General Linear Models for Testing Moderation: Single-Slope Interactions*

- Topics:
  - Ways of getting predicted outcomes given multiple predictors
  - Slopes of predictors within interactions: from unique main (marginal) effects to unique simple (conditional) effects
  - The 4 possible kinds of interactions: they make simple slopes more/less positive or more/less negative (and that's it)
  - Model-implied slopes as linear combinations of model slopes
  - Regions of significance for when simple slopes "turn on or off"

* Such as when testing an interaction among binary predictors or quantitative predictors described by a single linear slope

Creating Predicted Outcomes: 3 Options

- Figures of **predicted outcomes** will be essential in describing the results of any linear model (especially when interaction slopes are included)
- **Three ways to get them** (in order of most to least painful and inefficient):

1. In **excel**: input fixed effects, input predictor values, **write an equation** to create predicted outcomes for each combination of predictor values
   - Good for pedagogy, but is inefficient and error-prone (and SEs are harder)

2. Via **programming statements** (better for efficiency and accuracy):
   - Per prediction: Use SAS ESTIMATE or SPSS TEST → time-consuming ☹
   - For a range of predictor values: Use STATA MARGINS → way faster! ☺

3. Via **"fake people"** (most useful in SPSS and SAS without MARGINS)
   - Add cases to your data with desired predictor values (but no outcomes!)
   - Ask program to save predicted outcomes for all cases into your data
   - Fake cases won't contribute to model, but they will get predicted outcomes

Creating Predicted Outcomes: Option 1

#1

Fixed Effect Solution

Effect	Parameter	Estimate	StdErr	t Value	Pr > |t|
B0	Intercept	29.264	0.699	41.900	<.0001
B1	age85	-0.406	0.119	-3.410	0.001
B2	grip9	0.604	0.150	4.030	<.0001
B3	sexMW	-3.657	0.891	-4.100	<.0001
B4	demNF	-5.722	1.019	-5.610	<.0001
B5	demNC	-16.480	1.523	-10.820	<.0001

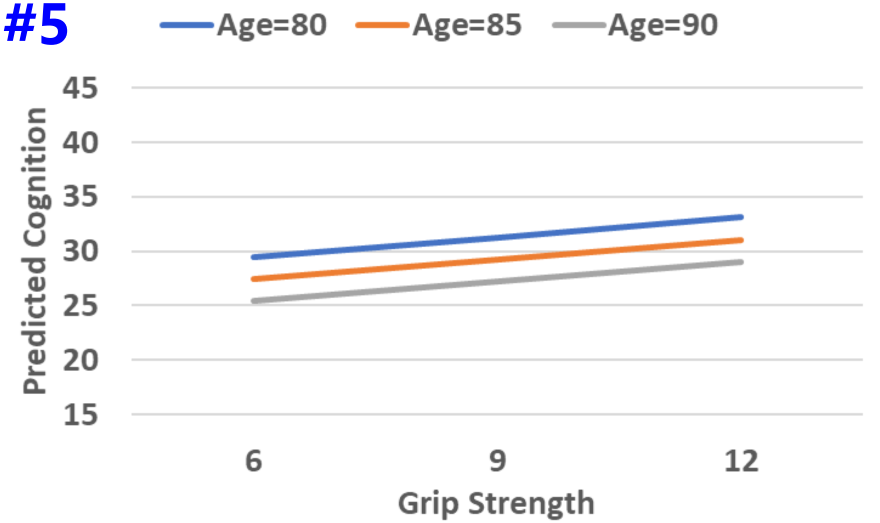
#2 Predictor Values / **#3** Pred

Age-85	Grip-9	SexMW	demNF	demNC	Pred Y-hat
-5	-3	0	0	0	29.480
-5	0	0	0	0	31.293
-5	3	0	0	0	33.106
0	-3	0	0	0	27.452
0	0	0	0	0	29.264
0	3	0	0	0	31.077
5	-3	0	0	0	25.423
5	0	0	0	0	27.236
5	3	0	0	0	29.048

