Example 4b: Random Slopes and Cross-Level Interactions of Hybrid Level-1 Predictors in General Multilevel Models for Two-Level Nested Outcomes (complete syntax, data, and output available for R electronically)

This example uses real data from a math test given at the end of 10th grade in a midwestern rectangular state. These analyses include 13,802 students from 94 schools, with 31–515 students in each school (M = 139). We will use "hybrid" models to examine how student lunch status (0 = pay full price for lunch, 1= receive reduced lunch, 2= receive free lunch) predicts student math test scores. We will use an atypical coding scheme to first distinguish paid lunch (0) from reduced or free lunch (1), and then further distinguish reduced lunch (−0.5) from free lunch (0.5). It uses several custom functions to process output and add computations, as shown below (all in the online files).

R Syntax for Packages; Importing and Preparing Data for Analysis

```
# Install and load required packages
if (!require(pacman)) install.packages("pacman")
pacman::p_load(readxl, TeachingDemos, nlme, lme4, lmerTest, ordinal, mclogit)
# readxl: import .xls or .xlsx files; TeachingDemos: save output to external file
# nlme: multilevel models with or without random effects; lme4: multilevel models
# lmerTest: DDF and p-values for lme4, also univ/Multiv Wald tests
# ordinal and mclogit: MLMs for ordinal or nominal outcomes, respectively

# Set working directory (to import and export files to)
setwd("C:/Dropbox/26_EDF9850/Unit4/Example4b")

# Load R functions for this class from R file in either place
source("../EDF9850_Functions.R") # Remove ../ for same folder
source("https://www.lesahoffman.com/EDF9850/EDF9850_Functions.R") # From web

#  Import example excel data file from sheet "grade10" and convert to data frame
Example4b = read_excel(path="Example4b_Data.xlsx", sheet="grade10")
Example4b = as.data.frame(Example4b)

# Labels for original variables in comments only
# schoolID=   "schoolID: School ID number"
# lunch=      "lunch: 0=Paid, 1=Reduced, 2=Free"
# math=       "math: Math Test Outcome"

# Filter to only cases complete on all variables to be used below (before cluster means)
Example4b = Example4b[complete.cases(Example4b[ , c("math","lunch")]),]

# Create new binary-coded predictors (start as blank, then recode)
Example4b$PvD=NA; Example4b$RvF=NA
Example4b$PvD[which(Example4b$lunch==0)]=0   # Replace paid lunch
Example4b$RvF[which(Example4b$lunch==0)]=0
Example4b$PvD[which(Example4b$lunch==1)]=1   # Replace reduced lunch
Example4b$RvF[which(Example4b$lunch==1)]=-0.5
Example4b$PvD[which(Example4b$lunch==2)]=1   # Replace free lunch
Example4b$RvF[which(Example4b$lunch==2)]=0.5
# PvD= "PvD: 0=Paid, 1=Discount(Reduced or Free Lunch)"
# RvF= "RvF: 0=Paid, -0.5=Reduced, 0.5=Free Lunch"
```

	PvD	RvF
1.Paid	0	0
2.Reduced	1	-0.5
3.Free	1	0.5

```
# Create binary indicator variables for demo purposes
Example4b$Paid=NA; Example4b$Redu=NA; Example4b$Free=NA
Example4b$Paid[which(Example4b$lunch==0)]=1   # Replace each for paid lunch
Example4b$Redu[which(Example4b$lunch==0)]=0
Example4b$Free[which(Example4b$lunch==0)]=0
```

```
Example4b$Paid[which(Example4b$lunch==1)]=0  # Replace each for reduced lunch
Example4b$Redu[which(Example4b$lunch==1)]=1
Example4b$Free[which(Example4b$lunch==1)]=0
Example4b$Paid[which(Example4b$lunch==2)]=0  # Replace each for free lunch
Example4b$Redu[which(Example4b$lunch==2)]=0
Example4b$Free[which(Example4b$lunch==2)]=1
# Paid= "Paid: 1=Paid Lunch"
# Redu= "Redu: 1=Reduced Lunch"
# Free= "Free: 1=Free Lunch"

# Make factor version of lunch variable for ordinal MLM with labeled levels
Example4b$lunchF = factor(Example4b$lunch, levels=c(0,1,2),
                          labels=c("0Paid","1Reduced","2Free"))

# Compute cluster means for level-1 variables
Example4b = addUnitMeans(data=Example4b, unitVariable="schoolID",
            meanVariables=c("math","PvD","RvF","Paid","Redu","Free"),
            newNames=c("CM_math","CM_PvD","CM_RvF","CM_Paid","CM_Redu","CM_Free"))

print("Descriptive Statistics in School-Level-2 Data")
Example4bMeans = unique(Example4b[,c("schoolID","NperschoolID","CM_math","CM_PvD",
                                     "CM_RvF","CM_Paid","CM_Redu","CM_Free")])
univSummary(data=Example4bMeans, vars=c("NperschoolID","CM_math","CM_PvD","CM_RvF",
                                        "CM_Paid","CM_Redu","CM_Free"))
```

```
Variable      N    Mean      SD  Variance    Min     Max
 NperschoolID 94 139.170 138.202 19099.713 31.000 515.000
 CM_math      94  47.725   6.970    48.586 29.451  61.614
 CM_PvD       94   0.301   0.213     0.045  0.000   0.803
 CM_RvF       94   0.044   0.098     0.010 -0.330   0.310
 CM_Paid      94   0.699   0.213     0.045  0.197   1.000
 CM_Redu      94   0.107   0.127     0.016  0.000   0.727
 CM_Free      94   0.195   0.160     0.026  0.000   0.678
```

```
# Center cluster means so ref school has 0paid=.70, 1red=.10, and 2free=.20
Example4b$CM_PvD30 = Example4b$CM_PvD - .30
Example4b$CM_RvF05 = Example4b$CM_RvF - .05 # ref=.20 in free group; 0=.15 instead
# Compute CM_RvF values as 0.5*(free-reduced) → 0.5(.20-.10)=.05 if free=.20

# Cluster-mean-center level-1 predictors for random slopes
Example4b$WC_PvD = Example4b$PvD - Example4b$CM_PvD
Example4b$WC_RvF = Example4b$RvF - Example4b$CM_RvF
# Cluster-mean-center level-1 math for correlations
Example4b$WC_math = Example4b$math - Example4b$CM_math
# WC_PvD=  "WC_PvD:  Within-Cluster Paid vs Reduced/Free Lunch"
# WC_RvF=  "WC_RvF:  Within-Cluster Paid/Reduced vs Free Lunch"
# WC_math= "WC_math: Within-Cluster Math Test Outcome"

print("Descriptive Statistics in Student-Level-1 Data")
freqSummary(data=Example4b, var="lunchF")
```

```
Variable Level       N Proportion
 lunchF   0Paid    9059      0.692
 lunchF   1Reduced 1140      0.087
 lunchF   2Free    2883      0.220
```

```
univSummary(data=Example4b, vars=c("PvD","RvF","WC_PvD","WC_RvF"))
```

```
Variable     N   Mean    SD Variance    Min   Max
 PvD      13082  0.308 0.461    0.213  0.000 1.000
 RvF      13082  0.067 0.269    0.072 -0.500 0.500
 WC_PvD   13082 -0.000 0.405    0.164 -0.803 0.989
 WC_RvF   13082  0.000 0.254    0.065 -0.810 0.830
```

```
corrSummary(data=Example4b, vars=c("math","PvD","WC_math","WC_PvD","CM_math","CM_PvD"),
            p.separate=TRUE) # No RvF because their correlations are not interpretable
```

```
Correlations:
 Variable    math     PvD WC_math WC_PvD CM_math CM_PvD
 math       1.000 -0.353   0.919 -0.215   0.395 -0.342
 PvD       -0.353  1.000  -0.205  0.877  -0.416  0.481
 WC_math    0.919 -0.205   1.000 -0.234  -0.000  0.000
 WC_PvD    -0.215  0.877  -0.234  1.000   0.000 -0.000
 CM_math    0.395 -0.416  -0.000  0.000   1.000 -0.865
 CM_PvD    -0.342  0.481   0.000 -0.000  -0.865  1.000
```

