predictor of Each Attitude"
display "Using SEM to create path analysis model estimated with ML on wide-format data"
sem
  (kidmarit mommarit dadmarit <- _cons)      /// All intercepts estimated (by default)
  (kidmarit <- kidboy kided12)              /// Regressions: y outcomes ON x predictors
  (mommarit <- momed12)                    ///
  (kidmarit mommarit dadmarit <- daded12),  ///
  var(e.kidmarit e.mommarit e.dadmarit)    /// All residual variances estimated (by default)
```



```

. sem, coeflegend // Print parameter labels, too (to use in lincom)
-----
|          Coef.  Legend
-----+-----
Structural
kidmarital <-
  kidboy | .2638938  _b[kidmarital:kidboy]
  kided12 | -.002641  _b[kidmarital:kided12]
  daded12 | -.0004795  _b[kidmarital:daded12]
  _cons  | 1.512271  _b[kidmarital:_cons]
-----
mommarital <-
  momed12 | -.0162593  _b[mommarital:momed12]
  daded12 | -.0016793  _b[mommarital:daded12]
  _cons  | 1.937305  _b[mommarital:_cons]
-----
dadmarital <-
  daded12 | -.0548368  _b[dadmarital:daded12]
  _cons  | 2.070718  _b[dadmarital:_cons]
-----
var(e.kidmarital) | .3091381  _b[var(e.kidmarital):_cons]
var(e.mommarital) | .3161529  _b[var(e.mommarital):_cons]
var(e.dadmarital) | .2938981  _b[var(e.dadmarital):_cons]
-----
cov(e.kidmarital,e.mommarital) | .0380059  _b[cov(e.kidmarital,e.mommarital):_cons]
cov(e.kidmarital,e.dadmarital) | .0761007  _b[cov(e.kidmarital,e.dadmarital):_cons]
cov(e.mommarital,e.dadmarital) | .0839167  _b[cov(e.mommarital,e.dadmarital):_cons]
-----
LR test of model vs. saturated: chi2(6) = 10.93, Prob > chi2 = 0.0906
. sem, standardized // Print fully standardized solution, too

```

This table from sem, coeflegend provides the parameter names for the LINCOS statements above.

```

Standardized Solution:
All variables M=0, SD=1
-----
|          Coef.      OIM          z  P>|z|  [95% Conf. Interval]
-----+-----
Structural
kidmarital <-
  kidboy | .2306503  .0770785  2.99  0.003  .0795792  .3817214
  kided12 | -.0090898  .0834042  -0.11  0.913  -.1725591  .1543794
  daded12 | -.0023406  .0861915  -0.03  0.978  -.1712728  .1665916
  _cons  | 2.645964  .2466043  10.73  0.000  2.162628  3.129299
-----
mommarital <-
  momed12 | -.0782303  .1019932  -0.77  0.443  -.2781333  .1216728
  daded12 | -.0083038  .1023122  -0.08  0.935  -.208832  .1922244
  _cons  | 3.433518  .2280257  15.06  0.000  2.986595  3.88044
-----
dadmarital <-
  daded12 | -.2716069  .0768234  -3.54  0.000  -.422178  -.1210357
  _cons  | 3.676054  .2200636  16.70  0.000  3.244738  4.107371
-----
var(e.kidmarital) | .9463698  .0355442  .8792068  1.018663
var(e.mommarital) | .9930701  .0139521  .9660976  1.020796
var(e.dadmarital) | .9262297  .0417315  .8479448  1.011742

```

These standardized <- paths are standardized regression coefficients.

```

-----
cov(e.kidmarital,e.mommarital) | .12157  .0835428  1.46  0.146  -.0421709  .2853109
cov(e.kidmarital,e.dadmarital) | .2524724  .0792209  3.19  0.001  .0972022  .4077426
cov(e.mommarital,e.dadmarital) | .2752969  .0801933  3.43  0.001  .1181209  .432473
-----

```

These standardized covariances are residual correlations (in RCORR).

```

. estat gof, stats(all) // Print fit statistics
-----
Fit statistic | Value  Description (from STATA!)  Notes from Lesa:
-----+-----
Likelihood ratio
  chi2_ms(6) | 10.929  model vs. saturated  This is -2*LL for our H0-H1
  p > chi2 | 0.091  Test of exact fit: NS is good!
  chi2_bs(15) | 52.998  baseline vs. saturated  This is -2*LL for H0-H1 if
  p > chi2 | 0.000  H0 had no paths at all
-----
Population error
  RMSEA | 0.077  Root mean squared error of approximation  Should be < .08 or so
  90% CI, lower bound | 0.000
  upper bound | 0.148
  pclose | 0.229  Probability RMSEA <= 0.05  Test of exact fit: NS is good!
-----

```

Information criteria					
AIC	2778.964	Akaike's information criterion		Does not match Mplus	
BIC	2823.089	Bayesian information criterion		Does not match Mplus	
Baseline comparison					
CFI	0.870	Comparative fit index		Should be > .9 or so	
TLI	0.676	Tucker-Lewis index		Should be > .9 or so	
Size of residuals					
SRMR	0.039	Standardized root mean squared residual		Should be < .05 or so	
CD	0.132	Coefficient of determination		Like an overall R2 across DVs	

