Example 5: General Linear Models with Single-DF and Multiple-DF Interactions
(complete data, syntax, and output in R available online)

The data for this example come from Hoffman (2015) chapter 2. Using this sample of 550 older adults (which was simulated based on real data from the Octogenarian Twin Study of Aging), we examine the extent to which cognition (as measured by the information test, a measure of crystallized general intelligence) can be predicted from quantitative age (centered at 85 years), quantitative grip strength (centered at 9 pounds per square inch), binary sex (with men as the reference), and subsequent three-category dementia diagnosis (none = 1, future = 2, and current = 3, with the none as the reference). After starting with a main-effects-only model, this example illustrates how to include and interpret interactions: first examining sex by age and sex by grip strength, then examining age by grip strength, and then examining sex by dementia status. This example will continue to estimate general linear models using the base R `lm` function. It will demonstrate how to create predicted outcomes using the `prediction` R package, how to estimate simple (conditional) slopes using the `glht` function from the `multcomp` R package and the `simple_slopes` function from the `reghelper` R package, and how to obtain regions of significance from the `interactions` R package.

Syntax for importing and preparing data for analysis (after loading packages `TeachingDemos, readxl, multcomp, supernova, prediction, lmhelpers, reghelper, and interactions`):

```
# Set working directory (to import and export files to)
# Paste in the folder address where your data file is saved in quotes
# Note the slashes are backwards relative to Windows file paths
setwd("C:/Dropbox/26_EDF9770/Example5/")

# Import Example5_Data.xlsx data from working directory -- path = file name
Example5 = read_excel(path="Example5_Data.xlsx", sheet="Data")
# Convert to data frame to use for analysis
Example5 = as.data.frame(Example5)

# Load R functions for this class from R file in working directory
source("EDF9770_Functions.R")
```

Descriptive statistics and creating new predictor variables:

```
# Select cases complete on all variables to be used
Example5 = Example5[complete.cases(Example5[ ,
            c("cognition","age","grip","sexMW","demgroup")]),]

print("R Descriptive Statistics for Quantitative Variables")
psych::describe(x=Example5[ , c("cognition","age","grip","sexMW")])

          vars   n  mean    sd median trimmed   mad   min   max range  skew kurtosis   se
cognition    1 550 24.82 10.99  25.00   25.18 11.86  0.00 44.00 44.00 -0.26    -0.62 0.47
age          2 550 84.93  3.43  84.33   84.49  2.88 80.02 96.97 16.95  1.10     0.72 0.15
grip         3 550  9.11  2.98   9.00    9.11  2.97  0.00 19.00 19.00 -0.01    -0.17 0.13
sexMW        4 550  0.59  0.49   1.00    0.61  0.00  0.00  1.00  1.00 -0.35    -1.88 0.02

# Center quantitative predictors near their means
Example5$age85=Example5$age-85  # age85: Age in Years (0=85)
Example5$grip9=Example5$grip-9  # grip9: Grip Strength in Pounds (0=9)

print("R Descriptive Statistics for Categorical Variables")
prop.table(table(x=Example5$sexMW,useNA="ifany"))  # 0=Men, 1=Women

     0      1
0.4127 0.5873
```

```
prop.table(table(x=Example5$demgroup,useNA="ifany"))  # 1=None, 2=Future, 3=Current

      1       2       3
0.72545 0.19818 0.07636

# Create 2 indicator-coded binary predictors for 3 dementia groups
Example5$demNF=NA; Example5$demNC=NA  # Create 2 new empty variables
Example5$demNF[which(Example5$demgroup==1)]=0  # 1=None
Example5$demNC[which(Example5$demgroup==1)]=0
Example5$demNF[which(Example5$demgroup==2)]=1  # 2=Future
Example5$demNC[which(Example5$demgroup==2)]=0
Example5$demNF[which(Example5$demgroup==3)]=0  # 3=Current
Example5$demNC[which(Example5$demgroup==3)]=1
# demNF: None=0 vs Future=1, demNC: None=0 vs Current=1
```

demgroup	demNF	demNC
1 = None	0	0
2 = Future	1	0
3 = Current	0	1

Bivariate relations—first, let's check age and grip strength for nonlinear relations:

$$Cognition_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Age_i - 85)^2 + e_i$$

```
print("Quadratic Age?")
ModelQuadAge = lm(data=Example5, formula=cognition~1+age85+I(age85^2))
obj=LMsummary(ModelQuadAge)  # Custom output

Fixed Effects Table
              Est    SE      t      p    LCI    UCI
Intercept  24.571 0.601 40.912 <0.001 23.392 25.751
age85      -0.609 0.178 -3.433 <0.001 -0.958 -0.261
I(age85^2)  0.018 0.032  0.549  0.583 -0.045  0.080
```

$$Cognition_i = \beta_0 + \beta_1(Grip_i - 9) + \beta_2(Grip_i - 9)^2 + e_i$$

```
print("Quadratic Grip?")
ModelQuadGrip = lm(data=Example5, formula=cognition~1+grip9+I(grip9^2))
obj=LMsummary(ModelQuadGrip)  # Custom output

Fixed Effects Table
              Est    SE      t      p    LCI    UCI
Intercept  24.579 0.566 43.420 <0.001 23.467 25.691
grip9       0.888 0.153  5.801 <0.001  0.587  1.188
I(grip9^2)  0.016 0.038  0.424  0.671 -0.058  0.090
```

Now that we have evidence that a linear trend for both age and grip is sufficient, let's examine their linear relations with cognition (and with binary sex, in which 0 = men and 1=women):

```
print("Pearson Correlation Matrix with p-values -- Must convert data frame to matrix")
Corrs=Hmisc::rcorr(x=as.matrix(Example5
                    [ ,c("cognition","age","grip","sexMW")]), type="pearson"); Corrs

          cognition   age  grip sexMW
cognition      1.00 -0.17  0.24 -0.24
age           -0.17  1.00 -0.18  0.05
grip           0.24 -0.18  1.00 -0.40
sexMW         -0.24  0.05 -0.40  1.00

n= 550
P
          cognition age    grip   sexMW
cognition           0.0000 0.0000 0.0000
age       0.0000           0.0000 0.2858
grip      0.0000    0.0000        0.0000
sexMW     0.0000    0.2858 0.0000
```

What can we conclude about the linear relationships between each pair of variables?

Cognition and age:

Cognition and grip:

Cognition and sex:

Age and grip:

Age and sex:

Grip and sex:

Our fourth predictor is three-category dementia group—we do not yet know if there is a significant bivariate relation of dementia group with cognition. A Pearson correlation would not have provided the desired interpretation of each binary predictor (i.e., it would have lumped together both groups coded 0 within each binary predictor). Instead, we can examine their bivariate relation by predicting cognition with dementia group in a GLM:

demgroup	demNF	demNC
1 = None	0	0
2 = Future	1	0
3 = Current	0	1

$$Cognition_i = \beta_0 + \beta_1(DemNF_i) + \beta_2(DemNC_i) + e_i$$

Linear combination for difference of future vs current dementia:

$$(\beta_0 + \beta_2) - (\beta_0 + \beta_1) = \beta_2 - \beta_1$$

```
print("Demgroup")
ModelDem = lm(data=Example5, formula=cognition~1+demNF+demNC)
obj=LMsummary(ModelDem)  # Custom output

Sums of Squares Table
             SS  DF       MS      F      p    R2
Model 11685.511   2 5842.755 58.523 <0.001 0.176
Error 54611.027 547   99.837
Total 66296.538 549  120.759

Fixed Effects Table
              Est    SE       t      p     LCI     UCI
Intercept  27.198 0.500  54.372 <0.001  26.215  28.181
demNF      -5.675 1.080  -5.255 <0.001  -7.796  -3.554
demNC     -16.388 1.621 -10.111 <0.001 -19.572 -13.205

print("R Ask for missing model-implied group difference as beta3-beta2")
glhtDem = multcomp::glht(model=ModelDem, linfct=rbind("Future vs Current"=c(0,-1,1)))
obj=glhtSummary(glhtObject=glhtDem)  # Custom output

Linear Combinations Table
                      Est    SE      t      p     LCI    UCI
Future vs Current -10.713 1.815 -5.904 <0.001 -14.278 -7.149

R2=Corrs[["r"]]^2; R2  # Print R2 values
          cognition      age    grip    sexMW
cognition   1.00000 0.029054 0.05848 0.055830
age         0.02905 1.000000 0.03391 0.002079
grip        0.05848 0.033906 1.00000 0.162605
sexMW       0.05583 0.002079 0.16260 1.000000
```

Given that bivariate relations can be expressed using R^2 more generally, here is the squared version of the r matrix for the other single-slope predictors

Model 1 with main effects only (Equation 2.8 in chapter 2): How do age in years, grip strength in pounds per square inch, binary sex, and dementia diagnosis each uniquely predict cognition?

