

# Higher-Order Factor and Method Factor (Bifactor) Models

- Topics:
  - The Big Picture
  - Identification and testing of higher-order models
  - Revisiting item construction
  - Method factor and bifactor models
  - Using constraints to support interpretation
  - Examples of abbreviating structural models

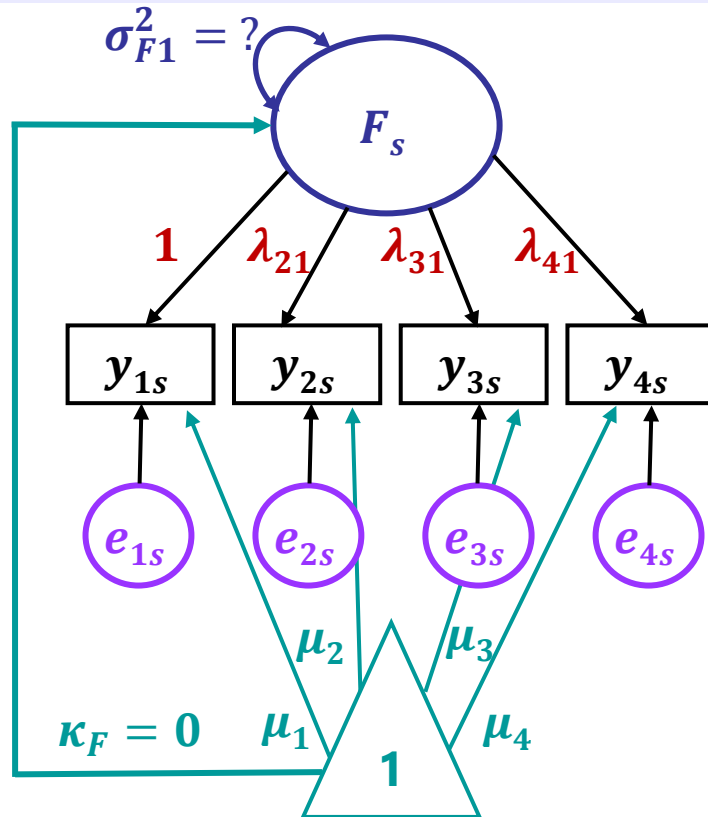
# Sequence of Steps in CFA or IFA

1. Specify your **measurement model**(s)
  - How many factors/thetas, which items load on which factors, and whether you need **any additional factors or error covariances**
  - For models with large numbers of items, you should start by modeling each factor in its own analysis to make sure \*each\* factor fits its items
2. Assess model fit, per factor, when possible (if 4+ indicators)
  - **Global model fit:** Does the hypothesized factor model adequately fit each set of indicators thought to measure the same latent construct?
  - **Local model fit:** Are any of the covariance discrepancies problematic? Any items not loading well (or are too redundant) that you might drop?
  - **Reliability/Info:** Are your standardized loadings practically meaningful?
3. Once the per-factor measurement models are good enough, then we consider the structural model → factors and their relations
  - e.g., Are the “subscale” factors indicators of a **higher-order factor**?
  - e.g., Should “**method**” or “**specific**” factors (in a “**bifactor**” model) replace error covariances due to positive/negative wording or other commonalities?

# Higher-Order Factor Models

- Purpose: What kind of higher-order factor structure best accounts for the **covariance among the measurement model *factors* (NOT the items)**?
  - In other words, what should the **structural model among the factors** look like?
  - Best-fitting baseline for the structural model has all possible covariances among the lower-order measurement model factors → **saturated structural model**
  - Just as the purpose of the measurement model factors is to predict covariance among the items, the **purpose of the higher-order factors is to predict covariance among the measurement model factors themselves**
  - **A single higher-order factor** would be suggested by **similar magnitude of correlations** across the measurement model lower-order factors
- Note that distinctions between CFA, IFA, and other measurement models for different item types are no longer relevant for the higher-order model!
  - Factors (thetas, traits) are all **multivariate normal latent variables**, so a higher-order model is like a CFA regardless of the measurement model for the items
  - Latent variables don't have means apart from their items, so those are irrelevant

# Recommended Measurement Model Scaling to fit Higher-Order Factors



## Use “marker Item” for factor loadings

- Fix 1 item loading to 1 per factor
- **Estimate** factor variance(s)

Because it will become “factor variance leftover” = “disturbance”, factor variance **should be freely** estimated (otherwise it can lead to estimation problems)

## Use “standardized factor” scaling for item intercepts or thresholds

- Fix factor mean(s) to 0
- Estimate all intercepts/thresholds

All the factor means will be 0 and you generally won't need to include them in the structural model anyway

# Identifying a 3-Factor Structural Model

## Option 1: 3 Correlated Factors

**Measurement Model for 12 Items:**  
*item variances, covariances, and means*

Possible DF =  $(12 \cdot 13) / 2 + 12 = 90$

Estimated DF =  $9\lambda_i + 12\mu_i + 12\sigma_{e_i}^2 = 33$

Leftover DF =  $90 - 33 = 57$

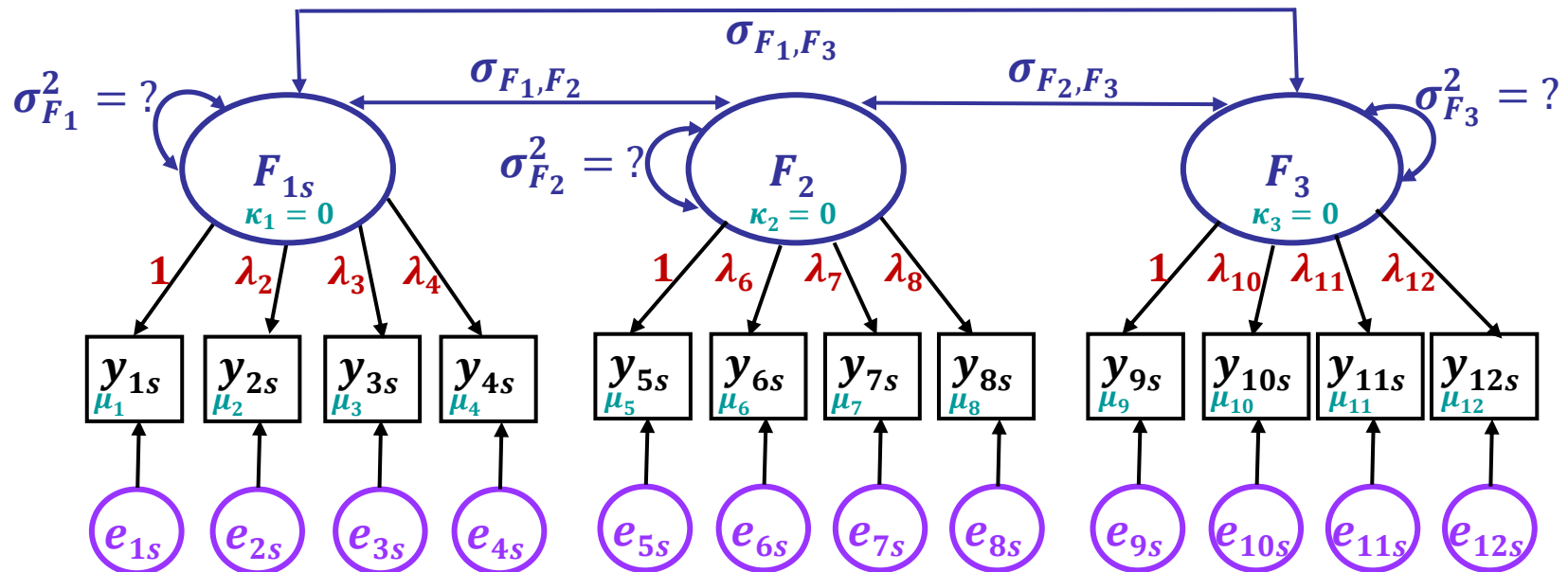
→ **over-identified**

**Structural Model for 3 Factors:**  
*factor variances and covariances, no means*

Possible DF =  $(3 \cdot 4) / 2 + 0 = 6$

Estimated DF = 3 variances + 3 covariances

Leftover DF =  $6 - 6 = 0 \rightarrow$  **just-identified**



# Option 2a: 3 Factor “Indicators” (Higher-Order Factor Variance = 1)

## Same Measurement Model for 12 Items:

Possible DF =  $(12 \cdot 13) / 2 + 12 = 90$

Estimated DF =  $9\lambda_i + 12\mu_i + 12\sigma_{e_i}^2 = 33$

Leftover DF =  $90 - 33 = 57$

→ **over-identified**

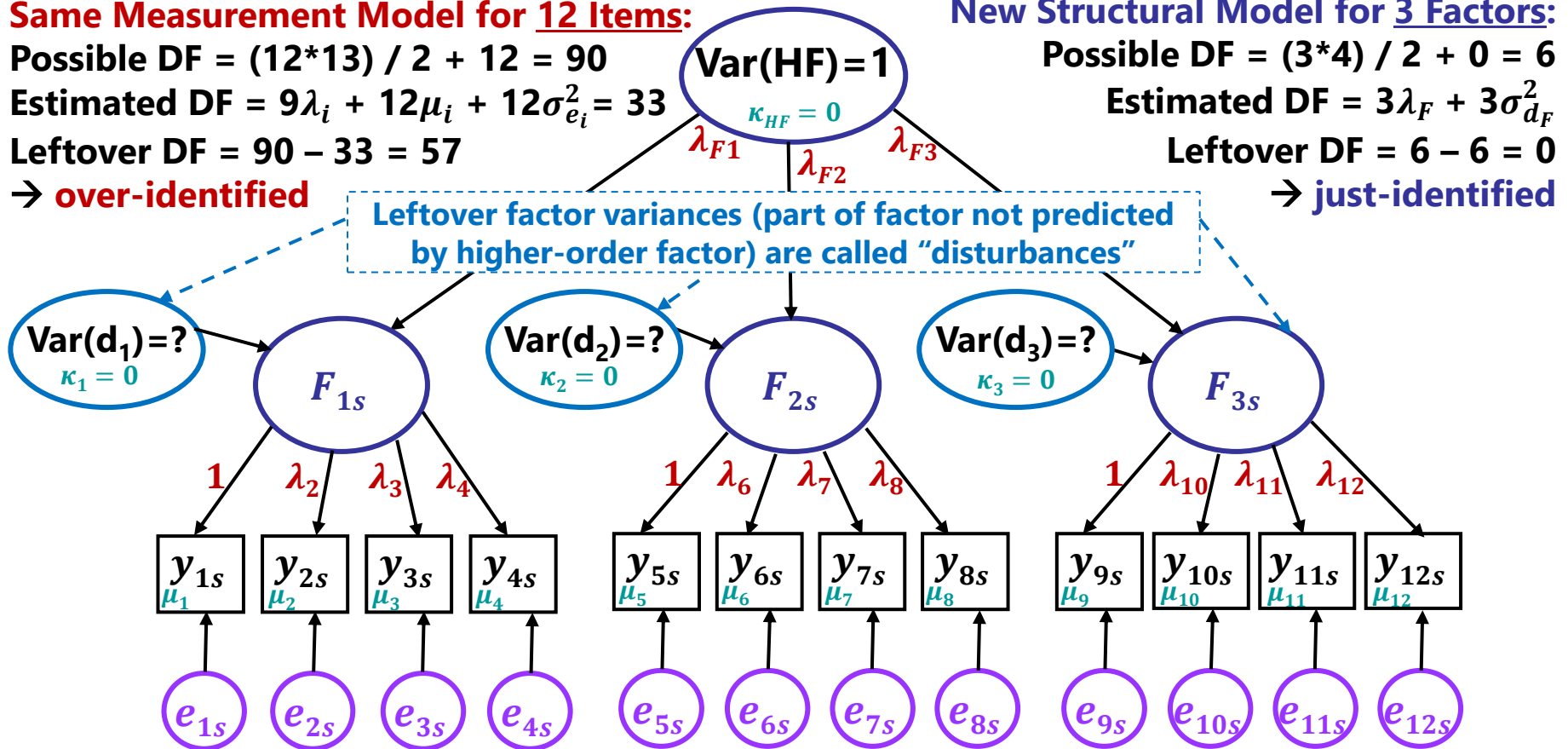
## New Structural Model for 3 Factors:

