Example 6a: Cross-Classified Models for Students Nested within Primary and Secondary Schools (complete syntax, data, and output available for R electronically)

Cross-classified models (also known as crossed random effects models) are useful in situations in which people belong to more than one type of cluster, but the types of clusters are not nested—they are crossed. To demonstrate, simulated data in the style of Hox et al. (2018) chapter 9 are analyzed below, in which the outcome is 9th grade academic achievement (*achieve*). There are 1000 level-1 children nested within 50 level-2 primary schools AND within 33 level-2 secondary schools, in which primary and secondary schools are crossed at level 2. We have predictors for whether the primary and secondary schools are denominational (i.e., religious; *Pdenom* and *Sdenom*) and child socio-economic status (*ses*). The number of children per unique crossing of primary by secondary school ranged from 1–6, which means we have a potential random interaction intercept AND three kinds of contextual effects! Note that these models are different than those in the text, in which contextual effects of child SES were not considered (i.e., it was smushed). The example uses several custom functions to process output and add additional computations, as shown below (all in the online files).

R Syntax for Packages; Importing and Preparing Data for Analysis

```
# Install and load required packages
if (!require(pacman)) install.packages("pacman")
pacman::p_load(readxl, TeachingDemos, lme4, lmerTest)
# readxl: import .xls or .xlsx files; TeachingDemos: save output to text file
# lme4/lmerTest: MLMs with crossed random effects (Satterthwaite DDF)

# Set working directory (to import and export files to)
setwd("C:/Dropbox/26_EDF9850/Unit6/Example6a")

# Load R functions for this class from R file in folder above working directory
source("../../EDF9850_Functions.R")  # Remove ../ for same folder
source("https://www.lesahoffman.com/EDF9850/EDF9850_Functions.R") # From the web

# Import Example 6a excel data file in sheet "Hox"
Example6a = read_excel(path="Example6a_Data.xlsx", sheet="Hox")
# Convert to data frame to use for analysis
Example6a = as.data.frame(Example6a)

# Filter to only cases complete on all variables (before cluster means)
Example6a = Example6a[complete.cases(Example6a[ ,
                      c("achieve","ses","Pdenom","Sdenom")]), ]

# Create unique ID for each primary/secondary school combination (the interaction)
Example6a$UniqueID = as.numeric(interaction(Example6a$PschoolID, Example6a$SschoolID,
                     drop=TRUE)) # drop=TRUE removes non-existent combinations

# Create cluster means of child variables (and cluster sizes) for each type of school
Example6a = addUnitMeans(data=Example6a, unitVariable="PschoolID",
               meanVariables=c("achieve","ses"), newNames=c("PMachieve","PMses"))
Example6a = addUnitMeans(data=Example6a, unitVariable="SschoolID",
               meanVariables=c("achieve","ses"), newNames=c("SMachieve","SMses"))
Example6a = addUnitMeans(data=Example6a, unitVariable="UniqueID",
               meanVariables=c("achieve","ses"), newNames=c("PSMachieve","PSMses"))
```

```
# Center child SES and each of its school means at 4 (the approximate grand mean)
Example6a$ses4    = Example6a$ses   - 4
Example6a$PMses4  = Example6a$PMses - 4
Example6a$SMses4  = Example6a$SMses - 4
Example6a$PSMses4 = Example6a$PSMses - 4

# Create cluster-mean-centered level-1 child SES predictors
Example6a$WPses  = Example6a$ses - Example6a$PMses    # Within primary school only
Example6a$WSses  = Example6a$ses - Example6a$SMses    # Within secondary school only
Example6a$WPSses = Example6a$ses - Example6a$PSMses   # Within unique primary/secondary

print("Descriptive Statistics in Child-Level-1 Data")
univSummary(data=Example6a, vars=c("achieve","ses","WPses","WSses","WPSses"))
```

```
 Variable    N   Mean    SD Variance    Min   Max
 achieve  1000  6.343 0.868    0.753  3.900 9.900
 ses      1000  4.098 1.398    1.954  1.000 6.000
 WPses    1000 -0.000 1.371    1.879 -3.684 2.529
 WSses    1000  0.000 1.374    1.888 -3.412 2.529
 WPSses   1000  0.000 0.793    0.629 -3.333 2.333
```

```
corrSummary(data=Example6a, p.separate=TRUE,
            vars=c("achieve","PMachieve","SMachieve","PSMachieve","ses",
                   "PMses","SMses","PSMses","WPses","WSses","WPSses"))
```

```
Correlations:
 Variable   achieve PMachieve SMachieve PSMachieve    ses  PMses  SMses PSMses  WPses  WSses WPSses
 achieve      1.000     0.521     0.357      0.885  0.174 -0.013  0.103  0.155  0.180  0.158  0.082
 PMachieve    0.521     1.000     0.084      0.588 -0.005 -0.025 -0.020 -0.006 -0.000 -0.001  0.000
 SMachieve    0.357     0.084     1.000      0.403  0.053 -0.026  0.288  0.064  0.059 -0.000  0.000
 PSMachieve   0.885     0.588     0.403      1.000  0.144 -0.015  0.116  0.175  0.150  0.125  0.000
 ses          0.174    -0.005     0.053      0.144  1.000  0.197  0.184  0.824  0.980  0.983  0.567
 PMses       -0.013    -0.025    -0.026     -0.015  0.197  1.000  0.015  0.239 -0.000  0.197  0.000
 SMses        0.103    -0.020     0.288      0.116  0.184  0.015  1.000  0.224  0.185  0.000 -0.000
 PSMses       0.155    -0.006     0.064      0.175  0.824  0.239  0.224  1.000  0.792  0.796 -0.000
 WPses        0.180    -0.000     0.059      0.150  0.980 -0.000  0.185  0.792  1.000  0.963  0.578
 WSses        0.158    -0.001    -0.000      0.125  0.983  0.197  0.000  0.796  0.963  1.000  0.577
 WPSses       0.082     0.000     0.000      0.000  0.567  0.000 -0.000 -0.000  0.578  0.577  1.000
```