Equation 1: $$Cognition_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + e_i$$

Linear combination for difference of future vs current dementia:
$$(\beta_0 + \beta_5) - (\beta_0 + \beta_4) = \beta_5 - \beta_4$$

```
print("R Main-Effects-Only Model Predicting Cognition (Equation 2.8)")
Model1 = lm(data=Example5, formula=cognition~1+age85+grip9+sexMW+demNF+demNC)
obj=LMsummary(Model1, explain=TRUE, effectsizes=TRUE)  # Custom output

Sums of Squares Table
             SS  DF       MS      F      p    R2
Model 18385.979   5 3677.196 41.753 <0.001 0.277
Error 47910.559 544   88.071
Total 66296.538 549  120.759

Fixed Effects Table
              Est    SE       t      p     LCI     UCI
Intercept  29.264 0.699  41.895 <0.001  27.892  30.636  beta0
age85      -0.406 0.119  -3.412 <0.001  -0.639  -0.172  beta1
grip9       0.604 0.150   4.034 <0.001   0.310   0.898  beta2
sexMW      -3.657 0.891  -4.103 <0.001  -5.408  -1.906  beta3
demNF      -5.722 1.019  -5.615 <0.001  -7.724  -3.720  beta4
demNC     -16.480 1.523 -10.822 <0.001 -19.471 -13.489  beta5

Effect Sizes for Fixed Effects Table
          Est    SE      p      d     pr   sR2
age85  -0.406 0.119 <0.001 -0.293 -0.145 0.015
grip9   0.604 0.150 <0.001  0.346  0.170 0.022
sexMW  -3.657 0.891 <0.001 -0.352 -0.173 0.022
demNF  -5.722 1.019 <0.001 -0.481 -0.234 0.042
demNC -16.480 1.523 <0.001 -0.928 -0.421 0.156

print("R Ask for missing model-implied group difference as beta5-beta4")
SlopesModel1 = multcomp::glht(model=Model1,
                              linfct=rbind("Future vs Current"=c(0,0,0,0,-1,1)))
obj=glhtSummary(glhtObject=SlopesModel1, effectsizes=TRUE) # Custom output

Linear Combinations Table
                      Est    SE      t      p     LCI    UCI
Future vs Current -10.758 1.708 -6.299 <0.001 -14.113 -7.403  beta5-beta4

Effect Sizes for Linear Combinations Table
                      Est    SE      p      d     pr   sR2
Future vs Current -10.758 1.708 <0.001 -0.540 -0.261 0.053

# Get F-test and effect sizes for change in R2 from full model for demgroup
Model1NoDem = lm(data=Example5, formula=cognition~1+age85+grip9+sexMW)
obj=R2compare(ReducedModel=Model1NoDem, FullModel=Model1, PredName="Demgroup")

F-Test and R2 Change for Demgroup Slopes
 R2reduced R2full R2diff DFnum DFden      F      p   pR2   sR2
     0.099  0.277  0.178     2   544 67.056 <0.001 0.198 0.178
```

	Bivariate R2	Unique sR2
age85	0.029	0.015
grip9	0.058	0.022
sexMW	0.056	0.022
demgroup	0.176	0.178

Let's compare effect sizes from the bivariate to unique versions of the same relations:

Example Results Section Part 1

Analytic Strategy

The extent to which cognition (as measured by the information test) could be predicted from age in years, grip strength in pounds per square inch, binary sex, and dementia group was examined in a series of general linear models. All analyses were conducted using the lm function in R v. 4.5.2. Predicted outcomes and linear combinations of the fixed effects were generated using the glht function within the multcomp package v. 1.4-29. To provide a meaningful intercept in the models that follow, we centered age such that 0 = 85 years and grip strength such that 0 = 9 pounds per square inch (i.e., near their sample means). Sex was coded such that 0 = men and 1 = women. Dementia group was represented by two binary contrasts to distinguish the three types, in which no diagnosis served as the reference to be compared to future diagnosis and current diagnosis.

Bivariate Relations

The predictors were first examined in separate models to obtain their bivariate relations (i.e., before controlling for the other predictors). Cognition was significantly higher in persons who were younger (R^2 = .029), with greater grip strength (R^2 = .058), and who were men (R^2 = .056). Although we examined the potential for quadratic effects of age and grip strength, neither quadratic slope was significant, indicating that linear slopes for age and grip strength were likely to be sufficient. In a separate analysis of variance, we examined the bivariate effect of dementia group and found significant mean differences in cognition across the three groups, $F(2, 547) = 58.52$, MSE = 99.84, $p < .001$, R^2 = .176. Relative to the reference group of persons with no diagnosis, cognition was significantly lower in both the future and current diagnosis groups; cognition was also significantly lower in the current than future diagnosis group).

Main Effects Model

The results from a combined model (as shown in Equation 1) with main effects for all four predictors are shown in Table 1. In addition to the partial *r* and Cohen's *d* effect sizes given in Table 1, semi-partial squared correlation (sR^2) effect sizes were also obtained to describe the amount of variance in cognition explained by each predictor variable. The model accounted for a significant amount of variance in cognition, $F(5, 544) = 41.75$, MSE = 88.07, $p < .0001$, R^2 = .277. The unique contribution of each predictor remained significant in the same direction as their bivariate effects. Cognition was predicted to be significantly lower by 0.41 per year of age (sR^2 = .015), significantly higher by 0.60 per pound per square inch of grip strength (sR^2 = .022), and significantly lower by 3.66 in women (sR^2 = .022). The overall effect of dementia group remained significant, $F(2, 544) = 67.06$, $p < .001$, sR^2 = .178. Relative to the reference group of persons with no diagnosis, cognition was significantly lower by 5.72 in the future diagnosis group, significantly lower by 16.48 in the current diagnosis group, and cognition was significantly lower by 10.76 in the future than current diagnosis group.

Table 1
Results from Main Effects Model in Equation 1

	Fixed Effect	Est	SE	*p* <	Cohen's *d*	Partial *r*
β_0	Intercept	29.264	0.699			
β_1	Age (0=85 years)	−0.406	0.119	.001	−0.293	−0.145
β_2	Grip (0=9 pounds)	0.604	0.150	.001	0.346	0.170
β_3	Sex (0=Men)	−3.657	0.891	.001	−0.352	−0.173
β_4	None vs Future Dementia	−5.722	1.019	.001	−0.481	−0.234
β_5	None vs Current Dementia	−16.480	1.523	.001	−0.928	−0.421
$\beta_5 - \beta_4$	Future vs Current Dementia	−10.758	1.708	.001	−0.540	−0.261

Note: Est = estimate, SE = standard error. Cohen's *d* and partial *r* effect sizes were computed from the slope of each *t* test-statistic as follows: $d = \frac{2t}{\sqrt{DF_{den}}}$; $r = \frac{t}{\sqrt{t^2+DF_{den}}}$.

FOR HW5 → Model 2 adding sex by age and sex by grip: Do the effects of age and grip strength differ between men and women?

Equation 2: $$Cognition_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(SexMW_i)(Age_i - 85) + \beta_7(SexMW_i)(Grip_i - 9) + e_i$$

```
print("Model 2: Add 2 Interactions -- Sex by Age, Sex by Grip")
Model2 = lm(data=Example5, formula=cognition~1+age85+grip9+sexMW+demNF+demNC
                                             +age85:sexMW +grip9:sexMW)

# Get F-test and effect sizes for change in R2 from full model for program
obj=R2compare(ReducedModel=Model1, FullModel=Model2, PredName="Sex*Age and Sex*Grip")

F-Test and R2 Change for Sex*Age and Sex*Grip Slopes
 R2reduced R2full R2diff DFnum DFden     F     p   pR2   sR2
     0.277  0.279  0.002     2   542 0.814 0.444 0.003 0.002

obj=LMsummary(Model2, effectsizes=TRUE)  # Custom output

Sums of Squares Table
             SS  DF       MS      F      p    R2
Model 18529.397   7 2647.057 30.035 <0.001 0.279
Error 47767.141 542   88.131
Total 66296.538 549  120.759

Fixed Effects Table
                Est    SE       t      p     LCI     UCI
Intercept    29.172 0.758  38.467 <0.001  27.682  30.662  beta0
age85        -0.232 0.189  -1.226  0.221  -0.604   0.140  beta1
grip9         0.697 0.239   2.918  0.004   0.228   1.166  beta2
sexMW        -3.627 0.911  -3.980 <0.001  -5.416  -1.837  beta3
demNF        -5.749 1.020  -5.635 <0.001  -7.753  -3.745  beta4
demNC       -16.495 1.523 -10.828 <0.001 -19.487 -13.502  beta5
age85:sexMW  -0.295 0.244  -1.208  0.227  -0.774   0.184  beta6
grip9:sexMW  -0.177 0.307  -0.576  0.565  -0.779   0.426  beta7
```

```
Effect Sizes for Fixed Effects Table
                 Est    SE      p      d     pr   sR2
age85         -0.232 0.189  0.221 -0.105 -0.053 0.002
grip9          0.697 0.239  0.004  0.251  0.124 0.011
sexMW         -3.627 0.911 <0.001 -0.342 -0.169 0.021
demNF         -5.749 1.020 <0.001 -0.484 -0.235 0.042
demNC        -16.495 1.523 <0.001 -0.930 -0.422 0.156
age85:sexMW   -0.295 0.244  0.227 -0.104 -0.052 0.002
grip9:sexMW   -0.177 0.307  0.565 -0.050 -0.025 0.000
```

Interpret β_1 for age85:

Interpret β_6 for age85*sexMW with sex as moderator:

Interpret β_2 for grip9:

Interpret β_7 for grip9*sexMW with sex as moderator:

Interpret β_3 for sexMW:

Interpret β_6 for age85*sexMW with age85 as moderator:

Interpret β_7 for grip9*sexMW with grip9 as moderator:

Computing "simple" (conditional) age and grip strength slopes for men and women:

$$\widehat{Cognition_i} = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(SexMW_i)(Age_i - 85) + \beta_7(SexMW_i)(Grip_i - 9)$$

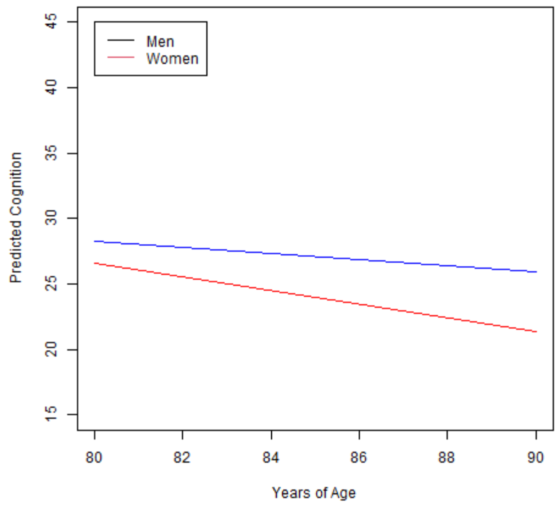
We can use the model equation to calculate the **simple age and grip slopes** for either sex (as the moderator):

