Example 2a: Predicting Binary Outcomes via SAS GLIMMIX, STATA LOGIT, and R GLM
(complete syntax, data, and output available for SAS, STATA, and R electronically))

The (fake) data for this example demonstrating "logistic regression" (i.e., using logit link function and Bernoulli conditional response distribution) came from: https://stats.idre.ucla.edu/sas/dae/ordinal-logistic-regression/. In this example we will predict a student's **binary decision** to apply to grad school (0=no, >0=pry) using undergraduate GPA (centered at 3.0), whether at least one of their parents has a graduate degree (0=no, 1=yes), and whether they attended a private university (0=no, 1=yes). I am using SAS GLIMMIX because it has more helpful options (even though these are not mixed-effects models). Further, because STATA LOGIT does not have denominator degrees of freedom, they were set to "none" in SAS GLIMMIX so that the SAS Wald test results (still labeled as t or F) will match those of STATA (using z or χ^2). I am using the R function GLM in all models for this example (also using z or χ^2).

SAS Syntax for Importing and Preparing Data for Analysis:

```
* Location for original files for these models - change this path;
* \\Client\ precedes path in Virtual Desktop outside H drive;
%LET filesave= C:\Dropbox\22_PSQF6270\PSQF6270_Example2ab;
LIBNAME filesave "&filesave.";

* Import data into work library, transform variables, apply labels;
DATA work.Example2; SET filesave.SAS_Example2;
* Create ID variable;
  PersonID=_N_;
* Rename 3-category outcome;
  apply3=apply;
* Create new binary outcome to demonstrate logistic regression;
  apply2=.; * New empty variable;
  IF apply3=0 THEN apply2=0;
  IF apply3>0 THEN apply2=1;
* Rename and center predictors;
  parD=pared; gpa3=gpa-3;
* Recode ref for public to create positive slope;
  priv=.;
  IF public=1 THEN priv=0;
  IF public=0 THEN priv=1;
* Label variables;
  LABEL apply3= "apply3: 0=Not, 1=Eh, 2=Very"
        apply2= "apply2: 0=No, 1=Pry"
        parD=   "parD: Parent Has Graduate Degree (0=N,1=Y)"
        priv=   "priv: Student Attends Private University (0=N,1=Y)"
        gpa3=   "gpa3: Student GPA (0=3)";
* Filter to only cases complete on all variables to be used below;
  IF NMISS(apply3, parD, priv, gpa3)>0 THEN DELETE;
RUN;

DATA work.FakePeople; * Create new blank dataset;
* INPUT: list variables in order of entry;
  INPUT PersonID gpa3 parD priv;
* Enter data -- each row is a fake person for which to create a predicted outcome;
  DATALINES;
-99 -1 0 0
-99  0 0 0
-99  1 0 0
-99 -1 0 1
-99  0 0 1
-99  1 0 1
-99 -1 1 0
-99  0 1 0
-99  1 1 0
-99 -1 1 1
-99  0 1 1
-99  1 1 1
```

```
; RUN;
* Add fake people to real data;
DATA work.Example2; SET work.FakePeople work.Example2; RUN;
```

STATA Syntax for Importing and Preparing Data for Analysis:

```
// Defining global variable for file location to be replaced in code below
// \\Client\ precedes path in Virtual Desktop outside H drive
global filesave "C:\Dropbox\22_PSQF6270\PSQF6270_Example2ab"

// Open Example 2 STATA dataset and clear away any existing data
use "$filesave\STATA_Example2.dta", clear // Has converted all variables to lower-case

// Create ID variable
gen PersonID = _n
// Rename 3-category outcome
gen apply3=apply

// Create new binary outcome to demonstrate logistic regression
gen apply2=.  // New empty variable
replace apply2=0 if apply==0
replace apply2=1 if apply>0

// Rename and center predictors
gen parD=pared
gen gpa3=gpa-3

// Recode ref for public to create positive slope
gen priv=.
replace priv=0 if public==1
replace priv=1 if public==0

// Label variables
label variable apply3 "apply3: 0=Not, 1=Eh, 2=Very"
label variable apply2 "apply2: 0=No, 1=Pry"
label variable parD   "parD: Parent Has Graduate Degree (0=N,1=Y)"
label variable priv   "priv: Student Attends Private University (0=N,1=Y)"
label variable gpa3   "gpa3: Student GPA (0=3)"

// Filter to only cases complete on all variables to be used below
egen nmiss=rowmiss(apply3 parD priv gpa3)
drop if nmiss>0
```

R Syntax for Importing and Preparing Data for Analysis:

```
# Define variables for working directory and data name
filesave = "C:\\Dropbox/22_PSQF6270/PSQF6270_Example2ab/"
filename = "SAS_Example2.sas7bdat"
setwd(dir=filesave)

# Import Example2 SAS data
Example2 = read_sas(data_file=paste0(filesave,filename))
# Convert to data frame without labels to use for analysis
Example2 = as.data.frame(Example2)

# Create ID variable
Example2$PersonID <- seq.int(nrow(Example2))

# Rename 3-category outcome
Example2$apply3=Example2$APPLY
```

```
# Create new binary outcome to demonstrate logistic regression
Example2$apply2 = NA  # New empty variable
Example2$apply2[which(Example2$APPLY==0)]=0
Example2$apply2[which(Example2$APPLY>0)]=1

# Rename and center predictors
Example2$parD=Example2$PARED
Example2$gpa3=Example2$GPA-3

# Recode ref for public to create positive slope
Example2$priv = NA  # New empty variable
Example2$priv[which(Example2$PUBLIC==1)]=0
Example2$priv[which(Example2$PUBLIC==0)]=1

# Label variables as comments only (not actually added to data)
#apply3= "apply3: 0=Not, 1=Eh, 2=Very"
#apply2= "apply2: 0=No, 1=Pry"
#parD=   "parD: Parent Has Graduate Degree (0=N,1=Y)"
#priv=   "priv: Student Attends Private University (0=N,1=Y)"
#gpa3=   "gpa3: Student GPA (0=3)"

# Filter to only cases complete on all variables to be used below
Example2 = Example2[complete.cases(Example2[ ,5:9]),]

# Create fake people for use in generating predicted outcomes
FakeGpa3 = c(-1,0,1,-1,0,1,-1,0,1,-1,0,1)
FakeParD = c( 0,0,0, 0,0,0, 1,1,1, 1,1,1)
FakePriv = c( 0,0,0, 1,1,1, 0,0,0, 1,1,1)
# Create dataset using just-created columns and constants for other model variables
FP = data.frame(gpa3=FakeGpa3, parD=FakeParD, priv=FakePriv)
```

Syntax and SAS Output for Descriptive Statistics:

```
TITLE1 "SAS Descriptive Statistics";
PROC MEANS NDEC=2 DATA=work.Example2;
     VAR gpa3 parD priv apply2;
RUN;

display "STATA Descriptive Statistics"
summarize gpa3 parD priv apply2

print("R Descriptive Statistics")
describe(x=Example2[ , c("gpa3","parD","priv","apply2")])
```

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
gpa3	gpa3: Student GPA (0=3)	400	-0.00	0.40	-1.10	1.00
parD	parD: Parent Has Graduate Degree (0=N,1=Y)	400	0.16	0.36	0.00	1.00
priv	priv: Student Attends Private University (0=N,1=Y)	400	0.86	0.35	0.00	1.00
apply2	apply2: 0=No, 1=Pry	400	**0.45**	0.50	0.00	1.00

So now we know that **55% of the respondents have apply2=0, and 45% have apply2=1**. This information will come in handy in making sure we understand which value our logistic regression models are predicting!

