Example 4a: Random Slopes and Cross-Level Interactions of Cluster-Mean-Centered Level-1 Predictors in General Multilevel Models for Two-Level Nested Outcomes (complete syntax, data, and output available for R electronically)

The data for this example come from the High School and Beyond dataset (HSB4) used in McNeish (2023) including 7,185 students from 160 schools. Following Examples 2 and 3, we will continue examining how student math can be predicted from student-level variables of SES and white versus nonwhite identity. This example demonstrates how to fit models with random slopes and cross-level interactions for cluster-mean-centered predictors. It uses several custom functions to process output and add computations, as shown below (all in the online files).

R Syntax for Packages; Importing and Preparing Data for Analysis

```
# Install and load required packages
if (!require(pacman)) install.packages("pacman")
pacman::p_load(readxl, TeachingDemos, nlme, lme4, lmerTest, prediction)
# readxl: import .xls or .xlsx files; TeachingDemos: save output to external file
# nlme: multilevel models with or without random effects; lme4: multilevel models
# lmerTest: DDF and p-values for lme4, also univ/Multiv Wald tests
# prediction: create predicted values using fixed effects for plotting

# Set working directory (to import and export files to)
setwd("C:/Dropbox/25_PSQF6272/PSQF6272_Example4a")

# Load R functions for this class from R file in either place
source("../EDF9850_Functions.R") # Remove ../ for same folder
source("https://www.lesahoffman.com/EDF9850/EDF9850_Functions.R") # From web

# Import Example 4a excel data file in sheet "HSB4"
Example4a = read_excel(path="Example4a_Data.xlsx", sheet="HSB4")
# Convert to data frame to use for analysis
Example4a = as.data.frame(Example4a)

# Filter to only cases complete on all variables used below (before cluster means)
Example4a = Example4a[complete.cases(Example4a[ , c("math","ses","nonwhite")]),]

# Add school means to student-level-1 data
Example4a = addUnitMeans(data=Example4a, unitVariable="schoolID",
                         meanVariables=c("math","ses","nonwhite"),
                         newNames=c("CMmath","CMses","CMnon"))

# Center L2 predictor at constant so 0 is meaningful
Example4a$CMnon30 = Example4a$CMnon - .30
# 0 is otherwise already meaningful in other predictors

# Center L1 predictors at cluster means
Example4a$WCses = Example4a$ses - Example4a$CMses
Example4a$WCnon = Example4a$nonwhite - Example4a$CMnon
```

Model 4c: Fixed Cluster-Mean-Centered Level-1 Slopes Only (Repeated from Example 3)

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + \beta_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20}$

```
print("Model 4c: Fixed Cluster-Mean-Centered Level-1 Slopes Only")
Model4c = lmer(data=Example4a, REML=TRUE,
               formula=math~1+WCses+WCnon+CMses+CMnon30+(1|schoolID))
lmerSummary(Model4c)

Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46404.554  46452.712 -23195.277  46390.554

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    2.559   1.600
 Residual              36.136   6.011
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model4c     4 292.047 218.054   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.642 0.147  152.081  86.087   <.001
 WCses       1.952 0.109 7020.170  17.931   <.001
 WCnon      -2.896 0.220 7020.170 -13.152   <.001
 CMses       5.327 0.403  150.020  13.230   <.001
 CMnon30    -1.544 0.552  148.270  -2.798   0.006
```

Try to label each row of the output with the corresponding symbol in the Model 4c equation.

Which fixed slopes directly adjust student predicted math?

Which fixed slopes indirectly adjust student predicted math through school math?

Model 5a: Add Random Slope for WCnon Only

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + \beta_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20} + U_{2c}$

How does the equation for Model 5a (+ random WCnon slope) differ from Model 4c (fixed WCnon slope)?

```
print("Model 5a: Add Random Slope for WCnon")
Model5a = lmer(data=Example4a, REML=TRUE,
               formula=math~1+WCses+WCnon+CMses+CMnon30+(1+WCnon|schoolID))
lmerSummary(Model5a, LRT=TRUE, randCI=TRUE, slopeRel=TRUE)
```

How does the lmer code for Model 5a (+ random WCnon slope) differ from Model 4c (fixed WCnon slope)?