Note the smushed correlation is in between the within correlation and much stronger between correlation (foreshadowing!)

Model 1: Empty Means, Random Intercept Model for the Math Outcome

Level 1: $Math_{pc} = \beta_{0c} + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

```
print("Model 1: Empty Means, Random Intercept for Math")
Model1 = lmer(data=Example4b, REML=TRUE, formula=math~1+(1|schoolID))
lmerSummary(Model1, ICC1=TRUE, ICC2=TRUE, Neff=TRUE, LRT=TRUE, randCI=TRUE)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
109795.717 109818.154 -54894.858 109789.717

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept   45.453   6.742
 Residual             253.176  15.912
Number of obs: 13082 , groups: schoolID, 94

Fixed Effects:
 Term          Est    SE     DF t.value p.value
 Intercept  47.756 0.723 93.903  66.051   <.001
```

```
ICC1 intraclass correlation:
    Adjusted ICC: 0.152
  Unadjusted ICC: 0.152
```

$$\text{ICC}_1 = \frac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e} = \frac{45.453}{45.453 + 253.176} = .\mathbf{152}$$

```
Design Effect and Effective Sample Size:
  Design Effect (using mean n = 139.170 per cluster): 22.030
  Effective N: 593.821

LRT for random effect(s):
 Random Effect Dropped  Npar     LL   -2LL    AIC    BIC      LRT DF p.value p.mix
 <none>                    3 -54895 109790 109796 109818
 Intercept | schoolID      2 -55825 111650 111654 111669 1860.211  1   <.001 <.001
```

```
ICC2 reliability of the cluster means
(using mean n = 139.170 per cluster):
0.962
```

$$\text{ICC}_2 = \frac{\tau^2_{U_0}}{\tau^2_{U_0} + \sigma^2_e/L1n} = \frac{45.453}{45.453 + 253.176/139.\mathbf{17}} = .\mathbf{962}$$

```
Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term      FixedEffect Variance    SD  Lower  Upper
 Intercept      47.756   45.453 6.742 34.542 60.970
```

Empty Model for the Level-1 Student Ordinal Lunch Predictor

```
print("Empty Means, Random Intercept for Ordinal Lunch using ordinal package")
EmptyLunch = ordinal::clmm(data=Example4b, link="logit", formula=lunchF~1+(1|schoolID))
clmmSummary(EmptyLunch, ICC1=TRUE, LRT=TRUE)
```

```
Model Fit for Predicting lunchF:
      AIC       BIC    logLik -2*logLik
17967.429 17989.866 -8980.714 17961.429

Model for the Variance:
 Groups    Name             Variance Std.Dev
 schoolID  Intercept           1.783   1.335
 Residual  (logit constant)  (3.290) (1.814)
Number of obs: 13082 , groups: schoolID, 94

Intercepts predicting the logit of the probability
of being in the HIGHER category after | :
 Term                    Est    SE z.value p.value  Prob Prob.SE
 0Paid|1Reduced,2Free  -1.203 0.143  -8.428   <.001 0.231   0.025
 0Paid,1Reduced|2Free  -1.767 0.143 -12.336   <.001 0.146   0.018

ICC1 intraclass correlation:
0.351

LRT for random effect(s):
 Random Effect Dropped  Npar     LL  -2LL   AIC   BIC      LRT DF p.value p.mix
 <none>                    3  -8981 17961 17967 17990
 Intercept | schoolID      2 -10471 20942 20946 20961 2980.755  1   <.001 <.001
```

Model 2: Add Uncentered Level-1 Student Lunch Fixed Slopes

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20}$

```
print("Model 2: Add L1 Student Lunch")
Model2 = lmer(data=Example4b, REML=TRUE, formula=math~1+PvD+RvF+(1|schoolID))
lmerSummary(Model2)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108962.804 109000.199 -54476.402 108952.804

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept   28.312   5.321
 Residual             238.161  15.432
Number of obs: 13082 , groups: schoolID, 94

Overall F-test for All Fixed Slopes:
 Model  NumDF     DenDF F.value p.value
 Model2     2 13028.726 435.910   <.001

Fixed Effects:
 Term          Est    SE        DF t.value p.value
 Intercept  50.532 0.590    96.393  85.708   <.001
 PvD        -8.489 0.351 12944.113 -24.164   <.001
 RvF        -4.433 0.561 13075.382  -7.907   <.001
```

Intercept γ_{00} =
PvD γ_{10} =
RvF γ_{20} =

```
PseudoR2(baseModel=Model1, altModel1=Model2)
```

```
Pseudo-R2
 Term       Model  Variance Pseudo.R2
 Intercept  Model1   45.453
 Intercept  Model2   28.312     0.377
 Residual   Model1  253.176
 Residual   Model2  238.161     0.059
```

What does this pattern of explained variance at each level tell us about the level-1 fixed slopes?

