PSQF 6249 Example 8 page 1 Example 8: Higher-Order Factor Models (CFA using MLR; then IFA using WLSMV) in M*plus* version 8.10 (Mplus syntax and output are available electronically; a partial Lavaan version of this example is available here)

Example data: 1,336 college students self-reporting on 49 items (measuring five factors) assessing <u>childhood maltreatment</u> on a 1–5 scale: 1=*Strongly Disagree*, 2=*Disagree*, 3=*Neutral*, 4=*Agree*, 5=*Strongly Agree*. The item responses are NOT normally distributed, so we'll use both CFA (with MLR) and IFA (with WLSMV) as two options to examine the fit of these models (i.e., only as an example of how to do each, NOT to compare between the model types).

- 1. Spurning: Verbal and nonverbal caregiver acts that reject and degrade a child
- 2. Terrorizing: Caregiver behaviors that threaten or are likely to physically hurt, kill, abandon, or place the child or the child's loved ones or objects in recognizably dangerous situations.
- 3. Isolating: Caregiver acts that consistently deny the child opportunities to meet needs for interacting or communicating with peers or adults inside or outside the home.
- 4. Corrupting: Caregiver acts that encourage the child to develop inappropriate behaviors (self-destructive, antisocial, criminal, deviant, or other maladaptive behaviors).
- 5. Ignoring: Emotional unresponsiveness includes caregiver acts that ignore the child's attempts and needs to interact (failing to express affection, caring, and love for the child) and show no emotion in interactions with the child

Here are the results from fitting the 5 factors separately to ensure their individual fit FIRST (see Mplus output files for details):

				ASE	SSMENT OF N	IODEL FIT US	SING MLR							
Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	ти	SRMR	RMSEA Estimate	RMSEA Lower Cl	RMSEA Higher Cl	RMSEA p-value
MLR Spurning	12	90	36	224.797	1.4009	54	<.0001	0.959	0.949	0.030	0.049	0.042	0.055	0.619
MLR Terror	9	54	27	189.816	1.5876	27	<.0001	0.918	0.891	0.040	0.067	0.058	0.076	0.001
MLR Isolate	6	27	18	80.355	1.4944	9	<.0001	0.916	0.961	0.041	0.077	0.062	0.093	0.002
MLR Corrupt	7	35	21	54.964	1.9075	14	<.0001	0.934	0.901	0.033	0.047	0.034	0.060	0.633
MLR Ignore	15	135	45	484.291	1.7921	90	<.0001	0.932	0.920	0.036	0.057	0.052	0.062	0.008
MLR 1 factor only	49	1274	147	6,183.985	1.4874	1127	<.0001	0.766	0.756	0.062	0.058	0.057	0.059	<.0001
MLR 5 correlated factors	49	1274	157	4,424.701	1.4645	1117	<.0001	0.847	0.839	0.057	0.047	0.046	0.049	1.000
MLR 5 factors + HO	49	1274	152	4,486.381	1.4681	1122	<.0001	0.844	0.837	0.058	0.047	0.046	0.049	0.999
MLR 5 factors + HO + 2 cov	49	1274	154	4,422.556	1.4669	1120	<.0001	0.847	0.840	0.057	0.047	0.046	0.048	1.000
				ASESS	MENT OF MO	DEL FIT USI	NG WLSMV							
	1	# Possible	# Free	Chi-Square	Chi-Square	Chi-Square	Chi-Square			RMSEA	RMSEA	RMSEA	RMSEA	
Model	# Items	Parms	Parms	Value	Scale Factor	DF	p-value	CFI	TU	SRMR	Estimate	Lower CI	Higher CI	p-value
WLSMV Spurning	12	126	60	294.706		54	<.0001	0.983	0.980	0.029	0.058	0.051	0.064	0.023
WLSMV Terror	9	81	45	263.155		27	<.0001	0.966	0.954	0.037	0.081	0.072	0.090	<.0001
WLSMV Isolate	6	45	30	129.828		9	<.0001	0.962	0.937	0.034	0.100	0.085	0.116	<.0001
WLSMV Corrupt	7	56	35	87.487		14	<.0001	0.976	0.964	0.029	0.063	0.050	0.076	0.044
WLSMV Ignore	15	180	75	897.689		90	<.0001	0.976	0.972	0.030	0.082	0.077	0.087	<.0001
WLSMV 1 factor only	49	1421	245	7,563.407		1127	<.0001	0.903	0.898	0.068	0.065	0.064	0.067	<.0001
WLSMV 5 correlated factors	49	1421	255	5,934.136		1117	<.0001	0.927	0.923	0.056	0.057	0.055	0.058	<.0001
WLSMV 5 factors + HO	49	1421	250	5,941.909		1122	<.0001	0.927	0.924	0.057	0.057	0.055	0.058	<.0001
WLSMV 5 factors + HO + 2 cov	49	1421	252	5,853.773		1122	<.0001	0.928	0.925	0.056	0.056	0.055	0.058	<.0001

Here are the standardized factor loadings for each item under each estimation method. Note that the WLSMV factor loadings are higher in this case—probably because of positive skewness in the original data (and thus the implausibility of a linear model).

