Longitudinal Invariance CFA (using MLR) Example in Mplus v. 8.4 (N = 151; 6 items over 3 occasions)

These data measuring a latent trait of social functioning were collected at a Psychiatric Rehabilitation center, in which time 1 was admittance, and times 2 and 3 were collected at six-month intervals. There were six subscales that were completed by the hospital staff for each patient, including positively-oriented measures of Social Competence, Social Interest, and Personal Neatness, and negatively-oriented measures of Psychoticism, Motor Retardation, and Irritability. The negatively-oriented subscales were reflected (*−1) prior to analysis. Initial models examined the fit of one-factor versus two-factor models given the two valences of the subscales, but the fit of the two-factor model was not a significant improvement, and thus a one-factor model with all six items was used here.

Mplus Code to Read in Data:

```
TITLE:       Longitudinal Invariance
DATA:        FILE = Example7b.csv;                 ! Don't need path if in same folder
             FORMAT = free; TYPE = INDIVIDUAL;    ! Defaults

VARIABLE:  NAMES = ID v1T1 v1T2 v1T3 v2T1 v2T2 v2T3        ! Every variable in data set
                   v3T1 v3T2 v3T3 v4T1 v4T2 v4T3
                   v5T1 v5T2 v5T3 v6T1 v6T2 v6T3;

           USEVARIABLES = v1T1 v1T2 v1T3 v2T1 v2T2 v2T3     ! Every variable in MODEL
                   v3T1 v3T2 v3T3 v4T1 v4T2 v4T3
                   v5T1 v5T2 v5T3 v6T1 v6T2 v6T3;

           MISSING = ALL (9999);     ! Specify all missing values
           IDVARIABLE = ID;          ! Specify person ID variable

! Reverse-coding items so that higher = better
DEFINE:      v4T1 = v4T1*(-1);
             v4T2 = v4T2*(-1);
             v4T3 = v4T3*(-1);
             v5T1 = v5T1*(-1);
             v5T2 = v5T2*(-1);
             v5T3 = v5T3*(-1);
             v6T1 = v6T1*(-1);
             v6T2 = v6T2*(-1);
             v6T3 = v6T3*(-1);

ANALYSIS: ESTIMATOR = MLR;    ! For continuous items whose residuals may not be normal

OUTPUT:    MODINDICES(3.84);  ! For modification indices of p<.05 for DF=1
           STDYX RESIDUAL;    ! Fully standardized solution, local model fit

MODEL:   ! Model syntax goes here, to be changed for each model
```

Model 1. Configural Longitudinal Invariance Model (everything separate across time)

```
MODEL:
!!!!! Model 1: Configural Longitudinal Invariance

! Factor loadings all freely estimated, not labeled
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1*;
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2*;
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3*;
! Item intercepts all freely estimated, not labeled
  [v1T1* v1T2* v1T3*]; [v2T1* v2T2* v2T3*];
  [v3T1* v3T2* v3T3*]; [v4T1* v4T2* v4T3*];
  [v5T1* v5T2* v5T3*]; [v6T1* v6T2* v6T3*];
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variances all fixed=1 for identification
  Time1@1 Time2@1 Time3@1;
! Factor means all fixed=0 for identification
  [Time1@0 Time2@0 Time3@0];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```



```
MODEL FIT INFORMATION

Number of Free Parameters                    75

Loglikelihood
        H0 Value                      -4430.302
        H0 Scaling Correction Factor     1.4617
          for MLR
        H1 Value                      -4284.045
        H1 Scaling Correction Factor     1.2029
          for MLR

Information Criteria
        Akaike (AIC)                   9010.604
        Bayesian (BIC)                 9236.900
        Sample-Size Adjusted BIC       8999.533
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                         283.247*
        Degrees of Freedom                 114
        P-Value                         0.0000
        Scaling Correction Factor       1.0327
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                        0.099
        90 Percent C.I.           0.085  0.114
        Probability RMSEA <= .05        0.000

CFI/TLI
        CFI                             0.903
        TLI                             0.870

Chi-Square Test of Model Fit for the Baseline Model
        Value                        1896.788
        Degrees of Freedom                 153
        P-Value                         0.0000

SRMR (Standardized Root Mean Square Residual)