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46396.177  46458.094 -23189.088  46378.177

Model for the Variance:
 Groups    Name      Variance Std.Dev  Corr
 schoolID  Intercept    2.574   1.605
           WCnon        2.427   1.558 0.367
 Residual              35.877   5.990
Number of obs: 7185 , groups: schoolID, 160
```

How does the interpretation of the random intercept variance change after adding a random WCnon slope?

```
Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model5a     4 177.402 192.164   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.632 0.147  151.957  85.913   <.001
 WCses       1.923 0.109 6985.711  17.631   <.001
 WCnon      -2.897 0.267  107.144 -10.839   <.001
 CMses       5.214 0.398  150.227  13.103   <.001
 CMnon30    -1.746 0.547  149.953  -3.189   0.002
```

Fixed Effects from Model 4c:

```
 Term          Est    SE
 Intercept  12.642 0.147
 WCses       1.952 0.109
 WCnon      -2.896 0.220
 CMses       5.327 0.403
 CMnon30    -1.544 0.552
```

```
LRT for random effect(s):
 Random Effect Dropped  Npar     LL  -2LL   AIC   BIC    LRT DF p.value p.mix
 <none>                    9 -23189 46378 46396 46458
 WCnon | schoolID          7 -23195 46391 46405 46453 12.378  2   0.002 0.001
```

What does it mean that the LRT for the new random slope variance is significant? Why does DF=2?

```
Random Slope Reliability (SR):
  SR = slope var / (slope var + res var / (mean L1n * var(X)))
 Term  SlopeVar ResVar Var.X    L1n    SR
 WCnon    2.427 35.877 0.107 44.906 0.245
```

What do you think a slope reliability of 0.245 implies?

```
Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term      FixedEffect Variance    SD  Lower  Upper
 Intercept      12.632    2.574 1.605  9.488 15.777
 WCnon          -2.897    2.427 1.558 -5.951  0.156
```

What does it mean that the WCnon random effect CI overlaps 0?

Model 5b: Add Cross-Level Interactions for WCnon

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + \beta_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20} + \gamma_{21}(\overline{SES}_c - 0) + \gamma_{22}(\overline{NW}_c - .30) + U_{2c}$

How does the equation for Model 5b (+ WCnon cross-level interactions) differ from Model 5a (random WCnon slope only)?

Composite equation written to replace the β placeholders with [ ]:

$$Math_{pc} = [\gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}] + e_{pc} + [\gamma_{10}](SES_{pc} - \overline{SES}_c) + [\gamma_{20} + \gamma_{21}(\overline{SES}_c - 0) + \gamma_{22}(\overline{NW}_c - .30) + U_{2c}](NW_{pc} - \overline{NW}_c)$$

Composite equation written to show interactions (with all fixed effects first by convention):

$$Math_{pc} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + \gamma_{10}(SES_{pc} - \overline{SES}_c) + \gamma_{20}(NW_{pc} - \overline{NW}_c) + \gamma_{21}(NW_{pc} - \overline{NW}_c)(\overline{SES}_c - 0) + \gamma_{22}(NW_{pc} - \overline{NW}_c)(\overline{NW}_c - .30) + U_{0c} + U_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}$$

```
print("Model 5b: Add Cross-Level Interactions for WCnon")
Model5b = lmer(data=Example4a, REML=TRUE,
               formula=math~1+WCses+WCnon+CMses+CMnon30
               +WCnon:CMnon30 +WCnon:CMses +(1+WCnon|schoolID))
lmerSummary(Model5b, LRT=TRUE)
```

How does the lmer code for Model 5b (+ WCnon cross-level interactions) differ from Model 5a (random WCnon slope only)?

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46389.795  46465.473 -23183.898  46367.795

Model for the Variance:
 Groups    Name      Variance Std.Dev  Corr
 schoolID  Intercept    2.572   1.604
           WCnon        2.002   1.415 0.304
 Residual              35.874   5.989
Number of obs: 7185 , groups: schoolID, 160
```

Which of the **three** variances should have been reduced by the WCnon cross-level interactions relative to Model 5a?

```
Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model5b     6 169.905 131.352   <.001
```