Empty Model Predicting Binary Apply 2:

$Logit(Apply2_i = 1) = \beta_0$ → $Probability(Apply2_i = 1) = \frac{exp(\beta_0)}{1+exp(\beta_0)}$

SAS Syntax and Output:

The GRADIENT option prints the partial first derivative with respect to each parameter, which is helpful in troubleshooting estimation problems.

```
TITLE1 "SAS Empty Model Predicting Binary Apply2 using MSPL=ML";
PROC GLIMMIX DATA=work.Example2 NOCLPRINT GRADIENT METHOD=MSPL;
* DESCENDING means predict prob=1 rather than prob=0;
  MODEL apply2 (DESCENDING) = / SOLUTION DDFM=NONE LINK=LOGIT DIST=BINARY;
* ILINK requests logit estimate to be transformed into probability;
  ESTIMATE "Intercept for y=1" int 1 / ILINK;
RUN; TITLE1;
```

The DESCENDING option makes the model predict the **probability of a 1** instead of the probability of a 0.

```
The GLIMMIX procedure is modeling the probability that apply2='1'.
```

Iteration History

Iteration	Restarts	Evaluations	Objective Function	Change	Max Gradient
0	0	4	275.34490191	.	4.200299
1	0	3	275.25552601	0.08937590	0.010221
2	0	3	**275.25552549**	0.00000053	5.275E-8

Convergence criterion (GCONV=1E-8) satisfied.

Hooray! The overall model converged! But we will still watch for crazy SEs and gradients ≠ 0 for problems with individual model parameters…

Fit Statistics

-2 Log Likelihood	**550.51** → **-2LL value for model** = 2*(-1)*275.256, last value above
AIC (smaller is better)	552.51
AICC (smaller is better)	552.52
BIC (smaller is better)	556.50
CAIC (smaller is better)	557.50
HQIC (smaller is better)	554.09
Pearson Chi-Square	400.00
Pearson Chi-Square / DF	1.00 → Indicates perfect distribution fit (always happens for binary)

$$\text{Probability of } (Apply2_i = 1) = \frac{exp(-0.2007)}{[1 + exp(-0.2007)]} = 0.450$$

Parameter Estimates

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient
Intercept	-0.2007	0.1005	Infty	-2.00	0.0465	5.275E-8

Estimates

Label	Estimate = predicted logit	Standard Error	DF	t Value	Pr > |t|	Mean = probability	Standard Error Mean
Intercept for y=1	-0.2007	0.1005	Infty	-2.00	0.0465	0.4500	0.02487 → **0.450 matches!**

STATA Syntax and Output:

To get STATA to compute −2LL for any model, add this code immediately after the procedure (e.g., logit here) that accesses the saved LL and multiplies by −2:

```
// Print -2LL for model
display "-2LL=" e(ll)*-2
```

```
display "STATA Empty Model Predicting Binary Apply2"
logit apply2
estat ic, n(400)  // AIC and BIC to match SAS
margins           // Intercept in probability
```

```
Logistic regression                             Number of obs     =        400
                                                LR chi2(0)        =       0.00
                                                Prob > chi2       =          .
Log likelihood = -275.25553                     Pseudo R2         =     0.0000

------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |  -.2006707   .1005038    -2.00   0.046    -.3976545   -.0036869 → In logits
------------------------------------------------------------------------------
```

STATA gives LL (so you need to *−2)

```
. estat ic, n(400)      // AIC and BIC to match SAS
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        400 -275.2555  -275.2555       1    552.5111   556.5025
-----------------------------------------------------------------------------
               Note: N=400 used in calculating BIC.

. margins               // Intercept in probability
Expression   : Pr(apply2), predict()
------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |        .45   .0248747    18.09   0.000     .4012465    .4987535 → In probability
------------------------------------------------------------------------------
```

R Syntax and Truncated Output:

```
print("R Empty Model Predicting Binary Apply2")
Model2Empty = glm(data=Example2, family=binomial, formula=apply2~1)
summary(Model2Empty); BIC(Model2Empty) # Get BIC too

print("Convert logits to probability to check interpretation")
Model2EmptyProb=1/(1+exp(-1*coefficients(Model2Empty))); Model2EmptyProb
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.0935  -1.0935  -1.0935   1.2637   1.2637

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.20067    0.10050 -1.9967  0.04586 → In logits

    Null deviance: 550.511  on 399  degrees of freedom → Is empty model -2LL
Residual deviance: 550.511  on 399  degrees of freedom → Is current model -2LL

AIC: 552.511

> BIC(Model2Empty)
[1] 556.50252

> Model2EmptyProb
      0.45 → In probability
```

Let's add three predictors, starting with main effects only...

$$Logit(Apply2_i = 1) = \beta_0 + \beta_1(GPA_i - 3) + \beta_2(ParentGD_i) + \beta_3(Private_i)$$

SAS Syntax and Output (condensed and re-arranged for convenience):

```
TITLE1 "SAS Main-Effects-Only Model Predicting Binary Apply2";
PROC GLIMMIX DATA=work.Example2 NOCLPRINT GRADIENT METHOD=MSPL;
MODEL apply2 (DESCENDING) = gpa3 parD priv
      / SOLUTION DDFM=NONE LINK=LOGIT DIST=BINARY ODDSRATIO(AT gpa3=0 LABEL);
CONTRAST "Multiv Wald Test of Model R2"  gpa3 1, parD 1, priv 1 / CHISQ;
* Get predicted outcomes (duplicates estimates below with less effort);
OUTPUT OUT=work.Pred2Main PRED(NOILINK)=Ylogit STDERR(NOILINK)=YlogitSE
                         PRED(ILINK)=Yprob    STDERR(ILINK)=YprobSE;
* Predicted outcomes in logits and ILINK=probability;
ESTIMATE "Yhat: Ndeg Pub GPA=2"  int 1 gpa3 -1 parD 0 priv 0 / ILINK;
ESTIMATE "Yhat: Ndeg Pub GPA=3"  int 1 gpa3  0 parD 0 priv 0 / ILINK;
ESTIMATE "Yhat: Ndeg Pub GPA=4"  int 1 gpa3  1 parD 0 priv 0 / ILINK;
ESTIMATE "Yhat: Ndeg Pri GPA=2"  int 1 gpa3 -1 parD 0 priv 1 / ILINK;
ESTIMATE "Yhat: Ndeg Pri GPA=3"  int 1 gpa3  0 parD 0 priv 1 / ILINK;
ESTIMATE "Yhat: Ndeg Pri GPA=4"  int 1 gpa3  1 parD 0 priv 1 / ILINK;
ESTIMATE "Yhat: Ydeg Pub GPA=2"  int 1 gpa3 -1 parD 1 priv 0 / ILINK;
ESTIMATE "Yhat: Ydeg Pub GPA=3"  int 1 gpa3  0 parD 1 priv 0 / ILINK;
ESTIMATE "Yhat: Ydeg Pub GPA=4"  int 1 gpa3  1 parD 1 priv 0 / ILINK;
ESTIMATE "Yhat: Ydeg Pri GPA=2"  int 1 gpa3 -1 parD 1 priv 1 / ILINK;
ESTIMATE "Yhat: Ydeg Pri GPA=3"  int 1 gpa3  0 parD 1 priv 1 / ILINK;
ESTIMATE "Yhat: Ydeg Pri GPA=4"  int 1 gpa3  1 parD 1 priv 1 / ILINK;
ESTIMATE "GPA Slope" gpa3 1 / ILINK; * Example of non-sense ILINK for a slope;
RUN;
TITLE1 "Predicted Logits and Probabilities for Fake People";
PROC PRINT NOOBS DATA=work.Pred2Main; WHERE PersonID=-99;
     VAR gpa3 parD priv Ylogit--YprobSE;
RUN; TITLE1;
```