```
. estat eqgof // Print R2 per variable
Equation-level goodness of fit
```

depvars	Variance		residual	R-squared	mc	mc2
	fitted	predicted				
observed						
kidmarital	.3266568	.0175187	.3091381	.0536302	.231582	.0536302
mommarital	.3183591	.0022062	.3161529	.0069299	.0832462	.0069299
dadmarital	.3173058	.0234077	.2938981	.0737703	.2716069	.0737703
overall				.1323532		

mc = correlation between depvar and its prediction
 mc2 = mc^2 is the Bentler-Raykov squared multiple correlation coefficient

```
. estat residuals // Print how far off each predicted covariance is
Residuals of observed variables
```

Mean residuals	kidmari~1	mommari~1	dadmari~1	kidboy	kided12	momed12	daded12
raw	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Above: the means are recovered perfectly because each outcome has its own intercept (and predictor means are not part of the model). Below: the bolded covariances indicate the biggest sources of misfit—it looks like momed12 needs to predict each outcome!

Covariance residuals	kidmari~1	mommari~1	dadmari~1	kidboy	kided12	momed12	daded12
kidmarital	0.002						
mommarital	0.003	0.002					
dadmarital	0.004	0.005	0.000				
kidboy	0.004	0.015	0.014	0.000			
kided12	-0.001	0.016	-0.008	0.000	0.000		
momed12	0.068	-0.072	-0.280	0.000	0.000	0.000	
daded12	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000

```
. estat mindices, minchi2(3.84) showpclass(all) // Print voodoo to improve model fit at p<.05
Modification indices
```

	MI	df	P>MI	EPC	Standard EPC	
Structural						
dadmarital <-						
mommarital	9.061	1	0.00	3.68633	3.692443	This is already in the model as a cov
momed12	9.061	1	0.00	-.0599371	-.2888608	This is MomEd → DadMarit

EPC = expected parameter change

R Syntax for Previous Model 3 as a Path Model (estimated with ML; regular SEs):

```
print("R Model 3: Own Education + Dad Education a Predictor of Each Attitude")
# Create model syntax as separate text object
Syntax3 = "
# Residual variances estimated separately (by default)
KidMarital ~~ KidMarital; MomMarital ~~ MomMarital; DadMarital ~~ DadMarital
# All possible pairwise residual covariances (not estimated by default)
KidMarital ~~ MomMarital + DadMarital; MomMarital ~~ DadMarital
# All intercepts estimated separately (by default)
```

```

KidMarital ~ 1; MomMarital ~ 1; DadMarital ~ 1
# Regressions: y outcomes ON x predictors (label to do math on later)
KidMarital ~ KidBoy + KidEd12
MomMarital ~ MomEd12
KidMarital ~ (DadEd2K)*DadEd12
MomMarital ~ (DadEd2M)*DadEd12
DadMarital ~ (DadEd2D)*DadEd12
# Getting differences in effect of DadEd for each person
KvMDadEd := DadEd2M - DadEd2K; # Kid v. Mom: Dad Educ Effect Diff
KvDDadEd := DadEd2D - DadEd2K; # Kid v. Dad: Dad Educ Effect Diff
MvDDadEd := DadEd2D - DadEd2M; # Mom v. Dad: Dad Educ Effect Diff
"
print("lavaan path analysis model estimated with ML on wide-format data")
PathModel3 = lavaan(data=Example5a_wide, model=Syntax3, estimator="ML", mimic="mplus")
summary(PathModel3, fit.measures=TRUE, rsquare=TRUE, standardized=TRUE, ci=TRUE)
print("Request sorted modification indices for p<.05 to troubleshoot local misfit")
modindices(object=PathModel3, sort=TRUE, minimum.value=3.84)
print("Request residual covariance matrix =leftover from observed minus predicted")
resid(object=PathModel3, type="raw") # also type="cor" for correlation matrix

```

Mplus Syntax and Output for Previous Model 3 as a Path Model (estimated with ML; regular SEs):

```

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
TITLE: Example 5a Model 3: Own Education + Dad Education a Predictor of Each Attitude;

DATA: FILE = PSQF6270_Example5aWide.csv; ! Can just list file name if in same folder;
      FORMAT = free; ! FREE (default) or FIXED format;
      TYPE = individual; ! Individual (default) or matrix data as input;

