Example 3: Fixed Slopes of Level-1 Predictors in General Multilevel Models for Two-Level Nested Outcomes (complete syntax, data, and output available for R electronically)

The data for this example come from the High School and Beyond dataset (HSB4) used in McNeish (2023). Using 7,185 students from 160 schools, we will examine how student math can be predicted from student-level variables of SES and white versus nonwhite identity. This example uses several custom functions to process output and add computations, as shown below (all in the online files).

R Syntax for Packages; Importing and Preparing Data for Analysis

```
# Install and load required packages
if (!require(pacman)) install.packages("pacman")
pacman::p_load(readxl, TeachingDemos, nlme, lme4, lmerTest, performance)
# readxl: import .xls or .xlsx files; TeachingDemos: save output to external file
# nlme: multilevel models with or without random effects; lme4: multilevel models
# lmerTest: DDF and p-values for lme4, also univ/Multiv Wald tests; performance: ICC1

# Set working directory (to import and export files to)
setwd("C:/Dropbox/25_PSQF6272/PSQF6272_Example3")

# Load R functions for this class from R file in folder above working directory
source("../EDF9850_Functions.R")  # Remove ../ for same folder

# Import Example 3 excel data file in sheet "HSB4"
Example3 = read_excel(path="Example3_Data.xlsx", sheet="HSB4")
# Convert to data frame to use for analysis
Example3 = as.data.frame(Example3)
# Filter to only cases complete on all variables to be used below (before cluster means)
Example3 = Example3[complete.cases(Example3[ , c("math","ses","nonwhite")]),]

# Add school means to student-level-1 data
Example3 = addUnitMeans(data=Example3, unitVariable="schoolID",
                        meanVariables=c("math","ses","nonwhite"),
                        newNames=c("CMmath","CMses","CMnon"))

# Create school-level-2 data frame with one row per school
Example3Means = unique(Example3[ , c("schoolID","CMmath","CMses","CMnon")])

print("Descriptive Statistics in School-Level-2 Data")
univSummary(data=Example3Means, vars=c("CMmath","CMses","CMnon"))

Variable   N   Mean    SD Variance    Min    Max
 CMmath   160 12.621 3.118    9.720  4.240 19.719
 CMses    160 -0.006 0.414    0.171 -1.194  0.825
 CMnon    160  0.275 0.301    0.091  0.000  1.000

print("Correlations in School-Level-2 Data")
corrSummary(data=Example3Means, p.separate=FALSE, vars=c("CMmath","CMses","CMnon"))

Correlations (and p-values):
 Variable  CMmath   CMses   CMnon
 CMmath     1.000   0.785  -0.491
                  (<.001) (<.001)
 CMses      0.785   1.000  -0.483
          (<.001)         (<.001)
 CMnon     -0.491  -0.483   1.000
          (<.001) (<.001)
```

```
# Center L2 predictor at constant so 0 is meaningful
Example3$CMnon30 = Example3$CMnon - .30
# 0 is otherwise already meaningful in other predictors

# Center L1 predictors at cluster means
Example3$WCses = Example3$ses - Example3$CMses
Example3$WCnon = Example3$nonwhite - Example3$CMnon
# Center outcome at cluster mean for correlations
Example3$WCmath = Example3$math - Example3$CMmath

print("Descriptive Statistics in Student-Level-1 Data")
univSummary(data=Example3,
            vars=c("math","WCmath","CMmath","ses","WCses","CMses",
                   "nonwhite","WCnon","CMnon"))
```

```
Variable    N   Mean    SD Variance     Min    Max
 math    7185 12.748 6.878   47.310  -2.832 24.993
 WCmath  7185  0.000 6.187   38.275 -19.675 17.969
 CMmath  7185 12.748 3.006    9.035   4.240 19.719
 ses     7185  0.000 0.779    0.607  -3.758  2.692
 WCses   7185  0.000 0.661    0.436  -3.651  2.856
 CMses   7185  0.000 0.414    0.171  -1.194  0.825
 nonwhite 7185 0.275 0.446    0.199   0.000  1.000
 WCnon   7185  0.000 0.327    0.107  -0.982  0.982
 CMnon   7185  0.275 0.304    0.093   0.000  1.000
```

```
print("Correlations in Student-Level-1 Data")
corrSummary(data=Example3, p.separate=TRUE,
            vars=c("math","ses","nonwhite","WCmath","WCses","WCnon",
                   "CMmath","CMses","CMnon"))
```

	Original L1 Vars			Cluster-MC L1 Vars			Cluster Mean L2 Vars		
	math	**ses**	**nonwhite**	**WCmath**	**WCses**	**WCnon**	**CMmath**	**CMses**	**CMnon**
math	1								
ses	0.36	1							
nonwhite	-0.27	-0.27	1						
WCmath	0.90	0.20	-0.14	1					
WCses	0.21	0.85	-0.12	0.23	1				
WCnon	-0.17	-0.14	0.73	-0.19	-0.17	1			
CMmath	0.44	0.42	-0.33	0	0	0	1		
CMses	0.34	0.53	-0.32	0	0	0	0.79	1	
CMnon	-0.21	-0.25	0.68	0	0	0	-0.49	-0.46	1

Below, notice three different answers for what the correlations are among math, SES, and nonwhite. Those from the "Cluster-MC L1 Vars" section reflect purely level-1 within-school (or between-student) relations, and those from the "Cluster Mean L2 Vars" section reflect purely level-2 between-school relations. Thus, those from the "Original L1 vars" section reflect smushed relations—a weighted combination of the within-level-1 and between-level-2 relations (but are more heavily weighted towards within-level-1 given its larger sample size).