Fit Statistics	
-2 Log Likelihood	**529.92**
AIC (smaller is better)	537.92
BIC (smaller is better)	553.89

Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
Multiv Wald Test of Model R2	3	Infty	**18.95**	6.32	0.0003	0.0003

Parameter Estimates

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient	
Intercept	-0.5388	0.2874	Infty	-1.87	0.0608	-7.76E-6	**Beta0**
gpa3	0.5482	0.2724	Infty	2.01	0.0442	-0.00002	**Beta1**
parD	1.0596	0.2974	Infty	3.56	0.0004	-0.00003	**Beta2**
priv	0.2006	0.3053	Infty	0.66	0.5113	-6.75E-6	**Beta3**

Interpret each fixed effect…

Intercept:

GPA3:

parentGD:

private:

Odds Ratio Estimates → from exp(logit)

Comparison	Estimate	DF	95% Confidence Limits		
unit change of gpa3 from gpa3=0	1.730	Infty	1.014	2.951	**exp(Beta1)**
unit change of parD from gpa3=0	2.885	Infty	1.611	5.168	**exp(Beta2)**
unit change of priv from gpa3=0	1.222	Infty	0.672	2.223	**exp(Beta3)**

Effects of continuous variables are assessed as units offsets from the reference value.

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Mean	Standard Error Mean	
Yhat: Ndeg Pub GPA=2	-1.0870	0.4312	Infty	-2.52	0.0117	0.2522	0.08132	
Yhat: Ndeg Pub GPA=3	-0.5388	0.2874	Infty	-1.87	0.0608	0.3685	0.06688	
Yhat: Ndeg Pub GPA=4	0.009454	0.3574	Infty	0.03	0.9789	0.5024	0.08934	
Yhat: Ndeg Pri GPA=2	-0.8865	0.2843	Infty	-3.12	0.0018	0.2918	0.05877	
Yhat: Ndeg Pri GPA=3	-0.3382	0.1187	Infty	-2.85	0.0044	0.4162	0.02885	
Yhat: Ndeg Pri GPA=4	0.2100	0.3095	Infty	0.68	0.4974	0.5523	0.07652	
Yhat: Ydeg Pub GPA=2	-0.02742	0.5123	Infty	-0.05	0.9573	0.4931	0.1280	
Yhat: Ydeg Pub GPA=3	0.5208	0.3714	Infty	1.40	0.1608	0.6273	0.08683	
Yhat: Ydeg Pub GPA=4	1.0691	0.4024	Infty	2.66	0.0079	0.7444	0.07656	
Yhat: Ydeg Pri GPA=2	0.1731	0.4078	Infty	0.42	0.6712	0.5432	0.1012	
Yhat: Ydeg Pri GPA=3	0.7214	0.2801	Infty	2.58	0.0100	0.6729	0.06164	
Yhat: Ydeg Pri GPA=4	1.2696	0.3728	Infty	3.41	0.0007	0.7807	0.06383	
GPA Slope	0.5482	0.2724	Infty	2.01	0.0442	**0.6337**	**0.06324**	**Nope!**

The last line illustrates why **you cannot "un-logit" a slope all the way back into probability**… the difference between the intercepts per unit GPA in logits is a constant 0.5482, but the corresponding difference in probability is not constant between GPA units. Similarly, the difference between the groups is constant in logits, but it is NOT constant in probability—it depends where you are on the probability scale.

Predicted Logits and Probabilities for Fake People

gpa3	parD	priv	Ylogit	Yprob	YlogitSE	YprobSE
-1	0	0	-1.08703	0.25218	0.43120	0.08132
0	0	0	-0.53879	0.36847	0.28742	0.06688
1	0	0	0.00945	0.50236	0.35738	0.08934
-1	0	1	-0.88648	0.29184	0.28435	0.05877
0	0	1	-0.33823	0.41624	0.11871	0.02885
1	0	1	0.21001	0.55231	0.30947	0.07652
-1	1	0	-0.02742	0.49314	0.51225	0.12804
0	1	0	0.52082	0.62734	0.37140	0.08683
1	1	0	1.06906	0.74442	0.40238	0.07656
-1	1	1	0.17313	0.54318	0.40785	0.10120
0	1	1	0.72138	0.67291	0.28007	0.06164
1	1	1	1.26962	0.78068	0.37281	0.06383

STATA Syntax and Output (condensed and re-arranged for convenience):

```
display "STATA Main-Effects-Only Model Predicting Binary Apply2"
logit apply2 c.gpa3 c.parD c.priv
estat ic, n(400)                        // AIC and BIC to match SAS
test (c.gpa3=0) (c.parD=0) (c.priv=0)  // Multiv Wald Test of Model R2
// For at, (from(by)to) for range of predictors
margins, at(c.gpa3=(-1(1)1) c.parD=(0(1)1) c.priv=(0(1)1)) predict(xb) // Yhat in logits
margins, at(c.gpa3=(-1(1)1) c.parD=(0(1)1) c.priv=(0(1)1))             // Yhat in probability

// Must re-estimate with 'or' added to first line to get odds ratios
display "STATA Main-Effects-Only Model Predicting Binary Apply2"
display "Get Odds Ratios Instead of Logit Fixed Effects"
logit apply2 c.gpa3 c.parD c.priv, or
```

```
Logistic regression                               Number of obs     =        400
                                                  LR chi2(3)        =      20.59 → From LRT
                                                  Prob > chi2       =     0.0001
Log likelihood =  -264.9624 * -2 = -2LL           Pseudo R2         =     0.0374
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        gpa3 |   .5482457   .2724341     2.01   0.044     .0142846    1.082207  Beta1
        parD |   1.059612   .2973854     3.56   0.000     .4767471    1.642476  Beta2
        priv |   .2005571   .3053354     0.66   0.511    -.3978894    .7990035  Beta3
       _cons |  -.5387909    .287416    -1.87   0.061    -1.102116    .0245341  Beta0
------------------------------------------------------------------------------

. estat ic, n(400)                        // AIC and BIC to match SAS
Akaike's information criterion and Bayesian information criterion
------------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+----------------------------------------------------------------
           . |        400 -275.2555  -264.9624       4    537.9248   553.8907
------------------------------------------------------------------------------
               Note: N=400 used in calculating BIC.

. test (c.gpa3=0) (c.parD=0) (c.priv=0)  // Multiv Wald Test of Model R2
 ( 1)  [apply2]gpa3 = 0
 ( 2)  [apply2]parD = 0
 ( 3)  [apply2]priv = 0

           chi2(  3) =   18.95
         Prob > chi2 =    0.0003
```