VARIABLE:
! List of ALL variables in original wide data file, in order;
! Mplus names must use 8 characters or fewer (so rename as needed);
NAMES = FamilyID KidBoy KidEd12 MomEd12 DadEd12 KidMarit MomMarit DadMarit;
! List of ALL variables used in model;
USEVARIABLES = KidBoy KidEd12 MomEd12 DadEd12 KidMarit MomMarit DadMarit;
! Missing data codes (here, -999);
MISSING = ALL (-999);

ANALYSIS: TYPE = GENERAL; ! Used for path models;
          ESTIMATOR = ML; ! Full-information maximum likelihood;

OUTPUT: CINTERVAL; ! Print confidence intervals;
        STDYX; ! Print fully standardized solution, too;
        RESIDUAL; ! Print how far off each predicted covariance is;
        MODINDICES (3.84); ! Print voodoo to improve our model fit at p<.05;

MODEL: ! * --> Estimated parameter (all listed below for clarity);

! All residual variances estimated separately (by default);
KidMarit* MomMarit* DadMarit*;

! All possible pairwise residual covariances (not estimated by default);
KidMarit MomMarit DadMarit WITH KidMarit* MomMarit* DadMarit*;

! All intercepts estimated separately (by default);
[KidMarit* MomMarit* DadMarit*];

! Regressions: y outcomes ON x predictors (label to do math on later);
KidMarit ON KidBoy* KidEd12*;
MomMarit ON MomEd12*;
KidMarit MomMarit DadMarit ON DadEd12* (DadEd2K DadEd2M DadEd2D);