The bold correlations in the first 4 columns are for the smushed correlation and level-specific correlations. The bold values in the later columsn indicate that only the *WPSses* within-primary AND within-secondary centered SES variable is completely uncorrelated with the primary school, secondary school, AND unique-combination SES means. Had each unique school combination had only one child, then centering using each school mean would have the same effect—the within predictor would be uncorrelated with each school mean.

```
print("Number of Children per Unique Primary/Secondary Combination")
# !duplicated keeps one row per combination, so each combination is counted once
freqSummary(data=Example6a[!duplicated(Example6a$UniqueID), ], vars="NperUniqueID")
```

```
 Variable     Level   N Proportion
 NperUniqueID 1     394      0.604
 NperUniqueID 2     185      0.284
 NperUniqueID 3      60      0.092
 NperUniqueID 4      10      0.015
 NperUniqueID 5       2      0.003
 NperUniqueID 6       1      0.002
```

60% of the unique combinations have only one child, whose variance will then go to any "unique" level-2 variable

Model 1a: Empty Means, Random Intercept for Primary School Only

$$Achieve_{cps} = \gamma_{000} + U_{0p0} + e_{cps}$$

For crossed models, a composite equation is easier!

Model 1a predicts 9th grade academic achievement for child *c* who previously went to primary school *p* and currently goes to secondary school *s*. Having only a random intercept for primary school creates an expected correlation only among children who went to the same primary school (so far).

```
print("Model 1a: Empty Means, Random Intercept for Primary")
Model1a = lmer(data=Example6a, REML=TRUE, formula=achieve~1+(1|PschoolID))
lmerSummary(Model1a, ICC1=TRUE, LRT=TRUE, randCI=TRUE)
```

```
Model Fit for Predicting achieve:
      AIC       BIC    logLik -2*logLik
 2393.736  2408.460 -1193.868  2387.736

Model for the Variance:
 Groups     Name       Variance Std.Dev
 PschoolID  Intercept    0.176   0.419
 Residual                0.577   0.760
Number of obs: 1000 , groups: PschoolID, 50

Fixed Effects:
 Term         Est    SE     DF t.value p.value
 Intercept  6.359 0.064 49.241  99.142   <.001

ICC1 intraclass correlation:
    Adjusted ICC: 0.233
  Unadjusted ICC: 0.233

LRT for random effect(s):
 Random Effect Dropped  Npar    LL -2LL  AIC  BIC     LRT DF p.value p.mix
 <none>                    3 -1194 2388 2394 2408
 Intercept | PschoolID     2 -1279 2558 2562 2572 170.632  1   <.001 <.001

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term                  FixedEffect Variance    SD Lower Upper
 Intercept | PschoolID       6.359    0.176 0.419 5.538 7.180
```

ICC1 = .233 is the expected correlation of children from the same primary school (i.e., 23.3% of the variance in achievement is between primary schools). The LRT for the random intercept variance tests whether children from the same primary school are more alike than children from different primary schools.