Model 3: Let's Fix It—Add L2 Cluster Means of Student Lunch Predictors

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20}$

```
print("Model 3: Add L2 Cluster Mean Student Lunch")
Model3 = lmer(data=Example4b, REML=TRUE,
              formula=math~1+PvD+RvF+CM_PvD30+CM_RvF05+(1|schoolID))
lmerSummary(Model3)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108904.022 108956.375 -54445.011 108890.022

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept   13.960   3.736
 Residual             238.225  15.435
Number of obs: 13082 , groups: schoolID, 94

Overall F-test for All Fixed Slopes:
 Model  NumDF   DenDF F.value p.value
 Model3     4 169.182 247.890   <.001

Fixed Effects:
 Term           Est    SE        DF t.value p.value
 Intercept   50.554 0.440    90.056 114.855   <.001
 PvD         -8.209 0.354 12978.590 -23.179   <.001
 RvF         -4.568 0.563 12978.590  -8.112   <.001
 CM_PvD30   -18.246 2.108    86.917  -8.656   <.001
 CM_RvF05     8.162 4.611    90.183   1.770   0.080
```

Results from previous Model 2:

```
Term          Est    SE
 Intercept  50.532 0.590
 PvD        -8.489 0.351
 RvF        -4.433 0.561
```

CM_PvD30 γ_{01} =
CM_RvF05 γ_{02} =

```
con1DTable(Model3, L=list(
  "L1 Within Paid v Redu Main Effect"     = c(0, 1,-.5, 0,  0),
  "L1 Within Paid v Free Main Effect"     = c(0, 1, .5, 0,  0),
  "L1 Within Redu v Free Main Effect"     = c(0, 0,  1, 0,  0),  # In model
  "L2 Contextual Paid v Redu Main Effect" = c(0, 0,  0, 1,-.5),
  "L2 Contextual Paid v Free Main Effect" = c(0, 0,  0, 1, .5),
  "L2 Contextual Redu v Free Main Effect" = c(0, 0,  0, 0,  1))) # In model
```

```
Requested Fixed Effects:
 Name                                        Est    SE        DF t.value p.value
 L1 Within Paid v Redu Main Effect        -5.925 0.521 12978.590 -11.368   <.001
 L1 Within Paid v Free Main Effect       -10.493 0.371 12978.590 -28.270   <.001
 L1 Within Redu v Free Main Effect        -4.568 0.563 12978.590  -8.112   <.001
 L2 Contextual Paid v Redu Main Effect -22.327 3.521    95.871  -6.341   <.001
 L2 Contextual Paid v Free Main Effect -14.165 2.669    77.942  -5.307   <.001
 L2 Contextual Redu v Free Main Effect   8.162 4.611    90.183   1.770   0.080
```

```
PseudoR2(baseModel=Model1, altModel1=Model2, altModel2=Model3)
```

```
Pseudo-R2 and Change in Pseudo-R2
 Term       Model  Variance Pseudo.R2 Change.R2
 Intercept  Model1   45.453
 Intercept  Model2   28.312     0.377
 Intercept  Model3   13.960     0.693     0.316
 Residual   Model1  253.176
 Residual   Model2  238.161     0.059
 Residual   Model3  238.225     0.059    -0.000
```

```
TotalR2(altModel1=Model2, altModel2=Model3)
```

```
Total-R2 and Change in Total-R2
 Model  Total.R2 Change.R2
 Model2    0.127
 Model3    0.168     0.040
```

Model 4a: Add Random Slopes for Cluster-Mean-Centered Student Lunch Predictors

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + \beta_{3c}(PvD_{pc} - \overline{PvD}_c) + \beta_{4c}(RvF_{pc} - \overline{RvF}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + U_{0c}$

$\beta_{1c} = \gamma_{10};\ \beta_{2c} = \gamma_{20};\ \beta_{3c} = U_{3c};\ \beta_{4c} = U_{4c}$

LRTdrop allows custom drop combinations; the last line allows random effect CIs for custom combinations of fixed + random effects (in paired order)

```
print("Model 4a: Add Random Cluster-MC Student Lunch")
Model4a = lmer(data=Example4b, REML=TRUE,
               formula=math~1+PvD+RvF+CM_PvD30+CM_RvF05
                      +(1+WC_PvD+WC_RvF|schoolID))
lmerSummary(Model4a, LRT=TRUE, LRTdrop=c("WC_PvD","WC_RvF"),
            LRTdropGroup="schoolID",randCI=TRUE, slopeRel=TRUE,
            randCIfixed=c("PvD","RvF"), randCIrandom=c("WC_PvD","WC_RvF"))
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108827.330 108917.078 -54401.665 108803.330

Model for the Variance:
 Groups    Name       Variance Std.Dev   Corr
 schoolID  Intercept   13.908   3.729
           WC_PvD      12.476   3.532 -0.508
           WC_RvF       0.695   0.834 -0.885 0.851
 Residual              235.706  15.353
Number of obs: 13082 , groups: schoolID, 94

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model4a     4 117.110 118.630   <.001
```

```
Fixed Effects:
 Term           Est    SE      DF t.value p.value
 Intercept   50.362 0.510  81.893  98.656   <.001
 PvD         -7.688 0.572  88.311 -13.445   <.001
 RvF         -4.392 0.581 558.762  -7.561   <.001
 CM_PvD30   -17.359 2.094  97.982  -8.290   <.001
 CM_RvF05     6.708 4.438  91.263   1.511   0.134

LRT for random effect(s):
 Random Effect Dropped       Npar     LL   -2LL    AIC    BIC    LRT DF p.value p.mix
 <none>                        12 -54402 108803 108827 108917
 WC_PvD | schoolID              9 -54440 108879 108897 108964 75.836  3   <.001 <.001
 WC_RvF | schoolID              9 -54403 108805 108823 108890  1.775  3   0.620 0.516
 WC_PvD + WC_RvF | schoolID     7 -54445 108890 108904 108956 86.692  5   <.001

Random Slope Reliability (SR):
  SR = slope var / (slope var + res var / (mean L1n * var(X)))
 Term   SlopeVar  ResVar Var.X     L1n    SR
 WC_PvD   12.476 235.706 0.164 139.170 0.547
 WC_RvF    0.695 235.706 0.065 139.170 0.026

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term                          FixedEffect Variance    SD   Lower  Upper
 Intercept | schoolID                50.362   13.908 3.729  43.052 57.671
 WC_PvD | schoolID (fixed: PvD)      -7.688   12.476 3.532 -14.611 -0.765
 WC_RvF | schoolID (fixed: RvF)      -4.392    0.695 0.834  -6.027 -2.758
```