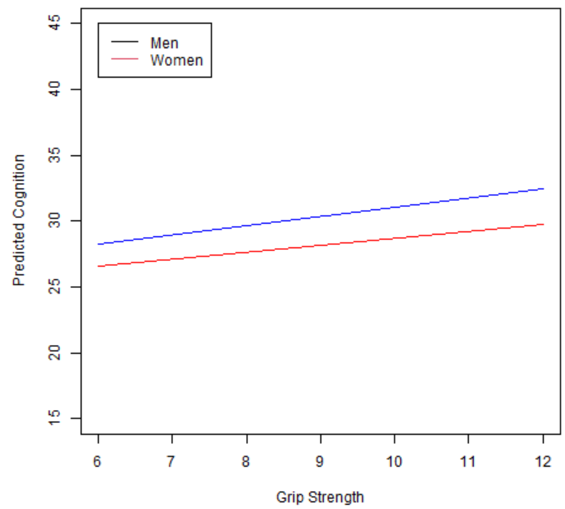
Simple Age Slope = $\beta_1(Age_i - 85) + \beta_6(SexMW_i)(Age_i - 85)$
= $[\beta_1 + \beta_6(SexMW_i)]$ that multiplies $(Age_i - 85)$

Simple Grip Slope = $\beta_2(Grip_i - 9) + \beta_7(SexMW_i)(Grip_i - 9)$
= $[\beta_2 + \beta_7(SexMW_i)]$ that multiplies $(Grip_i - 9)$

```
print("Simple slopes of age by sex, simple slopes of sex by age")
SlopesModel2 = multcomp::glht(model=Model2, linfct=rbind(
  "Age Slope for Men"      = c(0,1,0,0,0,0,0,0),  # Multipliers in order of fixed effects
  "Age Slope for Women"    = c(0,1,0,0,0,0,1,0),

  "Grip Slope for Men"     = c(0,0,1,0,0,0,0,0),
  "Grip Slope for Women"   = c(0,0,1,0,0,0,0,1)))
obj=glhtSummary(glhtObject=SlopesModel2, effectsizes=TRUE) # Custom output
```

```
Linear Combinations Table
                        Est    SE      t      p    LCI    UCI
Age Slope for Men    -0.232 0.189 -1.226  0.221 -0.604  0.140
Age Slope for Women  -0.527 0.154 -3.428 <0.001 -0.829 -0.225
Grip Slope for Men    0.697 0.239  2.918  0.004  0.228  1.166
Grip Slope for Women  0.520 0.193  2.694  0.007  0.141  0.899

Effect Sizes for Linear Combinations Table
                        Est    SE      p      d     pr   sR2
Age Slope for Men    -0.232 0.189  0.221 -0.105 -0.053 0.002
Age Slope for Women  -0.527 0.154 <0.001 -0.294 -0.146 0.016
Grip Slope for Men    0.697 0.239  0.004  0.251  0.124 0.011
Grip Slope for Women  0.520 0.193  0.007  0.231  0.115 0.010
```






```
print("Pred cognition outcomes holding demNF=none, and demNC=none")
print("Provides predicted outcomes from min,max,by=increment of predictors")
PredModel2 = prediction::prediction(model=Model2, type="response",
             at=list(demNF=0, demNC=0, grip9=seq(-3,3,by=6), age85=seq(-5,5,by=10),sexMW=0:1))
PlotModel2 = summary(PredModel2); print(PlotModel2, digits=6) # Save and print predictions for plotting

at(demNF) at(demNC) at(grip9) at(age85) at(sexMW) Prediction     SE     z           p lower upper
        0         0        -3        -5         0      28.24 1.5949 17.71   3.631e-70 25.12 31.37
        0         0         3        -5         0      32.42 1.1454 28.31 2.768e-176 30.18 34.67
        0         0        -3         5         0      25.92 1.5665 16.55   1.708e-61 22.85 28.99
        0         0         3         5         0      30.10 1.2769 23.57 7.188e-123 27.60 32.60
        0         0        -3        -5         1      26.62 1.1182 23.81 2.859e-125 24.43 28.81
        0         0         3        -5         1      29.74 1.1158 26.65 1.613e-156 27.55 31.93
        0         0        -3         5         1      21.35 0.9571 22.31 3.106e-110 19.47 23.23
        0         0         3         5         1      24.47 1.3380 18.29   1.006e-74 21.85 27.09

# Make and save plots of Predicted Outcomes
png(file = "Sex by Grip=x GLM Plot.png")  # open file
PlotModel2age80 = PlotModel2[which(PlotModel2$`at(age85)`==-5),] # Subset to age=80
plot(y=PlotModel2age80$Prediction, x=(PlotModel2age80$`at(grip9)`+9), type="n", # n= no dots
     ylim=c(15,45), xlim=c(6,12), xlab="Grip Strength",ylab="Predicted Cognition")
lines(x=(PlotModel2age80$`at(grip9)`+9)[1:2], y=PlotModel2age80$Prediction[1:2], type="l", col="blue1")
lines(x=(PlotModel2age80$`at(grip9)`+9)[3:4], y=PlotModel2age80$Prediction[3:4], type="l", col="red1")
legend(x=6, y=45, legend=c("Men","Women"), col=1:2, lty=1) # lty=linetype
dev.off()  # close file

png(file = "Sex by Age=x GLM Plot.png")  # open file
PlotModel2grip6 = PlotModel2[which(PlotModel2$`at(grip9)`==-3),] # Subset to grip=6
plot(y=PlotModel2grip6$Prediction, x=(PlotModel2grip6$`at(grip9)`+9), type="n",  # n= no dots
     ylim=c(15,45), xlim=c(80,90), xlab="Years of Age",ylab="Predicted Cognition")
lines(x=(PlotModel2grip6$`at(age85)`+85)[1:2], y=PlotModel2grip6$Prediction[1:2], type="l",
col="blue1")
lines(x=(PlotModel2grip6$`at(age85)`+85)[3:4], y=PlotModel2grip6$Prediction[3:4], type="l", col="red1")
legend(x=80, y=45, legend=c("Men","Women"), col=1:2, lty=1) # lty=linetype
dev.off()  # close file
```

Example Results Section Part 2

Interactions of Sex with Age and Grip Strength

We then estimated a new model (as shown in Equation 2) to examine the extent to which the slopes of age and grip strength differed between men and women. Although the augmented model accounted for a significant amount of variance in cognition, $F(7, 542) = 30.04$, MSE = 88.13, $p < .0001$, $R^2 = .280$, the addition of the two interactions did not significantly improve prediction relative to the main effects model, $F(2, 542) = 0.81$, $p = .444$, change in $R^2 = .002$. Results indicated that the effects of age and grip strength did not differ significantly between men and women, and so these nonsignificant interactions with sex were removed.

NONE OF WHAT FOLLOWS IS NEEDED FOR HW05,
but this model provides an example of a quantitative*quantitative predictor interaction…

Model 3 removing 2 sex interactions; add interaction of age by grip (Equation 2.9 in chapter 2):
Does the effect of age vary by grip strength (or does the effect of grip strength vary by age)?

Model 3: $$Cognition_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(Age_i - 85)(Grip_i - 9) + e_i$$

```
print("Model 3: Remove 2 Sex Interactions, Add Age by Grip Interaction (Equation 2.9)")
Model3 = lm(data=Example5, formula=cognition~1+age85+grip9+sexMW+demNF+demNC+age85:grip9)
obj=LMsummary(Model3, effectsizes=TRUE)  # Custom output

Sums of Squares Table
             SS  DF       MS      F      p    R2
Model 19185.041   6 3197.507 36.854 <0.001 0.289
Error 47111.497 543   86.762
Total 66296.538 549  120.759

Fixed Effects Table
                Est    SE       t      p     LCI     UCI
Intercept    29.408 0.695  42.319 <0.001  28.043  30.773
age85        -0.334 0.120  -2.775  0.006  -0.570  -0.098
grip9         0.619 0.149   4.164 <0.001   0.327   0.912
sexMW        -3.456 0.887  -3.895 <0.001  -5.199  -1.713
demNF        -5.923 1.014  -5.843 <0.001  -7.914  -3.931
demNC       -16.300 1.513 -10.777 <0.001 -19.272 -13.329
age85:grip9   0.123 0.041   3.035  0.003   0.043   0.203

Effect Sizes for Fixed Effects Table
                Est    SE      p      d     pr   sR2
age85        -0.334 0.120  0.006 -0.238 -0.118 0.010  beta0
grip9         0.619 0.149 <0.001  0.357  0.176 0.023  beta1
sexMW        -3.456 0.887 <0.001 -0.334 -0.165 0.020  beta2
demNF        -5.923 1.014 <0.001 -0.501 -0.243 0.045  beta3
demNC       -16.300 1.513 <0.001 -0.925 -0.420 0.152  beta4
age85:grip9   0.123 0.041  0.003  0.260  0.129 0.012  beta5
```

Interpret β_1 for age85:

Interpret β_6 for age85*grip9 with grip9 as moderator:

Interpret β_2 for grip9:

