Higher-Order Models (CFA using MLR; then IFA using WLSMV) in Mplus version 8.1

Example data: 1,336 college students self-reporting on 49 items (measuring five factors) assessing childhood maltreatment. Items are answered on a 1–5 scale: 1=Strongly Disagree, 2=Disagree, 3=Neutral, 4=Agree, 5=Strongly Agree. The item responses are NOT normally distributed, so we'll use both CFA with MLR and IFA with WLSMV as two options to examine the fit of these models (as an example of how to do each, but NOT to compare between estimators).

1. Spurning: Verbal and nonverbal caregiver acts that reject and degrade a child
2. Terrorizing: Caregiver behaviors that threaten or are likely to physically hurt, kill, abandon, or place the child or the child's loved ones or objects in
      recognizably dangerous situations.
3. Isolating: Caregiver acts that consistently deny the child opportunities to meet needs for interacting or communicating with peers or adults inside or outside
      the home.
4. Corrupting: Caregiver acts that encourage the child to develop inappropriate behaviors (self-destructive, antisocial, criminal, deviant, or other maladaptive
      behaviors).
5. Ignoring: Emotional unresponsiveness includes caregiver acts that ignore the child's attempts and needs to interact (failing to express affection, caring,
      and love for the child) and show no emotion in interactions with the child

Here are the results from fitting the 5 factors separately to ensure their individual fit FIRST (see Mplus output files for details):

ASESSMENT OF MODEL FIT USING MLR

Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
MLR Spurning	12	90	36	224.797	1.4009	54	<.0001	0.959	0.049	0.042	0.055	0.619
MLR Terror	9	54	27	189.815	1.5876	27	<.0001	0.918	0.067	0.058	0.076	0.001
MLR Isolate	6	27	18	80.354	1.4944	9	<.0001	0.916	0.077	0.062	0.093	0.002
MLR Corrupt	7	35	21	54.964	1.9075	14	<.0001	0.934	0.047	0.034	0.060	0.633
MLR Ignore	15	135	45	484.291	1.7921	90	<.0001	0.932	0.057	0.052	0.062	0.008
MLR 1 factor only	49	1274	147	6,183.986	1.4874	1127	<.0001	0.766	0.058	0.057	0.059	<.0001
MLR 5 correlated factors	49	1274	157	4,424.700	1.4645	1117	<.0001	0.847	0.047	0.046	0.049	1.000
MLR 5 factors + higher order	49	1274	152	4,486.382	1.4681	1122	<.0001	0.844	0.047	0.046	0.049	0.999
MLR 5 factors + HO + 2 res cov	49	1274	154	4,422.556	1.4669	1120	<.0001	0.847	0.047	0.046	0.048	1.000

ASESSMENT OF MODEL FIT USING WLSMV

Model	# Items	# Possible Parms	# Free Parms	Chi-Square Value	Chi-Square Scale Factor	Chi-Square DF	Chi-Square p-value	CFI	RMSEA Estimate	RMSEA Lower CI	RMSEA Higher CI	RMSEA p-value
WLSMV Spurning	12	126	60	294.707		54	<.0001	0.983	0.058	0.051	0.064	0.023
WLSMV Terror	9	81	45	263.156		27	<.0001	0.966	0.081	0.072	0.090	<.0001
WLSMV Isolate	6	45	30	129.827		9	<.0001	0.962	0.100	0.085	0.116	<.0001
WLSMV Corrupt	7	56	35	87.490		14	<.0001	0.976	0.063	0.050	0.076	0.044
WLSMV Ignore	15	180	75	897.694		90	<.0001	0.976	0.082	0.077	0.087	<.0001
WLSMV 1 factor only	49	1421	245	7,563.403		1127	<.0001	0.903	0.065	0.064	0.067	<.0001
WLSMV 5 correlated factors	49	1421	255	5,934.139		1117	<.0001	0.927	0.057	0.055	0.058	<.0001
WLSMV 5 factors + higher order	49	1421	250	5,941.909		1122	<.0001	0.927	0.057	0.055	0.058	<.0001
WLSMV 5 factors + HO + 2 res cov	49	1421	252	5,853.778		1122	<.0001	0.928	0.056	0.055	0.058	<.0001

Here are the standardized factor loadings for each item under each estimation method. Note that the WLSMV factor loadings are higher in this case – probably because of range restriction in the original data and thus the implausibility of a linear model.