Possible DF =  $(3 \cdot 4) / 2 + 0 = 6$

Estimated DF =  $3\lambda_F + 3\sigma_{d_F}^2$

Leftover DF =  $6 - 6 = 0$

→ **just-identified**



For only 3 factors, both models will fit the same—structural model is just-identified, and so the fit of a higher-order factor CANNOT be tested (without constraints)

# Option 2b: 3 Factor “Indicators” (using Marker Lower-Order Factor)

## Same Measurement Model for 12 Items:

Possible DF =  $(12 \cdot 13) / 2 + 12 = 90$

Estimated DF =  $9\lambda_i + 12\mu_i + 12\sigma_{e_i}^2 = 33$

Leftover DF =  $90 - 33 = 57$

→ **over-identified**

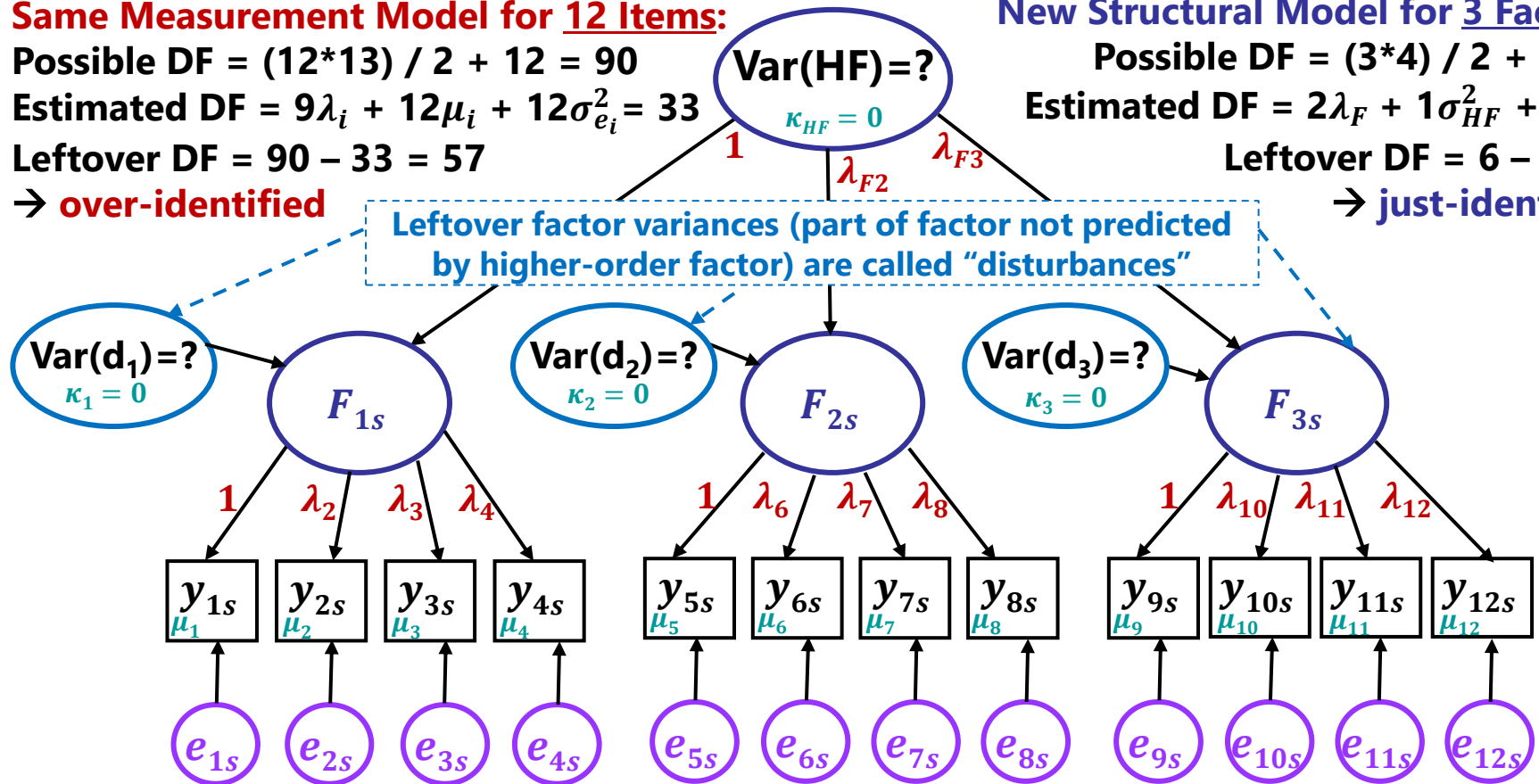
## New Structural Model for 3 Factors:

Possible DF =  $(3 \cdot 4) / 2 + 0 = 6$

Estimated DF =  $2\lambda_F + 1\sigma_{HF}^2 + 3\sigma_{d_F}^2$

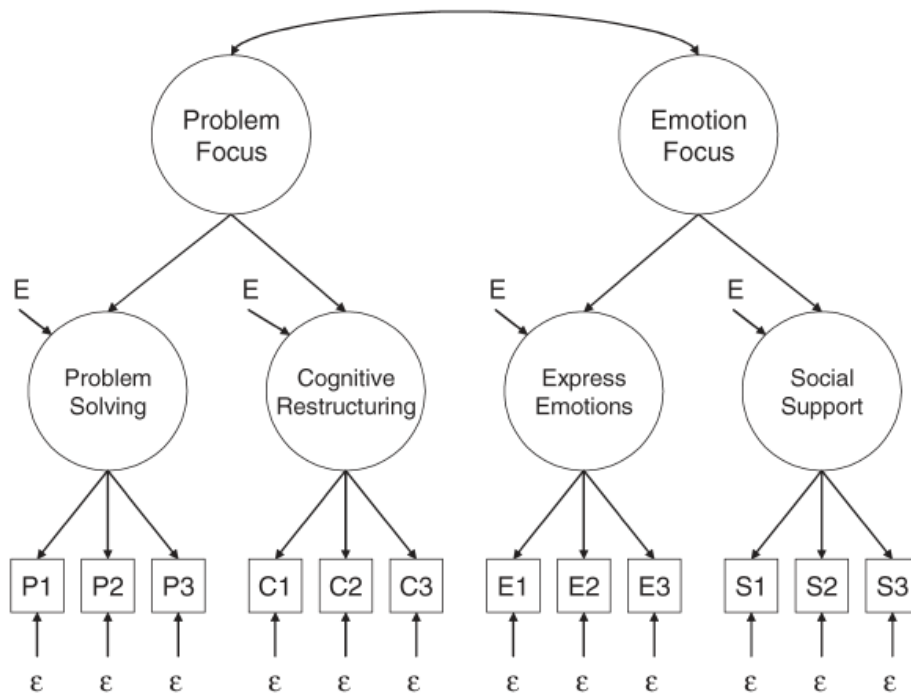
Leftover DF =  $6 - 6 = 0$

→ **just-identified**



For only 3 factors, both models will fit the same—structural model is just-identified, and so the fit of a higher-order factor CANNOT be tested (without constraints)

# Structural Model Identification: 4 Lower-Order Factor “Indicators”



From Brown (2015) Figure 8.1

## Measurement Model for 12 Items:

Possible DF =  $(12 \times 13) / 2 + 12 = 90$

Estimated DF =  $8\lambda_i + 12\mu_i + 12\sigma_{e_i}^2 = 32$

Leftover DF =  $90 - 32 = 58$

→ **over-identified**

## Structural Model for Factors:

Possible DF =  $(4 \times 5) / 2 + 0 = 10$

Estimated DF =  $4\lambda_F + 0\sigma_F^2 + 1\sigma_{F,F} + 4\sigma_{d_F}^2$   
— OR —

Estimated DF =  $2\lambda_F + 2\sigma_F^2 + 1\sigma_{F,F} + 4\sigma_{d_F}^2$

Leftover DF =  $10 - 9 = 1$

→ **over-identified**

However, this model requires a non-0 covariance between the higher-order factors to be structurally identified!



