Higher-Order Factor Models

- Topics:
  - The Big Picture
  - Identification of higher-order models
  - Measurement models for method effects
  - Equivalent models

Sequence of Steps in CFA or IFA

1. Specify your **measurement model**(s)
   - How many factors/thetas, which items load on which factors, and whether your need any method factors or error covariances
   - For models with large numbers of items, you should start by modeling each factor in its own analysis to make sure *each* factor fits its items

2. Assess model fit, per factor, when possible (if 4+ indicators)
   - **Global model fit**: Does a one-factor model adequately fit each set of indicators thought to measure the same latent construct?
   - **Local model fit**: Are any of the leftover covariances problematic? Any items not loading well (or are too redundant) that you might drop?
   - **Reliability/Info**: Are your standardized loadings practically meaningful?

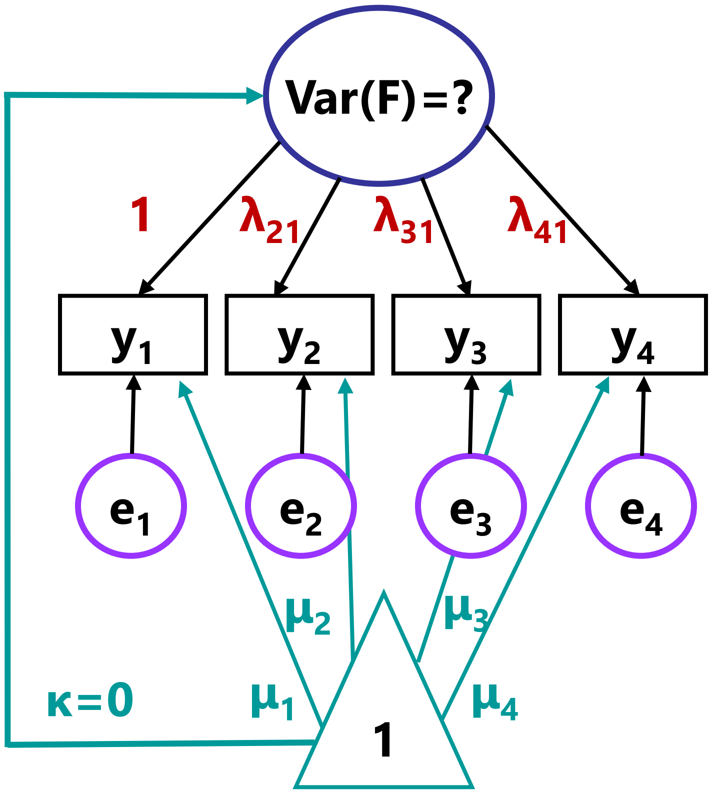
3. Once your single-factor measurement models are good, it’s time to consider the (higher-order) structural model

Higher-Order Factor Models

- Purpose: What kind of higher-order factor structure best accounts for the **covariance among the measurement model *factors* (not items)**?
  - In other words, what should the **structural model among the factors** look like?
  - Best-fitting baseline for the structural model has all possible covariances among the lower-order measurement model factors → **saturated structural model**
  - Just as the purpose of the measurement model factors is to predict covariance among the items, the **purpose of the higher-order factors is to predict covariance among the measurement model factors themselves**
  - **A single higher-order factor** would be suggested by similar magnitude of correlations across the measurement model factors

- Note that distinctions between CFA, IFA, and other measurement models for different item types are no longer relevant at this point
  - Factors and thetas are all **multivariate normal latent variables**, so a higher-order model is like a CFA regardless of the measurement model for the items
  - Latent variables don't have means apart from their items, so those are irrelevant

Necessary Measurement Model Scaling to fit Higher-Order Factors



"Marker Item" for factor loadings
→ Fix 1 item loading to 1
→ **Estimate** factor variance

Because it will become "factor variance leftover" = "disturbance", factor variance **can't be fixed** (it must be estimated)

"Z-Score" for item intercepts or thresholds
→ Fix factor mean to 0
→ Estimate all intercepts/thresholds

All the factor means will be 0 and you generally won't need to deal with them in the structural model anyway

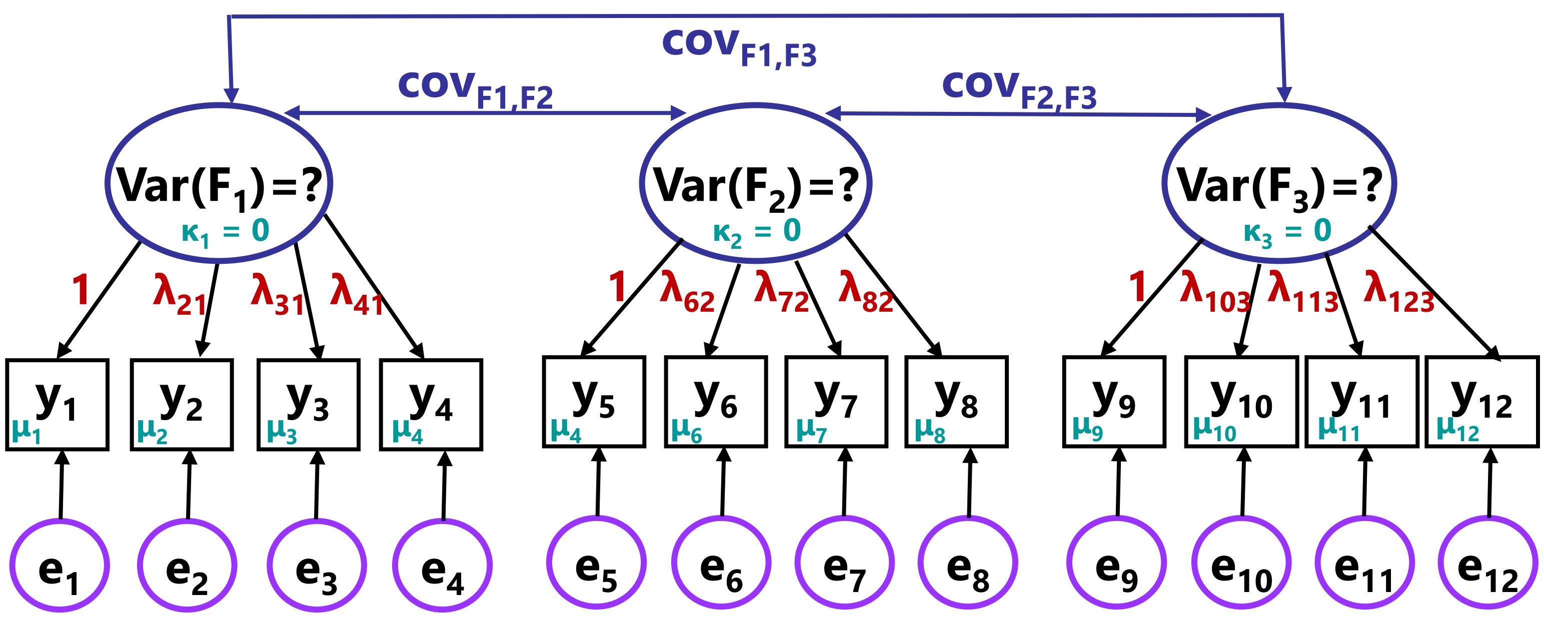
Identifying a 3-Factor Structural Model
Option 1: 3 Correlated Factors