MLR Spurning	WLSMV Spurning	MLR Terror	WLSMV Terror	MLR Isolate	WLSMV Isolate	MLR Corrupt	WLSMV Corrupt	MLR Ignore	WLSMV Ignore
0.599	0.660	0.512	0.617	0.521	0.696	0.589	0.739	0.672	0.813
0.457	0.528	0.673	0.771	0.550	0.630	0.545	0.713	0.654	0.749
0.769	0.837	0.451	0.713	0.545	0.685	0.375	0.523	0.657	0.748
0.526	0.597	0.612	0.721	0.540	0.629	0.545	0.854	0.724	0.801
0.607	0.677	0.571	0.787	0.563	0.726	0.631	0.826	0.445	0.540
0.816	0.865	0.554	0.617	0.752	0.822	0.580	0.708	0.745	0.833
0.835	0.907	0.685	0.805			0.646	0.840	0.847	0.913
0.465	0.538	0.643	0.743					0.713	0.813
0.516	0.728	0.732	0.815					0.808	0.891
0.655	0.744							0.749	0.845
0.674	0.756							0.656	0.795
0.610	0.680							0.830	0.904
								0.712	0.806
								0.739	0.815
								0.825	0.918

Strawman model: Syntax for single-factor CFA model estimated using MLR through 5 PERFECTLY correlated factors

```
DATA:    FILE IS abuse.csv;
VARIABLE:
NAMES ARE ID    ! All variables in DATA SET
  p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
  p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
  p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES ARE  ! All variables in MODEL
  p01 p02 p03 p04     p06 p07     p09 p10
  p11 p12 p13 p14     p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31     p33     p35 p36 p37 p39 p40
          p43 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;
IDVARIABLE IS ID;

ANALYSIS:    ESTIMATOR IS MLR;  ! For non-normal continuous items
OUTPUT:      STDYX              ! Standardized solution
             MODINDICES(3.84)   ! Voodoo for fixing the model
             RESIDUAL           ! Local fit info
             TECH4;             ! Factor correlation matrix
!SAVEDATA:   SAVE = FSCORES;          ! Save factor scores
!            FILE = Abuse_Thetas.dat; ! File of factor scores
!            MISSFLAG = 99;           ! Indicate missing values
!PLOT:       TYPE IS PLOT1 PLOT2 PLOT3; ! Pictures

MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariance (all fixed to 1 to create 1-factor model)
  Spurn  Terror  Isolate  Corrupt  Ignore  WITH
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
```

```
THE MODEL ESTIMATION TERMINATED NORMALLY
```

Because the factor covariances were fixed to 1, you will see the message below. In THIS CONTEXT ONLY, you can ignore it.

```
WARNING:  THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE.  THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE TERROR.

MODEL FIT INFORMATION
Number of Free Parameters                   147
Loglikelihood
        H0 Value                       -70386.526
        H0 Scaling Correction Factor     2.3983
          for MLR
        H1 Value                       -65787.405
        H1 Scaling Correction Factor     1.5925
          for MLR

Information Criteria
        Akaike (AIC)                   141067.051
        Bayesian (BIC)                 141831.074
        Sample-Size Adjusted BIC       141364.120
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                            6183.986*
        Degrees of Freedom                   1127
        P-Value                            0.0000
        Scaling Correction Factor          1.4874
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.058
        90 Percent C.I.                    0.057  0.059
        Probability RMSEA <= .05           0.000

CFI/TLI
        CFI                                0.766
        TLI                                0.756

SRMR (Standardized Root Mean Square Residual)
        Value                              0.062
```

#free parameters = 147 = 49 loadings + 49 intercepts + 49 residuals
  + 0 factor variances + 0 factor covariances = 147 parms USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 – 147 = 1127

Syntax for CFA model with MLR including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

```
MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore  BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
             p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariances (all estimated to allow 5-factor model)
  Spurn  Terror  Isolate  Corrupt  Ignore  WITH
  Spurn* Terror* Isolate* Corrupt* Ignore*;
```

Do we have one factor or five factors?

According to the −2ΔLL scaled difference relative to the previous single-factor model: −2ΔLL (10) = 671.689, p < .0001
Therefore, one factor does not capture the covariances for these 49 items.
Five factors (as hypothesized) does a significantly better job.