        Value                           0.089
```
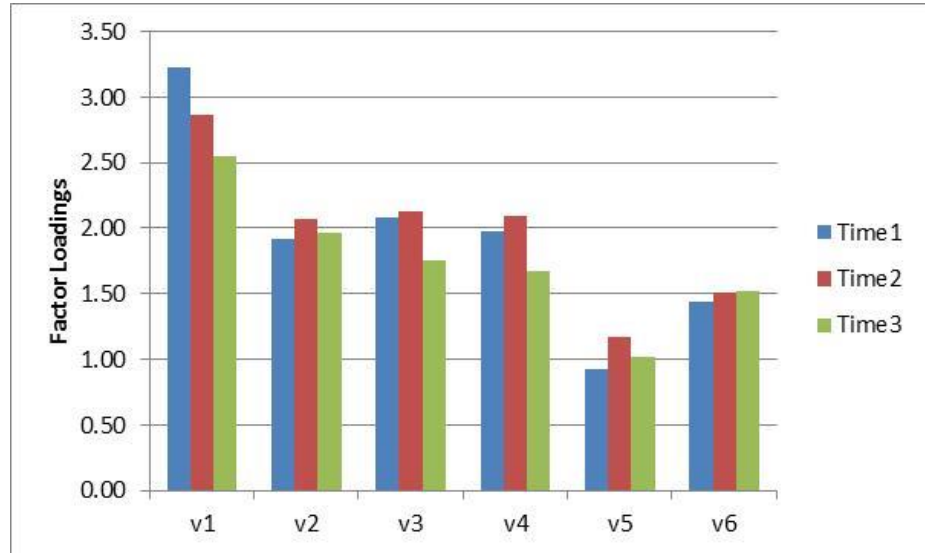
Although the fit is not great, attempts to improve it logically were unsuccessful, so we proceed from here with this as the configural invariance model. The plot of factor loadings on the left foreshadows what will happen when testing metric invariance next…

UNSTANDARDIZED MODEL RESULTS − NOTE ALL MEASUREMENT PARAMETERS DIFFER ACROSS TIME

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS PER OCCASION									
TIME1 BY					**Means (FACTOR MEANS FIXED=0 FOR IDENTIFICATION)**				
V1T1	3.222	0.267	12.063	0.000	TIME1	0.000	0.000	999.000	999.000
V2T1	1.915	0.274	6.997	0.000	TIME2	0.000	0.000	999.000	999.000
V3T1	2.080	0.209	9.956	0.000	TIME3	0.000	0.000	999.000	999.000
V4T1	1.975	0.271	7.298	0.000					
V5T1	0.931	0.148	6.281	0.000	**Intercepts (ARE EXPECTED OUTCOME WHEN FACTOR IS AT 0)**				
V6T1	1.441	0.119	12.101	0.000	V1T1	16.077	0.276	58.220	0.000
					V1T2	17.226	0.245	70.294	0.000
TIME2 BY					V1T3	17.756	0.220	80.620	0.000
V1T2	2.863	0.305	9.372	0.000	V2T1	8.672	0.298	29.132	0.000
V2T2	2.072	0.197	10.490	0.000	V2T2	9.981	0.263	37.921	0.000
V3T2	2.133	0.185	11.509	0.000	V2T3	10.442	0.281	37.204	0.000
V4T2	2.098	0.322	6.514	0.000	V3T1	11.970	0.225	53.108	0.000
V5T2	1.175	0.239	4.921	0.000	V3T2	12.467	0.218	57.264	0.000
V6T2	1.512	0.129	11.749	0.000	V3T3	13.029	0.213	61.157	0.000
					V4T1	-3.037	0.271	-11.216	0.000
TIME3 BY					V4T2	-3.211	0.260	-12.349	0.000
V1T3	2.550	0.288	8.865	0.000	V4T3	-2.738	0.249	-11.014	0.000
V2T3	1.961	0.230	8.539	0.000	V5T1	-1.283	0.138	-9.293	0.000
V3T3	1.751	0.210	8.323	0.000	V5T2	-1.664	0.200	-8.338	0.000
V4T3	1.678	0.260	6.448	0.000	V5T3	-1.247	0.166	-7.511	0.000
V5T3	1.021	0.170	6.012	0.000	V6T1	-2.871	0.164	-17.508	0.000
V6T3	1.523	0.159	9.574	0.000	V6T2	-2.413	0.158	-15.316	0.000
					V6T3	-2.075	0.152	-13.618	0.000
TIME1 WITH **(ESTIMATED FACTOR COVARIANCES)**									
TIME2	0.786	0.042	18.827	0.000	**Residual Variances (VARIANCE PER ITEM THAT IS NOT THE FACTOR)**				
TIME3	0.707	0.084	8.456	0.000	V1T1	0.241	0.395	0.610	0.542
					V1T2	0.511	0.268	1.907	0.056
TIME2 WITH					V1T3	0.523	0.349	1.497	0.134
TIME3	0.671	0.089	7.532	0.000	V2T1	8.672	1.022	8.484	0.000
					V2T2	5.913	0.617	9.581	0.000
***** Residual covariances among same item across time *****					V2T3	5.142	0.806	6.379	0.000
V1T1 WITH					V3T1	2.413	0.398	6.067	0.000
V1T2	-0.214	0.250	-0.855	0.393	V3T2	2.202	0.369	5.972	0.000
V1T3	-0.004	0.247	-0.016	0.987	V3T3	2.381	0.430	5.542	0.000
					V4T1	7.199	1.036	6.950	0.000
V1T2 WITH					V4T2	6.765	0.990	6.834	0.000
V1T3	0.113	0.231	0.488	0.626	V4T3	6.456	1.078	5.988	0.000
....					V5T1	1.824	0.446	4.093	0.000
					V5T2	4.676	1.439	3.251	0.001
Variances (FACTOR VARIANCES FIXED=1 FOR IDENTIFICATION)					V5T3	2.944	0.752	3.913	0.000
TIME1	1.000	0.000	999.000	999.000	V6T1	1.694	0.243	6.974	0.000
TIME2	1.000	0.000	999.000	999.000	V6T2	1.103	0.166	6.643	0.000
TIME3	1.000	0.000	999.000	999.000	V6T3	0.751	0.162	4.630	0.000

Model 2a. Metric Invariance Model (ALL loadings held equal across time – identified model using Time1 Factor Variance = 1)

MODEL:
!!!!! Model 2a: Metric Longitudinal Invariance

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts all freely estimated, not labeled
  [v1T1* v1T2* v1T3*]; [v2T1* v2T2* v2T3*];
  [v3T1* v3T2* v3T3*]; [v4T1* v4T2* v4T3*];
  [v5T1* v5T2* v5T3*]; [v6T1* v6T2* v6T3*];
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor means all fixed=0 for identification
  [Time1@0 Time2@0 Time3@0];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;

Does the metric model (2a) fit _worse_ than the configural model (1)?
Yes, −2ΔLL(df=10) = 19.14, _p_ =.04
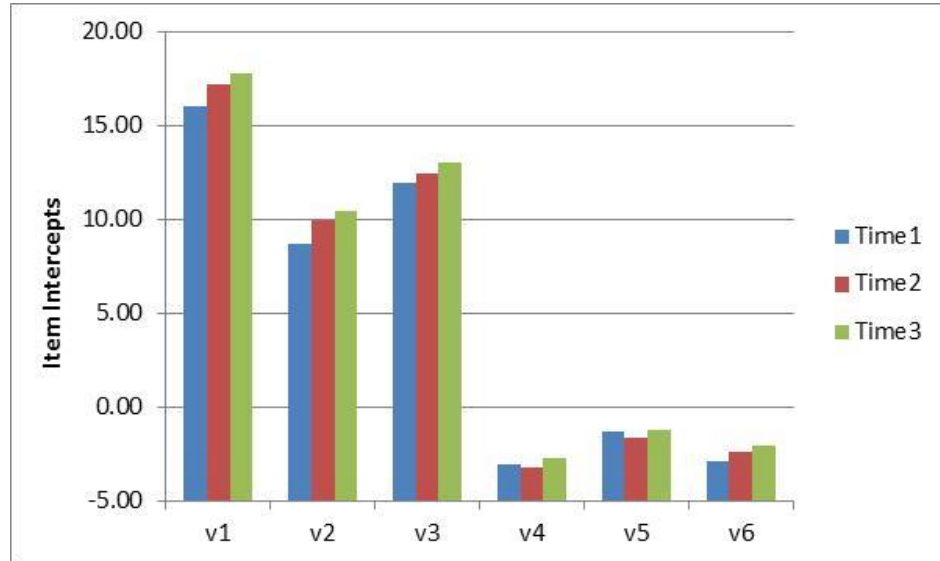
MODEL FIT INFORMATION
Number of Free Parameters   65

Loglikelihood
        H0 Value   -4442.401
        H0 Scaling Correction Factor   1.4921
            for MLR
        H1 Value   -4284.045
        H1 Scaling Correction Factor   1.2029
            for MLR

Information Criteria
        Akaike (AIC)   9014.803
        Bayesian (BIC)   9210.926
        Sample-Size Adjusted BIC   9005.208
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value   301.234*
        Degrees of Freedom   124
        P-Value   0.0000
        Scaling Correction Factor   1.0514
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate   0.097
        90 Percent C.I.   0.083 0.111
        Probability RMSEA <= .05   0.000

CFI/TLI
        CFI   0.898
        TLI   0.875

SRMR (Standardized Root Mean Square Residual)
        Value   0.094

MODEL MODIFICATION INDICES (relevant for testing invariance)
BY Statements
                      M.I.   E.P.C. Std E.P.C. StdYX E.P.C.
BY Statements
TIME1   BY V1T1   10.377   0.182   0.182   0.058
TIME1   BY V5T1   6.062   -0.054   -0.054   -0.033
TIME3   BY V6T3   7.603   0.201   0.175   0.105

Modification indices suggest that freeing the loading for v1 at Time1 would
help, and that matches our observations, so let's try that.

Model 2b. Partial Metric Invariance Model with loading for v1 at Time 1 free

```
MODEL:
! Model 2b: Partial Metric Invariance without v1T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts all freely estimated, not labeled
  [v1T1* v1T2* v1T3*]; [v2T1* v2T2* v2T3*];
  [v3T1* v3T2* v3T3*]; [v4T1* v4T2* v4T3*];
  [v5T1* v5T2* v5T3*]; [v6T1* v6T2* v6T3*];
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor means all fixed=0 for identification
  [Time1@0 Time2@0 Time3@0];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```



```
MODEL FIT INFORMATION
Number of Free Parameters                    66

Loglikelihood
        H0 Value                         -4435.669
        H0 Scaling Correction Factor        1.4980
          for MLR
        H1 Value                         -4284.045
        H1 Scaling Correction Factor        1.2029
          for MLR

Information Criteria
        Akaike (AIC)                      9003.337
        Bayesian (BIC)                    9202.478
        Sample-Size Adjusted BIC          8993.595
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                              290.301*
        Degrees of Freedom                     123
        P-Value                             0.0000
        Scaling Correction Factor           1.0446
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                            0.095
        90 Percent C.I.                     0.081  0.109
        Probability RMSEA <= .05            0.000

CFI/TLI
        CFI                                 0.904
        TLI                                 0.881

SRMR (Standardized Root Mean Square Residual)
        Value                               0.091
```

Does the partial metric model (2b) fit *better* than the full metric model (2a)? Yes, −2ΔLL(df=1) = 7.16, *p* < .01

Does the partial metric model (2b) fit *worse* than the configural model (1)? No, −2ΔLL(df=9) = 8.98, *p* = .44

No large invariance-related modification indices were found, so we'll call it good! Onto the next model! The plot of intercepts on the left foreshadow what we will find with testing scalar invariance…