# Higher-Order Factor Identification

- Possible structural df depends on # of measurement model **factor variances and covariances** (NOT # items)
  - **2 measurement model factors → Under-identified**
    - They can be correlated, which would be just-identified...
    - Higher-order factor be estimated if both lower-order loadings are held equal
  - **3 measurement model factors → Just-identified**
    - They can all be correlated OR a single higher-order factor can be fit
    - Same # variance/disturbances per factor (so, 3 total) in either option
    - Factor variances and covariances will be perfectly reproduced (so no fit test)
  - **4 measurement model factors → Can be over-identified**
    - They can all be correlated (6 correlations required; just-identified)
    - They can have a higher-order factor (4 loadings; over-identified)
    - **The fit of the higher-order factor can now be tested**

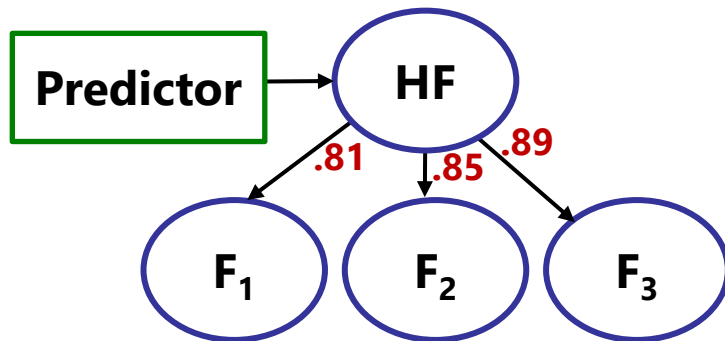
# Testing Higher-Order Factors

- Do I have a higher-order factor of my subscale factors?
  - If 4 or more subscale factors: Compare fit of alternative models
    - Saturated baseline: All 6 factor covariances estimated freely  
Alternative: 1 higher-order factor instead (so  $DF=2$ )—is model fit WORSE?
  - If 3 (or fewer) subscale factors: CANNOT BE DETERMINED
    - Saturated baseline and alternative models will fit equivalently (unless lower-order factor loadings or disturbance variances are constrained to save DF)
- Multiple higher-order factors may also be possible
  - e.g., [six indicators of general "school readiness" at age 5:](#)  
*approach to learning, math knowledge, language knowledge, social competence, emotional competence, behavior regulation*
    - Single higher-order factor → "academic" vs. "non-academic" readiness
  - Saturated structural model is still "answer key" for best fit
    - Higher-order factors are nested (use loadings to capture factor covariances)

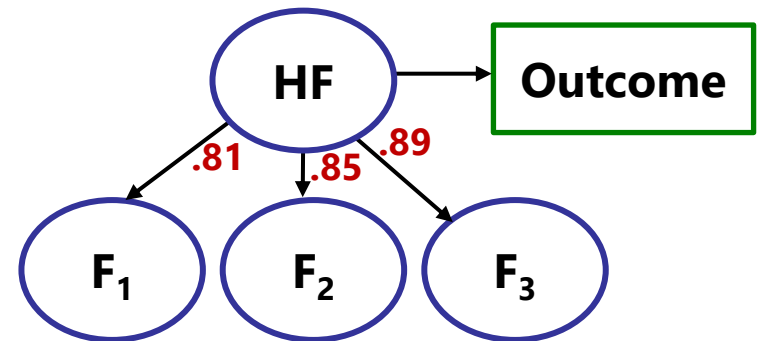
# Usefulness of Higher-Order Factors

- Whether or not higher-order factors are useful depends on the context of the rest of your structural model... and the lower-order factor correlations!
  - See examples below with standardized higher-order factor loadings...

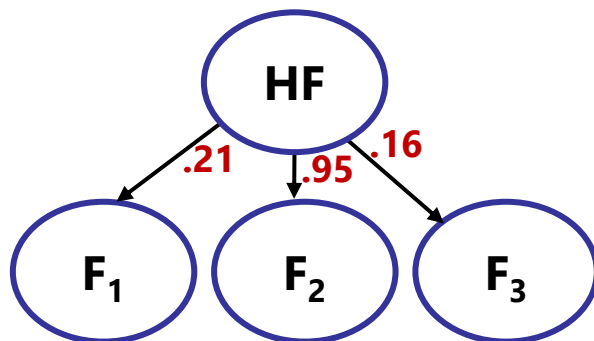
## Not that useful (just predict each)



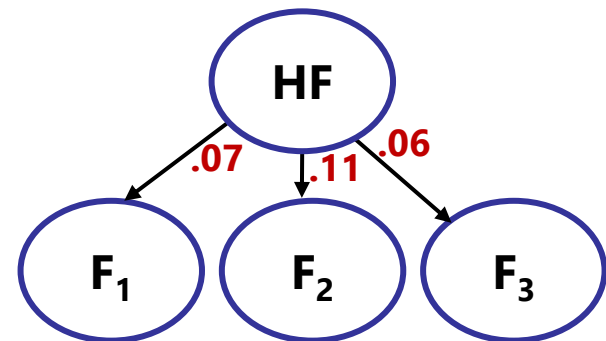
## More useful (common is predictor)



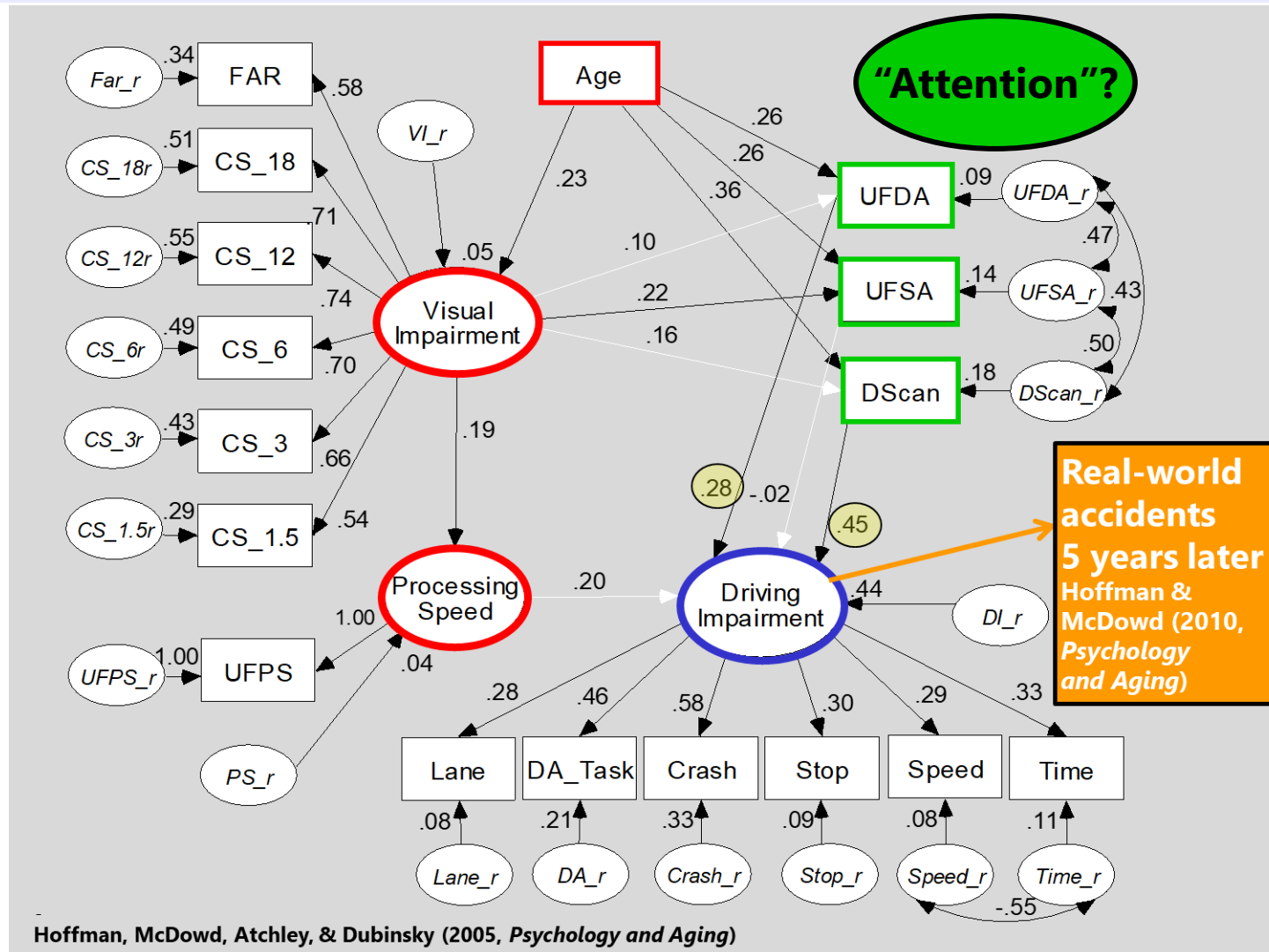
## Not that useful (b/c what is HF???)



## Not at all useful (HF is not a thing!)



# Another Example: A Factor is NOT Helpful if Focus is Indicators' Unique Prediction!



# Reconsidering Item Construction

- Latent variable measurement models (CFA, IFA/IRT, other)  
\*can\* represent multiple underlying constructs, but ***only to the extent that the items and their design can distinguish them***
- General principles for a single item:
  - Cover a **single concept** with a single, explicitly defined referent
  - Provide response options that are **applicable to all respondents**
  - Not too narrow, but not too broad... easier said than done!
- General principles for measuring multiple related constructs (i.e., subscales of a more general factor)
  - **Do not confound** any of the following: stem/valence/construct
  - You can avoid the need to reverse-code (or any other method-type confound) by re-writing items! **Yes, you are allowed to fix bad items!**

# Why Might These Items Have Problems?

- How important to you (on a scale of 1 to 5) is it that...
  - My family members have good relationships with extended family members (grandparents, in-laws, etc.).
  - My family is physically healthy.
- What is the quality of the relationship that you have with your children?
  - excellent                  very good                  good                  fair                  poor
- To what extent did others make it difficult for you to engage in various activities before your imprisonment?
  - never                  rarely                  often                  most of the time
- I rarely feel sad.
  - Strongly disagree                  disagree                  agree                  strongly agree

# Example: Confounded Valence and Construct

## Nonacceptance of Emotional Responses

- 11. When I'm upset, I become angry with myself for feeling that way.
- 12. When I'm upset, I become embarrassed for feeling that way.
- 21. When I'm upset, I feel ashamed with myself for feeling that way.
- 23. When I'm upset, I feel like I am weak.
- 25. When I'm upset, I feel guilty for feeling that way.
- 29. When I'm upset, I become irritated with myself for feeling that way.

## Difficulties in Engaging in Goal-Directed Behaviors

- 13. When I'm upset, I have difficulty getting work done.
- 18. When I'm upset, I have difficulty focusing on other things.
- 20. When I'm upset, I can still get things done. (R)
- 26. When I'm upset, I have difficulty concentrating.
- 33. When I'm upset, I have difficulty thinking about anything else.

## Impulse Control Difficulties

- 3. I experience my emotions as overwhelming and out of control.
- 14. When I'm upset, I become out of control.
- 19. When I'm upset, I feel out of control.
- 24. When I'm upset, I feel like I can remain in control of my behaviors. (R)
- 27. When I'm upset, I have difficulty controlling my behaviors.
- 32. When I'm upset, I lose control over my behaviors.