Does the F-test above tell us about the significance of the new WCnon cross-level interactions?

```
Fixed Effects:
 Term              Est    SE       DF t.value p.value
 Intercept      12.639 0.147  152.076  85.984   <.001
 WCses           1.929 0.109 6972.044  17.698   <.001
 WCnon          -3.018 0.266  118.357 -11.365   <.001
 CMses           5.329 0.403  150.030  13.222   <.001
 CMnon30        -1.542 0.553  148.301  -2.791   0.006
 WCnon:CMses     1.113 0.663   84.701   1.679   0.097
 WCnon:CMnon30   2.932 1.103  127.512   2.659   0.009
```

Interpret WCnon (γ_{20}):

Interpret WCnon:CMses (γ_{21}):

Interpret WCnon:CMnon30 (γ_{22}):

```
LRT for random effect(s):
 Random Effect Dropped  Npar     LL  -2LL   AIC   BIC   LRT DF p.value p.mix
 <none>                   11 -23184 46368 46390 46465
 WCnon | schoolID          9 -23188 46376 46394 46456 8.178  2   0.017 0.010
```

Now what does it mean that the LRT for the WCnon random slope variance is significant?

```
PseudoR2(baseModel=Model5a, altModel1=Model5b)
TotalR2(altModel1=Model4c, altModel2=Model5a, altModel3=Model5b)
```

```
Pseudo-R2
 Term       Model   Variance Pseudo.R2
 Intercept  Model5a    2.574
 Intercept  Model5b    2.572     0.001
 WCnon      Model5a    2.427
 WCnon      Model5b    2.002     0.175
 Residual   Model5a   35.877
 Residual   Model5b   35.874     0.000
```

Note that the baseline for computing pseudo-R^2 is the model in which the level-2 random WCnon slope was first added (by redistributing the level-1 residual variance). Consequently, if, like me, you don't believe that random slopes "explain" variance—and instead believe they form a new source of error that itself is to be predicted—then you should not compare back to the random intercept model (in which the variances do not have the same interpretation before adding the random slope as after).

```
Total-R2 and Change in Total-R2
 Model   Total.R2 Change.R2
 Model4c    0.184
 Model5a    0.184    -0.000
 Model5b    0.186     0.001
```

Why didn't Model 5a increase the total-R^2?

```
conMDSummary(Model5b, terms=c("WCnon:CMses","WCnon:CMnon30"))
```

```
Joint Test for: WCnon:CMses, WCnon:CMnon30
 Model   NumDF   DenDF F.value p.value
 Model5b     2 110.165   3.889   0.023
```

This new **conMDSummary** function uses contestMD from the lmerTest package to request a joint F-test for a set of slopes. This is conceptually equivalent to the F-test for the increase in R^2 for a set of predictors in GLMs. Here, we are testing whether the variance explained is > 0 in the WCnon random slope by adding the cross-level interactions—or alternatively, their change in the total-R^2.

Model 6a: Add Random Slope for WCses

Level 1: $$Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + \beta_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}$$

Level 2:
$$\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}$$
$$\beta_{1c} = \gamma_{10} + U_{1c}$$
$$\beta_{2c} = \gamma_{20} + \gamma_{21}(\overline{SES}_c - 0) + \gamma_{22}(\overline{NW}_c - .30) + U_{2c}$$

How does the equation for Model 6a (+ random WCses slope) differ from Model 5b (+ WCnon cross-level interactions)?

```
print("Model 6a: Add Random Slope for WCses")
Model6a = lmer(data=Example4a, REML=TRUE,
               formula=math~1+WCses+WCnon+CMses+CMnon30
               +WCnon:CMses +WCnon:CMnon30 +(1+WCnon+WCses|schoolID))
lmerSummary(Model6a, LRT=TRUE, randCI=TRUE, slopeRel=TRUE)
```

How does the lmer code for Model 6a (+ random WCses slope) differ from Model 5c (+ cross-level WCnon interactions)?