Long table that labels the predictors for each predicted value was omitted

```
------------------------------------------------------------------------------
Predicted    |            Delta-method
Logit        |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           1 |  -1.087037   .4312041    -2.52   0.012    -1.932181    -.241892
           2 |  -.8864795   .2843452    -3.12   0.002    -1.443786    -.329173
           3 |  -.0274248   .5122537    -0.05   0.957    -1.031424    .9765739
           4 |   .1731323   .4078464     0.42   0.671    -.6262321    .9724966
           5 |  -.5387909    .287416    -1.87   0.061    -1.102116    .0245341
           6 |  -.3382338   .1187131    -2.85   0.004    -.5709072   -.1055604
           7 |   .5208208   .3713982     1.40   0.161    -.2071063    1.248748
           8 |   .7213779   .2800735     2.58   0.010     .1724439    1.270312
           9 |   .0094548   .3573788     0.03   0.979    -.6909948    .7099044
          10 |   .2100119   .3094739     0.68   0.497    -.3965457    .8165695
          11 |   1.069067   .4023806     2.66   0.008      .280415    1.857718
          12 |   1.269624    .372806     3.41   0.001     .5389373     2.00031
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Predicted    |            Delta-method
Probability  |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           1 |   .2521767    .081318     3.10   0.002     .0927963    .4115571
           2 |   .2918369   .0587651     4.97   0.000     .1766594    .4070143
           3 |   .4931442   .1280393     3.85   0.000     .2421917    .7440967
           4 |   .5431753   .1012013     5.37   0.000     .3448243    .7415263
           5 |   .3684689   .0668816     5.51   0.000     .2373834    .4995544
           6 |   .4162386   .0288454    14.43   0.000     .3597027    .4727745
           7 |   .6273397   .0868272     7.23   0.000     .4571615    .7975178
           8 |   .6729104   .0616447    10.92   0.000     .5520889    .7937318
           9 |   .5023637   .0893427     5.62   0.000     .3272552    .6774722
          10 |   .5523108   .0765216     7.22   0.000     .4023312    .7022905
          11 |   .7444194   .0765566     9.72   0.000     .5943712    .8944675
          12 |   .7806783   .0638317    12.23   0.000     .6555704    .9057862
------------------------------------------------------------------------------
```

```
Logistic regression (ODDS RATIOS VERSION)       Number of obs     =        400
                                                LR chi2(3)        =      20.59
                                                Prob > chi2       =     0.0001
Log likelihood =  -264.9624                     Pseudo R2         =     0.0374
------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        gpa3 |   1.730215   .4713696     2.01   0.044     1.014387    2.951185  exp(Beta1)
        parD |   2.885251   .8580314     3.56   0.000     1.610826    5.167952  exp(Beta2)
        priv |   1.222083   .3731454     0.66   0.511     .6717363    2.223324  exp(Beta3)
       _cons |   .5834533   .1676938    -1.87   0.061     .3321675    1.024838  exp(Beta0)
------------------------------------------------------------------------------
```

R Syntax and Output (condensed for convenience):

```
print("R Main-Effects-Only Model Predicting Binary Apply2")
Model2Main = glm(data=Example2, family=binomial,
                 formula=apply2~1+gpa3+parD+priv)
summary(Model2Main); BIC(Model2Main) # Get BIC too

print("Odds ratios -- 95% CIs will not match SAS,STATA exactly")
exp(cbind(OR = coef(Model2Main), confint(Model2Main)))

print("Multiv Wald Test of Model R2")
Main2R2 = glht(model=Model2Main, linfct=c("gpa3=0","parD=0","priv=0"))
summary(Main2R2, test=Ftest()) # Joint F-test instead of separate
summary(Main2R2, test=Chisqtest()) # Joint chi-square test

print("Yhat in logits for specific values of predictors")
Main2Logits = prediction(model=Model2Main, type="link",
                         at=list(gpa3=-1:1,parD=0:1,priv=0:1))
summary(Main2Logits)
print("Yhat in probability for specific values of predictors")
Main2Probs = prediction(model=Model2Main, type="response",
                        at=list(gpa3=-1:1,parD=0:1,priv=0:1))
summary(Main2Probs)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-1.67299  -1.03536  -0.89593   1.27410   1.55108

Coefficients:
            Estimate Std. Error z value  Pr(>|z|)
(Intercept) -0.53879    0.28742 -1.8746 0.0608473  Beta0
gpa3         0.54825    0.27243  2.0124 0.0441780  Beta1
parD         1.05961    0.29739  3.5631 0.0003665  Beta2
priv         0.20056    0.30534  0.6568 0.5112826  Beta3

    Null deviance: 550.511  on 399  degrees of freedom → Is empty model -2LL
Residual deviance: 529.925  on 396  degrees of freedom → Is current model -2LL

AIC: 537.925

> BIC(Model2Main)
[1] 553.89066

[1] "Odds ratios -- 95% CIs will not match SAS,STATA exactly"
                    OR      2.5 %    97.5 %
(Intercept) 0.58345329 0.32846931 1.0192795  exp(Beta0)
gpa3        1.73021501 1.01774088 2.9675894  exp(Beta1)
parD        2.88525056 1.62940440 5.2560216  exp(Beta2)
priv        1.22208338 0.67428604 2.2423595  exp(Beta3)
```

```
[1] "Multiv Wald Test of Model R2"

         General Linear Hypotheses
Linear Hypotheses:
          Estimate
gpa3 == 0  0.54825
parD == 0  1.05961
priv == 0  0.20056

Global Test:
       F DF1 DF2     Pr(>F)
1 6.3157   3 396 0.00034174

Global Test:
   Chisq DF Pr(>Chisq)
1 18.947  3 0.00028037

[1] "Yhat in logits for specific values of predictors"

at(gpa3) at(parD) at(priv) Prediction     SE        z         p   lower    upper
       -1        0        0  -1.087037 0.4312 -2.52093 0.0117044 -1.9322 -0.24189
        0        0        0  -0.538791 0.2874 -1.87460 0.0608473 -1.1021  0.02453
        1        0        0   0.009455 0.3574  0.02646 0.9788937 -0.6910  0.70990
       -1        1        0  -0.027425 0.5123 -0.05354 0.9573036 -1.0314  0.97657
        0        1        0   0.520821 0.3714  1.40232 0.1608183 -0.2071  1.24875
        1        1        0   1.069067 0.4024  2.65685 0.0078873  0.2804  1.85772
       -1        0        1  -0.886479 0.2843 -3.11762 0.0018232 -1.4438 -0.32917
        0        0        1  -0.338234 0.1187 -2.84917 0.0043833 -0.5709 -0.10556
        1        0        1   0.210012 0.3095  0.67861 0.4973853 -0.3965  0.81657
       -1        1        1   0.173132 0.4078  0.42450 0.6711986 -0.6262  0.97250
        0        1        1   0.721378 0.2801  2.57567 0.0100045  0.1724  1.27031
        1        1        1   1.269624 0.3728  3.40559 0.0006602  0.5389  2.00031

[1] "Yhat in probability for specific values of predictors"

 at(gpa3) at(parD) at(priv) Prediction      SE      z         p  lower  upper
       -1        0        0     0.2522 0.08132  3.101 1.928e-03 0.0928 0.4116
        0        0        0     0.3685 0.06688  5.509 3.603e-08 0.2374 0.4996
        1        0        0     0.5024 0.08934  5.623 1.878e-08 0.3273 0.6775
       -1        1        0     0.4931 0.12804  3.852 1.174e-04 0.2422 0.7441
        0        1        0     0.6273 0.08683  7.225 5.005e-13 0.4572 0.7975
        1        1        0     0.7444 0.07656  9.724 2.388e-22 0.5944 0.8945
       -1        0        1     0.2918 0.05877  4.966 6.829e-07 0.1767 0.4070
        0        0        1     0.4162 0.02885 14.430 3.351e-47 0.3597 0.4728
        1        0        1     0.5523 0.07652  7.218 5.287e-13 0.4023 0.7023
       -1        1        1     0.5432 0.10120  5.367 7.994e-08 0.3448 0.7415
        0        1        1     0.6729 0.06164 10.916 9.672e-28 0.5521 0.7937
        1        1        1     0.7807 0.06383 12.230 2.143e-34 0.6556 0.9058
```