## Lack of Emotional Awareness

- 2. I pay attention to how I feel. (R)
- 6. I am attentive to my feelings. (R)
- 8. I care about what I am feeling. (R)
- 10. When I'm upset, I acknowledge my emotions. (R)
- 17. When I'm upset, I believe that my feelings are valid and important. (R)
- 34. When I'm upset, I take time to figure out what I'm really feeling. (R)

## Limited Access to Emotion Regulation Strategies

- 15. When I'm upset, I believe that I will remain that way for a long time.
- 16. When I'm upset, I believe that I'll end up feeling very depressed.
- 22. When I'm upset, I know that I can find a way to eventually feel better. (R)
- 28. When I'm upset, I believe there is nothing I can do to make myself feel better.
- 30. When I'm upset, I start to feel very bad about myself.
- 31. When I'm upset, I believe that wallowing in it is all I can do.
- 35. When I'm upset, it takes me a long time to feel better.
- 36. When I'm upset, my emotions feel overwhelming.

## Lack of Emotional Clarity

- 1. I am clear about my feelings. (R)
- 4. I have no idea how I am feeling.
- 5. I have difficulty making sense out of my feelings.
- 7. I know exactly how I am feeling. (R)
- 9. I am confused about how I feel.

**Difficulties in Emotion Regulation Scale (DERS):** The “lack of emotional awareness” subscale has only reverse-coded items. So these items could be correlated (i.e., seem to indicate a common trait) due to their content OR their valence—not good!

In addition, the first items on the scale do not have the referent “when I’m upset”—this could cause them to be responded to differently than the rest of the later scale items that have a different, more specific, referent.

# Davidson et al. (2016): How to Fix a Confound

**Table 1**

SPQ-BR original items (Cohen et al., 2010) plus 1st-person ("I") vs. 2nd-person ("You") pronoun.

SPQ-BR item	Factor	Sub-factor	I/you
1. Do you sometimes feel that people are talking about you?	CP	IR	You
2. Do you sometimes feel that other people are watching you?	CP	IR	You
3. When shopping, do you get the feeling that other people are taking notice of you?	CP	IR	You
4. I often feel that others have it in for me.	CP	SU	I
5. Do you sometimes get concerned that friends or co-workers are not really loyal or trustworthy?	CP	SU	You
6. Do you often have to keep an eye out to stop people from taking advantage of you?	CP	SU	You
7. Do you feel that you cannot get "close" to people?	IP	CF	You
8. I find it hard to be emotionally close to other people.	IP	CF	I
9. Do you feel that there is no one you are really close to outside of your immediate family, or people you can confide in or talk to about personal problems?	IP	CF	You
10. I tend to keep my feelings to myself.	IP	CA	I
11. I rarely laugh and smile.	IP	CA	I
12. I am not good at expressing my true feelings by the way I talk and look.	IP	CA	I
13. Other people see me as slightly eccentric (odd).	DO	EB	I
14. I am an odd, unusual person.	DO	EB	I
15. I have some eccentric (odd) habits.	DO	EB	I
16. People sometimes comment on my unusual mannerisms and habits.	DO	EB	I
17. Do you often feel nervous when you are in a group of unfamiliar people?	IP or SA	SA	You
18. I get anxious when meeting people for the first time.	IP or SA	SA	I
19. I feel very uncomfortable in social situations involving unfamiliar people.	IP or SA	SA	I
20. I sometimes avoid going to places where there will be many people because I will get anxious.	IP or SA	SA	I
21. Do you believe in telepathy (mind-reading)?	CP	MT	You
22. Do you believe in clairvoyance (psychic forces, fortune telling)?	CP	MT	You
23. Have you had experiences with astrology, seeing the future, UFO's, ESP, or a sixth sense?	CP	MT	You
24. Have you ever felt that you are communicating with another person telepathically (by mind-reading)?	CP	MT	You
25. I sometimes jump quickly from one topic to another when speaking.	DO	OS	I
26. Do you tend to wander off the topic when having a conversation?	DO	OS	You
27. I often ramble on too much when speaking.	DO	OS	I
28. I sometimes forget what I am trying to say.	DO	OS	I
29. I often hear a voice speaking my thoughts aloud.	CP	UP	I
30. When you look at a person or yourself in a mirror, have you ever seen the face change right before your eyes?	CP	UP	You
31. Are your thoughts sometimes so strong that you can almost hear them?	CP	UP	You
32. Do everyday things seem unusually large or small?	CP	UP	You

The CP scale had mostly "you" items. Changing all the items to "I" → better psychometrics.



# Avoid Wording Effects by Rewriting!

## Original

I feel I have a number of good qualities.

- Disagree strongly
- Disagree a little
- Neither agree nor disagree
- Agree a little
- Agree strongly

All in all, I am inclined to feel that I am a failure.

- Disagree strongly
- Disagree a little
- Neither agree nor disagree
- Agree a little
- Agree strongly

## Expanded

- I feel I have almost no good qualities.
- I feel I have a few good qualities.
- I feel I have some good qualities.
- I feel I have many good qualities.
- I feel I have a great many good qualities.
- All in all, I feel I am very much a failure.
- All in all, I feel I am somewhat a failure.
- All in all, I feel I am neither a failure nor a success.
- All in all, I feel I am somewhat a success.
- All in all, I feel I am very much a success.

## Item-Specific-F

How many good qualities do you feel you have?

- Almost none
- Few
- Some
- Many
- A great many

All in all, how much a failure/success do you feel you are?

- Very much a failure.
- Somewhat a failure.
- Neither a failure nor a success.
- Somewhat a success.
- Very much a success.

- See [Zhang, Zhou, & Savalei \(2022\)](#)
  - “Original” format artificially distinguishes positive and negative ends of trait  
→ two factors due wording difference
  - “Expanded” and “item-specific” formats include full trait range in answer choices  
→ one factor instead!
- Item design choices → experiments!

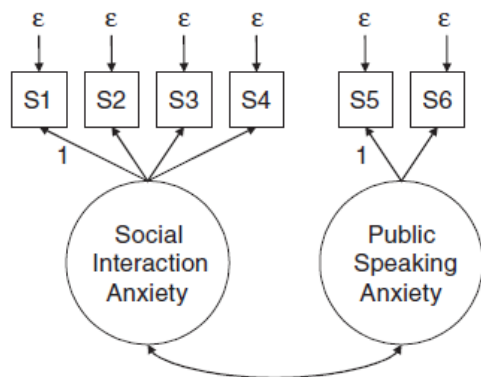
# Choices for “Method” Effects

- If your data are already collected (so you can't fix the items), then there are 3 basic choices for “method” effects:
  - **Error covariances** → allows a pair of items to have an extra correlation (for something else in common besides factor)
    - Not often used to support arguments about why items are still related
  - **Separate latent factors** that are method- and trait-specific
    - e.g., Our Example 4: one hypothesized factor for “forgiveness of situations” → two factors, one for positively worded forgiveness and one for negatively worded forgiveness
  - **“Bifactor” (“method factor”)** models → general factor with 1+ “specific” factors to capture correlation for other reasons
    - Loading constraints can help provide support for interpretation (e.g., not-worse fit of equal specific loadings for all reverse-coded supports that it's only capturing negative direction, not the content)

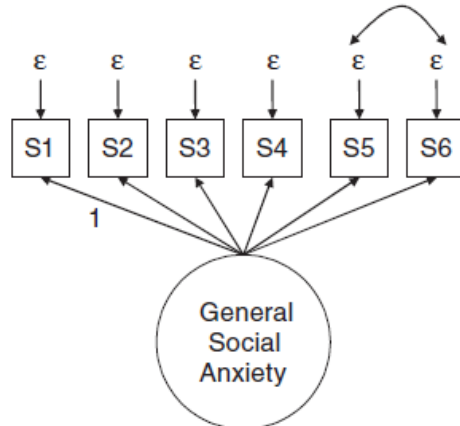
# Error Covariances vs. “Method” Factors

- Error covariances are only possible when there’s a saturated model
  - In all CFAs, but only in IFA/IRT when using limited-info estimation (WLSMV)
- Otherwise (e.g., IFA/IRT via MML), you need a “method” factor
  - Predict those two items with a method factor, fix both loadings=1 for a positive covariance, or fix loadings to  $-1/+1$  if for a negative covariance
  - Estimate its factor variance, which then induces an error covariance
- See equivalent models below (Brown, 2015 p. 181), but consider:
  - Do you want a “specific construct” or just “control for” extra correlation?

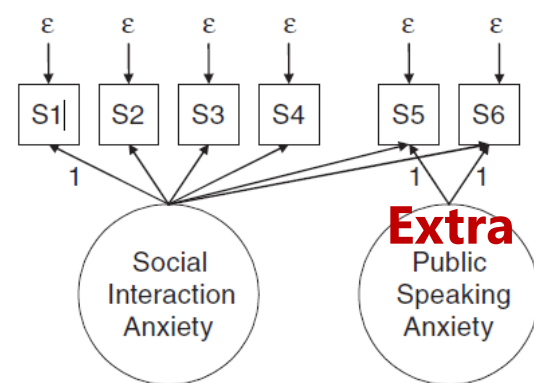
Two-Factor Models



Error Covariance



Factor + Method Factor



# Illustrative Example: “Life Orientation”

Table 2  
Means, Standard Deviations, and Correlations for E. C. Chang et al.'s (1994) Life Orientation Test Data

Item	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7
Item 1	1.00						
Item 2	.51	1.00					
Item 3	.44	.53	1.00				
Item 4	-.16	-.22	-.26	1.00			
Item 5	-.28	-.38	-.33	.50	1.00		
Item 6	-.24	-.29	-.30	.51	.70	1.00	
Item 7	-.22	-.35	-.30	.44	.54	.52	1.00
<i>M</i>	2.24	2.40	2.56	1.85	1.39	1.32	1.40
<i>SD</i>	1.00	0.99	0.99	1.05	1.03	1.00	1.07
Skewness	-0.12	-0.35	-0.57	0.25	0.63	0.68	0.71
Kurtosis	-0.65	-0.36	-0.11	-0.72	-0.14	0.01	-0.23

Note. *N* = 389.

Table 1  
Life Orientation Test (LOT) Items (E. C. Chang et al., 1994)

Item	Original item number
1. In uncertain times, I usually expect the best. (positive)	Item 1
2. I always look on the bright side of things. (positive)	Item 4
3. I'm always optimistic about my future. (positive)	Item 5
4. If something can go wrong for me, it will. (negative)	Item 3
5. I hardly ever expect things to go my way. (negative)	Item 8
6. Things never work out the way I want them to. (negative)	Item 9
7. I rarely count on good things happening to me. (negative)	Item 12

Note. The original item number is the order in which the item appears on the actual LOT questionnaire.