Here are the correlations among the latent factors we are now trying to account for—with models that replace them with a higher-order factor.

Saturated: 5-Factor All Covariances Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.929	1.000			
ISOLATE	.898	.876	1.000		
CORRUPT	.689	.792	.658	1.000	
IGNORE	.830	.767	.828	.630	1.000

NOTE: With respect to fit of the structural model, letting the 5 factors just be correlated is as good as it gets. This saturated structural model will be our "larger model" baseline with which to compare the fit of models that try to account for these correlations via a higher-order factor ("smaller models").

```
Number of Free Parameters                      157
Loglikelihood
        H0 Value                         -69027.431
        H0 Scaling Correction Factor        2.5033
            for MLR
        H1 Value                         -65787.405
        H1 Scaling Correction Factor        1.5925
            for MLR

Information Criteria
        Akaike (AIC)                     138368.862
        Bayesian (BIC)                   139184.860
        Sample-Size Adjusted BIC         138686.140
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                             4424.700*
        Degrees of Freedom                     1117
        P-Value                              0.0000
        Scaling Correction Factor           1.4645
            for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                            0.047
        90 Percent C.I.                     0.046   0.049
        Probability RMSEA <= .05            1.000

CFI/TLI
        CFI                                 0.847
        TLI                                 0.839

SRMR (Standardized Root Mean Square Residual)
        Value                               0.057
```

#free parameters = 157 = 49 loadings + 49 intercepts + 49 residuals
    + 0 factor variances + 10 factor covariances = 157 parms USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 – 157 = 1117

Now we can test the fit of a constrained structural model that posits a single higher-order "General Abuse" factor to account for the correlations among these 5 latent factors (shown on the left from TECH 4).

Syntax for CFA model with MLR and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

```
MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
          p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free – NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;
```

We can use a −2ΔLL scaled difference to test the fit of the higher-order factor model against the saturated structural model with all possible factor correlations. This higher-order factor model uses 5 fewer parameters: 5 higher-order loadings replace the 10 covariances among the factors. The −2ΔLL scaled difference is −2ΔLL (5) = 46.848, p < .0001.

So trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Why might this be the case? All the lower-order factors have large (enough) standardized loadings…

```
STDYX Standardization

                                        Two-Tailed
                 Estimate   S.E.   Est./S.E.   P-Value
ABUSE   BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN          0.971    0.010    101.941     0.000
    TERROR         0.952    0.011     88.191     0.000
    ISOLATE        0.933    0.016     59.159     0.000
    CORRUPT        0.745    0.027     27.312     0.000
    IGNORE         0.846    0.018     48.111     0.000
```

NOTE: With respect to fit of the structural model, we are now fitting a single higher-order factor INSTEAD OF covariances among the 5 factors.

```
Number of Free Parameters                    152
Loglikelihood
        H0 Value                       -69080.656
        H0 Scaling Correction Factor      2.5109
          for MLR
        H1 Value                       -65787.405
        H1 Scaling Correction Factor      1.5925
          for MLR


Information Criteria
        Akaike (AIC)                   138465.313
        Bayesian (BIC)                 139255.323
        Sample-Size Adjusted BIC       138772.486
          (n* = (n + 2) / 24)


Chi-Square Test of Model Fit
        Value                          4486.382*
        Degrees of Freedom                  1122
        P-Value                           0.0000
        Scaling Correction Factor         1.4681
          for MLR


RMSEA (Root Mean Square Error Of Approximation)
        Estimate                          0.047
        90 Percent C.I.                   0.046   0.049
        Probability RMSEA <= .05          0.999


CFI/TLI
        CFI                               0.844
        TLI                               0.837


SRMR (Standardized Root Mean Square Residual)
        Value                             0.058
```

#free parameters = 152 = 44 loadings + 49 intercepts + 49 residuals + 5 factor variances + 5 higher-order loadings = 152 parms USED

Possible = 49*50/2 + 49 = 1274
DF =1117 calculation: 1274 − 152 = 1122

Higher-Order Factor Model Output;
Comparison of Saturated versus
Higher-Order Factor Model predicted correlations:

```
MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index     3.840

                           M.I.     E.P.C.   Std E.P.C.  StdYX E.P.C.
TERROR    WITH SPURN       4.111    0.011     0.454       0.454
CORRUPT   WITH SPURN      18.864   -0.018    -0.451      -0.451
CORRUPT   WITH TERROR     44.080    0.021     0.595       0.595
CORRUPT   WITH ISOLATE     4.628   -0.006    -0.193      -0.193
IGNORE    WITH SPURN       4.800    0.010     0.248       0.248
IGNORE    WITH TERROR     31.774   -0.018    -0.510      -0.510
IGNORE    WITH ISOLATE    14.098    0.010     0.317       0.317
```