! Getting differences in effect of DadEd for each person;
MODEL CONSTRAINT:
NEW (KvMDadEd KvDDadEd MvDDadEd); ! List names of linear combinations here;
KvMDadEd = DadEd2M - DadEd2K; ! Kid v. Mom: Dad Educ Effect Diff;
KvDDadEd = DadEd2D - DadEd2K; ! Kid v. Dad: Dad Educ Effect Diff;
MvDDadEd = DadEd2D - DadEd2M; ! Mom v. Dad: Dad Educ Effect Diff;

```

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters	15	Notes from Lesa:
Loglikelihood		
H0 Value	-337.106	For OUR model: Larger is better
H1 Value	-331.641	For model with all possible paths estimated
Information Criteria		
Akaike (AIC)	704.211	For our model: Smaller is better
Bayesian (BIC)	748.336	
Sample-Size Adjusted BIC	700.878	
(n* = (n + 2) / 24)		
Chi-Square Test of Model Fit		
Value	10.929	This is -2ΔLL for our H0-H1
Degrees of Freedom	6	This is counting the covariances between X's and Y's too
P-Value	0.0906	Test of exact fit: Nonsignificant is good!
RMSEA (Root Mean Square Error Of Approximation)		
Estimate	0.077	Should be < .08 or so
90 Percent C.I.	0.000 0.148	
Probability RMSEA <= .05	0.229	Test of close fit: Nonsignificant is good!
CFI/TLI		
CFI	0.870	Should be > .9 or so
TLI	0.676	Should be > .9 or so
Chi-Square Test of Model Fit for the Baseline Model		
Value	52.998	This is -2ΔLL for H0-H1 if H0 had no paths at all
Degrees of Freedom	15	
P-Value	0.0000	
SRMR (Standardized Root Mean Square Residual)		
Value	0.046	Should be < .05 or so

MODEL RESULTS (UNSTANDARDIEZD SOLUTION; Mplus reorders them to list paths first)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	IN MIXED
KIDMARIT ON					
KIDBOY	0.264	0.091	2.886	0.004	B11
KIDED12	-0.003	0.024	-0.109	0.913	B31
DADED12	0.000	0.018	-0.027	0.978	B21
MOMMARIT ON					
MOMED12	-0.016	0.021	-0.767	0.443	B42
DADED12	-0.002	0.021	-0.081	0.935	B22
DADMARIT ON					
DADED12	-0.055	0.016	-3.339	0.001	B20
KIDMARIT WITH					
MOMMARIT	0.038	0.027	1.424	0.154	UN(2,1)
DADMARIT	0.076	0.026	2.893	0.004	UN(3,1)
MOMMARIT WITH					
DADMARIT	0.084	0.027	3.066	0.002	UN(3,2)
Intercepts					
KIDMARIT	1.512	0.099	15.290	0.000	B01
MOMMARIT	1.937	0.063	30.949	0.000	B02
DADMARIT	2.071	0.057	36.154	0.000	B00
Residual Variances					
KIDMARIT	0.309	0.037	8.365	0.000	UN(1,1)
MOMMARIT	0.316	0.038	8.339	0.000	UN(2,2)
DADMARIT	0.294	0.035	8.367	0.000	UN(3,3)
New/Additional Parameters (FROM MODEL CONSTRAINT, like ESTIMATE or LINCOM)					
KVMDADED	-0.001	0.026	-0.046	0.963	B22 - B21
KVDDADED	-0.054	0.021	-2.586	0.010	B20 - B21
MVDDADED	-0.053	0.023	-2.279	0.023	B20 - B22

These unstandardized ON paths are the fixed slopes from MIXED.

These unstandardized WITH covariances are residual covariances (in R).

Note that because we are using ML, the residual variances are smaller than in MIXED (that used REML instead to avoid this downward bias).

STANDARDIZED MODEL RESULTS - ALL VARIABLES HAVE MEAN=0, SD=1
STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
KIDMARIT ON				
KIDBOY	0.231	0.078	2.950	0.003
KIDED12	-0.009	0.083	-0.109	0.913
DADED12	-0.002	0.086	-0.027	0.978
MOMMARIT ON				
MOMED12	-0.078	0.102	-0.766	0.444
DADED12	-0.008	0.102	-0.081	0.935
DADMARIT ON				
DADED12	-0.272	0.078	-3.470	0.001
KIDMARIT WITH				
MOMMARIT	0.122	0.084	1.455	0.146
DADMARIT	0.252	0.079	3.187	0.001
MOMMARIT WITH				
DADMARIT	0.275	0.080	3.433	0.001

These standardized ON paths are standardized regression coefficients.

These standardized WITH covariances are residual correlations (in RCORR).

Intercepts				
KIDMARIT	2.646	0.247	10.723	0.000
MOMMARIT	3.434	0.228	15.057	0.000
DADMARIT	3.676	0.221	16.659	0.000
Residual Variances				
KIDMARIT	0.946	0.036	26.246	0.000
MOMMARIT	0.993	0.014	71.055	0.000
DADMARIT	0.926	0.043	21.782	0.000
R-SQUARE				
Observed				Two-Tailed
Variable	Estimate	S.E.	Est./S.E.	P-Value
KIDMARIT	0.054	0.036	1.487	0.137
MOMMARIT	0.007	0.014	0.496	0.620
DADMARIT	0.074	0.043	1.735	0.083

ESTIMATED MODEL AND RESIDUALS (OBSERVED - ESTIMATED)

The means are recovered perfectly because each outcome has its own intercept (and predictor means are not part of the model).

Residuals for Means						
KIDMARIT	MOMMARIT	DADMARIT	KIDBOY	KIDED12	MOMED12	DADED12
0.000	0.000	0.000	0.000	0.000	0.000	0.000

Residuals for Covariances							
	KIDMARIT	MOMMARIT	DADMARIT	KIDBOY	KIDED12	MOMED12	DADED12
KIDMARIT	0.002						
MOMMARIT	0.003	0.002					
DADMARIT	0.004	0.005	0.000				
KIDBOY	0.004	0.015	0.014	0.000			
KIDED12	-0.001	0.016	-0.008	0.000	0.000		
MOMED12	0.068	-0.072	-0.280	0.000	0.000	0.000	
DADED12	0.000	0.000	0.000	0.000	0.000	0.000	0.000

After shutting off the MODEL CONSTRAINT code and running it again, we get these “helpful” suggestions for how to improve model fit:

Minimum M.I. value for printing the modification index 3.840
M.I. E.P.C. Std E.P.C. StdYX E.P.C.

The bolded covariances indicate the biggest sources of misfit—it looks like momed12 needs to predict each outcome!

ON Statements						
DADMARIT ON MOMMARIT	9.062	3.687	3.687	3.693	This is already in the model as a cov	
DADMARIT ON MOMED12	9.061	-0.060	-0.060	-0.289	This is MomEd → DadMarit	
WITH Statements						
MOMED12 WITH DADMARIT	9.336	-0.294	-0.294	-0.200	This is MomEd ↔ DadMarit	
DADED12 WITH DADMARIT	8.134	0.491	0.491	0.324	This is already in the model as a path	

Model 4 in Univariate Software: DV-Specific Intercepts adding Mom Education as Predictor of Each Attitude uses long-format data, ML estimation, and robust standard errors to adjust for multivariate non-normality

$$\begin{aligned}
 \widehat{Marital}_{fi} = & \beta_{00}(Dad_{fi}) + \beta_{01}(Kid_{fi}) + \beta_{02}(Mom_{fi}) + \beta_{10}(Dad_{fi})(KidBoy_f) \\
 & + \beta_{20}(Dad_{fi})(DadEd_f - 12) + \beta_{31}(Kid_{fi})(KidEd_f - 12) + \beta_{42}(Mom_{fi})(MomEd_f - 12) \\
 & + \beta_{21}(Kid_{fi})(DadEd_f - 12) + \beta_{22}(Mom_{fi})(DadEd_f - 12) \\
 & + \beta_{41}(Kid_{fi})(MomEd_f - 12) + \beta_{40}(Dad_{fi})(MomEd_f - 12)
 \end{aligned}$$