Model 1: Empty Means, Random Intercept Model for the Math Outcome

Level 1: $Math_{pc} = \beta_{0c} + e_{pc}$
Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

The fixed intercept is the mean of the school means, which will differ from the overall mean (as is given without a random intercept variance) if clusters are of different sizes (i.e., are "unbalanced").

```
print("Model 1: Empty Means, Random Intercept for Math")
Model1 = lmer(data=Example3, REML=TRUE, formula=math~1+(1|schoolID))
lmerSummary(Model1, CI=TRUE, ICC1=TRUE, LRT=TRUE, randCI=TRUE)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 47122.793  47143.433 -23558.397  47116.793

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    8.614   2.935
 Residual              39.148   6.257
Number of obs: 7185 , groups: schoolID, 160

Fixed Effects (with 95% CI):
 Term          Est    SE      DF t.value p.value  Lower  Upper
 Intercept  12.637 0.244 156.647  51.707   <.001 12.154 13.120

ICC1 intraclass correlation:
    Adjusted ICC: 0.180
  Unadjusted ICC: 0.180

LRT for random effect(s):
               Npar     LL  -2LL   AIC   BIC     LRT DF p.value p.mix
<none>            3 -23558 47117 47123 47143
(1 | schoolID)    2 -24051 48103 48107 48121 986.124  1   <.001 <.001

Random Effect 95% CIs (Fixed Effect +/- 1.96*SD(random effect variance)):
 Term      FixedEffect Variance    SD Lower  Upper
 Intercept      12.637    8.614 2.935 6.884 18.390
```

Empty Means Models for the Level-1 Student Predictors: Quantitative SES and Binary Nonwhite

```
print("Empty Means, Random Intercept for SES")
EmptySES = lmer(data=Example3, REML=TRUE, formula=ses~1+(1|schoolID))
lmerSummary(EmptySES, CI=TRUE, ICC1=TRUE, LRT=TRUE, randCI=TRUE)
```

```
Model Fit for Predicting ses:
      AIC       BIC    logLik -2*logLik
15052.016 15072.655 -7523.008 15046.016

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    0.161   0.401
 Residual              0.446   0.668
Number of obs: 7185 , groups: schoolID, 160

Fixed Effects (with 95% CI):
 Term          Est    SE      DF t.value p.value  Lower Upper
 Intercept  -0.006 0.033 159.187  -0.173   0.863 -0.070 0.059
```

```
ICC1 intraclass correlation:
    Adjusted ICC: 0.265
  Unadjusted ICC: 0.265

LRT for random effect(s):
               Npar    LL  -2LL   AIC   BIC      LRT DF p.value p.mix
<none>            3 -7523 15046 15052 15073
(1 | schoolID)    2 -8407 16814 16818 16832 1768.398  1   <.001 <.001

Random Effect 95% CIs (Fixed Effect +/- 1.96*SD(random effect variance)):
 Term      FixedEffect Variance    SD  Lower Upper
 Intercept      -0.006    0.161 0.401 -0.792 0.780
```

For a binary variable (as predicted as an outcome below), we need to add a **logit** (i.e., **log-odds**) link function to keep predicted probabilities in bounds, as well as switch from a normal to Bernoulli conditional distribution at level 1 (the random effects always have a multivariate normal distribution at level 2, even in generalized multilevel models). Thus, an empty means, random intercept model equation for the **binary nonwhite variable** is as follows:

Level 1: $\log\left[\frac{\text{probability}(Nonwhite_{pc}=1)}{\text{probability}(Nonwhite_{pc}=0)}\right] = \beta_{0c}$

Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

Note that level-1 e_{pc} is not included because there is no separately estimated residual variance in the Bernoulli distribution—the variance is nonconstant and determined by the predicted probability instead.

```
print("Empty Means, Random Intercept for Binary Nonwhite")
print("Logit link, bernoulli level-1 distribution, 15 quadrature points")
EmptyNon = glmer(data=Example3, family=binomial(link="logit"), nAGQ=15,
                 nonwhite~1+(1|schoolID))
glmerSummary(EmptyNon, ICC1=TRUE, LRT=TRUE)
```

```
Model Fit for Predicting nonwhite:
      AIC       BIC    logLik -2*logLik
 5486.867  5500.626 -2741.433  5482.867

Model for the Variance:
 Groups    Name             Variance Std.Dev
 schoolID  Intercept           5.467   2.338
 Residual  (logit constant)  (3.290) (1.814)
Number of obs: 7185 , groups: schoolID, 160

Fixed Effects:
 Term          Est    SE z.value p.value
 Intercept  -1.620 0.195  -8.306   <.001

ICC1 intraclass correlation:
    Adjusted ICC: 0.624
  Unadjusted ICC: 0.624

LRTs for Random Effects (each removed one at a time):
               Model Npar    LL -2LL  AIC  BIC      LRT DF p.value p.mix
              <none>    2 -2741 5483 5487 5501
 1 in (1 | schoolID)    1 -4224 8448 8450 8457 2965.481  1   <.001 <.001
```

Btw, here are the results with only 1 quadrature point (Laplace approximation, as used by default)

```
Model for the Variance:
 Groups    Name             Variance Std.Dev
 schoolID  Intercept           5.322   2.307
 Residual  (logit constant)  (3.290) (1.814)
Number of obs: 7185 , groups: schoolID, 160

Fixed Effects:
 Term          Est    SE z.value p.value
 Intercept  -1.617 0.190  -8.496   <.001

ICC1 intraclass correlation:
    Adjusted ICC: 0.618
  Unadjusted ICC: 0.618
```

