

Example 7d: Testing Multiple-Group Measurement Invariance in Item Factor Models using ML in Mplus v. 8.10 (complete Mplus syntax and output is available electronically)

This example comes from the [Octogenarian Twin Study of Aging](#) in Sweden. The current analysis includes 634 older adults (age 80–100 years) on 7 items measuring the Instrumental Activities of Daily Living (IADL), as also used in Example 6a. We are examining differences for men ($N=214$) versus women ($N=420$) using **two** response options (for simplicity; ordinal data would proceed similarly), in which the proportion of $y=1$ responses is shown here for each item for men and women →

Item		Mean	
		Men	Women
1	Housework	0.596	0.657
2	Bedmaking	0.816	0.853
3	Cooking	0.687	0.814
4	Everyday shopping	0.743	0.622
5	Walking to places	0.751	0.603
6	Handling banking	0.767	0.716
7	Using a telephone	0.934	0.948

Multiple Group IFA Model Syntax and Truncated Output:

```

TITLE: Assess invariance for binary IADL items using ML
DATA: FILE = Example7cd.csv; ! Don't need path if data in same folder
        FORMAT = free;          ! Default
        TYPE = INDIVIDUAL;      ! Default

VARIABLE: NAMES = case female dial-dia7 cial-cia7; ! All vars in data
        USEVARIABLES = dial-dia7; ! All vars in model
        CATEGORICAL = dial-dia7; ! All ordinal outcomes
        CLASSES = group(2);      ! 2 fake latent classes
        KNOWNCLASS = group(female=0 1); ! Latent classes = gender
        MISSING = ALL (9999);    ! Missing value code
        IDVARIABLE = case;       ! Person ID variable

ANALYSIS: ESTIMATOR = ML; LINK = LOGIT; ! Full-info ML in logits
        CONVERGENCE = 0.0000001; ! For OS comparability
        TYPE = MIXTURE;          ! For latent classes
        ALGORITHM = INTEGRATION; ! Required for latent classes
!PLOT: TYPE = PLOT1 PLOT2 PLOT3; ! Get all the plots
OUTPUT: STDYX TECH10; ! Local misfit, no mod indices for IFA in ML
MODEL:

! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
%OVERALL% ! Needed for fake latent class model
! Proportion of men in logits
[group#1@-0.67428];
! Factor loadings all freely estimated, just labeled
IADL BY dial-dia7* (L1-L7);
! Item thresholds all freely estimated, just labeled
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

! WOMEN ALTERNATIVE GROUP 2 CONFIGURAL MODEL 1
%group#2% ! Needed for fake latent class model
! Factor loadings all freely estimated
IADL BY dial-dia7*;
! Item thresholds all freely estimated
[dial$1-dia7$1*];
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

```

MODEL FIT INFORMATION

```

Number of Free Parameters      28

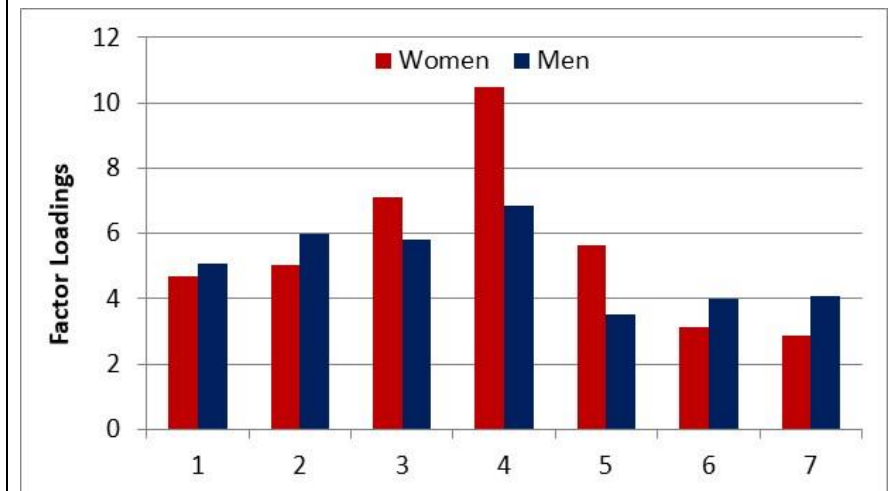
Loglikelihood
  H0 Value                    -1793.471

Information Criteria
  Akaike (AIC)                3642.942
  Bayesian (BIC)              3767.600
  Sample-Size Adjusted BIC    3687.703
                                (n* = (n + 2) / 24)

```

This will be our baseline configural model.