Measurement Model for <u>Items</u>:
item variances, covariances, and means

Possible df = (12*13) / 2 + 12 = 90
Estimated df = $9\lambda + 12\mu + 12\sigma^2_e$ = 33
df = 90 – 33 = 57 → over-identified

Structural Model for <u>Factors</u>:
factor variances and covariances, no means

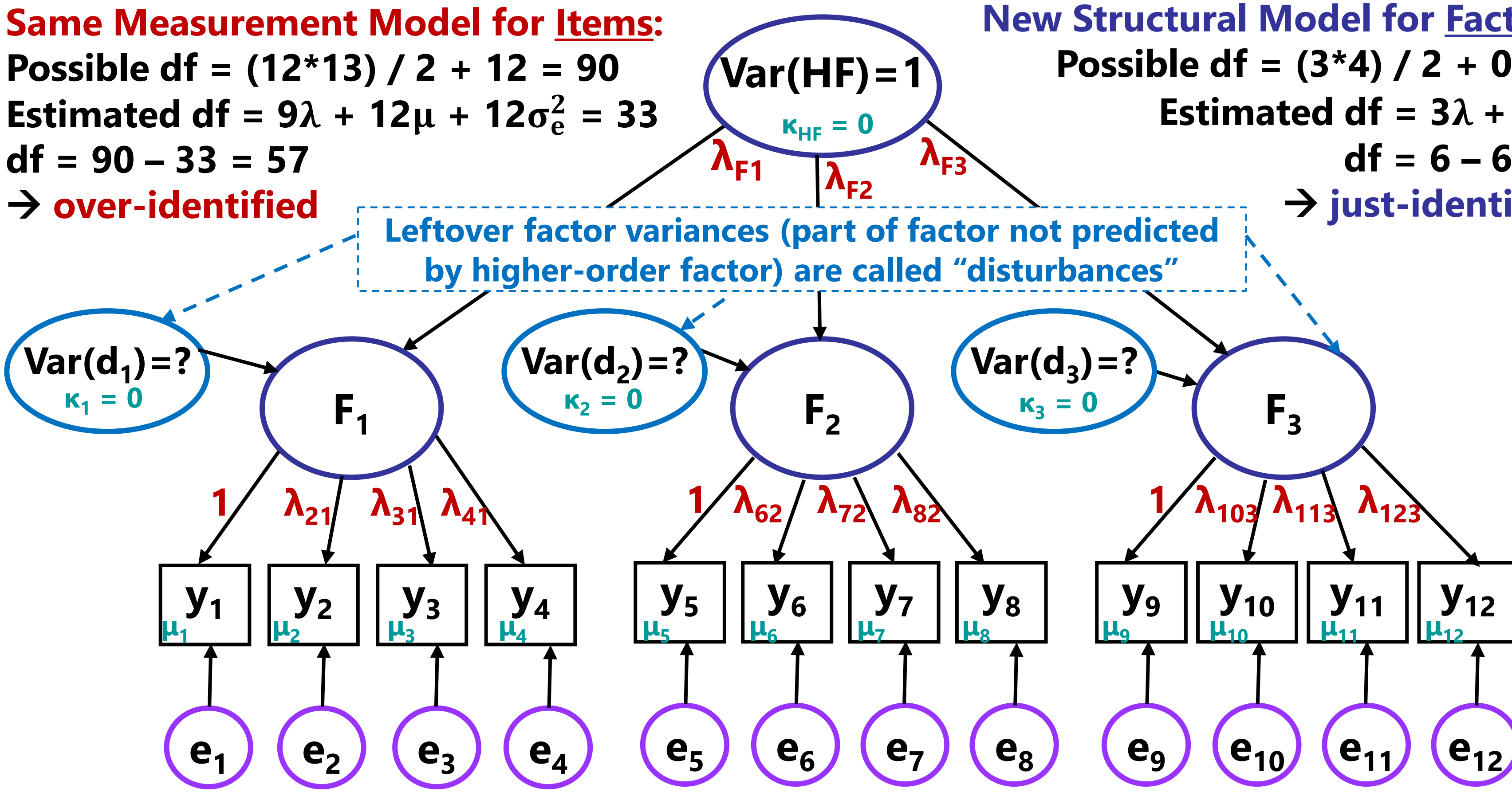
Possible df = (3*4) / 2 + 0 = 6
Estimated df = 3 variances + 3 covariances
df = 6 – 6 = 0 → just-identified



Option 2a: 3 Factor "Indicators" (Higher-Order Factor Variance = 1)

Same Measurement Model for <u>Items</u>:
Possible df = (12*13) / 2 + 12 = 90
Estimated df = $9\lambda + 12\mu + 12\sigma^2_e$ = 33
df = 90 – 33 = 57
→ over-identified

New Structural Model for <u>Factors</u>:
Possible df = (3*4) / 2 + 0 = 6
Estimated df = $3\lambda + 3\sigma^2_d$
df = 6 – 6 = 0
→ just-identified