Model 2a: Add Level-1 Cluster-Mean-Centered Student SES

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

$\beta_{1c} = \gamma_{10}$

Cluster-mean-centering removes all L2 mean differences from the L1 predictor, such that **WCses** contains within-school person-to-person variation only.

```
print("Model 2a: Add L1 Cluster-MC Student SES")
Model2a = lmer(data=Example3, REML=TRUE, formula=math~1+WCses+(1|schoolID))
lmerSummary(Model2a)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46731.996  46759.515 -23361.998  46723.996

Model for the Variance:
 Groups    Name       Variance Std.Dev
 schoolID  Intercept    8.672   2.945
 Residual              37.010   6.084
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF    DenDF F.value p.value
 Model2a     1 7022.024 406.684   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.636 0.244  156.741  51.684   <.001
 WCses       2.191 0.109 7022.024  20.166   <.001
```

Intercept γ_{00} =

WCses γ_{10} =

```
PseudoR2(baseModel=Model1, altModel1=Model2a)
```

```
Pseudo-R2
 Term       Model    Variance Pseudo.R2
 Intercept  Model1      8.614
 Intercept  Model2a     8.672    -0.007
 Residual   Model1     39.148
 Residual   Model2a    37.010     0.055
```

```
TotalR2(altModel1=Model2a)
```

```
Total R2
 Model   Total.R2
 Model2a    0.044
```

Pseudo-R^2 Results: The fixed slope of L1 **WCses** accounted for 5.5% of the L1 residual variance and NONE of the L2 random intercept variance—this is because **WCses** contains only L1 info! Consequently, the reduction in L1 σ_e^2 made L2 $\tau_{U_0}^2$ increase (= negative pseudo-$R_{U_0}^2$).

Total-R^2 Results: The fixed slope of L1 **WCses** accounted for 4.4% of the TOTAL variance in math across levels.

Model 2b: Add Level-2 Cluster Mean SES to Level-1 Cluster-Mean-Centered Student SES

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES}_c) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - \mathbf{0}) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

Because SES already had a mean ~0, the level-2 cluster mean SES predictor is left uncentered (i.e., centered at a constant of 0).

```
print("Model 2b: Add L2 Cluster Mean of Student SES")
Model2b = lmer(data=Example3, REML=TRUE, formula=math~1+WCses+CMses+(1|schoolID))
```

```
lmerSummary(Model2b)
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46578.584  46612.983 -23284.292  46568.584

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    2.693   1.641
 Residual              37.019   6.084
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model2b     2 298.343 334.812   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.683 0.149  153.652  84.906   <.001
 WCses       2.191 0.109 7021.509  20.164   <.001
 CMses       5.866 0.362  153.367  16.218   <.001
```

Intercept γ_{00} =

WCses γ_{10} =

CMses γ_{01} =

```
con1DSummary(contest1D(Model2b, L=c(0,-1,1)), name="L2 Contextual SES")

Fixed Effect: L2 Contextual SES
   Est    SE      DF t.value p.value
 3.675 0.378 182.286   9.731   <.001
```

```
PseudoR2(baseModel=Model1, altModel1=Model2a, altModel2=Model2b)

Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model1     8.614
 Intercept  Model2a    8.672    -0.007
 Intercept  Model2b    2.693     0.687     0.694
 Residual   Model1    39.148
 Residual   Model2a   37.010     0.055
 Residual   Model2b   37.019     0.054    -0.000
```

```
TotalR2(altModel1=Model2a, altModel2=Model2b)

Total-R2 and Change in Total-R2
 Model   Total.R2 Change.R2
 Model2a    0.044
 Model2b    0.162     0.118
```

Pseudo-R^2 Results: The fixed slope of L2 **CMses** accounted for 68.7% of the L2 random intercept variance (as pseudo-R^2)—this is because **CMses** contains only L2 info!

Total-R^2 Results: The fixed slope of L2 **CMses** accounted for 11.8% of the TOTAL variance in math across levels (as change in R^2).

Model 3a: What If We Had Just Used Grand-Mean-Centered Level-1 Student SES?

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - 0) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + U_{0c}$

$\beta_{1c} = \gamma_{10}$

The L1 predictor (centered at 0) still includes both L1 within-school and L2 between-school variance…

```
print("Model 3a: Use Smushed Main Effect of L1 Student SES Instead")
Model3a = lmer(data=Example3, REML=TRUE, formula=math~1+ses+(1|schoolID))
lmerSummary(Model3a)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46653.169  46680.688 -23322.585  46645.169

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    4.768   2.184
 Residual              37.034   6.086
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF    DenDF F.value p.value
 Model3a     1 6838.078 511.164   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.657 0.188  148.302  67.332   <.001
 ses         2.390 0.106 6838.078  22.609   <.001
```

Fixed effects from Model 2b:

```
Intercept 12.683
WCses      2.191
CMses      5.866
```

Intercept γ_{00} =

ses γ_{10} =

```
# Pseudo-R2 relative to empty model using custom function
PseudoR2(data=Example3, baseModel=Model1,
         model1=Model3a, name1="L1 Smushed SES")

PseudoR2(baseModel=Model1, altModel1=Model3a)
```

```
Pseudo-R2
 Term       Model    Variance Pseudo.R2
 Intercept  Model1      8.614
 Intercept  Model3a     4.768     0.446
 Residual   Model1     39.148
 Residual   Model3a    37.034     0.054

TotalR2(altModel1=Model3a)

Total R2
 Model   Total.R2
 Model3a    0.130
```