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46384.906  46481.223 -23178.453  46356.906

Model for the Variance:
 Groups    Name      Variance Std.Dev   Corr
 schoolID  Intercept    2.574   1.604
           WCnon        1.862   1.365  0.290
           WCses        0.436   0.660 -0.411 -0.824
 Residual              35.665   5.972
Number of obs: 7185 , groups: schoolID, 160
```

How does the interpretation of the random intercept variance further change after adding a random WCses slope?

```
Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model6a     6 138.358 111.332   <.001

Fixed Effects:
 Term             Est    SE      DF t.value p.value
 Intercept     12.626 0.147 151.359  85.942   <.001
 WCses          1.935 0.121 151.635  15.929   <.001
 WCnon         -3.003 0.264 116.935 -11.379   <.001
 CMses          5.259 0.398 148.987  13.206   <.001
```

```
 CMnon30        -1.954 0.545 146.397  -3.584   <.001
 WCnon:CMses     0.939 0.663  86.258   1.417   0.160
 WCnon:CMnon30   2.215 1.102 130.612   2.011   0.046

LRT for random effect(s):
 Random Effect Dropped  Npar     LL  -2LL   AIC   BIC    LRT DF p.value p.mix
 <none>                   14 -23178 46357 46385 46481
 WCnon | schoolID         11 -23183 46367 46389 46465 10.087  3   0.018 0.012
 WCses | schoolID         11 -23184 46368 46390 46465 10.889  3   0.012 0.008
```

What does it mean that the LRT for each random slope variance is significant? Why does DF=3?

```
Random Slope Reliability (SR):
  SR = slope var / (slope var + res var / (mean L1n * var(X)))
 Term  SlopeVar ResVar Var.X    L1n    SR
 WCnon    1.862 35.665 0.107 44.906 0.200
 WCses    0.436 35.665 0.436 44.906 0.193

Random Effect 95% CIs (Fixed Effect +/- 1.96*SQRT(random effect variance)):
 Term      FixedEffect Variance    SD  Lower  Upper
 Intercept      12.626    2.574 1.604  9.481 15.770
 WCnon          -3.003    1.862 1.365 -5.677 -0.328
 WCses           1.935    0.436 0.660  0.640  3.229
```

What is the conceptual difference between the two random slopes' LRT, reliability, and confidence intervals in what information they are conveying?

Model 6b: Add Cross-Level Interactions for WCses

Level 1: $\boldsymbol{Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + \beta_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}}$

Level 2:
$$\boldsymbol{\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}}$$
$$\boldsymbol{\beta_{1c} = \gamma_{10} + \gamma_{11}(\overline{SES}_c - 0) + \gamma_{12}(\overline{NW}_c - .30) + U_{1c}}$$
$$\boldsymbol{\beta_{2c} = \gamma_{20} + \gamma_{21}(\overline{SES}_c - 0) + \gamma_{22}(\overline{NW}_c - .30) + U_{2c}}$$

How does the equation for Model 6b (+ WCses cross-level interactions) differ from Model 6a (random WCses slope only)?

Composite equation written to replace the β placeholders with [ ]:

$$\begin{aligned}\boldsymbol{Math_{pc}} = &\boldsymbol{[\gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + U_{0c}] + e_{pc}} \\ &\boldsymbol{+ [\gamma_{10} + \gamma_{11}(\overline{SES}_c - 0) + \gamma_{12}(\overline{NW}_c - .30) + U_{1c}](SES_{pc} - \overline{SES}_c)} \\ &\boldsymbol{+ [\gamma_{20} + \gamma_{21}(\overline{SES}_c - 0) + \gamma_{22}(\overline{NW}_c - .30) + U_{2c}](NW_{pc} - \overline{NW}_c)}\end{aligned}$$

Composite equation written to show interactions (with all fixed effects first by convention):

$$\begin{aligned}\boldsymbol{Math_{pc}} = &\boldsymbol{\gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{NW}_c - .30) + \gamma_{10}(SES_{pc} - \overline{SES}_c)} \\ &\boldsymbol{+ \gamma_{11}(SES_{pc} - \overline{SES}_c)(\overline{SES}_c - 0) + \gamma_{12}(SES_{pc} - \overline{SES}_c)(\overline{NW}_c - .30)} \\ &\boldsymbol{+ \gamma_{20}(NW_{pc} - \overline{NW}_c) + \gamma_{21}(NW_{pc} - \overline{NW}_c)(\overline{SES}_c - 0)} \\ &\boldsymbol{+ \gamma_{22}(NW_{pc} - \overline{NW}_c)(\overline{NW}_c - .30)} \\ &\boldsymbol{+ U_{0c} + U_{1c}(SES_{pc} - \overline{SES}_c) + U_{2c}(NW_{pc} - \overline{NW}_c) + e_{pc}}\end{aligned}$$