2b UNSTANDARDIZED PARTIAL METRIC MODEL RESULTS – ALL FACTOR LOADINGS ARE HELD EQUAL EXCEPT v1T1

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value			Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
TIME1 BY						Means (FACTOR MEANS FIXED=0 FOR IDENTIFICATION)				
V1T1	**3.233**	**0.261**	**12.362**	**0.000**		TIME1	0.000	0.000	999.000	999.000
V2T1	1.950	0.201	9.706	0.000		TIME2	0.000	0.000	999.000	999.000
V3T1	1.967	0.198	9.910	0.000		TIME3	0.000	0.000	999.000	999.000
V4T1	1.899	0.224	8.481	0.000						
V5T1	0.968	0.137	7.055	0.000		Intercepts – SCALED SO SHOULD BE EQUAL ACROSS TIME				
V6T1	1.476	0.131	11.247	0.000		V1T1	16.078	0.276	58.267	0.000
						V1T2	17.225	0.245	70.282	0.000
TIME2 BY						V1T3	17.756	0.222	80.036	0.000
V1T2	**2.644**	**0.234**	**11.315**	**0.000**		V2T1	8.672	0.298	29.071	0.000
V2T2	1.950	0.201	9.706	0.000		V2T2	9.980	0.264	37.872	0.000
V3T2	1.967	0.198	9.910	0.000		V2T3	10.434	0.280	37.245	0.000
V4T2	1.899	0.224	8.481	0.000		V3T1	11.978	0.225	53.192	0.000
V5T2	0.968	0.137	7.055	0.000		V3T2	12.468	0.217	57.325	0.000
V6T2	1.476	0.131	11.247	0.000		V3T3	13.041	0.212	61.441	0.000
						V4T1	-3.034	0.267	-11.343	0.000
TIME3 BY						V4T2	-3.210	0.260	-12.365	0.000
V1T3	**2.644**	**0.234**	**11.315**	**0.000**		V4T3	-2.720	0.254	-10.720	0.000
V2T3	1.950	0.201	9.706	0.000		V5T1	-1.288	0.137	-9.377	0.000
V3T3	1.967	0.198	9.910	0.000		V5T2	-1.663	0.199	-8.340	0.000
V4T3	1.899	0.224	8.481	0.000		V5T3	-1.246	0.169	-7.373	0.000
V5T3	0.968	0.137	7.055	0.000		V6T1	-2.871	0.164	-17.506	0.000
V6T3	1.476	0.131	11.247	0.000		V6T2	-2.414	0.158	-15.319	0.000
						V6T3	-2.087	0.154	-13.571	0.000
TIME1 WITH										
TIME2	0.847	0.078	10.837	0.000		Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR				
TIME3	0.682	0.124	5.508	0.000		V1T1	0.170	0.374	0.454	0.650
						V1T2	0.548	0.265	2.070	0.038
TIME2 WITH						V1T3	0.509	0.314	1.618	0.106
TIME3	0.699	0.128	5.473	0.000		V2T1	8.702	1.026	8.483	0.000
						V2T2	5.895	0.605	9.746	0.000
*** Residual covariances among same item across time ****						V2T3	5.177	0.795	6.514	0.000
						V3T1	2.502	0.386	6.484	0.000
V1T1 WITH						V3T2	2.178	0.352	6.183	0.000
V1T2	-0.225	0.249	-0.904	0.366		V3T3	2.309	0.416	5.548	0.000
V1T3	-0.012	0.236	-0.049	0.961		V4T1	7.172	1.021	7.021	0.000
						V4T2	6.759	0.967	6.990	0.000
V1T2 WITH						V4T3	6.613	1.128	5.860	0.000
V1T3	0.132	0.230	0.573	0.566		V5T1	1.829	0.443	4.131	0.000
....						V5T2	4.678	1.430	3.272	0.001
						V5T3	2.944	0.760	3.872	0.000
Variances (FACTOR VARIANCE AT TIME1=1 FOR IDENTIFICATION)						V6T1	1.707	0.242	7.059	0.000
TIME1	1.000	0.000	999.000	999.000		V6T2	1.090	0.165	6.599	0.000
TIME2	1.162	0.185	6.270	0.000		V6T3	0.784	0.170	4.618	0.000
TIME3	0.941	0.157	5.999	0.000						

Model 3a. Scalar Invariance Model (all intercepts held equal across over time except v1T1); identified by Time1 mean=0

```
MODEL:
! Model 3a: Full Scalar Invariance without v1T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  [v1T1*]; [v1T2* v1T3*] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*    v4T2* v4T3*] (I4);
  [v5T1*    v5T2* v5T3*] (I5);
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

Does the full scalar model (3a) fit worse than the partial metric model (2b)? Yes, −2ΔLL(df=9) = 55.13, p <.01

Modification indices suggest that freeing these intercepts would help, so let's try v5T1 first (biggest χ^2 change suggested).

```
MODEL FIT INFORMATION
Number of Free Parameters                        57

Loglikelihood
            H0 Value                       -4461.842
            H0 Scaling Correction Factor      1.5846
               for MLR
            H1 Value                       -4284.045
            H1 Scaling Correction Factor      1.2029
               for MLR

Information Criteria
            Akaike (AIC)                    9037.685
            Bayesian (BIC)                  9209.670
            Sample-Size Adjusted BIC        9029.271
               (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
            Value                           342.530*
            Degrees of Freedom                   132
            P-Value                           0.0000
            Scaling Correction Factor         1.0381
               for MLR

RMSEA (Root Mean Square Error Of Approximation)
            Estimate                           0.103
            90 Percent C.I.             0.089   0.116
            Probability RMSEA <= .05           0.000

CFI/TLI
            CFI                                0.879
            TLI                                0.860

SRMR (Standardized Root Mean Square Residual)
            Value                              0.093


MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                       M.I.    E.P.C.  Std E.P.C.  StdYX E.P.C.
[ V2T1     ]         14.761    -0.696      -0.696        -0.189
[ V2T2     ]          5.578     0.307       0.307         0.094
[ V4T1     ]         10.400     0.366       0.366         0.113
[ V4T2     ]          5.167    -0.271      -0.271        -0.084
[ V5T1     ]         20.890    -0.027      -0.027        -0.017
[ V5T2     ]         14.191    -0.596      -0.596        -0.241
```

Model 3b. Partial Scalar Invariance Model (all intercepts held equal across over time except v1T1 and v5T1)

```
MODEL: ! Model 3b: Partial Scalar Invariance, no v1T1 v5T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME, no v1T1 v5T1
  [v1T1*]; [v1T2* v1T3*] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*    v4T2* v4T3*] (I4);
  [v5T1*]; [v5T2* v5T3*] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

Does the partial scalar model (3b) fit *better* than the full scalar model (3a)? Yes, −2ΔLL(df=1) = 15.16, *p* <.01

Does the partial scalar model (3b) fit *worse* than the partial metric model (2b)? Yes, −2ΔLL(df=8) = 27.84, *p* <.01

Modification indices still suggest that freeing these intercepts would help, so let's try v4T1 next (biggest χ^2 change suggested).

```
MODEL FIT INFORMATION
Number of Free Parameters                      58

Loglikelihood
        H0 Value                        -4450.001
        H0 Scaling Correction Factor        1.5626
          for MLR
        H1 Value                        -4284.045
        H1 Scaling Correction Factor        1.2029
          for MLR

Information Criteria
        Akaike (AIC)                     9016.001
        Bayesian (BIC)                   9191.004
        Sample-Size Adjusted BIC         9007.440
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                            318.018*
        Degrees of Freedom                    131
        P-Value                            0.0000
        Scaling Correction Factor          1.0437
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.097
        90 Percent C.I.              0.084  0.111
        Probability RMSEA <= .05           0.000

CFI/TLI
        CFI                                0.893
        TLI                                0.875

SRMR (Standardized Root Mean Square Residual)
        Value                              0.086

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds
```

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
[V2T1]	11.529	-0.599	-0.599	-0.164
[V2T2]	4.390	0.278	0.278	0.085
[V4T1]	**13.795**	**0.425**	**0.425**	**0.132**
[V4T2]	6.398	-0.306	-0.306	-0.096

Model 3c. Partial Scalar Invariance Model (all intercepts held equal across over time except v1T1, v5T1, v4T1)

```
MODEL: ! Model 3c: Partial Scalar Invariance, no v1T1 v5T1 v4T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1
  [v1T1*]; [v1T2* v1T3*] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*    v2T2* v2T3*] (I2);
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4); ! 3c: I4 applies only to 2 and 3
  [v5T1*]; [v5T2* v5T3*] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

Does the partial scalar model (3c) fit *better* than the partial scalar model (3b)? Yes, −2ΔLL(df=1) = 9.24, p <.01

Does the partial scalar model (3c) fit *worse* than the partial metric model (2b)? Eh, −2ΔLL(df=7) = 13.99, p =.05

Although fit is close to not worse than the partial metric model, there is a significant modification index for v2T1, suggesting localized strain. So let's see what happens if we free that one, too.