#4

	Grip Strength 6	9	12
Age=80	29.480	31.293	33.106
Age=85	27.452	29.264	31.077
Age=90	25.423	27.236	29.048

#5



Process using Excel (see above):

1. Enter fixed effect coefficients
2. Enter values of predictors
3. Calculate predicted y-hat values
4. Re-arrange into matrix of values
5. Request plot of matrix values

Creating Predicted Outcomes: Option 2 using SAS ESTIMATEs

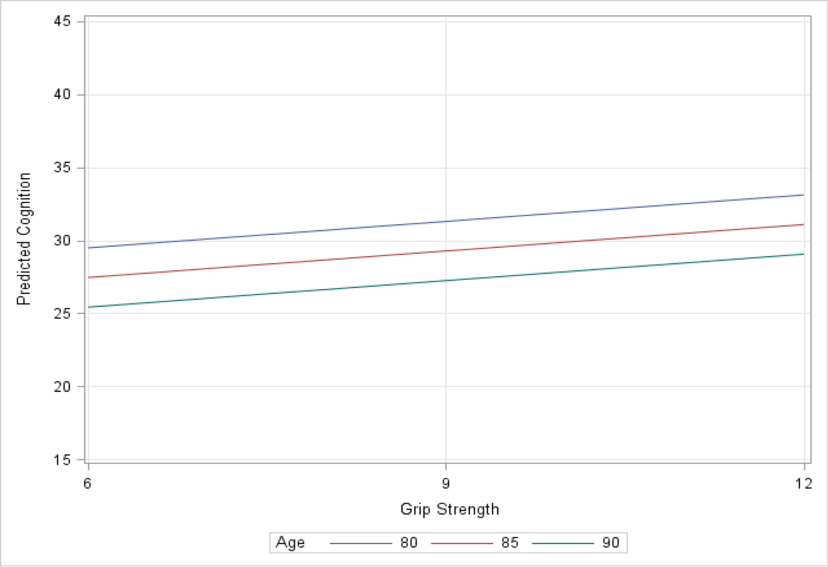
```
TITLE1 "SAS Combined Main Effects Only Model Predicting Cognition";
TITLE2 "Demonstrating how to get predicted outcomes using ESTIMATE statements";
PROC GLM DATA=work.Example6 NAMELEN=100;
     MODEL cognition = age85 grip9 sexMW demNF demNC / ALPHA=.05 CLPARM SOLUTION SS3 EFFECTSIZE;
     * Pred cognition outcomes holding sexMW=men, demNF=none, and demNC=none;
     ESTIMATE "Yhat for Age=80 Grip=6"  intercept 1 age85 -5 grip9 -3 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=80 Grip=9"  intercept 1 age85 -5 grip9  0 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=80 Grip=12" intercept 1 age85 -5 grip9  3 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=85 Grip=6"  intercept 1 age85  0 grip9 -3 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=85 Grip=9"  intercept 1 age85  0 grip9  0 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=85 Grip=12" intercept 1 age85  0 grip9  3 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=85 Grip=6"  intercept 1 age85  5 grip9 -3 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=85 Grip=9"  intercept 1 age85  5 grip9  0 sexMW 0 demNF 0 demNC 0;
     ESTIMATE "Yhat for Age=85 Grip=12" intercept 1 age85  5 grip9  3 sexMW 0 demNF 0 demNC 0;
     ODS OUTPUT Estimates=work.EstMainEffects; * Save ESTIMATEs to dataset for plotting;
RUN; QUIT; TITLE1; TITLE2;
```

