Testing Multiple-Group Measurement Invariance using WLSMV in Item Factor Models in Mplus version 8.1

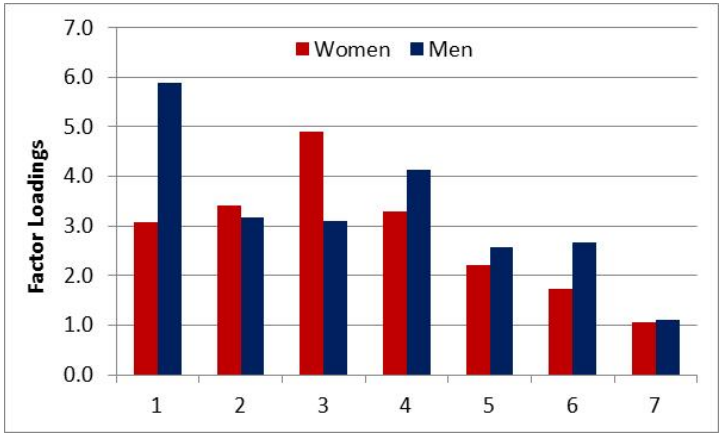
Example data: 634 older adults (age 80-100) self-reporting on 7 items assessing the Instrumental Activities of Daily Living (IADL) as follows. We are examining differences between men (*N*=214) and women (*N*=420). Each item has four response options: 0 = can't do it, 1= big problems doing it, 2 = some problems doing it, 3 = can do it. The items are: 1. Housework (cleaning and laundry), 2. Bedmaking, 3. Cooking, 4. Everyday shopping, 5. Getting to places outside of walking distance, 6. Handling banking and other business, and 7. Using the telephone.

Multiple Group IFA Model Syntax and Truncated Output:

```
TITLE:   Assess polytomous IADL items
DATA:    FILE IS ADL2.dat;
VARIABLE:  NAMES = case female cia1-cia7;
           USEVARIABLES = cia1-cia7;
           CATEGORICAL = cia1-cia7;
           GROUPING = female (0=Men 1=Women);
           IDVARIABLE = case;
           MISSING = .;

ANALYSIS:  ESTIMATOR=WLSMV; PARAMETERIZATION=THETA;

PLOT:      TYPE = PLOT1 PLOT2 PLOT3;

SAVEDATA:  DIFFTEST=Configural.dat; ! Save configural info

OUTPUT:    STDYX MODINDICES(3.84); ! Constraints to drop p<.05

!!! CONFIGURAL MODEL FOR MEN REFERENCE GROUP
MODEL:
! Factor loadings all estimated, just labeled
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free, just labeled
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

!!! CONFIGURAL MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings all estimated
    IADL BY cia1-cia7*;
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;
```

```
MODEL FIT INFORMATION
Number of Free Parameters                       56
Chi-Square Test of Model Fit
          Value                             72.921*
          Degrees of Freedom                   28
          P-Value                           0.0000
Chi-Square Contributions From Each Group
          MEN                               24.975
          WOMEN                             47.946
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.071
          90 Percent C.I.                    0.051  0.091
          Probability RMSEA <= .05           0.040
CFI/TLI
          CFI                                0.999
          TLI                                0.998
```

This will be our baseline configural model.
56 parameters estimated = 2*[7 loadings + 21 thresholds] = 56
Possible parameters = 2* ([7*(7+1)] / 2] + 21 thresholds) = 98
DF = 98 – 56 – 14 "residual variances" = 28



Model 1. Configural Invariance Model
(Everything separate across groups *except* for parameters needed to be constrained for identification)

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group MEN

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA				
IADL BY				
CIA1	5.894	1.487	3.963	0.000
CIA2	3.183	0.548	5.811	0.000
CIA3	3.089	0.445	6.948	0.000
CIA4	4.135	0.829	4.991	0.000
CIA5	2.576	0.374	6.893	0.000
CIA6	2.665	0.496	5.376	0.000
CIA7	1.115	0.218	5.120	0.000
Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-7.470	1.872	-3.990	0.000
CIA1$2	-5.149	1.385	-3.718	0.000
CIA1$3	-0.834	0.664	-1.255	0.210
CIA2$1	-4.480	0.638	-7.027	0.000
CIA2$2	-3.847	0.587	-6.558	0.000
CIA2$3	-2.263	0.488	-4.638	0.000
CIA3$1	-3.880	0.483	-8.027	0.000
CIA3$2	-2.810	0.451	-6.228	0.000
CIA3$3	-0.565	0.325	-1.737	0.082
CIA4$1	-5.178	0.923	-5.610	0.000
CIA4$2	-4.188	0.756	-5.542	0.000
CIA4$3	-2.172	0.566	-3.836	0.000
CIA5$1	-4.611	0.573	-8.050	0.000
CIA5$2	-2.620	0.374	-7.007	0.000
CIA5$3	-1.191	0.300	-3.974	0.000
CIA6$1	-4.606	0.710	-6.487	0.000
CIA6$2	-3.341	0.570	-5.859	0.000
CIA6$3	-2.235	0.474	-4.711	0.000
CIA7$1	-3.115	0.383	-8.125	0.000
CIA7$2	-2.375	0.279	-8.517	0.000
CIA7$3	-1.779	0.234	-7.592	0.000
Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group WOMEN

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA				
IADL BY				
CIA1	3.084	0.305	10.125	0.000
CIA2	3.414	0.463	7.374	0.000
CIA3	4.893	0.924	5.294	0.000
CIA4	3.283	0.330	9.946	0.000
CIA5	2.201	0.196	11.258	0.000
CIA6	1.741	0.172	10.102	0.000
CIA7	1.056	0.156	6.786	0.000
Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-4.581	0.400	-11.453	0.000
CIA1$2	-3.288	0.336	-9.797	0.000
CIA1$3	-0.706	0.221	-3.187	0.001
CIA2$1	-5.424	0.622	-8.721	0.000
CIA2$2	-4.451	0.545	-8.166	0.000
CIA2$3	-2.798	0.441	-6.341	0.000
CIA3$1	-7.214	1.194	-6.041	0.000
CIA3$2	-6.147	1.073	-5.732	0.000
CIA3$3	-3.885	0.835	-4.653	0.000
CIA4$1	-4.479	0.377	-11.867	0.000
CIA4$2	-2.802	0.306	-9.166	0.000
CIA4$3	-0.678	0.237	-2.857	0.004
CIA5$1	-3.745	0.281	-13.344	0.000
CIA5$2	-1.687	0.190	-8.882	0.000
CIA5$3	-0.118	0.151	-0.778	0.437
CIA6$1	-3.201	0.243	-13.153	0.000
CIA6$2	-2.115	0.190	-11.135	0.000
CIA6$3	-1.173	0.162	-7.255	0.000
CIA7$1	-3.407	0.325	-10.498	0.000
CIA7$2	-2.713	0.233	-11.638	0.000
CIA7$3	-1.747	0.165	-10.580	0.000
Variances: VARIANCE OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

Model 2a. Metric Invariance Model (IFA loadings held equal across groups – Mplus IRT discriminations still vary via factor variances)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=Configural.dat;   ! Compare against configural

SAVEDATA:   DIFFTEST=MetricA.dat;      ! Save full metric A info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 2 METRIC MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 for identification
! Factor variance NOW ESTIMATED
    [IADL@0]; IADL*;
```

```
Number of Free Parameters                       50

Chi-Square Test of Model Fit
          Value                             64.668*
          Degrees of Freedom                   34
          P-Value                           0.0012

Chi-Square Contributions From Each Group
          MEN                               30.856
          WOMEN                             33.812

THIS IS THE TEST OF CONFIGURAL VS. FULL METRIC A INVARIANCE
Chi-Square Test for Difference Testing
          Value                              9.395
          Degrees of Freedom                     6
          P-Value                           0.1525