Let's examine a model with two new interactions...

$$Logit(Apply2_i = 1) = \beta_0 + \beta_1(GPA_i - 3) + \beta_2(ParentGD_i) + \beta_3(Private_i) + \beta_4(GPA_i - 3)(ParentGD_i) + \beta_5(GPA_i - 3)(Private_i)$$

Model-implied GPA Slope: $\beta_1 + \beta_4(ParentGD_i) + \beta_5(Private_i)$

SAS Syntax and Output (condensed and re-arranged for convenience):

```
TITLE1 "SAS Interaction Model Predicting Binary Apply2";
PROC GLIMMIX DATA=work.Example2 NOCLPRINT GRADIENT METHOD=MSPL;
MODEL apply2 (DESCENDING) = gpa3 parD priv gpa3*parD gpa3*priv
      / SOLUTION DDFM=NONE LINK=LOGIT DIST=BINARY ODDSRATIO(AT gpa3=0 DIFF=ALL LABEL);
CONTRAST "Multiv Wald Test of R2 Change"  gpa3*parD 1, gpa3*priv 1 / CHISQ;
* Get predicted outcomes (duplicates estimates below with less effort;
OUTPUT OUT=work.Pred2Int PRED(NOILINK)=Ylogit STDERR(NOILINK)=YlogitSE
                         PRED(ILINK)=Yprob    STDERR(ILINK)=YprobSE;
* Predicted outcomes in logits and ILINK=probability;
ESTIMATE "Yhat: Ndeg Pub GPA=2"  int 1 gpa3 -1 parD 0 priv 0 gpa3*parD  0 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ndeg Pub GPA=3"  int 1 gpa3  0 parD 0 priv 0 gpa3*parD  0 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ndeg Pub GPA=4"  int 1 gpa3  1 parD 0 priv 0 gpa3*parD  0 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ndeg Pri GPA=2"  int 1 gpa3 -1 parD 0 priv 1 gpa3*parD  0 gpa3*priv -1 / ILINK;
ESTIMATE "Yhat: Ndeg Pri GPA=3"  int 1 gpa3  0 parD 0 priv 1 gpa3*parD  0 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ndeg Pri GPA=4"  int 1 gpa3  1 parD 0 priv 1 gpa3*parD  0 gpa3*priv  1 / ILINK;
ESTIMATE "Yhat: Ydeg Pub GPA=2"  int 1 gpa3 -1 parD 1 priv 0 gpa3*parD -1 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ydeg Pub GPA=3"  int 1 gpa3  0 parD 1 priv 0 gpa3*parD  0 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ydeg Pub GPA=4"  int 1 gpa3  1 parD 1 priv 0 gpa3*parD  1 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ydeg Pri GPA=2"  int 1 gpa3 -1 parD 1 priv 1 gpa3*parD -1 gpa3*priv -1 / ILINK;
ESTIMATE "Yhat: Ydeg Pri GPA=3"  int 1 gpa3  0 parD 1 priv 1 gpa3*parD  0 gpa3*priv  0 / ILINK;
ESTIMATE "Yhat: Ydeg Pri GPA=4"  int 1 gpa3  1 parD 1 priv 1 gpa3*parD  1 gpa3*priv  1 / ILINK;
* Simple slopes for GPA by moderators (EXP to get odds ratios, add CL for CIs);
ESTIMATE "GPA Slope: Ndeg Pub"  gpa3 1 gpa3*parD 0 gpa3*priv 0 / EXP;
ESTIMATE "GPA Slope: Ndeg Pri"  gpa3 1 gpa3*parD 0 gpa3*priv 1 / EXP;
ESTIMATE "GPA Slope: Ydeg Pub"  gpa3 1 gpa3*parD 1 gpa3*priv 0 / EXP;
ESTIMATE "GPA Slope: Ydeg Pri"  gpa3 1 gpa3*parD 1 gpa3*priv 1 / EXP;
RUN;
TITLE1 "Predicted Logits and Probabilities for Fake People";
PROC PRINT NOOBS DATA=work.Pred2Int; WHERE PersonID=-99;
     VAR gpa3 parD priv Ylogit--YprobSE;
RUN; TITLE1;
```

Fit Statistics

-2 Log Likelihood	**528.17**
AIC (smaller is better)	540.17
BIC (smaller is better)	564.12

Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
Multiv Wald Test of R2 Change	2	Infty	**1.70**	0.85	0.4272	0.4272

Parameter Estimates

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient	
Intercept	-0.6594	0.3374	Infty	-1.95	0.0506	3.151E-6	**Beta0**
gpa3	1.2564	0.7737	Infty	1.62	0.1044	-0.00001	**Beta1**
parD	1.1623	0.3197	Infty	3.64	0.0003	-0.00002	**Beta2**
priv	0.3198	0.3518	Infty	0.91	0.3634	-3.2E-6	**Beta3**
gpa3*parD	-0.8359	0.7696	Infty	-1.09	0.2774	-1.44E-6	**Beta4**
gpa3*priv	-0.6821	0.8077	Infty	-0.84	0.3984	-9E-6	**Beta5**

Interpret each simple effect and interaction…

GPA3:

parentGD:

private:

GPA3*parentGD:

GPA3*private:

This GPA odds ratio appears to be marginalized across interacting predictors

Odds Ratio Estimates

Comparison	Estimate	DF	95% Confidence Limits		
unit change of gpa3 from gpa3=0	1.714	Infty	0.998	2.944	**NOT = exp(Beta1)!**
unit change of parD from gpa3=0	3.197	Infty	1.709	5.983	**is = exp(Beta2)**
unit change of priv from gpa3=0	1.377	Infty	0.691	2.744	**is = exp(Beta3)**

Effects of continuous variables are assessed as units offsets from the reference value.