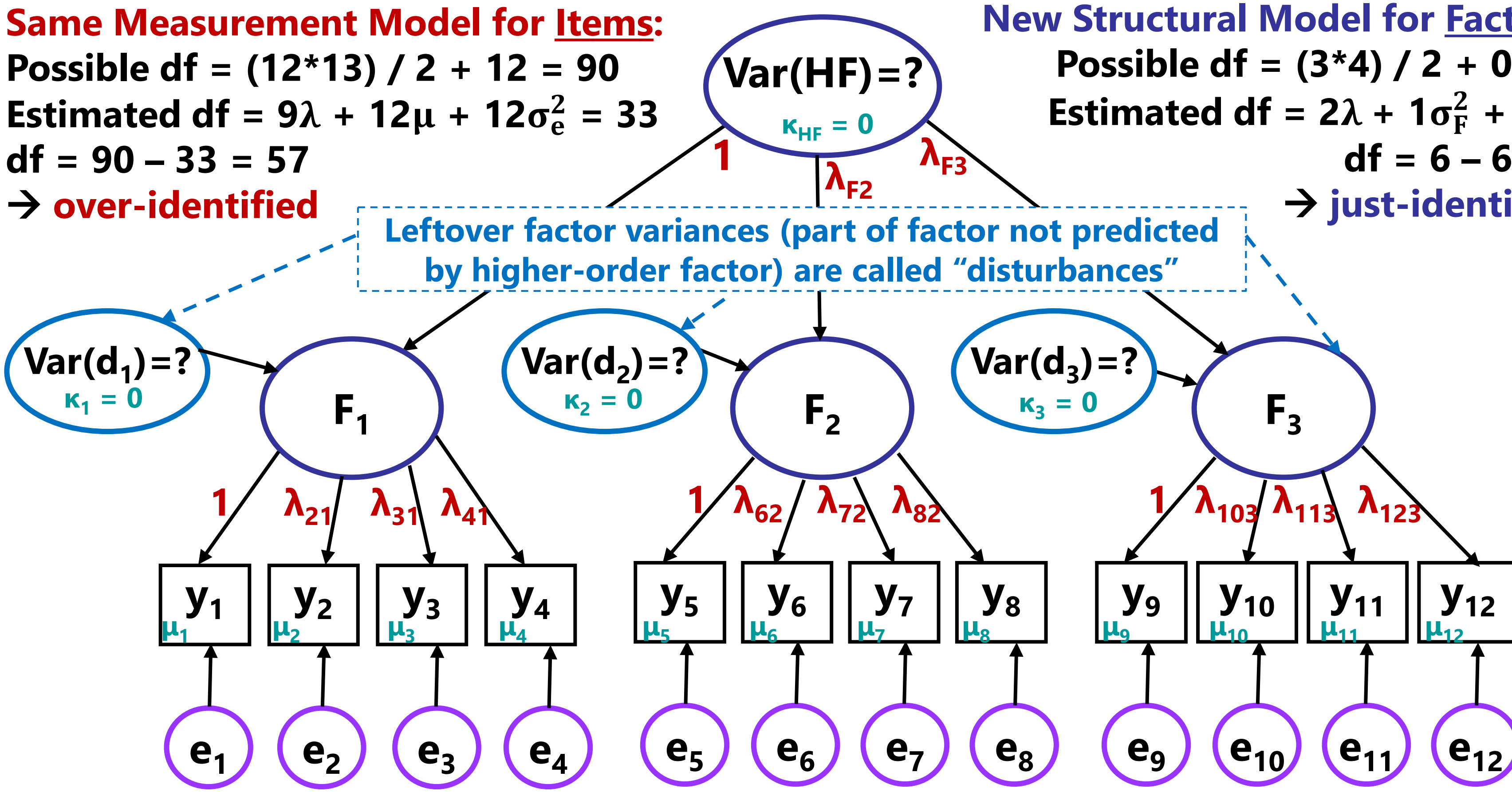


If you only have 3 factors, both models will fit the same—the structural model is just-identified, and thus <u>the fit of a higher-order factor CANNOT be tested</u>

Option 2b: 3 Factor “Indicators” (using Marker Lower-Order Factor)

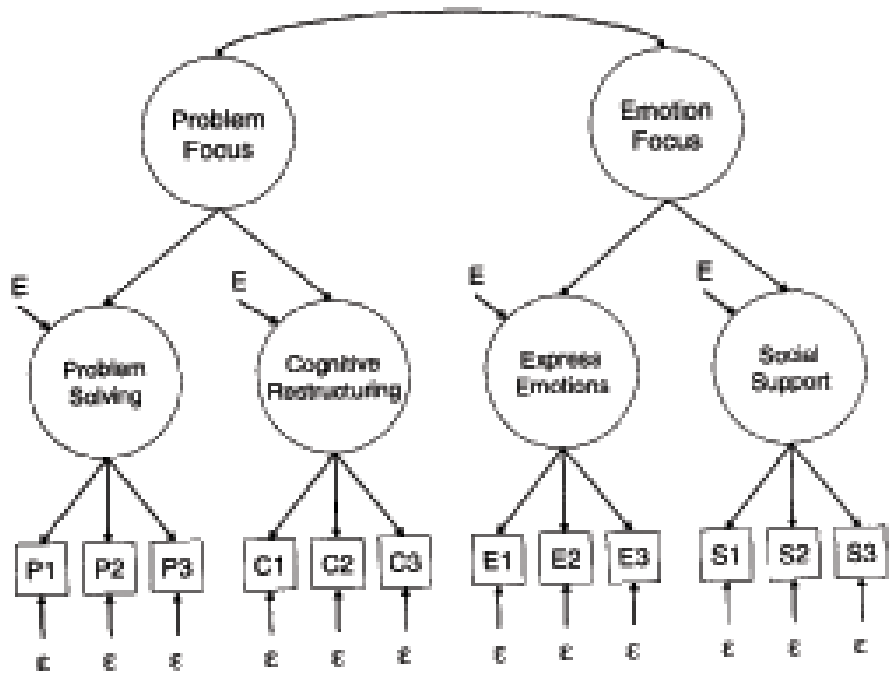
Same Measurement Model for <u>Items</u>:
Possible df = (12*13) / 2 + 12 = 90
Estimated df = $9\lambda + 12\mu + 12\sigma_e^2$ = 33
df = 90 – 33 = 57
→ over-identified

New Structural Model for <u>Factors</u>:
Possible df = (3*4) / 2 + 0 = 6
Estimated df = $2\lambda + 1\sigma_F^2 + 3\sigma_d^2$
df = 6 – 6 = 0
→ just-identified



If you only have 3 factors, both models will fit the same—the structural model is just-identified, and thus <u>the fit of a higher-order factor CANNOT be tested</u>

Structural Model Identification: 2 Factor "Indicators"



Measurement Model for Items:
Possible df = (12*13) / 2 + 12 = 90
Estimated df = 8λ + 12μ + 12σ^2_e = 32
df = 90 – 32 = 58 → over-identified

Structural Model for Factors:
Possible df = (4*5) / 2 + 0 = 10

Estimated df = 4λ + 0σ^2_F + 1$\sigma_{F,F}$ + 4σ^2_d
— OR —
Estimated df = 2λ + 2σ^2_F + 1$\sigma_{F,F}$ + 4σ^2_d

df = 10 – 9 = 1 → over-identified

However, this model will not be identified structurally unless there is a non-0 covariance between the higher-order factors