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.053
          90 Percent C.I.                    0.033  0.073
          Probability RMSEA <= .05           0.367

CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

Although the DIFFTEST chi-square is nonsignificant, the modification indices suggest freeing the loading for cia3 between groups:

```
MODEL MODIFICATION INDICES
                                    M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN

BY Statements
IADL     BY CIA3                   7.717    -2.375     -2.375       -0.487
IADL     BY CIA6                   5.858     0.338      0.338        0.136

Group WOMEN

BY Statements
IADL     BY CIA3                   7.674     1.524      1.269        0.310
IADL     BY CIA6                   5.858    -0.374     -0.311       -0.146
```

Model 2b. Metric Invariance Model (IFA loadings held equal across groups except cia3)

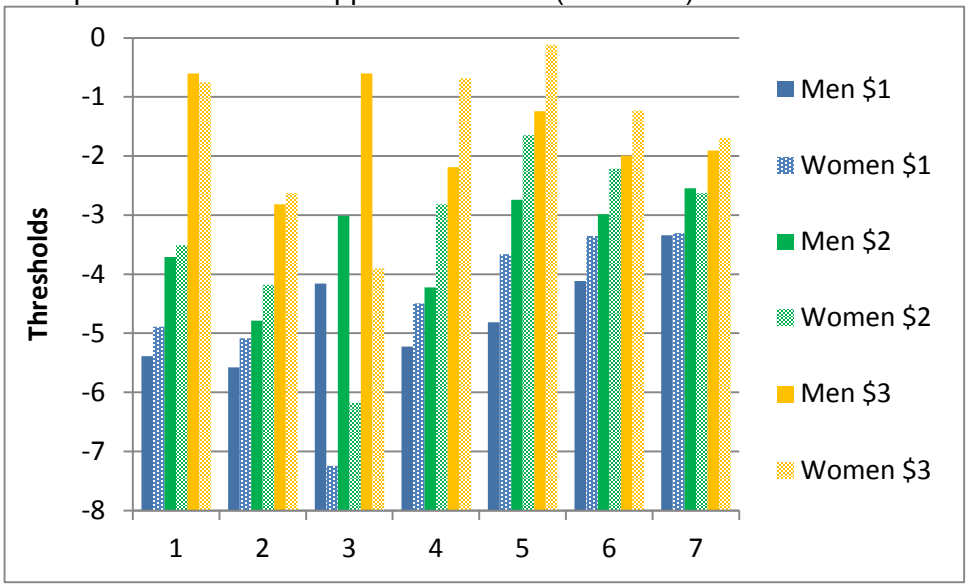
```
ANALYSIS:    ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
             DIFFTEST=Configural.dat;  ! Compare against configural

SAVEDATA:    DIFFTEST=MetricB.dat;     ! Save partial metric B info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 2 METRIC MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds all free
    [cia1$1-cia7$1*];
    [cia1$2-cia7$2*];
    [cia1$3-cia7$3*];
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 for identification
! Factor variance NOW ESTIMATED
    [IADL@0]; IADL*;
```

Here is the test comparing Metric B to Metric A.
Freeing cia3 helped:

```
Chi-Square Test for Difference Testing
          Value                              5.559
          Degrees of Freedom                     1
          P-Value                           0.0184
```

```
Number of Free Parameters                       51
Chi-Square Test of Model Fit
          Value                             58.606*
          Degrees of Freedom                    33
          P-Value                           0.0039
Chi-Square Contribution From Each Group
          MEN                               24.959
          WOMEN                             33.646

THIS IS THE TEST OF CONFIGURAL VS. PARTIAL METRIC B INVARIANCE
Chi-Square Test for Difference Testing
          Value                              5.521
          Degrees of Freedom                     5
          P-Value                           0.3556

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.049
          90 Percent C.I.                    0.028  0.070
          Probability RMSEA <= .05           0.490
CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

No new modification indices, so we are done with metric!
Let's preview what will happen with scalar (threshold) invariance:



Partial metric invariance solution (factor loadings constrained except cia3)

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group MEN				
IADL BY – CHANGE IN PROBIT FOR ONE-UNIT CHANGE IN THETA				
CIA1	4.189	0.454	9.221	0.000
CIA2	4.028	0.557	7.235	0.000
CIA3	**3.331**	**0.519**	**6.415**	**0.000**
CIA4	4.171	0.506	8.235	0.000
CIA5	2.710	0.300	9.020	0.000
CIA6	2.338	0.261	8.949	0.000
CIA7	1.256	0.174	7.228	0.000
Means MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000
Thresholds – EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-5.385	0.613	-8.779	0.000
CIA1$2	-3.709	0.551	-6.733	0.000
CIA1$3	-0.602	0.421	-1.430	0.153
CIA2$1	-5.577	0.674	-8.280	0.000
CIA2$2	-4.789	0.645	-7.428	0.000
CIA2$3	-2.816	0.530	-5.311	0.000
CIA3$1	-4.158	0.548	-7.584	0.000
CIA3$2	-3.011	0.505	-5.964	0.000
CIA3$3	-0.605	0.352	-1.718	0.086
CIA4$1	-5.224	0.606	-8.622	0.000
CIA4$2	-4.223	0.555	-7.607	0.000
CIA4$3	-2.191	0.474	-4.620	0.000
CIA5$1	-4.816	0.488	-9.861	0.000
CIA5$2	-2.739	0.351	-7.803	0.000
CIA5$3	-1.244	0.290	-4.285	0.000
CIA6$1	-4.113	0.415	-9.906	0.000
CIA6$2	-2.985	0.339	-8.809	0.000
CIA6$3	-1.995	0.303	-6.594	0.000
CIA7$1	-3.340	0.355	-9.397	0.000
CIA7$2	-2.547	0.252	-10.086	0.000
CIA7$3	-1.907	0.219	-8.717	0.000
Variances VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group WOMEN				
IADL BY				
CIA1	4.189	0.454	9.221	0.000
CIA2	4.028	0.557	7.235	0.000
CIA3	**6.224**	**1.252**	**4.970**	**0.000**
CIA4	4.171	0.506	8.235	0.000
CIA5	2.710	0.300	9.020	0.000
CIA6	2.338	0.261	8.949	0.000
CIA7	1.256	0.174	7.228	0.000
Means MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000
Thresholds				
CIA1$1	-4.885	0.428	-11.407	0.000
CIA1$2	-3.506	0.360	-9.746	0.000
CIA1$3	-0.752	0.236	-3.186	0.001
CIA2$1	-5.083	0.487	-10.446	0.000
CIA2$2	-4.176	0.432	-9.678	0.000
CIA2$3	-2.624	0.361	-7.268	0.000
CIA3$1	-7.242	1.230	-5.890	0.000
CIA3$2	-6.174	1.103	-5.600	0.000
CIA3$3	-3.902	0.854	-4.570	0.000
CIA4$1	-4.493	0.368	-12.197	0.000
CIA4$2	-2.812	0.302	-9.324	0.000
CIA4$3	-0.680	0.236	-2.881	0.004
CIA5$1	-3.660	0.259	-14.134	0.000
CIA5$2	-1.648	0.179	-9.202	0.000
CIA5$3	-0.115	0.147	-0.780	0.435
CIA6$1	-3.350	0.256	-13.091	0.000
CIA6$2	-2.213	0.197	-11.206	0.000
CIA6$3	-1.227	0.168	-7.321	0.000
CIA7$1	-3.299	0.287	-11.504	0.000
CIA7$2	-2.627	0.200	-13.110	0.000
CIA7$3	-1.692	0.142	-11.899	0.000
Variances VARIANCE OF THETA NOW ESTIMATED				
IADL	**0.624**	**0.139**	**4.473**	**0.000**
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

Model 3a. Full Threshold Invariance Model (all IFA thresholds held equal across groups – Mplus IRT difficulties will still vary)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricB.dat; ! Compare against partial metric B

SAVEDATA:   DIFFTEST=ScalarA.dat; ! Save full scalar A info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds all CONSTRAINED EQUAL TO MEN BY OMISSION



! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean NOW ESTIMATED
! Factor variance NOW ESTIMATED
    [IADL*]; IADL*;
```