29 parameters estimated = $2 \times [7 \text{ loadings} + 7 \text{ thresholds}] = 28$
 Here are the logit factor loadings by group (foreshadowing).



Model 1. Configural Invariance Model**(Everything separate across groups *except* for parameters needed to be constrained for identification)**

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)					UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class 1 (0) - refers to female=0 from KNOWNCLASS					Latent Class 2 (1) - refers to female=1 from KNOWNCLASS				
IADL BY -- FACTOR LOADINGS: dif in logit for unit dif in theta					IADL BY -- FACTOR LOADINGS: dif in logit for unit dif in theta				
DIA1	5.076	1.291	3.932	0.000	DIA1	4.665	0.730	6.391	0.000
DIA2	5.974	1.772	3.371	0.001	DIA2	5.020	0.993	5.055	0.000
DIA3	5.794	1.552	3.733	0.000	DIA3	7.095	1.660	4.274	0.000
DIA4	6.835	2.126	3.214	0.001	DIA4	10.466	6.770	1.546	0.122!!!
DIA5	3.504	0.712	4.918	0.000	DIA5	5.627	1.114	5.052	0.000
DIA6	4.006	0.882	4.544	0.000	DIA6	3.141	0.422	7.440	0.000
DIA7	4.060	1.388	2.925	0.003	DIA7	2.856	0.610	4.681	0.000
Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION					Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000	IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED LOGIT(Y=0) IF THETA=0					Thresholds: EXPECTED LOGIT(Y=0) IF THETA=0				
DIA1\$1	-1.382	0.614	-2.250	0.024	DIA1\$1	-2.189	0.425	-5.150	0.000
DIA2\$1	-5.602	1.580	-3.546	0.000	DIA2\$1	-5.705	1.017	-5.608	0.000
DIA3\$1	-3.024	0.918	-3.295	0.001	DIA3\$1	-6.710	1.490	-4.505	0.000
DIA4\$1	-4.449	1.372	-3.243	0.001	DIA4\$1	-3.687	2.471	-1.492	0.136
DIA5\$1	-2.736	0.553	-4.946	0.000	DIA5\$1	-1.803	0.493	-3.658	0.000
DIA6\$1	-3.347	0.708	-4.726	0.000	DIA6\$1	-2.185	0.314	-6.953	0.000
DIA7\$1	-6.816	1.973	-3.454	0.001	DIA7\$1	-5.578	0.886	-6.296	0.000
Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION					Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000	IADL	1.000	0.000	999.000	999.000
IRT PARAMETERIZATION					IRT PARAMETERIZATION				
Item Discriminations: slope where prob(y=1)=0.5					Item Discriminations: slope where prob(y=1)=0.5				
IADL BY					IADL BY				
DIA1	5.076	1.291	3.932	0.000	DIA1	4.665	0.730	6.391	0.000
DIA2	5.974	1.772	3.371	0.001	DIA2	5.020	0.993	5.055	0.000
DIA3	5.794	1.552	3.733	0.000	DIA3	7.095	1.660	4.274	0.000
DIA4	6.835	2.126	3.214	0.001	DIA4	10.466	6.770	1.546	0.122!!!
DIA5	3.504	0.712	4.918	0.000	DIA5	5.627	1.114	5.052	0.000
DIA6	4.006	0.882	4.544	0.000	DIA6	3.141	0.422	7.440	0.000
DIA7	4.060	1.388	2.925	0.003	DIA7	2.856	0.610	4.681	0.000
Item Difficulties: theta needed for prob(y=1)=0.5					Item Difficulties: theta needed for prob(y=1)=0.5				
DIA1	-0.272	0.091	-2.994	0.003	DIA1	-0.469	0.062	-7.588	0.000
DIA2	-0.938	0.103	-9.111	0.000	DIA2	-1.136	0.081	-14.007	0.000
DIA3	-0.522	0.089	-5.877	0.000	DIA3	-0.946	0.068	-13.839	0.000
DIA4	-0.651	0.091	-7.140	0.000	DIA4	-0.352	0.051	-6.915	0.000
DIA5	-0.781	0.107	-7.316	0.000	DIA5	-0.320	0.061	-5.257	0.000
DIA6	-0.835	0.105	-7.937	0.000	DIA6	-0.695	0.073	-9.574	0.000
DIA7	-1.679	0.179	-9.387	0.000	DIA7	-1.953	0.176	-11.105	0.000
Categorical Latent Variables									
Means : Proportion of cases in female=0 in logits → prob = .338									
GROUP#1	-0.674	0.000	999.000	999.000					