Higher-Order Factor Identification

- Possible structural df depends on # of measurement model **factor variances and covariances** (NOT # items)
  - **2 measurement model factors → Under-identified**
    - They can be correlated, which would be just-identified… that's it
  - **3 measurement model factors → Just-identified**
    - They can all be correlated OR a single higher-order factor can be fit
    - Some # variance/disturbances per factor (so, 3 total) in either option
    - Factor variances and covariances will be perfectly reproduced
  - **4 measurement model factors → Can be over-identified**
    - They can all be correlated (6 correlations required; just-identified)
    - They can have a higher-order factor (4 loadings; over-identified)
    - **The fit of the higher-order factor can now be tested**

Examples of Structural Model Hypothesis Testing

- Do I have a higher-order factor of my subscale factors?
  - If 4 or more subscale factors: Compare fit of alternative models
    - Saturated Baseline: All 6 factor covariances estimated freely
      Alternative: 1 higher-order factor instead (so df=2)—is model fit WORSE?
  - If 3 (or fewer) subscale factors: CANNOT BE DETERMINED
    - Saturated baseline and alternative models will fit equivalently
- Do I need a residual covariance, but I'm doing IFA in ML?
  - Predict those two items with a factor, fix both loadings=1 if you need a positive covariance or −1/+1 if you need a negative covariance
  - Estimate its factor variance, which then becomes the residual covariance
- Do I have need additional "method factors"?
  - Some examples…

Illustrative Example: “Life Orientation”

Table 2
Means, Standard Deviations, and Correlations for E. C. Chang et al.’s (1994) Life Orientation Test Data

Item	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7
Item 1	1.00						
Item 2	.51	1.00					
Item 3	.44	.53	1.00				
Item 4	−.16	−.22	−.26	1.00			
Item 5	−.28	−.38	−.33	.50	1.00		
Item 6	−.24	−.29	−.30	.51	.70	1.00	
Item 7	−.22	−.35	−.30	.44	.54	.52	1.00
M	2.24	2.40	2.56	1.85	1.39	1.32	1.40
SD	1.00	0.99	0.99	1.05	1.03	1.00	1.07
Skewness	−0.12	−0.35	−0.57	0.25	0.63	0.68	0.71
Kurtosis	−0.65	−0.36	−0.11	−0.72	−0.14	0.01	−0.23

Note. $N = 389$.

Table 1
Life Orientation Test (LOT) Items (E. C. Chang et al., 1994)

Item	Original item number
1. In uncertain times, I usually expect the best. (positive)	Item 1
2. I always look on the bright side of things. (positive)	Item 4
3. I’m always optimistic about my future. (positive)	Item 5
4. If something can go wrong for me, it will. (negative)	Item 3
5. I hardly ever expect things to go my way. (negative)	Item 8
6. Things never work out the way I want them to. (negative)	Item 9
7. I rarely count on good things happening to me. (negative)	Item 12

Note. The original item number is the order in which the item appears on the actual LOT questionnaire.

Maydeu-Olivares & Coffman (Psychologicial Methods, 2006) present 4 models by which to measure a latent factor of optimism using the 3 positively and 4 negatively worded items shown below

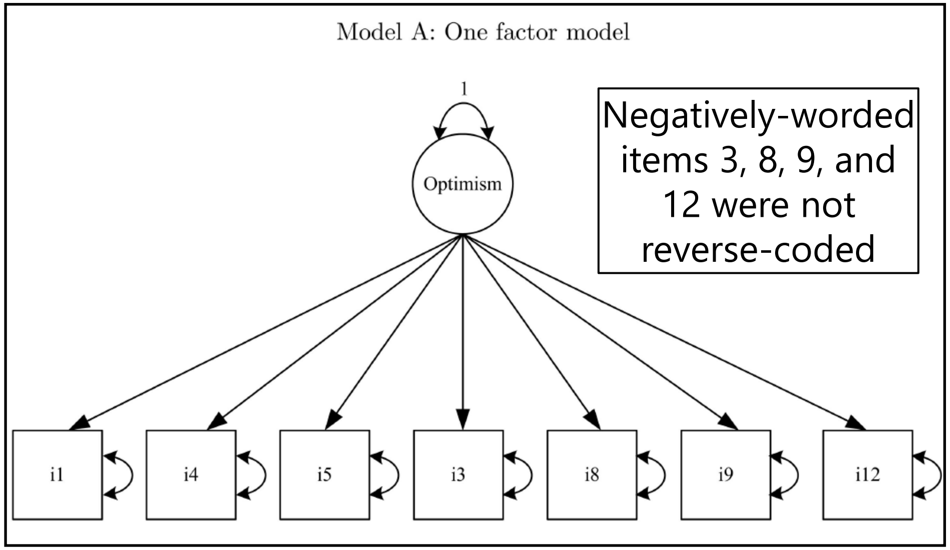
A: Single factor (df = 14)

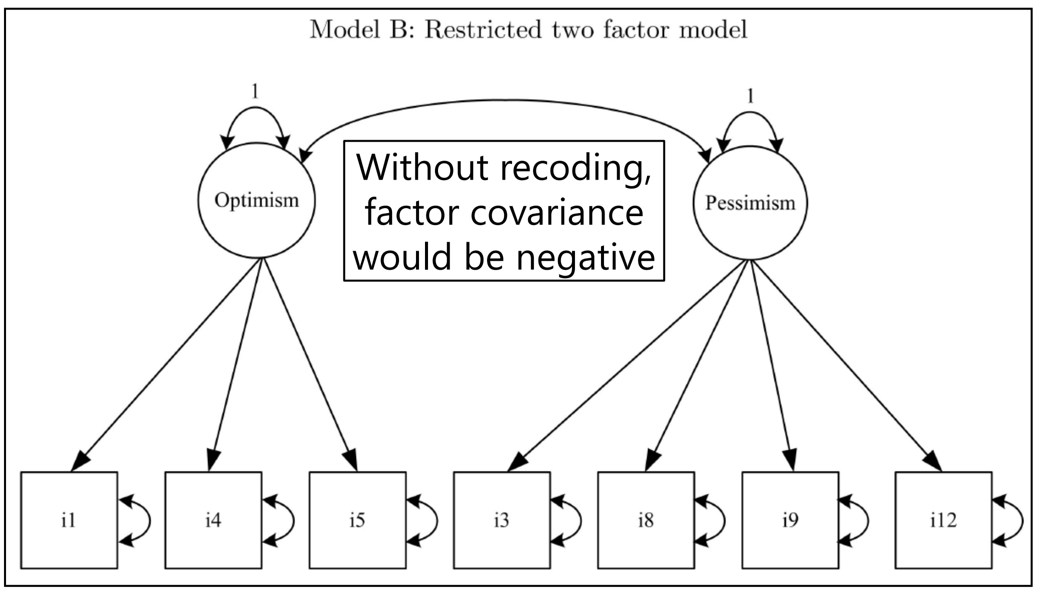
B: Two wording factors (df = 13)

C: Three-factor “Bifactor” model (df = 7)

D: “Random Intercept” 2-factor model (df = 13)

What to do with method effects?