The DIFFTEST chi-square is significant, and the modification indices suggest that cia3 threshold 3 is the biggest problem. This is not surprising given that its loading differed, too (but I thought I'd try to constrain cia3's thresholds anyway given that there are only seven items). Given the other mention of the "intercept" for cia3, let's see what happens when we free all the cia3 thresholds between groups.

```
MODEL FIT INFORMATION
Number of Free Parameters                       31

Chi-Square Test of Model Fit
          Value                             134.476*
          Degrees of Freedom                    53
          P-Value                           0.0000

Chi-Square Contributions From Each Group
          MEN                               88.185
          WOMEN                             46.291

THIS IS THE TEST OF PARTIAL METRIC B VS. FULL SCALAR A INVARIANCE
Chi-Square Test for Difference Testing
          Value                             91.663
          Degrees of Freedom                    20
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.070
          90 Percent C.I.                    0.055  0.084
          Probability RMSEA <= .05           0.014

CFI/TLI
          CFI                                0.998
          TLI                                0.998

MODEL MODIFICATION INDICES
                              M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Means/Intercepts/Thresholds
[ CIA1     ]                 10.619    -1.055     -1.055       -0.267
[ CIA2     ]                  4.043    -0.703     -0.703       -0.170
[ CIA3     ]                 40.682    -9.940     -9.940       -1.361
[ CIA4     ]                  4.390     0.712      0.712        0.170
[ CIA5     ]                 16.374     0.927      0.927        0.331
[ CIA6     ]                  4.015     0.427      0.427        0.172
[ IADL     ]                 40.339    -4.584     -4.584       -4.584
[ CIA1$3   ]                  6.828     0.628      0.628        0.159
[ CIA3$1   ]                  6.321    -2.568     -2.568       -0.352
[ CIA3$3   ]                 26.579     3.043      3.043        0.417
[ CIA4$3   ]                  4.568    -0.544     -0.544       -0.130
[ CIA5$2   ]                  4.494    -0.409     -0.409       -0.146
[ CIA5$3   ]                  8.185    -0.460     -0.460       -0.164
```

Note: As of Mplus v 7, there is an "automatic" option to do invariance testing. For WLSMV, at this step (scalar) their version simultaneously constrains the thresholds while freeing the residual variances in the alternative group (which are now possible to be separately identified). I disagree with this logic, and thus these results will not match those of the automated testing. Instead, I test scalar first, and then test residual variances in a separate step. Reasonable people may disagree with this choice.

Model 3b. Partial Threshold Invariance Model (freeing all cia3 thresholds between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricB.dat; ! Compare against partial metric B

SAVEDATA:   DIFFTEST=ScalarB.dat; ! Save partial scalar B info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds otherwise CONSTRAINED EQUAL TO MEN BY OMISSION
    [cia3$1* cia3$2* cia3$3*];  ! 3b


! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean NOW ESTIMATED
! Factor variance NOW ESTIMATED
    [IADL*]; IADL*;
```

Threshold for Men…				
CIA3$1	-4.107	0.525	-7.820	0.000
CIA3$2	-2.978	0.487	-6.119	0.000
CIA3$3	-0.599	0.346	-1.729	0.084
Threshold for Women…				
CIA3$1	-8.826	1.576	-5.599	0.000
CIA3$2	-7.742	1.447	-5.349	0.000
CIA3$3	-5.445	1.208	-4.508	0.000

MODEL FIT INFORMATION

Number of Free Parameters	34

Chi-Square Test of Model Fit

Value	96.673*
Degrees of Freedom	50
P-Value	0.0001

Chi-Square Contributions From Each Group

MEN	53.397
WOMEN	43.276

THIS IS THE TEST OF PARTIAL METRIC B VS. PARTIAL SCALAR B INVARIANCE

Chi-Square Test for Difference Testing

Value	44.797
Degrees of Freedom	17
P-Value	0.0003

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.054	
90 Percent C.I.	0.038	0.070
Probability RMSEA <= .05	0.315	

CFI/TLI

CFI	0.999
TLI	0.999

MODEL MODIFICATION INDICES

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
Group MEN				
Means/Intercepts/Thresholds				
[CIA1]	13.848	-1.287	-1.287	-0.295
[CIA2]	4.520	-0.730	-0.730	-0.173
[CIA5]	12.330	0.816	0.816	0.279
[CIA1$3]	10.577	0.855	0.855	0.196
[CIA5$3]	5.001	-0.373	-0.373	-0.127

The DIFFTEST chi-square is still significant, and the modification indices suggest that cia1 threshold 3 is the next to go…

Model 3c. Partial Threshold Invariance Model (also freeing cia1 threshold 3 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricB.dat; ! Compare against partial metric B

SAVEDATA:   DIFFTEST=ScalarC.dat; ! Save partial scalar C info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds otherwise CONSTRAINED EQUAL TO MEN BY OMISSION
    [cia3$1* cia3$2* cia3$3*];  ! 3b
    [cia1$3*]; ! 3c

! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean NOW ESTIMATED
! Factor variance NOW ESTIMATED
    [IADL*]; IADL*;
```

Thresholds for Men…

CIA1$1	-5.909	0.584	-10.124	0.000
CIA1$2	-4.469	0.531	-8.424	0.000
CIA1$3	-0.620	0.434	-1.429	0.153

Thresholds for Women…

CIA1$1	-5.909	0.584	-10.124	0.000
CIA1$2	-4.469	0.531	-8.424	0.000
CIA1$3	-1.937	0.462	-4.193	0.000

MODEL FIT INFORMATION

Number of Free Parameters 35

Chi-Square Test of Model Fit

Value	87.761*
Degrees of Freedom	49
P-Value	0.0006

Chi-Square Contributions From Each Group

MEN	46.376
WOMEN	41.385

THIS IS THE TEST OF PARTIAL METRIC B VS. PARTIAL SCALAR C INVARIANCE

Chi-Square Test for Difference Testing

Value	32.461
Degrees of Freedom	16
P-Value	0.0087

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.050	
90 Percent C.I.	0.033	0.067
Probability RMSEA <= .05	0.480	

CFI/TLI

CFI	0.999
TLI	0.999

MODEL MODIFICATION INDICES

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
Group MEN				
Means/Intercepts/Thresholds				
[CIA1]	4.651	-0.956	-0.956	-0.215
[CIA2]	6.370	-0.884	-0.884	-0.205
[CIA5]	8.713	0.705	0.705	0.236
[CIA2$3]	**5.146**	**0.635**	**0.635**	**0.147**

The DIFFTEST chi-square is still significant, and the modification indices suggest that cia2 threshold 3 is the next to go…

Model 3d. Partial Threshold Invariance Model (also freeing cia2 threshold 3 between groups)

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=MetricB.dat; ! Compare against partial metric B

SAVEDATA:   DIFFTEST=ScalarD.dat; ! Save partial scalar D info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds otherwise CONSTRAINED EQUAL TO MEN BY OMISSION
    [cia3$1* cia3$2* cia3$3*];  ! 3b
    [cia1$3*]; ! 3c
    [cia2$3*]; ! 3d

! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean NOW ESTIMATED
! Factor variance NOW ESTIMATED
    [IADL*]; IADL*;
```

Thresholds for Men…

CIA2$1	-6.161	0.682	-9.027	0.000
CIA2$2	-5.289	0.640	-8.270	0.000
CIA2$3	-2.929	0.554	-5.282	0.000

Thresholds for Women…

CIA2$1	-6.161	0.682	-9.027	0.000
CIA2$2	-5.289	0.640	-8.270	0.000
CIA2$3	-3.852	0.587	-6.567	0.000

MODEL FIT INFORMATION

Number of Free Parameters 36

Chi-Square Test of Model Fit

Value	82.974*
Degrees of Freedom	48
P-Value	0.0013

Chi-Square Contributions From Each Group

MEN	42.539
WOMEN	40.435

THIS IS THE TEST OF PARTIAL METRIC B VS. PARTIAL SCALAR D INVARIANCE

Chi-Square Test for Difference Testing

Value	26.168
Degrees of Freedom	15
P-Value	0.0363

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.048	
90 Percent C.I.	0.030	0.065
Probability RMSEA <= .05	0.556	

CFI/TLI

CFI	0.999
TLI	0.999

MODEL MODIFICATION INDICES

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
Group MEN				
Means/Intercepts/Thresholds				
[CIA1]	5.844	-1.082	-1.082	-0.243
[CIA5]	6.734	0.631	0.631	0.210
[IADL]	999.000	0.000	0.000	0.000
[CIA1$2]	**4.630**	**0.721**	**0.721**	**0.162**

So close… but the DIFFTEST chi-square is still significant, and the modification indices suggest that cia1 threshold 2 is the next to go…

Model 3e. Partial Threshold Invariance Model (also freeing cia1 threshold 2 between groups)

```
ANALYSIS:    ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
             DIFFTEST=MetricB.dat; ! Compare against partial metric B

SAVEDATA:    DIFFTEST=ScalarE.dat; ! Save partial scalar D info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 3 SCALAR MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds otherwise CONSTRAINED EQUAL TO MEN BY OMISSION
    [cia3$1* cia3$2* cia3$3*];  ! 3b
    [cia1$3*]; ! 3c
    [cia2$3*]; ! 3d
    [cia1$2*]; ! 3e

! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean NOW ESTIMATED
! Factor variance NOW ESTIMATED
    [IADL*]; IADL*;
```

```
Thresholds for Men…
    CIA1$1            -6.035      0.591    -10.215      0.000
    CIA1$2            -3.832      0.570     -6.728      0.000
    CIA1$3            -0.622      0.435     -1.431      0.152

Thresholds for Women…
    CIA1$1            -6.035      0.591    -10.215      0.000
    CIA1$2            -4.823      0.544     -8.868      0.000
    CIA1$3            -2.103      0.468     -4.493      0.000
```

```
MODEL FIT INFORMATION

Number of Free Parameters                       37

Chi-Square Test of Model Fit
          Value                             78.437*
          Degrees of Freedom                    47
          P-Value                           0.0027

Chi-Square Contributions From Each Group
          MEN                               38.829
          WOMEN                             39.578

THIS IS THE TEST OF PARTIAL METRIC B VS. PARTIAL SCALAR E INVARIANCE
Chi-Square Test for Difference Testing
          Value                             20.323
          Degrees of Freedom                    14
          P-Value                           0.1203

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.046
          90 Percent C.I.                    0.027  0.063
          Probability RMSEA <= .05           0.627

CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

No more modification indices! Scalar is finally done!

The next step is to see if the “residual” variances need to be different between groups. This is only possible (i.e., the model with free residuals is only identified) given at least partial metric and scalar invariance per item.

So far the residual variances have been fixed=1 in both groups for identification. We now proceed backwards: we estimated a “bigger” model, in which the residual variances are freed in the alternative women group for the items with at least partial scalar invariance (so all but cia3), and compare it back to our Scalar E model (in which they were fixed=1).

Partial Scalar E invariance solution

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group MEN				
IADL BY – CHANGE IN PROBIT FOR ONE-UNIT CHANGE IN THETA				
CIA1	4.332	0.476	9.091	0.000
CIA2	4.193	0.585	7.163	0.000
CIA3	**3.239**	**0.474**	**6.826**	**0.000**
CIA4	4.324	0.522	8.286	0.000
CIA5	2.820	0.316	8.932	0.000
CIA6	2.404	0.272	8.851	0.000
CIA7	1.307	0.182	7.183	0.000
Means MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	**0.000**	**0.000**	**999.000**	**999.000**
Thresholds – EXPECTED PROBIT OF Y=0 IF THETA=0				
CIA1$1	-6.035	0.591	-10.215	0.000
CIA1$2	**-3.832**	**0.570**	**-6.728**	**0.000**
CIA1$3	**-0.622**	**0.435**	**-1.431**	**0.152**
CIA2$1	-6.215	0.686	-9.054	0.000
CIA2$2	-5.343	0.643	-8.305	0.000
CIA2$3	**-2.926**	**0.554**	**-5.278**	**0.000**
CIA3$1	**-4.052**	**0.508**	**-7.970**	**0.000**
CIA3$2	**-2.933**	**0.472**	**-6.208**	**0.000**
CIA3$3	**-0.589**	**0.341**	**-1.726**	**0.084**
CIA4$1	-5.705	0.561	-10.167	0.000
CIA4$2	-4.188	0.504	-8.309	0.000
CIA4$3	-2.081	0.458	-4.547	0.000
CIA5$1	-4.622	0.368	-12.554	0.000
CIA5$2	-2.590	0.314	-8.239	0.000
CIA5$3	-1.066	0.285	-3.744	0.000
CIA6$1	-4.092	0.337	-12.127	0.000
CIA6$2	-2.962	0.298	-9.926	0.000
CIA6$3	-1.982	0.277	-7.149	0.000
CIA7$1	-3.594	0.295	-12.170	0.000
CIA7$2	-2.880	0.234	-12.322	0.000
CIA7$3	-2.058	0.194	-10.601	0.000
Variances VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Group WOMEN				
IADL BY				
CIA1	4.332	0.476	9.091	0.000
CIA2	4.193	0.585	7.163	0.000
CIA3	**6.717**	**1.374**	**4.889**	**0.000**
CIA4	4.324	0.522	8.286	0.000
CIA5	2.820	0.316	8.932	0.000
CIA6	2.404	0.272	8.851	0.000
CIA7	1.307	0.182	7.183	0.000
Means MEAN OF THETA NOW ESTIMATED				
IADL	**-0.314**	**0.097**	**-3.239**	**0.001**
Thresholds				
CIA1$1	-6.035	0.591	-10.215	0.000
CIA1$2	**-4.823**	**0.544**	**-8.868**	**0.000**
CIA1$3	**-2.103**	**0.468**	**-4.493**	**0.000**
CIA2$1	-6.215	0.686	-9.054	0.000
CIA2$2	-5.343	0.643	-8.305	0.000
CIA2$3	**-3.925**	**0.591**	**-6.645**	**0.000**
CIA3$1	**-9.550**	**1.785**	**-5.349**	**0.000**
CIA3$2	**-8.458**	**1.649**	**-5.128**	**0.000**
CIA3$3	**-6.121**	**1.389**	**-4.406**	**0.000**
CIA4$1	-5.705	0.561	-10.167	0.000
CIA4$2	-4.188	0.504	-8.309	0.000
CIA4$3	-2.081	0.458	-4.547	0.000
CIA5$1	-4.622	0.368	-12.554	0.000
CIA5$2	-2.590	0.314	-8.239	0.000
CIA5$3	-1.066	0.285	-3.744	0.000
CIA6$1	-4.092	0.337	-12.127	0.000
CIA6$2	-2.962	0.298	-9.926	0.000
CIA6$3	-1.982	0.277	-7.149	0.000
CIA7$1	-3.594	0.295	-12.170	0.000
CIA7$2	-2.880	0.234	-12.322	0.000
CIA7$3	-2.058	0.194	-10.601	0.000
Variances VARIANCE OF THETA ESTIMATED				
IADL	**0.567**	**0.125**	**4.530**	**0.000**
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

Model 4a. Partial Residual Variance Invariance Model (thresholds unconstrained between groups from Scalar E)
This last step for testing measurement invariance proceeds backwards.
Because freeing the residual variances is adding parameters, we must estimate this model with free residual variances FIRST.