```
print("Model 6b: Add Cross-Level Interactions for WCses")
Model6b = lmer(data=Example4a, REML=TRUE,
               formula=math~1+WCses+WCnon+CMses+CMnon30
               +WCnon:CMses +WCnon:CMnon30 +WCses:CMses +WCses:CMnon30
               +(1+WCnon+WCses|schoolID))
lmerSummary(Model6b, LRT=TRUE)
```

How does the lmer code for Model 6b (+ WCses cross-level interactions) differ from Model 6a (random WCses slope only)?

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46364.424  46474.500 -23166.212  46332.424

Model for the Variance:
 Groups    Name       Variance Std.Dev   Corr
 schoolID  Intercept     2.565   1.602
           WCnon         1.962   1.401  0.289
           WCses         0.195   0.442 -0.528 -0.965
 Residual               35.627   5.969
Number of obs: 7185 , groups: schoolID, 160
```

```
This model generated an error
message: boundary (singular) fit:
see help('isSingular')
```

This was likely due to the random slopes' correlation near −1. For details see here

Which of the **four** variances should have been reduced by the WCses cross-level interactions relative to Model 6a?

```
Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model6b     8 195.702  91.584   <.001
```

Does the F-test above tell us about the significance of the new WCses cross-level interactions?

```
Fixed Effects:
 Term              Est    SE      DF t.value p.value
 Intercept      12.637 0.147 152.205  86.128   <.001
 WCses           1.893 0.115 329.231  16.439   <.001
 WCnon          -3.022 0.265 127.826 -11.394   <.001
 CMses           5.332 0.402 150.166  13.252   <.001
 CMnon30        -1.543 0.552 148.443  -2.797   0.006
 WCnon:CMses     1.001 0.668  93.731   1.500   0.137
 WCnon:CMnon30   2.385 1.108 141.119   2.152   0.033
 WCses:CMses    -0.326 0.324 359.457  -1.007   0.314
 WCses:CMnon30  -2.079 0.423 285.383  -4.914   <.001
```

Interpret WCses (γ_{10}):

Interpret WCses:CMses (γ_{11}):

Interpret WCses:CMnon30 (γ_{12}):

```
LRT for random effect(s):
 Random Effect Dropped  Npar     LL  -2LL   AIC   BIC    LRT DF p.value p.mix
 <none>                   16 -23166 46332 46364 46474
 WCnon | schoolID         13 -23172 46344 46370 46459 11.264  3   0.010 0.007
 WCses | schoolID         13 -23170 46339 46365 46455  7.036  3   0.071 0.050
```

Now what does it mean that the LRT for the WCses random slope variance is marginally significant?

```
PseudoR2(baseModel=Model6a, altModel1=Model6b)
TotalR2(altModel1=Model4c, altModel2=Model5b, altModel3=Model6b)
```

```
Pseudo-R2
 Term       Model   Variance Pseudo.R2
 Intercept  Model6a    2.574
 Intercept  Model6b    2.565     0.003
 WCnon      Model6a    1.862
 WCnon      Model6b    1.962    -0.054
 WCses      Model6a    0.436
 WCses      Model6b    0.195     0.553
 Residual   Model6a   35.665
 Residual   Model6b   35.627     0.001

Total-R2 and Change in Total-R2
 Model   Total.R2 Change.R2
 Model4c    0.184
 Model5b    0.186     0.001
 Model6b    0.189     0.003
```

```
conMDSummary(Model6b, terms=c("WCses:CMses","WCses:CMnon30"))
```

```
Joint Test for: WCses:CMses, WCses:CMnon30
 Model   NumDF   DenDF F.value p.value
 Model6b     2 320.444  13.128   <.001
```

What does the F-test above indicate?