Model 4b: Remove Random Slope for Cluster-Mean-Centered RvF Lunch Predictor

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + \beta_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + U_{0c}$

$\beta_{1c} = \gamma_{10}$; $\beta_{2c} = \gamma_{20}$; $\beta_{3c} = U_{3c}$

```
print("Model 4b: Remove Random Cluster-MC RvF Student Lunch")
Model4b = lmer(data=Example4b, REML=TRUE,
               formula=math~1+PvD+RvF+CM_PvD30+CM_RvF05
                     +(1+WC_PvD|schoolID))
lmerSummary(Model4b, LRT=TRUE, randCI=TRUE, slopeRel=TRUE,
            randCIfixed=c("PvD"), randCIrandom=c("WC_PvD"))
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108823.105 108890.416 -54402.553 108805.105

Model for the Variance:
 Groups    Name      Variance Std.Dev   Corr
 schoolID  Intercept   13.887   3.727
           WC_PvD      13.436   3.666 -0.518
 Residual             235.736  15.354
Number of obs: 13082 , groups: schoolID, 94

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model4b     4 146.601 122.780   <.001

Fixed Effects:
 Term           Est    SE       DF t.value p.value
 Intercept   50.366 0.510   82.430  98.820   <.001
 PvD         -7.688 0.584   93.895 -13.172   <.001
```

```
 RvF          -4.450 0.568 12481.252  -7.834   <.001
 CM_PvD30    -17.541 2.106    99.843  -8.330   <.001
 CM_RvF05      7.512 4.402    89.442   1.706   0.091

LRT for random effect(s):
 Random Effect Dropped  Npar     LL   -2LL    AIC    BIC    LRT DF p.value p.mix
 <none>                    9 -54403 108805 108823 108890
 WC_PvD | schoolID         7 -54445 108890 108904 108956 84.916  2   <.001 <.001

Random Slope Reliability (SR):
  SR = slope var / (slope var + res var / (mean L1n * var(X)))
 Term   SlopeVar  ResVar Var.X     L1n    SR
 WC_PvD   13.436 235.736 0.164 139.170 0.565

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term                           FixedEffect Variance    SD   Lower  Upper
 Intercept | schoolID                50.366   13.887 3.727  43.062 57.670
 WC_PvD | schoolID (fixed: PvD)      -7.688   13.436 3.666 -14.872 -0.503
```

Recap: We already knew that schools differed from each other in their mean math scores, but now we know that schools also differ from each other in the student disadvantage for receiving discounted lunch. Next, we try to predict those school slope differences by school lunch composition—does the lunch-related disadvantage depend on how many students are also disadvantaged in the school?

Model 5: Add Cross-Level Interactions with PvD Lunch Predictor

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + \beta_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + U_{0c}$

$\beta_{1c} = \gamma_{10} + \gamma_{11}(\overline{PvD}_c - .30) + \gamma_{12}(\overline{RvF}_c - .05)$

$\beta_{2c} = \gamma_{20}; \ \beta_{3c} = U_{3c}$

Composite equation written to show interactions (with all fixed effects first by convention):

$$Math_{pc} = \gamma_{00} + \gamma_{10}(PvD_{pc}) + \gamma_{20}(RvF_{pc}) + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + \gamma_{11}(PvD_{pc})(\overline{PvD}_c - .30) + \gamma_{12}(PvD_{pc})(\overline{RvF}_c - .05) + U_{0c} + U_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$$

```
print("Model 5: Add Cross-Level Interactions with Random PvD")
Model5 = lmer(data=Example4b, REML=TRUE,
              formula=math~1+PvD+RvF+CM_PvD30+CM_RvF05
              +PvD:CM_PvD30 +PvD:CM_RvF05 +(1+WC_PvD|schoolID))
lmerSummary(Model5)

Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108815.134 108897.402 -54396.567 108793.134

Model for the Variance:
 Groups    Name       Variance Std.Dev   Corr
 schoolID  Intercept   14.130   3.759
           WC_PvD      12.412   3.523 -0.502
 Residual             235.737  15.354
Number of obs: 13082 , groups: schoolID, 94
```

```
Overall F-test for All Fixed Slopes:
 Model  NumDF   DenDF F.value p.value
 Model5     6 145.890  83.276   <.001

Fixed Effects:
 Term              Est    SE        DF t.value p.value
 Intercept      50.247 0.514    83.622  97.832   <.001
 PvD            -7.941 0.591   105.127 -13.442   <.001
 RvF            -4.429 0.574 12982.445  -7.718   <.001
 CM_PvD30      -20.430 2.771    94.826  -7.372   <.001
 CM_RvF05        9.941 6.961    92.240   1.428   0.157
 PvD:CM_PvD30    4.733 2.795   145.609   1.693   0.093
 PvD:CM_RvF05   -2.499 5.977   139.320  -0.418   0.676
```

What kind of pattern of moderation by school composition does this model assume?

Model 6: Let's Fix It—Add Level-2 Interactions to Unsmush the Cross-Level Interactions

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + \beta_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$

Level 2:
$$\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + \gamma_{03}(\overline{PvD}_c - .30)^2 + \gamma_{04}(\overline{RvF}_c - .05)(\overline{PvD}_c - .30) + U_{0c}$$
$$\beta_{1c} = \gamma_{10} + \gamma_{11}(\overline{PvD}_c - .30) + \gamma_{12}(\overline{RvF}_c - .05)$$
$$\beta_{2c} = \gamma_{20};\ \ \beta_{3c} = U_{3c}$$

Composite equation written to show interactions (with all fixed effects first by convention):

$$Math_{pc} = \gamma_{00} + \gamma_{10}(PvD_{pc}) + \gamma_{20}(RvF_{pc}) + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + \gamma_{11}(PvD_{pc})(\overline{PvD}_c - .30) + \gamma_{12}(PvD_{pc})(\overline{RvF}_c - .05) + \gamma_{03}(\overline{PvD}_c - .30)^2 + \gamma_{04}(\overline{RvF}_c - .05)(\overline{PvD}_c - .30) + U_{0c} + U_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$$