Parameter	Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits	
Yhat for Age=80 Grip=6	29.4803185	1.15590606	25.50	<.0001	27.2097326	31.7509045
Yhat for Age=80 Grip=9	31.2929952	0.92090860	33.98	<.0001	29.4840228	33.1019676
Yhat for Age=80 Grip=12	33.1056719	0.87396571	37.88	<.0001	31.3889110	34.8224327
Yhat for Age=85 Grip=6	27.4516487	0.93731216	29.29	<.0001	25.6104543	29.2928432
Yhat for Age=85 Grip=9	29.2643254	0.69850792	41.90	<.0001	27.8922223	30.6364285
Yhat for Age=85 Grip=12	31.0770021	0.70785742	43.90	<.0001	29.6865335	32.4674707
Yhat for Age=90 Grip=6	25.4229789	1.06198691	23.94	<.0001	23.3368816	27.5090763
Yhat for Age=90 Grip=9	27.2356556	0.91355395	29.81	<.0001	25.4411302	29.0301810
Yhat for Age=90 Grip=12	29.0483323	0.97218055	29.88	<.0001	27.1386447	30.9580199

Creating Predicted Outcomes: Option 2 using SAS ESTIMATE

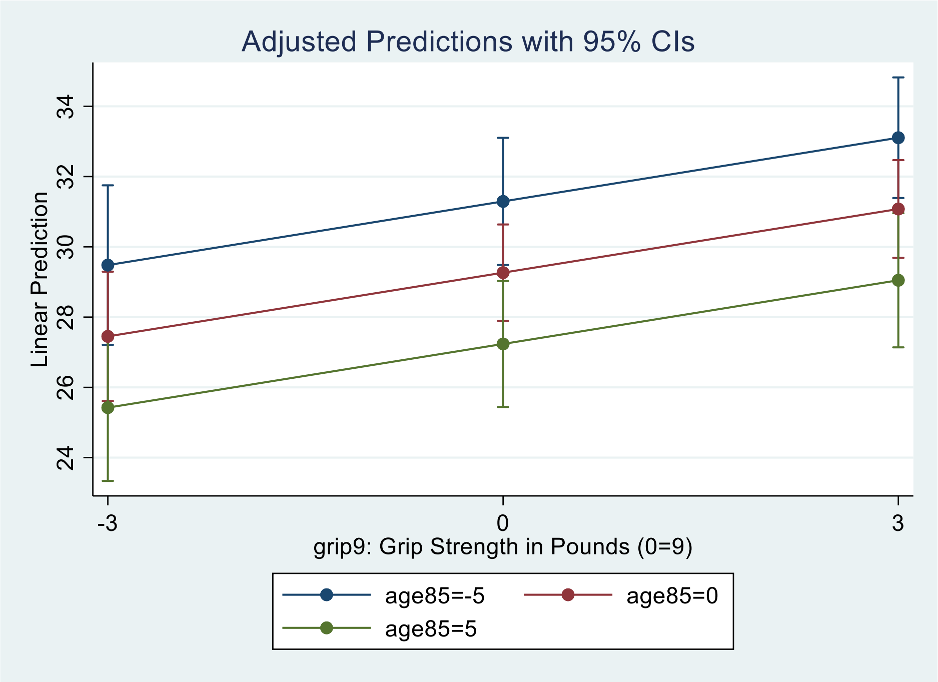
```
* Labeling saved ESTIMATES for use in plot;
* INDEX finds value in parentheses for that column;
DATA work.EstMainEffects; SET work.EstMainEffects;
IF INDEX(Parameter,"Age=80")>0  THEN age=80;
IF INDEX(Parameter,"Age=85")>0  THEN age=85;
IF INDEX(Parameter,"Age=90")>0  THEN age=90;
IF INDEX(Parameter,"Grip=6")>0  THEN grip=6;
IF INDEX(Parameter,"Grip=9")>0  THEN grip=9;
IF INDEX(Parameter,"Grip=12")>0 THEN grip=12;
RUN;


* Plot ESTIMATES (same as for option 3);
* grip as X by age;
PROC SGPLOT DATA=work.EstMainEffects;
SERIES x=grip y=Estimate / GROUP=age;
XAXIS GRID LABEL="Grip Strength"
           VALUES=(3 TO 15 BY 3);
YAXIS GRID LABEL="Predicted Cognition"
           VALUES=(15 TO 45 BY 5);
RUN; QUIT;
```