Interpret β_6 for age85*grip9 with age85 as moderator:

```
# Get F-test and effect sizes for change in R2 from full model for demgroup
Model3NoDem = lm(data=Example5, formula=cognition~1+age85+grip9+sexMW+age85:grip9)

obj=R2compare(ReducedModel=Model3NoDem, FullModel=Model3, PredName="Demgroup")

F-Test and R2 Change for Demgroup Slopes
 R2reduced R2full R2diff DFnum DFden      F      p   pR2   sR2
     0.112  0.289  0.177     2   543 67.701 <0.001 0.200 0.177

print("R Ask for missing model-implied group difference as beta5-beta4")
SlopesModel3 = multcomp::glht(model=Model3,
                              linfct=rbind("Future vs Current"=c(0,0,0,0,-1,1,0)))
obj=glhtSummary(glhtObject=SlopesModel3, effectsizes=TRUE) # Custom output
```

```
Linear Combinations Table
                           Est    SE      t      p     LCI    UCI
Future vs Current -10.378 1.700 -6.105 <0.001 -13.717 -7.039

Effect Sizes for Linear Combinations Table
                           Est    SE      p      d     pr   sR2
Future vs Current -10.378 1.700 <0.001 -0.524 -0.253 0.049
```

Computing simple (conditional) age and grip strength slopes at quantitative moderator values:

$$\widehat{Cognition_i} = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(Age_i - 85)(Grip_i - 9)$$

We can use the model equation to calculate the **simple age slope** at any *grip strength* (as the moderator):

Simple Age Slope = $\beta_1(Age_i - 85) + \beta_6(Age_i - 85)(Grip_i - 9)$ = $[\beta_1 + \beta_6(Grip_i - 9)]\ (Age_i - 85)$

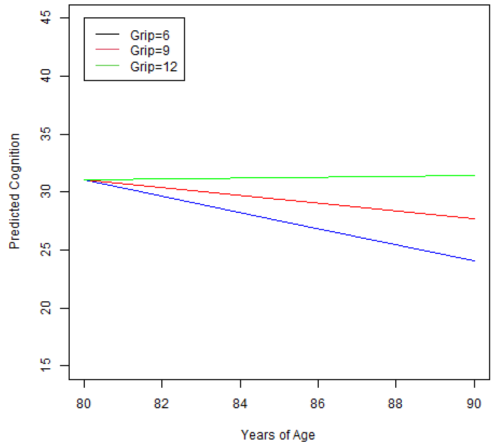
We can also use the model equation to calculate the **simple grip strength slope** at any *age* (as the moderator):

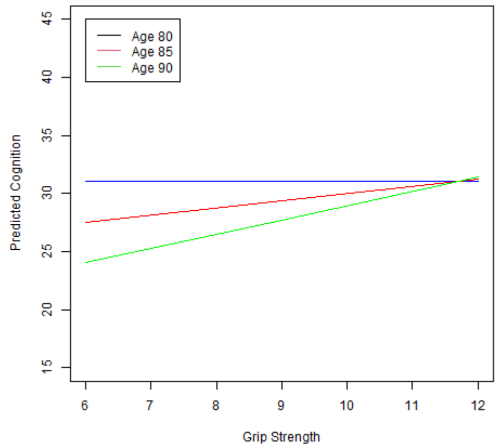
Simple Grip Slope = $\beta_2(Grip_i - 9) + \beta_6(Age_i - 85)(Grip_i - 9)$= $[\beta_2 + \beta_6(Age_i - 85)]\ (Grip_i - 9)$

```
SlopesModel3 = multcomp::glht(model=Model3, linfct=rbind(
  "Age Slope at Grip = 6"  = c(0,1,0,0,0,0,-3),  # Multipliers in order of fixed effects
  "Age Slope at Grip = 9"  = c(0,1,0,0,0,0, 0),
  "Age Slope at Grip = 12" = c(0,1,0,0,0,0, 3),

  "Grip Slope at Age = 80" = c(0,0,1,0,0,0,-5),
  "Grip Slope at Age = 85" = c(0,0,1,0,0,0, 0),
  "Grip Slope at Age = 90" = c(0,0,1,0,0,0, 5)))
obj=glhtSummary(glhtObject=SlopesModel3, effectsizes=TRUE) # Custom output
```

```
Linear Combinations Table
                           Est    SE      t      p    LCI    UCI
Age Slope at Grip = 6   -0.703 0.153 -4.584 <0.001 -1.004 -0.402
Age Slope at Grip = 9   -0.334 0.120 -2.775  0.006 -0.570 -0.098
Age Slope at Grip = 12   0.035 0.187  0.188  0.851 -0.333  0.403

Grip Slope at Age = 80   0.004 0.247  0.017  0.986 -0.482  0.490
Grip Slope at Age = 85   0.619 0.149  4.164 <0.001  0.327  0.912
Grip Slope at Age = 90   1.235 0.255  4.833 <0.001  0.733  1.736

Effect Sizes for Linear Combinations Table
                           Est    SE      p      d     pr   sR2
Age Slope at Grip = 6   -0.703 0.153 <0.001 -0.393 -0.193 0.027
Age Slope at Grip = 9   -0.334 0.120  0.006 -0.238 -0.118 0.010
Age Slope at Grip = 12   0.035 0.187  0.851  0.016  0.008 0.000

Grip Slope at Age = 80   0.004 0.247  0.986  0.002  0.001 0.000
Grip Slope at Age = 85   0.619 0.149 <0.001  0.357  0.176 0.023
Grip Slope at Age = 90   1.235 0.255 <0.001  0.415  0.203 0.031
```

(Figure 1 and 2 for the results)






```
print("Simple slopes over range of moderator values using reghelper package instead")
reghelper::simple_slopes(model=Model3,
                         levels=list(age85=c(-5,0,5,'sstest'),grip9=c(-3,0,3,'sstest')))

   age85  grip9 Test Estimate Std. Error t value  df   Pr(>|t|)
1 sstest     -3        -0.703      0.153  -4.584 543 0.00000567
2 sstest      0        -0.334      0.120  -2.775 543    0.00571
3 sstest      3         0.035      0.187   0.188 543    0.85132
4     -5 sstest         0.004      0.247   0.017 543    0.98605
5      0 sstest         0.619      0.149   4.164 543 0.00003631
6      5 sstest         1.235      0.255   4.833 543 0.00000175
```

Computing and plotting predicted outcomes:

```
print("Pred cognition outcomes holding sexMW=0, demNF=none, and demNC=none")
print("Provides predicted outcomes from min,max,by=increment of predictors")
PredModel3 = prediction(model=Model3, type="response",
                        at=list(sexMW=0, demNF=0, demNC=0, grip9=seq(-3,3,by=3), age85=seq(-5,5,by=5)))
PlotModel3 = summary(PredModel3); print(PlotModel3, digits=6) # Save and print predictions for plotting

at(sexMW) at(demNF) at(demNC) at(grip9) at(age85) Prediction     SE     z         p lower upper
        0         0         0        -3        -5      31.06 1.2605 24.65 4.141e-134 28.59 33.54
        0         0         0         0        -5      31.08 0.9168 33.90 7.049e-252 29.28 32.87
        0         0         0         3        -5      31.09 1.0924 28.46 3.602e-178 28.95 33.23
        0         0         0        -3         0      27.55 0.9309 29.60 1.720e-192 25.73 29.37
        0         0         0         0         0      29.41 0.6949 42.32  0.000e+00 28.05 30.77
        0         0         0         3         0      31.27 0.7053 44.33  0.000e+00 29.88 32.65
        0         0         0        -3         5      24.03 1.1491 20.92  3.808e-97 21.78 26.29
        0         0         0         0         5      27.74 0.9217 30.09 5.867e-199 25.93 29.54
        0         0         0         3         5      31.44 1.2462 25.23 1.861e-140 29.00 33.88
```

```
# Make and save plots
png(file = "Age by Grip=x GLM Plot.png")  # open file
PlotModel3 = PlotModel3[order(PlotModel3$`at(age85)`), ] # 3 rows per age
plot(y=PlotModel3$Prediction, x=(PlotModel3$`at(grip9)`+9), type="n",  # n= no points
     ylim=c(15,45), xlim=c(6,12), xlab="Grip Strength",ylab="Predicted Cognition")
lines(x=(PlotModel3$`at(grip9)`+9)[1:3], y=PlotModel3$Prediction[1:3], type="l", col="blue1")
lines(x=(PlotModel3$`at(grip9)`+9)[4:6], y=PlotModel3$Prediction[4:6], type="l", col="red1")
lines(x=(PlotModel3$`at(grip9)`+9)[7:9], y=PlotModel3$Prediction[7:9], type="l", col="green1")
legend(x=6, y=45, legend=c("Age 80","Age 85","Age 90"), col=1:3, lty=1) # lty=linetype
dev.off()  # close file

png(file = "Grip by Age=x GLM Plot.png")  # open file
PlotModel3 = PlotModel3[order(PlotModel3$`at(grip9)`), ]  # 3 rows per grip
plot(y=PlotModel3$Prediction, x=(PlotModel3$`at(age85)`+85), type="n",  # n= no points
     ylim=c(15,45), xlim=c(80,90), xlab="Years of Age",ylab="Predicted Cognition")
lines(x=(PlotModel3$`at(age85)`+85)[1:3], y=PlotModel3$Prediction[1:3], type="l", col="blue1")
lines(x=(PlotModel3$`at(age85)`+85)[4:6], y=PlotModel3$Prediction[4:6], type="l", col="red1")
lines(x=(PlotModel3$`at(age85)`+85)[7:9], y=PlotModel3$Prediction[7:9], type="l", col="green1")
legend(x=80, y=45, legend = c("Grip=6", "Grip=9", "Grip=12"), col=1:3, lty=1) # lty=linetype
dev.off()  # close file
```

Computing and plotting regions of significance for age and grip strength slopes:

