Testing Multiple-Group Measurement Invariance using ML in Item Factor Models in Mplus version 8.1

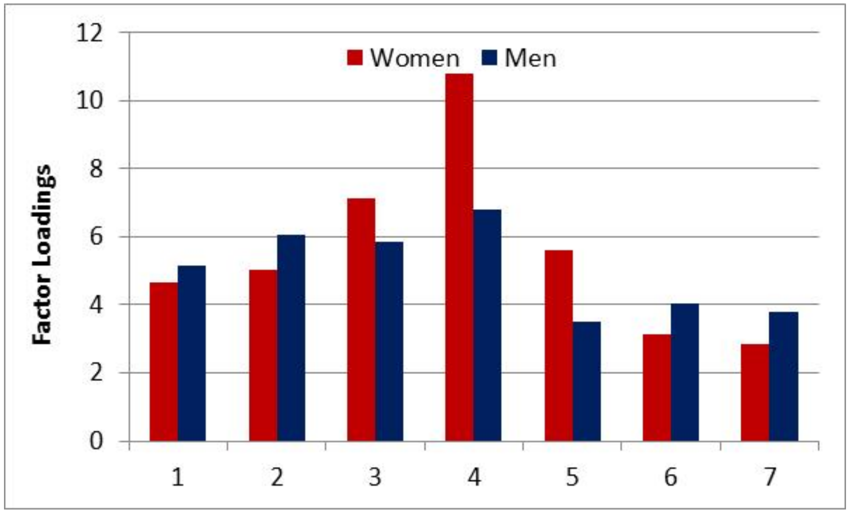
Example data: 634 older adults (age 80-100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows. We are examining differences between men (*N*=214) and women (*N*=420). Each item has **two** response options: 0 = needs help, 1 = does not need help. The items are: 1. Housework (cleaning and laundry), 2. Bedmaking, 3. Cooking, 4. Everyday shopping, 5. Getting to places outside of walking distance, 6. Handling banking and other business, and 7. Using the telephone.

Multiple Group IFA Model Syntax and Truncated Output:

```
TITLE:  Assess binary IADL items using ML
DATA:   FILE = ADL2.dat;
VARIABLE:  NAMES = case female cia1-cia7;
           USEVARIABLES = dia1-dia7;
           CATEGORICAL = dia1-dia7;
           CLASSES = group(2);             ! 2 fake latent classes
           KNOWNCLASS = group(female=0 1); ! Latent classes = gender
           IDVARIABLE = case;
           MISSING = .;

ANALYSIS:  ESTIMATOR = ML; LINK = LOGIT;   ! Full info estimator
           TYPE = MIXTURE; ALGORITHM = INTEGRATION; ! Latent class

PLOT:      TYPE = PLOT1 PLOT2 PLOT3;

OUTPUT:    STDYX RESIDUAL; ! No voo-doo for IFA in ML

! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
MODEL:
%OVERALL%  ! Needed for fake latent class model
! Factor loadings all estimated, just labeled
    IADL BY dia1-dia7* (L1-L7);
! Item thresholds all free, just labeled
    [dia1$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! CONFIGURAL MODEL FOR WOMEN ALTERNATIVE GROUP 2
%group#2%  ! Needed for fake latent class model
! Factor loadings all estimated
    IADL BY dia1-dia7*;
! Item thresholds all free
    [dia1$1-dia7$1*];
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       29

Loglikelihood
          H0 Value                       -1796.106

Information Criteria
          Akaike (AIC)                    3650.212
          Bayesian (BIC)                  3779.367
          Sample-Size Adjusted BIC        3687.295
            (n* = (n + 2) / 24)
```

This will be our baseline configural model.
29 parameters estimated = 2*[7 loadings + 7 thresholds] = 28
1 extra parameter is estimated as the logit of the proportion of the sample in group#1 (the female=0 group).



Model 1. Configural Invariance Model
(Everything separate across groups *except* for parameters needed to be constrained for identification)

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

```
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Latent Class 1 (0) – refers to female=0 from KNOWNCLASS

 IADL     BY -- FACTOR LOADINGS
    DIA1               5.138      1.321      3.890      0.000
    DIA2               6.066      1.830      3.315      0.001
    DIA3               5.828      1.566      3.722      0.000
    DIA4               6.792      2.086      3.256      0.001
    DIA5               3.520      0.717      4.907      0.000
    DIA6               4.026      0.890      4.525      0.000
    DIA7               3.789      1.211      3.129      0.002

 Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION
    IADL               0.000      0.000    999.000    999.000

 Thresholds: EXPECTED LOGIT OF Y=0 IF THETA=0
    DIA1$1            -1.372      0.622     -2.204      0.028
    DIA2$1            -5.632      1.614     -3.489      0.000
    DIA3$1            -3.005      0.919     -3.270      0.001
    DIA4$1            -4.378      1.337     -3.274      0.001
    DIA5$1            -2.722      0.552     -4.928      0.000
    DIA6$1            -3.311      0.707     -4.684      0.000
    DIA7$1            -6.398      1.701     -3.762      0.000

 Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION
    IADL               1.000      0.000    999.000    999.000
```

Although Mplus does give IRT a-parameters and b-parameters for binary items, it rescales them to assume a theta mean=0 and variance=1 in both groups. Thus, they will not be invariant even when the loadings and thresholds are invariant. For this reason, they are not shown here (but one can calculate the non-invariant versions using MODEL CONSTRAINT or excel).