$\gamma_{03}(\overline{PvD}_c - .30)^2$ **unsmushes** γ_{11}; $\gamma_{04}(\overline{RvF}_c - .05)(\overline{PvD}_c - .30)$ **unsmushes** γ_{12}

```
print("Model 6: Add L2 Interactions with Random PvD")
Model6 = lmer(data=Example4b, REML=TRUE,
              formula=math~1+PvD+RvF+CM_PvD30+CM_RvF05
              +PvD:CM_PvD30 +PvD:CM_RvF05 +CM_PvD30:CM_RvF05 +I(CM_PvD30^2)
              +(1+WC_PvD|schoolID))
lmerSummary(Model6)

Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108801.788 108899.015 -54387.894 108775.788

Model for the Variance:
 Groups    Name       Variance Std.Dev   Corr
 schoolID  Intercept   14.105   3.756
           WC_PvD      12.206   3.494 -0.489
 Residual             235.738  15.354
Number of obs: 13082 , groups: schoolID, 94

Overall F-test for All Fixed Slopes:
 Model  NumDF   DenDF F.value p.value
 Model6     8 122.111  62.800   <.001
```

```
Fixed Effects:
 Term                   Est     SE        DF t.value p.value
 Intercept           50.722  0.660    95.864  76.864   <.001
 PvD                 -8.045  0.596   105.306 -13.489   <.001
 RvF                 -4.409  0.574 12972.668  -7.674   <.001
 CM_PvD30           -17.604  3.650   103.357  -4.823   <.001
 CM_RvF05            -1.179 13.448    98.248  -0.088   0.930
 I(CM_PvD30^2)      -15.324 10.569   104.263  -1.450   0.150 → Not last as in my code!
 PvD:CM_PvD30         6.166  2.997   114.940   2.057   0.042
 PvD:CM_RvF05        -3.398  6.054   130.541  -0.561   0.576
 CM_PvD30:CM_RvF05   27.627 32.223    92.259   0.857   0.393
```

```
PseudoR2(baseModel=Model4b, altModel1=Model5, altModel2=Model6)

Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model4b   13.887
 Intercept  Model5    14.130    -0.018
 Intercept  Model6    14.105    -0.016     0.002
 WC_PvD     Model4b   13.436
 WC_PvD     Model5    12.412     0.076
 WC_PvD     Model6    12.206     0.092     0.015
 Residual   Model4b  235.736
 Residual   Model5   235.737    -0.000
 Residual   Model6   235.738    -0.000    -0.000
```

Model 7: Let's Simplify—Drop Unneeded Nonsignificant Interactions

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(PvD_{pc}) + \beta_{2c}(RvF_{pc}) + \beta_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + \gamma_{03}(\overline{PvD}_c - .30)^2 + U_{0c}$

$\beta_{1c} = \gamma_{10} + \gamma_{11}(\overline{PvD}_c - .30)$; $\beta_{2c} = \gamma_{20}$; $\beta_{3c} = U_{3c}$

Composite equation written to show interactions (with all fixed effects first by convention):

$$Math_{pc} = \beta_{0c} + \gamma_{10}(PvD_{pc}) + \gamma_{20}(RvF_{pc}) + \gamma_{01}(\overline{PvD}_c - .30) + \gamma_{02}(\overline{RvF}_c - .05) + \gamma_{11}(PvD_{pc})(\overline{PvD}_c - .30) + \gamma_{03}(\overline{PvD}_c - .30)^2 + U_{0c} + U_{3c}(PvD_{pc} - \overline{PvD}_c) + e_{pc}$$

```
print("Model 7: Remove Nonsignificant Unnecessary Interactions")
Model7 = lmer(data=Example4b, REML=TRUE,
              formula=math~1+PvD+RvF+CM_PvD30+CM_RvF05
              +I(CM_PvD30^2) +PvD:CM_PvD30 +(1+WC_PvD|schoolID))
lmerSummary(Model7)

Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
108812.878 108895.147 -54395.439 108790.878

Model for the Variance:
 Groups    Name       Variance Std.Dev   Corr
 schoolID  Intercept   13.934   3.733
           WC_PvD      12.221   3.496 -0.477
 Residual             235.726  15.353
Number of obs: 13082 , groups: schoolID, 94

Overall F-test for All Fixed Slopes:
 Model  NumDF   DenDF F.value p.value
 Model7     6 126.573  83.586   <.001
```

```
Fixed Effects:
 Term              Est    SE        DF t.value p.value
 Intercept      50.759 0.656    96.907  77.401   <.001
 PvD            -8.066 0.596   106.814 -13.523   <.001
 RvF            -4.465 0.568 12452.849  -7.865   <.001
 CM_PvD30      -19.160 2.688    94.273  -7.128   <.001
 CM_RvF05        6.477 4.573    85.645   1.416   0.160
 I(CM_PvD30^2) -11.814 9.646    97.918  -1.225   0.224
 PvD:CM_PvD30    5.675 2.821    92.824   2.011   0.047
```

Because the quadratic CM_PvD30 slope is half the rate of acceleration of the curve it creates, the **quadratic slope gets multiplied by 2** to convey how the linear CM_PvD30 slope changes per unit of CM_PvD30

PvD $\boldsymbol{\gamma_{10}}$ =

PvD*:CM_PvD30 $\boldsymbol{\gamma_{11}}$ =

CM_PvD30 $\boldsymbol{\gamma_{01}}$ =

CM_PvD30^2 $\boldsymbol{\gamma_{03}}$ =

RvF $\boldsymbol{\gamma_{20}}$ =

CM_RvF05 $\boldsymbol{\gamma_{02}}$ =

```
con1DTable(Model7, L=list(
  "L1 Within Paid v Redu Main Effect"          = c(0, 1,-.5, 0,  0, 0, 0),
  "L1 Within Paid v Free Main Effect"          = c(0, 1, .5, 0,  0, 0, 0),
  "L1 Within Redu v Free Main Effect"          = c(0, 0,  1, 0,  0, 0, 0),  # In model
  "L2 Contextual Paid v Redu Main Effect"      = c(0, 0,  0, 1,-.5, 0, 0),
  "L2 Contextual Paid v Free Main Effect"      = c(0, 0,  0, 1, .5, 0, 0),
  "L2 Contextual Redu v Free Main Effect"      = c(0, 0,  0, 0,  1, 0, 0))) # In model
```

```
Requested Fixed Effects:
 Name                                    Est    SE        DF t.value p.value
 L1 Within Paid v Redu Main Effect    -5.834 0.701   208.425  -8.320   <.001
 L1 Within Paid v Free Main Effect   -10.299 0.617   116.981 -16.682   <.001
 L1 Within Redu v Free Main Effect    -4.465 0.568 12452.849  -7.865   <.001
 L2 Contextual Paid v Redu Main Effect -22.398 3.985   101.078  -5.621   <.001
 L2 Contextual Paid v Free Main Effect -15.921 3.004    91.435  -5.299   <.001
 L2 Contextual Redu v Free Main Effect   6.477 4.573    85.645   1.416   0.160
```

```
PseudoR2(baseModel=Model4b, altModel1=Model5, altModel2=Model7)
```