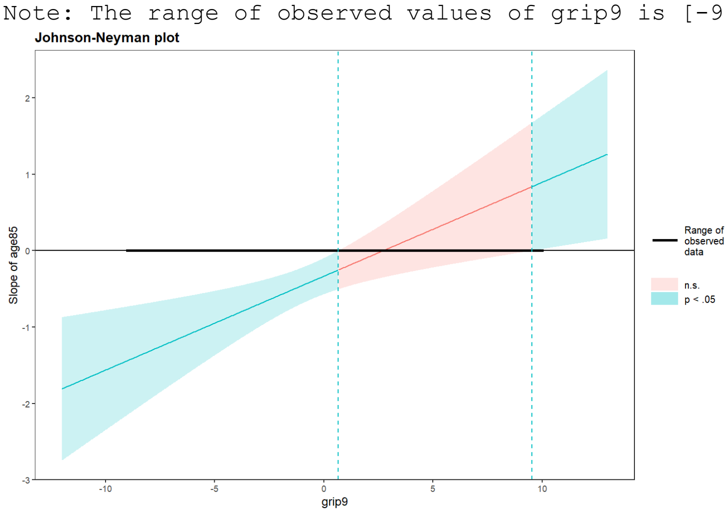
```
print("Values for regions of significance")
vcov(Model3) # Asymptotic covariance matrix of fixed effects for regions
             (Intercept)       age85      grip9     sexMW     demNF      demNC age85:grip9
(Intercept)    0.4828946   0.0004536 -0.0307533 -0.450672 -0.182029 -0.2263470   0.0019165
age85          0.0004536   0.0144857  0.0033170  0.005024 -0.004131 -0.0011511   0.0009587
grip9         -0.0307533   0.0033170  0.0221243  0.053743 -0.013388 -0.0003011   0.0002029
sexMW         -0.4506716   0.0050240  0.0537427  0.787257 -0.071016  0.0237081   0.0026947
demNF         -0.1820286  -0.0041309 -0.0133877 -0.071016  1.027449  0.2129117  -0.0026791
demNC         -0.2263470  -0.0011511 -0.0003011  0.023708  0.212912  2.2877993   0.0023964
age85:grip9    0.0019165   0.0009587  0.0002029  0.002695 -0.002679  0.0023964   0.0016432
```

In the above "**asymptotic covariance matrix of the fixed effects**" the diagonal is the SE^2 (sampling variance) for each slope, and the off-diagonals hold the covariances among the slope SE values. Bold values are needed to compute regions of significance for the age and grip strength slopes, shown next (or see excel spreadsheet).

```
print(Model3[["coefficients"]], digits=8) # Get fixed effects with more precision
(Intercept)         age85          grip9          sexMW          demNF          demNC   age85:grip9
 29.40780315   -0.33396058     0.61941863    -3.45563720    -5.92254309   -16.30040485    0.12301848
interactions::johnson_neyman(model=Model3, pred="age85", modx="grip9", digits=3, plot=TRUE)
```

```
JOHNSON-NEYMAN INTERVAL
```
→ **[9.665, 18.521] pounds per square in original metric**

```
When grip9 is OUTSIDE the interval [0.665, 9.521], the slope of age85 is p < .05.
Note: The range of observed values of grip9 is [-9.000, 10.000]
```

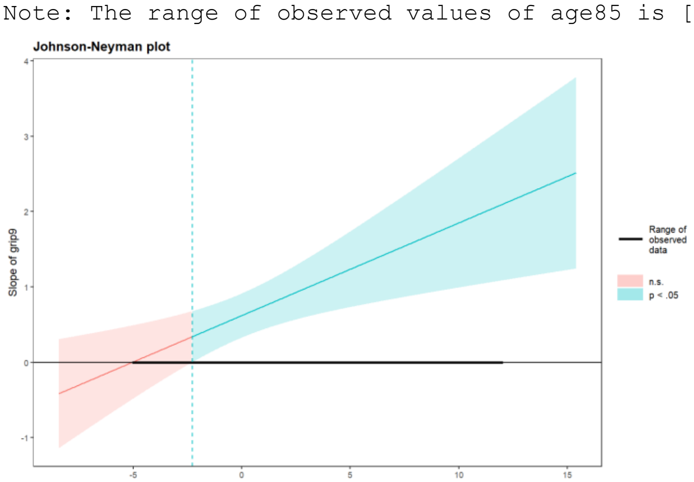


This result indicates that the age slope will be significantly negative below grip = 9.665 pounds, nonsignificant between grip = 9.665 and 18.521 pounds, and significantly positive after grip = 18.521 pounds.

```
interactions::johnson_neyman(model=Model3, pred="grip9", modx="age85", digits=3, plot=TRUE)
```

```
JOHNSON-NEYMAN INTERVAL
```
→ **[70.127, 82.719] years in original metric**

```
When age85 is OUTSIDE the interval [-14.873, -2.281], the slope of grip9 is p < .05.
Note: The range of observed values of age85 is [-4.984, 11.967]
```



This result indicates that the grip strength slope will be significantly negative below age = 70.127 years, nonsignificant between age = 70.127 and 82.719 years, and significantly positive after age = 82.719 years.

```
print("Simple slope boundaries for age and grip given by regions of significance")
RegionSlopesModel3 = multcomp::glht(model=Model3, linfct=rbind(
  "Age Slope at Grip =  9.665"  = c(0,1,0,0,0,0, 0.665),  # Multipliers in order of fixed effects
  "Age Slope at Grip = 18.521"  = c(0,1,0,0,0,0, 9.521),
  "Grip Slope at Age = 70.127"  = c(0,0,1,0,0,0,-14.873),
  "Grip Slope at Age = 82.719"  = c(0,0,1,0,0,0, -2.281)))
obj=glhtSummary(glhtObject=RegionSlopesModel3, effectsizes=TRUE) # Custom output

Linear Combinations Table
                              Est    SE      t     p    LCI    UCI
Age Slope at Grip =  9.665 -0.252 0.128 -1.964 0.050 -0.504  0.000
Age Slope at Grip = 18.521  0.837 0.426  1.964 0.050 -0.000  1.675
Grip Slope at Age = 70.127 -1.210 0.616 -1.964 0.050 -2.420 -0.000
Grip Slope at Age = 82.719  0.339 0.172  1.964 0.050  0.000  0.678

Effect Sizes for Linear Combinations Table
                              Est    SE     p      d     pr   sR2
Age Slope at Grip =  9.665 -0.252 0.128 0.050 -0.169 -0.084 0.005
Age Slope at Grip = 18.521  0.837 0.426 0.050  0.169  0.084 0.005
Grip Slope at Age = 70.127 -1.210 0.616 0.050 -0.169 -0.084 0.005
Grip Slope at Age = 82.719  0.339 0.172 0.050  0.169  0.084 0.005
```

Example Results Section Part 3

Age by Grip Strength Interaction

We then estimated a new model (as shown in Equation 3) adding an interaction between age and grip strength to examine the extent to which the slope of age varied by grip strength (or equivalently, how the slope of grip strength varied by age). The augmented model accounted for a significant amount of variance in cognition, $F(6, 543) = 36.85$, MSE = 86.67, $p < .0001$, $R^2 = .289$. Cognition was predicted to be significantly lower by 3.46 in women ($sR^2 = .020$). The overall effect of dementia group remained significant, $F(2, 543) = 67.70$, $p < .001$, $sR^2 = .177$. Relative to the reference group of persons with no diagnosis, cognition was significantly lower by 5.92 in the future diagnosis group, significantly lower by 16.30 in the current diagnosis group, and cognition was significantly lower by 10.38 in the future than current diagnosis group. The age by grip strength interaction was significant and added .012 to the R^2 relative to the main-effects-only Model 1. The pattern of the age by grip strength interaction is described below.

The simple slope of age given directly by the model indicated that cognition was predicted to be significantly lower by 0.33 per additional year of age (in persons with a grip strength of 9 pounds per square inch). The simple slope of grip strength given directly by the model indicated that cognition was predicted to be significantly greater by 0.62 per additional pound of grip strength (in persons who are age 85). As shown in Figure 1, the age by grip strength interaction indicated the age slope predicting cognition became significantly less negative by 0.12 per additional pound per square inch of grip strength (as shown by the differences in the slopes of the lines). Equivalently, the grip strength slope predicting cognition became significantly more positive by 0.12 per additional year of age (as shown by the differences in the vertical distance between the lines in Figure 1, or the differences in the slopes of the lines in Figure 2).

To further describe the age by grip strength interaction, the regions along each moderator through which the other main effect is expected to be significant were then calculated using the fixed effect estimates and their asymptotic covariance matrix (see Hoffman, 2015). For the effect of age, the threshold values of grip strength were 9.67 and 18.52 pounds per square inch. Given the range of grip strength of 0–19 pounds per square inch in the current sample (M ≈ 9 pounds per square inch), the slope of age is expected to be negative for about half the sample (< 9.67 pounds per square inch), to be nonsignificant for the other half (9.67–18.52 pounds per square inch), and to be positive for almost no one (> 18.52 pounds per square inch). Similarly, for the effect of grip strength, the threshold values of age were 70.13 and 82.72 years. Given the range of age of 80–97 years in the sample (M ≈ 85 years), the slope of grip strength is expected to be negative for no one (< 70.18 years), to be nonsignificant for a small part of the sample (70.18–82.71 years), and to be positive for the majority of the sample (> 82.71 years).

Table 2

*Results from Age*Grip Strength Model in Equation 3*

	Fixed Effect	Est	SE	*p* <	Cohen's *d*	Partial *r*
β_0	Intercept	29.408	0.695			
β_1	Age (0=85 years)	−0.334	0.120	.006	−0.238	−0.118
β_2	Grip (0=9 pounds)	0.619	0.149	.001	0.357	0.176
β_6	Age * Grip	0.123	0.041	.003	0.260	0.129
β_3	Sex (0=Men)	−3.456	0.887	.001	−0.334	−0.165
β_4	None vs Future Dementia	−5.923	1.014	.001	−0.501	−0.243
β_5	None vs Current Dementia	−16.300	1.513	.001	−0.925	−0.420
$\beta_5 - \beta_4$	Future vs Current Dementia	−10.378	1.700	.001	−0.524	−0.253

Note: Est = estimate, SE = standard error. Cohen's *d* and partial *r* effect sizes were computed from the slope of each *t* test-statistic as follows: $d = \frac{2t}{\sqrt{DF_{den}}}$; $r = \frac{t}{\sqrt{t^2+DF_{den}}}$.