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations, it appears we need to allow two more relationships among the factor disturbances, as follows:

```
MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
           p45* p46* p51* p52* p57*;

! Factor Variances (all must be free – NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;


! Add disturbance covariances suggested by voo-doo
  Corrupt WITH Terror*;
  Ignore  WITH Terror*;
```

MLR Solutions				
Saturated: 5-Factor All Covariances Model				
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.929	1.000			
ISOLATE	.898	.876	1.000		
CORRUPT	.689	.792	.658	1.000	
IGNORE	.830	.767	.828	.630	1.000

Predicted 1: 5-Factor + Higher-Order Factor Model				
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.925	1.000			
ISOLATE	.906	.889	1.000		
CORRUPT	.724	.710	.696	1.000	
IGNORE	.821	.806	.790	.631	1.000

Discrepancy: Saturated - Predicted 1				
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.004				
ISOLATE	-.008	-.013			
CORRUPT	-.035	**.082**	-.038		
IGNORE	.009	**-.039**	.038	-.001	

Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model				
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.923	1.000			
ISOLATE	.898	.894	1.000		
CORRUPT	.690	.794	.668	1.000	
IGNORE	.838	.766	.812	.623	1.000

Discrepancy: Saturated - Predicted 2				
	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.006				
ISOLATE	.000	-.018			
CORRUPT	-.001	-.002	-.010		
IGNORE	-.008	.001	.016	.007	

```
MODEL FIT INFORMATION
Number of Free Parameters                    154

Loglikelihood
        H0 Value                      -69031.180
        H0 Scaling Correction Factor      2.5060
          for MLR
        H1 Value                      -65787.405
        H1 Scaling Correction Factor      1.5925
          for MLR

Information Criteria
        Akaike (AIC)                  138370.360
        Bayesian (BIC)                139170.765
        Sample-Size Adjusted BIC      138681.575
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit
        Value                           4422.556*
        Degrees of Freedom                   1120
        P-Value                            0.0000
        Scaling Correction Factor         1.4669
          for MLR

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.047
        90 Percent C.I.               0.046  0.048
        Probability RMSEA <= .05           1.000

CFI/TLI
        CFI                                0.847
        TLI                                0.840

SRMR (Standardized Root Mean Square Residual)
        Value                              0.057

STDYX Standardization

                                           Two-Tailed
                Estimate    S.E.  Est./S.E.  P-Value
ABUSE   BY (HIGHER-ORDER STANDARDIZED LOADINGS)
   SPURN      0.963      0.011    87.824     0.000
   TERROR     0.958      0.012    77.198     0.000
   ISOLATE    0.933      0.016    58.212     0.000
   CORRUPT    0.716      0.028    25.150     0.000
   IGNORE     0.870      0.019    45.845     0.000

 CORRUPT  WITH
   TERROR     0.540      0.097     5.550     0.000
 IGNORE   WITH
   TERROR    -0.483      0.172    -2.811     0.005
```

Next we will duplicate these analyses using WLSMV, which requires starting with the biggest model first…

Two comparisons are relevant.

First, did we help the higher-order factor model by adding two covariances among the factor disturbances? −2ΔLL (2) = 46.378, p < .0001, so yes, model fit is better.

Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? −2ΔLL (3) = 3.171, p = .3660, so yes, our revised model captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

Example results section for CFA using MLR:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. Robust maximum likelihood (MLR) estimation was used to estimate all models using Mplus v. 8.1 (Muthén & Muthén, 1998–2017), and differences in fit between nested models were evaluated using −2* rescaled difference in the model log-likelihood values. The fit of each model referenced below is shown in Table 1.