Creating Predicted Outcomes: Option 2 using STATA Margins

```
display "STATA Combined Main Effects Only Model Predicting Cognition"
regress cognition c.age85 c.grip9 c.sexmw c.demnf c.demnc, level(95)
// Pred cognition outcomes holding sexMW=men, demNF=none, and demNC=none
// one margins replaces 9 ESTIMATEs in SAS
// vsquish compresses output empty lines
// predictor=(from(by)to), c.=quantitative predictor
   margins, at(c.age85=(-5(5)5) c.grip9=(-3(3)3) ///
            c.sexmw=0 c.demnf=0 c.demnc=0) vsquish
// Get plot of predicted outcomes
   marginsplot, xdimension(grip9)
```



Further customization of the plot can be done through marginsplots options or in the graph editor window

Creating Predicted Outcomes: Option 3 using SAS Fake People

```
* Demonstrating how to get predicted
  outcomes using "fake people";
* Each row is a fake person for
  which to create a pred outcome;
DATA work.FakePeople;
* List variables;
INPUT PersonID age grip
      sexMW demNF demNC;
* Center predictors;
age85=age-85; grip9=grip-9;
* Enter data;
  DATALINES;
-99 80  6  0  0  0
-99 80  9  0  0  0
-99 80 12  0  0  0
-99 85  6  0  0  0
-99 85  9  0  0  0
-99 85 12  0  0  0
-99 90  6  0  0  0
-99 90  9  0  0  0
-99 90 12  0  0  0
; RUN;

* Merge with real data;
DATA work.Example6;
SET work.FakePeople work.Example6;
RUN;
```

```
TITLE1 "SAS Combined Main Effects Only
        Model Predicting Cognition";
TITLE2 "Using dataset with fake
        people to get predicted
        outcomes as saved variable";
PROC GLM DATA=work.Example6 NAMELEN=100;
MODEL cognition = age85 grip9 sexMW
                  demNF demNC
            / ALPHA=.05 CLPARM SOLUTION
              SS3 EFFECTSIZE;
* Request pred outcome and SE for all;
OUTPUT OUT=work.PredOutcomes
       PREDICTED=Yhat STDP=SEyhat;
RUN; QUIT; TITLE1; TITLE2;
```

SAS Code to generate the plot and the resulting plot are then the same as SAS option 2

GLM with an Interaction: $y_i = \beta_0 + \beta_1(x_i) + \beta_2(z_i) + \boxed{\beta_3(x_i)(z_i)} + e_i$

- **Interaction slopes (β_3 here) test "Moderation":** whether a predictor's slope **depends** on the value of an interacting predictor
  - Either predictor can be "the moderator" (is interpretive distinction only)
- **Interactions can always be evaluated** for any combination of categorical and quantitative predictors, although traditionally…
  - **In "ANOVA":** By default (in SPSS), all possible interactions are estimated
    - Oddly, nonsignificant interactions are usually kept in the model (even if only significant interactions are interpreted)
  - **In "ANCOVA":** Quantitative predictors ("covariates") are not included in interaction terms → this is the "homogeneity of regression assumption"
    - But you don't have to assume this—it is always a testable hypothesis!
  - **In "Regression":** No default—effects of predictors are as you specify
    - Requires most thought, but gets annoying in regression-specific programs when you have to manually create the interaction variable:
    - e.g., XZinteraction = X * Z;