Model 1b: Empty Means, Primary by Secondary Crossed Random Intercepts

$$Achieve_{cps} = \gamma_{000} + U_{0p0} + U_{00s} + e_{cps}$$

In model 1b, the secondary school random intercept U_{00s} introduces a separate correlation of children from the same secondary school (beyond the primary school random intercept U_{0p0}).

```
print("Model 1b: Empty Means, Crossed Random Intercepts
  for Primary and Secondary")
Model1b = lmer(data=Example6a, REML=TRUE,
          formula=achieve~1+(1|PschoolID)+(1|SschoolID))
lmerSummary(Model1b, ICC1=TRUE, LRT=TRUE, randCI=TRUE)
```

The random intercepts are uncorrelated (crossed) by using separate ( ) terms

```
Model Fit for Predicting achieve:
      AIC       BIC    logLik -2*logLik
 2320.269  2339.900 -1156.134  2312.269

Model for the Variance:
 Groups     Name      Variance Std.Dev
 PschoolID  Intercept    0.175   0.418
 SschoolID  Intercept    0.079   0.281
 Residual                0.505   0.711
Number of obs: 1000 , groups: PschoolID, 50; SschoolID, 33

Fixed Effects:
 Term         Est    SE     DF t.value p.value
 Intercept  6.342 0.081 66.879  78.468   <.001

ICC1 intraclass correlation:
    Adjusted ICC: 0.334
  Unadjusted ICC: 0.334

Proportion of Variance by Source (adj = adjusted after fixed effects):
 Source        Variance Prop.Adj Prop.Unadj
 PschoolID        0.175    0.230      0.230
 SschoolID        0.079    0.104      0.104
 Residual         0.505    0.666      0.666
 Fixed Effects    0.000               0.000
```

All sources of variance are orthogonal, so we can sum them to compute the proportion of variance due to each sampling dimension, as shown in the table above. ICC1 for children in the same primary school but different secondary schools = .230; ICC1 for children in the same secondary school but different primary schools = .104; and ICC1 for children in the same primary AND same secondary school = .230 + .104 = .334 (the ICC1 above). The LRTs below show that each random intercept variance is indeed needed.

```
LRT for random effect(s):
 Random Effect Dropped  Npar    LL -2LL  AIC  BIC     LRT DF p.value p.mix
 <none>                    4 -1156 2312 2320 2340
 Intercept | PschoolID     3 -1250 2501 2507 2521 188.309  1   <.001 <.001
 Intercept | SschoolID     3 -1194 2388 2394 2408  75.468  1   <.001 <.001

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term                  FixedEffect Variance    SD Lower Upper
 Intercept | PschoolID       6.342    0.175 0.418 5.523 7.161
 Intercept | SschoolID       6.342    0.079 0.281 5.792 6.892
```

The random effect 95% CIs show that 95% of primary schools are predicted to have mean achievement between 5.52—7.16, whereas 95% of secondary schools are predicted to have mean achievement between 5.79—6.89.

Model 1c: Model 1b + Random Intercept for the Unique Primary by Secondary Combination

$$\boldsymbol{Achieve_{cps} = \gamma_{000} + U_{0p0} + U_{00s} + U_{0ps} + e_{cps}}$$

Given 1–6 children per unique combination of primary and secondary school, we can test for an extra correlation among children who have both schools in common—a random interaction intercept U_{0ps}!

```
print("Model 1c: Model 1b + Random Intercept for Unique Primary/Secondary Combination")
Model1c = lmer(data=Example6a, REML=TRUE,
               formula=achieve~1+(1|PschoolID)+(1|SschoolID)+(1|UniqueID))
```

```
lmerSummary(Model1c, ICC1=TRUE, LRT=TRUE)

Model Fit for Predicting achieve:
      AIC       BIC    logLik -2*logLik
 2321.284  2345.822 -1155.642  2311.284

Model for the Variance:
 Groups     Name      Variance Std.Dev
 UniqueID   Intercept    0.027   0.164
 PschoolID  Intercept    0.174   0.417
 SschoolID  Intercept    0.078   0.279
 Residual                0.480   0.693
Number of obs: 1000 , groups: UniqueID, 652; PschoolID, 50; SschoolID, 33

Fixed Effects:
 Term         Est    SE     DF t.value p.value
 Intercept  6.344 0.081 66.345  78.627   <.001

ICC1 intraclass correlation:
    Adjusted ICC: 0.367
  Unadjusted ICC: 0.367

Proportion of Variance by Source (adjusted: random intercepts and residual; unadjusted:
also the fixed effects):
 Source        Variance Prop.Adj Prop.Unadj
 UniqueID         0.027    0.036      0.036
 PschoolID        0.174    0.229      0.229
 SschoolID        0.078    0.103      0.103
 Residual         0.480    0.633      0.633
 Fixed Effects    0.000               0.000
```