We first established the need for 5 factors by showing a significant decrease in fit for a single-factor model relative to that of the 5-factor model, −2ΔLL (10) = 671.689, p < .0001. As shown in Table 1, the fit of the model with five correlated factors was acceptable by the RMSEA (.047), but not by the CFI (.847). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations of .6 or higher were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was testing by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, −2ΔLL(5) = 46.848, p < .0001, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the 5-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was over-estimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit, −2ΔLL(2) = 46.378, p < .0001. Further, the revised model successfully accounted for the pattern of correlation among the 5 factors, as indicated by a nonsignificant decrease in model fit relative to the model with all 10 factor correlations estimated directly, −2ΔLL(3) = 3.171, p = .3660.

Syntax for IFA model with WLSMV including all 5 non-perfectly correlated factors ("saturated structural model") for comparison:

```
DATA:   FILE IS abuse.csv;
VARIABLE:
NAMES ARE ID    ! All variables in DATA SET
  p01 p02 p03 p04 p05 p06 p07 p08 p09 p10
  p11 p12 p13 p14 p15 p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31 p32 p33 p34 p35 p36 p37 p38 p39 p40
  p41 p42 p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;

USEVARIABLES ARE  ! All variables in MODEL
  p01 p02 p03 p04     p06 p07     p09 p10
  p11 p12 p13 p14     p16 p17 p18 p19 p20
  p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
  p31     p33     p35 p36 p37 p39 p40
      p43 p44 p45 p46 p47 p48 p49 p50
  p51 p52 p53 p54 p55 p56 p57;
IDVARIABLE IS ID;

CATEGORICAL ARE  ! All variables for IFA
  p01 p02 p03 p04     p06 p07     p09 p10
 p11 p12 p13 p14     p16 p17 p18 p19 p20
 p21 p22 p23 p24 p25 p26 p27 p28 p29 p30
 p31     p33     p35 p36 p37 p39 p40
      p43 p44 p45 p46 p47 p48 p49 p50
 p51 p52 p53 p54 p55 p56 p57;

OUTPUT:    STDYX           ! Standardized solution
           MODINDICES(3.84)  ! Voodoo for fixing the model
           RESIDUAL          ! Local fit info
           TECH4;            ! Factor correlation matrix
PLOT:      TYPE IS PLOT1 PLOT2 PLOT3; ! Pictures

ANALYSIS:  ESTIMATOR IS WLSMV;        ! Limited info estimator
           PARAMETERIZATION IS THETA;

SAVEDATA:  DIFFTEST=5factor.dat;      ! Save fit of 5 factor model
```

#free parameters = 255 = 49 loadings + 49*4=196 thresholds
    + 0 factor variances + 10 factor covariances = 255 parms USED

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 255 – 49 "residual variances" = 1117

```
MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
            p45* p46* p51* p52* p57*;! Factor Variances (all must be
fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariances (all estimated to allow 5-factor model)
  Spurn  Terror  Isolate  Corrupt  Ignore  WITH
  Spurn* Terror* Isolate* Corrupt* Ignore*;
```

NOTE: With respect to fit of the structural model, letting the 5 factors just be correlated is as good as it gets. This saturated structural model will be our "larger model" baseline with which to compare the fit of models that try to account for these correlations via a higher-order factor ("smaller models").

```
MODEL FIT INFORMATION
Number of Free Parameters                 255

Chi-Square Test of Model Fit
        Value                        5934.139*
        Degrees of Freedom              1117
        P-Value                       0.0000

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                      0.057
        90 Percent C.I.               0.055  0.058
        Probability RMSEA <= .05      0.000

CFI/TLI
        CFI                           0.927
        TLI                           0.923

SRMR (Standardized Root Mean Square Residual)
        Value                         0.056
```

Strawman model: Syntax for single-factor IFA model estimated using WLSMV through 5 PERFECTLY correlated factors

```
ANALYSIS:    ESTIMATOR IS WLSMV;          ! Limited info estimator
             PARAMETERIZATION IS THETA;
             DIFFTEST=5factor.dat;        ! Test fit against saturated

! (no SAVEDATA needed)

MODEL: ! (To be changed below for each model)

! 5 Factors (loadings for first item are estimated)
! 12-Item Spurning
  Spurn BY p06* p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07* p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01* p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09* p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02* p03* p04* p21* p22* p30* p31* p37* p40* p44*
           p45* p46* p51* p52* p57*;

! Factor Variances (all must be fixed to 1 for identification)
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];
! Factor Covariance (all fixed to 1 to create 1-factor model)
  Spurn   Terror   Isolate   Corrupt   Ignore   WITH
  Spurn@1 Terror@1 Isolate@1 Corrupt@1 Ignore@1;
```