```
MODEL FIT INFORMATION
Number of Free Parameters                    59

Loglikelihood
        H0 Value                       -4442.214
        H0 Scaling Correction Factor      1.5647
            for MLR
        H1 Value                       -4284.045
        H1 Scaling Correction Factor      1.2029
            for MLR

Information Criteria
        Akaike (AIC)                    9002.427
        Bayesian (BIC)                  9180.447
        Sample-Size Adjusted BIC        8993.718
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                           304.537*
        Degrees of Freedom                   130
        P-Value                           0.0000
        Scaling Correction Factor         1.0387
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.094
        90 Percent C.I.              0.081  0.108
        Probability RMSEA <= .05           0.000

CFI/TLI
        CFI                                0.900
        TLI                                0.882

SRMR (Standardized Root Mean Square Residual)
        Value                              0.092

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                     M.I.   E.P.C.  Std E.P.C.  StdYX E.P.C.
[ V2T1     ]        8.560   -0.497    -0.497       -0.137
```

Model 3d. Partial Scalar Invariance Model (all intercepts held equal across over time except v1T1, v5T1, v4T1, v2T1)

```
MODEL: ! Model 3d: Partial Scalar Invariance,
       ! no v1T1 v5T1 v4T1 v2T1

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*]; [v2T2* v2T3] (I2); ! 3d: I2 applies only to 2 and 3
  [v3T1*    v3T2* v3T3] (I3);
  [v4T1*]; [v4T2* v4T3] (I4); ! 3c: I4 applies only to 2 and 3
  [v5T1*]; [v5T2* v5T3] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*    v6T2* v6T3] (I6);
! Residual variances all freely estimated, not labeled
  v1T1* v1T2* v1T3*; v2T1* v2T2* v2T3*;
  v3T1* v3T2* v3T3*; v4T1* v4T2* v4T3*;
  v5T1* v5T2* v5T3*; v6T1* v6T2* v6T3*;
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```
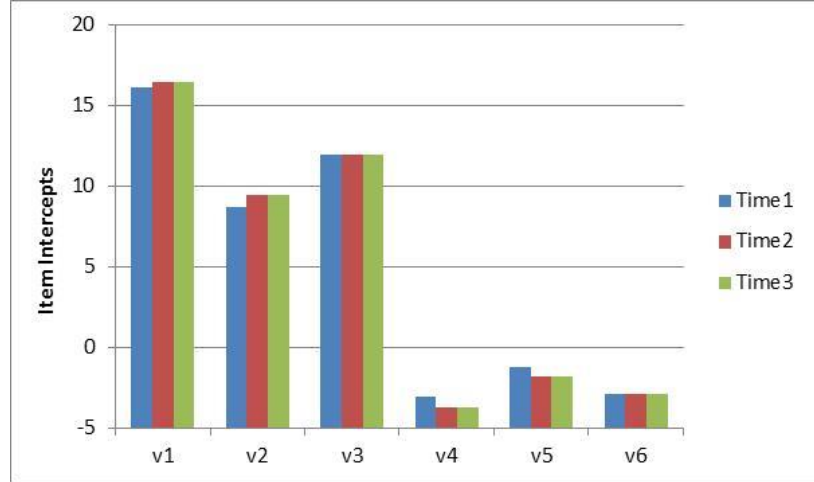


```
MODEL FIT INFORMATION
Number of Free Parameters                    60

Loglikelihood
        H0 Value                       -4437.665
        H0 Scaling Correction Factor      1.5560
          for MLR
        H1 Value                       -4284.045
        H1 Scaling Correction Factor      1.2029
          for MLR

Information Criteria
        Akaike (AIC)                    8995.330
        Bayesian (BIC)                  9176.366
        Sample-Size Adjusted BIC        8986.473
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                           295.789*
        Degrees of Freedom                   129
        P-Value                           0.0000
        Scaling Correction Factor         1.0387
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                          0.093
        90 Percent C.I.                   0.079  0.106
        Probability RMSEA <= .05          0.000
CFI/TLI
        CFI                               0.904
        TLI                               0.887

Chi-Square Test of Model Fit for the Baseline Model
        Value                          1896.788
        Degrees of Freedom                  153
        P-Value                           0.0000
SRMR (Standardized Root Mean Square Residual)
        Value                             0.091
```

Does the partial scalar model (3d) fit *better* than the partial scalar model (3c)? Yes, −2ΔLL(df=1) = 8.73, *p* <.01

Does the partial scalar model (3d) fit *worse* than the partial metric model (2b)? No, −2ΔLL(df=6) = 4.35, *p* =.63

No invariance-related modification indices remain, so we are done! The intercepts at the end of Model 3d are shown on the left.

3d UNSTANDARDIZED PARTIAL SCALAR MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
TIME1 BY					**Means (FACTOR MEAN AT TIME1 FIXED=0 FOR IDENTIFICATION)**				
V1T1	**3.231**	**0.262**	**12.330**	**0.000**	TIME1	0.000	0.000	999.000	999.000
V2T1	1.953	0.201	9.739	0.000	TIME2	0.293	0.081	3.625	0.000
V3T1	1.974	0.198	9.989	0.000	TIME3	0.521	0.093	5.612	0.000
V4T1	1.904	0.220	8.656	0.000					
V5T1	0.983	0.138	7.097	0.000	**Intercepts**				
V6T1	1.477	0.130	11.353	0.000	**V1T1**	**16.090**	**0.274**	**58.684**	**0.000**
					V1T2	16.425	0.281	58.364	0.000
TIME2 BY					V1T3	16.425	0.281	58.364	0.000
V1T2	2.629	0.232	11.317	0.000	**V2T1**	**8.674**	**0.294**	**29.540**	**0.000**
V2T2	1.953	0.201	9.739	0.000	V2T2	9.413	0.261	36.036	0.000
V3T2	1.974	0.198	9.989	0.000	V2T3	9.413	0.261	36.036	0.000
V4T2	1.904	0.220	8.656	0.000	V3T1	11.950	0.225	53.099	0.000
V5T2	0.983	0.138	7.097	0.000	V3T2	11.950	0.225	53.099	0.000
V6T2	1.477	0.130	11.353	0.000	V3T3	11.950	0.225	53.099	0.000
					V4T1	**-3.024**	**0.267**	**-11.334**	**0.000**
TIME3 BY					V4T2	-3.744	0.299	-12.535	0.000
V1T3	2.629	0.232	11.317	0.000	V4T3	-3.744	0.299	-12.535	0.000
V2T3	1.953	0.201	9.739	0.000	**V5T1**	**-1.215**	**0.131**	**-9.277**	**0.000**
V3T3	1.974	0.198	9.989	0.000	V5T2	-1.802	0.207	-8.706	0.000
V4T3	1.904	0.220	8.656	0.000	V5T3	-1.802	0.207	-8.706	0.000
V5T3	0.983	0.138	7.097	0.000	V6T1	-2.854	0.161	-17.688	0.000
V6T3	1.477	0.130	11.353	0.000	V6T2	-2.854	0.161	-17.688	0.000
					V6T3	-2.854	0.161	-17.688	0.000
TIME1 WITH									
TIME2	0.850	0.079	10.809	0.000	**Residual Variances (ITEM VARIANCE THAT IS NOT THE FACTOR)**				
TIME3	0.686	0.124	5.543	0.000	V1T1	0.170	0.374	0.454	0.650
					V1T2	0.548	0.265	2.070	0.038
TIME2 WITH					V1T3	0.509	0.314	1.618	0.106
TIME3	0.706	0.128	5.519	0.000	V2T1	8.702	1.026	8.483	0.000
					V2T2	5.895	0.605	9.746	0.000
*** Residual covariances among same item across time ****					V2T3	5.177	0.795	6.514	0.000
					V3T1	2.502	0.386	6.484	0.000
V1T1 WITH					V3T2	2.178	0.352	6.183	0.000
V1T2	-0.206	0.246	-0.838	0.402	V3T3	2.309	0.416	5.548	0.000
V1T3	-0.010	0.233	-0.043	0.966	V4T1	7.172	1.021	7.021	0.000
					V4T2	6.759	0.967	6.990	0.000
V1T2 WITH					V4T3	6.613	1.128	5.860	0.000
V1T3	0.130	0.231	0.561	0.575	V5T1	1.829	0.443	4.131	0.000
....					V5T2	4.678	1.430	3.272	0.001
					V5T3	2.944	0.760	3.872	0.000
Variances (FACTOR VARIANCE AT TIME1=1 FOR IDENTIFICATION)					V6T1	1.707	0.242	7.059	0.000
TIME1	1.000	0.000	999.000	999.000	V6T2	1.090	0.165	6.599	0.000
TIME2	1.167	0.187	6.252	0.000	V6T3	0.784	0.170	4.618	0.000
TIME3	0.947	0.156	6.054	0.000					

Model 4a. Residual Variance Invariance Model (error variances held equal for all except non-invariant items)

```
MODEL: ! Model 4a: Residual Variances
       ! except for non-invariant items

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*]; [v2T2* v2T3*] (I2); ! 3d: I2 applies only to 2 and 3
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4); ! 3c: I4 applies only to 2 and 3
  [v5T1*]; [v5T2* v5T3*] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE)
  v1T1*; v1T2* v1T3* (E1); !4a: E1 applies only to 2 and 3
  v2T1*; v2T2* v2T3* (E2); !4a: E2 applies only to 2 and 3
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4); !4a: E4 applies only to 2 and 3
  v5T1*; v5T2* v5T3* (E5); !4a: E5 applies only to 2 and 3
  v6T1*  v6T2* v6T3* (E6);
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

Does the full residual model (4a) fit *worse* than the partial scalar model (3d)? Yes, $-2\Delta LL(df=8) = 24.72$, $p < .01$

Modification indices suggest freeing v5 across Time2 and Time3, so let's try that next.