The LRT indicates we do not need the random interaction variance, −2ΔLL(1) = 0.99, p = .321, so we will remove it and use the model 1b structure. If it were retained, the UniqueID random intercept variance would contribute extra to the both-same-school ICC1 (.036 + .229 + .103 = .367, instead of .334 ignoring the random interaction).

```
LRT for random effect(s):
 Random Effect Dropped  Npar    LL -2LL  AIC  BIC     LRT DF p.value p.mix
 <none>                    5 -1156 2311 2321 2346
 Intercept | PschoolID     4 -1232 2465 2473 2492 153.233  1   <.001 <.001
 Intercept | SschoolID     4 -1188 2376 2384 2404  64.961  1   <.001 <.001
 Intercept | UniqueID      4 -1156 2312 2320 2340   0.985  1   0.321 0.160
```

Empty Means, Crossed Random Intercepts for the Level-1 SES Predictor

$$SES_{cps} = \gamma_{000} + U_{0p0} + U_{00s} + U_{0ps} + e_{cps}$$

Before adding child SES as a predictor, we can check how much of its variance is at each type of school crossing, as this determines whether contextual (school mean) effects of SES are possible.

```
print("Empty Means, Crossed Random Intercepts for Child SES (Level-1 Predictor)")
EmptySES = lmer(data=Example6a, REML=TRUE,
                formula=ses~1+(1|PschoolID)+(1|SschoolID)+(1|UniqueID))
lmerSummary(EmptySES, ICC1=TRUE, LRT=TRUE)

Model Fit for Predicting ses:
      AIC       BIC    logLik -2*logLik
 3521.238  3545.777 -1755.619  3511.238
```

```
Model for the Variance:
 Groups     Name      Variance Std.Dev
 UniqueID   Intercept    0.032   0.179
 PschoolID  Intercept    0.000   0.000
 SschoolID  Intercept    0.000   0.000
 Residual                1.922   1.386

Fixed Effects:
 Term         Est    SE      DF t.value p.value
 Intercept  4.098 0.045 469.461  92.004   <.001

ICC1 intraclass correlation:
    Adjusted ICC: 0.016
  Unadjusted ICC: 0.016

Proportion of Variance by Source (adjusted: random intercepts and residual; unadjusted:
also the fixed effects):
 Source        Variance Prop.Adj Prop.Unadj
 UniqueID         0.032    0.016      0.016
 PschoolID        0.000    0.000      0.000
 SschoolID        0.000    0.000      0.000
 Residual         1.922    0.984      0.984
 Fixed Effects    0.000               0.000

LRT for random effect(s):
Random Effect Dropped  Npar    LL -2LL  AIC  BIC    LRT DF p.value p.mix
 <none>                   5 -1756 3511 3521 3546
 Intercept | PschoolID    4 -1756 3511 3519 3539 -0.000  1   1.000 1.000
 Intercept | SschoolID    4 -1756 3511 3519 3539 -0.000  1   1.000 1.000
 Intercept | UniqueID     4 -1756 3511 3519 3539  0.096  1   0.756 0.378
```

There is no detectable SES variance across primary schools or across secondary schools (both variances were estimated at 0.000, LRT p = 1.000), and the unique-combination variance (0.032; ICC1 = .016) was also not significant (p = .756). Child SES is thus nearly all level-1 variation, so the school means of SES will be unreliable estimates of any school-level SES context—this is proof these data are simulated!