Previous results from Model 2b with WCses instead:

```
 Term       Model    Variance Pseudo.R2 Change.R2
 Intercept  Model1      8.614
 Intercept  Model2a     8.672    -0.007
 Intercept  Model2b     2.693     0.687     0.694
 Residual   Model1     39.148
 Residual   Model2a    37.010     0.055
 Residual   Model2b    37.019     0.054    -0.000

 Model   Total.R2
 Model2a    0.044
```

Pseudo-R^2 Results: The fixed slope of our L1 **`ses`** predictor accounted for **5.40%** of the L1 residual variance (which was **5.44%** for **`WCses`** before). However, L1 **`ses`** also accounted for **44.7%** of the L2 random intercept variance—this is because L1 SES still contains L2 info! But this L2 model assumes no contextual effect—that the WC and BC slopes are equal—and so it does not account for the correct amount of L2 variance: **69.4%**.

Model 3b: Let's Fix It—Add Level-2 Cluster Mean SES to Grand-Mean-Centered Level-1 Student SES

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - 0) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

The L1 predictor (centered at 0) still includes both L1 within-school and L2 between-school variance… but now we've controlled for the L2 between-school variance explicitly (i.e., **the model is unsmushed**)!

```
print("Model 3b: L1 Grand-MC Student SES + L2 Cluster Mean SES")
Model3b = lmer(data=Example3, REML=TRUE, formula=math~1+ses+CMses+(1|schoolID))
lmerSummary(Model3b)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46578.584  46612.983 -23284.292  46568.584

Model for the Variance:
 Groups    Name       Variance Std.Dev
 schoolID  Intercept    2.693   1.641
 Residual              37.019   6.084
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model3b     2 362.097 334.812   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.683 0.149  153.652  84.906   <.001
 ses         2.191 0.109 7021.509  20.164   <.001
 CMses       3.675 0.378  182.286   9.731   <.001
```

Fixed effects from equivalent model 2b:

```
Intercept 12.683
WCses      2.191
CMses      5.866
```

Intercept γ_{00} =

ses γ_{10} =

CMses γ_{01} =

```
con1DSummary(contest1D(Model3b, L=c(0,1,1)), name="L2 Between")
```

```
Fixed Effect: L2 Between SES
   Est    SE      DF t.value p.value
 5.866 0.362 153.367  16.218   <.001
```

```
PseudoR2(baseModel=Model1, altModel1=Model3a, altModel2=Model3b)
```

```
Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model1     8.614
 Intercept  Model3a    4.768     0.446
 Intercept  Model3b    2.693     0.687     0.241
 Residual   Model1    39.148
 Residual   Model3a   37.034     0.054
 Residual   Model3b   37.019     0.054     0.000
```

The L2 contextual slope accounted for another 24.1% of the L2 random intercept variance—model 3b is fully equivalent to model 2b that used cluster-mean-centered L1 SES instead.

```
TotalR2(altModel1=Model3a, altModel2=Model3b)
```

```
Total-R2 and Change in Total-R2
 Model   Total.R2 Change.R2
 Model3a    0.130
 Model3b    0.162     0.032
```

Model 4a: Add Binary Student Nonwhite Predictor (to Model 3b)

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - 0) + \beta_{2c}(Nonwhite_{pc}) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20}$

```
print("Model 4a: Add L1 Binary Student Nonwhite")
Model4a = lmer(data=Example3, REML=TRUE,
formula=math~1+ses+CMses+nonwhite+(1|schoolID))
lmerSummary(Model4a)
```

nonwhite: 0=white, 1=nonwhite

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46408.480  46449.759 -23198.240  46396.480

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    2.638   1.624
 Residual              36.139   6.012
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model4a     3 542.008 286.922   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  13.424 0.158  191.567  84.955   <.001
 ses         1.967 0.109 7063.565  18.102   <.001
 CMses       2.953 0.378  186.919   7.822   <.001
 nonwhite   -2.714 0.205 4400.818 -13.249   <.001
```

```
PseudoR2(baseModel=Model1, altModel1=Model3b, altModel2=Model4a)
```

```
Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model1     8.614
 Intercept  Model3b    2.693     0.687
 Intercept  Model4a    2.638     0.694     0.006
 Residual   Model1    39.148
 Residual   Model3b   37.019     0.054
 Residual   Model4a   36.139     0.077     0.022
```

Intercept γ_{00} =

ses γ_{10} =

CMses γ_{01} =

nonwhite γ_{20} =

Model 4b: Let's Fix It—Add Level-2 Cluster Mean of Nonwhite to Grand-Mean-Centered Level-1 Student Nonwhite

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - 0) + \beta_{2c}(Nonwhite_{pc}) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES}_c - 0) + \gamma_{02}(\overline{Nonwhite}_c - .30) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20}$

The L2 Nonwhite predictor was centered at .30 to prevent weirdness in interpreting its L1 slope (which would be impossible at `CMnon` = 0).