```

TITLE1 "SAS Model 4: DV-Specific Intercepts -- Add Mom Educ (Controlling for Own+Dad Educ)";
TITLE2 "To match path model, switch to ML estimation, robust SEs via EMPIRICAL";
TITLE3 "Satterthwaite DF not allowed with EMPIRICAL, so switch to residual (N-k)";
PROC MIXED DATA=work.Example5a NOCLPRINT COVTEST NAMELEN=100 IC METHOD=ML EMPIRICAL;
  CLASS FamilyID DV; * CLASS is for ID variables;
  MODEL marital = kid mom dad kid*KidBoy kid*KidEd12 mom*MomEd12 dad*DadEd12
    kid*DadEd12 mom*DadEd12 kid*MomEd12 dad*MomEd12
    / NOINT SOLUTION DDFM= RESIDUAL /* OUTPM saves y-hat from fixed effects */
    OUTPM=work.PredFinal RESIDUAL; * RESIDUAL requests plots of residuals;
  REPEATED DV / R RCORR TYPE=UN SUBJECT=FamilyID;
  ESTIMATE "Kid vs. Mom: DadEd12 Slope Diff" kid*DadEd12 -1 mom*DadEd12 1;
  ESTIMATE "Kid vs. Dad: DadEd12 Slope Diff" kid*DadEd12 -1 dad*DadEd12 1;
  ESTIMATE "Mom vs. Dad: DadEd12 Slope Diff" mom*DadEd12 -1 dad*DadEd12 1;
  
```

```

ESTIMATE "Kid vs. Mom: MomEd12 Slope Diff" kid*MomEd12 -1 mom*MomEd12 1;
ESTIMATE "Kid vs. Dad: MomEd12 Slope Diff" kid*MomEd12 -1 dad*MomEd12 1;
ESTIMATE "Mom vs. Dad: MomEd12 Slope Diff" mom*MomEd12 -1 dad*MomEd12 1;
RUN; TITLE1; TITLE2;
* Save corr of pred and actual marital attitudes to square as R2;
TITLE "Correlation of Predicted and Actual Marital Attitudes by DV";
PROC SORT DATA=work.PredFinal; BY DV FamilyID; RUN;
PROC CORR NOSIMPLE DATA=work.PredFinal OUT=work.Rpred;
    BY DV; VAR marital; WITH pred; RUN;
* Compute R2 in saved output;
DATA work.Rpred; SET work.Rpred;
    WHERE _TYPE_="CORR"; R2=marital*marital; RUN;
TITLE "R2 of Predicted and Actual Marital Attitudes by DV";
PROC PRINT NOOBS DATA=work.Rpred; ID DV; VAR R2; RUN; TITLE;

display "STATA Model 4: DV-Specific Intercepts -- Add Mom Educ (Controlling for Own+Dad Educ)"
display "To match path model in Part 2, switch to ML estimation, robust SEs"
display "Satterthwaite DF not allowed with EMPIRICAL, so switch to residual (N-k)"
mixed marital c.kid c.mom c.dad c.kid#c.kidboy c.kid#c.kided12 c.mom#c.momed12 ///
    c.dad#c.daded12 c.kid#c.daded12 c.mom#c.daded12 ///
    c.kid#c.momed12 c.dad#c.momed12, noconstant ///
    || familyid: , noconstant /// This NOCONSTANT removes family random intercept
    variance mle residuals(unstructured,t(DV)) /// Unstructured R matrix by DV
    vce(robust) // Use robust SEs, so no denominator DF allowed
display "-2LL=" e(l1)*-2 // Print -2LL for model
estat wcorrelation, covariance // R matrix
estat wcorrelation // RCORR matrix
predict pred, xb // Add column pred of predicted outcomes to data
lincom c.kid#c.daded12*-1 + c.mom#c.daded12*1, small // Kid vs. Mom: DadEd12 Slope Diff
lincom c.kid#c.daded12*-1 + c.dad#c.daded12*1, small // Kid vs. Dad: DadEd12 Slope Diff
lincom c.mom#c.daded12*-1 + c.dad#c.daded12*1, small // Mom vs. Dad: DadEd12 Slope Diff
lincom c.kid#c.momed12*-1 + c.mom#c.momed12*1, small // Kid vs. Mom: MomEd12 Slope Diff
lincom c.kid#c.momed12*-1 + c.dad#c.momed12*1, small // Kid vs. Dad: MomEd12 Slope Diff
lincom c.mom#c.momed12*-1 + c.dad#c.momed12*1, small // Mom vs. Dad: MomEd12 Slope Diff
// Get correlation of actual and predicted outcomes to form R2
pwcrr marital pred if DV==1, sig
display "DV=1 Kid R2= " r(rho)^2 // Print R2 relative to empty model
pwcrr marital pred if DV==2, sig
display "DV=2 Mom R2= " r(rho)^2 // Print R2 relative to empty model
pwcrr marital pred if DV==3, sig
display "DV=3 Dad R2= " r(rho)^2 // Print R2 relative to empty model