MLR	WLSMV	MLR	WLSMV	MLR	WLSMV		MLR	WLSMV		MLR	WLSMV
Spurning	Spurning	Terror	Terror	Isolate	Isolate	_	Corrupt	Corrupt	_	Ignore	Ignore
0.599	0.660	0.512	0.617	0.521	0.695		0.589	0.739	-	0.672	0.813
0.457	0.528	0.673	0.771	0.550	0.630		0.545	0.713		0.654	0.749
0.769	0.837	0.451	0.713	0.545	0.685		0.375	0.523		0.657	0.748
0.526	0.597	0.612	0.721	0.540	0.629		0.545	0.854		0.724	0.801
0.607	0.677	0.571	0.787	0.563	0.726		0.631	0.826		0.445	0.540
0.816	0.865	0.554	0.617	0.752	0.822		0.580	0.708		0.745	0.833
0.835	0.907	0.685	0.805				0.646	0.840		0.847	0.913
0.465	0.538	0.643	0.743							0.713	0.813
0.516	0.728	0.732	0.815							0.808	0.891
0.655	0.744									0.749	0.845
0.674	0.756									0.656	0.795
0.610	0.680									0.830	0.904
										0.712	0.806
										0.739	0.815
										0.825	0.918

Strawman model: Syntax for single-factor CFA model estimated using MLR through 5 PERFECTLY correlated factors

DATA: FILE = abuse.csv; ! Don't need path if in same folder as input	THE MODEL ESTIMATION TERMINATED NORMALLY
TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults	
VARIABLE:	Because the factor covariances were fixed to 1, you will see the
NAMES = ID ! All variables in DATA SET	
p01 p02 p03 p04 p05 p06 p07 p08 p09 p10	message below. In THIS CONTEXT ONLY, you can ignore it.
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20	
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	WARNING: THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40	DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50	FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
p11 p12 p13 p14 p15 p16 p17 p16 p19 p30 p51 p52 p53 p54 p55 p56 p57;	TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
psi psz pss ps4 pss ps0 ps7,	LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
USEVARIABLES = ! All variables in MODEL	PROBLEM INVOLVING VARIABLE TERROR.
p01 p02 p03 p04 p06 p07 p09 p10	
p11 p12 p13 p14 p16 p17 p18 p19 p20	MODEL FIT INFORMATION
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	Number of Free Parameters 147
p31 p33 p35 p36 p37 p39 p40	Loglikelihood
p43 p44 p45 p46 p47 p48 p49 p50	H0 Value -70386.526
p51 p52 p53 p54 p55 p56 p57;	H0 Scaling Correction Factor 2.3983
IDVARIABLE = ID; ! Person ID variable	for MLR
	H1 Value -65787.405
MISSING = ALL (99999); ! Missing data value used	H1 Scaling Correction Factor 1.5925
	for MLR
ANALYSIS: ESTIMATOR = MLR; ! For non-normal continuous items	
OUTPUT: STDYX ! Standardized solution	Information Criteria
MODINDICES(3.84) ! Cheat codes for fixing the model	
RESIDUAL ! Local fit info	Akaike (AIC) 141067.051
TECH4; ! Factor correlation matrix	Bayesian (BIC) 141831.074
!SAVEDATA: SAVE = FSCORES; ! Save factor scores	Sample-Size Adjusted BIC 141364.120
! FILE = Abuse_Thetas.dat; ! File of factor scores	$(n^* = (n + 2) / 24)$
! MISSFLAG = 99; ! Indicate missing values	
PLOT: TYPE = PLOT1 PLOT2 PLOT3; ! For pictures	Chi-Square Test of Model Fit
	Value 6183.986*
MODEL: ! (To be changed below for each model)	Degrees of Freedom 1127
	P-Value 0.0000
! 5 Factors (loadings for first item are estimated)	Scaling Correction Factor 1.4874
! 12-Item Spurning	for MLR
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	
9-Item Terrorizing	RMSEA (Root Mean Square Error Of Approximation)
Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;	Estimate 0.058
! 6-Item Isolating	90 Percent C.I. 0.057 0.059
Isolate BY p01* p18* p19* p23* p39* p43*;	Probability RMSEA <= .05 0.000
! 7-Item Corrupting	CFI/TLI
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	CFI 0.766
! 15-Item Ignoring	TLI 0.756
Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	0.750
p45* p46* p51* p52* p57*;	(DND (Standard) and Dast Man Course Dasidual)
	SRMR (Standardized Root Mean Square Residual)
! Factor Variances (all must be fixed to 1 for identification)	Value 0.062
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	
! Factor Means (all fixed = 0 by default)	#free parameters = 147 = 49 loadings + 49 intercepts + 49 error variances
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	+ 0 factor variances + 0 factor covariances = 147 parameters USED
! Factor Covariance (all fixed to 1 to create 1-factor model)	
Spurn Terror Isolate Corrupt Ignore WITH	
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	Possible = $49*50/2 + 49 = 1274$
	DF =1117 calculation: 1274 – 147 = 1127
L	

Syntax for CFA model with MLR including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