Sample Results Section starts here—see the last page and online files for figures that could be included!

[indicates notes about what to customize or also include; note that SE and p-values are not needed if you provide tables for the model solutions; these are wordy on purpose to help illustrate how to interpret things]

The extent to which student math outcomes could be predicted from student-level (and corresponding school-level) variables of socio-economic status (SES) and white versus nonwhite identity was examined in a series of multilevel models in which the 7,185 students were modeled as nested within their 160 schools. Restricted Maximum likelihood (REML) within the lmer function in the lme4 R package (v. 2.0.6; Bates et al., 2015) was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with univariate Wald tests using Satterthwaite denominator degrees of freedom. Alpha was chosen as .05. Model-implied fixed effects were requested via contest1D statements, and joint tests of multiple fixed slopes were requested via contestMD statements (v. 3.2.1; Kuznetsova et al., 2017). The significance of new random effects was evaluated using likelihood ratio tests (i.e., −2ΔLL = the difference in −2LL between nested models with the same fixed effects). The −2ΔLL test statistics were evaluated using χ^2 distributions with a 50–50 mixture of (DF; DF−1) given the 0 boundary of the new random effect variance. Effect size for the fixed effects was evaluated via pseduo-R^2 values for the proportion reduction in each variance component relative to a nested model without the predictors in question, as well as with total-R^2, the squared correlation between the actual math outcomes and those predicted by the model fixed effects. Random slope reliability (SR), as adapted from Willett (1989), was computed as SR = level-2 random variance / (level-2 random variance + [level-1 residual variance / {mean cluster size * var(predictor)}). For descriptive purposes, 95% random effect confidence intervals (CIs) were also computed as CI = fixed effect ± 1.96*SQRT(level-2 random variance).

[Example 3 results would go here]

{Model 5a} We then estimated a series of models to examine school-level differences in the within-school effects of cluster-mean-centered student nonwhite identify and student SES. First, a level-2 random variance for the student nonwhite slope (and its covariance with the school random intercept) significantly improved model fit, $-2\Delta LL(1;2) = 12.378$, $p = .001$, indicating significant school-to-school differences in the effect of within-school nonwhite identity. However, slope reliability = 0.245, indicating weak slope heterogeneity. A 95% random slope CI = −5.951 to 0.156 indicated that while most schools were expected to have a negative slope for within-school nonwhite identity, the slope was expected to be 0 or slightly positive for at least some schools.

{Model 5b} We then examined moderation of the within-school nonwhite identity slope by school mean SES and proportion nonwhite students. These two cross-level interactions explained 17.5% of the level-2 school random nonwhite slope variance (and an additional 0.1% of the total math variance), which was significant, $F(2, 100) = 3.889$, $p = .023$. The negative within-school nonwhite slope became nonsignificantly less negative by 1.113 per unit higher school mean SES and significantly less negative by 0.293 per 10% additional nonwhite students.

{Model 6a} Next, a level-2 random variance for the student SES slope (and its covariances with the school random intercept and nonwhite slope) significantly improved model fit, $-2\Delta LL(2;3) = 10.889$, $p = .008$, indicating significant school-to-school differences in the effect of within-school SES. However, slope reliability = 0.193, indicating weak slope heterogeneity. A 95% random slope CI = 0.640 to 3.229 indicated that most schools were expected to have a positive slope for within-school SES.

{Model 6b} Finally, we examined moderation of the within-school SES slope by school mean SES and proportion nonwhite students. These two cross-level interactions explained 55.3% of the random SES slope variance (and an additional 0.3% of the total math variance), which was significant, $F(2, 320) = 13.128$, $p < .001$. The positive within-school SES slope became nonsignificantly less positive by 0.326 per unit higher school mean SES and significantly less positive by 0.208 per 10% additional nonwhite students.