#free parameters = 245 = 49 loadings + 49*4=196 thresholds
   + 0 factor variances + 0 factor covariances = 245 parms USED

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 245 – 49 "residual variances" = 1127

```
THE MODEL ESTIMATION TERMINATED NORMALLY
```

Because the factor covariances were fixed to 1, you will see the message below. In THIS CONTEXT ONLY, you can ignore it.

```
WARNING:  THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE.  THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE
FOR A LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN
TWO LATENT VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO
LATENT VARIABLES. CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE TERROR.


MODEL FIT INFORMATION
Number of Free Parameters                     245

Chi-Square Test of Model Fit
        Value                             7563.403*
        Degrees of Freedom                    1127
        P-Value                             0.0000
```

Chi-Square Test for Difference Testing
**   Value   769.754***
**   Degrees of Freedom   10**
**   P-Value   0.0000**

```
RMSEA (Root Mean Square Error Of Approximation)
        Estimate                            0.065
        90 Percent C.I.                     0.064  0.067
        Probability RMSEA <= .05            0.000

CFI/TLI
        CFI                                 0.903
        TLI                                 0.898

SRMR (Standardized Root Mean Square Residual)
        Value                               0.068
```

Do we have one factor or five factors?

According to the DIFFTEST relative to the previous 5-factor model:
$\chi^2(10) = 769.754$, $p < .0001$

Therefore, one factor does not capture the covariances for these 49 items. Five factors (as hypothesized) does a significantly better job.

Syntax for IFA model with WLSMV and a higher-order factor instead of correlations among 5 factors ("smaller/bigger model" for comparison):

```
ANALYSIS:    ESTIMATOR IS WLSMV;         ! Limited info estimator
             PARAMETERIZATION IS THETA;
             DIFFTEST=5factor.dat;       ! Test fit against saturated

SAVEDATA:    DIFFTEST=HigherOrder.dat;  ! Save fit of higher-order model


MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
  Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
  Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
  Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
  Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
  Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
          p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free - NOW ARE "DISTURBANCES")
  Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
  [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
  Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
  [Abuse@0]; Abuse@1;
```

We can use DIFFTEST to test the fit of the higher-order factor model against the saturated structural model with all possible factor correlations. This higher-order factor model uses 5 fewer parameters: 5 higher-order loadings replace the 10 covariances among the factors. The difference is $\chi^2(5) = 92.048$, $p < .0001$.

So trying to reproduce the 5 factor covariances with a single higher-order factor results in a significant decrease in fit. Why might this be the case? All the lower-order factors have large (enough) standardized loadings…

NOTE: With respect to fit of the structural model, we are now fitting a single higher-order factor INSTEAD OF covariances among the 5 factors.

```
MODEL FIT INFORMATION
Number of Free Parameters                  250

Chi-Square Test of Model Fit
      Value                             5941.909*
      Degrees of Freedom                     1122
      P-Value                              0.0000

Chi-Square Test for Difference Testing
      Value                               92.048*
      Degrees of Freedom                        5
      P-Value                              0.0000

RMSEA (Root Mean Square Error Of Approximation)
      Estimate                             0.057
      90 Percent C.I.                      0.055  0.058
      Probability RMSEA <= .05             0.000

CFI/TLI
      CFI                                  0.927
      TLI                                  0.924

SRMR (Standardized Root Mean Square Residual)
      Value                                0.057
```

#free parameters = 250 = 44 loadings + 49*4=196 thresholds
+ 5 factor variances + 5 higher-order loadings = 250 parms USED

Possible = 49*50/2 + 49*4 = 1421
DF =1117 calculation: 1421 – 250 – 49 "residual variances" = 1122

```
STDYX Standardization

                                                    Two-Tailed
                      Estimate    S.E.  Est./S.E.    P-Value
ABUSE    BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN             0.990     0.005   204.056      0.000
    TERROR            0.948     0.007   139.928      0.000
    ISOLATE           0.951     0.009   106.595      0.000
    CORRUPT           0.835     0.014    60.998      0.000
    IGNORE            0.885     0.009    93.999      0.000
```