MODEL: ! (To be changed below for each model)	NOTE: With respect to fit of the structural model, letting the 5 factors just be				
L. F. Testere (1 - Aliene for first item and estimate A)	correlated is as good as it gets. This saturated structural model will be our				
<pre>! 5 Factors (loadings for first item are estimated) ! 12-Item Spurning</pre>	"larger model" baseline with which to compare the fit of models that try to				
Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	account for these correlations via a higher-order factor (a "smaller model").				
9-Item Terrorizing					
Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;	Number of Free Parameters 157				
! 6-Item Isolating	Loglikelihood				
Isolate BY p01* p18* p19* p23* p39* p43*;	HO Value -69027.431				
! 7-Item Corrupting	HO Scaling Correction Factor 2.5033				
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	for MLR				
! 15-Item Ignoring Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	H1 Value -65787.405 H1 Scaling Correction Factor 1.5925				
p45* p46* p51* p52* p57*;	for MLR				
! Factor Variances (all must be fixed to 1 for identification)	Information Criteria				
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	Akaike (AIC) 138368.862				
! Factor Means (all fixed = 0 by default)	Bayesian (BIC) 139184.860				
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	Sample-Size Adjusted BIC 138686.140				
! Factor Covariances (all estimated to allow 5-factor model) Spurn Terror Isolate Corrupt Ignore WITH	$(n^* = (n + 2) / 24)$				
Spurn* Terror* Isolate* Corrupt* Ignore*;	Chi-Square Test of Model Fit				
,	Value 4424.701*				
So do we have one factor or five factors?	Degrees of Freedom 1117				
	P-Value 0.0000				
According to the $-2\Delta LL$ scaled difference relative to the previous single-	Scaling Correction Factor 1.4645				
factor model: $-2\Delta LL(10) = 671.689$, $p < .0001$	for MLR				
120001	RMSEA (Root Mean Square Error Of Approximation)				
	Estimate 0.047				
Therefore, one factor does not capture the covariances among these 49	90 Percent C.I. 0.046 0.049				
items. Five factors (as hypothesized) does a significantly better job.	Probability RMSEA <= .05 1.000				
Here are the correlations among the latent factors we are now trying to	CFI/TLI				
account for-with models that replace them with a higher-order factor!	CFI 0.847 TLI 0.839				
	101 0.039				
	SRMR (Standardized Root Mean Square Residual)				
Saturated: 5-Factor All Covariances Model	Value 0.057				
SPURN TERROR ISOLATE CORRUPT IGNORE	#free parameters = 157 = 49 loadings + 49 intercepts + 49 error variances				
SPURN 1.000	+ 0 factor variances + 10 factor covariances = 157 parameters USED				
TERROR .929 1.000					
	Possible = $49*50/2 + 49 = 1274$				
ISOLATE .898 .876 1.000	DF =1117 calculation: 1274 – 157 = 1117				
CORRUPT .689 .792 .658 1.000					
IGNORE .830 .767 .828 .630 1.000	Now we can test the fit of a constrained structural model that posits a single				
10101L .030 .707 .020 .030 1.000	higher-order "General Abuse" factor to account for the correlations among				
	these 5 latent factors (shown on the left from TECH 4).				

Syntax for CFA model with MLR and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

<pre>! 12-Item Spurning Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; ! 9-Item Terrorizing Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*; ! 6-Item Isolating</pre>	H0 Value -69080.656
Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; ! 9-Item Terrorizing Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*; ! 6-Item Isolating	HO Value -69080.656
<pre>! 9-Item Terrorizing Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*; ! 6-Item Isolating</pre>	H0 Value -69080.656
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*; ! 6-Item Isolating	
! 6-Item Isolating	
	HO Scaling Correction Factor 2.5109
Isolate BY p01@1 p18* p19* p23* p39* p43*;	for MLR
! 7-Item Corrupting	H1 Value -65787.405
Corrupt BY p0901 p12* p16* p20* p28* p47* p50*;	H1 Scaling Correction Factor 1.5925
! 15-Item Ignoring	for MLR
Tanore BY p0201 p03* p04* p21* p22* p30* p31* p37* p40* p44*	
p45* p46* p51* p52* p57*;	ation Criteria
	Akaike (AIC) 138465.313
! Factor Variances (all must be free - NOW ARE "DISTURBANCES")	Bayesian (BIC) 139255.323
Spurn* Terror* Isolate* Corrupt* Ignore*;	Sample-Size Adjusted BIC 138772.486
! Factor Means (all fixed = 0 by default)	$(n^* = (n + 2) / 24)$
[Spurp(0] Terror(0] Isolate(0) Corrupt(0) Ignore(0):	
Chi-Squ	lare Test of Model Fit
! Higher-Order Factor (estimate higher-order factor loadings)	Value 4486.381*
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;	Degrees of Freedom 1122
! Fix higher-order factor mean=0 & variance=1	P-Value 0.0000
[Abuse@0]; Abuse@1;	Scaling Correction Factor 1.4681
	for MLR
We can use a $-2\Delta LL$ scaled difference to test the fit of the higher-order	(Root Mean Square Error Of Approximation)
	Estimate 0.047
factor model against the saturated structural model with all possible factor	90 Percent C.I. 0.046 0.049
correlations. This higher-order factor model uses 5 fewer parameters: the	Probability RMSEA <= .05 0.999
5 higher-order loadings replace the 10 covariances among the factors	
The $-2\Delta LL$ scaled difference is $-2\Delta LL(5) = 46.848$, $p < .0001$.	CFI 0.844
	TLI 0.837
So trying to reproduce the 5 factor covariances with a single higher-order	Standardized Root Mean Square Residual)
factor results in a significant decrease in fit. Why might this be the case? All	Value 0.058
the lower-order factors have large (enough) standardized loadings	
	arameters = 152 = 44 loadings + 49 intercepts + 49 error variances
Two-Tailed +5	factor variances + 5 higher-order loadings = 152 parameters USED
Estimate S.E. Est./S.E. P-Value	
ABUSE BY (HIGHER-ORDER STANDARDIZED LOADINGS) POSSIBL	le = 49*50/2 + 49 = 1274
	17 calculation: $1274 - 152 = 1122$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	117 calculation. $1274 = 102 = 1122$
ISOLATE 0.933 0.016 59.159 0.000	
CORRUPT 0.745 0.027 27.312 0.000	
IGNORE 0.846 0.018 48.111 0.000	