Btw, if we had added cross-level interactions with RvF, which variance would they have explained?

```
Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model4b   13.887
 Intercept  Model5    14.130    -0.018
 Intercept  Model7    13.934    -0.003     0.014
 WC_PvD     Model4b   13.436
 WC_PvD     Model5    12.412     0.076
 WC_PvD     Model7    12.221     0.090     0.014
 Residual   Model4b  235.736
 Residual   Model5   235.737    -0.000
 Residual   Model7   235.726     0.000     0.000
```

```
TotalR2(altModel1=Model3, altModel2=Model7)
```

```
Total-R2 and Change in Total-R2
 Model  Total.R2 Change.R2
 Model3    0.168
 Model7    0.169     0.001
```

Sample Results Section starts here—see the last page and online files for figures that could be included!

[indicates notes about what to customize or also include; note that SE and p-values are not needed if you provide tables for the model solutions; these are wordy on purpose to help illustrate how to interpret things]

[Note that the smushed results are not reported, and results are combined across models to give all fixed slopes of interest (so not all models are reported). Models are identified for reference in { }]

The extent to which student math outcomes could be predicted from student-level (and corresponding school-level) variables of ordinal lunch status (fully paid, reduced-price, or free) was examined in a series of multilevel models in which the 13,802 students were modeled as nested within their 94 schools. Restricted Maximum likelihood (REML) within the lmer function in the lme4 R package (v. 2.0.6; Bates et al., 2015) was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with univariate Wald tests using Satterthwaite denominator degrees of freedom. Alpha was chosen as .05. Model-implied fixed effects were requested via contest1D statements (v. 3.2.1; Kuznetsova et al., 2017). The significance of new random effects was evaluated using likelihood ratio tests (i.e., −2ΔLL = the difference in −2LL between nested models with the same fixed effects). The −2ΔLL test statistics were evaluated using $\chi 2$ distributions with a 50–50 mixture of (DF; DF−1) given the 0 boundary of the new random effect variance. Effect size for the fixed effects was evaluated via pseduo-R2 values for the proportion reduction in each variance component relative to a nested model without the predictors in question, as well as with total-R2, the squared correlation between the actual math outcomes and those predicted by the model fixed effects. For descriptive purposes, 95% random effect confidence intervals (CIs) were also computed as CI = fixed effect ± 1.96*SQRT(level-2 random variance).

{Model 1} As derived from an empty means, random intercept model, student math had an intraclass correlation of ICC1 = .152, indicating that 15.2% of the variance in student math was due to mean differences between schools, a significant amount, −2ΔLL(1) = 1860.20, $p < .0001$. Given an average of 139 students per school in this sample, ICC1 = .152 translated into a design effect = 22 (and effective sample size ≈ 593), further indicating the need for a multilevel analysis. The school mean math outcomes had strong reliability, as evidenced by an ICC2 = .962. The fixed intercept was 47.756 (SE = 0.723), which represented the expected average school mean math outcome. A random intercept confidence interval (computed as the fixed intercept ± 1.96*SQRT[random intercept variance]) indicated that 95% of the schools were expected to have school mean math outcomes between 34.542 and 60.970 (around the average of 47.756).

{EmptyLunch} For the ordinal student lunch variable, an ordinal two-level model (i.e., with a cumulative logit link function and a multinomial level-1 conditional distribution) was estimated instead. Using $\pi 2 / 3$ for the logit-scale residual variance, ICC1 = .351, which was also significantly greater than 0, −2ΔLL(1) = 2,981.76, $p < .0001$. Consequently, the effects of student lunch were modeled at both levels. For level-1 students, the ordinal lunch variable was represented via two predictors. The first (PvD) distinguished discounted lunches (reduced or free = 1) from fully paid = 0. The second (RvF) further distinguished free = 0.5 from reduced = −0.5 (with paid = 0). School means were then computed for these predictors and centered by subtracing .30 and .05, respectively, to create a reference school with 70% paid lunch, 10% reduced lunch, and 20% free lunch.

{Model 3} We then examined the fixed slopes for the level-1 student and level-2 school lunch predictors, which accounted for 5.9% of the level-1 residual variance in math and 69.3% of the level-2 random intercept variance in math, respectively (total-R2 = .059). The fixed intercept of 50.554 (SE = 0.440), which represented the expected math outcome for a student with paid lunch from the reference school described above. At level 1, the within-school PvD lunch slope was significantly negative (Est = −8.209, SE = 0.354, $p < .001$), indicating that student math was expected to be significantly lower by 8.209 for students who received discounted lunch as

compared to paid lunch. Further, the within-school RvF lunch slope was also significantly negative (Est = −4.568, SE = 0.563, p < .001), indicating that student math was expected to be significantly lower by 4.568 for students who received free lunch as compared to reduced lunch. Relative to paid lunch, model-implied differences indicated significantly lower student math by 5.925 (SE = 0.521, p < .001) or 10.493 (SE = 0.371, p < .001) for reduced or free lunch, respectively. At level 2, the contextual PvD lunch slope was also significantly negative (Est = −18.246, SE = 2.108, p < .001), indicating that after controlling for student lunch status, school mean math was expected to be significantly lower by 1.825 for every 10% more students with discounted lunch in that school. However, the contextual RvF lunch slope was nonsignificantly positive (Est = 8.162, SE = 4.611, p = .080), indicating that after controlling for student lunch status, school mean math was expected to be nonsignificantly higher by 0.816 for every 10% more students with free lunch in that school. Model-implied contextual effects differentiating reduced or free lunch proportions from paid lunch proportions were −22.327 (SE = 3.251, p < .001) and −14.165 (SE = 2.669, p < .001), respectively.