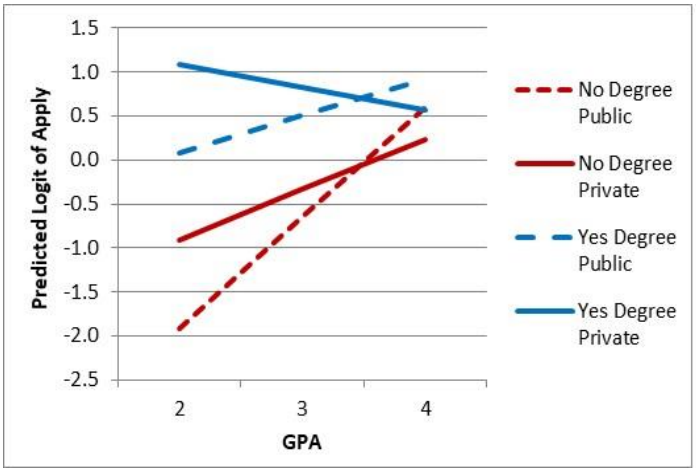
Estimates

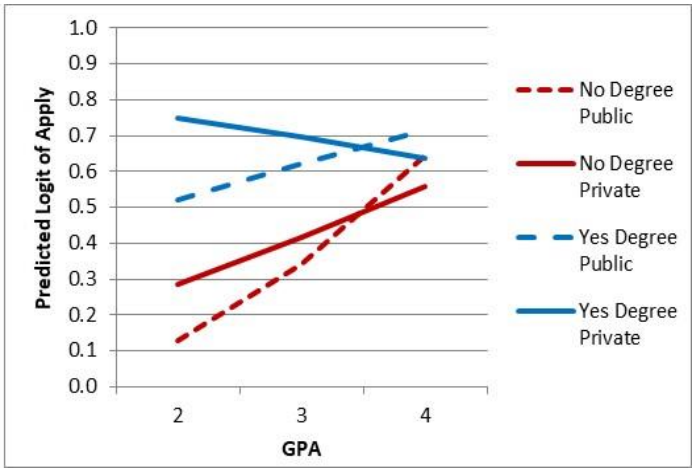
Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Mean	Standard Error Mean	Exponentiated Estimate
Yhat: Ndeg Pub GPA=2	-1.9159	0.9908	Infty	-1.93	0.0532	0.1283	0.1108	.
Yhat: Ndeg Pub GPA=3	-0.6594	0.3374	Infty	-1.95	0.0506	0.3409	0.07580	.
Yhat: Ndeg Pub GPA=4	0.5970	0.6656	Infty	0.90	0.3698	0.6450	0.1524	.
Yhat: Ndeg Pri GPA=2	-0.9140	0.3194	Infty	-2.86	0.0042	0.2862	0.06525	.
Yhat: Ndeg Pri GPA=3	-0.3397	0.1187	Infty	-2.86	0.0042	0.4159	0.02884	.
Yhat: Ndeg Pri GPA=4	0.2347	0.3422	Infty	0.69	0.4929	0.5584	0.08439	.
Yhat: Ydeg Pub GPA=2	0.08233	1.2300	Infty	0.07	0.9466	0.5206	0.3070	.
Yhat: Ydeg Pub GPA=3	0.5029	0.4289	Infty	1.17	0.2410	0.6231	0.1007	.
Yhat: Ydeg Pub GPA=4	0.9234	0.8068	Infty	1.14	0.2524	0.7157	0.1641	.
Yhat: Ydeg Pri GPA=2	1.0842	0.8967	Infty	1.21	0.2266	0.7473	0.1693	.
Yhat: Ydeg Pri GPA=3	0.8227	0.3034	Infty	2.71	0.0067	0.6948	0.06433	.
Yhat: Ydeg Pri GPA=4	0.5611	0.6796	Infty	0.83	0.4090	0.6367	0.1572	.
GPA Slope: Ndeg Pub	1.2564	0.7737	Infty	1.62	0.1044	Non-est	.	3.5129
GPA Slope: Ndeg Pri	0.5743	0.3090	Infty	1.86	0.0631	Non-est	.	1.7759
GPA Slope: Ydeg Pub	0.4206	0.9476	Infty	0.44	0.6572	Non-est	.	1.5228
GPA Slope: Ydeg Pri	-0.2616	0.7355	Infty	-0.36	0.7221	Non-est	.	0.7699

Here are the "simple" GPA odds ratios (for each interacting predictor value)

Predicted Logits and Probabilities for Fake People

gpa3	parD	priv	Ylogit	Yprob	YlogitSE	YprobSE
-1	0	0	-1.91589	0.12832	0.99084	0.11083
0	0	0	-0.65944	0.34086	0.33740	0.07580
1	0	0	0.59700	0.64497	0.66562	0.15242
-1	0	1	-0.91401	0.28618	0.31943	0.06525
0	0	1	-0.33967	0.41589	0.11874	0.02884
1	0	1	0.23466	0.55840	0.34224	0.08439
-1	1	0	0.08233	0.52057	1.23003	0.30699
0	1	0	0.50289	0.62314	0.42891	0.10072
1	1	0	0.92345	0.71574	0.80679	0.16415
-1	1	1	1.08421	0.74729	0.89666	0.16933
0	1	1	0.82266	0.69480	0.30336	0.06433
1	1	1	0.56111	0.63671	0.67958	0.15719





The model provides direct tests of the differences in logits amongst the degree and school conditions, as well as for the simple slopes of GPA for each degree and school type. Model-predicted logit outcomes can then be converted through an inverse link ("un-logit") into predicted probabilities for ease of interpretation, but the slopes or mean differences themselves cannot be converted in differences in probabilities, only odds ratios.

STATA Syntax and Output (condensed and re-arranged for convenience):

```
display "STATA Interaction Model Predicting Binary Apply2"
logit apply2 c.gpa3 c.parD c.priv c.gpa3#c.parD c.gpa3#c.priv
estat ic, n(400)                          // AIC and BIC to match SAS
test (c.gpa3#c.parD=0) (c.gpa3#c.priv=0)  // Multiv Wald Test of R2 Change
// For at, (from(by)to) for range of predictors
margins, at(c.gpa3=(-1(1)1) c.parD=(0(1)1) c.priv=(0(1)1)) predict(xb) // Yhat in logits
margins, at(c.gpa3=(-1(1)1) c.parD=(0(1)1) c.priv=(0(1)1))             // Yhat in probability
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*0  // GPA slope: Ndeg Pub
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*1  // GPA slope: Ndeg Pri
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*0  // GPA slope: Ydeg Pub
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*1  // GPA slope: Ydeg Pri

// Must re-estimate with 'or' added to first line to get odds ratios
display "STATA Interaction Model Predicting Binary Apply2"
display "Get Odds Ratios Instead of Logit Fixed Effects"
logit apply2 c.gpa3 c.parD c.priv c.gpa3#c.parD c.gpa3#c.priv, or
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*0, or  // GPA slope: Ndeg Pub
lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*1, or  // GPA slope: Ndeg Pri
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*0, or  // GPA slope: Ydeg Pub
lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*1, or  // GPA slope: Ydeg Pri
```

```
Logistic regression                             Number of obs     =        400
                                                LR chi2(5)        =      22.34 → Is LRT
                                                Prob > chi2       =     0.0005
Log likelihood = -264.08684 * -2 = -2LL         Pseudo R2         =     0.0406
------------------------------------------------------------------------------
       apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
         gpa3 |    1.25645   .7736689     1.62   0.104    -.2599127    2.772814  Beta1
         parD |   1.162334   .3196688     3.64   0.000     .5357943    1.788873  Beta2
         priv |   .3197742    .351815     0.91   0.363    -.3697705    1.009319  Beta3
c.gpa3#c.parD |  -.8358888   .7695577    -1.09   0.277    -2.344194    .6724166  Beta4
c.gpa3#c.priv |  -.6821162   .8077466    -0.84   0.398    -2.265271    .9010381  Beta5
        _cons |  -.6594466   .3373972    -1.95   0.051    -1.320733    .0018397  Beta0
------------------------------------------------------------------------------
```

```
. estat ic, n(400)                          // AIC and BIC to match SAS
Akaike's information criterion and Bayesian information criterion
-------------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+-----------------------------------------------------------------
           . |        400 -275.2555  -264.0868       6    540.1737   564.1225
-------------------------------------------------------------------------------
               Note: N=400 used in calculating BIC.

. test (c.gpa3#c.parD=0) (c.gpa3#c.priv=0)  // Multiv Wald Test of R2 Change

 ( 1)  [apply2]c.gpa3#c.parD = 0
 ( 2)  [apply2]c.gpa3#c.priv = 0

           chi2(  2) =    1.70
         Prob > chi2 =    0.4272
```