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

MODEL MODIFICAT	TION INDICES				
Minimum M.I. va	3.840				
		M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
TERROR WITH		4.111	0.011	0.454	0.454
TERROR WITH 3	SPURN	4.111	0.011	0.454	0.454
CORRUPT WITH S	SPURN	18.864	-0.018	-0.451	-0.451
CORRUPT WITH T	TERROR	44.080	0.021	0.595	0.595
CORRUPT WITH I	ISOLATE	4.628	-0.006	-0.193	-0.193
IGNORE WITH S	SPURN	4.800	0.010	0.248	0.248
IGNORE WITH T	TERROR	31.774	-0.018	-0.510	-0.510
IGNORE WITH I	ISOLATE	14.098	0.010	0.317	0.317

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations), it appears we need to allow two more relationships among the factor disturbances, as follows:

MODEL: ! (To be changed below for each model)

- ! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
- ! 12-Item Spurning
- Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; ! 9-Item Terrorizing
- Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
- ! 6-Item Isolating
- Isolate BY p01@1 p18* p19* p23* p39* p43*;
- ! 7-Item Corrupting Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
- ! 15-Item Ignoring
 - Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44* p45* p46* p51* p52* p57*;
- ! Factor Variances (all must be free NOW ARE "DISTURBANCES")
 Spurn* Terror* Isolate* Corrupt* Ignore*;
- ! Factor Means (all fixed = 0 by default)
 [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
- ! Higher-Order Factor (estimate higher-order factor loadings) Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
- ! Fix higher-order factor mean=0 & variance=1
 [Abuse@0]; Abuse@1;
- ! Add disturbance covariances suggested by mod indices Corrupt WITH Terror*; Ignore WITH Terror*;

MLR Solutions								
Saturated: 5-Factor All Covariances Model								
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE			
SPURN	1.000							
TERROR	.929	1.000						
ISOLATE	.898	.876	1.000					
CORRUPT	.689	.792	.658	1.000				
IGNORE	.830	.767	.828	.630	1.000			
P	redicted 1: 5		_					
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE			
SPURN	1.000							
TERROR	.925	1.000						
ISOLATE	.906	.889	1.000					
CORRUPT	.724	.710	.696	1.000				
IGNORE	.821	.806	.790	.631	1.000			
	_	oancy: Satu						
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE			
SPURN								
TERROR	.004							
ISOLATE	008	013						
CORRUPT	035	.082	038					
IGNORE	.009	039	.038	001				
Predicte	ed 2: 5-Facto	_						
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE			
SPURN	1.000							
TERROR	.923	1.000						
ISOLATE	.898	.894	1.000					
CORRUPT	.690	.794	.668	1.000				
IGNORE	.838	.766	.812	.623	1.000			
		<u> </u>						
	_	bancy: Satu						
CDUDAL	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE			
SPURN	0000							
TERROR	.006	.						
ISOLATE	.000	018						
CORRUPT	001	002	010					