{Model 4a and 4b} We then examined the extent of school differences in the within-school lunch-related math disadvantage by adding level-2 random slopes (and their correlations with the level-2 random intercept); we used a cluster-mean-centered version of each level-1 lunch predictor to prevent random slope conflation. Model fit improved significantly for a random PvD slope, −2ΔLL(2;3) = 75.836, p < .001, indicating significant between-school heterogeneity in the gap between students with paid or discounted lunch. However, the random RvF slope was not significant, −2ΔLL(2;3) = 1.775, p = .620, indicating a lack of school variation in the gap between students with reduced or free lunch. Accordingly, the nonsignificant random slope was removed from the model before proceeding further. The random PvD slope remained significant, −2ΔLL(2;3) = 84.916, p < .001. The level-1 within-school fixed PvD slope was then −7.688 (SE = 0.584), which represented the average paid–discount lunch gap in student math across schools. A random slope confidence interval (computed as the fixed slope ± 1.96*SQRT[random slope variance]) indicated that 95% of the schools were expected to have paid–discount lunch gaps between 0.503 and 14.872, warranting cross-level interactions to explain these slope differences.

{Models 6 and 7} We then examined moderation of the paid–discount lunch gap by school lunch composition by allowing the within-school and contextual PvD lunch slopes to interact with each school mean lunch predictor. However, the interactions of school mean RvF with the PvD predictors were nonsignificant and were thus removed. In the revised model, the significant PvD by school mean PvD interaction explained 9.00% of the random PvD lunch slope variance, whereas the nonsignificant school mean PvD quadratic slope explained ~0% of the remaining random intercept variance across schools (but was retained to avoid conflating the cross-level interaction). Results for the model below are shown in Table 1 and can be interpreted as follows.

Final Model (repeated for reference from Model 7):

Level 1: $$\boldsymbol{Math_{pc} = \beta_{0c} + \beta_{1c}\left(PvD_{pc}\right) + \beta_{2c}\left(RvF_{pc}\right) + \beta_{3c}\left(PvD_{pc} - \overline{PvD}_{c}\right) + e_{pc}}$$

Level 2: $$\boldsymbol{\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{PvD}_{c} - .30) + \gamma_{02}(\overline{RvF}_{c} - .05) + \gamma_{03}(\overline{PvD}_{c} - .30)^{2} + U_{0c}}$$

$$\boldsymbol{\beta_{1c} = \gamma_{10} + \gamma_{11}(\overline{PvD}_{c} - .30);\ \ \beta_{2c} = \gamma_{20};\ \ \beta_{3c} = U_{3c}}$$

Table 1 *(Note: Model-implied group differences found as linear combinations could also be included, as could the Cohen's d or partial r effect size columns)*

Fixed effects		Estimate	SE	$p <$
γ_{00}	Intercept	50.759	0.656	.001
	PvD: Paid = 0 versus Discounted (Reduced or Free Lunch) = 1			
γ_{10}	Within-School Simple Main Effect	−8.066	0.596	.001
$\gamma_{10} - 0.5\gamma_{30}$	Paid versus Reduced Lunch	−5.834	0.701	.001
$\gamma_{10} + 0.5\gamma_{30}$	Paid versus Free Lunch	−10.299	0.617	.001
γ_{01}	Linear Contextual Main Effect	−19.160	2.688	.001
γ_{02}	Quadratic Contextual Main Effect	−11.814	9.646	.224
γ_{11}	Within-School by Linear Contextual Interaction	5.675	2.821	.047
	RvF: Paid = 0; Reduced = −0.5 versus Free Lunch = 0.5			
γ_{20}	Within-School Main Effect	−4.465	0.568	.001
γ_{03}	Linear Contextual Main Effect	6.477	4.573	.160

Model for the Variance		Estimate
$\tau^2_{U_0}$	Level-2 School Random Intercept Variance	13.934
$\tau^2_{U_3}$	Level-2 School Random Within-School PvsRF Slope Variance	12.221
	Level-2 Random Effects Correlation	−0.477
σ^2_e	Level-1 Residual Variance	235.726

The intercept (50.759) is the expected math outcome for a student who pays for lunch and who attends a school where 70% of students pay for lunch, 10% receive reduced-price lunch, and 20% receive free lunch. Let us first consider differences between students with discounted lunch relative to paid lunch. At level 1, relative to their peers with paid lunch at the same school, students with discounted lunch were predicted to have significantly lower math by 8.066 (specifically for schools in which 30% of students received discounted lunch, given its cross-level interaction with the school mean predictor). Further, student math was expected to be significantly lower by 4.465 for students who received free lunch as compared to reduced lunch (unconditional, given the lack of interactions with the RvF predictor). Relative to paid lunch, model-implied differences indicated significantly lower student math by 5.834 or 10.299 for reduced or free lunch, respectively. At level 2, school mean math was significantly lower by 1.916 per 10% more children who received discounted rather than paid lunch (specifically for students with paid lunch given the cross-level interaction, and specifically for schools in which 30% of students received discounted lunch given the level-2 interaction). The cross-level interaction (5.675) indicated that the within-school paid–discount lunch math gap was significantly less negative (weaker) by 0.568 for every 10% more students who received discounted in that school. The level-2 interaction (−11.814) indicated that the contextual PvD lunch slope became nonsignificantly more negative (stronger) by twice the slope value of 1.181 for every 10% more students who received discounted lunch in that school. Finally, school mean math was nonsignificantly higher by 0.648 per 10% more children who received free rather than reduced lunch.

These results are also summarized in Figures 1–3. [Note: predicted values are shown to the left for pedagogical purposes but would normally not be included in a figure. Also, see the online files for how these plots can be recreated in R using the PlotPredicted R file.]

First, as shown by the x-axis in all three figures, students who paid full-price for lunch had the highest predicted math outcomes, followed by students who paid reduced-price lunch, who in turn had higher predicted math outcomes than students who received free lunch.

Figure 1 holds constant the proportion of students receiving free lunch at .10. In addition, as shown by the growing vertical distance between the lines, schools with a greater proportion of students receiving reduced-price lunch (rather than paid lunch) had lower average math outcomes, and this effect became nonsignificantly stronger as the school proportion reduced lunch increased. In addition, as shown by the difference in slope from paid to reduced-price lunch on the x-axis, that student disadvantage was significantly smaller (shallower slopes) in schools with more children receiving reduced-price lunch. However, there was no model-predicted difference in slope from reduced-price to free lunch by school composition (i.e., no moderation), and thus the lines are parallel from reduced to free. Figure 2 holds constant the proportion of students receiving reduced-price lunch at .10. It shows the same pattern of growing negative contextual effects for schools with a higher proportion of students receiving free lunch (rather than paid lunch).

