Multivariate Multilevel Models for Longitudinal Data (in SAS and Mplus)

- Topics:
  - **Univariate vs. multivariate approaches for modeling time-varying (or any lower-level) predictors**
  - Multivariate relations of change (per level of analysis)
  - Multivariate tests of differences in effect size and their specification in univariate MLM software
  - What not to do: smushed effects path models for longitudinal data
  - Single-level SEM for multivariate multilevel models

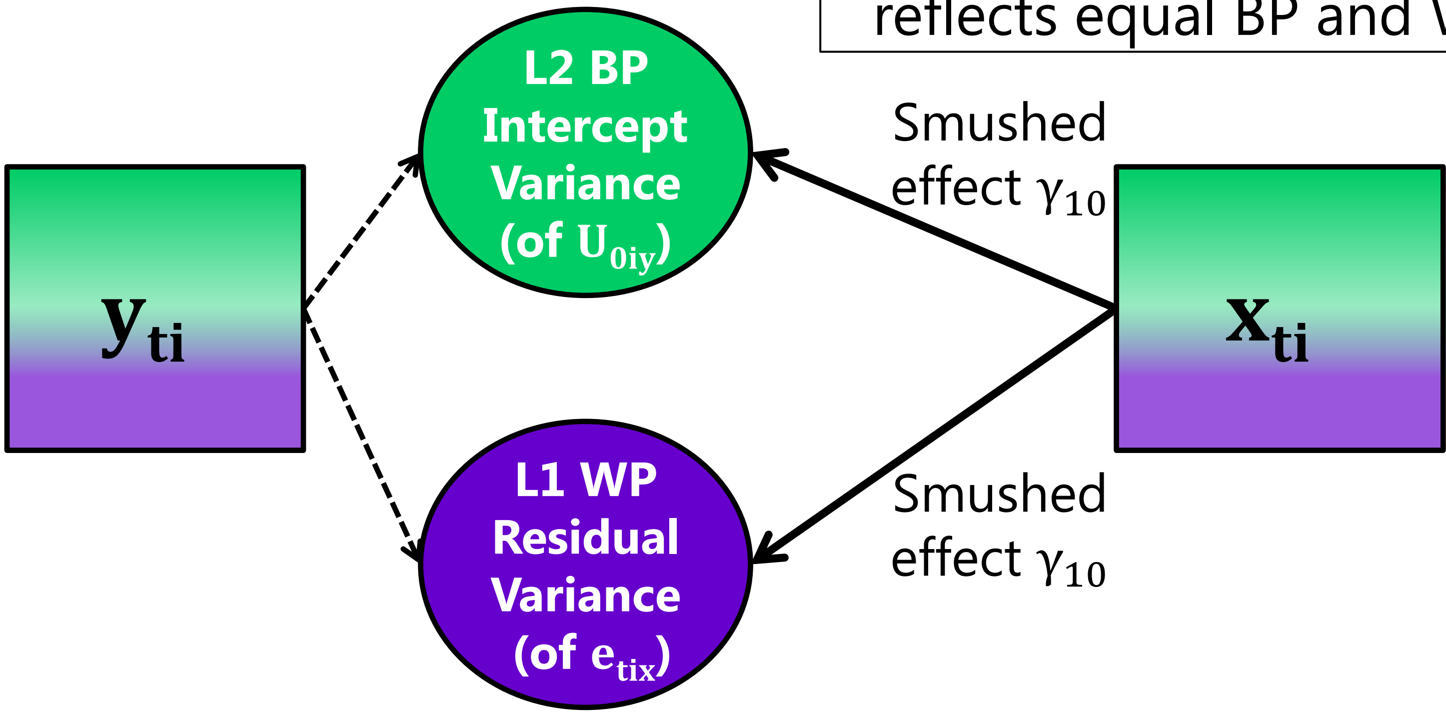
Univariate MLM for Specifying Time-Varying Predictors

- "Univariate" approach to MLM is appropriate for time-varying predictors that *fluctuate* over time (and lower-level predictors with only mean differences across higher levels in general)
- Level-1 predictor can be created two different ways:
  - Easier to understand is variable-based-centering: $\mathbf{WPx_{ti}} = \mathbf{x_{ti}} - \mathbf{\bar{X}_i}$
    - Directly isolates level-1 within variance, so $\mathbf{WPx_{ti}}$ → within effects
  - More common is constant-based-centering: $\mathbf{TVx_{ti}} = \mathbf{x_{ti}} - \boldsymbol{C}$
    - Does NOT isolate level-1 within variance, so $\mathbf{TVx_{ti}}$ will have smushed between/within effects unless it is paired with level-2 predictor analog
- Level-2 predictor is always constant-centered: $\mathbf{PMx_i} = \mathbf{\bar{X}_i} - \boldsymbol{C}$
  - $\mathbf{PMx_i}$ indicates *total* between effect when paired with $\mathbf{WPx_{ti}}$
  - $\mathbf{PMx_i}$ indicates contextual between effect when paired with $\mathbf{TVx_{ti}}$
    - Within + Contextual Between = Total Between

Univariate: Constant-Based Centering Without Level-2 Predictor = Smushing

Model-based partitioning of level-1 y_{ti} outcome variance into **variance components**:

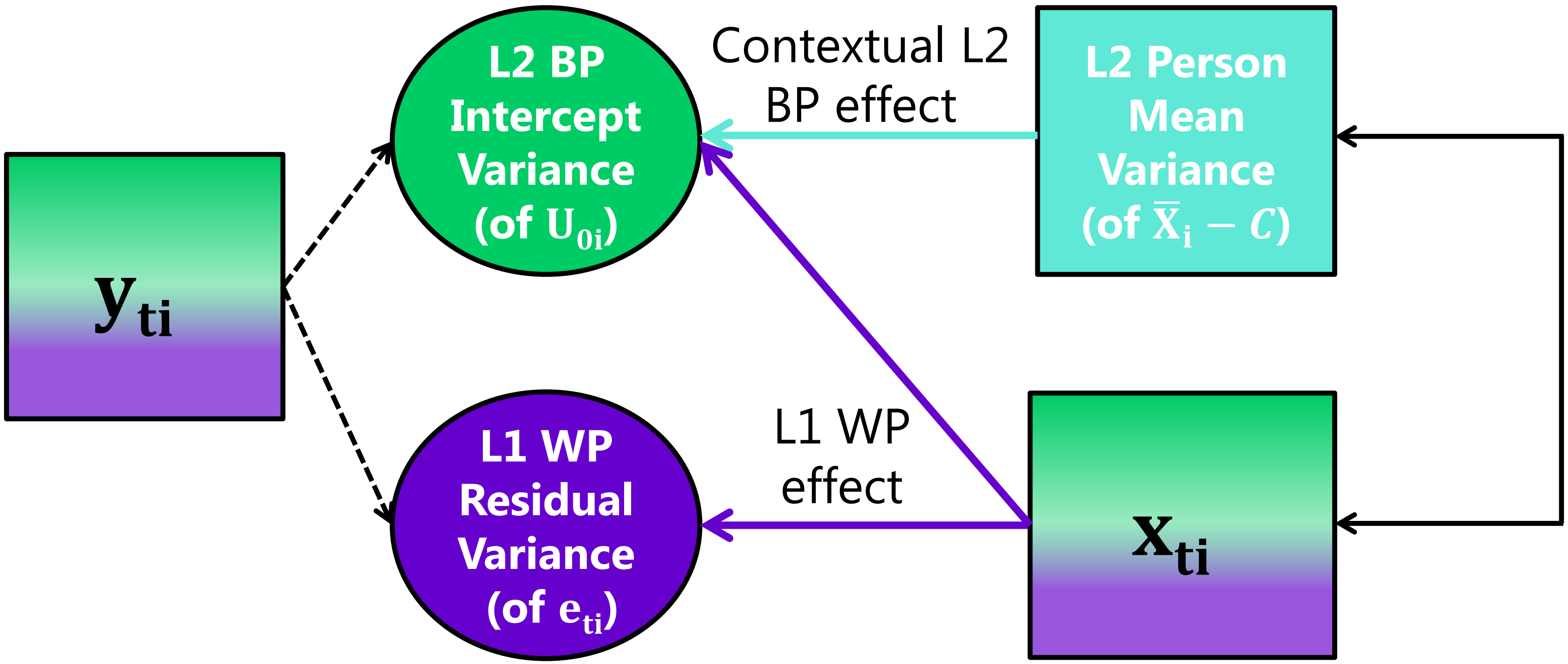
Constant-centered level-1 x_{ti} has not been partitioned – AND – it has only **one fixed effect** in the model. Thus, that smushed effect reflects equal BP and WP effects.