```
MODEL FIT INFORMATION
Number of Free Parameters                        52

Loglikelihood
         H0 Value                         -4454.592
         H0 Scaling Correction Factor        1.5487
           for MLR
         H1 Value                         -4284.045
         H1 Scaling Correction Factor        1.2029
           for MLR


Information Criteria
         Akaike (AIC)                      9013.185
         Bayesian (BIC)                    9170.083
         Sample-Size Adjusted BIC          9005.509
           (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
         Value                             318.280*
         Degrees of Freedom                     137
         P-Value                             0.0000
         Scaling Correction Factor           1.0717
           for MLR

RMSEA (Root Mean Square Error Of Approximation)
         Estimate                             0.094
         90 Percent C.I.               0.080  0.107
         Probability RMSEA <= .05             0.000

CFI/TLI
         CFI                                  0.896
         TLI                                  0.884

SRMR (Standardized Root Mean Square Residual)
         Value                                0.095

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds
```

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
Variances/Residual Variances				
V5T2	**12.739**	**0.755**	**0.755**	**0.153**
V5T3	**12.740**	**-1.125**	**-1.125**	**-0.238**
V6T1	13.740	0.421	0.421	0.124
V6T3	7.815	-0.393	-0.393	-0.124

Model 4b. Partial Residual Variance Invariance Model (error variances held equal for all except non-invariant items and v5T2/T3)

```
MODEL: ! Model 4b: Residual Variances
       ! except for non-invariant items, v5T2-v5T3

! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*]; [v2T2* v2T3*] (I2); ! 3d: I2 applies only to 2 and 3
  [v3T1*   v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4); ! 3c: I4 applies only to 2 and 3
  [v5T1*]; [v5T2* v5T3*] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*   v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE) except v5T2-v5T3
  v1T1*; v1T2* v1T3* (E1); !4a: E1 applies only to 2 and 3
  v2T1*; v2T2* v2T3* (E2); !4a: E2 applies only to 2 and 3
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4); !4a: E4 applies only to 2 and 3
  v5T1*; v5T2*; v5T3*;     !4b: 2 and 3 now also separate
  v6T1*  v6T2* v6T3* (E6);
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

Does the partial residual model (4b) fit *better* than the full residual model (4a)? Yes, $-2\Delta LL(df=1) = 10.06$, $p < .01$

Does the partial residual model (4b) fit *worse* than the partial scalar model (3d)? Eh, $-2\Delta LL(df=7) = 14.14$, $p = .05$
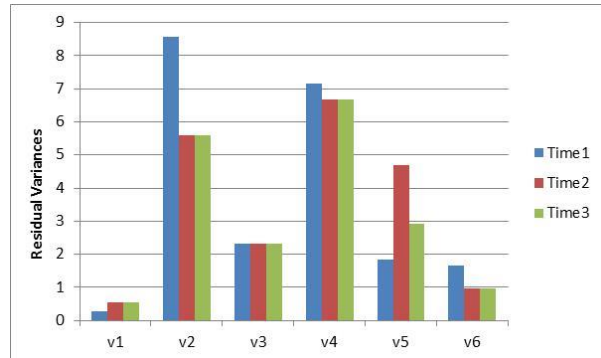
Modification indices suggest freeing v6 from Time1, so let's try that next.

```
MODEL FIT INFORMATION
Number of Free Parameters                    53

Loglikelihood
        H0 Value                      -4447.259
        H0 Scaling Correction Factor     1.5823
           for MLR
        H1 Value                      -4284.045
        H1 Scaling Correction Factor     1.2029
           for MLR


Information Criteria
        Akaike (AIC)                   9000.518
        Bayesian (BIC)                 9160.434
        Sample-Size Adjusted BIC       8992.694
           (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                           309.384*
        Degrees of Freedom                   136
        P-Value                           0.0000
        Scaling Correction Factor         1.0551
           for MLR


RMSEA (Root Mean Square Error Of Approximation)
        Estimate                          0.092
        90 Percent C.I.            0.078  0.105
        Probability RMSEA <= .05          0.000

CFI/TLI
        CFI                               0.901
        TLI                               0.888

SRMR (Standardized Root Mean Square Residual)
        Value                             0.093

MODEL MODIFICATION INDICES (relevant for invariance testing)
Means/Intercepts/Thresholds

                  M.I.   E.P.C.  Std E.P.C.  StdYX E.P.C.
Variances/Residual Variances

V6T1             13.772   0.419      0.419         0.125
V6T3              7.149  -0.373     -0.373        -0.118
```

Model 4c. Partial Residual Variance Invariance Model (error variances held equal for all except non-invariant items, v5T2/T3, v6T1)

```
MODEL: ! Model 4c: Residual Variances
        ! except for non-invariant items, v5T2-v5T3, v6T1
! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1 v2T1
  [v1T1*]; [v1T2* v1T3*] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*]; [v2T2* v2T3*] (I2); ! 3d: I2 applies only to 2 and 3
  [v3T1*    v3T2* v3T3*] (I3);
  [v4T1*]; [v4T2* v4T3*] (I4); ! 3c: I4 applies only to 2 and 3
  [v5T1*]; [v5T2* v5T3*] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*    v6T2* v6T3*] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE) except v5T2-v5T3, v6T1
  v1T1*; v1T2* v1T3* (E1); !4a: E1 applies only to 2 and 3
  v2T1*; v2T2* v2T3* (E2); !4a: E2 applies only to 2 and 3
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4); !4a: E4 applies only to 2 and 3
  v5T1*; v5T2*; v5T3*;     !4b: 2 and 3 now also separate
  v6T1*; v6T2* v6T3* (E6); !4c: E6 applies only to 2 and 3
! Factor variance AT TIME 1 fixed=1 for identification
  Time1@1 Time2* Time3*;
! Factor mean AT TIME 1 fixed=0 for identification
  [Time1@0 Time2* Time3*];
! Factor covariances all freely estimated
  Time1 Time2 Time3 WITH Time1* Time2* Time3*;
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```



```
MODEL FIT INFORMATION
Number of Free Parameters                    54

Loglikelihood
        H0 Value                        -4439.971
        H0 Scaling Correction Factor       1.5771
            for MLR
        H1 Value                        -4284.045
        H1 Scaling Correction Factor       1.2029
            for MLR

Information Criteria
        Akaike (AIC)                     8987.942
        Bayesian (BIC)                   9150.876
        Sample-Size Adjusted BIC         8979.971
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                             296.084*
        Degrees of Freedom                     135
        P-Value                            0.0000
        Scaling Correction Factor          1.0533
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.089
        90 Percent C.I.                    0.075  0.103
        Probability RMSEA <= .05           0.000