Maydeu-Olivares & Coffman (2006) present 4 ways to measure a latent factor of optimism with 3 positively and 4 negatively worded items

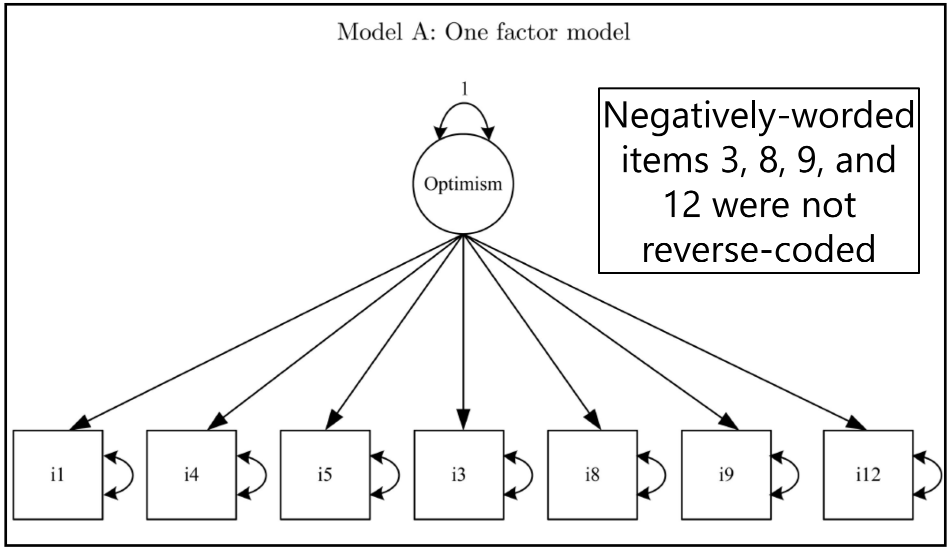
A: Single "optimism" factor (which doesn't fit well)

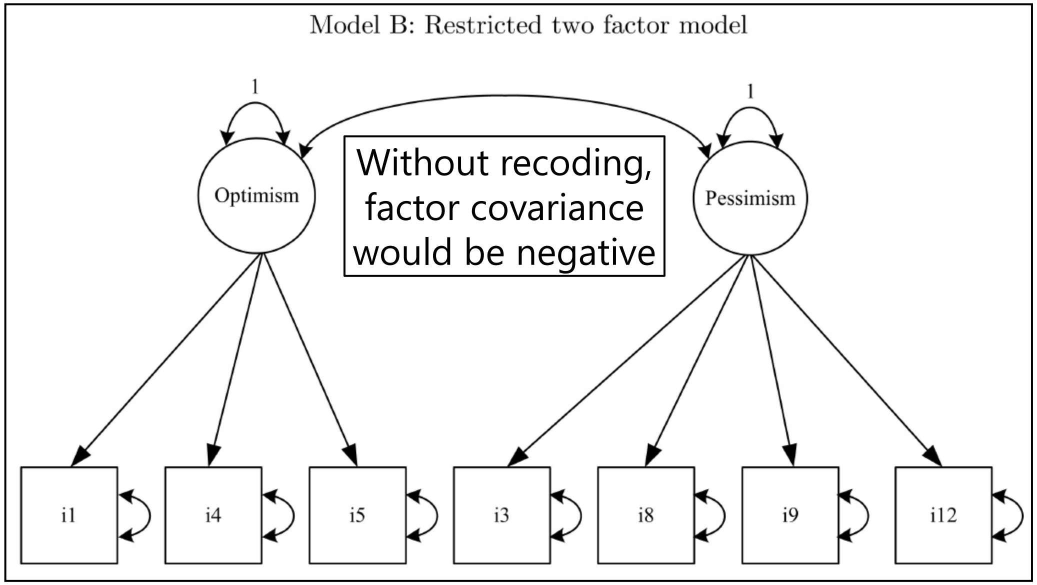
```
Opt BY i1* i4* i5*
       i3* i8* i9* i12*;
Opt@1; [Opt@0];
```

B: "Optimism" and "Pessimism" two-factor model (fits better)

```
Opt BY i1* i4* i5*;
Pes BY i3* i8* i9* i12*;
Opt WITH Pes*;
Opt@1; [Opt@0];
Pes@1; [Pes@0];
```