print("R Model 4: DV-Specific Intercepts -- Add Mom Educ (Controlling for Own+Dad Educ)")
print("To match path model, switch to ML estimation, but robust SEs not directly available")
Model4 = gls(data=Example5a, method="ML",
    model=marital~0+kid+mom+dad+ kid:KidBoy +kid:KidEd12+mom:MomEd12+dad:DadEd12
    +kid:DadEd12+mom:DadEd12 +kid:MomEd12+dad:MomEd12,
    correlation=corSymm(form=~DVnum|FamilyID), # Unstructured correlations
    weights=varIdent(form=~1|DV)) # Separate variance by DV
print("Print -2LL and Results") # Btw, AIC and BIC are incorrect (match STATA)
-2*logLik(Model4); summary(Model4)
print("Show R and RCORR matrices for first family in the data")
getVarCov(Model4, individual="3996");
corMatrix(Model4$modelStruct$corStruct)[[3]] # 3=Dimensions of R here

print("DadEd Slope Diff -- Had to give it correct Denominator DF")
summary(glht(model=Model4, df=135, linfct=rbind(
    "Kid vs. Mom: DadEd12 Slope Diff" = c(0,0,0,0,0,0,0,-1,1,0,0), # in order of fixed effects
    "Kid vs. Dad: DadEd12 Slope Diff" = c(0,0,0,0,0,0,1,-1,0,0,0),
    "Mom vs. Dad: DadEd12 Slope Diff" = c(0,0,0,0,0,0,1,0,-1,0,0),
    "Kid vs. Mom: MomEd12 Slope Diff" = c(0,0,0,0,0,1,0,0,0,-1,0),
    "Kid vs. Dad: MomEd12 Slope Diff" = c(0,0,0,0,0,0,0,0,-1,1),
    "Mom vs. Dad: MomEd12 Slope Diff" = c(0,0,0,0,0,-1,0,0,0,0,1))), test=adjusted("none"))

print("Save predicted marital attitudes and correlate with actual marital attitudes")
Example5a$Pred = predict(Model4, type="response")

```



```

KidMarit MomMarit DadMarit ON MomEd12* (MomEd2K MomEd2M MomEd2D); ! New effects here;

! Getting differences in effect of DadEd for each person;
MODEL CONSTRAINT: ! List names of linear combinations here;
NEW (KvMDadEd KvDDadEd MvDDadEd KvMMomEd KvDMomEd MvDMomEd);
KvMDadEd = DadEd2M - DadEd2K;      ! Kid v. Mom: Dad Educ Effect Diff;
KvDDadEd = DadEd2D - DadEd2K;      ! Kid v. Dad: Dad Educ Effect Diff;
MvDDadEd = DadEd2D - DadEd2M;      ! Mom v. Dad: Dad Educ Effect Diff;
KvMMomEd = MomEd2M - MomEd2K;      ! Kid v. Mom: Mom Educ Effect Diff;
KvDMomEd = MomEd2D - MomEd2K;      ! Kid v. Dad: Mom Educ Effect Diff;
MvDMomEd = MomEd2D - MomEd2M;      ! Mom v. Dad: Mom Educ Effect Diff;

print("R Model 4: Own + Dad + Mom Education a Predictor of Each Attitude")
# Create model syntax as separate text object
Syntax4 = "
# Residual variances estimated separately (by default)
KidMarital ~ KidMarital; MomMarital ~ MomMarital; DadMarital ~ DadMarital
# All possible pairwise residual covariances (not estimated by default)
KidMarital ~ MomMarital + DadMarital; MomMarital ~ DadMarital
# All intercepts estimated separately (by default)
KidMarital ~ 1; MomMarital ~ 1; DadMarital ~ 1
# Regressions: y outcomes ON x predictors (label to do math on later)
KidMarital ~ KidBoy + KidEd12
KidMarital ~ (DadEd2K)*DadEd12
MomMarital ~ (DadEd2M)*DadEd12
DadMarital ~ (DadEd2D)*DadEd12
# New effects here
KidMarital ~ (MomEd2K)*MomEd12
MomMarital ~ (MomEd2M)*MomEd12
DadMarital ~ (MomEd2D)*MomEd12