Univariate: Constant-Based Centering WITH Level-2 Predictor = OK NOW!

Model-based partitioning of y_{ti} outcome variance into **variance components**:

Level-1 x_{ti} is still not partitioned, but person mean $\bar{X}_i - C$ is added to allow an extra (different) effect at L2.

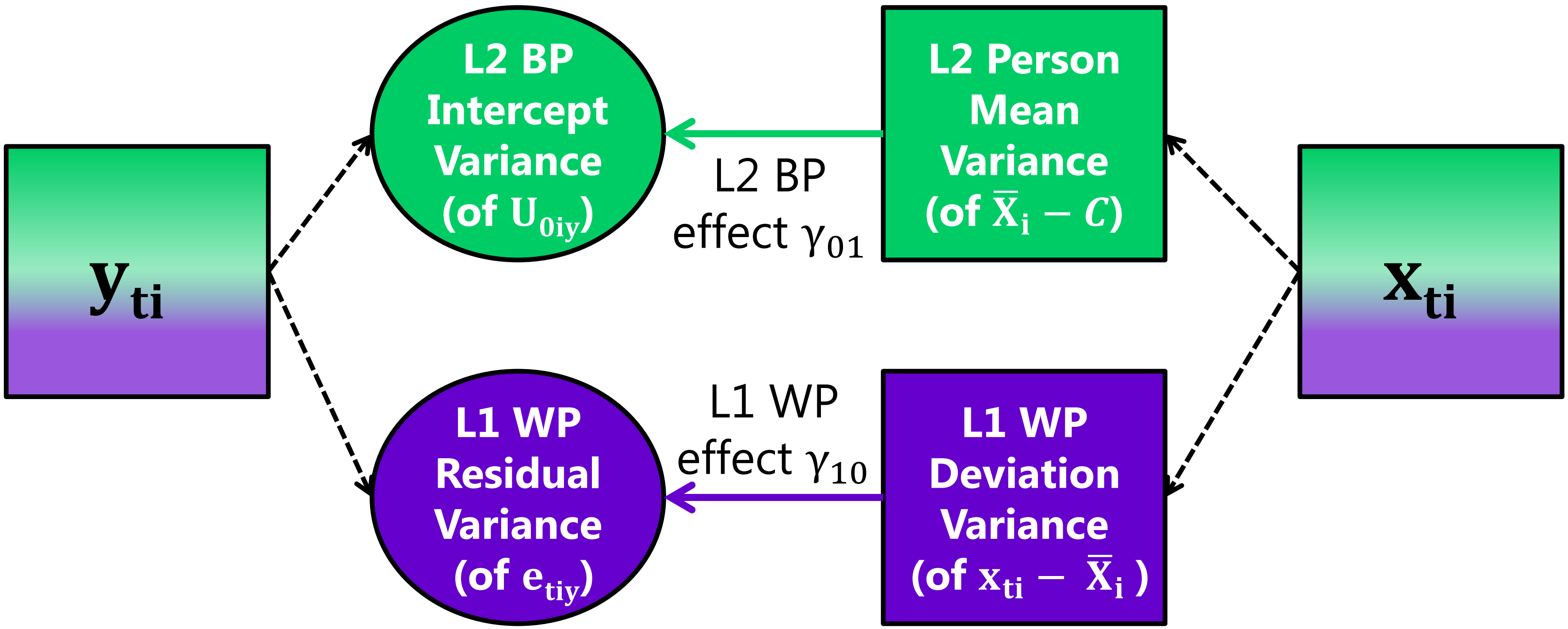


Because original x_{ti} still has BP variance, it still carries *part* of the BP effect…

Univariate: Variable-Based-Centering

Model-based partitioning of level-1 y_{ti} outcome variance into **variance components**:

Brute-force partitioning of level-1 x_{ti} predictor variance into **observed variables**:

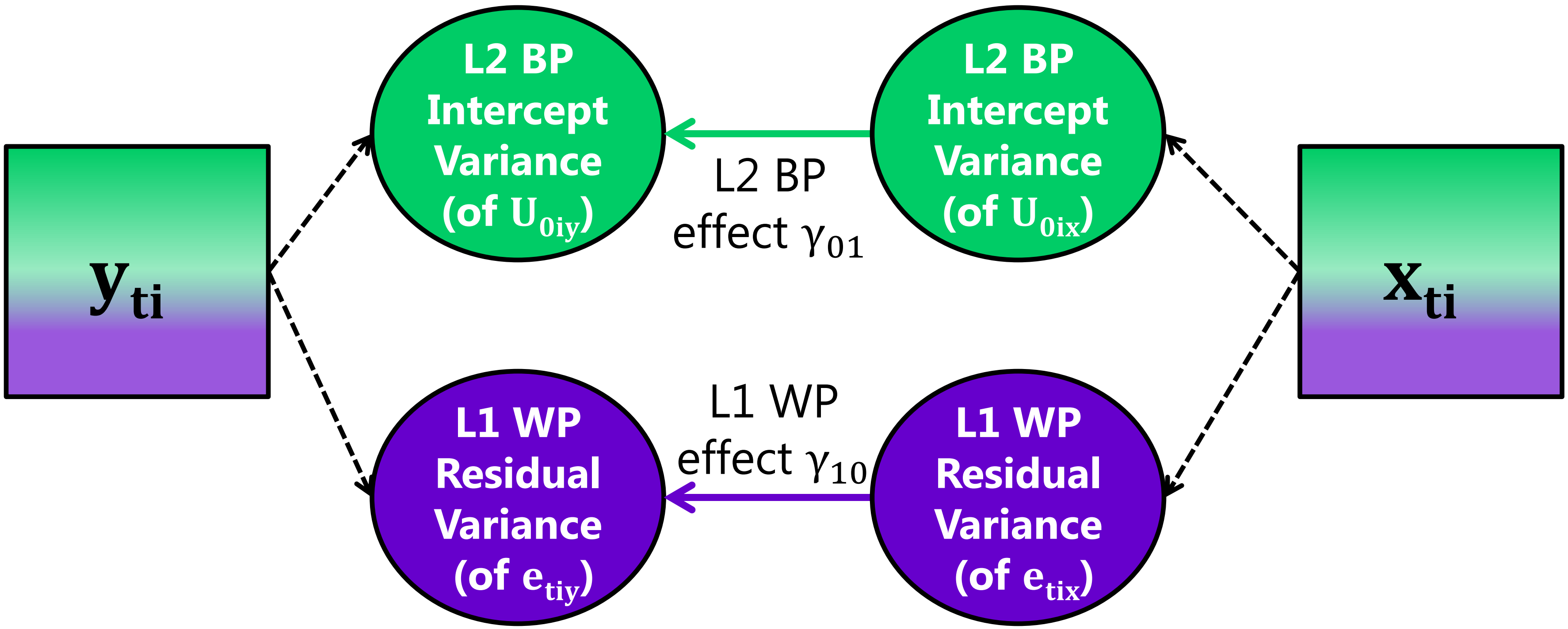


Why not let the model make variance components for x_{ti}, too?
This is the basis of multivariate MLM (or "multilevel SEM" = M-SEM).