IGNORE

-.008

.001

.016

.007

					PSQF 6249 Example 8 page 7
MODEL FIT INF					Two comparisons are relevant:
Number of Fre	e Parameters		154		
Loglikelihood	1				First, did we help the higher-order factor model by adding two covariances among
2	Value		-69031.180		the factor disturbances? $-2\Delta LL(2) = 46.378$, $p < .0001$, so yes, model fit is better.
	Scaling Correction	. Factor	2.5060		The factor distributions $= 2\Delta EE(2) = 40.076$, $\beta < 0.007$, so yes, model in is better.
	for MLR	ridecor	2.0000		
	Value		-65787.405		Second, does the revised higher-order factor model fit nonsignificantly worse
	Scaling Correction	n Factor	1.5925		than the saturated structural model with all 10 correlations among the 5 factors?
	for MLR				$-2\Delta LL(3) = 3.171$, $p = .3660$, so yes, our revised model captures those 10
					correlations using 3 fewer parameters (5 loadings + 2 covariances).
Information C	Criteria				correlations using 5 rewer parameters (5 loadings + 2 covariances).
Aka	ike (AIC)		138370.360		
-	vesian (BIC)		139170.765		Example results section for CFA using MLR:
	ple-Size Adjusted		138681.575		
($(n^* = (n + 2) / 24)$)			After examining the fit of each of the five factors individually, as described previously,
					a combined model was estimated in which all five factors were fit simultaneously with
-	est of Model Fit		4400 550		
Val			4422.556		covariances estimated freely among them. A total of 49 items were thus included.
	grees of Freedom Value		1120 0.0000		Robust maximum likelihood (MLR) estimation was used to estimate all models using
	alue Aling Correction Fa	actor	1.4669		Mplus v. 8.10 (Muthén & Muthén, 1998–2017), and differences in fit between nested
	for MLR	10001	1.1005		models were evaluated using -2^* rescaled difference in the model log-likelihood
_					values. The fit of each model referenced below is shown in Table 1.
RMSEA (Root M	lean Square Error (Of Approx	imation)		
	imate		0.047		We first established the need for five factors by chaving a similarity despect of fit
90	Percent C.I.		0.046	0.048	We first established the need for five factors by showing a significant decrease in fit
Pro	bability RMSEA <=	.05	1.000		for a single-factor model relative to that of the five-factor model, $-2\Delta LL(10) = 671.689$,
					p < .0001. As shown in Table 1, the fit of the model with five correlated factors was
CFI/TLI					acceptable by the RMSEA (.047), but not by the CFI (.847). Standardized model
CFI			0.847		parameters (loadings, intercepts, and error variances) are shown in Table 2.
TLI	-		0.840		
CDMD (Chander	diand Deat Mana Co	Dee			Correlations \geq .6 were found amongst the five factors, suggesting evidence that the
Val	dized Root Mean So	quare kes	0.057		five factors may be indicators of a single higher-order factor. This idea was tested by
Val	ue		0.037		eliminating the covariances among the factors and instead estimating loadings for the
					five factors from a single higher-order factor (whose variance was fixed to 1).
STDYX Standar	dization				Although the fit of the higher-order factor model remained marginal overall (see Table
				Two-Tailed	1), a nested model comparison revealed a significant decrease in fit, $-2\Delta LL(5) =$
	Estimate	S.E.	Est./S.E.	P-Value	
ABUSE BY	HIGHER-ORDER STAN	DARDIZED	LOADINGS)		46.848, $p < .0001$, indicating that a single factor did not appear adequate to describe
SPURN	0.963	0.011	87.824	0.000	the pattern of correlation amongst the five factors. Inspection of the discrepancy
TERROR	0.958	0.012		0.000	between the factor correlations from the saturated five-factor model and those
ISOLATE	0.933	0.016		0.000	predicted by the higher-order factor indicated two sources of misfit-the correlation
CORRUPT	0.716	0.028		0.000	between Corrupt and Terror was under-estimated, whereas the correlation between
IGNORE	0.870	0.019	45.845	0.000	Ignore and Terror was over-estimated. These discrepancies were captured via two
CORRUPT WIT	ים				
TERROR	0.540	0.097	5.550	0.000	additional covariances among those lower-order factor disturbances, resulting in a
IGNORE WIT		0.097	5.550	0.000	significant improvement in fit, $-2\Delta LL(2) = 46.378$, $p < .0001$. Further, the revised
TERROR	-0.483	0.172	-2.811	0.005	model successfully accounted for the pattern of correlation among the five factors, as
	0.100		2.011		indicated by a nonsignificant decrease in model fit relative to the model with all 10
				01 0/	factor correlations estimated directly, $-2\Delta LL(3) = 3.171$, $p = .3660$.
Next we will	duplicate these	analyse	es using W	SMV, which	

requires starting with the biggest model first instead...
Syntax for IFA model with WLSMV including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