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;

SAVEDATA:   DIFFTEST=ResidualFreeA.dat;  ! Save free residual info

!!! MODEL FOR MEN REFERENCE GROUP (same as configural)
MODEL:
! Factor loadings all estimated
    IADL BY cia1-cia7* (L1-L7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_1-T1_7);
    [cia1$2-cia7$2*] (T2_1-T2_7);
    [cia1$3-cia7$3*] (T3_1-T3_7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;


!!! 4 RESIDUAL VARIANCE MODEL FOR WOMEN ALTERNATIVE GROUP
MODEL WOMEN:
! Factor loadings NOW CONSTRAINTED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L1-L2 L3a L4-L7);
! Item thresholds otherwise CONSTRAINED EQUAL TO MEN BY OMISSION
    [cia3$1* cia3$2* cia3$3*];  ! 3b
    [cia1$3*]; ! 3c
    [cia2$3*]; ! 3d
    [cia1$2*]; ! 3e
! Item residual variances NOW FREE EXCEPT cia3
    cia1-cia7*;
    cia3@1;
! Factor mean STILL ESTIMATED
! Factor variance STILL ESTIMATED
    [IADL*]; IADL*;
```

```
(Number of Free Parameters                       43

Chi-Square Test of Model Fit
          Value                             72.070*
          Degrees of Freedom                   41
          P-Value                           0.0019

Chi-Square Contributions From Each Group
          MEN                               29.082
          WOMEN                             42.987

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.049
          90 Percent C.I.                    0.029  0.067
          Probability RMSEA <= .05           0.515

CFI/TLI
          CFI                                0.999
          TLI                                0.999

Residual Variances for Men…
    CIA1               1.000      0.000    999.000    999.000
    CIA2               1.000      0.000    999.000    999.000
    CIA3               1.000      0.000    999.000    999.000
    CIA4               1.000      0.000    999.000    999.000
    CIA5               1.000      0.000    999.000    999.000
    CIA6               1.000      0.000    999.000    999.000
    CIA7               1.000      0.000    999.000    999.000

Residual Variances for Women…
    CIA1               1.697      0.955      1.777      0.076
    CIA2               0.387      0.143      2.719      0.007
    CIA3               1.000      0.000    999.000    999.000
    CIA4               0.718      0.308      2.334      0.020
    CIA5               0.698      0.244      2.856      0.004
    CIA6               1.113      0.390      2.856      0.004
    CIA7               0.552      0.146      3.783      0.000
```

Model 4b. Residual Variance Invariance Model (testing all items except cia3)

We now constrain the residual variances to be equal between groups and test decrease in fit (left side is just Scalar E again).

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=ResidualFreeA.dat;  ! Compare to free residual

SAVEDATA:   DIFFTEST=ResidualFixedB.dat; ! Save fixed B residual info

!!! 4B MODEL IS THE SAME AS SCALAR E

MODEL FIT INFORMATION
Number of Free Parameters                       37

Chi-Square Test of Model Fit
          Value                             78.437*
          Degrees of Freedom                   47
          P-Value                           0.0027

Chi-Square Contribution From Each Group
          MEN                               38.859
          WOMEN                             39.578

THIS IS THE TEST OF FULL RESIDUAL B VS. PARTIAL SCALAR E INVARIANCE
Chi-Square Test for Difference Testing
          Value                             11.016
          Degrees of Freedom                    6
          P-Value                           0.0879

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.046
          90 Percent C.I.                    0.027  0.063
          Probability RMSEA <= .05           0.627

CFI/TLI
          CFI                                0.999
          TLI                                0.999


MODEL MODIFICATION INDICES
                          M.I.     E.P.C.  Std E.P.C.  StdYX E.P.C.
Group MEN
Variances/Residual Variances

CIA2                     5.009     1.006      1.006        0.054
CIA7                     9.405     0.632      0.632        0.233
```

Although DIFFTEST is nonsignificant, it looks like the residual variance for cia7 wants to be free, too…

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=ResidualFreeA.dat;  ! Compare to free residual

SAVEDATA:   DIFFTEST=ResidualFixedC.dat; ! Save fixed C residual info

!!! 4C MODEL IS THE SAME AS RESIDUAL 4B=SCALAR E EXCEPT:
    cia7*;

MODEL FIT INFORMATION for 4c
Number of Free Parameters                       38
Chi-Square Test of Model Fit
          Value                             74.740*
          Degrees of Freedom                   46
          P-Value                           0.0047
Chi-Square Contribution From Each Group
          MEN                               35.043
          WOMEN                             39.696

THIS IS THE TEST OF PARTIAL RESIDUAL C VS. PARTIAL SCALAR E INVARIANCE
Chi-Square Test for Difference Testing
          Value                              6.854
          Degrees of Freedom                     5
          P-Value                           0.2317
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                           0.044
          90 Percent C.I.                    0.025  0.062
          Probability RMSEA <= .05           0.676
CFI/TLI
          CFI                                0.999
          TLI                                0.999
```

No more modification indices! Residual variance is now done!
The last step is to test structural invariance to see if the factor variance can be constrained across groups:

```
ANALYSIS:   ESTIMATOR IS WLSMV; PARAMETERIZATION=THETA;
            DIFFTEST=ResidualFixedC.dat;  ! Compare to fixed C residual

SAVEDATA:   DIFFTEST=FactorVariance.dat;  ! Save fixed variance info

!!! MODEL IS THE SAME AS RESIDUAL 4C EXCEPT:
    IADL@1;