Interaction variables are made on the fly in GLM! ☺

Main Effects of Predictors within Interactions

- **"Main effect" slopes** of predictors that are included in interaction terms should always remain in the model regardless of their significance
  - e.g., given $\boldsymbol{\beta_3}(\boldsymbol{x_i})(\boldsymbol{z_i})$, you must keep $\boldsymbol{\beta_1}(\boldsymbol{x_i})$ and $\boldsymbol{\beta_2}(\boldsymbol{z_i})$ in the model, too
  - Why? Because an interaction term creates an over-additive (enhancing) or under-additive (dampening) effect, so *what it is additive to* must be included for the interaction to actually represent an "interaction"

- **The role of a two-way interaction is to <u>adjust</u> the "main effect" slopes of the two predictors involved… (in one of four possible ways)**
  - But the idea of a "marginal" main effect slope (that holds for everyone) no longer applies: the main effect slopes become ***simple main effect slopes*** that are ***conditional*** each interacting predictor = 0

- Note that this is a different type of conditionality than just "holding the other predictors constant" (which means constant at **any value**)
  - Simple main effect slopes are held constant (conditional on) the **0 value** of the interacting predictor(s)—these slopes would be different if 0 were defined differently by centering the interacting predictor elsewhere
  - This language can be confusing, so here is a taxonomy that may help…

A Taxonomy of Fixed Effect Interpretations

- In the most common statistical models, fixed effects will be either:
  - an **intercept** that provides an expected (conditional) y_i outcome,
  - or **a slope** for the difference in y_i per unit difference in x_i predictor

- **All slopes** can be described as falling within one of three categories: ***bivariate marginal***, ***unique marginal***, or ***unique conditional***
  - In models with only **one fixed slope**, that slope's main effect is ***bivariate marginal*** (is uncontrolled and applies across all persons)
  - In models with **more than one fixed slope**, each slope's main effect is ***unique*** (it controls for the overlap in contribution with each other slope)
    - If a predictor is not part of an interaction term, its ***unique effect is marginal*** (it controls for the other slopes, but its effect still applies across all persons)
    - NEW → If a predictor is part of one or more interaction terms, its ***unique effect is conditional***, which means it is **specific to each interacting predictor = 0**
      - **Unique conditional** effects are also called **"simple main effects" (simple slopes)**

Practice Labeling Fixed Slopes—Choices:

bivariate marginal, unique marginal, or unique conditional

Model: $\boldsymbol{y_i} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1}(\boldsymbol{w_i}) + \boldsymbol{e_i}$

- Label for $\boldsymbol{\beta_1}$ slope of $\boldsymbol{w_i}$ =

Model: $\boldsymbol{y_i} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1}(\boldsymbol{w_i}) + \boldsymbol{\beta_2}(\boldsymbol{x_i}) + \boldsymbol{\beta_3}(\boldsymbol{z_i}) + \boldsymbol{\beta_4}(\boldsymbol{x_i})(\boldsymbol{z_i}) + \boldsymbol{e_i}$

- Label for $\boldsymbol{\beta_1}$ slope of $\boldsymbol{w_i}$ =
- Label for $\boldsymbol{\beta_2}$ slope of $\boldsymbol{x_i}$ =
- Label for $\boldsymbol{\beta_3}$ slope of $\boldsymbol{z_i}$ =
- Label for $\boldsymbol{\beta_4}$ slope of $\boldsymbol{x_i z_i}$ interaction term=

The 4 Possible Kinds of Interactions

- There are only 4 kinds of interactions: they make each of their main effect slopes more/less positive/negative
  - **More** positive or more negative → effect becomes **stronger**, known as "over-additive" interaction
  - **Less** positive or less negative → effect becomes **weaker**, known as "under-additive" interaction

- Model: $\boldsymbol{y_i} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1}(\boldsymbol{w_i}) + \boldsymbol{\beta_2}(\boldsymbol{x_i}) + \boldsymbol{\beta_3}(\boldsymbol{z_i}) + \boldsymbol{\beta_4}(\boldsymbol{x_i})(\boldsymbol{z_i}) + \boldsymbol{e_i}$

Slope of x_i is β_2=	Interaction Slope is β_4=	So β_4 makes effect of x_i ??? per unit higher z_i
10	2	
10	-2	
-10	-2	
-10	2	