DATA: FILE = abuse.csv; ! Don't need path if in same folder as input	
TYPE = INDIVIDUAL; FORMAT = FREE; ! Defaults	MODEL: ! (To be changed below for each model)
VARIABLE:	
NAMES = ID ! All variables in DATA SET	! 5 Factors (loadings for first item are estimated)
p1 p02 p03 p04 p05 p06 p07 p08 p09 p10	! 12-Item Spurning
p11 p12 p13 p14 p15 p16 p17 p18 p19 p20	Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	9-Item Terrorizing
p31 p32 p33 p34 p35 p36 p37 p38 p39 p40	Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
p41 p42 p43 p44 p45 p46 p47 p48 p49 p50	! 6-Item Isolating
p51 p52 p53 p54 p55 p56 p57;	Isolate BY p01* p18* p19* p23* p39* p43*;
	! 7-Item Corrupting
USEVARIABLES = ! All variables in MODEL	Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
p01 p02 p03 p04 p06 p07 p09 p10	! 15-Item Ignoring
p11 p12 p13 p14 p16 p17 p18 p19 p20	Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	p45* p46* p51* p52* p57*;
p31 p33 p35 p36 p37 p39 p40	
p43 p44 p45 p46 p47 p48 p49 p50	! Factor Variances (all must be fixed to 1 for identification)
p51 p52 p53 p54 p55 p56 p57;	Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
	! Factor Means (all fixed = 0 by default)
IDVARIABLE = ID; ! Person ID variable	[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
MISSING = ALL (99999); ! Missing data value used	! Factor Covariances (all estimated to allow 5-factor model)
MISSING - ALL (55555), · · MISSING data value used	Spurn Terror Isolate Corrupt Ignore WITH
CATEGORICAL = ! All ordinal outcomes for IFA	<pre>Spurn* Terror* Isolate* Corrupt* Ignore*;</pre>
p01 p02 p03 p04 p06 p07 p09 p10	
p11 p12 p13 p14 p16 p17 p18 p19 p20	
p21 p22 p23 p24 p25 p26 p27 p28 p29 p30	NOTE: With respect to fit of the structured model. Letting the E factors just be
p31 p33 p35 p36 p37 p39 p40	<u>NOTE:</u> With respect to fit of the structural model, letting the 5 factors just be
p43 p44 p45 p46 p47 p48 p49 p50	correlated is as good as it gets. This saturated structural model will be our
p51 p52 p53 p54 p55 p56 p57;	"larger model" baseline with which to compare the fit of models that try to
OUTPUT: STDYX ! Standardized solution	account for these correlations via a higher-order factor ("smaller models").
MODINDICES (3.84) ! Cheat codes for fixing the model	
RESIDUAL ! Local fit info	MODEL FIT INFORMATION
TECH4; ! Factor correlation matrix	Number of Free Parameters 255
PLOT: TYPE = PLOT1 PLOT2 PLOT3; ! For pictures	
	Chi-Square Test of Model Fit
ANALYSIS: ESTIMATOR = WLSMV; ! Limited-info in probits	Value 5934.136*
PARAMETERIZATION = THETA;	Degrees of Freedom 1117
CONVERGENCE = 0.0000001; ! For OS comparability	P-Value 0.0000
CONVERGENCE = 0.0000001; ! FOR US COmparability	
	RMSEA (Root Mean Square Error Of Approximation)
SAVEDATA: DIFFTEST=5factor.dat; ! Save fit of 5-factor model	Estimate 0.057
1	90 Percent C.I. 0.055 0.058
	Probability RMSEA <= .05 0.000
	FIODADIIILY MAER <05 0.000
#free parameters = $255 = 49$ loadings + $49*4=196$ thresholds	CFI/TLI
+ 0 factor variances + 10 factor covariances = 255 parameters USED	
+ 0 lactor variances + 10 lactor covariances = 200 parameters USED	CFI 0.927
	TLI 0.923
Possible = $49*50/2 + 49*4 = 1421$	
DF =1117 calculation: $1421 - 255 - 49$ "residual variances" = 1117	SRMR (Standardized Root Mean Square Residual)
$D_1 = 1117$ calculation. $1421 = 200 = 43$ residual valiances = 1117	Value 0.056

Strawman model: Syntax for single-factor IFA model estimated using WLSMV through 5 PERFECTLY correlated factors

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model	THE MODEL ESTIMATION TERMINATED NORMAL	LLY			
! (no SAVEDATA needed)	Because the factor covariances we	ere fixed to 1, you will see the			
MODEL: ! (To be changed below for each model)	message below. In THIS CONTEXT	ONLY, you can ignore it.			
! 5 Factors (loadings for first item are estimated)	WARNING: THE LATENT VARIABLE COVARIAN DEFINITE. THIS COULD INDICATE A NEGAS				
! 12-Item Spurning Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	FOR A LATENT VARIABLE, A CORRELATION (
9-Item Terrorizing	TWO LATENT VARIABLES, OR A LINEAR DEPH				
Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;	LATENT VARIABLES. CHECK THE TECH4 OUT				
! 6-Item Isolating	PROBLEM INVOLVING VARIABLE TERROR.				
Isolate BY p01* p18* p19* p23* p39* p43*;					
! 7-Item Corrupting	MODEL FIT INFORMATION				
Corrupt BY p09* p12* p16* p20* p28* p47* p50*;	Number of Free Parameters	245			
! 15-Item Ignoring	Chi-Square Test of Model Fit				
Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*	Value	7563.407*			
p45* p46* p51* p52* p57*;	Degrees of Freedom	1127			
! Factor Variances (all must be fixed to 1 for identification)	P-Value	0.0000			
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;					
<pre>! Factor Means (all fixed = 0 by default)</pre>	Chi-Square Test for Difference Testing				
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	Value	769.755*			
! Factor Covariance (all fixed to 1 to create 1-factor model)	Degrees of Freedom	10			
Spurn Terror Isolate Corrupt Ignore WITH	P-Value	0.0000			
Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;	DMOED (Deet Meen Omiene Enner Of Jerry				
	RMSEA (Root Mean Square Error Of Appro Estimate	0.065			
	90 Percent C.I.	0.064 0.067			
#free parameters = $245 = 49$ loadings + $49*4=196$ thresholds	Probability RMSEA <= .05	0.000			
+ 0 factor variances + 0 factor covariances = 245 parameters USED					
	CFI/TLI				
Possible = 49*50/2 + 49*4 = 1421	CFI	0.903			
DF =1117 calculation: 1421 – 245 – 49 "error variances" = 1127	TLI	0.898			
	SRMR (Standardized Root Mean Square Re	esidual)			
	Value	0.068			
	Do we have one factor or five factors?				
	According to the DIFFTEST relative to the previous 5-factor model: $\chi^2(10) = 769.755$, $p < .0001$				
	Therefore, one factor does not capture the covariances for these 4 Five factors (as hypothesized) does a significantly better job.				