"Truly" Multivariate Multilevel Modeling

Model-based partitioning of level-1 y_{ti} outcome variance into **variance components**:

Model-based partitioning of level-1 x_{ti} outcome variance into **variance components**:



Univariate MLM software can do multivariate MLM if the relationships between X and Y at each level are phrased as covariances, but if you want directed regressions (or moderators thereof), you need **"M-SEM"**

Univariate vs. Truly Multivariate MLM

- If your time-varying predictors have only BP intercept variance, their piles of variance can be reasonably approximated in univariate MLM OR by truly multivariate MLMs (so-called Multilevel SEM, or M-SEM)
  - It's called "SEM" because random effects = latent variables, but there is no latent variable measurement model as in traditional SEM, so that's why I don't like the term M-SEM, and prefer "(Truly) Multivariate MLM" (where "truly" distinguishes which software is used)

- <u>Pros of Truly Multivariate MLMs (M-SEM):</u>
  - Univariate MLM uses observed variables for variance in X, but fits a model for the variance in Y; truly multivariate MLMs fit a model for both X and Y, which makes more sense
  - Simulations suggest that L2 fixed effects in M-SEM are less biased (because person means are not perfectly reliable as assumed), but they also less precise (because there are more parameters to estimate), particularly for variables with lower ICCs (little intercept info)

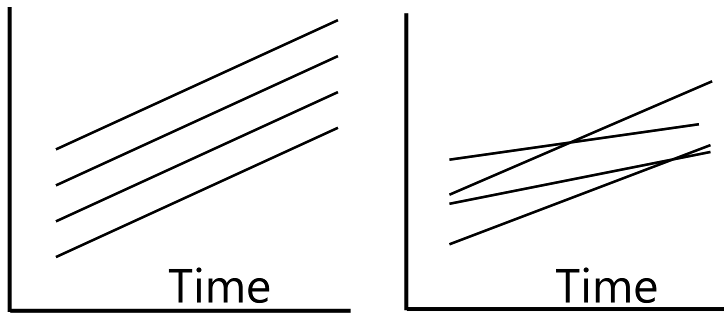
- <u>Cons of Truly Multivariate MLMs (M-SEM):</u>
  - Current software does not have REML or denominator DF → not good for small samples
  - Interactions among what used to be person means in univariate MLM instead become interactions among latent variables (random effects) in multivariate MLM
  - Latent variable interactions in ML require computationally intense numeric integration, which may limit the number of interactions that can be tested at once

Time-Varying Predictors that Change **Need** Multivariate Multilevel Models

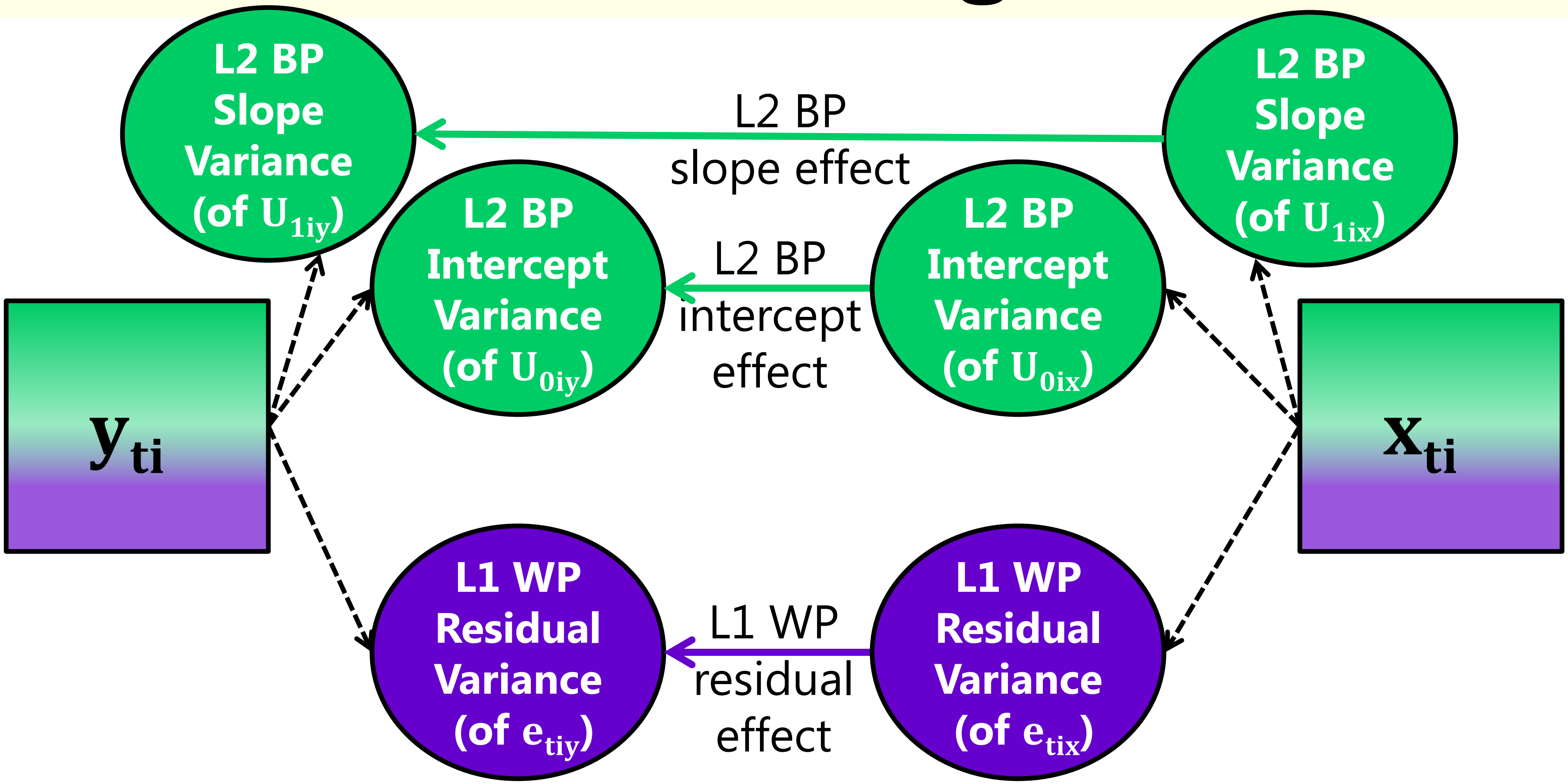
- Univariate MLMs for time-varying predictors can still be reasonable if a time-varying predictor has only a **fixed effect of time**
  - Adding fixed effects of time creates "unique" effects controlling for time

- But if a time-varying predictor has ***individual differences* in change**, univariate MLM (variable-based-centering) cannot provide a reasonable separation of its between and within variance:
  - There are then **at least two "kinds" of BP variance** to be concerned with: intercept and time slope (and possibly more for other kinds of change)
  - The level-1 predictor has both individual differences in change (U_{1i}) and residual deviations from change (e_{ti}), which should each have their own relationship to Y, otherwise they are **smushed into the level-1 WP effect**