Model 2a. Metric Invariance Model (IFA loadings held equal across groups – but IRT discriminations still vary via factor variances)

MODEL:

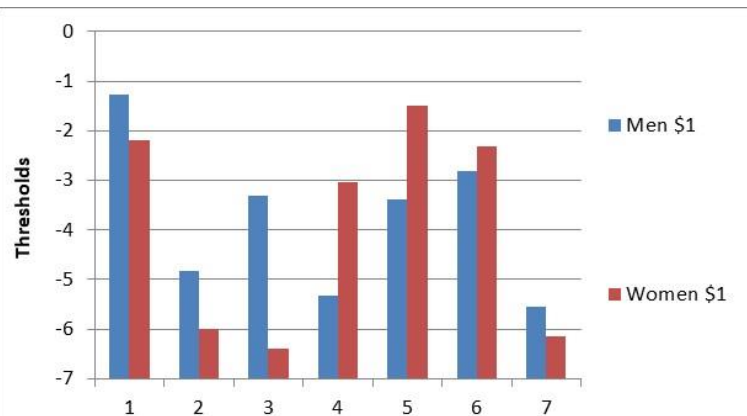
```
! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
%OVERALL% ! Needed for fake latent class model
! Proportion of men in logits
[group#1@-0.67428];
! Factor loadings all freely estimated, just labeled
IADL BY dial-dia7* (L1-L7);
! Item thresholds all freely estimated, just labeled
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;

! WOMEN ALTERNATIVE GROUP 2 METRIC MODEL 2
%group#2% ! Needed for fake latent class model
! Factor loadings NOW CONSTRAINED EQUAL TO MEN VIA SAME LABELS
IADL BY dial-dia7* (L1-L7);
! Item thresholds all freely estimated
[dial$1-dia7$1*];
! Factor variance NOW ESTIMATED
IADL*;
! Factor mean fixed to 0 for identification
[IADL@0];
```

```
Number of Free Parameters      22
Loglikelihood
  H0 Value                    -1797.600

Information Criteria
  Akaike (AIC)                3639.200
  Bayesian (BIC)              3737.145
  Sample-Size Adjusted BIC    3667.298
```

Does the full metric invariance model (2a) fit significantly worse than the configural model (1)? Nope, $-2\Delta LL(df=6) = 8.26, p = .22$



	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class 1 (0) – refers to female=0 from KNOWNCLASS				
IADL BY -- FACTOR LOADINGS: dif in logit(y=1) for unit dif in theta				
DIA1	4.415	0.660	6.691	0.000
DIA2	4.926	0.871	5.652	0.000
DIA3	6.249	1.206	5.182	0.000
DIA4	8.101	2.195	3.690	0.000
DIA5	4.403	0.635	6.930	0.000
DIA6	3.159	0.434	7.275	0.000
DIA7	3.028	0.591	5.121	0.000
Means: Factor mean fixed=0 for identification				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED LOGIT(Y=0) IF THETA=0				
DIA1\$1	-1.268	0.452	-2.804	0.005
DIA2\$1	-4.816	0.853	-5.646	0.000
DIA3\$1	-3.320	0.803	-4.131	0.000
DIA4\$1	-5.313	1.483	-3.583	0.000
DIA5\$1	-3.382	0.573	-5.904	0.000
DIA6\$1	-2.825	0.432	-6.543	0.000
DIA7\$1	-5.549	0.902	-6.152	0.000
Variances: Factor variance fixed=1 for identification				
IADL	1.000	0.000	999.000	999.000
Latent Class 2 (1) – refers to female=1 from KNOWNCLASS				
IADL BY -- FACTOR LOADINGS: dif in logit(y=1) for unit dif in theta				
DIA1	4.415	0.660	6.691	0.000
DIA2	4.926	0.871	5.652	0.000
DIA3	6.249	1.206	5.182	0.000
DIA4	8.101	2.195	3.690	0.000
DIA5	4.403	0.635	6.930	0.000
DIA6	3.159	0.434	7.275	0.000
DIA7	3.028	0.591	5.121	0.000
Means: Factor mean fixed=0 for identification				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED LOGIT(Y=0) IF THETA=0				
DIA1\$1	-2.200	0.397	-5.539	0.000
DIA2\$1	-5.989	0.908	-6.598	0.000
DIA3\$1	-6.395	1.114	-5.738	0.000
DIA4\$1	-3.034	0.950	-3.195	0.001
DIA5\$1	-1.495	0.360	-4.154	0.000
DIA6\$1	-2.314	0.314	-7.361	0.000
DIA7\$1	-6.150	0.885	-6.945	0.000
Variances: Factor variance now estimated				
IADL	1.192	0.268	4.441	0.000