Model 2: Model 1b + Primary by Secondary Denomination (0 = not religious, 1 = religious)

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}(Pdenom_p) + \gamma_{001}(Sdenom_s) + \gamma_{011}(Pdenom_p)(Sdenom_s) + U_{0p0} + U_{00s} + e_{cps}$$

```
print("Model 2: Model 1b + Primary by Secondary School Denomination")
Model2 = lmer(data=Example6a, REML=TRUE,
              formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom
                              +(1|PschoolID)+(1|SschoolID))
lmerSummary(Model2)
```

```
Model Fit for Predicting achieve:
      AIC       BIC    logLik -2*logLik
 2326.533  2360.887 -1156.267  2312.533

Model for the Variance:
 Groups     Name      Variance Std.Dev
 PschoolID  Intercept    0.172   0.415
 SschoolID  Intercept    0.073   0.269
 Residual                0.504   0.710
```

```
Overall F-test for All Fixed Slopes:
 Model  NumDF   DenDF F.value p.value
 Model2     3 128.976   2.450   0.066

Fixed Effects:
 Term             Est    SE      DF t.value p.value
 Intercept      6.167 0.137  78.411  44.862   <.001
 Pdenom         0.073 0.144  75.523   0.506   0.614
 Sdenom         0.101 0.126  45.008   0.798   0.429
 Pdenom:Sdenom  0.161 0.101 949.193   1.599   0.110
```

```
PseudoR2(baseModel=Model1b, altModel1=Model2)

Pseudo-R2
 Term                   Model   Variance Pseudo.R2
 Intercept | PschoolID  Model1b    0.175
 Intercept | PschoolID  Model2     0.172     0.015
 Intercept | SschoolID  Model1b    0.079
 Intercept | SschoolID  Model2     0.073     0.079
 Residual               Model1b    0.505
 Residual               Model2     0.504     0.002
```

Which fixed slope should have caused the reduction in each pile of variance?

Model 3a: Add Level-1 Child SES (centered at 4)

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}(Pdenom_p) + \gamma_{001}(Sdenom_s) + \gamma_{011}(Pdenom_p)(Sdenom_s) + \gamma_{100}(SES_{cps} - 4) + U_{0p0} + U_{00s} + e_{cps}$$

```
print("Model 3a: Add Child SES (Smushed, Centered at 4)")
Model3a = lmer(data=Example6a, REML=TRUE,
               formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom+ses4
                           +(1|PschoolID)+(1|SschoolID))
lmerSummary(Model3a)

Model Fit for Predicting achieve:
      AIC       BIC    logLik -2*logLik
 2292.859  2332.121 -1138.429  2276.859

Model for the Variance:
 Groups     Name      Variance Std.Dev
 PschoolID  Intercept    0.173   0.415
 SschoolID  Intercept    0.068   0.260
 Residual                0.483   0.695

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model3a     4 162.926  12.661   <.001

Fixed Effects:
 Term             Est    SE      DF t.value p.value
 Intercept      6.171 0.135  79.189  45.568   <.001
 Pdenom         0.052 0.144  74.533   0.359   0.721
 Sdenom         0.073 0.123  45.283   0.592   0.557
 ses4           0.107 0.016 939.146   6.556   <.001
 Pdenom:Sdenom  0.199 0.099 947.930   2.016   0.044
```

What does the slope for ses4 indicate?

```
PseudoR2(baseModel=Model1b, altModel1=Model2, altModel2=Model3a)
```

```
Pseudo-R2 and Change in Pseudo-R2
 Term                  Model   Variance Pseudo.R2 Change.R2
 Intercept | PschoolID Model1b    0.175
 Intercept | PschoolID Model2     0.172     0.015
 Intercept | PschoolID Model3a    0.173     0.012    -0.003
 Intercept | SschoolID Model1b    0.079
 Intercept | SschoolID Model2     0.073     0.079
 Intercept | SschoolID Model3a    0.068     0.138     0.060
 Residual              Model1b    0.505
 Residual              Model2     0.504     0.002
 Residual              Model3a    0.483     0.043     0.042
```

Which variance should have been reduced by child SES?

Model 3b: Add Contextual SES Slopes (each centered at 4)

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}(Pdenom_p) + \gamma_{001}(Sdenom_s) + \gamma_{011}(Pdenom_p)(Sdenom_s) + \gamma_{100}(SES_{cps} - 4) + \gamma_{020}(\overline{SES}_p - 4) + \gamma_{002}(\overline{SES}_s - 4) + \gamma_{022}(\overline{SES}_{ps} - 4) + U_{0p0} + U_{00s} + e_{cps}$$