[Maydeu-Olivares & Coffman, \(Psychological Methods, 2006\)](#) present 4 models by which to measure a latent factor of optimism using the 3 positively and 4 negatively worded items shown below

A: Single factor  
(DF = 14)

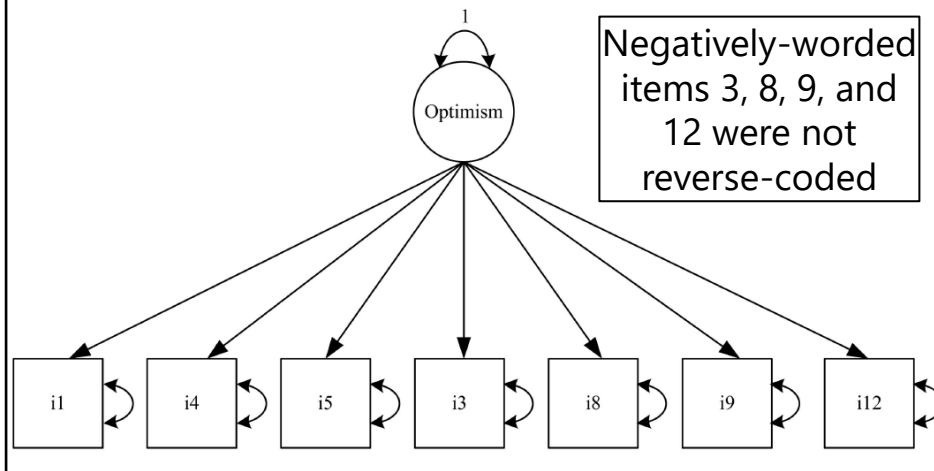
B: Two factors due to wording (DF = 13)

C: Three-factor  
“Bifactor” model  
(DF = 7)

D: “Random Intercept”  
2-factor model  
(DF = 13)

# What to Do about Method Effects?

Model A: One factor model

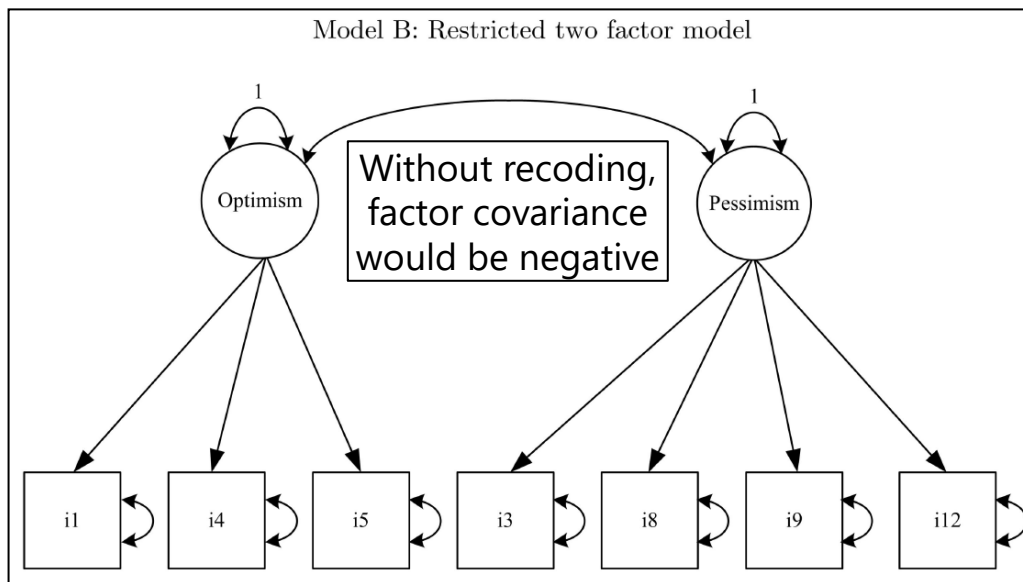


Maydeu-Olivares & Coffman (2006) present 4 ways to measure a latent factor of optimism with 3 positively and 4 negatively worded items

**A: Single “optimism” factor (which doesn’t fit well)**

```
Opt BY i1* i4* i5*
      i3* i8* i9* i12*;
Opt@1; [Opt@0];
```

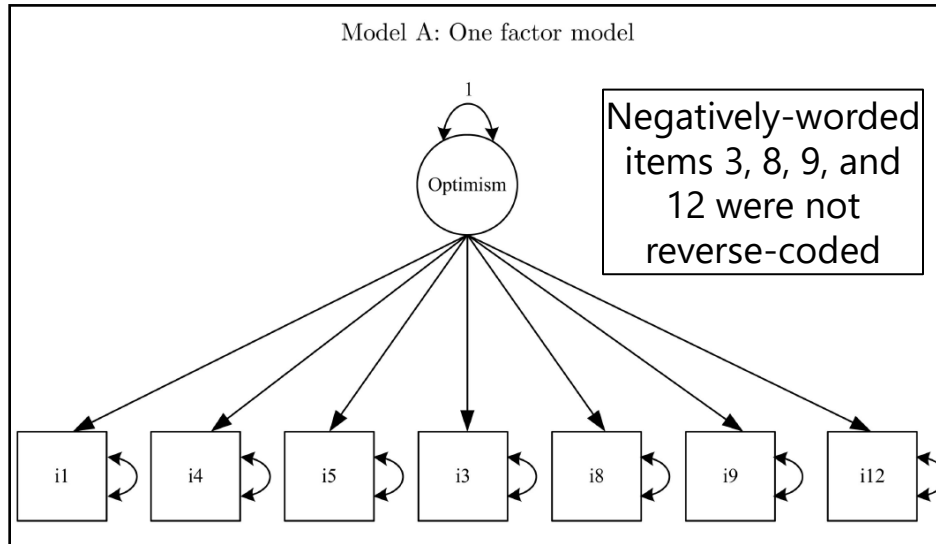
Model B: Restricted two factor model



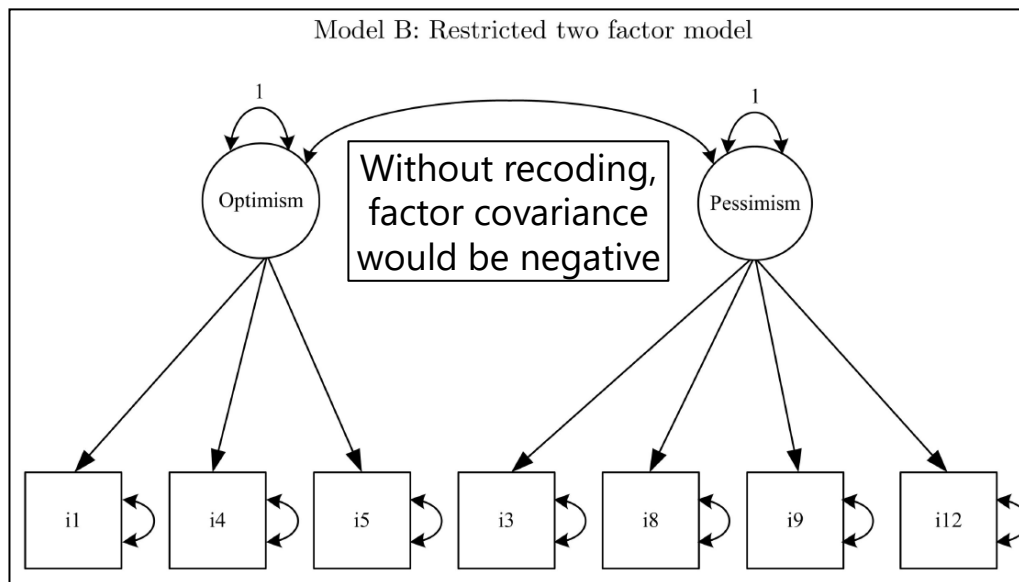
**B: “Optimism” and “Pessimism” two-factor model (fits better)**

```
Opt BY i1* i4* i5*;
Pes BY i3* i8* i9* i12*;
Opt WITH Pes*;
Opt@1; [Opt@0];
Pes@1; [Pes@0];
```

# One- vs. Two-Factor Models (Two wins)

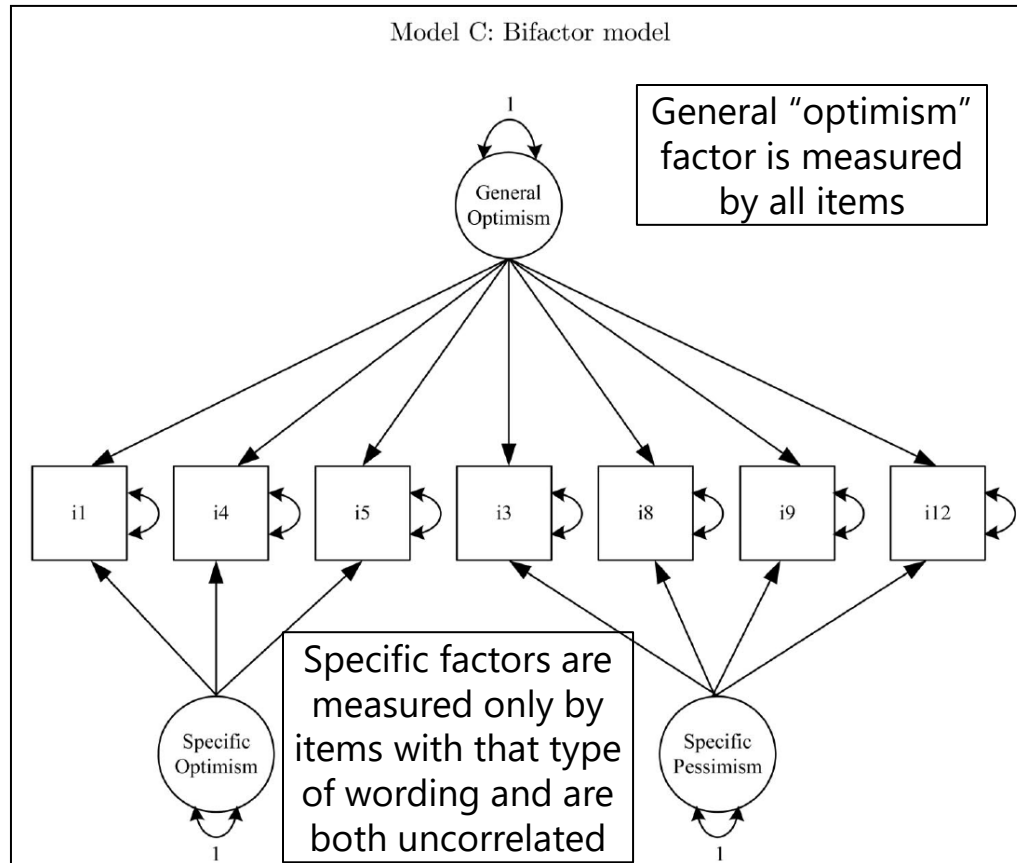


Item	One-factor model: Optimism	Two-factor model	
		Optimism	Pessimism
Item 1	0.38 (0.05)	0.64 (0.05)	0
Item 2	0.48 (0.05)	0.78 (0.05)	0
Item 3	0.46 (0.05)	0.68 (0.05)	0
Item 4	-0.64 (0.05)	0	0.65 (0.05)
Item 5	-0.86 (0.05)	0	0.87 (0.05)
Item 6	-0.79 (0.05)	0	0.82 (0.05)
Item 7	-0.70 (0.05)	0	0.70 (0.05)



Note: a higher-order factor could be included if both loadings were fixed to 1, but it would fit the same as just allowing the two wording factors to covary.