CFI/TLI
        CFI                                0.908
        TLI                                0.895

SRMR (Standardized Root Mean Square Residual)
        Value                              0.092
```

Does the partial residual model (4c) fit *better* than the partial residual model (4b)? Yes, −2ΔLL(df=1) = 11.20, *p* <.01

Does the partial residual model (4c) fit *worse* than the partial scalar model (3d)? No, −2ΔLL(df=6) = 3.38, *p* =.76

No invariance-related modification indices remain, so we are done! The residual variances at the end of Model 4c are shown on the left. Next is structural invariance.

4c UNSTANDARDIZED FINAL MEASUREMENT INVARIANCE SOLUTION

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value			Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
TIME1 BY						Means (FACTOR MEAN AT TIME1 FIXED=0 FOR IDENTIFICATION)				
V1T1	3.214	0.259	12.409	0.000		TIME1	0.000	0.000	999.000	999.000
V2T1	1.945	0.200	9.735	0.000		TIME2	0.295	0.081	3.654	0.000
V3T1	1.983	0.196	10.094	0.000		TIME3	0.520	0.092	5.668	0.000
V4T1	1.913	0.219	8.741	0.000						
V5T1	0.987	0.138	7.154	0.000		Intercepts – V3 AND V6 ARE HOLDING THIS TOGETHER WITH TIME1				
V6T1	1.470	0.123	11.975	0.000		**V1T1**	**16.089**	**0.275**	**58.597**	**0.000**
						V1T2	16.418	0.283	58.056	0.000
TIME2 BY						V1T3	16.418	0.283	58.056	0.000
V1T2	2.644	0.230	11.473	0.000		**V2T1**	**8.675**	**0.294**	**29.523**	**0.000**
V2T2	1.945	0.200	9.735	0.000		V2T2	9.416	0.262	35.991	0.000
V3T2	1.983	0.196	10.094	0.000		V2T3	9.416	0.262	35.991	0.000
V4T2	1.913	0.219	8.741	0.000		V3T1	11.950	0.225	53.170	0.000
V5T2	0.987	0.138	7.154	0.000		V3T2	11.950	0.225	53.170	0.000
V6T2	1.470	0.123	11.975	0.000		V3T3	11.950	0.225	53.170	0.000
						V4T1	**-3.024**	**0.266**	**-11.352**	**0.000**
TIME3 BY						V4T2	-3.750	0.298	-12.565	0.000
V1T3	2.644	0.230	11.473	0.000		V4T3	-3.750	0.298	-12.565	0.000
V2T3	1.945	0.200	9.735	0.000		**V5T1**	**-1.213**	**0.131**	**-9.275**	**0.000**
V3T3	1.983	0.196	10.094	0.000		V5T2	-1.803	0.207	-8.720	0.000
V4T3	1.913	0.219	8.741	0.000		V5T3	-1.803	0.207	-8.720	0.000
V5T3	0.987	0.138	7.154	0.000		V6T1	-2.851	0.160	-17.815	0.000
V6T3	1.470	0.123	11.975	0.000		V6T2	-2.851	0.160	-17.815	0.000
						V6T3	-2.851	0.160	-17.815	0.000
TIME1 WITH										
TIME2	0.843	0.078	10.745	0.000		Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR				
TIME3	0.683	0.124	5.505	0.000		**V1T1**	**0.285**	**0.342**	**0.831**	**0.406**
						V1T2	0.539	0.233	2.316	0.021
TIME2 WITH						V1T3	0.539	0.233	2.316	0.021
TIME3	0.692	0.126	5.489	0.000		**V2T1**	**8.562**	**1.004**	**8.526**	**0.000**
						V2T2	5.592	0.502	11.132	0.000
*** Residual covariances among same item across time ****						V2T3	5.592	0.502	11.132	0.000
						V3T1	2.312	0.271	8.534	0.000
V1T1 WITH						V3T2	2.312	0.271	8.534	0.000
V1T2	-0.165	0.230	-0.716	0.474		V3T3	2.312	0.271	8.534	0.000
V1T3	0.014	0.212	0.066	0.948		**V4T1**	**7.139**	**1.043**	**6.842**	**0.000**
						V4T2	6.686	0.870	7.684	0.000
V1T2 WITH						V4T3	6.686	0.870	7.684	0.000
V1T3	0.153	0.230	0.667	0.505		**V5T1**	**1.829**	**0.448**	**4.078**	**0.000**
....						**V5T2**	**4.705**	**1.455**	**3.233**	**0.001**
						V5T3	**2.908**	**0.749**	**3.881**	**0.000**
Variances (FACTOR VARIANCE AT TIME1=1 FOR IDENTIFICATION)						**V6T1**	**1.664**	**0.233**	**7.138**	**0.000**
TIME1	1.000	0.000	999.000	999.000		V6T2	0.957	0.136	7.039	0.000
TIME2	1.159	0.186	6.231	0.000		V6T3	0.957	0.136	7.039	0.000
TIME3	0.934	0.151	6.171	0.000						

STRUCTURAL INVARIANCE TESTS

Model 5a. Factor Variance Invariance Model	Model 6a. Factor Covariance Invariance Model

Model 5a. Factor Variance Invariance Model

```
MODEL: ! Model 5a: Factor Variance Invariance
(rest of code before and after is same as 4c)

! Model 5a: Factor Variance Invariance (all fixed to 1 now)
    Time1@1 Time2@1 Time3@1;

MODEL FIT INFORMATION
Number of Free Parameters                      52
Loglikelihood
        H0 Value                        -4441.238
        H0 Scaling Correction Factor      1.5848
          for MLR
        H1 Value                        -4284.045
        H1 Scaling Correction Factor      1.2029
          for MLR
Information Criteria
        Akaike (AIC)                     8986.475
        Bayesian (BIC)                   9143.374
        Sample-Size Adjusted BIC         8978.799
          (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
        Value                             297.152*
        Degrees of Freedom                     137
        P-Value                            0.0000
        Scaling Correction Factor          1.0580
          for MLR
RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.088
        90 Percent C.I.                    0.074  0.102
        Probability RMSEA <= .05           0.000
CFI/TLI
        CFI                                0.908
        TLI                                0.897
SRMR (Standardized Root Mean Square Residual)
        Value                              0.100
```

Does the factor variance model (5a) fit *worse* than the partial residual model (4c)? No, $-2\Delta LL(df=2) = 1.84$, $p = .40$

```
Factor Covariances...
TIME1    WITH
    TIME2             0.778     0.042    18.375     0.000
    TIME3             0.713     0.087     8.214     0.000
 TIME2    WITH
    TIME3             0.662     0.095     6.929     0.000
```

Model 6a. Factor Covariance Invariance Model

```
MODEL: ! Model 6a: Factor Covariance Invariance
(rest of code before and after is same as 5a)

! Model 6a: Factor Covariance Invariance (all fixed equal)
Time1 Time2 Time3 WITH Time1* Time2* Time3* (Fcov);

MODEL FIT INFORMATION
Number of Free Parameters                      50
Loglikelihood
        H0 Value                        -4443.654
        H0 Scaling Correction Factor      1.5649
          for MLR
        H1 Value                        -4284.045
        H1 Scaling Correction Factor      1.2029
          for MLR
Information Criteria
        Akaike (AIC)                     8987.308
        Bayesian (BIC)                   9138.172
        Sample-Size Adjusted BIC         8979.927
          (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
        Value                             297.568*
        Degrees of Freedom                     139
        P-Value                            0.0000
        Scaling Correction Factor          1.0728
          for MLR
RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.087
        90 Percent C.I.                    0.073  0.101
        Probability RMSEA <= .05           0.000
CFI/TLI
        CFI                                0.909
        TLI                                0.900
SRMR (Standardized Root Mean Square Residual)
        Value                              0.100
```

Does the factor covariance model (6a) fit *worse* than the factor variance model (5a)? No, $-2\Delta LL(df=2) = 2.32$, $p = .31$

```
FACTOR COVARIANCES FROM MODEL 6a (REPRESENT CORRELATIONS):
TIME1 WITH TIME2   0.724     0.053    13.748     0.000
TIME1 WITH TIME3   0.724     0.053    13.748     0.000
TIME2 WITH TIME3   0.724     0.053    13.748     0.000
FACTOR MEANS FROM MODEL 6a (REPRESENT MEAN DIFFERENCES):
    TIME1          0.000     0.000   999.000   999.000
    TIME2          0.284     0.079     3.605     0.000
    TIME3          0.520     0.091     5.700     0.000
```
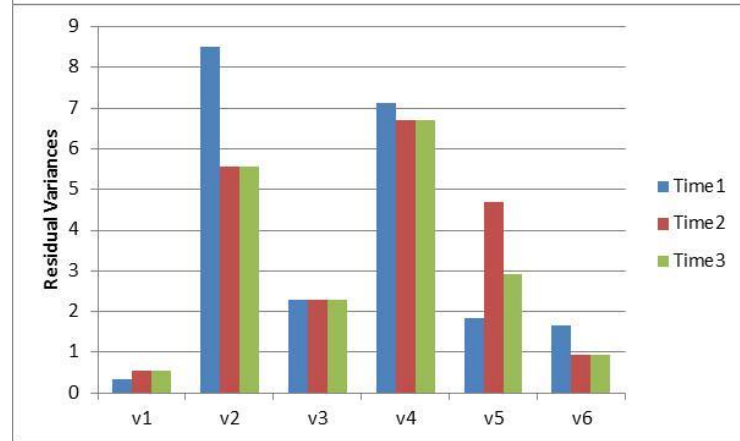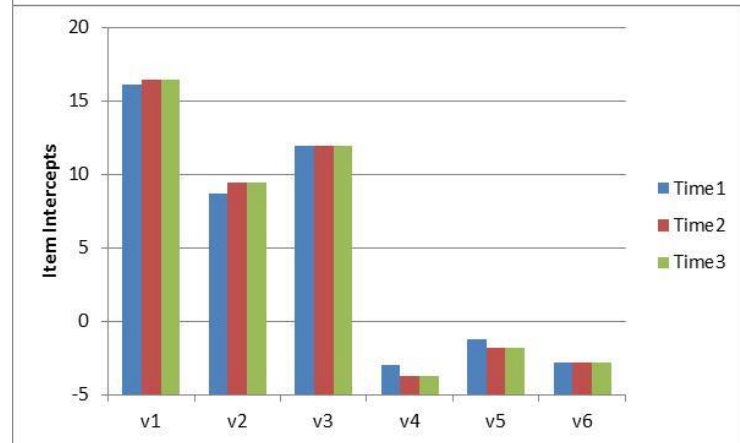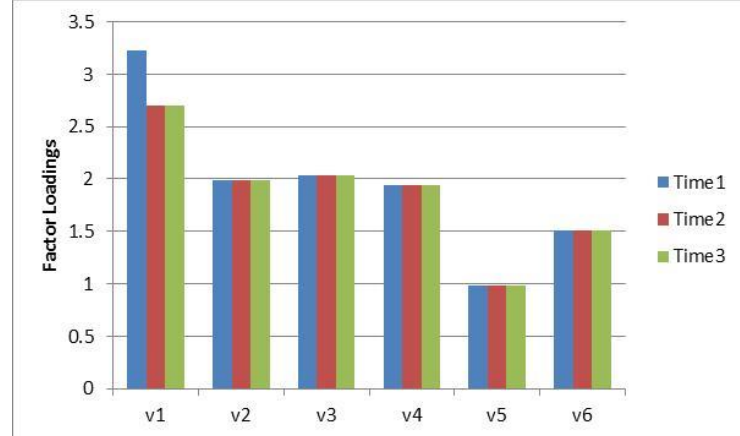
Model 7a. Factor Mean Invariance Model