Syntax for IFA model with WLSMV and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model	NOTE: With respect to fit of the stru		0
SAVEDATA: DIFFTEST=HigherOrder.dat; ! Save fit of higher-order model	single higher-order factor INSTEAD	OOF covariances am	ong the 5 factors.
MODEL: ! (To be changed below for each model)	MODEL FIT INFORMATION		
MODEL: ! (10 be changed below for each model)	Number of Free Parameters	250	
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)			
! 12-Item Spurning	Chi-Square Test of Model Fit		
Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;	Value	5941.909*	
9-Item Terrorizing	Degrees of Freedom	1122	
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;	P-Value	0.0000	
! 6-Item Isolating			
Isolate BY p01@1 p18* p19* p23* p39* p43*;	Chi-Square Test for Difference T	-	
! 7-Item Corrupting	Value Desmos of Freedom	92.048*	
Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;	Degrees of Freedom P-Value	د 0.0000	
! 15-Item Ignoring	P-value	0.0000	
Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*	RMSEA (Root Mean Square Error Of	Approximation	
p45* p46* p51* p52* p57*;	Estimate	Approximation) 0.057	
	90 Percent C.I.	0.055	0 059
! Factor Variances (all must be free - NOW ARE "DISTURBANCES")			0.058
Spurn* Terror* Isolate* Corrupt* Ignore*;	Probability RMSEA <= .	0.000	
! Factor Means (all fixed = 0 by default)			
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];	CFI/TLI	0.927	
	CFI		
! Higher-Order Factor (estimate higher-order factor loadings)	TLI	0.924	
Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;	ODVD (Observational) - and Death Margare Com		
! Fix higher-order factor mean=0 & variance=1	SRMR (Standardized Root Mean Squ Value		
[Abuse@0]; Abuse@1;	value	0.057	
	#free neverneters OFO 44 leading	and 10*1 100 three	halda
	#free parameters = 250 = 44 loadir		
	+ 5 factor variances + 5 higher-	-order loadings = 250	parameters USED
We can use DIFFTEST to test the fit of the higher-order factor model against			
the saturated structural model with all possible factor correlations. This higher-	Possible = $49*50/2 + 49*4 = 1421$		
order factor model uses 5 fewer parameters: 5 higher-order loadings replace		40 "residual varian	
	DF =1117 calculation: 1421 – 250 -	- 49 residual variand	es = 1122
the 10 covariances among the factors. The difference is $\chi^2(5) = 92.048$, $p < 100$			
.0001.	STDYX Standardization		
			Two-Tailed
	Estimate	S.E. Est./S.E.	P-Value
Contrained to reproduce the E factor equationed with a single higher order		RDIZED LOADINGS)	
So trying to reproduce the 5 factor covariances with a single higher-order	ABUSE BY (HIGHER-ORDER STANDA		
So trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Why might this be the case? All	SPURN 0.990	0.005 204.056	0.000
factor results in a significant decrease in fit. Why might this be the case? All		0.007 139.928	0.000 0.000
	SPURN 0.990		
factor results in a significant decrease in fit. Why might this be the case? All	SPURN 0.990 TERROR 0.948	0.007 139.928	0.000

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

MODEL MO Minimum	3.840					
			M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
TERROR	WITH	SPURN	8.776	0.018	0.558	0.558
ISOLATE	WITH	SPURN	11.743	-0.025	-0.742	-0.742
ISOLATE	WITH	TERROR	5.966	-0.022	-0.256	-0.256
CORRUPT	WITH	SPURN	39.197	-0.056	-0.762	-0.762
CORRUPT	WITH	TERROR	122.583	0.116	0.627	0.627
IGNORE	WITH	SPURN	25.058	0.050	0.596	0.596
IGNORE	WITH	TERROR	82.830	-0.100	-0.471	-0.471
IGNORE	WITH	ISOLATE	42.440	0.080	0.372	0.372
IGNORE	WITH	CORRUPT	6.035	-0.036	-0.077	-0.077

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations), it appears we need to allow two more relationships among the factor disturbances, as follows:

ANALYSIS: DIFFTEST=5factor.dat; ! Test fit against 5-factor model							
SAVEDATA: DIFFTEST=HigherOrder2.dat; ! Save fit of higher-order2 model							
MODEL: ! (To be changed below for each model)							
! 5 Lower-Order Factors (loadings for first item NOW FIXED =1) ! 12-Item Spurning							
Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*; 9 - Item Terrorizing							
Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;							
! 6-Item Isolating Isolate BY p0101 p18* p19* p23* p39* p43*;							
! 7-Item Corrupting Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;							
! 15-Item Ignoring Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44* p45* p46* p51* p52* p57*;							
! Factor Variances (all must be free - NOW ARE "DISTURBANCES")							
Spurn* Terror* Isolate* Corrupt* Ignore*; ! Factor Means (all fixed = 0 by default)							
[Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];							
<pre>! Higher-Order Factor (estimate higher-order factor loadings) Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;</pre>							
! Fix higher-order factor mean=0 & variance=1							
[Abuse@0]; Abuse@1;							
<pre>! Add disturbance covariances suggested by mod indices Corrupt WITH Terror*;</pre>							
Ignore WITH Terror*;							