# Bifactor Model C Fits Well, BUT:



## 2 problems in interpreting these factors as desired:

- 1) "Specific" positive loadings > "general" loadings
- 2) Specific negative loadings are weak or non-significant (indicating model is over-parameterized)

```
Gen BY i1* i4* i5*
      i3* i8* i9* i12*;
Opt BY i1* i4* i5*;
      i3* i8* i9* i12*;
Pes BY i3* i8* i9* i12*;
Gen@1; Opt@1; Pes@1;
[Gen@0 Opt@0 Pes@0];
Gen WITH Opt@0 Pes@0;
Opt WITH Pes@0;
```

Bifactor model		
Overall optimism	Specific optimism	Specific pessimism
0.35	0.56	0
(0.07)	(0.07)	
0.49	0.61	0
(0.08)	(0.07)	
0.44	0.51	0
(0.07)	(0.07)	
-0.59	0	0.26 <sup>a</sup>
(0.09)		(0.18)
-0.76	0	0.38
(0.10)		(0.23)
-0.63	0	0.64 <sup>a</sup>
(0.11)		(0.16)
-0.73	0	0.15 <sup>a</sup>
(0.08)		(0.18)

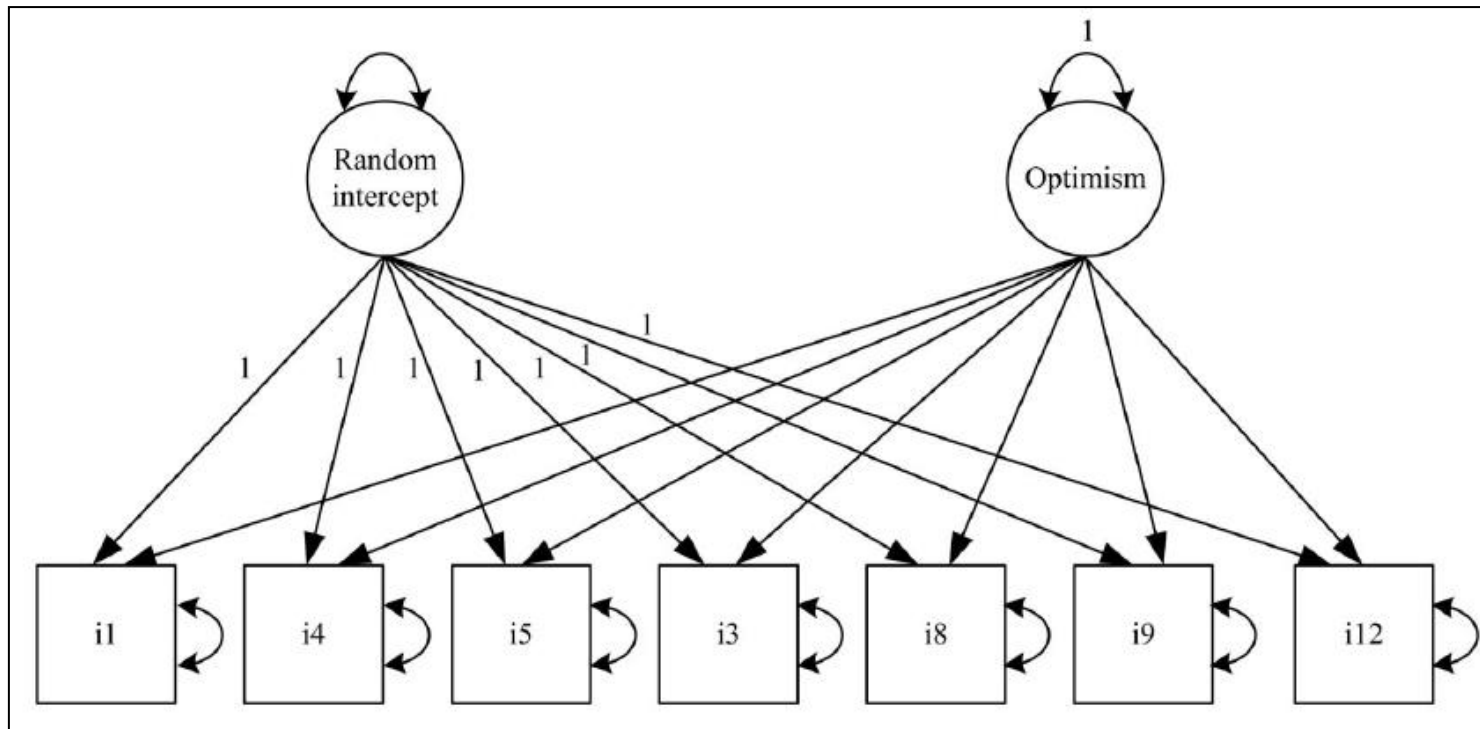


# Random Intercept Factor Fits Well...

- General "optimism" factor is measured by all items (all loadings estimated)
- New "random intercept" factor allows for constant person shifts across items (e.g., due to different response scale interpretations); Variance = 0.13 here

```

Opt BY i1* i4* i5*
      i3* i8* i9* i12*;
Opt@1; [Opt@0];
Int BY i1@1 i4@1 i5@1
      i3@1 i8@1 i9@1 i12@1;
Int*; [Int@0];
Opt WITH Int@0;
    
```



One-factor random intercept: Optimism
0.54
(0.05)
0.66
(0.05)
0.61
(0.05)
-0.56
(0.05)
-0.78
(0.05)
-0.71
(0.05)
-0.65
(0.05)



# Heartland Forgiveness Scale (HFS)

[Yamhure Thompson, L., Snyder, C.R., Hoffman, L., Michael, S.T., Rasmussen, H.N., Billings, L.S., et al. \(2005\). Dispositional forgiveness of self, others, and situations. \*Journal of Personality\*, 73\(2\), 313-360.](#)

**Baseline:** Six correlated lower-order factors for positive and negative self, other, and situation “forgiveness” and “not unforgiveness” (reverse-coded)

**Total possible df for 18 items = 189**

$$\frac{v * (v + 1)}{2} + v = \frac{18 * 19}{2} + 18 = 189$$

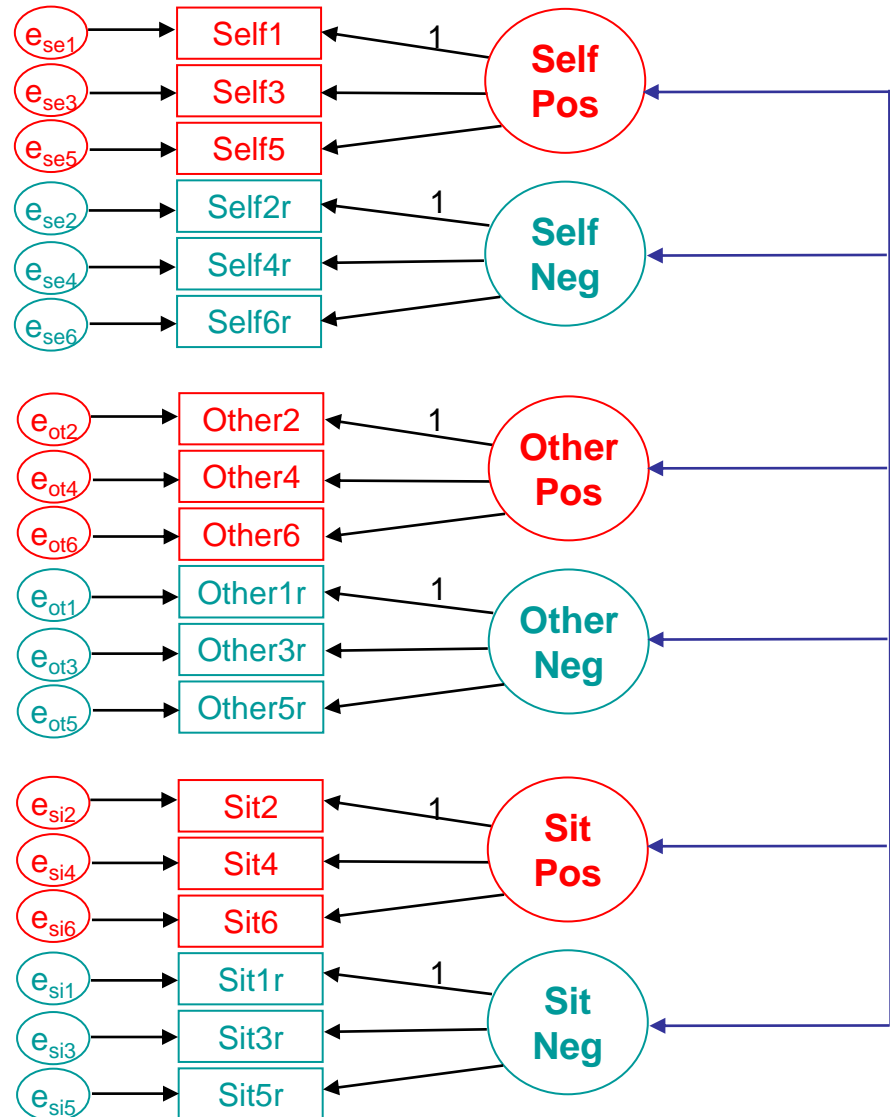
**Measurement Model = 48 parameters**

$$12\lambda_i + 18\mu_i + 18\sigma_{e_i}^2$$

**Structural Model = 21 parameters**

$6\sigma_F^2$ , 15 factor covariances (all possible, abbreviated with arrows from line)

**Total model DF = 189 – 69 = 120**



# HFS Structural Model with Constraints

**Figure 1.** Six lower-order factors for positive and negative self, other, and situation forgiveness and not unforgiveness as before, but now with three higher-order correlated factors of Self, Other, and Situation, and two uncorrelated wording factors

**Structural Model = 8 parms**

(DF = 21 – 8 = 13)

! Constant Method Effects

Pos BY SelfPos\* (L)  
OtherPos\* (L)  
SitPos\* (L);