```
print("Model 4b: L1 Binary Student Nonwhite + L2 Cluster Mean Nonwhite")
Model4b = lmer(data=Example3, REML=TRUE,
               formula=math~1+ses+CMses+nonwhite+CMnon30+(1|schoolID))
lmerSummary(Model4b)
```

```
Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46404.554  46452.712 -23195.277  46390.554

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    2.559   1.600
 Residual              36.136   6.011
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model4b     4 399.685 218.054   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  13.511 0.161  219.691  83.906   <.001
 ses         1.952 0.109 7020.170  17.931   <.001
 CMses       3.375 0.417  172.761   8.090   <.001
 nonwhite   -2.896 0.220 7020.170 -13.152   <.001
 CMnon30     1.351 0.594  199.136   2.274   0.024
```

Intercept γ_{00} =

ses γ_{10} =

CMses γ_{01} =

nonwhite γ_{20} =

CMnon30 γ_{02} =

```
print("Computing between = within + contextual")
con1DTable(Model4b,
           L=list("SES L2 Between"      =c(0,1,1,0,0),  # gamma10 + gamma01
                  "Nonwhite L2 Between" =c(0,0,0,1,1))) # gamma20 + gamma02
```

```
Requested Fixed Effects:
 Name                  Est    SE      DF t.value p.value
 SES L2 Between      5.327 0.403 150.020  13.230   <.001
 Nonwhite L2 Between -1.544 0.552 148.270  -2.798   0.006
```

```
PseudoR2(baseModel=Model1, altModel1=Model4a, altModel2=Model4b)
Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model1     8.614
 Intercept  Model4a    2.638     0.694
 Intercept  Model4b    2.559     0.703     0.009
 Residual   Model1    39.148
 Residual   Model4a   36.139     0.077
 Residual   Model4b   36.136     0.077     0.000
```

Note the small increase in pseudo-R^2 for the random intercept after adding a contextual effect.

Model 4c: Switch to Cluster-Mean-Centered Versions of Level-1 Predictors Instead

Level 1: $Math_{pc} = \beta_{0c} + \beta_{1c}(SES_{pc} - \overline{SES_c}) + \beta_{2c}(Nonwhite_{pc} - \overline{Nonwhite_c}) + e_{pc}$

Level 2: $\beta_{0c} = \gamma_{00} + \gamma_{01}(\overline{SES_c} - 0) + \gamma_{02}(\overline{Nonwhite_c} - .30) + U_{0c}$

$\beta_{1c} = \gamma_{10}$

$\beta_{2c} = \gamma_{20}$

```
print("Model 4c: Use Cluster-MC Versions of L1 Predictors Instead")
Model4c = lmer(data=Example3, REML=TRUE,
               formula=math~1+WCses+CMses+WCnon+CMnon30+(1|schoolID))
lmerSummary(Model4c)

Model Fit for Predicting math:
       AIC        BIC     logLik  -2*logLik
 46404.554  46452.712 -23195.277  46390.554

Model for the Variance:
 Groups    Name      Variance Std.Dev
 schoolID  Intercept    2.559   1.600
 Residual              36.136   6.011
Number of obs: 7185 , groups: schoolID, 160

Overall F-test for All Fixed Slopes:
 Model   NumDF   DenDF F.value p.value
 Model4c     4 292.047 218.054   <.001

Fixed Effects:
 Term          Est    SE       DF t.value p.value
 Intercept  12.642 0.147  152.081  86.087   <.001
 WCses       1.952 0.109 7020.170  17.931   <.001
 CMses       5.327 0.403  150.020  13.230   <.001
 WCnon      -2.896 0.220 7020.170 -13.152   <.001
 CMnon30    -1.544 0.552  148.270  -2.798   0.006
```

Fixed effects from equivalent Model 4b:

```
Intercept 13.511
ses        1.952
CMses      3.375
nonwhite  -2.896
CMnon30    1.351
```

Which slopes changed relative to model 4b, and why?

```
print("Computing contextual = between - within")
con1DTable(Model4c,
           L=list("SES L2 Contextual"              =c(0,-1, 1,    0,  0),
                  "Nonwhite L2 Contextual"         =c(0, 0, 0,   -1,  1),
                  "Nonwhite L2 Between per 10%"    =c(0, 0, 0,    0,1/10),
                  "Nonwhite L2 Contextual per 10%" =c(0, 0, 0,-1/10,1/10)))

Requested Fixed Effects:
 Name                            Est    SE      DF t.value p.value
 SES L2 Contextual             3.375 0.417 172.761   8.090   <.001
 Nonwhite L2 Contextual        1.351 0.594 199.136   2.274   0.024
 Nonwhite L2 Between per 10%  -0.154 0.055 148.270  -2.798   0.006
 Nonwhite L2 Contextual per 10%  0.135 0.059 199.136   2.274   0.024
```

```
PseudoR2(baseModel=Model1, altModel1=Model2b, altModel2=Model3b, altModel3=Model4c)
```

```
Pseudo-R2 and Change in Pseudo-R2
 Term       Model   Variance Pseudo.R2 Change.R2
 Intercept  Model1     8.614
 Intercept  Model2b    2.693     0.687
 Intercept  Model3b    2.693     0.687    -0.000
 Intercept  Model4c    2.559     0.703     0.016
 Residual   Model1    39.148
 Residual   Model2b   37.019     0.054
 Residual   Model3b   37.019     0.054     0.000
 Residual   Model4c   36.136     0.077     0.023
```