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software, 67*(1), 1–48. https://doi.org/10.18637/jss.v067.i01

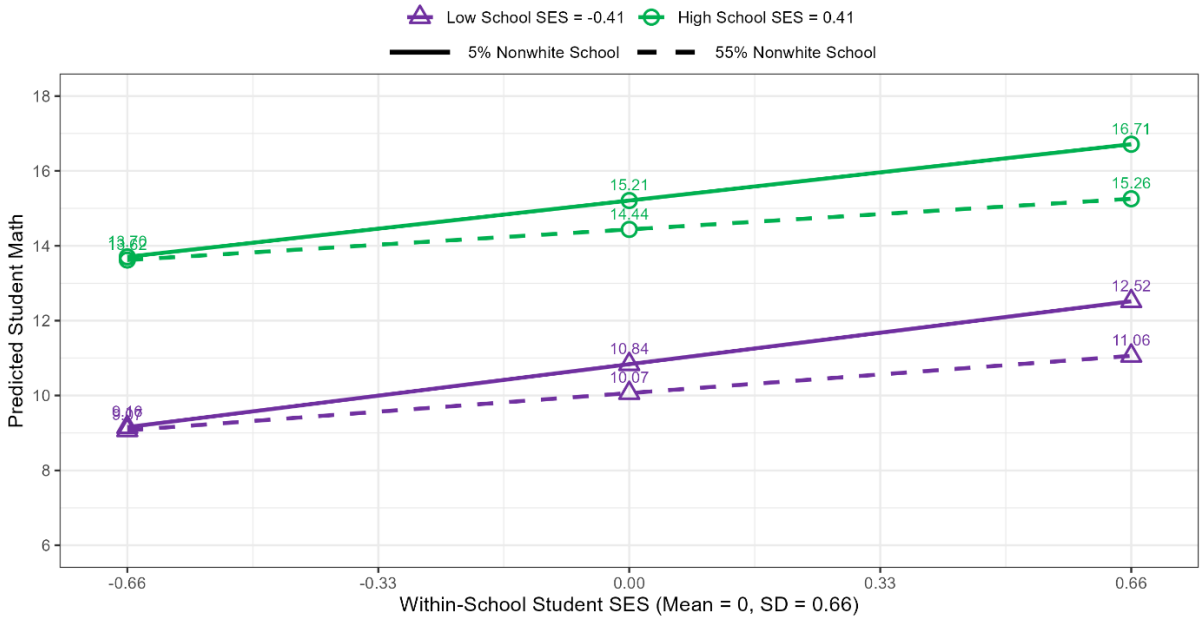
Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software, 82*(13), 1–26. https://doi.org/10.18637/jss.v082.i13

Willett, J.B. (1989). Some results on reliability for the longitudinal measurement of change: Implications for the design of studies of individual growth. *Educational and Psychological Measurement, 49*, 587–602. https://doi.org/10.1177%2F001316448904900309

Figures using Model 6c Results—see Example 4a files online for R syntax for generating predictions and making plots!

Figure 1 below shows between-school differences holding within-school student nonwhite = 0, in which the slope of the lines shows the positive within-school SES effect for each combination of schools. The significant conditional between-school main effects of SES (by color; more prominent) and % nonwhite (by line type; less prominent) are represented by vertical distances at within-school SES = 0. The significant cross-level interaction is shown by the shallower slopes for 55% vs 5% nonwhite (dashed vs solid), whereas the slopes for high vs low SES (green vs purple) are largely parallel (nonsignificant cross-level interaction). (See the online files for Figure 2, in which conditioning on original student SES = 0 would show the contextual main effect instead, which is likely to be confusing given the model is not phrased that way).

Figure 1: Within-School SES Slope by School SES and School %Nonwhite



Likewise, Figure 3 below shows between-school differences holding within-school SES = 0, in which the slope of the lines shows the negative within-school nonwhite identity effect for each combination of schools. The significant conditional between-school main effects of SES (by color; more prominent) and % nonwhite (by line type; less prominent) are represented by vertical distances at within-school nonwhite = 0, whereas the significant cross-level interaction is shown by the shallower slopes for 55% vs 5% nonwhite (dashed vs solid), whereas the slopes for high vs low SES (green vs purple) are largely parallel (nonsignificant cross-level interaction). (See the online files for Figure 4, in which conditioning on original student nonwhite = 0 would show the contextual main effect instead, which is likely to be confusing given the model is not phrased that way).

Figure 3: Within-School Nonwhite Slope by School SES and School %Nonwhite