Higher-Order Factor Model Output; Comparison of Saturated versus Higher-Order Factor Model predicted correlations:

```
MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index     3.840

                            M.I.      E.P.C.    Std E.P.C.   StdYX E.P.C.
TERROR    WITH SPURN        8.776     0.011      0.558        0.558
ISOLATE   WITH SPURN       11.742    -0.014     -0.742       -0.742
ISOLATE   WITH TERROR       5.966    -0.012     -0.256       -0.256
CORRUPT   WITH SPURN       39.197    -0.028     -0.762       -0.762
CORRUPT   WITH TERROR     122.582     0.056      0.627        0.627
IGNORE    WITH SPURN       25.058     0.021      0.596        0.596
IGNORE    WITH TERROR      82.830    -0.040     -0.471       -0.471
IGNORE    WITH ISOLATE     42.439     0.031      0.372        0.372
IGNORE    WITH CORRUPT      6.036    -0.013     -0.077       -0.077
```

Based on the modification indices (which are picking up on the discrepancies between the saturated model and higher-order factor model in the factor correlations, it appears we need to allow two more relationships among the factor disturbances, as follows:

```
ANALYSIS:    ESTIMATOR IS WLSMV;         ! Limited info estimator
             PARAMETERIZATION IS THETA;
             DIFFTEST=5factor.dat;       ! Test fit against saturated

SAVEDATA:    DIFFTEST=HigherOrder2.dat;  ! Save fit of higher-order2 model

MODEL: ! (To be changed below for each model)

! 5 Lower-Order Factors (loadings for first item NOW FIXED =1)
! 12-Item Spurning
   Spurn BY p06@1 p10* p14* p25* p27* p29* p33* p35* p48* p49* p53* p54*;
! 9-Item Terrorizing
   Terror BY p07@1 p11* p13* p17* p24* p26* p36* p55* p56*;
! 6-Item Isolating
   Isolate BY p01@1 p18* p19* p23* p39* p43*;
! 7-Item Corrupting
   Corrupt BY p09@1 p12* p16* p20* p28* p47* p50*;
! 15-Item Ignoring
   Ignore BY p02@1 p03* p04* p21* p22* p30* p31* p37* p40* p44*
           p45*  p46* p51* p52* p57*;

! Factor Variances (all must be free – NOW ARE "DISTURBANCES")
   Spurn* Terror* Isolate* Corrupt* Ignore*;
! Factor Means (all fixed = 0 by default)
   [Spurn@0 Terror@0 Isolate@0 Corrupt@0 Ignore@0];

! Higher-Order Factor (estimate higher-order factor loadings)
   Abuse BY Spurn* Terror* Isolate* Corrupt* Ignore*;
! Fix higher-order factor mean=0 & variance=1
   [Abuse@0]; Abuse@1;

! Add disturbance covariances suggested by voo-doo
   Corrupt WITH Terror*;
   Ignore  WITH Terror*;
```

WLSMV Solutions

Saturated: 5-Factor All Covariances Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.947	1.000			
ISOLATE	.925	.885	1.000		
CORRUPT	.791	.866	.776	1.000	
IGNORE	.882	.817	.863	.729	1.000

Predicted 1: 5-Factor + Higher-Order Factor Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.938	1.000			
ISOLATE	.941	.902	1.000		
CORRUPT	.826	.791	.794	1.000	
IGNORE	.876	.839	.841	.738	1.000

Discrepancy: Saturated – Predicted 1

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.009				
ISOLATE	-.016	-.017			
CORRUPT	-.035	.075	-.018		
IGNORE	.006	-.022	.022	-.009	

Predicted 2: 5-Factor + Higher-Order Factor + 2 Fact Cov Model

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN	1.000				
TERROR	.939	1.000			
ISOLATE	.927	.907	1.000		
CORRUPT	.792	.866	.765	1.000	
IGNORE	.885	.817	.855	.730	1.000

