

Bonus Coding Example: 3x3 Two-Way ANOVA as a Linear Model, Two Approaches

Means:	1	2	3	
A	*a1*	a2	a3	Marginal A
B	b1	b2	b3	Marginal B
C	c1	c2	c3	Marginal C
	Marginal 1	Marginal 2	Marginal 3	Grand Mean

Method 1 Data Coding: Create 2 contrasts for each 3-category predictor (a1 = intercept)

Letter	Number	AvB	AvC	1v2	1v3
A	1	0	0	0	0
A	2	0	0	1	0
A	3	0	0	0	1
B	1	1	0	0	0
B	2	1	0	1	0
B	3	1	0	0	1
C	1	0	1	0	0
C	2	0	1	1	0
C	3	0	1	0	1

$$\text{Model 1 } \hat{y}_i = \beta_0 + \beta_1(AvB_i) + \beta_2(AvC_i) + \beta_3(1v2_i) + \beta_4(1v3_i) \\ + \beta_5(AvB_i)(1v2_i) + \beta_6(AvC_i)(1v2_i) + \beta_7(AvB_i)(1v3_i) + \beta_8(AvC_i)(1v3_i)$$

Typical two-way ANOVA results for model 1 would include:

- the marginal DF=2 F-test for letter (averaged over numbers)
- the marginal DF=2 F-test for number (averaged over letters)
- the DF=4 F-test of the interaction
- The marginal main effects are not useful if any part of the interaction is significant, and the DF=4 interaction test is too general to tell you what is really going on... instead we need to create DF=1 simple main effects and their DF=1 simple interaction contrasts as follows...

	1	2	3	
Means:				
A	β_0	$\beta_0 + \beta_3$	$\beta_0 + \beta_4$	
B	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_3 + \beta_5$	$\beta_0 + \beta_1 + \beta_4 + \beta_7$	
C	$\beta_0 + \beta_2$	$\beta_0 + \beta_2 + \beta_3 + \beta_6$	$\beta_0 + \beta_2 + \beta_4 + \beta_8$	

Effects of Interest, All Given by Model 1 (1 intercept + 8 slopes recreate all 9 cell means):

- 9 possible letter simple mean differences (3 per number)
- 9 possible number simple mean differences (3 per letter)
- 9 possible single-DF interaction contrasts of simple mean differences:
 - Does AvB differ from 1 to 2? OR Does 1v2 differ from A to B? $\rightarrow \beta_5$
 - Does AvC differ from 1 to 2? OR Does 1v2 differ from A to C? $\rightarrow \beta_6$
 - Does BvC differ from 1 to 2? OR Does 1v2 differ from B to C? $\rightarrow \beta_6 - \beta_5$

 - Does AvB differ from 1 to 3? OR Does 1v3 differ from A to B? $\rightarrow \beta_7$
 - Does AvC differ from 1 to 3? OR Does 1v3 differ from A to C? $\rightarrow \beta_8$
 - Does BvC differ from 1 to 3? OR Does 1v3 differ from B to C? $\rightarrow \beta_8 - \beta_7$

 - Does AvB differ from 2 to 3? OR Does 2v3 differ from A to B? $\rightarrow \beta_7 - \beta_5$
 - Does AvC differ from 2 to 3? OR Does 2v3 differ from A to C? $\rightarrow \beta_8 - \beta_6$
 - Does BvC differ from 2 to 3? OR Does 2v3 differ from B to C? $\rightarrow (\beta_6 - \beta_5) - (\beta_8 - \beta_7)$

Sound complicated? It is... here is a simpler approach that can be especially useful in not-fully-crossed designs (when one or more of the cells is missing by design)—create cell-specific dummy coded predictors (=1 if it's that cell, =0 otherwise) to create cell-specific intercepts and use those instead of a global intercept and differences:

$$\text{Model 2 } \hat{y}_i = \beta_{a1}(a1_i) + \beta_{a2}(a2_i) + \beta_{a3}(a3_i) + \\ \beta_{b1}(b1_i) + \beta_{b2}(b2_i) + \beta_{b3}(b3_i) + \\ \beta_{c1}(c1_i) + \beta_{c2}(c2_i) + \beta_{c3}(c3_i)$$

Effects of Interest, All Given by Model 2 (9 cell-specific intercepts recreate all 9 cell means):

- 9 possible letter simple mean differences (3 per number)
- 9 possible number simple mean differences (3 per letter)
- 9 possible single-DF interaction contrasts of simple mean differences:
 - Does AvB differ from 1 to 2? OR Does 1v2 differ from A to B? $\rightarrow (\beta_{b2} - \beta_{a2}) - (\beta_{b1} - \beta_{a1})$
 - Does AvC differ from 1 to 2? OR Does 1v2 differ from A to C? $\rightarrow (\beta_{c2} - \beta_{a2}) - (\beta_{c1} - \beta_{a1})$
 - Does BvC differ from 1 to 2? OR Does 1v2 differ from B to C? $\rightarrow (\beta_{c2} - \beta_{b2}) - (\beta_{c1} - \beta_{b1})$
 - Does AvB differ from 1 to 3? OR Does 1v3 differ from A to B? $\rightarrow (\beta_{b3} - \beta_{a3}) - (\beta_{b1} - \beta_{a1})$
 - Does AvC differ from 1 to 3? OR Does 1v3 differ from A to C? $\rightarrow (\beta_{c3} - \beta_{a3}) - (\beta_{c1} - \beta_{a1})$
 - Does BvC differ from 1 to 3? OR Does 1v3 differ from B to C? $\rightarrow (\beta_{c3} - \beta_{b3}) - (\beta_{c1} - \beta_{b1})$
 - Does AvB differ from 2 to 3? OR Does 2v3 differ from A to B? $\rightarrow (\beta_{b3} - \beta_{a3}) - (\beta_{b2} - \beta_{a2})$
 - Does AvC differ from 2 to 3? OR Does 2v3 differ from A to C? $\rightarrow (\beta_{c3} - \beta_{a3}) - (\beta_{c2} - \beta_{a2})$
 - Does BvC differ from 2 to 3? OR Does 2v3 differ from B to C? $\rightarrow (\beta_{c3} - \beta_{b3}) - (\beta_{c2} - \beta_{b2})$