Chi-Square Test for Difference Testing: Keep the factor variance different!
          Value                              8.818
          Degrees of Freedom                     1
          P-Value                           0.0030
```

Final Model: Partial Measurement Invariance (solution from Model 4c)

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group MEN

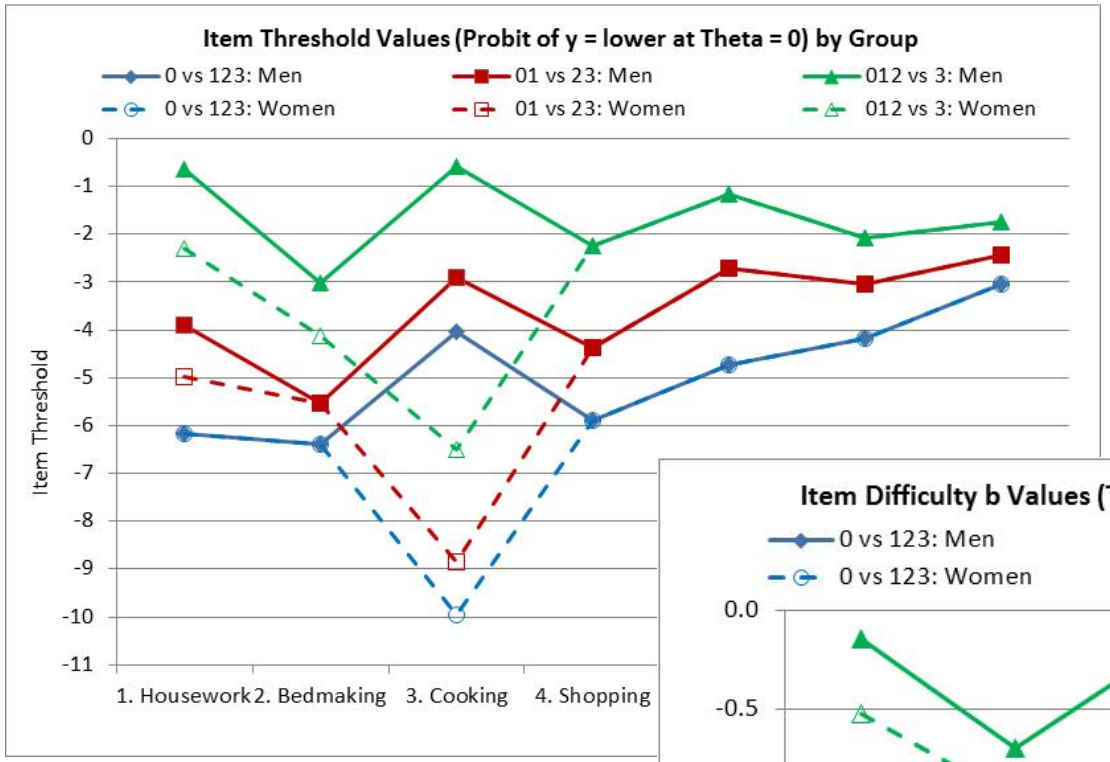
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA				
IADL BY				
CIA1	4.418	0.499	8.860	0.000
CIA2	4.312	0.614	7.029	0.000
CIA3	**3.214**	**0.463**	**6.948**	**0.000**
CIA4	4.445	0.549	8.102	0.000
CIA5	2.898	0.335	8.640	0.000
CIA6	2.451	0.283	8.649	0.000
CIA7	1.109	0.170	6.537	0.000
Means: MEAN OF THETA FIXED=0 FOR IDENTIFICATION				
IADL	0.000	0.000	999.000	999.000
Thresholds: EXPECTED PROBIT OF LOWER CATEGORY WHEN THETA=0				
CIA1$1	-6.175	0.607	-10.166	0.000
CIA1$2	**-3.903**	**0.585**	**-6.673**	**0.000**
CIA1$3	**-0.633**	**0.443**	**-1.429**	**0.153**
CIA2$1	-6.400	0.716	-8.944	0.000
CIA2$2	-5.524	0.673	-8.209	0.000
CIA2$3	**-3.005**	**0.574**	**-5.234**	**0.000**
CIA3$1	**-4.024**	**0.498**	**-8.085**	**0.000**
CIA3$2	**-2.912**	**0.464**	**-6.276**	**0.000**
CIA3$3	**-0.586**	**0.338**	**-1.734**	**0.083**
CIA4$1	-5.890	0.585	-10.062	0.000
CIA4$2	-4.365	0.528	-8.272	0.000
CIA4$3	-2.253	0.480	-4.692	0.000
CIA5$1	-4.740	0.388	-12.220	0.000
CIA5$2	-2.704	0.330	-8.184	0.000
CIA5$3	-1.174	0.300	-3.909	0.000
CIA6$1	-4.165	0.347	-12.008	0.000
CIA6$2	-3.040	0.308	-9.869	0.000
CIA6$3	-2.063	0.287	-7.177	0.000
CIA7$1	-3.035	0.353	-8.587	0.000
CIA7$2	-2.440	0.283	-8.623	0.000
CIA7$3	-1.739	0.220	-7.899	0.000
Variances: VARIANCE OF THETA FIXED=1 FOR IDENTIFICATION				
IADL	1.000	0.000	999.000	999.000
Residual Variances ALL FIXED=1 FOR IDENTIFICATION				
CIA1	1.000	0.000	999.000	999.000
CIA2	1.000	0.000	999.000	999.000
CIA3	1.000	0.000	999.000	999.000
CIA4	1.000	0.000	999.000	999.000
CIA5	1.000	0.000	999.000	999.000
CIA6	1.000	0.000	999.000	999.000
CIA7	1.000	0.000	999.000	999.000

UNSTANDARDIZED MODEL RESULTS (IFA MODEL SOLUTION)

Group WOMEN

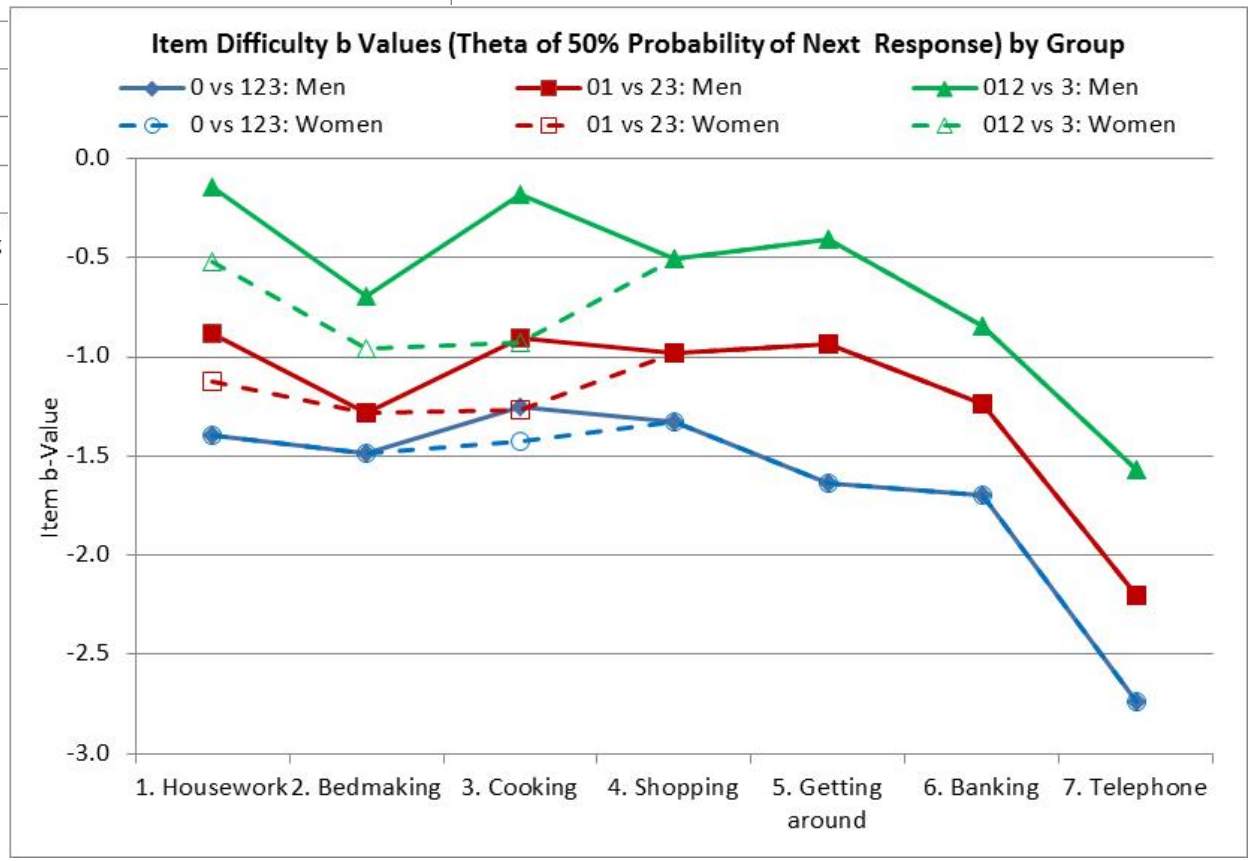
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
FACTOR LOADINGS: CHANGE IN PROBIT FOR 1-UNIT CHANGE IN THETA					
IADL BY					
CIA1	4.418	0.499	8.860	0.000	
CIA2	4.312	0.614	7.029	0.000	
CIA3	**6.992**	**1.455**	**4.807**	**0.000**	**cooking**
CIA4	4.445	0.549	8.102	0.000	
CIA5	2.898	0.335	8.640	0.000	
CIA6	2.451	0.283	8.649	0.000	
CIA7	1.109	0.170	6.537	0.000	
Means: MEAN DIFFERENCE OF THETA IN WOMEN					
IADL	**-0.352**	**0.096**	**-3.677**	**0.000**	
Thresholds: NEGATIVE OF EXPECTED PROBIT WHEN THETA=0					
CIA1$1	-6.175	0.607	-10.166	0.000	housework
CIA1$2	**-4.980**	**0.562**	**-8.866**	**0.000**	
CIA1$3	**-2.289**	**0.486**	**-4.711**	**0.000**	
CIA2$1	-6.400	0.716	-8.944	0.000	bed making
CIA2$2	-5.524	0.673	-8.209	0.000	
CIA2$3	**-4.115**	**0.619**	**-6.652**	**0.000**	
CIA3$1	**-9.964**	**1.889**	**-5.274**	**0.000**	**cooking**
CIA3$2	**-8.862**	**1.749**	**-5.068**	**0.000**	
CIA3$3	**-6.504**	**1.482**	**-4.389**	**0.000**	
CIA4$1	-5.890	0.585	-10.062	0.000	shopping
CIA4$2	-4.365	0.528	-8.272	0.000	
CIA4$3	-2.253	0.480	-4.692	0.000	
CIA5$1	-4.740	0.388	-12.220	0.000	get around
CIA5$2	-2.704	0.330	-8.184	0.000	
CIA5$3	-1.174	0.300	-3.909	0.000	
CIA6$1	-4.165	0.347	-12.008	0.000	banking
CIA6$2	-3.040	0.308	-9.869	0.000	
CIA6$3	-2.063	0.287	-7.177	0.000	
CIA7$1	-3.035	0.353	-8.587	0.000	telephone
CIA7$2	-2.440	0.283	-8.623	0.000	
CIA7$3	-1.739	0.220	-7.899	0.000	
Variances: VARIANCE OF THETA					
IADL	0.533	0.120	4.425	0.000	
Residual Variances (NOT ALL FIXED=1)					
CIA1	1.000	0.000	999.000	999.000	
CIA2	1.000	0.000	999.000	999.000	
CIA3	1.000	0.000	999.000	999.000	
CIA4	1.000	0.000	999.000	999.000	
CIA5	1.000	0.000	999.000	999.000	
CIA6	1.000	0.000	999.000	999.000	
CIA7	0.588	0.165	3.555	0.000	

Figures from Partial Measurement Invariance Model 4c (see next page for Mplus code and output for IRT a and b parameters):



The **thresholds** (shown on the left) predict the probit of y=lower response for someone with Theta=0. At theta=0, women (dashed lines) have a *lower* probability than men of not being able to do housework, make the bed, or cook (items 1–3).

The **b-values** (=threshold/loading; shown below) predict the trait level at which the next response becomes 50%. So in women, it takes less amount of the latent trait to be more able to perform these items.



The difference in the loading for cia3 (not pictured) also indicates that cooking is less discriminating (lower a-value) in women than in men (i.e., it is less important for measuring ability to live independently).

In addition, women are less variable but less able to live independently than men (as given by the factor variance and factor mean differences, respectively.)