NONE OF WHAT FOLLOWS IS NEEDED FOR HW05, but this model provides an example of a categorical*binary (or quantitative) predictor interaction…

Model 4 adding interaction of sex by demgroup (Equation 2.13 in chapter 2): Does the effect of sex on cognition vary by dementia group (or does the effect of dementia on cognition vary by sex)?

Adjusted means holding age=85 and grip=9:

Dementia Group	**Men**	Women	Marginal Mean
None	29.07	26.20	27.63
Future	23.01	20.30	21.66
Current	17.10	6.35	11.72
Marginal Mean	23.03	17.62	

Equation 4: $$\boldsymbol{Cognition_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(Age_i - 85)(Grip_i - 9) + \beta_7(SexMW_i)(DemNF_i) + \beta_8(SexMW_i)(DemNC_i) + e_i}$$

```
print("Model 4: Add Sex by Dementia Group Interaction (Equation 2.13)")
print("Binary Predictors for Sex (0=Men) and Demgroup (0=None)")
Model4 = lm(data=Example5, formula=cognition~1+age85+grip9+sexMW+demNF+demNC +age85:grip9
                                              +sexMW:demNF +sexMW:demNC)

# Get F-test and effect sizes for change in R2 for two new interactions
obj=R2compare(ReducedModel=Model3, FullModel=Model4, PredName="Sex*Demgroup")

F-Test and R2 Change for Sex*Demgroup Slopes
 R2reduced R2full R2diff DFnum DFden     F     p   pR2   sR2
     0.289  0.298  0.009     2   541 3.492 0.031 0.013 0.009

# Get F-test and effect sizes for change in R2 for age, grip, and age*grip
Model4NoAgeGrip = lm(data=Example5, formula=cognition~1+sexMW+sexMW:demNF +sexMW:demNC)
obj=R2compare(ReducedModel=Model4NoAgeGrip, FullModel=Model4,
              PredName="Age, Grip, and Age*Grip")

F-Test and R2 Change for Age, Grip, and Age*Grip Slopes
 R2reduced R2full R2diff DFnum DFden      F      p   pR2   sR2
     0.193  0.298  0.106     5   541 16.329 <0.001 0.131 0.106
```

```
obj=LMsummary(Model4, effectsizes=TRUE)  # Custom output

Sums of Squares Table
             SS  DF       MS      F      p    R2
Model 19785.461   8 2473.183 28.767 <0.001 0.298
Error 46511.077 541   85.972
Total 66296.538 549  120.759

Fixed Effects Table
                Est    SE      t      p     LCI    UCI
Intercept    29.070 0.748 38.838 <0.001  27.600 30.540  beta0
age85        -0.335 0.120 -2.793  0.005  -0.570 -0.099  beta1
grip9         0.618 0.148  4.173 <0.001   0.327  0.909  beta2
sexMW        -2.876 1.011 -2.844  0.005  -4.862 -0.889  beta3
demNF        -6.056 1.635 -3.704 <0.001  -9.268 -2.844  beta4
demNC       -11.971 2.245 -5.332 <0.001 -16.381 -7.561  beta5
age85:grip9   0.122 0.040  3.027  0.003   0.043  0.201  beta6
sexMW:demNF   0.164 2.070  0.079  0.937  -3.903  4.231  beta7
sexMW:demNC  -7.875 3.025 -2.604  0.009 -13.816 -1.934  beta8

Effect Sizes for Fixed Effects Table
                Est    SE      p      d     pr   sR2
age85        -0.335 0.120  0.005 -0.240 -0.119 0.010
grip9         0.618 0.148 <0.001  0.359  0.177 0.023
sexMW        -2.876 1.011  0.005 -0.245 -0.121 0.010
demNF        -6.056 1.635 <0.001 -0.318 -0.157 0.018
demNC       -11.971 2.245 <0.001 -0.459 -0.223 0.037
age85:grip9   0.122 0.040  0.003  0.260  0.129 0.012
sexMW:demNF   0.164 2.070  0.937  0.007  0.003 0.000
sexMW:demNC  -7.875 3.025  0.009 -0.224 -0.111 0.000
```

Dementia Group	**Men**	Women	Marginal Mean
None	29.07	26.20	27.63
Future	23.01	20.30	21.66
Current	17.10	6.35	11.72
Marginal Mean	23.03	17.62	

Interpret β_3 for sexMW:

Interpret β_7 for sexMW*demNF with demNF as moderator:

Interpret β_8 for sexMW*demNF with demNC as moderator:

Interpret β_4 for demNF:

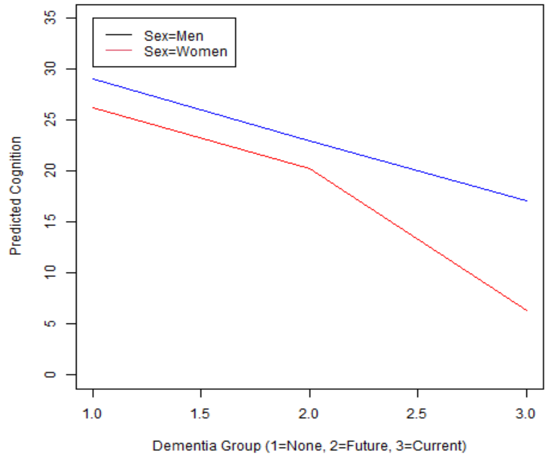
Interpret β_7 for sexMW*demNF with sexMW as moderator:

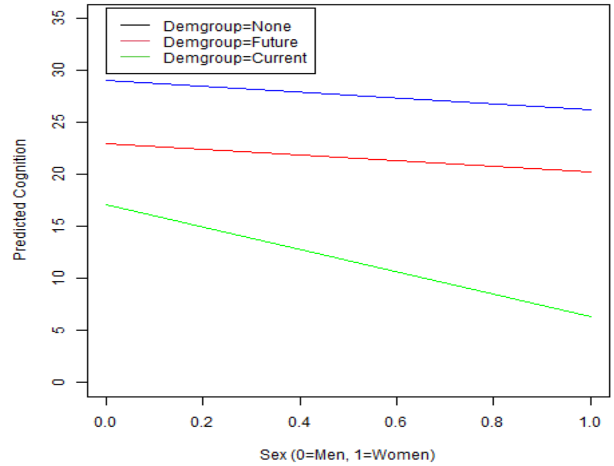
Interpret β_5 for demNC:

Interpret β_8 for sexMW*demNC with sexMW as moderator:

What simple slopes are we missing?

(Figure 3 and 4 for the results)





Computing and plotting predicted outcomes:

```
print("Pred cognition outcomes --adjusted cell means-- holding age=85 and grip=9")
print("Will need to ignore impossible combinations of demNF and demNC for min:max")
PredModel4 = summary(prediction(model=Model4, type="response", at=list(age85=0, grip9=0,
                        sexMW=0:1, demNF=0:1, demNC=0:1))); print(PredModel4, digits=6)

at(age85) at(grip9) at(sexMW) at(demNF) at(demNC) Prediction     SE       z         p  lower upper
        0         0         0         0         0    29.0701 0.7485 38.8379 0.000e+00 27.603 30.54
        0         0         1         0         0    26.1946 0.6388 41.0037 0.000e+00 24.942 27.45
        0         0         0         1         0    23.0142 1.4928 15.4172 1.253e-53 20.088 25.94
        0         0         1         1         0    20.3029 1.1186 18.1498 1.290e-73 18.110 22.50
        0         0         0         0         1    17.0994 2.1402  7.9896 1.354e-15 12.905 21.29
        0         0         1         0         1     6.3487 1.9479  3.2593 1.117e-03  2.531 10.17
        0         0         0         1         1    11.0435 2.6964  4.0956 4.211e-05  5.759 16.33
        0         0         1         1         1     0.4571 2.3179  0.1972 8.437e-01 -4.086  5.00

print("Create data frame for plotting and remove last 2 unneeded rows")
PredModel4 = data.frame(PredModel4) # first remove () from variable names
PredModel4$sum = PredModel4$at.demNF.+PredModel4$at.demNC. # sum dummy codes
PredModel4 = subset(x=PredModel4, PredModel4$sum<2) # keep if sum<2
# Make demgroup combined variable for plot
PredModel4$demgroup=NA # Make new empty variable to be recoded
PredModel4$demgroup[which(PredModel4$at.demNF.==0 & PredModel4$at.demNC.==0)]=1
PredModel4$demgroup[which(PredModel4$at.demNF.==1 & PredModel4$at.demNC.==0)]=2
PredModel4$demgroup[which(PredModel4$at.demNF.==0 & PredModel4$at.demNC.==1)]=3
# Make and save plots
png(file = "Sex by Demgroup=x GLM Plot.png")  # open file
plot(y=PredModel4$Prediction, x=PredModel4$demgroup, type="n", ylim=c(0,35), xlim=c(1,3),
     xlab="Dementia Group (1=None, 2=Future, 3=Current)", ylab="Predicted Cognition")
PredModel4 = PredModel4[order(PredModel4$at.sexMW.), ] # 3 rows per sexMW now
lines(x=PredModel4$demgroup[1:3], y=PredModel4$Prediction[1:3], type="l", col="blue1")
lines(x=PredModel4$demgroup[4:6], y=PredModel4$Prediction[4:6], type="l", col="red1")
legend(x=1, y=35, legend = c("Sex=Men", "Sex=Women"), col=1:2, lty=1) #lty=linetype
dev.off()  # close file

png(file = "Demgroup by Sex=x GLM Plot.png")  # open file
plot(y=PredModel4$Prediction, x=PredModel4$at.sexMW., type="n", ylim=c(0,35), xlim=c(0,1),
     xlab="Sex (0=Men, 1=Women)", ylab="Predicted Cognition")
PredModel4 = PredModel4[order(PredModel4$demgroup), ] # 2 rows per demgroup now
lines(x=PredModel4$at.sexMW.[1:2], y=PredModel4$Prediction[1:2], type="l", col="blue1")
lines(x=PredModel4$at.sexMW.[3:4], y=PredModel4$Prediction[3:4], type="l", col="red1")
lines(x=PredModel4$at.sexMW.[5:6], y=PredModel4$Prediction[5:6], type="l", col="green1")
legend(x=0, y=36, legend = c("Demgroup=None", "Demgroup=Future", "Demgroup=Current"),
       col=1:3, lty=1) #lty=linetype
dev.off()  # close file
```