```
TotalR2(altModel1=Model2b, altModel2=Model3b, altModel3=Model4c)
```

```
Total-R2 and Change in Total-R2
 Model   Total.R2 Change.R2
 Model2b    0.162
 Model3b    0.162    -0.000
 Model4c    0.184     0.022
```

Total-R^2 Results: The two fixed slopes for SES accounted for 16.24% of the total math variance. The two fixed slopes for nonwhite accounted for an additional 2.2% of the total math variance (up to total-R^2 = .184).

Sample Results Section starts here—see last two pages for figures that could be included!

[indicates notes about what to customize or also include; note that SE and p-values are not needed if you provide tables for the model solutions; these are wordy on purpose to help illustrate how to interpret things]

Note that the smushed results are not reported, and two versions of the results are given based on type of level-1 centering {so not all models are reported, but I've labeled the ones that are}…

The extent to which student math outcomes could be predicted from student-level (and corresponding school-level) variables of socio-economic status (SES) and white versus nonwhite identity was examined in a series of multilevel models in which the 7,185 students were modeled as nested within their 160 schools. Restricted Maximum likelihood (REML) within the lmer function in the lme4 R package (v. 2.0.6; Bates et al., 2015) was used in estimating and reporting all model parameters. The significance of fixed effects was evaluated with univariate Wald tests using Satterthwaite denominator degrees of freedom. Alpha was chosen as .01. Model-implied fixed effects were requested via contest1D statements (v. 3.2.1; Kuznetsova et al., 2017). Effect size for the fixed effects was evaluated via pseduo-R^2 values for the proportion reduction in each variance component relative to a nested model without the predictors in question, as well as with total-R^2, the squared correlation between the actual math outcomes and those predicted by the model fixed effects.

As derived from an empty means, random intercept model, student math had an intraclass correlation of ICC_1 = .180, indicating that 18.0% of the variance in student math was between schools, a significant amount, $-2\Delta LL(1)$ = 986.12, $p < .0001$. Likewise, student SES had an intraclass correlation of ICC_1 = .265, which was significantly greater than 0, $-2\Delta LL(1)$ = 1,768.40, $p < .001$. For the binary student nonwhite identity variable, a logistic two-level model (i.e., with a logit link function and a Bernoulli level-1 conditional distribution) was estimated instead. Using $\pi^2 / 3$ for the model-scale residual variance, the ICC_1 = .624, which was also significantly greater than 0, $-2\Delta LL(1)$ = 2,965.48, $p < .001$. Consequently, cluster-mean-centering (i.e., school-mean-centering here) was used to isolate the level-1 student variability in each predictor, whereas the cluster mean was used to represent the level-2 school variability in each predictor. The school mean of SES was left uncentered given its mean near 0, whereas the cluster mean of nonwhite identity (i.e., the proportion of students who identified as nonwhite at each school) was centered at .30 to facilitate interpretation of the other fixed effects.

Models with Cluster-Mean-Centered Student-Level-1 Predictors

{Model 2b} We first examined the effects of level-1 student SES and level-2 school SES, which accounted for 5.4% of the level-1 residual variance in math and 68.7% of the level-2 random intercept variance in math, respectively (total-R^2 = .162). The fixed intercept was 12.683 (SE = 0.149), which represented the expected math outcome for a student with average SES for their school from a school whose average SES = 0 (near the sample mean). At level 1, the within-school slope for student SES was significantly positive, indicating that student math was expected to be higher by 2.191 (SE = 0.109, p < .001) per additional unit of SES among students from the same school. At level 2, the between-school slope for school mean SES was also significantly positive, indicating that school mean math was expected to be higher by 5.866 (SE = 0.362, p < .001) per additional unit of school mean SES. The level-2 between-school slope was significantly larger (more positive) than the level-1 within-school slope, as indicated by the level-2 contextual slope for their difference (Est = 3.675). Said differently, the level-2 contextual slope indicated that, after controlling for the effect of student SES, school mean math was expected to be higher by 3.675 (SE = 0.378, p < .001) per additional unit of school mean SES.