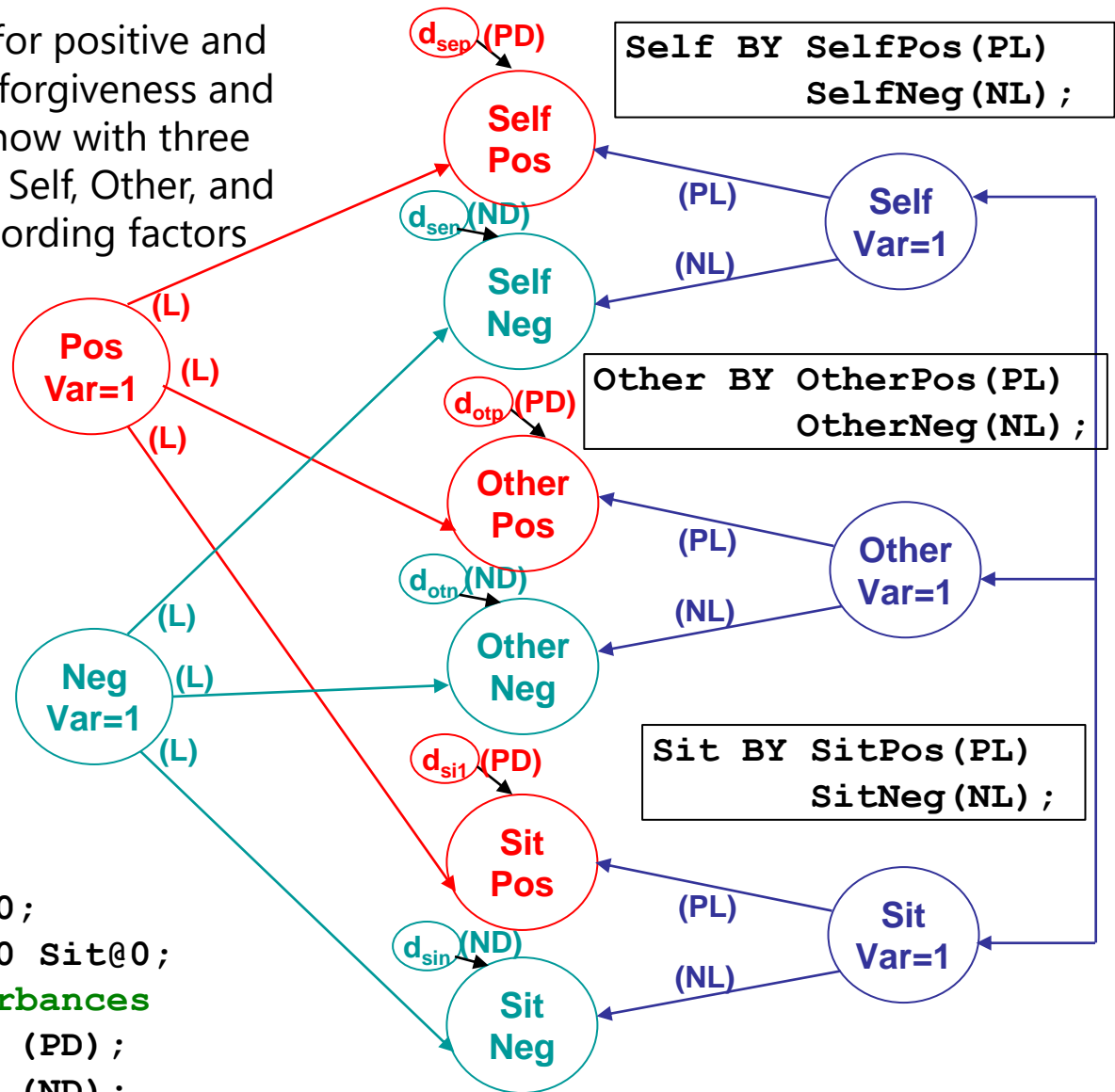
Neg BY SelfNeg\* (L)  
OtherNeg\* (L)  
SitNeg\* (L);

! No method factor covs

Self@1 Other@1 Sit@1;  
Self WITH Other\* Sit\*;  
Other WITH Sit\*;  
Pos@1 Neg@1; Pos WITH Neg@0;  
Pos Neg WITH Self@0 Other@0 Sit@0;

! Constrained factor disturbances

SelfPos\* OtherPos\* SitPos\* (PD);  
SelfNeg\* OtherNeg\* SitNeg\* (ND);



# HFS “Unforgiveness” Alternative Model

Reviewer 3 insisted that the trait factors were “Forgiveness” and “Not Unforgiveness” (positive and negative wording), and that the method factors were Self, Other, and Situation... tested via new constraints!

## Structural Model = 8 parms

(DF = 21 – 8 = 13)

### ! Constant Method Effects

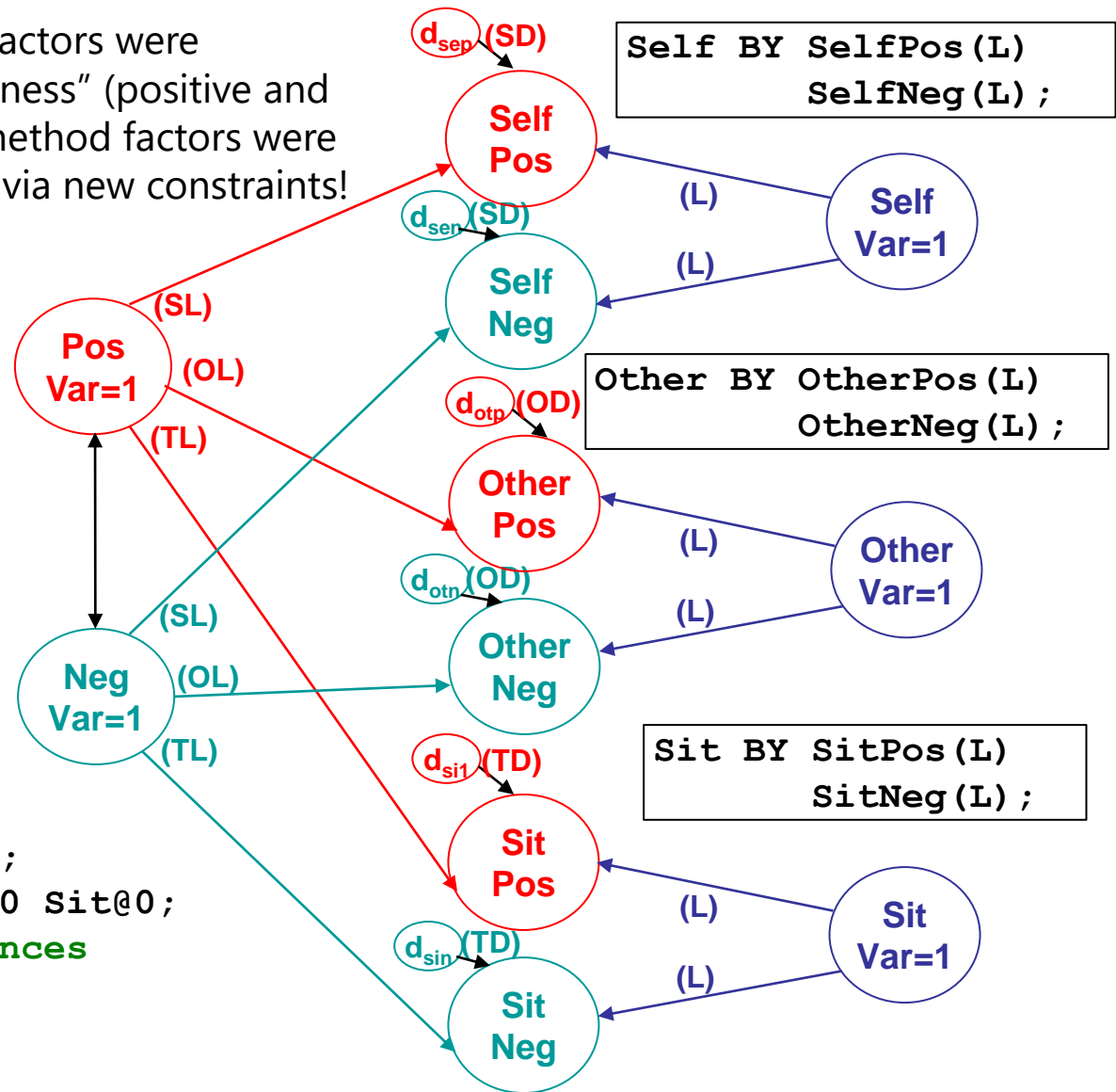
```
Pos BY SelfPos* (SL)
      OtherPos* (OL)
      SitPos* (TL);
Neg BY SelfNeg* (SL)
      OtherNeg* (OL)
      SitNeg* (TL);
```

### ! No method factor covs

```
Self@1 Other@1 Sit@1;
Self WITH Other@0 Sit@0;
Other WITH Sit@0;
Pos@1 Neg@1; Pos WITH Neg*;
Pos Neg WITH Self@0 Other@0 Sit@0;
```

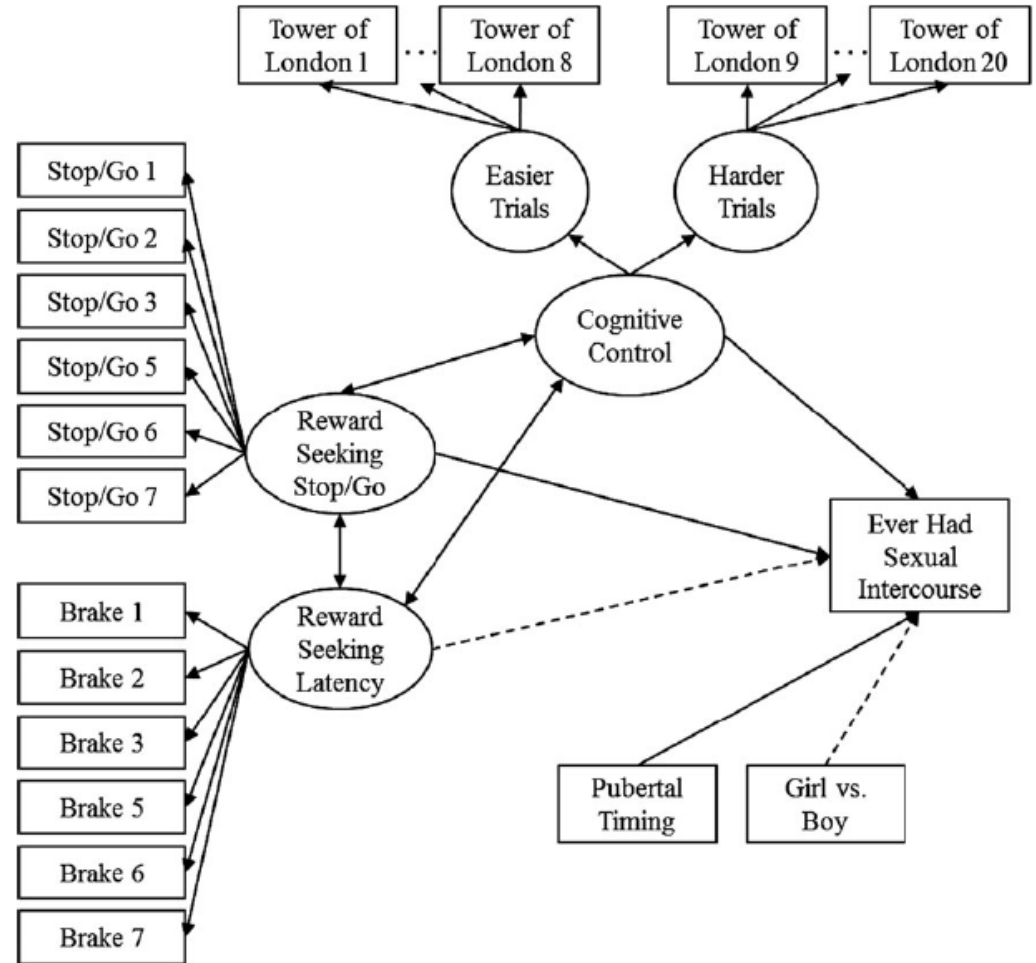
### ! Constant factor disturbances

```
SelfPos* SelfNeg* (SD);
OtherPos* OtherNeg* (OD);
SitPos* SitNeg* (TD);
```