Figure 1: Contextual Effect of School Proportion of Students Receving Reduced Lunch (Rather than Paid)
CM_PvD = .2 .3 .4 .5 → CM_PvD30 = −.1 0 .1 .2
CM_RvF = .5(free−reduced), so CM_RvF = 0 −.05 −.10 −.15 → CM_RvF05 = −.05 −.10 −.15 −.20

Holding Constant School Proportion Free Lunch at 10%:			
School Composition:	**Paid**	**Reduced**	**Free**
80% Paid, 10% Reduced	52.233	45.832	41.367
70% Paid, 20% Reduced	50.111	44.278	39.813
60% Paid, 30% Reduced	47.753	42.487	38.022
50% Paid, 40% Reduced	45.159	40.461	35.996
Disadvantages:		**PvR:**	**RvF:**
80% Paid, 10% Reduced		-6.401	-4.465
70% Paid, 20% Reduced		-5.834	-4.465
60% Paid, 30% Reduced		-5.266	-4.465
50% Paid, 40% Reduced		-4.699	-4.465

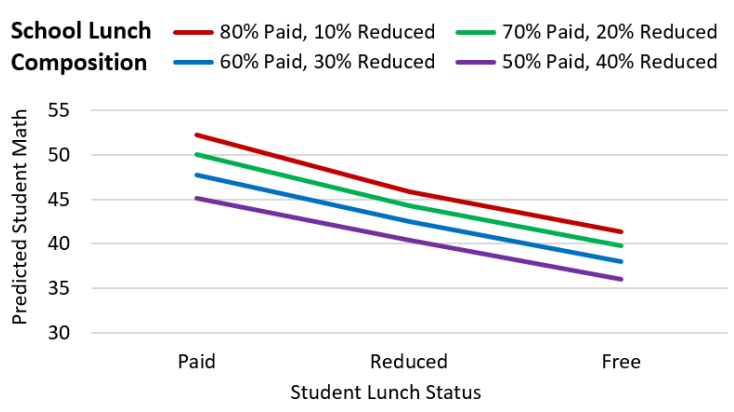


Figure 2: Contextual Effect of School Proportion of Students Receving Free Lunch (Rather than Paid)
CM_PvD = .2 .3 .4 .5 → CM_PvD30 = −.1 0 .1 .2
CM_RvF = .5(free−reduced), so CM_RvF = 0 .05 .10 .15 → CM_RvF05 = −.05 0 .05 .10

Holding Constant School Proportion Reduced Lunch at 10%:			
School Composition:	**Paid**	**Reduced**	**Free**
80% Paid, 10% Free	52.233	45.832	41.367
70% Paid, 20% Free	50.759	44.926	40.461
60% Paid, 30% Free	49.049	43.783	39.318
50% Paid, 40% Free	47.102	42.404	37.939
Disadvantages:		**PvR:**	**RvF:**
80% Paid, 10% Free		-6.401	-4.465
70% Paid, 20% Free		-5.834	-4.465
60% Paid, 30% Free		-5.266	-4.465
50% Paid, 40% Free		-4.699	-4.465

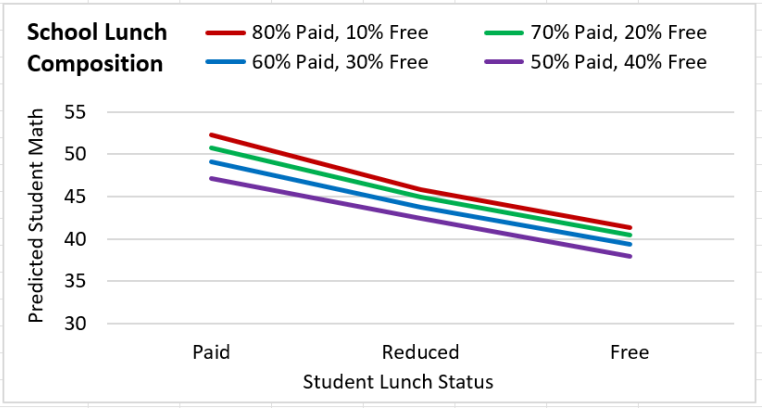
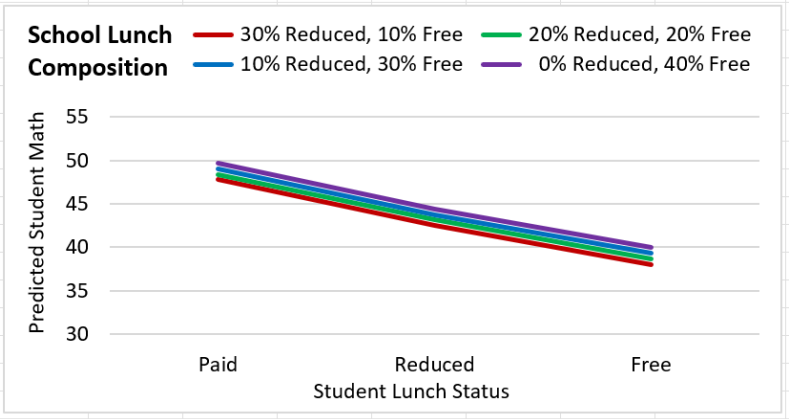


Figure 3 holds constant the proportion of students receiving paid lunch at .60. However, as shown by the lack of vertical distance between the lines, a nonsignificant tendency in the opposite direction was found for the proportion of students receiving free lunch instead of reduced lunch—a small positive and constant contextual effect instead (holding constant the proportion of students receiving paid lunch). Thus, at the school level, the proportion of students with paid lunch is the most salient distinction in predicting math outcomes.

Figure 3: Contextual Effect of School Proportion of Students Receving Free Lunch (Rather than Reduced)
CM_PvD = .4 → CM_PvD30 = .1
CM_RvF = .5(free−reduced), so CM_RvF = −.10 0 .10 .20 → CM_RvF05 = −.15 −.05 .05 .15

Holding Constant School Proportion Paid Lunch at 60%:			
School Composition:	**Paid**	**Reduced**	**Free**
30% Reduced, 10% Free	47.753	42.487	38.022
20% Reduced, 20% Free	48.401	43.135	38.670
10% Reduced, 30% Free	49.049	43.783	39.318
0% Reduced, 40% Free	49.696	44.430	39.965
Disadvantages:		**PvR:**	**RvF:**
30% Reduced, 10% Free		-5.266	-4.465
20% Reduced, 20% Free		-5.266	-4.465
10% Reduced, 30% Free		-5.266	-4.465
0% Reduced, 40% Free		-5.266	-4.465



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