```
MODEL:  !!! FINAL MODEL 4c FOR MEN REFERENCE GROUP

! Factor loadings all estimated
    IADL BY cia1-cia7* (L_I1-L_I7);
! Item thresholds all free
    [cia1$1-cia7$1*] (T1_I1-T1_I7);
    [cia1$2-cia7$2*] (T2_I1-T2_I7);
    [cia1$3-cia7$3*] (T3_I1-T3_I7);
! Item residual variances all fixed=1
    cia1-cia7@1;
! Factor mean=0 and variance=1 for identification
    [IADL@0]; IADL@1;

MODEL WOMEN: !!! FINAL MODEL 4c FOR WOMEN ALTERNATIVE GROUP
! Factor loadings CONSTRAINED EQUAL TO MEN EXCEPT 3
    IADL BY cia1-cia7* (L_I1 L_I2 L_I3w L_I4 L_I5 L_I6 L_I7);
! Item thresholds CONSTRAINED EQUAL TO MEN BY LABELS EXCEPT "w"
    [cia1$1-cia7$1*] (T1_I1  T1_I2  T1_I3w T1_I4 T1_I5 T1_I6 T1_I7);
    [cia1$2-cia7$2*] (T2_I1w T2_I2  T2_I3w T2_I4 T2_I5 T2_I6 T2_I7);
    [cia1$3-cia7$3*] (T3_I1w T3_I2w T3_I3w T3_I4 T3_I5 T3_I6 T3_I7);
! Item residual variances HELD EQUAL EXCEPT 7
     cia1-cia6@1; cia7*;
! Factor mean and variance ESTIMATED
    [IADL*]; IADL*;