And, if people change differently over time, then BP differences between people depend on time, too



Multivariate Modeling of Time-Varying Predictors that *Change* over Time



Univariate MLM software can do multivariate MLM if the relationships between X and Y at each level are phrased as covariances, but if you want directed regressions (or moderators thereof), you need **"M-SEM"**

Estimation of Multivariate Multilevel Models: Current Interface of Software and Models

- **Multivariate**":
  - Multiple kinds of level-1 outcomes (DVs) per level-2 unit (e.g., person)
- "**Multilevel**":
  - Two+ dimensions of sampling (e.g., time in persons, persons in groups)

- **Three types of software using maximum likelihood (ML)**:
  - "Univariate" MLM, as in SAS MIXED, SPSS MIXED, STATA MIXED, R LME4
    - Pro: also offers REML estimation (as well as denominator DF options in some)
  - "Truly" multivariate MLM, as in Mplus %BETWEEN% / %WITHIN%
    - Also called "Multilevel Structural Equation Modeling" (M-SEM) by others (not me)
  - Single-level SEM, as in Mplus, AMOS, LISREL, EQS, STATA SEM, R lavaan…

- These options differ in the extent to which certain model types are possible, as well as the ease with which they can be specified
  - Seems to be more confusion in single-level SEM for time-varying predictors

Why Use Multivariate Multilevel Models?

- Examine **relations across outcomes** at multiple levels of analysis, especially when the "predictor" has more than one kind of BP variance (random intercepts and slopes)
  - In univariate MLM, this can only be done via covariances in L2 G and L1 R (by tricking it into a multivariate model, stay tuned)
  - In "truly" multivariate MLM/M-SEM and single-level SEM, this can also be done via directed regressions (as in multilevel mediation)

- Examine **differences in predictor effects** across outcomes
  - This part can be done using any of the three software options
  - Outcomes should be transformed to common scale if not same already
  - Common question in "doubly" multivariate designs where all outcomes are DVs only (i.e., as in repeated measures experiments)
  - As a better alternative to difference score models

Multivariate Multilevel Models for Longitudinal Data (as in SAS and Mplus)

- Topics:
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  - **Multivariate relations of change (per level of analysis)**
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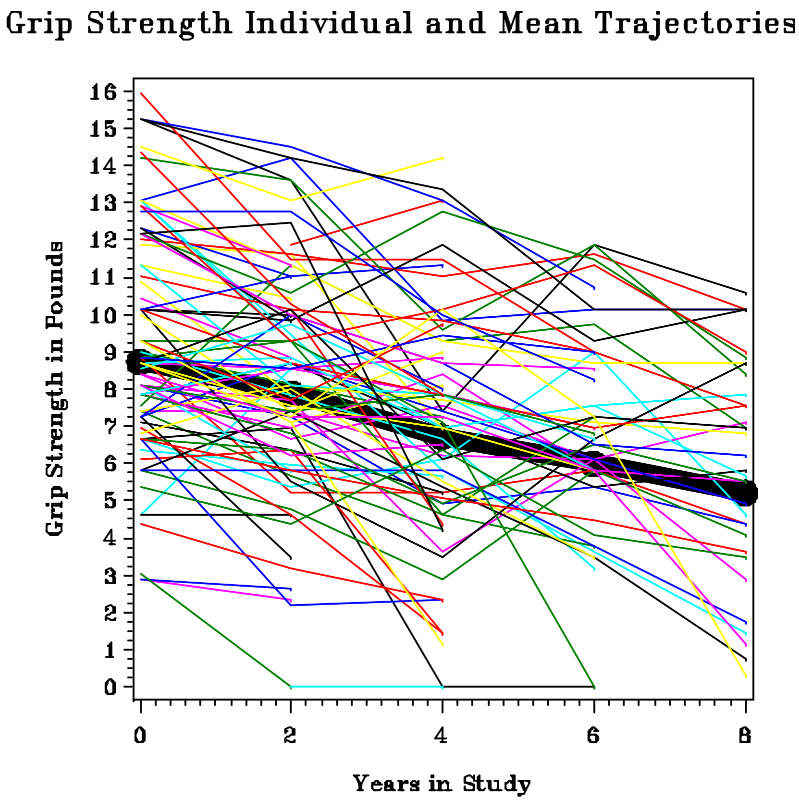
Multivariate Relations of Change: BP

- Multivariate questions about **fixed effects**:
  Does change appear similar **on average** across DVs?
  - Are the fixed effects for the overall sample heading in the same direction or of equal magnitude?
  - Tells us about average change, but says nothing about individuals

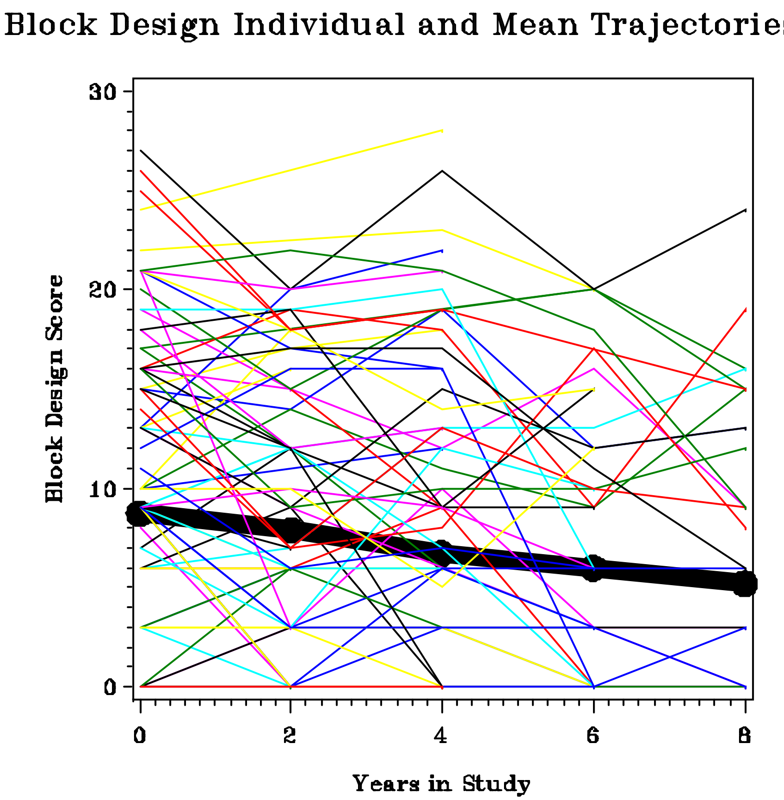
- Multivariate questions about **random effects**:
  Are **individual differences** in change related across DVs?
  - Is level (intercept) on one DV related to level (intercept) on another DV (at the centering point)?
  - Is magnitude of change (slope) on one DV related to magnitude of change (slope) on another DV?
  - These are **Between-Person** relations, relative to other people