One- vs. Two-Factor Models

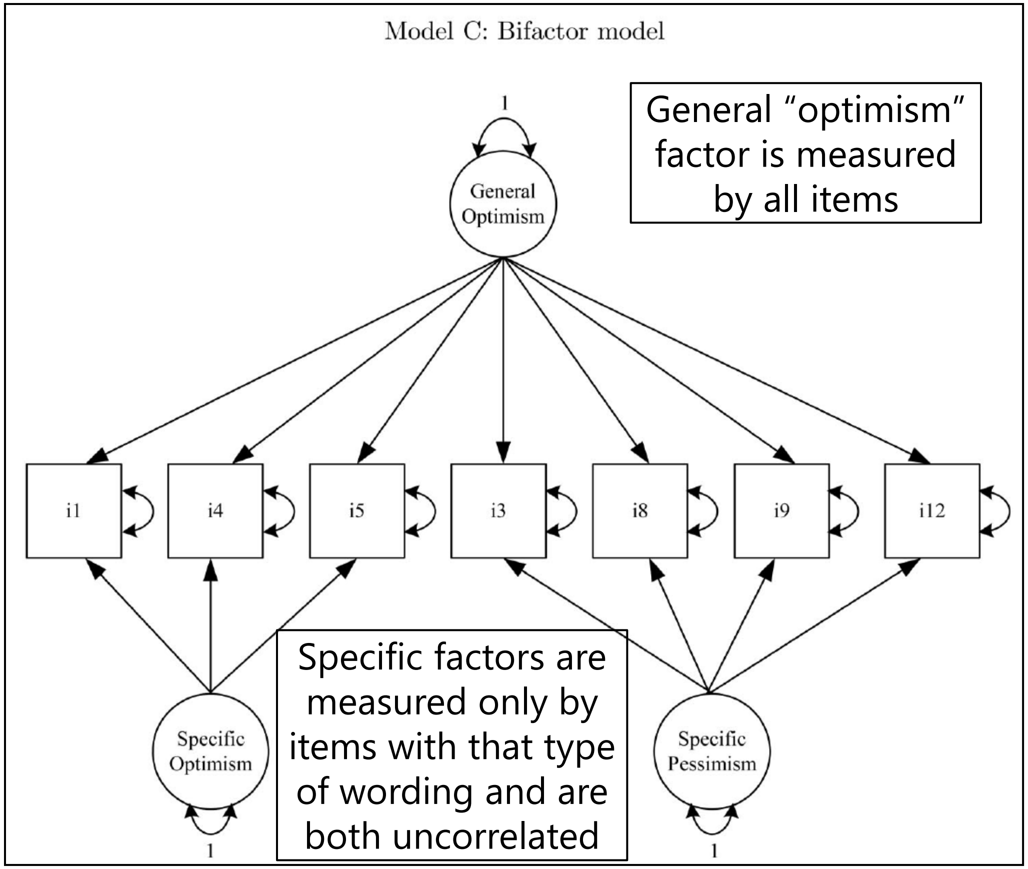




Item	One-factor model: Optimism	Two-factor model	
		Optimism	Pessimism
Item 1	0.38 (0.05)	0.64 (0.05)	0
Item 2	0.48 (0.05)	0.78 (0.05)	0
Item 3	0.46 (0.05)	0.68 (0.05)	0
Item 4	−0.64 (0.05)	0	0.65 (0.05)
Item 5	−0.86 (0.05)	0	0.87 0.05)
Item 6	−0.79 (0.05)	0	0.82 (0.05)
Item 7	−0.70 (0.05)	0	0.70 (0.05)

Note: a higher-order factor could be included if both loadings were fixed to 1, but it would fit the same as just allowing the two wording factors to covary.

Bifactor Model Fits Well…




```
 Gen BY i1* i4* i5*
        i3* i8* i9* i12*;
 Opt BY i1* i4* i5*;
 Pes BY i3* i8* i9* i12*;
 Gen@1; Opt@1; Pes@1;
[Gen@0  Opt@0  Pes@0];
 Gen WITH Opt@0 Pes@0;
 Opt WITH Pes@0;
```

Bifactor model		
Overall optimism	Specific optimism	Specific pessimism
0.35	0.56	0
(0.07)	(0.07)	
0.49	0.61	0
(0.08)	(0.07)	
0.44	0.51	0
(0.07)	(0.07)	
−0.59	0	0.26[a]
(0.09)		(0.18)
−0.76	0	0.38
(0.10)		(0.23)
−0.63	0	0.64[a]
(0.11)		(0.16)
−0.73	0	0.15[a]
(0.08)		(0.18)

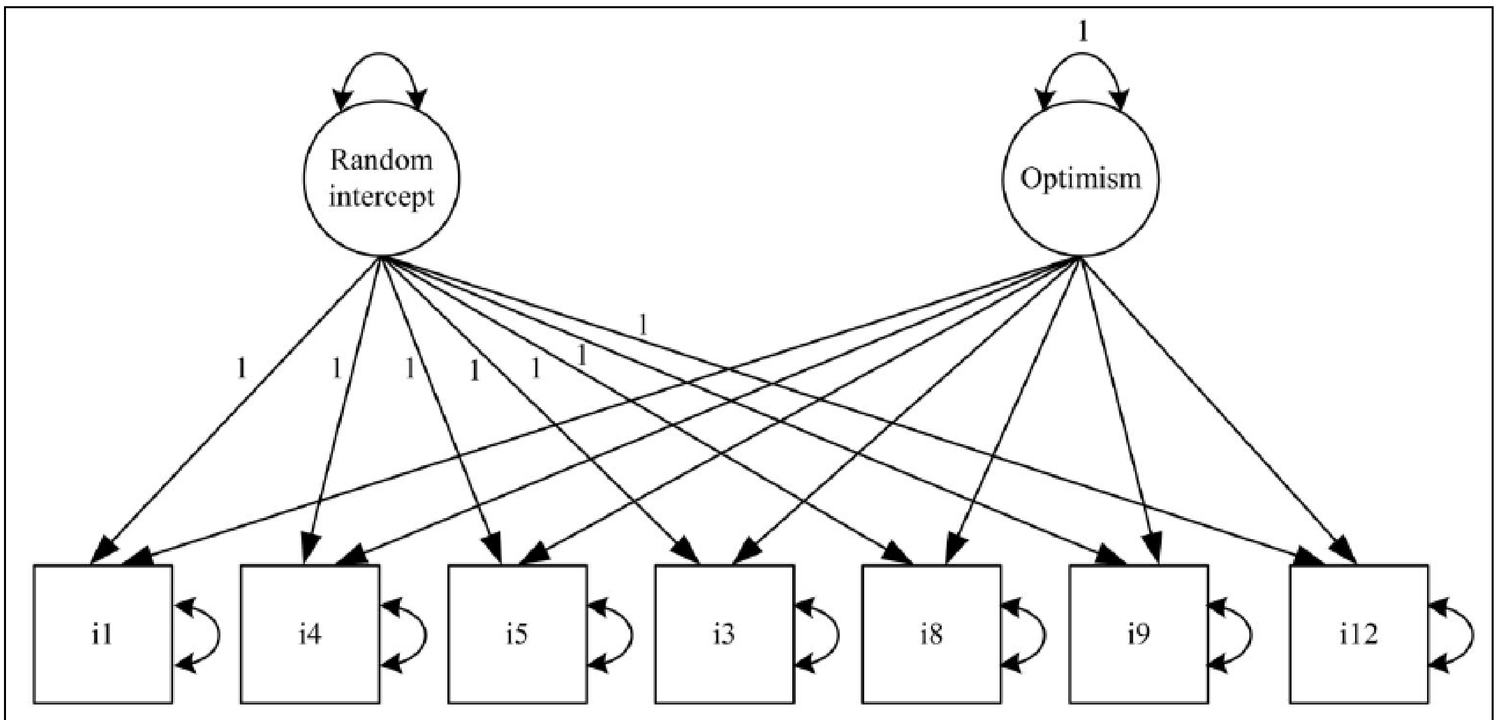
2 problems in interpreting these factors as desired:

1) “Specific” positive loadings > “general” loadings
2) Specific negative loadings are weak or non-significant (indicating model is over-parameterized)

Random Intercept Factor Fits Well…

- General "optimism" factor is measured by all items (all loadings estimated)
- New "random intercept" factor allows for constant person shifts across items (e.g., due to different response scale interpretations); Variance = 0.13 here

```
Opt BY i1* i4* i5*
       i3* i8* i9* i12*;
Opt@1; [Opt@0];
Int BY i1@1 i4@1 i5@1
       i3@1 i8@1 i9@1 i12@1;
Int*; [Int@0];
Opt WITH Int@0;
```



One-factor random intercept: Optimism
0.54
(0.05)
0.66
(0.05)
0.61
(0.05)
−0.56
(0.05)
−0.78
(0.05)
−0.71
(0.05)
−0.65
(0.05)

Heartland Forgiveness Scale (HFS)

Yamhure Thompson, L., Snyder, C.R., **Hoffman, L.,** Michael, S.T., Rasmussen, H.N., Billings, L.S., et al. (2005). Dispositional forgiveness of self, others, and situations. *Journal of Personality, 73*(2), 313-360.