# Getting differences in effect of DadEd for each person
KvMDadEd := DadEd2M - DadEd2K; # Kid v. Mom: Dad Educ Effect Diff
KvDDadEd := DadEd2D - DadEd2K; # Kid v. Dad: Dad Educ Effect Diff
MvDDadEd := DadEd2D - DadEd2M; # Mom v. Dad: Dad Educ Effect Diff
KvMMomEd := MomEd2M - MomEd2K; # Kid v. Mom: Mom Educ Effect Diff
KvDMomEd := MomEd2D - MomEd2K; # Kid v. Dad: Mom Educ Effect Diff
MvDMomEd := MomEd2D - MomEd2M; # Mom v. Dad: Mom Educ Effect Diff
"

print("lavaan path analysis model estimated with ML on wide-format data")
PathModel4 = lavaan(data=Example5a_wide, model=Syntax4, estimator="ML", mimic="mplus")
summary(PathModel4, fit.measures=TRUE, rsquare=TRUE, standardized=TRUE, ci=TRUE)
print("Request sorted modification indices for p<.05 to troubleshoot local misfit")
modindices(object=PathModel4, sort=TRUE, minimum.value=3.84)
print("Request residual correlation matrix =leftover from observed minus predicted")
resid(object=PathModel4, type="raw") # also type="cor" for correlation matrix

```

R LAVAAN Output—shows both regular ML and “robust” ML fit statistics:

Estimator	ML		
Optimization method	NLMINB		
Number of model parameters	17		
Number of observations	140		
Number of missing patterns	1		
Model Test User Model:			
	Standard	Robust	
Test Statistic	1.034	1.026	This is $-2\Delta LL$ for our H_0-H_1
Degrees of freedom	4	4	
P-value (Chi-square)	0.905	0.906	
Scaling correction factor		1.007	
Yuan-Bentler correction (Mplus variant)			
Model Test Baseline Model:			
Test statistic	52.998	52.902	
Degrees of freedom	15	15	
P-value	0.000	0.000	
Scaling correction factor		1.002	

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000	1.000	Want close to 1
Tucker-Lewis Index (TLI)	1.293	1.294	
Robust Comparative Fit Index (CFI)		1.000	
Robust Tucker-Lewis Index (TLI)		1.296	

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-332.158	-332.158	For our model: Larger is better
Scaling correction factor for the MLR correction		1.007	1=multivariate normality (so not bad!)
Loglikelihood unrestricted model (H1)	-331.641	-331.641	For model with all paths estimated
Scaling correction factor for the MLR correction		1.007	
Akaike (AIC)	698.316	698.316	For our model: Smaller is better
Bayesian (BIC)	748.324	748.324	For our model: Smaller is better
Sample-size adjusted Bayesian (BIC)	694.538	694.538	For our model: Smaller is better

Root Mean Square Error of Approximation:

RMSEA	0.000	0.000	Want close to 0
90 Percent confidence interval - lower	0.000	0.000	
90 Percent confidence interval - upper	0.052	0.051	
P-value RMSEA <= 0.05	0.947	0.948	Test of RMSEA <= .05
Robust RMSEA		0.000	
90 Percent confidence interval - lower		0.000	
90 Percent confidence interval - upper		0.052	

Standardized Root Mean Square Residual:

SRMR	0.016	0.016	Want close to 0
------	-------	-------	-----------------

Parameter estimates, their SEs, and standardized estimates would be Table 1

Regressions: -- THESE ARE THE FIXED SLOPES FROM MIXED

	Estimate	Std.Err	z-value	P(> z)	ci.lower	ci.upper	Std.lv	Std.all	STDYX IN MPLUS
KidMarital ~									
KidBoy	0.258	0.093	2.786	0.005	0.076	0.439	0.258	0.225	B11
KidEd12	-0.011	0.024	-0.441	0.659	-0.058	0.037	-0.011	-0.037	B31
DadEd12 (DE2K)	-0.007	0.020	-0.367	0.714	-0.046	0.032	-0.007	-0.035	B21
MomMarital ~									
DadEd12 (DE2M)	0.006	0.020	0.316	0.752	-0.033	0.046	0.006	0.031	B22
DadMarital ~									
DadEd12 (DE2D)	-0.024	0.017	-1.388	0.165	-0.057	0.010	-0.024	-0.117	B20
KidMarital ~									
MomEd12 (ME2K)	0.015	0.022	0.681	0.496	-0.028	0.059	0.015	0.072	B41
MomMarital ~									
MomEd12 (ME2M)	-0.031	0.022	-1.412	0.158	-0.073	0.012	-0.031	-0.148	B42
DadMarital ~									
MomEd12 (ME2D)	-0.056	0.019	-2.974	0.003	-0.094	-0.019	-0.056	-0.272	B40

Covariances: -- THESE ARE RESIDUAL COVARIANCES FROM R MATRIX OFF-DIAGONALS