Individual Relations of Functional and Cognitive Change in Old Age

Functional Change

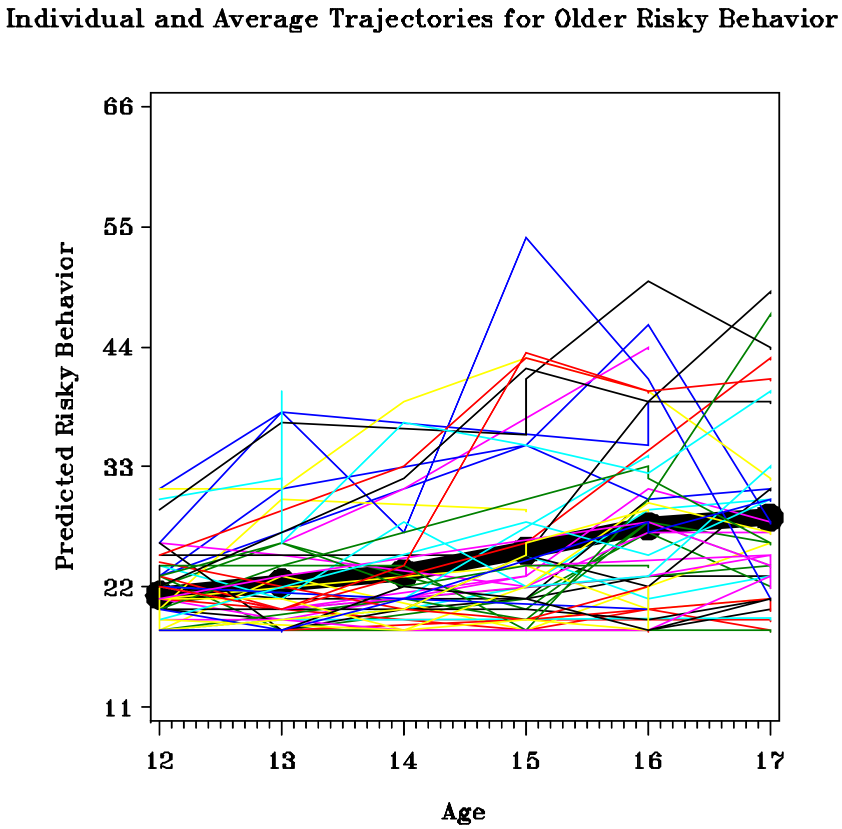


Cognitive Change

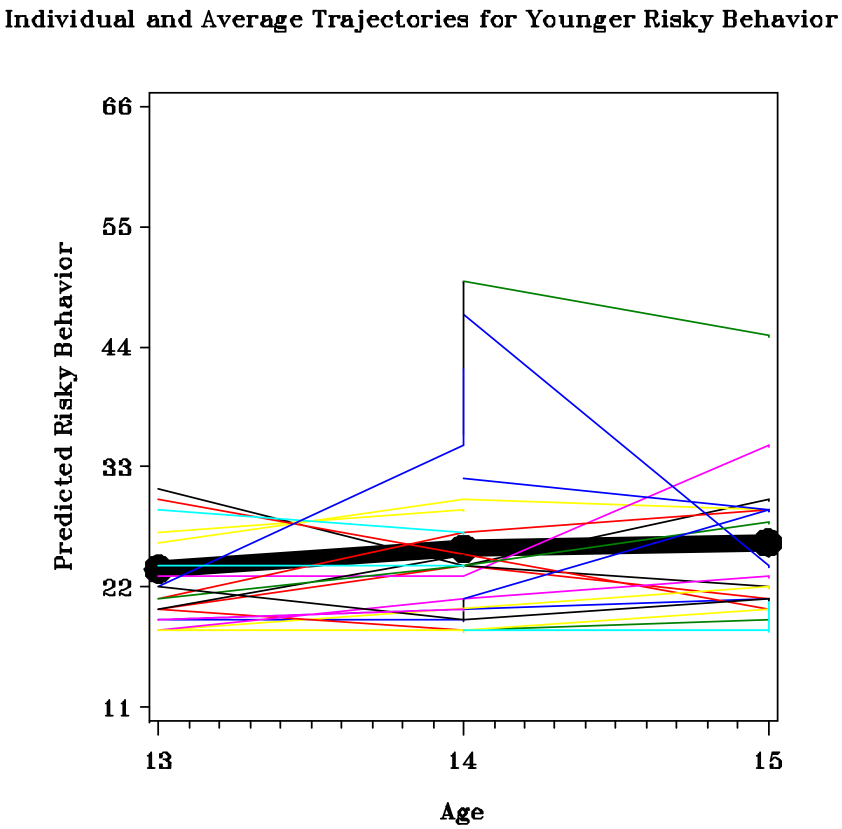


Individual Relations of Change in Risky Behavior Across Siblings

Older Siblings

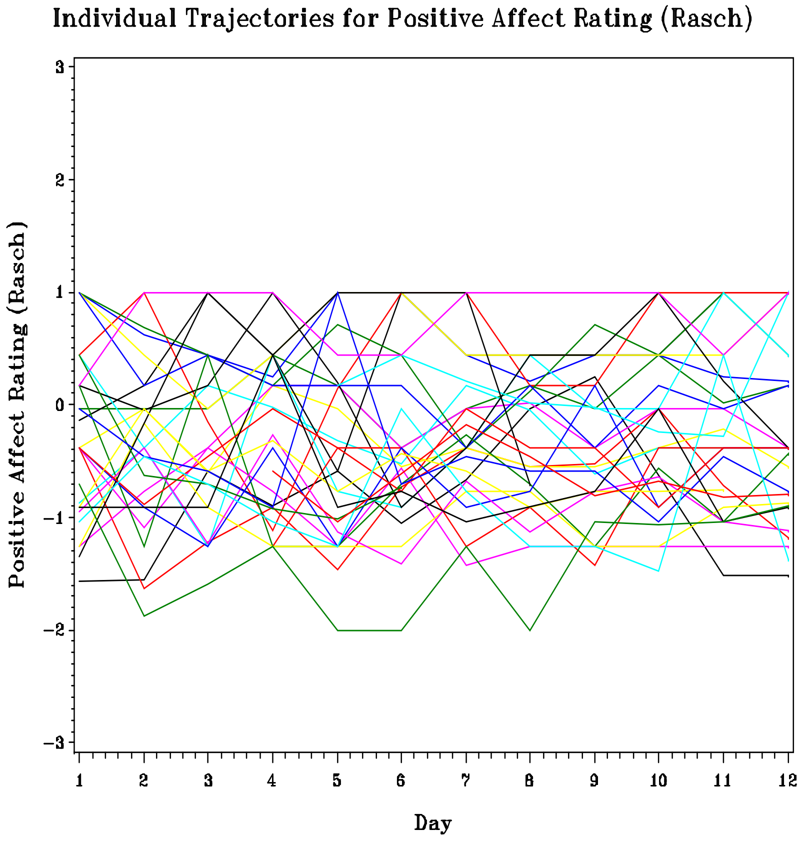


Younger Siblings

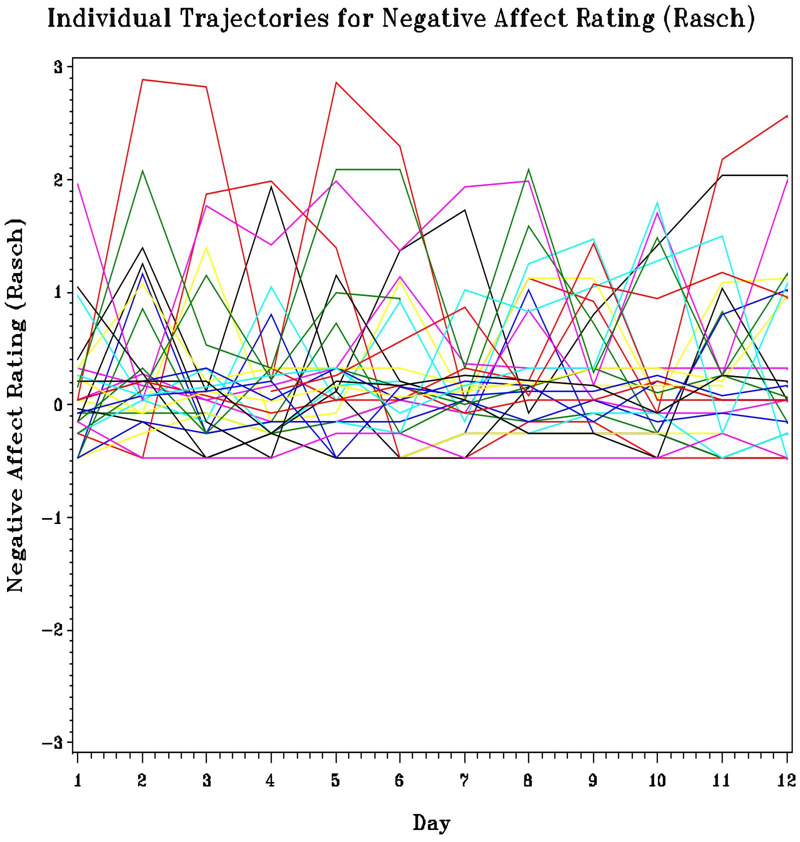


Daily Covariation in Rated Positive and Negative Affect

Rated Positive Affect



Rated Negative Affect



Multivariate Model Level-2 G Matrix

G Matrix for Between-Person Random Effects Variances:
Estimate intercept and slope variances **per DV** and all covariances

	Int DV A	Int DV B	Slope DV A	Slope DV B
Int DV A	$\tau^2_{U_{0a}}$			
Int DV B	$\tau_{U_{0b0a}}$	$\tau^2_{U_{0b}}$		
Slope DV A	$\tau_{U_{1a0a}}$	$\tau_{U_{1a0b}}$	$\tau^2_{U_{1a}}$	
Slope DV B	$\tau_{U_{1b0a}}$	$\tau_{U_{1b0b}}$	$\tau_{U_{1b1a}}$	$\tau^2_{U_{1b}}$

Random Intercept Variances

Random Time Slope Variances

DV = A

DV = B

Int-Int and Slope-Slope Covariances

To estimate this model *directly* in univariate software, there will be no general random intercept nor random "main" effects of predictors (i.e., as listed by themselves). Instead, all random effects will be tied to a DV via an "interaction" term (that actually creates nested versions of all fixed effects). Stay tuned…

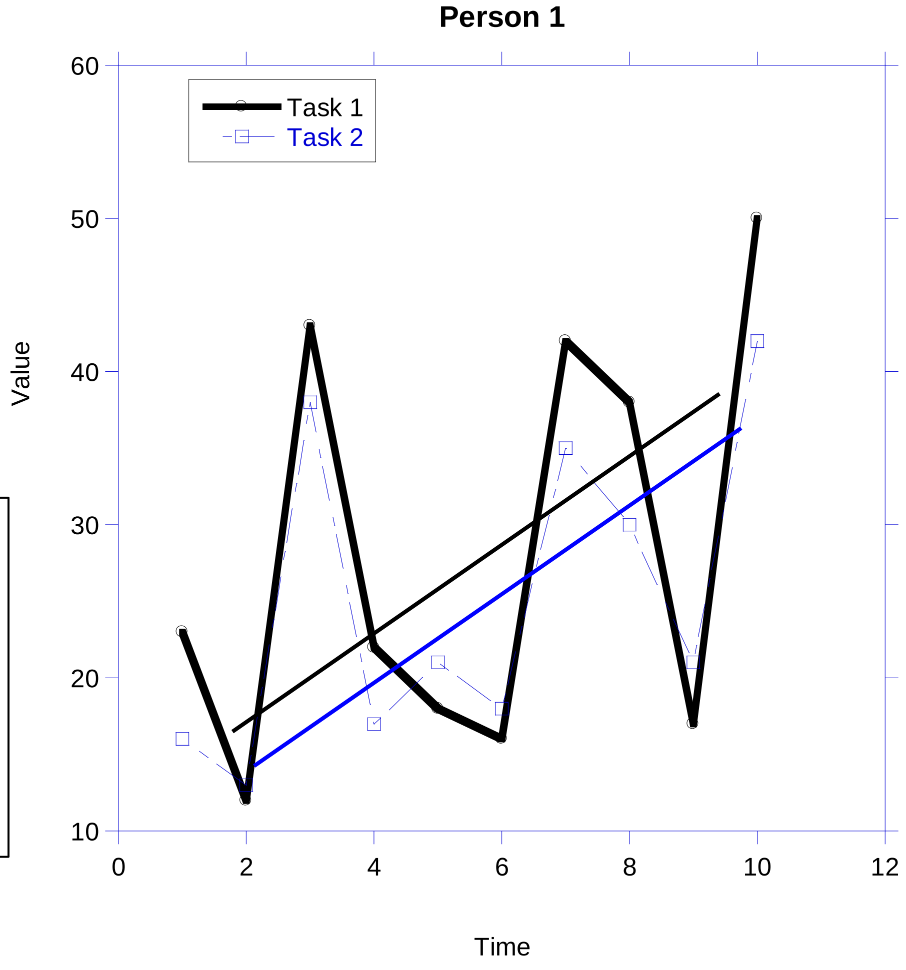
Caveats about Correlated Random Effects in Multivariate Longitudinal Models

- Random effects structure doesn't have to match across DVs, but it's helpful if it does for their clearer interpretation
  - e.g., DV A has random intercept and slope, DV B has random intercept only → then random intercept is conditional on slope=0 only for DV A