Three contextual SES effects are added: the primary school mean SES ($\overline{SES}_p$), the secondary school mean SES ($\overline{SES}_s$), and the mean SES of the unique primary by secondary combination ($\overline{SES}_{ps}$), each centered at 4. The level-1 child SES slope then represents the pure within-schools effect.

```
print("Model 3b: Add Contextual Effects of Child SES (Each Centered at 4)")
Model3b = lmer(data=Example6a, REML=TRUE,
               formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom+ses4+PMses4+SMses4+PSMses4
                              +(1|PschoolID)+(1|SschoolID))
lmerSummary(Model3b)
```

```
Model Fit for Predicting achieve:
      AIC       BIC    logLik -2*logLik
 2304.899  2358.884 -1141.450  2282.899

Model for the Variance:
 Groups     Name      Variance Std.Dev
 PschoolID  Intercept    0.177   0.421
 SschoolID  Intercept    0.066   0.258
 Residual                0.484   0.695

Overall F-test for All Fixed Slopes:
 Model   NumDF  DenDF F.value p.value
 Model3b     7 95.393   7.468   <.001

Fixed Effects:
 Term              Est    SE      DF t.value p.value
 Intercept       6.166 0.138  77.484  44.592   <.001
 Pdenom          0.050 0.145  72.328   0.342   0.733
 Sdenom          0.054 0.123  42.604   0.442   0.661
 ses4            0.090 0.028 914.494   3.237   0.001
 PMses4         -0.036 0.230  48.395  -0.157   0.876
 SMses4          0.179 0.176  38.457   1.020   0.314
 PSMses4         0.023 0.034 915.094   0.680   0.497
 Pdenom:Sdenom   0.200 0.099 945.777   2.028   0.043
```

What do we conclude about the effect of child SES at the school level?

```
PseudoR2(baseModel=Model1b, altModel1=Model3a, altModel2=Model3b)
```

```
Pseudo-R2 and Change in Pseudo-R2
 Term                   Model   Variance Pseudo.R2 Change.R2
 Intercept | PschoolID  Model1b    0.175
 Intercept | PschoolID  Model3a    0.173     0.012
 Intercept | PschoolID  Model3b    0.177    -0.014    -0.026
 Intercept | SschoolID  Model1b    0.079
 Intercept | SschoolID  Model3a    0.068     0.138
 Intercept | SschoolID  Model3b    0.066     0.155     0.017
 Residual               Model1b    0.505
 Residual               Model3a    0.483     0.043
 Residual               Model3b    0.484     0.042    -0.001
```

```
TotalR2(altModel1=Model2, altModel2=Model3a, altModel3=Model3b)
```

```
Total-R2 and Change in Total-R2
 Model   Total.R2 Change.R2
 Model2     0.022
 Model3a    0.053     0.031
 Model3b    0.058     0.005
```

Next we can examine how the effect of child SES varies over primary or secondary schools. The within-both-schools centered SES predictor (*WPSses*) is used for the random slopes because it is uncorrelated with all of the school-level SES means, and thus the random slope variance is not smushed.

Model 4a: Model 3b + Random Within-Both-Centered Child SES Slope across Primary

$$\begin{aligned} \boldsymbol{Achieve_{cps}} = \boldsymbol{\gamma_{000}} + \boldsymbol{\gamma_{010}}(\boldsymbol{Pdenom_p}) + \boldsymbol{\gamma_{001}}(\boldsymbol{Sdenom_s}) + \boldsymbol{\gamma_{011}}(\boldsymbol{Pdenom_p})(\boldsymbol{Sdenom_s}) \\ + \boldsymbol{\gamma_{100}}\left(\boldsymbol{SES_{cps}} - \boldsymbol{4}\right) + \boldsymbol{\gamma_{020}}\left(\overline{\boldsymbol{SES}}_{\boldsymbol{p}} - \boldsymbol{4}\right) + \boldsymbol{\gamma_{002}}\left(\overline{\boldsymbol{SES}}_{\boldsymbol{s}} - \boldsymbol{4}\right) \\ + \boldsymbol{\gamma_{022}}\left(\overline{\boldsymbol{SES}}_{\boldsymbol{ps}} - \boldsymbol{4}\right) + \boldsymbol{U_{0p0}} + \boldsymbol{U_{2p0}}\left(\boldsymbol{SES_{cps}} - \overline{\boldsymbol{SES}}_{\boldsymbol{ps}}\right) + \boldsymbol{U_{00s}} + \boldsymbol{e_{cps}} \end{aligned}$$

```
print("Model 4a: Add Random Within-Both-Centered Child SES Slope across Primary")
Model4a = lmer(data=Example6a, REML=TRUE,
               formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom+ses4+PMses4+SMses4+PSMses4
                          +(1+WPSses|PschoolID)+(1|SschoolID))
lmerSummary(Model4a, LRT=TRUE, randCI=TRUE, randCIfixed="ses4", randCIrandom="WPSses")
```

```
Model for the Variance:
 Groups     Name       Variance Std.Dev  Corr
 PschoolID  Intercept    0.177   0.421
            WPSses       0.000   0.010 1.000
 SschoolID  Intercept    0.066   0.258
 Residual                0.484   0.695

LRT for random effect(s):
 Random Effect Dropped     Npar    LL -2LL  AIC  BIC    LRT DF p.value p.mix
 <none>                      13 -1141 2283 2309 2373
 WPSses | PschoolID          11 -1141 2283 2305 2359  0.106  2   0.948 0.846
 Intercept | SschoolID       12 -1170 2341 2365 2423 57.724  1   <.001 <.001

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term                              FixedEffect Variance    SD Lower Upper
 Intercept | PschoolID                  6.166    0.177 0.421 5.341 6.990
 Intercept | SschoolID                  6.166    0.066 0.258 5.660 6.671
 WPSses | PschoolID (fixed: ses4)       0.090    0.000 0.010 0.071 0.110
```