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

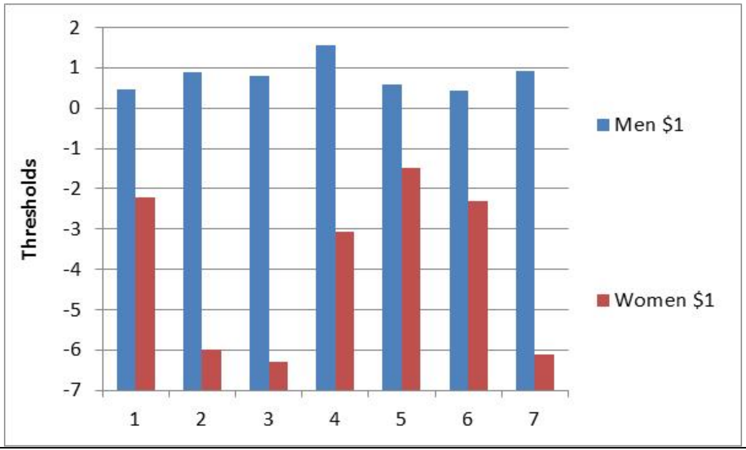
```
                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Latent Class 2 (1) – refers to female=1 from KNOWNCLASS

 IADL     BY -- FACTOR LOADINGS
    DIA1               4.664      0.731      6.377      0.000
    DIA2               5.014      0.998      5.026      0.000
    DIA3               7.108      1.670      4.257      0.000
    DIA4              10.774      8.194      1.315      0.189 !!!
    DIA5               5.611      1.145      4.902      0.000
    DIA6               3.143      0.422      7.440      0.000
    DIA7               2.857      0.610      4.681      0.000

 Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION
    IADL               0.000      0.000    999.000    999.000

 Thresholds
    DIA1$1            -2.188      0.423     -5.177      0.000
    DIA2$1            -5.694      1.020     -5.583      0.000
    DIA3$1            -6.715      1.494     -4.496      0.000
    DIA4$1            -3.791      2.941     -1.289      0.197
    DIA5$1            -1.798      0.491     -3.659      0.000
    DIA6$1            -2.184      0.313     -6.976      0.000
    DIA7$1            -5.577      0.886     -6.298      0.000

 Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION

    IADL               1.000      0.000    999.000    999.000

Categorical Latent Variables

 Means = logit of proportion of men (group#1) = .34
    GROUP#1           -0.670      0.084     -7.985      0.000
```

Model 2a. Metric Invariance Model (IFA loadings held equal across groups – Mplus IRT discriminations still vary via factor variances)

```
! MEN REFERENCE GROUP CONFIGURAL MODEL (will stay the same)
MODEL:
%OVERALL%  ! Needed for fake latent class model
! Factor loadings all estimated, just labeled
    IADL BY dia1-dia7* (L1-L7);
! Item thresholds all free, just labeled
    [dia1$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! 2 METRIC MODEL FOR WOMEN ALTERNATIVE GROUP 2
%group#2%  ! Needed for fake latent class model
! Factor loadings NOW CONSTRAINTED EQUAL TO MEN
    IADL BY dia1-dia7* (L1-L7);
! Item thresholds all free
    [dia1$1-dia7$1*];
! Factor mean=0 for identification
! Factor variance NOW ESTIMATED
    [IADL@0]; IADL*;
```

```
MODEL FIT INFORMATION
Number of Free Parameters                        23

Loglikelihood
          H0 Value                        -1799.928

Information Criteria
          Akaike (AIC)                     3645.855
          Bayesian (BIC)                   3748.288
          Sample-Size Adjusted BIC         3675.266
            (n* = (n + 2) / 24)
```