Model 4. Six correlated lower-order factors for positive and negative self, other, and situation "forgiveness" and "not unforgiveness" (reverse-coded)

Total possible df for 18 items = 189

$$\frac{v * (v+1)}{2} + v = \frac{18 * 19}{2} + 18 = 189$$

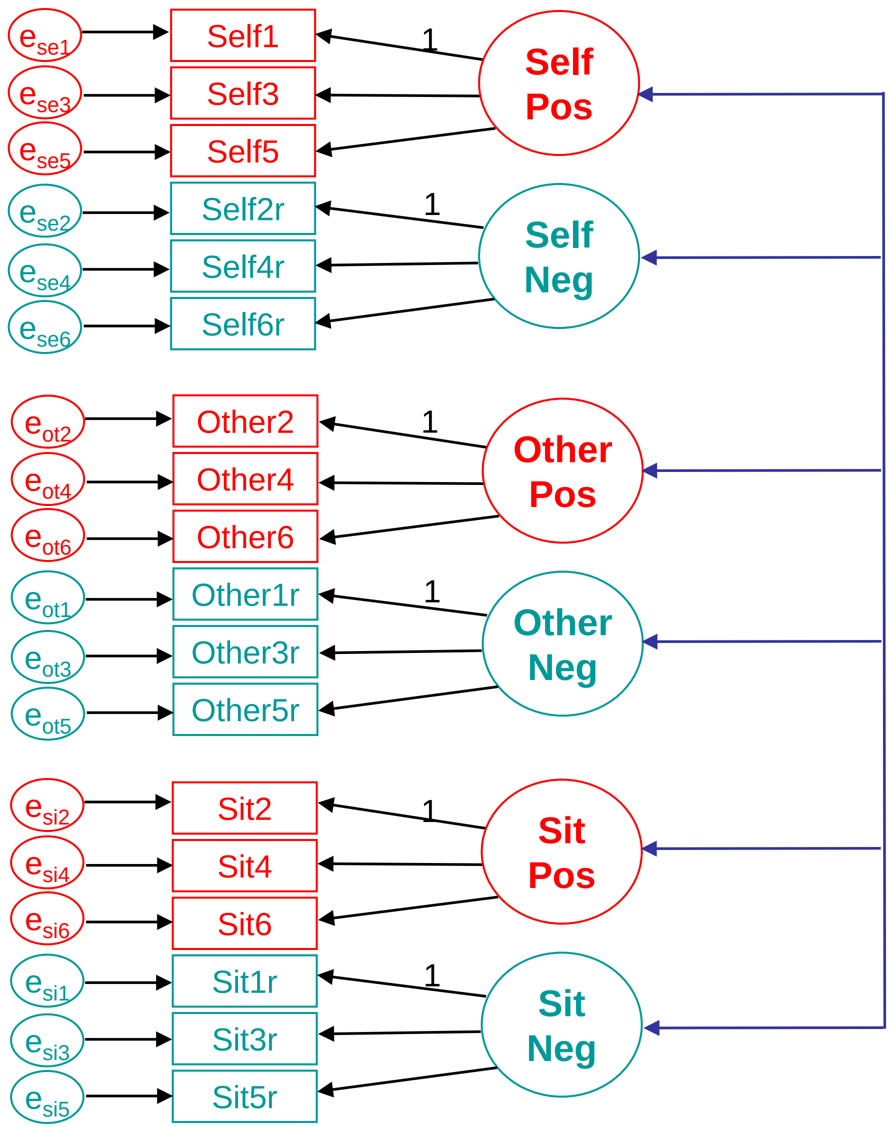
Measurement Model = 48 parameters

$12\lambda + 18\mu + 18\sigma^2_e$

Structural Model = 21 parameters

$6\sigma^2_F$, 15 factor covariances (all possible)