Fixed Effects: Why Centering Matters

- y_i = Student achievement (GPA as percentage out of 100)
  x_i = Parent **attitudes** about education (measured on 1–5 scale)
  z_i = Parent **education** level (measured in years of education)

$$GPA_i = \beta_0 + \beta_1(Att_i) + \beta_2(Ed_i) + \beta_3(Att_i)(Ed_i) + e_i$$
$$GPA_i = 30 + 1(Att_i) + 2(Ed_i) + 0.5(Att_i)(Ed_i) + e_i$$

- **Interpret β_0:**
- **Interpret β_1:**
- **Interpret β_2:**
- **Interpret β_3: Attitude as Moderator:**

  Education as Moderator:

- **Predicted GPA** for **attitude = 3** and **Ed = 12**?

  75 = 30 + 1*(3) + 2*(12) + 0.5*(3)*(12)

How Centering Changes Fixed Effects

- $\boldsymbol{y_i}$ = Student achievement (GPA as percentage out of 100)
  $\boldsymbol{x_i}$ = Parent **attitudes** about education (now centered at **3**)
  $\boldsymbol{z_i}$ = Parent years of **education** (now centered at **12**)

$$GPA_i = \beta_0 + \beta_1(Att_i - 3) + \beta_2(Ed_i - 12) + \beta_3(Att_i - 3)(Ed_i - 12) + e_i$$
$$GPA_i = 75 + 7(Att_i - 3) + 3.5(Ed_i - 12) + 0.5(Att_i - 3)(Ed_i - 12) + e_i$$

- **Interpret β_0:**
- **Interpret β_1:**
- **Interpret β_2:**
- **Interpret β_3: Attitude as Moderator:**

  Education as Moderator:

- But how did I know what the new fixed effects would be???

Model-Implied Predicted Outcomes

- **Predicted outcomes = expected outcomes = intercepts**
  - Need to start with `"intercept 1"` or `"_const*1"` (need 1 $\boldsymbol{\beta}_0$)
  - ALL model effects must be included (or else are assumed = 0)

$$\widehat{GPA}_i = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1(\boldsymbol{Att}_i - \mathbf{3}) + \boldsymbol{\beta}_2(\boldsymbol{Ed}_i - \mathbf{12}) + \boldsymbol{\beta}_3(\boldsymbol{Att}_i - \mathbf{3})(\boldsymbol{Ed}_i - \mathbf{12})$$

SAS: Each line starts with ESTIMATE

```
"Yhat: Att=5 Ed=16" intercept 1 att __ ed __ att*ed __
"Yhat: Att=1 Ed=12" intercept 1 att __ ed __ att*ed __
"Yhat: Att=3 Ed=20" intercept 1 att __ ed __ att*ed __
```

STATA: Each line starts with lincom, title moved to end of line after //

```
"Yhat: Att=5 Ed=16" _cons*1 + att*__ + ed*__ + att#ed*__
"Yhat: Att=1 Ed=12" _cons*1 + att*__ + ed*__ + att#ed*__
"Yhat: Att=3 Ed=20" _cons*1 + att*__ + ed*__ + att#ed*__
```

Model-Implied Predictor Simple Slopes

- Example equation for <u>predicted GPA</u> using centered predictors:
  $\widehat{GPA}_i = \beta_0 + \beta_1(Att_i - 3) + \beta_2(Ed_i - 12) + \beta_3(Att_i - 3)(Ed_i - 12)$

- This model equation provides predictions for:
  - Expected outcome given any combination of predictor values
  - Any conditional (simple) main effect slopes implied by interaction term
  - **Any slope can be found as: what it is + what *modifies* it**

- Three steps to get any model-implied simple main effect slope:

1. **Identify** all terms in model involving the predictor of interest
2. **Factor out** common predictor variable to find slope linear combination
3. **Calculate** estimate and SE for slope linear combination
   - By "calculate" I of course mean "ask a program to do this for you"