Does the full metric invariance model (2a) fit significantly worse than the configural model (1)? Nope, $-2\Delta LL(df=6) = 7.64$, $p = .27$




```
                                                     Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Latent Class 1 (0)

 IADL     BY
    DIA1               4.691      0.714      6.570      0.000
    DIA2               5.209      0.938      5.550      0.000
    DIA3               6.517      1.258      5.179      0.000
    DIA4               8.631      2.433      3.548      0.000
    DIA5               4.611      0.669      6.893      0.000
    DIA6               3.337      0.465      7.181      0.000
    DIA7               3.195      0.629      5.082      0.000

 Means: Factor mean fixed=0 for identification
    IADL               0.000      0.000    999.000    999.000

 Thresholds
    DIA1$1            -1.267      0.473     -2.681      0.007
    DIA2$1            -4.872      0.876     -5.563      0.000
    DIA3$1            -3.308      0.811     -4.078      0.000
    DIA4$1            -5.408      1.559     -3.468      0.001
    DIA5$1            -3.387      0.580     -5.838      0.000
    DIA6$1            -2.828      0.441     -6.409      0.000
    DIA7$1            -5.626      0.924     -6.091      0.000

 Variances: Factor variance fixed=1 for identification
    IADL               1.000      0.000    999.000    999.000

Latent Class 2 (1)

 IADL     BY
    DIA1               4.691      0.714      6.570      0.000
    DIA2               5.209      0.938      5.550      0.000
    DIA3               6.517      1.258      5.179      0.000
    DIA4               8.631      2.433      3.548      0.000
    DIA5               4.611      0.669      6.893      0.000
    DIA6               3.337      0.465      7.181      0.000
    DIA7               3.195      0.629      5.082      0.000

 Means: Factor mean fixed=0 for identification
    IADL               0.000      0.000    999.000    999.000

 Thresholds
    DIA1$1            -2.220      0.402     -5.520      0.000
    DIA2$1            -5.983      0.913     -6.555      0.000
    DIA3$1            -6.310      1.089     -5.794      0.000
    DIA4$1            -3.079      0.995     -3.095      0.002
    DIA5$1            -1.495      0.358     -4.179      0.000
    DIA6$1            -2.317      0.315     -7.348      0.000
    DIA7$1            -6.127      0.882     -6.949      0.000

 Variances:  Factor variance now estimated
    IADL               1.056      0.240      4.409      0.000
```

Model 3a. Full Threshold Invariance Model (all IFA thresholds held equal across groups – Mplus IRT difficulties can still vary)

```
! MEN REFERENCE GROUP CONFIGURAL MODEL (stays the same)
MODEL:
%OVERALL%  ! Needed for fake latent class model
! Factor loadings all estimated, just labeled
    IADL BY dia1-dia7* (L1-L7);
! Item thresholds all free, just labeled
    [dia1$1-dia7$1*] (T1-T7);
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP 2
%group#2%  ! Needed for fake latent class model
! Factor loadings NOW CONSTRAINED EQUAL TO MEN
    IADL BY dia1-dia7* (L1-L7);
! Item thresholds NOW CONSTRAINED EQUAL TO MEN
    [dia1$1-dia7$1*] (T1-T7);
! Factor mean=0 NOW ESTIMATED
! Factor variance NOW ESTIMATED
    [IADL*]; IADL*;
```

```
MODEL FIT INFORMATION
Number of Free Parameters                        17

Loglikelihood
          H0 Value                        -1860.402
Information Criteria

          Akaike (AIC)                     3754.804
          Bayesian (BIC)                   3830.515
          Sample-Size Adjusted BIC         3776.542
```

Does the full scalar invariance model (3a) fit significantly worse than the full metric model (2a)? Yep, $-2\Delta LL(df=6) = 120.95$, $p < .01$

Based on the local misfit of the univariate distributions (from TECH10 output, see right), it looks like dia3 is the biggest problem… so the same steps would be followed as in our other examples. That is, try to free problematic thresholds until the scalar model fits not worse than the metric model (or the configural model, if you choose to constrain both loadings and thresholds at the same time).

```
RESIDUAL OUTPUT


     UNIVARIATE DISTRIBUTION FIT FOR CLASS 1

     Variable         Observed   Estimated Residual (Obs.-Est.) Stand. Residual
     DIA1
       Category 1        0.404        0.352             0.052            1.591
       Category 2        0.596        0.648            -0.052           -1.591
     DIA2
       Category 1        0.184        0.168             0.017            0.664
       Category 2        0.816        0.832            -0.017           -0.664
     DIA3
       Category 1        0.313        0.229             0.084            2.939
       Category 2        0.687        0.771            -0.084           -2.939
     DIA4
       Category 1        0.257        0.333            -0.076           -2.362
       Category 2        0.743        0.667             0.076            2.362
     DIA5
       Category 1        0.249        0.335            -0.086           -2.683
       Category 2        0.751        0.665             0.086            2.683
     DIA6
       Category 1        0.233        0.261            -0.028           -0.946
       Category 2        0.767        0.739             0.028            0.946
     DIA7
       Category 1        0.066        0.062             0.004            0.233
       Category 2        0.934        0.938            -0.004           -0.233

     UNIVARIATE DISTRIBUTION FIT FOR CLASS 2

     Variable         Observed    Estimated  Residual (Obs.-Est.)  Stand. Residual
     DIA1
       Category 1        0.343         0.370            -0.027           -1.132
       Category 2        0.657         0.630             0.027            1.132
     DIA2
       Category 1        0.147         0.162            -0.015           -0.813
       Category 2        0.853         0.838             0.015            0.813
     DIA3
       Category 1        0.186         0.230            -0.044           -2.165
       Category 2        0.814         0.770             0.044            2.165
     DIA4
       Category 1        0.378         0.348             0.030            1.298
       Category 2        0.622         0.652            -0.030           -1.298
     DIA5
       Category 1        0.397         0.350             0.046            1.985
       Category 2        0.603         0.650            -0.046           -1.985
     DIA6
       Category 1        0.284         0.267             0.017            0.785
       Category 2        0.716         0.733            -0.017           -0.785
     DIA7
       Category 1        0.052         0.053            -0.001           -0.078
       Category 2        0.948         0.947             0.001            0.078
```