Total model df = 189 – 69 = 120



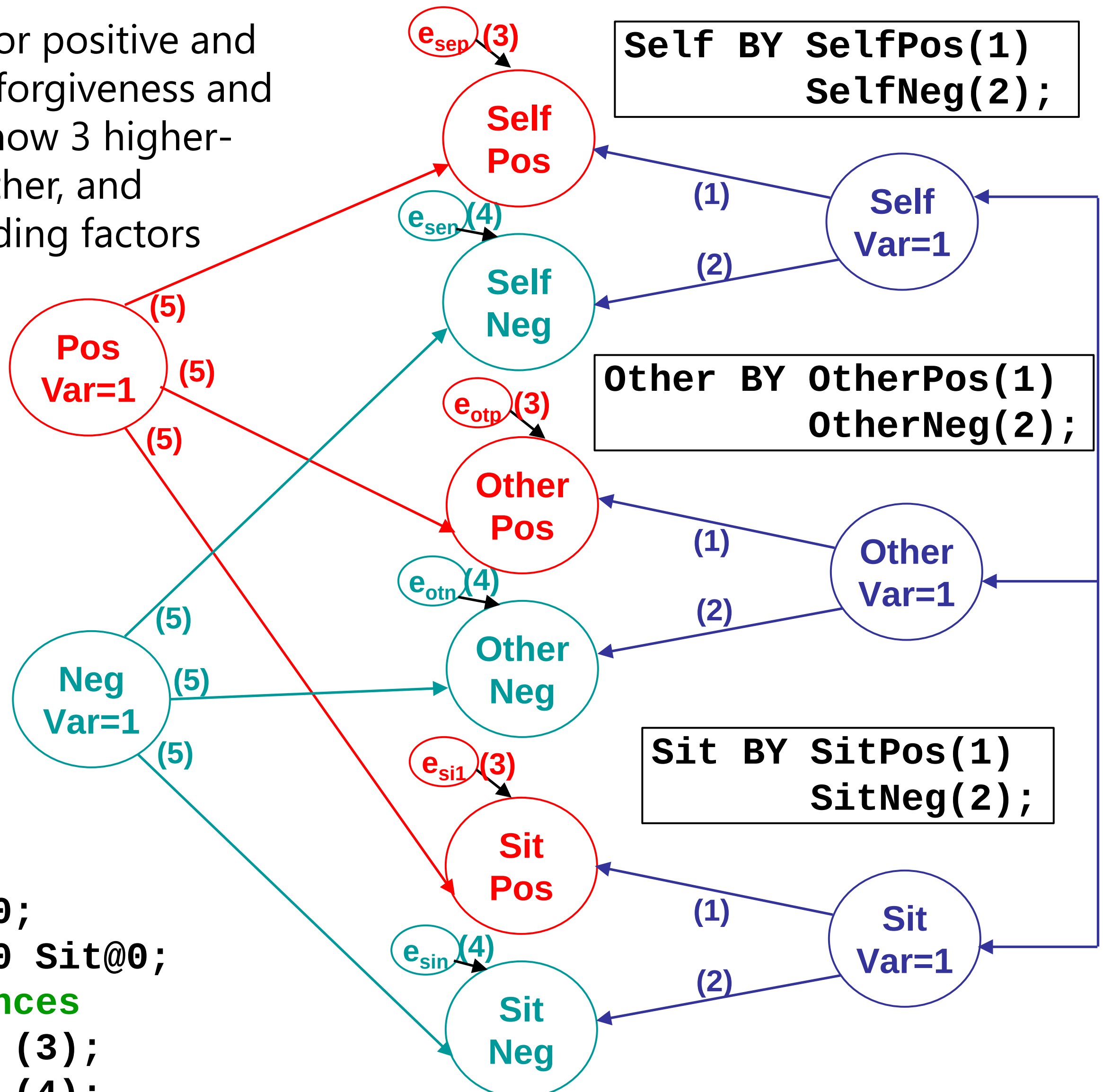
HFS Structural Model

Model 5. Six lower-order factors for positive and negative self, other, and situation forgiveness and not unforgiveness as before, but now 3 higher-order correlated factors of Self, Other, and Situation, and 2 uncorrelated wording factors

Structural Model = 8 parms

$(\mathrm{DF} = 21 - 8 = 13)$

```
! Constant Method Effects
Pos BY SelfPos*   (5)
       OtherPos* (5)
       SitPos*   (5);
Neg BY SelfNeg*   (5)
       OtherNeg* (5)
       SitNeg*   (5);
! No method factor cov.
Self@1 Other@1 Sit@1;
Self  WITH Other* Sit*;
Other WITH Sit*;
Pos@1 Neg@1; Pos WITH Neg@0;
Pos Neg WITH Self@0 Other@0 Sit@0;
! Constant factor disturbances
SelfPos* OtherPos* SitPos* (3);
SelfNeg* OtherNeg* SitNeg* (4);
```

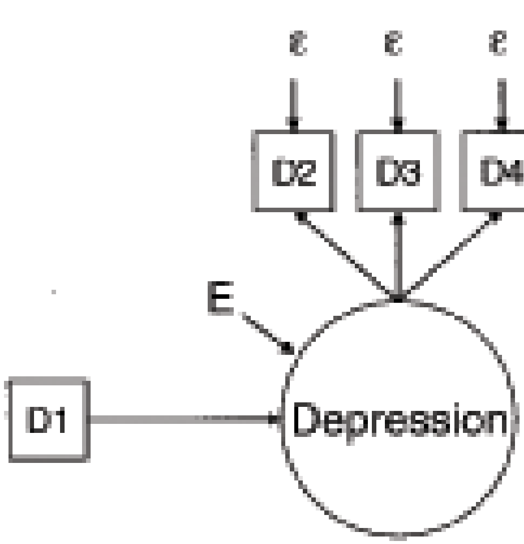


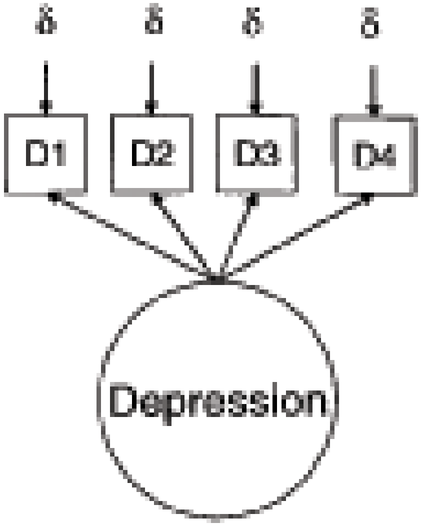
Equivalency across Models

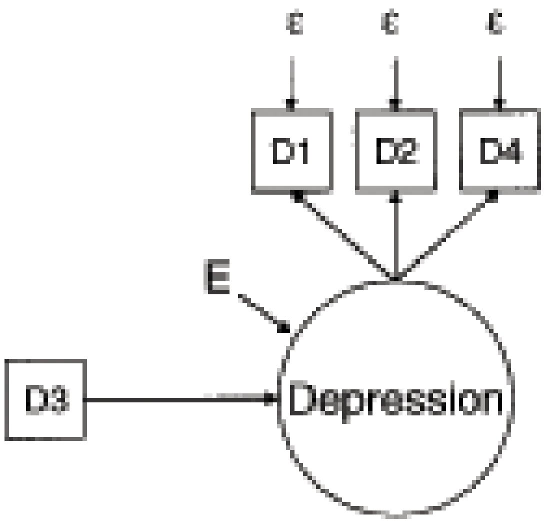
- Remember, the purpose of a measurement model is to reproduce the observed covariance matrix and means of the items
- This means that models that generate the same predicted covariance matrix and means are equivalent models
- This will often not be comforting, but it is the truth…
- Here's an example: These models make very different theoretical statements, but they will nevertheless fit equivalently

- Generally speaking, the fewer df left over (i.e., the more complicated the model), the more equivalent alternative solutions there are

More Equivalent Models…







Top: One can think these 4 items as "effects" (indicators) of depression…

Left: One can think of any one item as "causing" depression and the others as "effects" of depression…

Point of the story: CFA/SEM cannot give you TRUTH. Contrary to what it's often called, SEM is not really "causal" modeling

Wrapping Up…

- Fitting measurement and structural models are two separate issues:
  - **Measurement model**: Do my lower-order factors account for the *observed covariances among my ITEMS*?
  - **Structural model**: Do higher-order factors account for the *estimated covariances among my measurement model FACTORS/THETAS*?
    - A higher-order factor is NOT the same thing as a 'total score' though

- Figure out the measurement models FIRST, then structural models
  - Recommend fitting measurement models separately per factor, then bringing them together once you have each factor/theta settled
  - This will help to pinpoint the source of misfit in complex models

- Keep in mind that structural models may not be 'unique'
  - Mathematically equivalent models can make very different theoretical statements, so there's no real way to choose between them if so…