```
MODEL: ! Model 7a: Factor Mean Invariance
       ! Testing Diff between Time2 and Time3
! Factor loadings NOW CONSTRAINED EQUAL ACROSS TIME EXCEPT v1T1
  Time1 BY v1T1* v2T1* v3T1* v4T1* v5T1* v6T1* (L1a L2-L6);
  Time2 BY v1T2* v2T2* v3T2* v4T2* v5T2* v6T2* (L1-L6);
  Time3 BY v1T3* v2T3* v3T3* v4T3* v5T3* v6T3* (L1-l6);
! Item intercepts NOW CONSTRAINED EQUAL ACROSS TIME,
! no v1T1 v5T1 v4T1
  [v1T1*]; [v1T2* v1T3] (I1); ! 3a: I1 applies only to 2 and 3
  [v2T1*]; [v2T2* v2T3] (I2); ! 3d: I2 applies only to 2 and 3
  [v3T1*    v3T2* v3T3] (I3);
  [v4T1*]; [v4T2* v4T3] (I4); ! 3c: I4 applies only to 2 and 3
  [v5T1*]; [v5T2* v5T3] (I5); ! 3b: I5 applies only to 2 and 3
  [v6T1*    v6T2* v6T3] (I6);
! Residual variances NOW CONSTRAINED EQUAL ACROSS TIME
  (WHEN POSSIBLE) except v5T2-v5T3, v6T1
  v1T1*; v1T2* v1T3* (E1); !4a: E1 applies only to 2 and 3
  v2T1*; v2T2* v2T3* (E2); !4a: E2 applies only to 2 and 3
  v3T1*  v3T2* v3T3* (E3);
  v4T1*; v4T2* v4T3* (E4); !4a: E4 applies only to 2 and 3
  v5T1*; v5T2*; v5T3*;       !4b: 2 and 3 now also separate
  v6T1*; v6T2* v6T3* (E6); !4c: E6 applies only to 2 and 3
! Factor variance fixed=1 for structural invariance
  Time1@1 Time2@1 Time3@1;
! Testing factor mean difference between Time2 and Time3
  [Time1@0]; [Time2* Time3*] (Fmean); ! NEW CONSTRAINT
! Factor covariances held equal for structural invariance
  Time1 Time2 Time3 WITH Time1* Time2* Time3* (Fcov);
! Residual covariances estimated for same item across time
  v1T1 v1T2 v1T3 WITH v1T1* v1T2* v1T3*;
  v2T1 v2T2 v2T3 WITH v2T1* v2T2* v2T3*;
  v3T1 v3T2 v3T3 WITH v3T1* v3T2* v3T3*;
  v4T1 v4T2 v4T3 WITH v4T1* v4T2* v4T3*;
  v5T1 v5T2 v5T3 WITH v5T1* v5T2* v5T3*;
  v6T1 v6T2 v6T3 WITH v6T1* v6T2* v6T3*;
```

Does the factor mean model (7a) fit *worse* than the factor covariance model (6a)? Yes, $-2\Delta LL(df=1) = 11.15$, $p < .01$, so we keep Model 6a instead.