Model-Implied Predictor Simple Slopes

- Example equation for predicted GPA using centered predictors:
  $\widehat{GPA}_i = \beta_0 + \beta_1(Att_i - 3) + \beta_2(Ed_i - 12) + \beta_3(Att_i - 3)(Ed_i - 12)$

1. **Identify** all slopes in model involving the predictor of interest

   To get attitudes slope: $Est = \beta_1(Att_i - 3) + \beta_3(Att_i - 3)(Ed_i - 12)$

   To get education slope: $Est = \beta_2(Ed_i - 12) + \beta_3(Att_i - 3)(Ed_i - 12)$

2. **Factor out** predictor of interest to find slope linear combination

   To get attitudes slope: $Est = [\beta_1 + \beta_3(Ed_i - 12)]$ **that will multiply** $(Att_i - 3)$

   To get education slope: $Est = [\beta_2 + \beta_3(Att_i - 3)]$ **that will multiply** $(Ed_i - 12)$

- Btw, the SEs for the new slopes provided by the program come from:
  - SE^2 = sampling variance of slope estimate → e.g., $Var(\beta_1) = SE^2_{\beta_1}$

  attitudes slope: $SE^2 = Var(\beta_1) + Var(\beta_3)(Ed_i - 12) + 2Cov(\beta_1, \beta_3)(Ed_i - 12)$

  education slope: $SE^2 = Var(\beta_2) + Var(\beta_3)(Att_i - 3) + 2Cov(\beta_2, \beta_3)(Att_i - 3)$

Model-Implied Predictor Simple Slopes

- **To request <u>predicted simple slopes</u> (= simple main effects):**
  - **DO NOT include the intercept** (β_0 does **not** contribute to slopes)
  - **Include ONLY** the fixed effects that contain the predictor of interest

$$\widehat{GPA}_i = \beta_0 + \beta_1(Att_i - 3) + \beta_2(Ed_i - 12) + \beta_3(Att_i - 3)(Ed_i - 12)$$

→ attitudes slope: $Est = [\beta_1 + \beta_3\,(Ed_i - 12)]$ that multiplies $(Att_i - 3)$
→ education slope: $Est = [\beta_2 + \beta_3(Att_i - 3)]$ that multiplies $(Ed_i - 12)$

SAS: Each line starts with ESTIMATE

```
"Att Slope if Ed=10" intercept 0 att __ ed __ att*ed __
"Att Slope if Ed=18" intercept 0 att __ ed __ att*ed __
"Ed Slope  if Att=2" intercept 0 att __ ed __ att*ed __
"Ed Slope  if Att=5" intercept 0 att __ ed __ att*ed __
```

STATA: Each line starts with lincom, title moved to end of line after //

```
"Att Slope if Ed=10" _cons*0 + att*__ + ed*__ + att#ed*__
"Att Slope if Ed=18" _cons*0 + att*__ + ed*__ + att#ed*__
"Ed Slope  if Att=2" _cons*0 + att*__ + ed*__ + att#ed*__
"Ed Slope  if Att=5" _cons*0 + att*__ + ed*__ + att#ed*__
```

Regions of Significance for Simple Slopes

- For quantitative predictors, there may not be specific values of the moderator at which you want to know the slope's significance…

- For example, with age*woman (in which 0=man, 1=woman here):

$$\hat{y}_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Woman_i) + \beta_3(Age_i - 85)(Woman_i)$$

→ **age slope:** $Est =$ ______________________ that multiplies $(Age_i - 85)$

→ **gender slope:** $Est =$ ______________________ that multiplies $(Woman_i)$

- Age slopes are only relevant for two specific values of *woman*:

```
"Age Slope for Men"    age85 __ woman __ age85*woman __
"Age Slope for Women"  age85 __ woman __ age85*woman __
```