{Model 4c} We then added the effects of level-1 student nonwhite identity and level-2 school proportion nonwhite identity (i.e., the proportion of of students who identified as nonwhite at each school, centered at .30), which accounted for another 2.3% of the level-1 residual variance in math and another 1.6% of the level-2 random intercept variance in math, respectively (change in total-R^2 = .022). The fixed intercept was 12.642 (SE = 0.149), which represented the expected math outcome for a student with average SES for their school, and an average proportion of nonwhite students for their school, who attended a school whose average SES = 0 (near the sample mean) and with 30% non-white-identifying students (near the sample mean). The effects of SES remained significant with patterns as previously described, and so we focus on the new effects of nonwhite identity. At level 1, the within-school slope for student nonwhite identity was significantly negative, indicating that student math was expected to be lower by 2.896 (SE = 0.220, p < .001) for students identifying as nonwhite relative to white. At level 2, the between-school slope for school mean nonwhite identity was also significantly negative (i.e., Est = −1.544 for the difference between proportion = 0 or 1), indicating that school mean math was expected to be lower by 0.154 (SE = 0.055, p = .006) for each additional 10% of students identifying as nonwhite. The level-2 between-school slope was nonsignificantly weaker (less negative) than the level-1 within-school slope, as indicated by the level-2 contextual-school slope for their difference (Est = 1.351). Said differently, the level-2 contextual-school slope indicated that, after controlling for the effect of student nonwhite identity, school mean math was expected to be nonsignificantly higher by 0.135 (SE = 0.059, p = .024) for each additional 10% of students identifying as nonwhite.

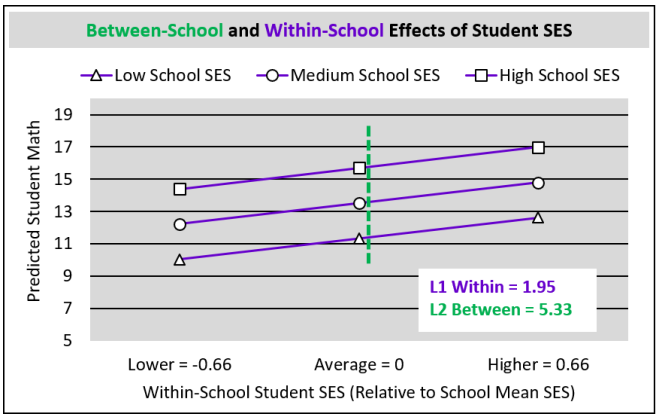
Models with Constant-Centered Student-Level-1 Predictors [remove references to cluster-MC above]

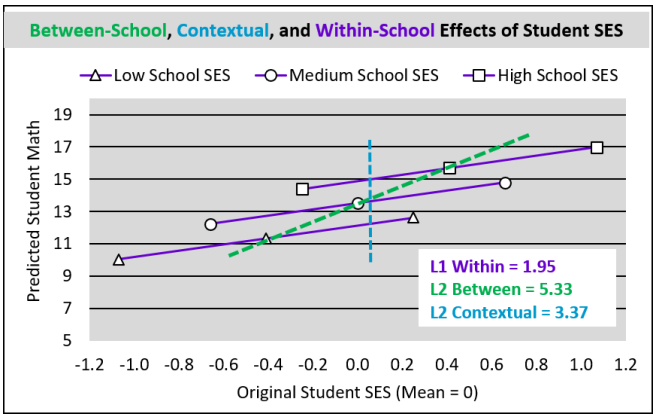
{Model 3b} We first examined the effects of level-1 student SES and level-2 school SES, which accounted for 5.4% of the level-1 residual variance in math and 68.7% of the level-2 random intercept variance in math, respectively (total-R^2 = .162). The fixed intercept was 12.683 (SE = 0.149), which represented the expected math outcome for a student with SES = 0 from a school whose average SES = 0 (near the mean of the sample). At level 1, the within-school slope for student SES was significantly positive, indicating that student math was expected to be higher by 2.191 (SE = 0.109, p < .001) per additional unit of SES among students from the same school. At level 2, the contextual-school slope indicated that, after controlling for the effect of student SES, school mean math was expected to be higher by 3.675 (SE = 0.378, p < .001) per additional unit of school mean SES. Said differently, the level-2 contextual-school slope also indicated that the level-2 between-school slope for school mean SES was also significantly positive, indicating that school mean math was expected to be higher by 5.866 (SE = 0.362, p < .0001) per additional unit of school mean SES, without controlling for level-1 student SES.

{Model 4b} We then added the effects of level-1 student nonwhite identity and level-2 school proportion nonwhite identity (i.e., the proportion of of students who identified as nonwhite at each school, centered at .30), which accounted for another 2.3% of the level-1 residual variance in math and another 1.6% of the level-2 random intercept variance in math, respectively (change in total-R^2 = .022). The fixed intercept was 13.511 (SE = 0.161), which represented the expected math outcome for a student with SES = 0, who identifies as white (i.e., nonwhite = 0), who attended a school whose average SES = 0 (near the sample mean) and with 30% non-white-identifying students (near the sample mean). The effects of SES remained significant with patterns as previously described, and so we focus on the new effects of nonwhite identity. At level 1, the within-school slope for student nonwhite identity was significantly negative, indicating that student math was expected to be lower by 2.896 (SE = 0.220, p < .001) for students identifying as nonwhite relative to white. At level 2, level-2 contextual-school slope indicated that, after controlling for the effect of student nonwhite identity, school mean math was expected to be nonsignificantly higher by 0.135 (SE = 0.059, p = .024) for each additional 10% of students identifying as nonwhite. Said differently, the level-2 contextual-school slope also indicated that the level-2 between-school slope for school mean nonwhite identity was significantly negative (i.e., Est = −1.544 for the difference between proportion = 0 or 1), indicating that school mean math was expected to be lower by 0.154 (SE = 0.055, p = .006) for each additional 10% of students identifying as nonwhite, without controlling for level-1 student nonwhite identity.

Figures to Illustrate SES Effects using Model 4c Results—see Example 3 files online for R versions!