Discrepancy: Saturated – Predicted 2

	SPURN	TERROR	ISOLATE	CORRUPT	IGNORE
SPURN					
TERROR	.008				
ISOLATE	-.002	-.022			
CORRUPT	-.001	.000	.011		
IGNORE	-.003	.000	.008	-.001	

```
MODEL FIT INFORMATION
Number of Free Parameters                    252

Chi-Square Test of Model Fit
        Value                           5853.778*
        Degrees of Freedom                  1120
        P-Value                           0.0000

Chi-Square Test for Difference Testing
        Value                             8.483*
        Degrees of Freedom                     3
        P-Value                           0.0370

RMSEA (Root Mean Square Error Of Approximation)
        Estimate                           0.056
        90 Percent C.I.                    0.055  0.058
        Probability RMSEA <= .05           0.000

CFI/TLI
        CFI                                0.928
        TLI                                0.925

SRMR (Standardized Root Mean Square Residual)
        Value                              0.056


STDYX Standardization

                                                 Two-Tailed
                    Estimate    S.E.   Est./S.E.   P-Value
ABUSE   BY (HIGHER-ORDER STANDARDIZED LOADINGS)
    SPURN          0.980      0.006    173.657      0.000
    TERROR         0.959      0.007    130.093      0.000
    ISOLATE        0.946      0.009    105.339      0.000
    CORRUPT        0.809      0.015     54.055      0.000
    IGNORE         0.903      0.009     97.384      0.000

 CORRUPT  WITH
    TERROR         0.544      0.068      7.983      0.000
 IGNORE   WITH
    TERROR        -0.406      0.102     -3.991      0.000

MODEL MODIFICATION INDICES
Minimum M.I. value for printing the modification index   3.840

                    M.I.    E.P.C.    Std E.P.C.   StdYX E.P.C.

TERROR   WITH SPURN    13.424    0.031     0.757      0.757
ISOLATE  WITH TERROR   13.420   -0.036    -0.454     -0.454
IGNORE   WITH SPURN     5.964   -0.029    -0.271     -0.271
IGNORE   WITH ISOLATE   8.177    0.039     0.186      0.186
```

It looks like we could add a 1-2 more covariances to ensure not worse fit than the saturated (all 10 correlations) model, but which should be added seems somewhat arbitrary…

Two comparisons are relevant. First, did we help the higher-order factor model by adding two covariances among the factor disturbances? This comparison is not shown here, but yes, $\chi^2(2) = 88.343$, $p < .0001$, so yes, model fit is better. Second, does the revised higher-order factor model fit nonsignificantly worse than the saturated structural model with all 10 correlations among the 5 factors? Almost: $\chi^2(3) = 8.483$, $p = .0370$. So our revised model almost captures those 10 correlations using 3 fewer parameters (5 loadings + 2 covariances).

Example results section for IFA using WLSMV:

After examining the fit of each of the five factors individually, as described previously, a combined model was estimated in which all five factors were fit simultaneously with covariances estimated freely among them. A total of 49 items were thus included. WLSMV estimation (i.e., diagonally weighted least squares) in Mplus v 8.1 including a probit link and the THETA parameterization (such that all item residual variances were constrained to 1) was used to estimate all models (Muthén & Muthén, 1998–2017). Thus, model fit statistics describe the fit of the item factor model to the polychoric correlation matrix among the items. The fit of each model referenced below is shown in Table 1. Nested model comparisons were conducted using the Mplus DIFFTEST procedure.

We first established the need for 5 factors by showing a significant decrease in fit for a single-factor model relative to that of the 5-factor model, $\chi^2(10) = 769.754$, $p < .0001$. As shown in Table 1, the fit of the model with five correlated factors was marginally acceptable by both the RMSEA (.057) and the CFI (.927). Standardized model parameters (loadings, intercepts, and residual variances) are shown in Table 2. Correlations of .7 or higher were found amongst the five factors, suggesting evidence that the five factors may indicate a single higher-order factor. This idea was testing by eliminating the covariances among the factors and instead estimating loadings for the five factors from a single higher-order factor (whose variance was fixed to 1). Although the fit of the higher-order factor model remained marginal (see Table 1), a nested model comparison revealed a significant decrease in fit, $\chi^2(5) = 92.048$, $p < .0001$, indicating that a single factor did not appear adequate to describe the pattern of correlation amongst the five factors. Inspection of the discrepancy between the factor correlations from the 5-factor model and those predicted by the higher-order factor indicated two sources of misfit—the correlation between Corrupt and Terror was under-estimated, whereas the correlation between Ignore and Terror was over-estimated. These discrepancies were captured via two additional covariances among those lower-order factor disturbances, resulting in a significant improvement in fit, $\chi^2(2) = 88.343$, $p < .0001$. However, the revised model did not completely account for the pattern of correlation among the 5 factors, as indicated by a significant decrease in model fit relative to the model with all 10 factor correlations estimated directly, $\chi^2(3) = 8.483$, $p = .0370$.