MODEL CONSTRAINT: ! IRT parms using current factor mean and variance
! DO (begin, end), replace # with index
! A = discrimination, B1=y>0, B2=y>1, B3=y>2
! Ref group (men) parms
  NEW(Am_I1-Am_I7 B1m_I1-B1m_I7 B2m_I1-B2m_I7 B3m_I1-B3m_I7);
  ! Discriminations
    DO (1,7) Am_I# = L_I#;
  ! Difficulties
    DO (1,7) B1m_I# = T1_I# / L_I#;
    DO (1,7) B2m_I# = T2_I# / L_I#;
    DO (1,7) B3m_I# = T3_I# / L_I#;
! Alt group (women) parms
  NEW(Aw_I1-Aw_I7 B1w_I1-B1w_I7 B2w_I1-B2w_I7 B3w_I1-B3w_I7);
  ! Discriminations
    DO (1,2) Aw_I# = L_I#;
             Aw_I3 = L_I3w;
    DO (4,7) Aw_I# = L_I#;
  ! Difficulties
  ! Thresh 1
    DO (1,2) B1w_I# = T1_I#  / L_I#;
             B1w_I3 = T1_I3w / L_I3w;
    DO (4,7) B1w_I# = T1_I#  / L_I#;
  ! Thresh 2
             B2w_I1 = T2_I1w / L_I1;
             B2w_I2 = T2_I2  / L_I2;
             B2w_I3 = T2_I3w / L_I3w;
    DO (4,7) B2w_I# = T2_I#  / L_I#;
  ! Thresh 3
             B3w_I1 = T3_I1w / L_I1;
             B3w_I2 = T3_I2w / L_I2;
             B3w_I3 = T3_I3w / L_I3w;
    DO (4,7) B3w_I# = T2_I#  / L_I#;
```

New/Additional Parameters

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
AM_I1	4.418	0.499	8.860	0.000
AM_I2	4.312	0.614	7.029	0.000
AM_I3	3.214	0.463	6.948	0.000
AM_I4	4.445	0.549	8.101	0.000
AM_I5	2.898	0.335	8.640	0.000
AM_I6	2.451	0.283	8.649	0.000
AM_I7	1.109	0.170	6.537	0.000
B1M_I1	-1.398	0.118	-11.855	0.000
B1M_I2	-1.484	0.125	-11.861	0.000
B1M_I3	-1.252	0.136	-9.227	0.000
B1M_I4	-1.325	0.114	-11.597	0.000
B1M_I5	-1.635	0.141	-11.636	0.000
B1M_I6	-1.700	0.143	-11.855	0.000
B1M_I7	-2.737	0.294	-9.310	0.000
B2M_I1	-0.884	0.111	-7.973	0.000
B2M_I2	-1.281	0.110	-11.622	0.000
B2M_I3	-0.906	0.115	-7.867	0.000
B2M_I4	-0.982	0.096	-10.180	0.000
B2M_I5	-0.933	0.095	-9.820	0.000
B2M_I6	-1.240	0.110	-11.303	0.000
B2M_I7	-2.200	0.223	-9.873	0.000
B3M_I1	-0.143	0.096	-1.494	0.135
B3M_I2	-0.697	0.099	-7.029	0.000
B3M_I3	-0.182	0.098	-1.865	0.062
B3M_I4	-0.507	0.085	-5.969	0.000
B3M_I5	-0.405	0.086	-4.704	0.000
B3M_I6	-0.842	0.091	-9.245	0.000
B3M_I7	-1.569	0.153	-10.279	0.000
AW_I1	4.418	0.499	8.860	0.000
AW_I2	4.312	0.614	7.029	0.000
AW_I3	6.993	1.455	4.806	0.000
AW_I4	4.445	0.549	8.101	0.000
AW_I5	2.898	0.335	8.640	0.000
AW_I6	2.451	0.283	8.649	0.000
AW_I7	1.109	0.170	6.537	0.000
B1W_I1	-1.398	0.118	-11.855	0.000
B1W_I2	-1.484	0.125	-11.861	0.000
B1W_I3	-1.425	0.120	-11.827	0.000
B1W_I4	-1.325	0.114	-11.597	0.000
B1W_I5	-1.635	0.141	-11.636	0.000
B1W_I6	-1.700	0.143	-11.855	0.000
B1W_I7	-2.737	0.294	-9.310	0.000
B2W_I1	-1.127	0.104	-10.891	0.000
B2W_I2	-1.281	0.110	-11.622	0.000
B2W_I3	-1.267	0.111	-11.428	0.000
B2W_I4	-0.982	0.096	-10.180	0.000
B2W_I5	-0.933	0.095	-9.820	0.000
B2W_I6	-1.240	0.110	-11.303	0.000
B2W_I7	-2.200	0.223	-9.873	0.000
B3W_I1	-0.518	0.088	-5.880	0.000
B3W_I2	-0.954	0.097	-9.825	0.000
B3W_I3	-0.930	0.094	-9.850	0.000
B3W_I4	-0.982	0.096	-10.180	0.000
B3W_I5	-0.933	0.095	-9.820	0.000
B3W_I6	-1.240	0.110	-11.303	0.000
B3W_I7	-2.200	0.223	-9.873	0.000

Example write-up of these IFA analyses:

The extent to which am item factor model measuring independent daily living (with seven observed items) exhibited measurement invariance and structural invariance between men and women was examined using Mplus v. 8.1 (Muthén & Muthén, 1998–2017). WLSMV limited-information estimation (i.e., diagonally weighted least squares) including a probit link and the THETA parameterization was used for all models. Thus, model fit statistics describe the fit of the item factor model to the item polychoric correlation matrix for each group. Nested model comparisons were conducted using the DIFFTEST procedure. A configural invariance model was initially specified in which a single factor was estimated simultaneously in each group. The factor variance was fixed to 1 and the factor mean was fixed to 0 in each group for identification, such that all item factor loadings (one per item) and thresholds (three per item given four response options) were then estimated. The residual variances are not uniquely identified in the configural invariance model and as such were all constrained to 1 in both groups. As shown in Table 1, the configural invariance model had good fit. The analysis proceeded by applying parameter constraints in successive models to examine potential decreases in fit resulting from measurement or structural non-invariance between men and women, with men as the reference group.

Equality of the unstandardized item factor loadings between groups was then examined in a metric invariance model. The factor variance was fixed to 1 in men for identification but was freely estimated in women; the factor mean was fixed to 0 in both groups for identification. All factor loadings were constrained equal across groups, all item thresholds were estimated, and all residual variances were constrained to 1 across groups. Although the metric invariance model did not fit significantly worse than the configural invariance model, DIFFTEST (6) = 9.40, $p = .15$, the modification indices suggested localized misfit for the constrained loading of item 3 (cooking). After freeing its loading between groups, the partial metric invariance model did not fit significantly worse than the configural invariance model, DIFFTEST (5) = 5.52, $p = .36$, and no loadings were indicated as problematic. The fact that partial metric invariance (i.e., “weak invariance”) held indicates that the items were related to the latent factor equivalently across groups, or more simply, that the same latent factor was being measured in each group. The except is for the cooking item, which was less related to the trait of independent daily living in women.

Equality of the unstandardized item thresholds across groups was then examined in a scalar invariance model. The factor mean and variance were fixed to 0 and 1, respectively, in men for identification, but the factor mean and variance were then estimated for women. All factor loadings (except item 3) and all item thresholds were constrained equal across groups; all residual variances were still constrained equal to 1 in both groups. The full scalar invariance model A fit significantly worse than the partial metric invariance model, DIFFTEST (20) = 91.66, $p < .01$. Not surprisingly given its differential loading across groups, the modification indices suggested that item 3 was the largest source of the misfit. After freeing item 3’s thresholds, the partial scalar invariance model B still had significantly worse fit than the partial metric invariance model, DIFFTEST (17) = 44.80, $p < 01$. The modification indices suggested that threshold 3 of item 1 was the next largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model C (with all thresholds for item 3 and threshold 3 for item 1 freed) still fit significantly worse than the partial metric invariance model, DIFFTEST (16) = 32.46, $p < .01$. The modification indices suggested that threshold 3 of item 2 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model D (with all thresholds for item 3 and threshold 3 for items 1 and 2 freed) still fit significantly worse than the partial metric invariance model, DIFFTEST (15) = 26.17, $p < .04$. The modification indices suggested that threshold 2 of item 1 was the largest remaining source of the misfit and should be freed. After doing so, the new partial scalar invariance model E (with all thresholds for items 3, thresholds 2 and 3 for item 1, and threshold 3 for item 2 freed) did not fit significantly worse than the partial metric invariance model, DIFFTEST (14) = 20.32, $p = .12$. The factor that partial scalar invariance (i.e., “strong invariance”) held indicates that items 4, 5, 6, and 7 have the same expected response at the same absolute level of the trait, or more simply, that the observed differences in the proportion of responses in each category for those items was due to factor mean

differences only. However, at the same absolute level of the IADL factor, thresholds 2 and 3 for item 1, threshold3 for item 2, and all thresholds for item 3 were more difficult for men, indicating that men have a greater probability of not being able to do housework, make the bed, or cook.

Equality of the unstandardized residual variances across groups was then examined in a residual variance invariance model. The model comparison at this step proceeded backwards, such that a model with all residual variances freely estimated in the women (except for item 3, which was non-invariant) was fitted first, and then compared with a model in which all residual variances were fixed to 1 in the women. The residual variances in the men were all fixed to 1 for identification in both models, and the rest of the model parameters were estimated as described for the last partial scalar invariance model E. Although the model with the residual variances constrained to 1 (to be equal to the men) fit nonsignificantly worse than the model with those residual variances freed, DIFFTEST (6) = 11.02, $p = .09$, the modification indices suggested that the residual variance for item 7 the a source of local misfit and should be freed. After doing so, the new partial residual variance invariance model B did not fit significantly worse than the model with free residual variances in the women group, DIFFTEST (5) = 6.85, $p = .23$, indicating that residual variance for item 7 was significantly smaller for women than men. The fact that partial residual variance invariance (i.e., “strict invariance”) held indicates that the amount of item variance not accounted for by the factor was the same across groups in all other items.

After achieving partial measurement invariance as was just described, structural invariance was then tested with one additional model. The factor variance in the women (which had been estimated freely) was constrained to 1 (i.e., to be equal to the factor variance in men), resulting in a significant decrease in fit relative to the last partial residual invariance model B, DIFFTEST (1) = 8.82, $p < .01$. Thus, women showed significantly less variability in ability to live independently (factor variance of 0.53) than did men (factor variance fixed = 1). The factor mean for women in the partial measurement invariance model was significantly different from 0 (difference = −0.35, SE = 0.10, $p < .01$), indicating that women were marginally less able to live independently than men (factor mean fixed = 0).

In conclusion, these analyses showed that partial measurement invariance was obtained across men and women—that is, the relationships of the items to the latent factor of independent living were equivalent between men women. However, items 1, 2, and 3 (housework, bedmaking, and cooking) were systematically more difficult for men than women at the same level of the latent trait. Structural invariance was not obtained, such that women were less variable and less able on average than men. Model parameters from the final model are given in Table 2.

(see Example 7c spreadsheet for Table 1 and figures; Table 2 would have unstandardized and standardized estimates and their SEs)

References: Muthén, L. K., & Muthén, B.O. (1998–2017). Mplus User’s Guide (Eighth Edition). Los Angeles, CA: Muthén & Muthén.