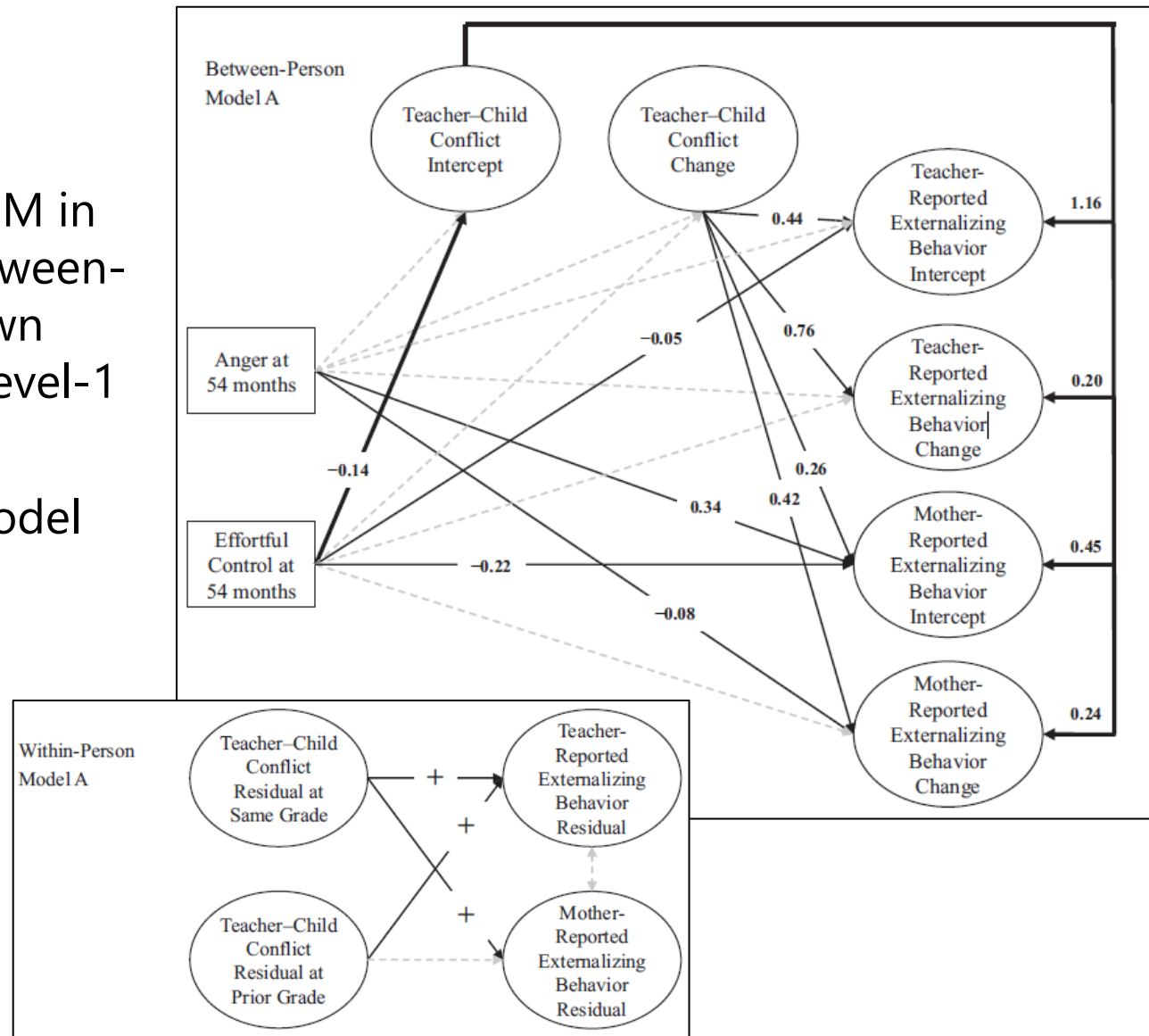
# Abbreviating Structural Model Diagrams

- Structural model diagrams can become very cluttered! Here are some examples from my work on how to abbreviate them...
- Right: Figure 1 of [Wasserman et al. \(2017\)](#)
  - For the Tower or London, trials 2–7 and 10–19 are implied but not shown
  - Coefficients are given in tables instead



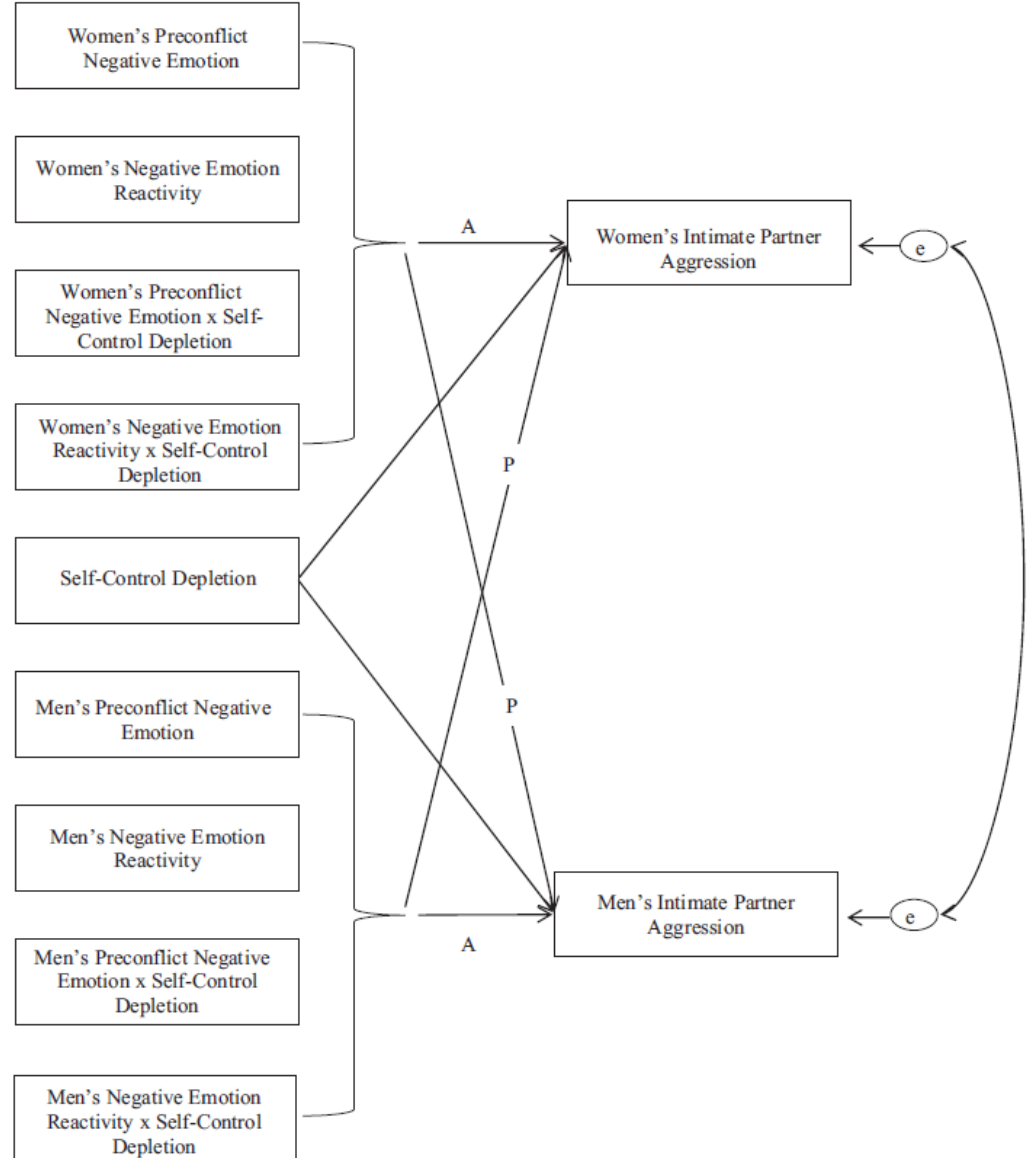
# Abbreviating Structural Model Diagrams

- Right: Figure 1 of [Crockett et al. \(2018\)](#)
- This is a multilevel SEM in which the level-2 between-person model is shown separately from the level-1 within-person model
- The measurement model for the intercept and change factors was described in the text but was not shown



# Abbreviating Structural Model Diagrams

- Right: Figure 1 of [Watkins et al. 2015](#)
- This is an “actor–partner” model estimated as a path analysis (no latent variables)
- Brackets are used to group the predictors from the same person together
- The “actor” paths (my X → my Y) are abbreviated with the A paths
- The “partner” paths (my X → your Y) are abbreviated with the P paths
- Coefficients are given in tables instead



# Wrapping Up...

- Fitting measurement and structural models are two separate issues:
  - **Measurement model:** Do my lower-order factor loadings predict the *observed covariances among my ITEMS*?
  - **Structural model:** Do higher-order factor loadings predict the *estimated covariances among my measurement model FACTORS/THETAS*?
    - A higher-order factor is NOT the same thing as a total score, but it is a way to rescue a multidimensional trait that you want to think of as unidimensional in how it relates to other constructs (i.e., those relations can be specified with just higher-order factor)
- Figure out the measurement models FIRST, then structural models
  - I recommend fitting measurement models separately per factor, then bringing them together once you have the items for each factor/theta fitting well
  - This will help to limit the source of misfit in complex models to across factors
- “Bifactor” or “method factor” models need loadings that make sense
  - Constraints can (and should) be used to test alternative interpretations!