- But there are many ages to request gender differences for…

```
"Gender Diff at Age=80" age85 __ woman __ age85*woman __
"Gender Diff at Age=90" age85 __ woman __ age85*woman __
```

Regions of Significance for Simple Slopes

- An alternative approach for continuous moderators is known as **regions of significance** (see Hoffman 2015 chapter 2 for refs)
- Rather than asking if the simple main effect of gender is still significant at a particular age, we can find the **boundary ages** at which the gender effect becomes non-significant
- We know that: EST / SE = *t*-value → if |*t*| > |1.96|, then *p*<.05
- So we work backwards to find the EST and SE such that:

$$\pm \text{t} = \pm 1.96 = \frac{\text{Slope Estimate}}{\sqrt{\text{Variance of Slope Estimate}}}, \text{ where:}$$

$$\text{Gender Slope (Gender Difference) Estimate} = \beta_2 + \beta_3 (\text{Age} - 85)$$

$$\text{Variance of Slope Estimate} = \text{Var}(\beta_2) + \boxed{2\text{Cov}(\beta_2 \beta_3)(\text{Age} - 85)} + \text{Var}(\beta_3)(\text{Age} - 85)^2$$

- Need to request "asymptotic covariance matrix" (COVB)
  - Covariance matrix of fixed effect estimates (SE^2 on diagonal)

Regions of Significance for Simple Slopes

$$\pm t = \pm 1.96 = \frac{\text{Slope Estimate}}{\sqrt{\text{Variance of Slope Estimate}}}, \text{ where:}$$

$$\text{Gender Slope (Gender Difference) Estimate} = \beta_2 + \beta_3(\text{Age} - 85)$$

$$\text{Variance of Slope Estimate} = \text{Var}(\beta_2) + 2\text{Cov}(\beta_2\beta_3)(\text{Age} - 85) + \text{Var}(\beta_3)(\text{Age} - 85)^2$$

- For example, age*woman (0=man, 1=woman), age = moderator:

$$\hat{y}_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Woman_i) + \beta_3(Age_i - 85)(Woman_i)$$

- **$\beta_2 = -0.5306$* at age=85, $Var(\beta_2)$ → SE^2 for β_2 was 0.06008**
- **$\beta_3 = -0.1104$* unconditionally, $Var(\beta_3)$ → SE^2 for β_3 was 0.00178**
- **Covariance of $\beta_2\ SE$ and $\beta_3\ SE$ was 0.00111**

- **Regions of Significance for Moderator of Age = 60.16 to 79.52**
  - The gender effect β_2 is predicted to be significantly negative above age 79.52, non-significant from ages 79.52 to 60.16, and significantly positive below age 60.16 (because non-parallel lines will cross eventually).

Modeling Interactions: Summary

- Interactions create "**moderation**": the idea that the effect (slope) of one predictor **depends** upon the value of another predictor
  - The current "**single slope**" interaction examples show one form of moderation—that each predictor's **slope increases linearly** with the value of the interacting predictor
  - But other forms of moderation that involve 2+ slopes can be tested using the specifications shown in Lecture 4 (e.g., for interactions with 3+ groups, quadratic, or piecewise effects)
- Predictors' **main effect slopes** will change once they are included in an interaction term, because **they now mean different things**:
  - Former "marginal main effect slopes" become "conditional (or simple) effect slopes" specifically when the interacting predictor = 0
  - Need to have **0 as a meaningful value** for each predictor for that reason
- **Rules for interpreting conditional (or simple) fixed slopes:**
  - Predicted outcomes are conditional on (get adjusted by) main effect slopes
    - Positive slopes create higher outcomes; negative slopes create lower outcomes
  - Main effect slopes are conditional (get adjusted by) on two-way interactions
    - Interactions make main effect slopes more/less positive or more/less negative
    - Btw, three-way interactions do the same thing to two-way interactions
  - Highest-order interaction slope is unconditional—it will stay the same regardless of centering (i.e., extent of moderation is unconditional)