Computing predicted outcomes, all possible conditional main effect ("simple") slopes (3 for sex, 6 for dementia), and all possible interaction slopes (3):

$$\widehat{Cognition}_i = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(Age_i - 85)(Grip_i - 9) + \beta_7(SexMW_i)(DemNF_i) + \beta_8(SexMW_i)(DemNC_i)$$

```
# Here is another way of using GLHT create predicted outcomes
print("GLHT pred cognition outcomes --adjusted cell means-- holding age=85 and grip=9")
Pred2Model4 = multcomp::glht(model=Model4, linfct=rbind(
  "Yhat for Men   None"     = c(1,0,0,0,0,0,0,0,0),  # in order of fixed effects
  "Yhat for Women None"     = c(1,0,0,1,0,0,0,0,0),
  "Yhat for Men Future"     = c(1,0,0,0,1,0,0,0,0),
  "Yhat for Women Future"   = c(1,0,0,1,1,0,0,1,0),
  "Yhat for Men Current"    = c(1,0,0,0,0,1,0,0,0),
  "Yhat for Women Current" = c(1,0,0,1,0,1,0,0,1)))
obj=glhtSummary(glhtObject=Pred2Model4) # Custom output

Linear Combinations Table
                          Est    SE      t      p    LCI    UCI
Yhat for Men   None    29.070 0.748 38.838 <0.001 27.600 30.540
Yhat for Women None    26.195 0.639 41.004 <0.001 24.940 27.449
Yhat for Men Future    23.014 1.493 15.417 <0.001 20.082 25.947
Yhat for Women Future  20.303 1.119 18.150 <0.001 18.106 22.500
Yhat for Men Current   17.099 2.140  7.990 <0.001 12.895 21.304
Yhat for Women Current  6.349 1.948  3.259  0.001  2.522 10.175

print("DF=1 simple slopes for sex per demgroup, demgroup per sex, and interactions")
SlopesModel4 = multcomp::glht(model=Model4, linfct=rbind(
```

We can use the model equation to calculate the **simple sex slope** for any *dementia group* (as the moderator):

Simple Sex Slope = $\beta_3(SexMW_i) + \beta_7(SexMW_i)(DemNF_i) + \beta_8(SexMW_i)(DemNC_i)$
= $[\beta_3 + \beta_7(DemNF_i) + \beta_8(DemNC_i)]$ that multiplies $(SexMW_i)$

```
  "Sex Diff for No Dementia"      = c(0,0,0,1, 0,0,0, 0,0),  # in order of fixed effects
  "Sex Diff for Future Dementia"  = c(0,0,0,1, 0,0,0, 1,0),
  "Sex Diff for Current Dementia" = c(0,0,0,1, 0,0,0, 0,1),
```

We can use the model equation to calculate the **simple dementia slope** for any *sex* (as the moderator):

Simple None vs. Future Slope = $\beta_4(DemNF_i) + \beta_7(SexMW_i)(DemNF_i)$
= $[\beta_4 + \beta_7(SexMW_i)]$ that multiplies $(DemNF_i)$

Simple None vs. Current Slope = $\beta_5(DemNC_i) + \beta_8(SexMW_i)(DemNC_i)$
= $[\beta_5 + \beta_8(SexMW_i)]$ that multiplies $(DemNC_i)$

Simple Future vs. Current Slope = $[\beta_5 + \beta_8(SexMW_i)] - [\beta_4 + \beta_7(SexMW_i)]$

```
  "None-Future Diff for Men"      = c(0,0,0,0, 1,0,0, 0,0),
  "None-Future Diff for Women"    = c(0,0,0,0, 1,0,0, 1,0),

  "None-Current Diff for Men"     = c(0,0,0,0, 0,1,0, 0,0),
  "None-Current Diff for Women"   = c(0,0,0,0, 0,1,0, 0,1),

  "Future-Current Diff for Men"   = c(0,0,0,0,-1,1,0, 0,0),
  "Future-Current Diff for Women" = c(0,0,0,0,-1,1,0,-1,1),
```

The interaction slopes are then computed as the difference between the relevant simple slopes—A and B are directly given as model fixed slopes, whereas C is found as a linear combination (each explained two ways):

```
  "A: Sex effect differ btw None and Future?"          = c(0,0,0,0,0,0,0, 1,0),
  "A: None-Future effect differ btw Men and Women?"    = c(0,0,0,0,0,0,0, 1,0),

  "B: Sex effect differ btw None and Current?"         = c(0,0,0,0,0,0,0, 0,1),
  "B: None-Current effect differ btw Men and Women?"   = c(0,0,0,0,0,0,0, 0,1),

  "C: Sex effect differ btw Future and Current?"       = c(0,0,0,0,0,0,0,-1,1),
  "C: Future-Current effect differ btw Men and Women?" = c(0,0,0,0,0,0,0,-1,1)))

obj=glhtSummary(glhtObject=SlopesModel4, effectsizes=TRUE) # Custom output
```

```
Linear Combinations Table
                                                        Est    SE      t      p     LCI     UCI
Sex Diff for No Dementia                             -2.876 1.011 -2.844  0.005  -4.862  -0.889
Sex Diff for Future Dementia                         -2.711 1.874 -1.447  0.149  -6.393   0.970
Sex Diff for Current Dementia                       -10.751 2.899 -3.708 <0.001 -16.446  -5.055

None-Future Diff for Men                             -6.056 1.635 -3.704 <0.001  -9.268  -2.844
None-Future Diff for Women                           -5.892 1.278 -4.611 <0.001  -8.402  -3.382
None-Current Diff for Men                           -11.971 2.245 -5.332 <0.001 -16.381  -7.561
None-Current Diff for Women                         -19.846 2.029 -9.783 <0.001 -23.831 -15.861
Future-Current Diff for Men                          -5.915 2.587 -2.287  0.023 -10.996  -0.834
Future-Current Diff for Women                       -13.954 2.239 -6.233 <0.001 -18.352  -9.556

A: Sex effect differ btw None and Future?             0.164 2.070  0.079  0.937  -3.903   4.231
A: None-Future effect differ btw Men and Women?       0.164 2.070  0.079  0.937  -3.903   4.231
B: Sex effect differ btw None and Current?           -7.875 3.025 -2.604  0.009 -13.816  -1.934
B: None-Current effect differ btw Men and Women?     -7.875 3.025 -2.604  0.009 -13.816  -1.934
C: Sex effect differ btw Future and Current?         -8.039 3.415 -2.354  0.019 -14.748  -1.331
C: Future-Current effect differ btw Men and Women?   -8.039 3.415 -2.354  0.019 -14.748  -1.331

Effect Sizes for Linear Combinations Table
                                                        Est    SE      p      d     pr   sR2
Sex Diff for No Dementia                             -2.876 1.011  0.005 -0.245 -0.121 0.010
Sex Diff for Future Dementia                         -2.711 1.874  0.149 -0.124 -0.062 0.003
Sex Diff for Current Dementia                       -10.751 2.899 <0.001 -0.319 -0.157 0.018

None-Future Diff for Men                             -6.056 1.635 <0.001 -0.318 -0.157 0.018
None-Future Diff for Women                           -5.892 1.278 <0.001 -0.396 -0.194 0.028
None-Current Diff for Men                           -11.971 2.245 <0.001 -0.459 -0.223 0.037
None-Current Diff for Women                         -19.846 2.029 <0.001 -0.841 -0.388 0.124
Future-Current Diff for Men                          -5.915 2.587  0.023 -0.197 -0.098 0.007
Future-Current Diff for Women                       -13.954 2.239 <0.001 -0.536 -0.259 0.050


A: Sex effect differ btw None and Future?             0.164 2.070  0.937  0.007  0.003 0.000
A: None-Future effect differ btw Men and Women?       0.164 2.070  0.937  0.007  0.003 0.000
B: Sex effect differ btw None and Current?           -7.875 3.025  0.009 -0.224 -0.111 0.009
B: None-Current effect differ btw Men and Women?     -7.875 3.025  0.009 -0.224 -0.111 0.009
C: Sex effect differ btw Future and Current?         -8.039 3.415  0.019 -0.202 -0.101 0.007
C: Future-Current effect differ btw Men and Women?   -8.039 3.415  0.019 -0.202 -0.101 0.007
```

Computing “omnibus” tests for the overall dementia group difference by sex:

$$\widehat{Cognition_i} = \beta_0 + \beta_1(Age_i - 85) + \beta_2(Grip_i - 9) + \beta_3(SexMW_i) + \beta_4(DemNF_i) + \beta_5(DemNC_i) + \beta_6(Age_i - 85)(Grip_i - 9) + \beta_7(SexMW_i)(DemNF_i) + \beta_8(SexMW_i)(DemNC_i)$$

```
print("Omnibus DF=2 F-test for Dementia Simple Main Effect for Men")
print("Comma separates each fixed effect to be tested jointly, already in model here")
DemforM = multcomp::glht(model=Model4,
                         linfct=rbind(c(0,0,0,0,1,0,0,0,0),c(0,0,0,0,0,1,0,0,0)))
print(summary(DemforM, test=Ftest()), digits=4) # joint hypothesis test instead of separate
```

```
Global Test:
      F DF1 DF2          Pr(>F)
1 18.69   2 541 0.00000001419
```

```
print("Omnibus DF=2 F-test for Dementia Simple Main Effect for Women")
print("Comma separates each fixed effect to be tested jointly, as a linear combination")
DemforW = multcomp::glht(model=Model4,
                         linfct=rbind(c(0,0,0,0,1,0,0,1,0),c(0,0,0,0,0,1,0,0,1)))
print(summary(DemforM, test=Ftest()), digits=4) # joint hypothesis test instead of separate
```

```
Global Test:
      F DF1 DF2     Pr(>F)
1 53.16   2 541 8.378e-22
```