Note that a correlation for the new random WPSses slope was estimated only with the primary school random intercept, not with the secondary school random intercept. The random slope variance is decidedly not there, but what kind of fixed effects *would* have explained a WPSses random slope variance over primary schools?

Model 4b: Model 3b + Random Within-Both-Centered Child SES Slope across Secondary

$$Achieve_{cps} = \gamma_{000} + \gamma_{010}(Pdenom_p) + \gamma_{001}(Sdenom_s) + \gamma_{011}(Pdenom_p)(Sdenom_s) + \gamma_{100}(SES_{cps} - 4) + \gamma_{020}(\overline{SES}_p - 4) + \gamma_{002}(\overline{SES}_s - 4) + \gamma_{022}(\overline{SES}_{ps} - 4) + U_{0p0} + U_{00s} + U_{20s}(SES_{cps} - \overline{SES}_{ps}) + e_{cps}$$

```
print("Model 4b: Add Random Within-Both-Centered Child SES Slope across Secondary")
Model4b = lmer(data=Example6a, REML=TRUE,
               formula=achieve~1+Pdenom+Sdenom+Pdenom:Sdenom+ses4+PMses4+SMses4+PSMses4
                           +(1|PschoolID)+(1+WPSses|SschoolID))
lmerSummary(Model4b, LRT=TRUE, randCI=TRUE, randCIfixed="ses4", randCIrandom="WPSses")
```

```
Model for the Variance:
 Groups     Name       Variance Std.Dev  Corr
 PschoolID  Intercept    0.177   0.421
 SschoolID  Intercept    0.067   0.258
            WPSses       0.000   0.012 1.000
 Residual                0.484   0.695

Random Effect Dropped  Npar    LL -2LL  AIC  BIC     LRT DF p.value p.mix
 <none>                  13 -1141 2283 2309 2373
 Intercept | PschoolID   12 -1238 2476 2500 2559 193.003  1   <.001 <.001
 WPSses | SschoolID      11 -1141 2283 2305 2359   0.132  2   0.936 0.827

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term                              FixedEffect Variance    SD Lower Upper
 Intercept | PschoolID                   6.164    0.177 0.421 5.340 6.989
 Intercept | SschoolID                   6.164    0.067 0.258 5.659 6.670
 WPSses | SschoolID (fixed: ses4)        0.089    0.000 0.012 0.067 0.112
```

Note that a correlation for the new random WPSses slope was estimated only with the secondary school random intercept, not with the primary school random intercept. The random slope variance is decidedly not there, but what kind of fixed effects *would* have explained a WPSses random slope variance over secondary schools?

Sample Results Section starts here

[indicates notes about what to customize or also include; note that SE and p-values are not needed if you provide tables for the model solutions; these are wordy on purpose to help show how to interpret each effect]

The extent to which 9th grade academic achievement could be predicted from school denomination and child socio-economic status (SES) was examined in a series of multilevel models with crossed random effects (i.e., for child cross-classification). Specifically, the 1,000 children at level 1 were modeled as nested within their 50 primary schools at level 2, as well as within their 33 secondary schools at level 2, such that primary and secondary schools were crossed sampling dimensions at level 2. Restricted maximum likelihood (REML) within the lmer function in the lme4 R package (v. 2.0.6; Bates et al., 2015) was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with Wald tests using Satterthwaite

denominator degrees of freedom (computed using the lmerTest R package, v. 3.2.1; Kuznetsova et al., 2017), whereas random effects were evaluated via likelihood ratio tests (i.e., −2ΔLL with degrees of freedom equal to the number of new random effects variances and covariances). Alpha was chosen as .01. Effect size was evaluated via pseudo-R^2 values for the proportion reduction in each variance component, as well as with total-R^2, the proportion of the total variance explained by the model fixed effects.