	Estimate	Std.Err	z-value	P(> z)	ci.lower	ci.upper	Std.lv	Std.all	
.KidMarital ~~									
.MomMarital	0.039	0.028	1.400	0.162	-0.016	0.094	0.039	0.126	UN(1,2)
.DadMarital	0.080	0.024	3.296	0.001	0.033	0.128	0.080	0.274	UN(1,3)
.MomMarital ~~									
.DadMarital	0.080	0.020	4.011	0.000	0.041	0.119	0.080	0.270	UN(2,3)

Intercepts: -- THESE ARE THE FIXED INTERCEPTS FROM MIXED

	Estimate	Std.Err	z-value	P(> z)	ci.lower	ci.upper	Std.lv	Std.all	
.KidMarital	1.522	0.101	15.125	0.000	1.325	1.719	1.522	2.664	B01
.MomMarital	1.951	0.063	30.825	0.000	1.827	2.075	1.951	3.445	B02
.DadMarital	2.123	0.060	35.574	0.000	2.006	2.240	2.123	3.769	B00

Variances: -- THESE ARE THE RESIDUAL VARIANCES FROM R MATRIX DIAGONAL

	Estimate	Std.Err	z-value	P(> z)	ci.lower	ci.upper	Std.lv	Std.all	
.KidMarital	0.308	0.031	10.096	0.000	0.248	0.368	0.308	0.944	UN(1,1)
.MomMarital	0.315	0.045	7.081	0.000	0.228	0.402	0.315	0.983	UN(2,2)
.DadMarital	0.278	0.034	8.092	0.000	0.211	0.345	0.278	0.876	UN(3,3)

R-Square: -- THESE ARE CLOSE TO BUT NOT THE SAME AS WAS FOUND IN THE UNIVARIATE MODELS

	Estimate
KidMarital	0.056
MomMarital	0.017
DadMarital	0.124

Defined Parameters: -- THESE ARE ESTIMATE/LINCOM/GLHT/MODEL CONSTRAINT LINEAR COMBINATIONS

	Estimate	Std.Err	z-value	P(> z)	ci.lower	ci.upper	Std.lv	Std.all	
KvMDadEd	0.014	0.026	0.522	0.602	-0.038	0.065	0.014	0.067	B22 - B21
KvDDadEd	-0.016	0.022	-0.741	0.459	-0.059	0.027	-0.016	-0.081	B20 - B21
MvDDadEd	-0.030	0.025	-1.212	0.225	-0.078	0.018	-0.030	-0.148	B20 - B22
KvMMomEd	-0.046	0.029	-1.569	0.117	-0.103	0.011	-0.046	-0.219	B42 - B41
KvDMomEd	-0.072	0.027	-2.657	0.008	-0.124	-0.019	-0.072	-0.344	B40 - B41
MvDMomEd	-0.026	0.025	-1.041	0.298	-0.074	0.023	-0.026	-0.124	B40 - B42

\$cov - THESE ARE THE DISCREPANCIES FOR OBSERVED MINUS PREDICTED COVARIANCES

	KdMrtl	MmMrtl	DdMrtl	KidBoy	KdEd12	DdEd12	MmEd12
KidMarital	0.002						
MomMarital	0.004	0.000					
DadMarital	0.003	0.000	0.000				
KidBoy	0.005	0.015	0.015	0.000			
KidEd12	0.013	0.029	0.043	0.000	0.000		
DadEd12	0.000	0.000	0.000	0.000	0.000	0.000	
MomEd12	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Only the kid predictors on the mom and dad outcomes have leftover covariance, and no single added paths would help the model.

\$mean

	KidMarital	MomMarital	DadMarital	KidBoy	KidEd12	DadEd12	MomEd12
	0	0	0	0	0	0	0

Example results section for Part 2 Models 3–4 (picking up from Part 1; using R LAVAAN output):

Next, we examined whether father’s education incrementally predicted the marital attitudes of the mother or adult child after controlling for their own education, but neither effect was significant (and the effect of father’s education on his own attitudes was significantly larger). The effect of father’s education on his own attitudes remained significant, while the effect of education on their own attitudes for the adult child and mother remained nonsignificant).

Finally, we examined the incremental effects of mother’s education on marital attitudes, and results from this final model are shown in Table 1. For every additional year of mother’s education, father’s attitudes were expected to be significantly less conservative by 0.056 ($p = .003$). The effect of mother’s education on the adult child attitudes was nonsignificant and significantly smaller than its effect on father’s attitudes.

We re-estimated the final model as a path analysis in the R package lavaan (using robust maximum likelihood) in order to obtain indices of absolute model fit. The model had excellent fit, $\chi^2(4) = 1.026, p = .906, RMSEA = .00$ [CI = .00–.051], CFI = 1.00, indicating that no further paths were needed. This final model is depicted in Figure 1 below.

Figure 1 (line types used to help visually distinguish the paths; standardized coefficients may also be added)