- If random effects variances are small or nonsignificant, covariances between them may not be estimated very well
  - Can always try it anyway if you do get some variance estimates in the first place (i.e., numbers as opposed to dots)
  - Random effects solution may be unstable: numerically large correlations may not be statistically significant due to large SEs for covariances
  - More DVs at once = more random effects → harder to estimate

Multivariate Relations of Change: WP

- Outcomes can be related **within persons** as well
- Correlated (Coupled) Residuals:
  - Do two DVs travel together over time?
  - Are you off your line in the same way for each DV at a given occasion?
    - (Yes, in this picture)

Note: allowing correlated residuals only makes sense for designs in which the occasions for each DV occur at the same time.



Multivariate Model Level-1 **R** Matrix

<u>**R** Matrix for Within-Person Residual Variances</u>: Estimate residual variance **per DV** and covariance between DVs *if at same occasion;* else estimate separate residual variances per DV without covariance

	Res DV A	Res DV B
Res DV A	$\sigma^2_{e_a}$	
Res DV B	$\sigma_{e_{ab}}$	$\sigma^2_{e_b}$

DV = A Residual Variance

DV = B Residual Variance

Res-Res Covariance: = covariance remaining after accounting for any *individual* effects of time

The categorical version of DV is used to structure the **R** matrix as **per occasion, per person**. This assumes equal residual variance with no covariance over time WITHIN EACH DV, but residuals at the same occasion have a covariance across DVs.

<u>**Example SAS code:**</u>

REPEATED DV / R RCORR TYPE=UN SUBJECT=Wave*Person

What about Multivariate Alternative Covariance Structures Models?