We first examined the extent of dependency due to mean differences by including a random intercept variance for each type of school. Relative to a model assuming independent children (i.e., with only a single model residual), adding a random intercept variance across primary schools significantly improved model fit, $-2\Delta LL(1) = 170.63$, $p < .001$. Adding another random intercept variance across secondary schools also significantly improved model fit, $-2\Delta LL(1) = 75.47$, $p < .001$, providing empirical support for the need to model the cross-classification of children within primary and secondary schools. Given 1–6 children within each unique combination of primary and secondary schools, we also examined the need for a random primary by secondary interaction. It was removed given that it did not significantly improve model fit, $-2\Delta LL(1) = 0.99$, $p = .321$, indicating no extra correlation of children from the same unique combination. Of the total variation in child achievement, 23.0% reflected mean differences between primary schools, 10.4% reflected mean differences between secondary schools, and 66.6% reflected remaining between-children differences after controlling for primary and secondary school additive effects. Using an empty means model with two random intercepts, a 95% random effects confidence interval was calculated for each source of intercept variation as the fixed intercept ± 1.96*SQRT(random intercept variance). Results indicated that 95% of the primary schools were predicted to have intercepts for school mean achievement between 5.52 and 7.16, whereas 95% of the secondary schools were predicted to have intercepts for school mean achievement between 5.79 and 6.89.

We then added the effects for the denomination status (0 = not religious, 1 = religious) for the primary school and for the secondary school, as well as for their interaction. Both main effects indicated nonsignificantly greater achievement outcomes for denominational schools, with no significant interaction. Primary school denomination captured 1.5% of the primary school random intercept variance, secondary school denomination captured 7.9% of the secondary school random intercept variance, and their interaction captured 0.2% of the level-1 residual variance. However, all three denomination predictors were retained in the model as control variables.

We then considered the effect of child SES (centered at 4). Its fixed slope was significantly positive, such that child achievement was expected to be larger by 0.107 per unit SES, $p < .001$. Relative to the model with only denomination, it captured an additional 6.0% of the secondary school random intercept variance and 4.2% of the level-1 residual variance, but none of the primary school random intercept variance. However, the inclusion of a single fixed slope for child SES assumes no contextual effects of any kind. To test this assumption, and to ensure proper interpretation of the child-level SES fixed effect as the within-schools effect, we added three level-2 contextual SES slopes: primary school mean SES, secondary school mean SES, and the mean SES of the unique combination of primary by secondary school (each centered at 4).

The level-1 SES slope—now representing the pure within-schools effect—was significantly positive, and it indicated that child achievement was expected to be larger by 0.090 per unit greater SES than the mean of the child's primary and secondary school combination, $p = .001$. The following level-2 contextual effects are each interpreted as the incremental contribution of the school after controlling for child SES. The level-2 contextual SES effect for primary schools indicated that primary school mean achievement was nonsignificantly lower by 0.036 per unit higher primary mean SES, $p = .876$. Likewise, the level-2 contextual SES effect for secondary schools indicated that secondary school mean achievement was nonsignificantly higher by 0.179 per unit higher secondary mean SES, $p = .314$. Finally, the level-2 contextual SES effect for the unique combination of primary and secondary schools indicated that child achievement was expected to be nonsignificantly higher by 0.023 per

unit higher school combination mean SES, p = .497. Relative to the model with only the child SES slope, the contextual effects in total accounted for none of the primary school random intercept variance, an additional 1.7% of the secondary school random intercept variance, and none of the level-1 residual variance. The denomination and SES fixed effects together accounted for 5.8% of the total variance in achievement.

Lastly, we considered the potential for random slopes for the child SES effect (using a within-unique-combination centered predictor to avoid conflated random slopes). Within-school child SES slope variation was nonsignificant across primary schools, $-2\Delta LL(2) = 0.106$, p = .948, as well as across secondary schools, $-2\Delta LL(2) = 0.132$, p = .936, indicating that the size of the relative child SES advantage did not differ significantly across each type of school.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software, 67*(1), 1–48.

Hox, J. J., Moerbeek, M., & van de Schoot, R. (2018). *Multilevel analysis: Techniques and applications* (3rd ed.). Routledge.

Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software, 82*(13), 1–26.