The figures below demonstrate the SES effects holding the nonwhite predictors constant: at original student nonwhite = 0 and school proportion nonwhite students = .30. Given the use of cluster-mean-centering for level-1 student nonwhite, that translates into its level-1 predictor being held constant at −.30 The **slope of the purple lines shows the positive within-school SES effect** in both figures, but their differing x-axes translate into different level-2 effects being shown!



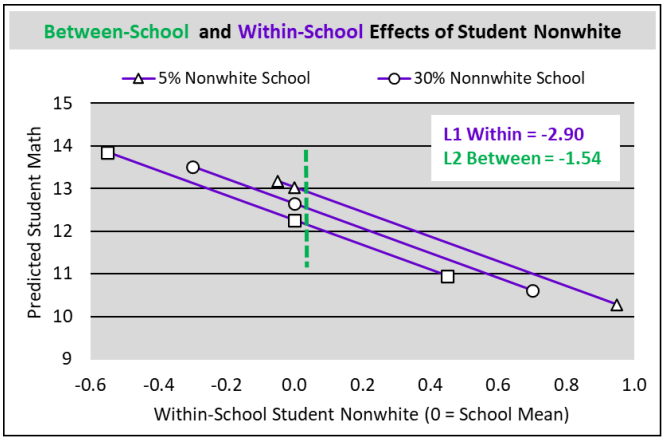


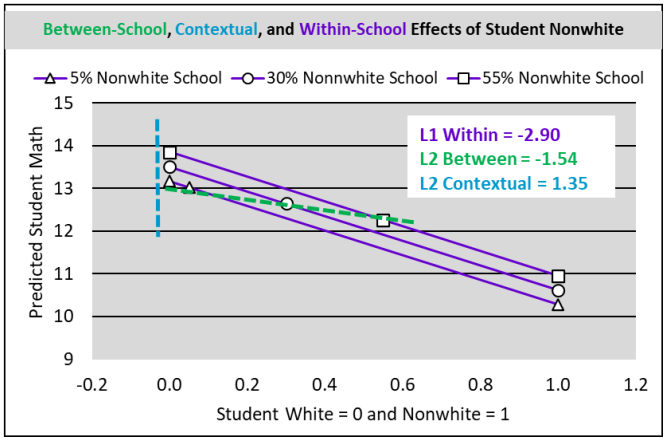
In the left plot above, the x-axis shows level-1 within-school student SES relative to the school's mean (in which lower = −0.66 as −1 SD of school mean, average = 0 when at the school mean, and higher = 0.66 as +1 SD of the school mean for within-school SES). Consequently, the **positive level-2 between-school slope** is shown by the vertical distance between the purple lines (in which low = −0.41 as −1 SD, medium = 0 as near the mean, and high = 0.41 as +1 SD for school mean SES)—it must be the "between" slope because it holds *within-school* student SES constant at 0, which is at the school's mean, whose actual value varies by line! It is not possible to show the positive level-2 contextual slope in the left figure because there is no shown common point at which the *original* student SES predictor is held constant across schools—the x-axis only shows variation in student SES *relative to school mean SES*.

In the right plot above, the x-axis shows original level-1 student SES, in which the values plotted are the same as on the left (to show relatively lower, average, and higher student SES within each school). Consequently, the **positive level-2 contextual slope** is now shown by the vertical distance between the lines (in which low = −0.41 as −1 SD, medium = 0 as near the mean, and high = 0.41 as +1 SD for school mean SES) holding *original* student SES constant (at 0 here, but it could be any constant given the lack of an interaction term between the SES predictors). The **positive level-2 between-school slope** is then shown by diagonal distance between the lines at within-school SES = 0 (when a student is at their school's mean SES).

Figures to Illustrate Nonwhite Effects using Model 4c Results

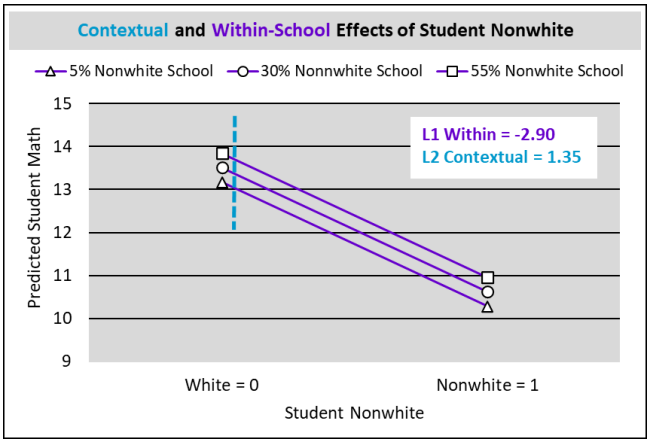
The figures below demonstrate the nonwhite effects holding the SES predictors constant: at student SES = 0 and school mean SES = 0 (i.e., an average SES student in an average SES school). The slope of the purple lines shows the negative within-school nonwhite effect in both figures, but their differing x-axes means showing different level-2 effects!





In the left panel above, the x-axis shows level-1 within-school student nonwhite relative to the school's mean (in which the lowest = white, the middle = school mean, and the highest = nonwhite, respectively). Consequently, the positive level-2 between-school slope is shown by the vertical distance between the lines (in which low = 0.05, medium = 0.30, and high = 0.55 for school proportion nonwhite, chosen to be a smaller range than ±1 SD = .30 to avoid a school without any nonwhite students)—it must be the "between" slope because it holds *within-school* student nonwhite constant at the value for the school's mean, which varies by line. It is not possible to show the positive level-2 contextual slope in the left figure because there is no shown common point at which the *original* student nonwhite predictor is held constant across schools—the x-axis shows student nonwhite relative to school mean nonwhite only (so no longer using the original 0 and 1 values, which will seem odd).

In the right panel above, the x-axis shows original level-1 student nonwhite, in which the values plotted are the same as on the left (to show white, school mean, and nonwhite students within each school). Consequently, the positive level-2 contextual slope is now shown by the vertical distance between the lines (in which low = 0.05, medium = 0.30, and high = 0.55 for school proportion nonwhite, chosen to be a smaller range than ± 1 SD = .30 to avoid a school without any nonwhite students) holding *original* student nonwhite constant (at 0 here, but it could be any constant given the lack of an interaction term between the nonwhite predictors). The negative level-2 between-school slope is then shown by diagonal distance between the lines when within-school nonwhite = 0 (at school mean). But this figure is likely to break your readers' brains because the intermediate values—of student nonwhite = school mean—are not possible! Therefore, the figure below omitting that intermediate value may be more understandable, but then it cannot be used to show the negative between-school slope.



Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software, 67*(1), 1–48. https://doi.org/10.18637/jss.v067.i01

Kuznetsova, A., Brockhoff, P. B., & Christensen, R. H. B. (2017). lmerTest package: Tests in linear mixed effects models. *Journal of Statistical Software, 82*(13), 1–26. https://doi.org/10.18637/jss.v082.i13