- We will examine multivariate random effects models. Are there multivariate versions of alternative covariance structures?
- Yes, but they are much more limited—3 real options in SAS:
  - Direct product structures: TYPE= UN@UN, UN@AR1
    - Assumes equal variances across time
    - Assumes same pattern of autocorrelation holds for each DV!
    - See REPEATED statement in SAS manual for further explanation
  - Completely unstructured multivariate
    - Specify DV*occasion on REPEATED statement
    - Estimates all possible variances and covariances separately, so it will fit the best, but with the least parsimony in doing so
    - No random effects = no between- and within-person separation
  - Just specify a random intercept (i.e., assume compound symmetry)
    - Not optimal, but it's the best I can come up with (for now)

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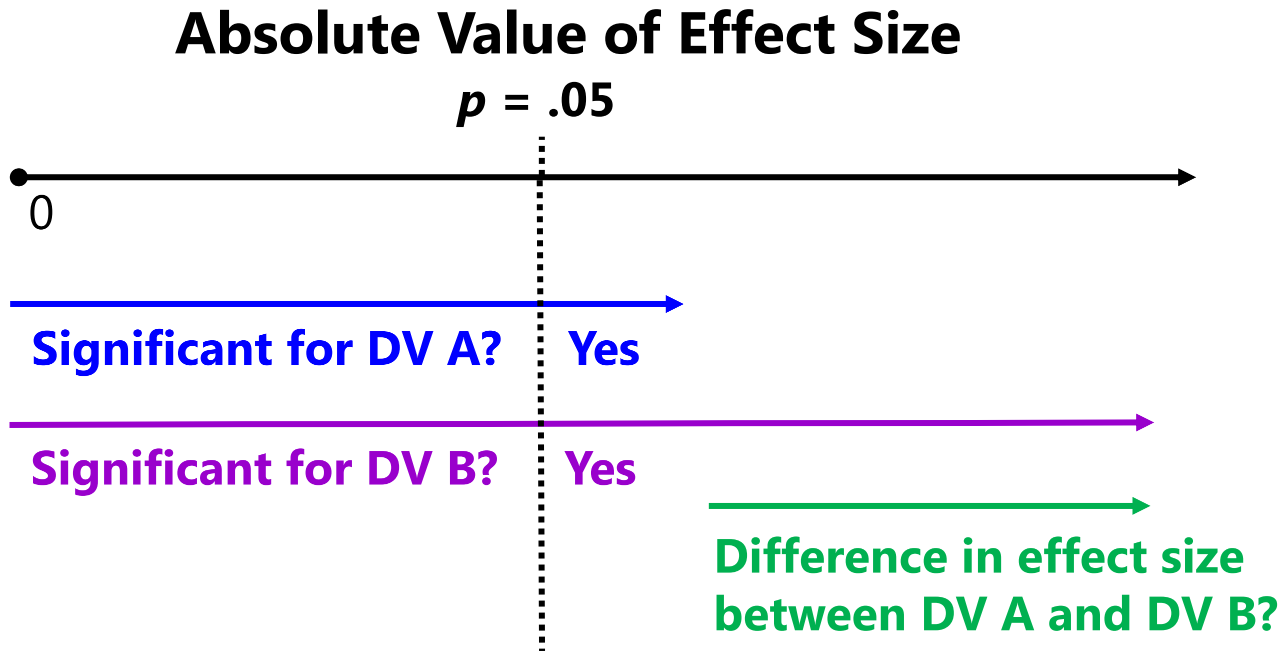
Differences in Effect Size across DVs

Scenario 1: Fixed effect is significant for both DVs:

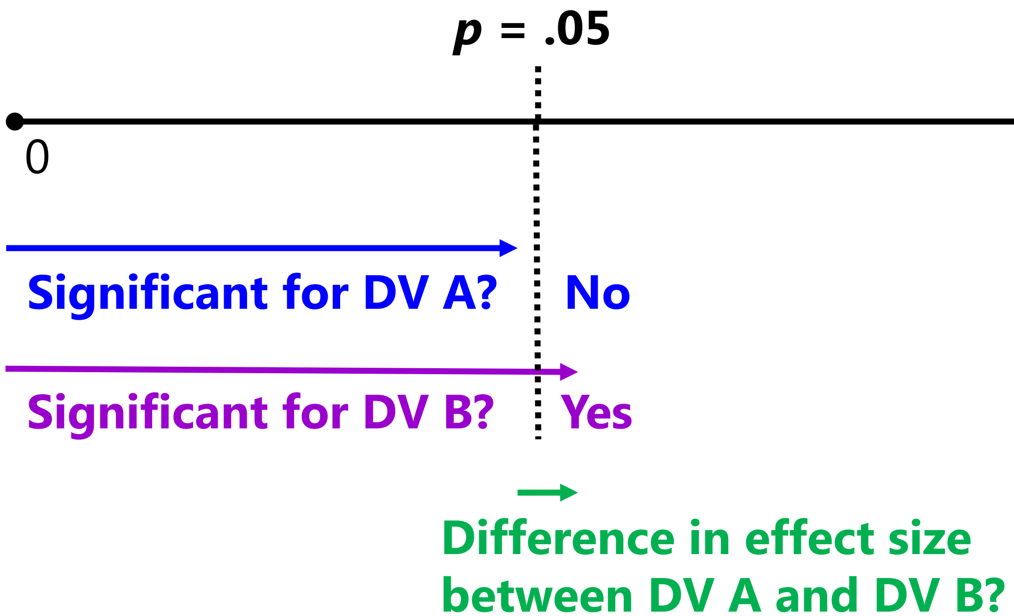


Just because a predictor is **significant for both DVs** does not mean it has the **same magnitude** of relationship across DVs!

Differences in Effect Size across DVs

Scenario 1: Fixed effect is significant for DV B only:

Absolute Value of Effect Size



Also, just because a predictor is **non-significant for one DV but significant for another DV** does not mean it has **different magnitudes** of relationships across DVs!

Why multivariate models should be used to test differences in effect sizes:

- Testing **differences in effect size of predictors** requires all DVs in the same model!
- But if the effects are the same, you can specify a **single effect** across DVs to reduce the number of estimated parameters.
- Hypotheses about **difference scores** are best tested using the original outcomes that created the difference in a multivariate model so that information about **absolute amount** is also provided.
- If DVs have missing data but are correlated, then tests of fixed effects may have **more power** in a multivariate model.
- Keep in mind that these models test differences in **unstandardized** fixed effects, so the DVs need to be on the **same scale** (or should be transformed onto the same scale before-hand otherwise).

Tricking Univariate MLM Software into Estimating Multivariate MLMs (here, 2 DVs)

Outcome	DV	dvA	dvB	Wave
Y_{i1a}	A	1	0	1
Y_{i2a}	A	1	0	2
Y_{i3a}	A	1	0	3
Y_{i4a}	A	1	0	4
Y_{i5a}	A	1	0	5
Y_{i6a}	A	1	0	6
Y_{i1b}	B	0	1	1
Y_{i2b}	B	0	1	2
Y_{i3b}	B	0	1	3
Y_{i4b}	B	0	1	4
Y_{i5b}	B	0	1	5
Y_{i6b}	B	0	1	6

1. Double-stack all DVs into a single outcome
2. Create a categorical predictor for which DV is which (e.g., A,B)
3. Create a dummy variable for each
   dvA= (1,0)
   dvB= (0,1)
4. Keep all other variables

This shows data for 1 person, 2 outcomes, over 6 waves.

We'll use "DV" to structure the **G** and **R** matrices, and "dvA" and "dvB" to create DV-specific fixed effects in the model for the means.