```
MODEL FIT INFORMATION
Number of Free Parameters                     49
Loglikelihood
          H0 Value                      -4448.472
          H0 Scaling Correction Factor     1.5792
              for MLR
Means
    TIME1            0.000      0.000    999.000    999.000
    TIME2            0.378      0.075      5.014      0.000
    TIME3            0.378      0.075      5.014      0.000
```

6a UNSTANDARDIZED FINAL STRUCTURAL INVARIANCE SOLUTION

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value			Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
TIME1 BY						Means (FACTOR MEAN AT TIME1 FIXED=0 FOR IDENTIFICATION)				
V1T1	3.229	0.243	13.272	0.000		TIME1	0.000	0.000	999.000	999.000
V2T1	1.993	0.170	11.754	0.000		TIME2	0.284	0.079	3.605	0.000
V3T1	2.029	0.169	12.022	0.000		TIME3	0.520	0.091	5.700	0.000
V4T1	1.939	0.214	9.077	0.000						
V5T1	0.986	0.147	6.701	0.000		Intercepts – V3 AND V6 ARE HOLDING THIS TOGETHER WITH TIME1				
V6T1	1.508	0.109	13.821	0.000		**V1T1**	**16.099**	**0.271**	**59.420**	**0.000**
						V1T2	16.428	0.281	58.488	0.000
TIME2 BY						V1T3	16.428	0.281	58.488	0.000
V1T2	2.704	0.232	11.677	0.000		**V2T1**	**8.681**	**0.292**	**29.694**	**0.000**
V2T2	1.993	0.170	11.754	0.000		V2T2	9.423	0.259	36.368	0.000
V3T2	2.029	0.169	12.022	0.000		V2T3	9.423	0.259	36.368	0.000
V4T2	1.939	0.214	9.077	0.000		V3T1	11.956	0.223	53.706	0.000
V5T2	0.986	0.147	6.701	0.000		V3T2	11.956	0.223	53.706	0.000
V6T2	1.508	0.109	13.821	0.000		V3T3	11.956	0.223	53.706	0.000
						V4T1	**-3.018**	**0.263**	**-11.463**	**0.000**
TIME3 BY						V4T2	-3.737	0.292	-12.784	0.000
V1T3	2.704	0.232	11.677	0.000		V4T3	-3.737	0.292	-12.784	0.000
V2T3	1.993	0.170	11.754	0.000		**V5T1**	**-1.210**	**0.131**	**-9.269**	**0.000**
V3T3	2.029	0.169	12.022	0.000		V5T2	-1.791	0.203	-8.807	0.000
V4T3	1.939	0.214	9.077	0.000		V5T3	-1.791	0.203	-8.807	0.000
V5T3	0.986	0.147	6.701	0.000		V6T1	-2.847	0.159	-17.889	0.000
V6T3	1.508	0.109	13.821	0.000		V6T2	-2.847	0.159	-17.889	0.000
						V6T3	-2.847	0.159	-17.889	0.000
TIME1 WITH										
TIME2	0.724	0.053	13.748	0.000		Residual Variances – ITEM VARIANCE THAT IS NOT THE FACTOR				
TIME3	0.724	0.053	13.748	0.000		**V1T1**	**0.351**	**0.331**	**1.060**	**0.289**
						V1T2	0.562	0.231	2.432	0.015
TIME2 WITH						V1T3	0.562	0.231	2.432	0.015
TIME3	0.724	0.053	13.748	0.000		**V2T1**	**8.506**	**0.999**	**8.511**	**0.000**
						V2T2	5.563	0.494	11.261	0.000
*** Residual covariances among same item across time ****						V2T3	5.563	0.494	11.261	0.000
						V3T1	2.288	0.269	8.507	0.000
V1T1 WITH						V3T2	2.288	0.269	8.507	0.000
V1T2	-0.106	0.225	-0.471	0.638		V3T3	2.288	0.269	8.507	0.000
V1T3	0.038	0.215	0.175	0.861		**V4T1**	**7.134**	**1.041**	**6.853**	**0.000**
						V4T2	6.694	0.873	7.666	0.000
V1T2 WITH						V4T3	6.694	0.873	7.666	0.000
V1T3	0.130	0.243	0.534			**V5T1**	**1.825**	**0.446**	**4.092**	**0.000**
0.593..........						**V5T2**	**4.705**	**1.454**	**3.235**	**0.001**
						V5T3	**2.921**	**0.752**	**3.887**	**0.000**
Variances (FACTOR VARIANCES CONSTRAINED EQUAL)						**V6T1**	**1.656**	**0.235**	**7.054**	**0.000**
TIME1	1.000	0.000	999.000	999.000		V6T2	0.942	0.131	7.188	0.000
TIME2	1.000	0.000	999.000	999.000		V6T3	0.942	0.131	7.188	0.000
TIME3	1.000	0.000	999.000	999.000						

Example results section for these analyses:

The extent to which a confirmatory factor model measuring social functioning (with six observed indicators) exhibited measurement invariance and structural invariance over time (i.e., across three occasions taken at six-month intervals) was examined using M*plus* v. 8.4 (Muthén & Muthén, 1998–2017). Robust maximum likelihood (MLR) estimation was used for all analyses; accordingly, nested model comparisons were conducted using the rescaled difference in the model −2LL values as a function of the difference in model degrees of freedom. A configural invariance model was initially specified in which three correlated factors (i.e., the factor at the three occasions) were estimated simultaneously; all factor means were fixed to 0 and all factor variances were fixed to 1 for identification. Residual covariances between the same indicators across occasions were estimated as well. As shown in Table 1, although the configural invariance model had marginal fit, theoretically reasonable attempts to improve the fit were unsuccessful. Thus, the analysis proceeded by applying parameter constraints in successive models to examine potential decreases in fit resulting from measurement or structural non-invariance over the three occasions.

Equality of the unstandardized indicator factor loadings across occasions was then examined in a metric invariance model. The factor variance was fixed to 1 at time 1 but was freely estimated at times 2 and 3. All factor loadings were constrained equal across time; all intercepts and residual variances were still permitted to vary across time. Factor covariances and residual covariances were estimated as described previously. The metric invariance model fit significantly worse than the configural invariance model $-2\Delta LL(10) = 19.14$, $p = .04$. The modification indices suggested that the loading of indicator 1 at time 1 was a source of misfit and should be freed. After doing so, the partial metric invariance model fit significantly better than the full metric invariance model, $-2\Delta LL(1) = 7.16$, $p < .001$, and the partial metric invariance model did not fit worse than the configural invariance model $-2\Delta LL(9) = 8.98$, $p = .44$. The fact that partial metric invariance (i.e., "weak invariance") held indicates that the indicators were related to the latent factor equivalently across time, or more simply, that the same latent factor was being measured at each of occasion (with the exception of indicator 1, which was more related to the factor at time 1 than at times 2 or 3).

Equality of the unstandardized indicator intercepts across time was then examined in a scalar invariance model. The factor mean and variance were fixed to 0 and 1, respectively, at time 1 for identification, but the factor mean and variance were then estimated at times 2 and 3. All factor loadings and indicator intercepts were constrained equal across time (except for indicator 1 at time 1); all residual variances were still permitted to differ across time. Factor covariances and residual covariances were estimated as described previously. The scalar invariance model fit significantly worse than the partial metric invariance model, $-2\Delta LL(9) = 55.13$, $p < .01$. The modification indices suggested that the intercept of indicator 5 at time 1 was the largest source of the misfit and should be freed. After doing so, although the partial scalar invariance model had significantly better fit than the full scalar invariance model, $-2\Delta LL(1) = 15.16$, $p < .01$, it still fit worse than the partial metric invariance model, $-2\Delta LL(8) = 27.84$, $p < 001$. The modification indices suggested that the intercept of indicator 4 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, although the new partial scalar invariance model (with the intercepts for indicators 1, 4, and 5 freed at time 1) fit significantly better than the previous partial scalar invariance model (without the intercept for indicator 4 freed at time 1), $-2\Delta LL(1) = 9.24$, $p < .01$, it still fit marginally worse than the partial metric invariance model, $-2\Delta LL(7) = 13.99$, $p = 05$. The modification indices suggested that the intercept of indicator 2 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model (with the intercepts for indicators 1, 2, 4 and 5 freed at time 1) fit significantly better than the previous partial scalar invariance model (without the intercept for indicator 2 freed at time 1), $-2\Delta LL(1) = 8.73$, $p < .01$, and it did not fit significantly worse than the partial metric invariance model, $-2\Delta LL(6) = 4.35$, $p = .63$. The fact that partial scalar invariance (i.e., "strong invariance") held indicates that times 2 and 3 have the same expected response for each indicator at the same absolute level of the trait, or more simply, that the observed differences in the indicator means between times 2 and 3 is due to factor mean differences only. However, indicators 1 and 2 had a lower expected indicator response at the same absolute level of social functioning at time 1 than at time 2 or 3, while indicators 4 and 5 had a higher expected response.

Equality of the unstandardized residual variances across time was then examined in a residual variance invariance model. As in the partial scalar invariance model, the factor mean and variance were fixed to 0 and 1, respectively, for identification at time 1, but the factor mean and variance were still estimated at times 2 and 3. All factor loadings (except for indicator 1 at time 1), item intercepts (except for indicators 1, 2, 4, and 5 at time 1), and all residual variances (except for indicators 1, 2, 4, and 5 at time 1) were constrained to be equal across groups. Factor covariances and residual covariances were estimated as described previously. The residual variance invariance model fit significantly worse than the last partial scalar invariance model, $-2\Delta LL(8) = 24.72$, $p < .01$. The modification indices suggested that the residual variance of indicator 5 at time 2 versus time 3 was the largest remaining source of the misfit and should be

freed. After doing so, the partial residual variance invariance model fit significantly better than the residual invariance model, −2ΔLL(1) = 10.06, $p < .01$, but still fit marginally worse than the last partial scalar invariance model, −2ΔLL(7) = 14.14, $p = .05$. The modification indices suggested that the residual variance of indicator 6 at time 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial residual variance invariance model (with residual variances for indicators 1, 2, 4, 5, and 6 free at time 1; indicator 5 free at times 2 and 3 also) fit significantly better than the partial residual invariance model (without the residual variance for indicator 6 at time 1 freed), −2ΔLL(1) = 11.20, $p < .01$, and did not fit worse than the last partial scalar invariance model, −2ΔLL(6) = 3.38, $p = .76$. The fact that partial residual variance invariance (i.e., "strict invariance") held indicates that the amount of indicator variance not accounted for by the factor was the same across times 2 and 3 (except for indicator 5, for which there was more residual variance at time 2). However, 5 out of 6 indicators did not have residual variance invariance at time 1 (although this was required because of a lack of metric or scalar invariance for indicators 1, 2, 4, and 5).

After achieving partial measurement invariance as was just described, structural invariance was then tested with three additional models. First, the factor variance at times 2 and 3 (which had been estimated freely) was constrained to 1 (i.e., to be equal to the factor variance at time 1), resulting in a nonsignificant decrease in fit relative to the last partial residual invariance model, −2ΔLL(2) = 1.84, $p = .40$. Thus, equivalent amounts of individual differences in social functioning were found across time. Second, the factor covariances across time were constrained to be equal (which become factor correlations given a variance of 1 for each factor across time), resulting in a nonsignificant decrease in fit relative to the factor variance invariance model, −2ΔLL(2) = 2.32, $p = .31$. Third, the factor means at times 2 and 3 (which had been estimated freely) was constrained to be equal to each other, resulting in a significant decrease in fit relative to the factor covariance invariance model −2ΔLL(1) = 11.15, $p < .01$, indicating that the factor mean at time 3 was significantly higher than at time 2. The factor mean at time 2 was already significantly different from 0 (the factor mean at time 1), and thus, the three factor means were significantly different, increasing over time.

In conclusion, these analyses showed that partial measurement invariance was obtained over time—that is, the relationships of the indicators to the latent factor of social functioning were equivalent at times 2 and 3, although primarily not equivalent at time 1, as previous described. These analyses also showed that partial structural invariance was obtained over time, such that the same amount of individual differences variance in social functioning was observed with equal covariance over time across occasions (i.e., compound symmetry of the latent factor), although the amount of social functioning on average increased significantly over time. Model parameters from the final model are given in Table 2.

Reference: Muthén, L. K., & Muthén, B.O. (1998–2017). *Mplus user's guide* (8th ed.). Los Angeles, CA: Muthén & Muthén.

(see excel worksheet for Table 1; Table 2 would have unstandardized and standardized estimates and their SEs)

You might also replace all the nested model comparisons tests in the text with a table that provides them instead.