Computing regions of significance for age and grip strength slopes from Model 4:

```
Fixed Effects Table
                Est    SE       t      p     LCI    UCI
Intercept    29.070 0.748  38.838 <0.001  27.600 30.540  beta0
age85        -0.335 0.120  -2.793  0.005  -0.570 -0.099  beta1
grip9         0.618 0.148   4.173 <0.001   0.327  0.909  beta2
sexMW        -2.876 1.011  -2.844  0.005  -4.862 -0.889  beta3
demNF        -6.056 1.635  -3.704 <0.001  -9.268 -2.844  beta4
demNC       -11.971 2.245  -5.332 <0.001 -16.381 -7.561  beta5
age85:grip9   0.122 0.040   3.027  0.003   0.043  0.201  beta6
sexMW:demNF   0.164 2.070   0.079  0.937  -3.903  4.231  beta7
sexMW:demNC  -7.875 3.025  -2.604  0.009 -13.816 -1.934  beta8
```

```
johnson_neyman(model=Model4, pred="age85", modx="grip9", digits=3, plot=FALSE)
```

JOHNSON-NEYMAN INTERVAL → **[9.680, 18.638] pounds per square inch in original metric**

```
When grip9 is OUTSIDE the interval [0.680, 9.521], the slope of age85 is p < .05.
Note: The range of observed values of grip9 is [-9.000, 10.000]
```

This result indicates that the age slope will be significantly negative below grip = 9.680 pounds, nonsignificant between grip = 9.680 and 18.638 pounds, and significantly positive after grip = 18.638 pounds.

```
johnson_neyman(model=Model4, pred="grip9", modx="age85", digits=3, plot=FALSE)
```

JOHNSON-NEYMAN INTERVAL → **[69.997, 82.707] years in original metric**

```
When age85 is OUTSIDE the interval [-15.003, -2.293], the slope of grip9 is p < .05.
Note: The range of observed values of age85 is [-4.984, 11.967]
```

This result indicates that the grip strength slope will be significantly negative below age = 69.997 years, nonsignificant between age = 69.997 and 82.707 years, and significantly positive after age = 82.707 years.

Example Results Section Part 4

Sex by Dementia Diagnosis Interaction

We estimated a new model (as shown in Equation 4) adding an interaction between sex and dementia diagnosis to examine the extent to which the dementia group slopes differed between men and women (or equivalently, how the sex difference differed across dementia groups). The augmented model accounted for a significant amount of variance in cognition, $F(8, 541) = 28.77$, MSE = 85.97, $p < .0001$, $R^2 = .298$. Table 3 provides the model results, including the fixed effects estimated directly in the model, as well as their linear combinations that provide simple slopes by which to describe the sex by dementia group interaction. Effect sizes in Cohen's *d* (standardized mean difference) and partial *r* (correlation metric) are also provided in Table 3.

Results from Model 4 can be interpreted as follows; for clarity, model fixed effects and their linear combinations are also provided in Table 3. The intercept 29.07 is the expected cognition outcome for an 85-year-old man with 9 pounds per square inch of grip strength who will not be diagnosed with dementia later in the study. Age, grip strength, and their interaction contributed $sR^2 = .106$ to the full model R^2. The simple slope of age given directly by the model indicated that cognition is predicted to be significantly lower by 0.33 per additional year of age (in persons with grip strength of 9 pounds per square inch). The simple slope of grip strength given directly by the model indicated that cognition is predicted to be significantly greater by 0.62 per additional pound per square inch of grip strength (in persons who are age 85). As shown in [figure generated from Model 4 specifically], the age by grip strength interaction indicated that the age slope predicting cognition became significantly less negative by 0.12 per additional pound per square inch of grip strength (as shown by the difference in slope across the lines). Equivalently, the grip strength slope predicting cognition became significantly more positive by 0.12 per additional year of age (as shown by the difference in the vertical distance between the lines). To further describe the age by grip strength interaction, the regions along each moderator through which the other main effect is expected to be significant were then calculated using the fixed effect estimates and their asymptotic covariance matrix (see Hoffman, 2015). For the effect of age, the threshold values of grip strength were 9.68 and 18.64 pounds per square inch. Given the range of grip strength of 0–19 pounds per square inch in the current sample (M ≈ 9 pounds per square inch), the effect of age is expected to be negative for about half the sample (< 9.68 pounds per square inch), the effect of age is expected to be nonsignificant for the other half (9.68–18.64 pounds per square inch), and the effect of age is expected to be positive for almost no one (> 18.64 pounds per square inch). Similarly, for the effect of grip strength, the threshold values of age were 70.00 and 82.71 years. Given the range of age of 80–97 years in the sample (M ≈ 85 years), the effect of grip strength is expected to be negative for no one (< 70.00 years), the effect of grip strength is expected to be nonsignificant for a small part of the sample (70.00–82.71 years), and the effect of grip strength is expected to be positive for the majority of the sample (> 82.71 years).

The main and interactive effects of sex by dementia group are presented next, as illustrated in Figure 3, in which the sex differences are shown by the vertical distance between the lines, and the dementia group differences are shown by the difference within the lines. [Figure 4 could also be used instead.] Given the significant sex by dementia group interaction, $F(2, 541) = 3.49$, $p = .031$, $sR^2 = .009$, simple slopes and their differences (i.e., interaction contrasts) for both sex and dementia group are reported next.

First, there was a significant simple main effect of sex in the no dementia group, in which cognition was significantly lower by 2.88 in women than in men. The sex difference in cognition was equivalent in no dementia and future dementia groups, as shown by the nonsignificant sex by no dementia vs. future dementia interaction = 0.16. However, the resulting sex difference in cognition favoring men in the future dementia group (of −2.88 + 0.16 = −2.71) was not significant, likely a result of the small number of persons with future dementia (only 20% of the sample). In addition, the sex difference in cognition was significantly larger in the current dementia group than in the no dementia group, as shown by the significant sex by no dementia vs. current dementia interaction

= −7.88, and the resulting sex difference in the current dementia group (−2.88 − 7.88 = −10.75) was also significant. The sex difference in cognition was also significantly larger in the current dementia group than in the future dementia group (−7.88 − 0.16 = −8.04).

Second, with respect to differences among dementia groups, a significant omnibus group difference was found both in men, $F(2, 541) = 18.69$, $p < .001$, and in women, $F(2, 541) = 53.16$, $p < .001$. More specifically, cognition was significantly lower in the future dementia than no dementia group, both in men (−6.06) and in women (−6.06 + 0.16 = −5.89). This group difference was equivalent across sexes, as indicated by the nonsignificant sex by no dementia vs. future dementia interaction = 0.16. Cognition was also significantly lower in the current dementia than no dementia group, both in men (−11.97) and women (−11.97 − 7.88 = −19.85). This group difference was significantly larger in women, as indicated by the sex by no dementia vs. current dementia interaction = −7.88. Finally, cognition was also significantly lower in the current dementia group than future diagnosis group, both in men (−11.97 + 6.06 = −5.91) and women (−11.97 − 7.88 + 6.06 + 0.16 = −13.95). This dementia group difference was significantly larger in women, as indicated by the additional interaction contrast (−7.88 − 0.16 = −8.04).

Table 3

*Results from Age*Grip Strength and Sex*Dementia Diagnosis Model in Equation 4*

Fixed Effect		Est	SE	*p* <	Cohen's *d*	Partial *r*
β_0	Intercept	29.07	0.75	.001		
β_1	Age Slope	−0.33	0.12	.005	−.240	−.119
β_2	Grip Strength Slope	0.62	0.15	.001	.359	.177
β_6	Age by Grip Interaction	0.12	0.04	.003	.260	.129
	Sex (0 = Men, 1 = Women) Differences:					
β_3	No Diagnosis	−2.88	1.01	.005	−.245	−.121
$\beta_3 + \beta_7$	Future Diagnosis	−2.71	1.87	.149	−.124	−.062
$\beta_3 + \beta_8$	Current Diagnosis	−10.75	2.90	.001	−.319	−.157
	Dementia Group Differences:					
	None vs. Future Diagnosis					
β_4	Men	−6.06	1.64	.001	−.318	−.157
$\beta_4 + \beta_7$	Women	−5.89	1.28	.001	−.396	−.194
β_7	Sex by None vs. Future	0.16	2.07	.937	.007	.003
	None vs. Current Diagnosis					
β_5	Men	−11.97	2.25	.001	−.459	−.223
$\beta_5 + \beta_8$	Women	−19.85	2.03	.001	−.841	−.388
β_8	Sex by None vs. Current	−7.88	3.03	.010	−.224	−.111
	Future vs. Current Diagnosis					
$\beta_5 - \beta_4$	Men	−5.91	2.59	.023	−.197	−.098
$\beta_5 - \beta_4 + \beta_8 - \beta_7$	Women	−13.95	2.24	.001	−.536	−.259
$\beta_8 - \beta_7$	Sex by Future vs. Current	−8.04	3.42	.019	−.202	−.101

Note: Est = estimate, SE = standard error. Cohen's *d* and partial *r* effect sizes were computed from the slope of each *t* test-statistic as follows: $d = \frac{2t}{\sqrt{DF_{den}}}$; $r = \frac{t}{\sqrt{t^2+DF_{den}}}$.