rder Fact	tor Mode	•		lations:					
WLSMV Solutions									
Saturated: 5-Factor All Covariances Model									
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
SPURN	1.000								
TERROR	.947	1.000							
ISOLATE	.925	.885	1.000						
CORRUPT	.791	.866	.776	1.000					
IGNORE	.882	.817	.863	.729	1.000				
Predicted 1: 5-Factor + Higher-Order Factor Model									
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
SPURN	1.000								
TERROR	.938	1.000							
ISOLATE	.941	.902	1.000						
CORRUPT	.826	.791	.794	1.000					
IGNORE	.876	.839	.841	.738	1.000				
	Discrepa	ancy: Satur	ated - Pred	dicted 1					
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
SPURN									
TERROR	.009								
ISOLATE	016	017							
CORRUPT	035	.075	018						
IGNORE	.006	022	.022	009					
Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model									
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
SPURN	1.000								
TERROR	.939	1.000							
ISOLATE	.927	.907	1.000						
CORRUPT	.792	.866	.765	1.000					
IGNORE	.885	.817	.855	.730	1.000				
	Discrepa	ancy: Satur	ated - Pred	dicted 2					
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE				
SPURN									
TERROR	.008								
ISOLATE	002	022							
CORRUPT	001	.000	.011						

MODEL FT	I INFORMATION								
	f Free Paramet	ters		252					
Chi-Squar	re Test of Mod	del Fit							
	Value			5853.773*					
	Degrees of 1	Freedom		1120					
	P-Value			0.0000					
Chi-Sour	Chi-Square Test for Difference Testing								
CIII Squar	Value	TTETENCE	restring	8.483*					
	Degrees of 1	Freedom		3					
	P-Value			0.0370					
RMSEA (Ro	oot Mean Squa	re Error C	of Approxi						
	Estimate			0.056					
	90 Percent (0.5		0.058				
	Probability	RMSEA <=	.05	0.000					
CFI/TLI									
CT T/ THT	CFI			0.928					
	TLI			0.925					
SRMR (Sta	andardized Roo	ot Mean Sq	uare Resi	dual)					
	Value			0.056					
STDIX STA	andardization				Two-Tailed				
	E	stimate	S.E.	Est./S.E.	P-Value				
ABUSE	BY (HIGHER-O				1 14140				
SPURI			0.006		0.000				
TERRO	OR	0.959	0.007	130.093					
ISOLA	ATE	0.946	0.009		0.000				
CORRI		0.809	0.015		0.000				
IGNO	RE	0.903	0.009	97.384	0.000				
CODDUDE	MT DI								
CORRUPT TERRO		0.544	0.068	7.984	0.000				
IGNORE		0.011	0.000	1.504	0.000				
TERRO		-0.406	0.102	-3.991	0.000				
MODEL MOI	DIFICATION IN	DICES							
Minimum N	M.I. value for	r printing	the modi	fication ind	dex 3.840				
					o				
E D C		M.1.	E.P.C.	Std E.P.C.	StdYX				
E.P.C.	WITH SPURN	13.421	0.031	0.757	0.757				
	WITH TERROR	13.421	-0.031		-0.454				
	WITH SPURN	5.964	-0.029		-0.271				
	WITH ISOLATE		0.039		0.186				
It looks like we could add a 1–2 more covariances to ensure not									
worse fit than the saturated (all 10 correlations) model, but which									
	e added seer								
		iis somew		ary 50111					
done.									

PSQF 6249 Example 8 page 12 Two comparisons are relevant. First, did we help the higher-order factor model by adding two covariances among the factor disturbances? This comparison is not shown here (had to re-estimate the model without them and then compare against the model with them), but yes, $\chi^2(2) = 88.343$, p < .0001, so model fit is better. Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? Almost: $\chi^2(3) = 8.483$, p = .0370. So our revised model almost captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

Example results section for IFA using WLSMV:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. WLSMV estimation (i.e., diagonally weighted least squares) in Mplus v 8.10 including a probit link and the THETA parameterization (in which all item residual variances were constrained to 1 for identification) was used to estimate all models (Muthén & Muthén, 1998–2017). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items. The fit of each model referenced below is shown in Table 1. Nested model comparisons were conducted using the Mplus DIFFTEST procedure.

We first established the need for 5 factors by showing a significant decrease in fit for a single-factor model relative to that of the five-factor model, $\chi^2(10) = 769.755$, p < .0001. As shown in Table 1, the fit of the model with five correlated factors was marginally acceptable by both the RMSEA (.057) and the CFI (.927). Standardized model parameters (loadings and thresholds) are shown in Table 2 [not given here]. Correlations \geq .7 were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was tested by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, $\chi^2(5) = 92.048$, p < .0001, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the 5-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was overestimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit, χ^2 (2) = 88.343, p < .0001. However, the revised model did not completely account for the pattern of correlation among the 5 factors, as indicated by a significant decrease in model fit relative to the model with all 10 factor correlations estimated directly, $\chi^2(3)$ = 8.483, p = .0370.