Long table that labels the predictors for each predicted value was omitted

```
------------------------------------------------------------------------------
Predicted    |            Delta-method
Logit        |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           1 |  -1.915897   .9908357    -1.93   0.053    -3.857899    .0261053
           2 |  -.9140066   .3194316    -2.86   0.004    -1.540081   -.2879321
           3 |   .0823253   1.230028     0.07   0.947    -2.328486    2.493137
           4 |   1.084216   .8966626     1.21   0.227    -.6732107    2.841642
           5 |  -.6594466   .3373972    -1.95   0.051    -1.320733    .0018397
           6 |  -.3396724   .1187386    -2.86   0.004    -.5723958    -.106949
           7 |    .502887   .4289053     1.17   0.241    -.3377518    1.343526
           8 |   .8226612   .3033583     2.71   0.007     .2280899    1.417233
           9 |   .5970039   .6656165     0.90   0.370    -.7075806    1.901588
          10 |   .2346618   .3422373     0.69   0.493    -.4361109    .9054345
          11 |   .9234487   .8067913     1.14   0.252    -.6578331    2.504731
          12 |   .5611067   .6795829     0.83   0.409    -.7708513    1.893065
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Predicted    |            Delta-method
Probability  |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           1 |   .1283198   .1108288     1.16   0.247    -.0889006    .3455402
           2 |   .2861807   .0652539     4.39   0.000     .1582854     .414076
           3 |   .5205697   .3069867     1.70   0.090    -.0811131    1.122253
           4 |    .747291   .1693322     4.41   0.000     .4154059    1.079176
           5 |   .3408639   .0758049     4.50   0.000      .192289    .4894389
           6 |   .4158891   .0288446    14.42   0.000     .3593547    .4724235
           7 |   .6231376   .1007229     6.19   0.000     .4257243    .8205508
           8 |    .694801   .0643279    10.80   0.000     .5687206    .8208813
           9 |   .6449705   .1524152     4.23   0.000     .3462422    .9436989
          10 |   .5583977   .0843922     6.62   0.000     .3929921    .7238034
          11 |   .7157443   .1641452     4.36   0.000     .3940255    1.037463
          12 |   .6367086   .1571948     4.05   0.000     .3286123    .9448048
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*0  // GPA slope: Ndeg Pub
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |    1.25645   .7736689     1.62   0.104    -.2599127    2.772814
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*1  // GPA slope: Ndeg Pri
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .5743342   .3090026     1.86   0.063    -.0312998    1.179968
------------------------------------------------------------------------------
```

```
. lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*0  // GPA slope: Ydeg Pub
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .4205617   .9476188     0.44   0.657    -1.436737     2.27786
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*1  // GPA slope: Ydeg Pri
------------------------------------------------------------------------------
      apply2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -.2615545   .7354537    -0.36   0.722    -1.703017    1.179908
------------------------------------------------------------------------------

Logistic regression (ODDS RATIOS VERSION)       Number of obs     =        400
                                                LR chi2(5)        =      22.34
                                                Prob > chi2       =     0.0005
Log likelihood = -264.08684                     Pseudo R2         =     0.0406
------------------------------------------------------------------------------
       apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
         gpa3 |    3.51293   2.717845     1.62   0.104     .7711189     16.0036  exp(Beta1)
         parD |   3.197386   1.022105     3.64   0.000     1.708805    5.982706  exp(Beta2)
         priv |   1.376817   .4843848     0.91   0.363     .6908928    2.743732  exp(Beta3)
c.gpa3#c.parD |    .433489   .3335948    -1.09   0.277     .0959245    1.958966  exp(Beta4)
c.gpa3#c.priv |    .505546   .4083531    -0.84   0.398     .1038019    2.462158  exp(Beta5)
        _cons |   .5171374   .1744807    -1.95   0.051     .2669396    1.001841  exp(Beta0)
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*0, or  // GPA slope: Ndeg Pub
------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |    3.51293   2.717845     1.62   0.104     .7711189     16.0036
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*0 + c.gpa3#c.priv*1, or  // GPA slope: Ndeg Pri
------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   1.775948   .5487725     1.86   0.063      .969185    3.254271
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*0, or  // GPA slope: Ydeg Pub
------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   1.522817    1.44305     0.44   0.657     .2377021    9.755784
------------------------------------------------------------------------------

. lincom c.gpa3*1 + c.gpa3#c.parD*1 + c.gpa3#c.priv*1, or  // GPA slope: Ydeg Pri
------------------------------------------------------------------------------
      apply2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .7698539   .5661919    -0.36   0.722     .1821331    3.254075
------------------------------------------------------------------------------
```