Model 3a. Full Threshold Invariance Model (all IFA thresholds held equal across groups – but IRT difficulties still vary)**MODEL:**

```
! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
%OVERALL% ! Needed for fake latent class model
! Proportion of men in logits
[group#1@-0.67428];
! Factor loadings all freely estimated, just labeled
IADL BY dial-dia7* (L1-L7);
! Item thresholds all freely estimated, just labeled
[dial$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
[IADL@0]; IADL@1;
```

```
! WOMEN ALTERNATIVE GROUP 2 SCALAR MODEL 3
%group#2% ! Needed for fake latent class model
! Factor loadings still constrained equal to men
IADL BY dial-dia7* (L1-L7);
! Item thresholds NOW CONSTRAINED EQUAL TO MEN (SAME LABEL)
[dial$1-dia7$1*] (T1-T7);
! Factor mean NOW ESTIMATED
! Factor variance still estimated
[IADL*]; IADL*;
```

MODEL FIT INFORMATION

Number of Free Parameters 16

Loglikelihood

H0 Value -1857.971

Information Criteria

Akaike (AIC) 3747.943
Bayesian (BIC) 3819.176
Sample-Size Adjusted BIC 3768.377

Does the full scalar invariance model (3a) fit significantly worse than the full metric model (2a)?

Yep, $-2\Delta LL(df=6) = 120.74, p < .001$

Based on the local misfit of the univariate distributions (from TECH10 output, see right), it looks like item 3 is the biggest problem... so the same steps would be followed as in our other examples. That is, we would try to free problematic thresholds until the scalar model fits not worse than the metric model (or the configural model, if you choose to constrain both loadings and thresholds at the same time).

(See example 7c for a sample results section using WLSMV and polytomous versions of these items)

RESIDUAL OUTPUT**UNIVARIATE DISTRIBUTION FIT FOR CLASS 1 – refers to female=0 from KNOWNCLASS**

Variable	Observed	Estimated	Residual (Obs.-Est.)	Stand. Residual
DIA1				
Category 1	0.404	0.350	0.054	1.651
Category 2	0.596	0.650	-0.054	-1.651
DIA2				
Category 1	0.184	0.166	0.018	0.724
Category 2	0.816	0.834	-0.018	-0.724
DIA3				
Category 1	0.313	0.227	0.086	3.001
Category 2	0.687	0.773	-0.086	-3.001
DIA4				
Category 1	0.257	0.331	-0.074	-2.296
Category 2	0.743	0.669	0.074	2.296
DIA5				
Category 1	0.249	0.333	-0.084	-2.619
Category 2	0.751	0.667	0.084	2.619
DIA6				
Category 1	0.229	0.259	-0.030	-0.993
Category 2	0.771	0.741	0.030	0.993
DIA7				
Category 1	0.066	0.061	0.005	0.277
Category 2	0.934	0.939	-0.005	-0.277

UNIVARIATE DISTRIBUTION FIT FOR CLASS 2 – refers to female=1 from KNOWNCLASS

Variable	Observed	Estimated	Residual (Obs.-Est.)	Stand. Residual
DIA1				
Category 1	0.343	0.370	-0.027	-1.138
Category 2	0.657	0.630	0.027	1.138
DIA2				
Category 1	0.147	0.162	-0.015	-0.818
Category 2	0.853	0.838	0.015	0.818
DIA3				
Category 1	0.186	0.231	-0.045	-2.170
Category 2	0.814	0.769	0.045	2.170
DIA4				
Category 1	0.378	0.348	0.030	1.292
Category 2	0.622	0.652	-0.030	-1.292
DIA5				
Category 1	0.397	0.351	0.046	1.979
Category 2	0.603	0.649	-0.046	-1.979
DIA6				
Category 1	0.284	0.266	0.017	0.803
Category 2	0.716	0.734	-0.017	-0.803
DIA7				
Category 1	0.052	0.053	-0.001	-0.084
Category 2	0.948	0.947	0.001	0.084