R Syntax and Output (condensed and re-arranged for convenience):

```
print("R Interaction Model Predicting Binary Apply2")
Model2Int = glm(data=Example2, family=binomial,
                formula=apply2~1+gpa3+parD+priv+gpa3:parD+gpa3:priv)
summary(Model2Int); BIC(Model2Int) # Get BIC too

print("Get odds ratios -- 95% CIs will not match SAS,STATA exactly")
exp(cbind(OR = coef(Model2Int), confint(Model2Int)))

print("Multiv Wald Test of R2 Change")
Int2CR2 = glht(model=Model2Int, linfct=c("gpa3:parD=0","gpa3:priv=0"))
summary(Int2CR2, test=Chisqtest()) # Joint chi-square test

print("Yhat in logits for specific values of predictors")
Int2Logits = prediction(model=Model2Int, type="link",
                        at=list(gpa3=-1:1,parD=0:1,priv=0:1))
summary(Int2Logits)
print("Yhat in probability for specific values of predictors")
Int2Probs = prediction(model=Model2Int, type="response",
                       at=list(gpa3=-1:1,parD=0:1,priv=0:1))
summary(Int2Probs)

print("Simple slopes for GPA by moderators")
Int2Slopes = (summary(glht(model=Model2Int, linfct=rbind(
        "GPA Slope: Ndeg Pub" = c(0,1,0,0,0,0),  # in order of fixed effects
        "GPA Slope: Ndeg Pri" = c(0,1,0,0,0,1),
        "GPA Slope: Ydeg Pub" = c(0,1,0,0,1,0),
        "GPA Slope: Ydeg Pri" = c(0,1,0,0,1,1))),test=adjusted("none")))
Int2Slopes; print("Odds ratios for simple slopes")
data.frame(OR=exp(Int2Slopes$test$coefficients))
```

```
Deviance Residuals:
     Min        1Q    Median        3Q       Max
-1.62915  -1.04275  -0.87951   1.26659   1.78376

Coefficients:
            Estimate Std. Error z value  Pr(>|z|)
(Intercept) -0.65945    0.33740 -1.9545 0.0506406  Beta0
gpa3         1.25645    0.77367  1.6240 0.1043722  Beta1
parD         1.16233    0.31967  3.6361 0.0002768  Beta2
priv         0.31977    0.35181  0.9089 0.3633882  Beta3
gpa3:parD   -0.83589    0.76956 -1.0862 0.2773930  Beta4
gpa3:priv   -0.68212    0.80775 -0.8445 0.3984075  Beta5

    Null deviance: 550.511  on 399  degrees of freedom → Is empty model -2LL
Residual deviance: 528.174  on 394  degrees of freedom → Is current model -2LL

AIC: 540.174

> BIC(Model2Int)
[1] 564.12247

[1] "Get odds ratios -- 95% CIs will not match SAS,STATA exactly"

                    OR       2.5 %      97.5 %
(Intercept) 0.51713738 0.257469434  0.97967885  exp(Beta0)
gpa3        3.51293103 0.813326564 17.54149233  exp(Beta1)
parD        3.19738627 1.740136424  6.15935755  exp(Beta2)
priv        1.37681699 0.704453915  2.83251229  exp(Beta3)
gpa3:parD   0.43348900 0.092168868  1.93822999  exp(Beta4)
gpa3:priv   0.50554586 0.095682692  2.34652419  exp(Beta5)
```

```
[1] "Multiv Wald Test of R2 Change"

         General Linear Hypotheses
Linear Hypotheses:
               Estimate
gpa3:parD == 0 -0.83589
gpa3:priv == 0 -0.68212

Global Test:
  Chisq DF Pr(>Chisq)
1 1.701  2    0.42721

[1] "Yhat in logits for specific values of predictors"

at(gpa3) at(parD) at(priv) Prediction     SE        z        p   lower     upper
       -1        0        0   -1.91590 0.9908 -1.93362 0.053160 -3.8579  0.026104
        0        0        0   -0.65945 0.3374 -1.95451 0.050641 -1.3207  0.001839
        1        0        0    0.59700 0.6656  0.89692 0.369762 -0.7076  1.901588
       -1        1        0    0.08233 1.2300  0.06693 0.946638 -2.3285  2.493135
        0        1        0    0.50289 0.4289  1.17249 0.241000 -0.3378  1.343526
        1        1        0    0.92345 0.8068  1.14459 0.252377 -0.6578  2.504730
       -1        0        1   -0.91401 0.3194 -2.86135 0.004218 -1.5401 -0.287932
        0        0        1   -0.33967 0.1187 -2.86067 0.004227 -0.5724 -0.106949
        1        0        1    0.23466 0.3422  0.68567 0.492921 -0.4361  0.905434
       -1        1        1    1.08422 0.8967  1.20917 0.226598 -0.6732  2.841641
        0        1        1    0.82266 0.3034  2.71185 0.006691  0.2281  1.417232
        1        1        1    0.56111 0.6796  0.82566 0.408995 -0.7709  1.893064

[1] "Yhat in probability for specific values of predictors"

at(gpa3) at(parD) at(priv) Prediction      SE      z         p    lower  upper
       -1        0        0     0.1283 0.11083  1.158 2.469e-01 -0.08890 0.3455
        0        0        0     0.3409 0.07580  4.497 6.905e-06  0.19229 0.4894
        1        0        0     0.6450 0.15242  4.232 2.320e-05  0.34624 0.9437
       -1        1        0     0.5206 0.30699  1.696 8.993e-02 -0.08111 1.1223
        0        1        0     0.6231 0.10072  6.187 6.145e-10  0.42572 0.8206
        1        1        0     0.7157 0.16415  4.360 1.298e-05  0.39403 1.0375
       -1        0        1     0.2862 0.06525  4.386 1.156e-05  0.15829 0.4141
        0        0        1     0.4159 0.02884 14.418 3.972e-47  0.35935 0.4724
        1        0        1     0.5584 0.08439  6.617 3.673e-11  0.39299 0.7238
       -1        1        1     0.7473 0.16933  4.413 1.019e-05  0.41541 1.0792
        0        1        1     0.6948 0.06433 10.801 3.407e-27  0.56872 0.8209
        1        1        1     0.6367 0.15719  4.050 5.112e-05  0.32861 0.9448

[1] "Simple slopes for GPA by moderators"

         Simultaneous Tests for General Linear Hypotheses

                        Estimate Std. Error z value Pr(>|z|)
GPA Slope: Ndeg Pub == 0  1.25645    0.77367  1.6240  0.10437
GPA Slope: Ndeg Pri == 0  0.57433    0.30900  1.8587  0.06307
GPA Slope: Ydeg Pub == 0  0.42056    0.94762  0.4438  0.65718
GPA Slope: Ydeg Pri == 0 -0.26155    0.73545 -0.3556  0.72211
(Adjusted p values reported -- none method)

[1] "Odds ratios for simple slopes"

                           OR
GPA Slope: Ndeg Pub 3.5129310
GPA Slope: Ndeg Pri 1.7759477
GPA Slope: Ydeg Pub 1.5228169
GPA Slope: Ydeg Pri 0.7698538
```

Sample results section using SAS output (can replace χ^2 with *F* when using denominator DF):

We examined the extent to which a binary decision to apply to graduate school (55.00% 0=No, 45.00% 1=Pry) could be predicted by a student's undergraduate GPA (M = 3.0, SD = 0.40, range = 1.90 to 4.00), whether at least one of their parents has a graduate degree (15.75% 0=No, 84.25% 1=Yes), and whether they attended a private university (14.25% 0=No, 85.75% 1=Yes). Specifically, we estimated generalized linear models using maximum likelihood in SAS GLIMMIX, in which the conditional probability of applying to graduate school was predicted using a logit link function and a conditional Bernoulli distribution (i.e., logistic regressions). The GPA predictor was centered such that 0 indicated a GPA = 3. Effect sizes are provided using odds ratios (OR), in which OR values between 0 and 1 indicate negative effects, 1 indicates no effect, and values above 1 indicate positive effects. SAS ESTIMATE and LSMEANS statements were used to request simple effects and model-implied predicted outcomes.

The first model examined only the main effects of the three predictors, which together resulted in a significant model, $\chi^2(3) = 18.95$, $p = .0003$. GPA had a significantly positive effect, such that for every unit greater GPA, the logit of applying to graduate school was greater by 0.548 (SE = 0.272; OR = 1.730). Likewise, the logit of applying to graduate school was significantly greater for students for whom at least one parent had a graduate degree by 1.060 (SE = 0.297, OR = 2.882). However, the logit of applying to graduate school was nonsignificantly greater for students who attended a private university by 0.200 (SE = 0.305, OR = 1.222).

The second model then included two-way interactions of GPA with parent graduate degree and GPA with university type. This augmented model was not a significant improvement over the main effects model, $\chi^2(2) = 1.70$, $p = .427$. Neither individual interaction term was significant, nor was the simple slope of GPA significant in any of the four subgroups (i.e., formed by parent graduate degree by university type).