"Direct Effects" Multivariate Model

Within-Person Level 1: *t* = Time crossed with *d* = DV

$$y_{tid} = dvA[\beta_{0ia} + \beta_{1ia}(Time_{tia}) + e_{tia}]$$
$$dvB[\beta_{0ib} + \beta_{1ib}(Time_{tib}) + e_{tib}]$$

If DV=A, the β_{ia} are awake
If DV=B, the β_{ib} are awake

Between-Person Level 2: *i* = individual crossed with *d* = DV

$$\left.\begin{aligned}\beta_{0ia} &= \gamma_{00a} + \gamma_{01a}(Pred_i) + U_{0ia} \\ \beta_{1ia} &= \gamma_{10a} + \gamma_{11a}(Pred_i) + U_{1ia}\end{aligned}\right\}$$

Intercept and time slope for DV=A

$$\left.\begin{aligned}\beta_{0ib} &= \gamma_{00b} + \gamma_{01b}(Pred_i) + U_{0ib} \\ \beta_{1ib} &= \gamma_{10b} + \gamma_{11b}(Pred_i) + U_{1ib}\end{aligned}\right\}$$

Intercept and time slope for DV=B

To estimate this model *directly* in univariate software, there will be no general fixed intercept (via option NOINT) nor "main" effects of predictors (i.e., as listed by themselves). Instead, all fixed effects will be tied to a DV via an "interaction" term (that actually creates nested versions of all fixed effects). Let's see how…

Multivariate MLM in Univariate MLM Software: "Direct Effects" Version

```
* "Outcome" variable holds both DV A and DV B in one column;
* IMPORTANT: NOINT is needed to shut off general intercept,
   so that dvA and dvB become the intercepts per DV;
PROC MIXED DATA=work.multivstacked COVTEST NOCLPRINT NAMELEN=100 IC METHOD=REML;
* Level-2 ID, Level-1 ID, DV ID;
   CLASS PersonID Wave DV;

* This version lists all fixed and random effects being estimated,
  where the dv interactions specify each effect per DV;
   MODEL outcome = dvA dvB dvA*time dvB*time dvA*pred dvB*pred
                   dvA*time*pred dvB*time*pred / NOINT DDFM=Satterthwaite;
   RANDOM dvA dvB dvA*time dvB*time / G GCORR TYPE=UN SUBJECT=PersonID;

* This version does the exact same thing with less code;
   MODEL outcome = DV DV*time DV*pred DV*time*pred / NOINT DDFM=Satterthwaite;
   RANDOM DV DV*time / G GCORR TYPE=UN SUBJECT=PersonID;

* This line adds separate residual variances per DV and covariance;
   REPEATED DV / R RCORR TYPE=UN SUBJECT=PersonID*Wave;
* If you do not want a residual covariance, do this instead, which
  still allows separate residual variances per DV via first diagonal;
   REPEATED DV / R RCORR TYPE=TOEPH(1) SUBJECT=PersonID*Wave;
```

Multivariate MLM in Univariate MLM Software: "Direct Effects" Version

- Pros of previous "direct effects" version of model:
  - Fixed effects solution gives significance test for every effect per DV
  - Type 3 Tests (multivariate Wald tests automatic in SAS and SPSS) gives significance test for each effect combined across DVs
  - Is easier to do correctly, particularly if not all effects are included per DV
    - `MODEL outcome = dvA dvB dvA*pred` says no effect of pred for dv B
- Cons of "direct effects" version of model:
  - Does NOT give you tests of differences in effects across DVs, so you will need to write ESTIMATE or CONTRAST statements to obtain those
- To get the significance of the differences in effects across DVs automatically, switch to the "differences in effects" version
  - But then you only get significance of fixed effects for the reference DV

Multivariate MLM in Univariate MLM Software: "Difference in Effects" Version

```
* Everything else is the same (PROC MIXED, CLASS, REPEATED), but
  the removal of NOINT changes the interpretation of what DV does;

* NOW the effect of DV indicates the difference between DVs (relative to a
  reference DV, highest numerically or last alphabetically) in each effect;
   MODEL outcome = DV time DV*time pred DV*pred
                   time*pred DV*time*pred / DDFM=Satterthwaite;
```

Type 3 Tests (multivariate Wald tests automatic in SAS and SPSS) will now give combined significance test differences in effects across all DVs

```
* You can still use the direct effects version for random effects if you want;
   RANDOM DV DV*time / G GCORR TYPE=UN SUBJECT=PersonID;

* Or this version gives you the difference between DVs as random effects,
  which may be harder to estimate in some cases than the direct effects version;
   RANDOM INTERCEPT DV time DV*time / G GCORR TYPE=UN SUBJECT=PersonID;
```

Summary: Multivariate MLMs permit…

- Tests of hypotheses about BP relations (among intercepts and slopes) and WP relations (among time-specific residuals)
  - BP: Does intercept on one DV correlate with level on another DV?
  - BP: Does change on one DV correlate with change on another DV?
  - WP: Do two DVs 'travel together' over time within persons?
  - Questions involving <u>directed relationships</u> among DVs instead require "truly" multivariate MLM software instead of tricking univariate MLM software (which can only phrase these relationships as covariances in L2 G and L1 R)

- Tests about differences in effect size of predictors across DVs
  - Is the effect of the predictor significant per DV?
  - Is the effect of the predictor significantly *different* across DVs?
  - These questions can be answered in any kind of MLM software

- Multivariate multilevel models (or "Multilevel-SEM") can usually be phrased similarly using measurement models for latent variables within a single-level **SEM** framework… we'll see how this works.

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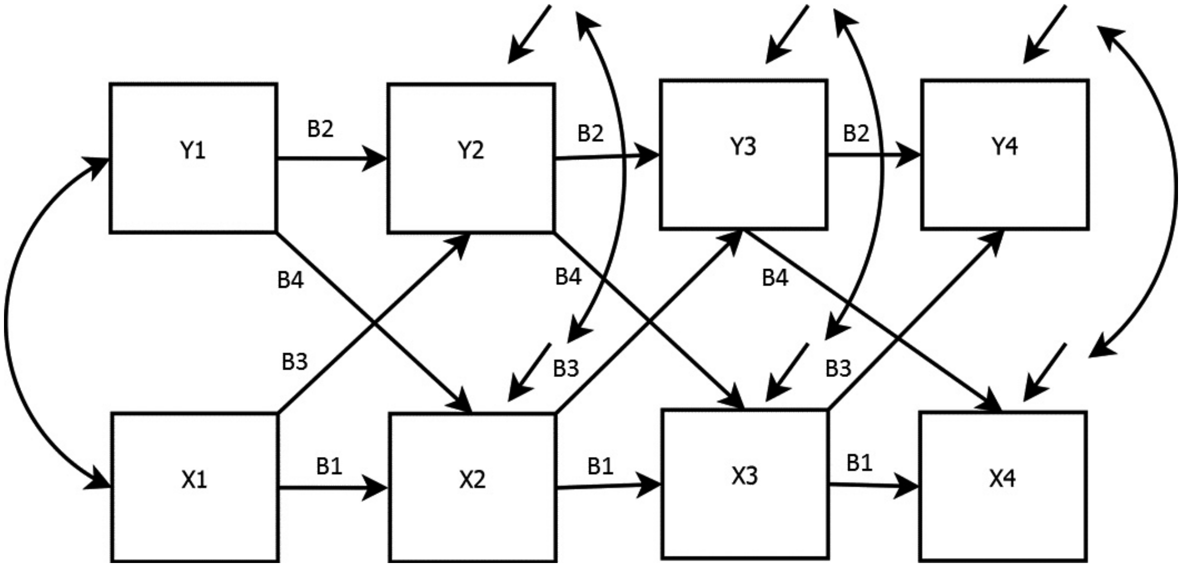
  - **What not to do: smushed effects path models for longitudinal data**
  - **Single-level SEM for multivariate multilevel models**

What *Not* to Do with Longitudinal Data

- Mis-specified path models (involving observed variables only) for longitudinal data are still far too common
  - These models include auto-regressive effects, cross-lagged effects, and observed variable mediation models involving different variables each measured on two or more occasions
  - Common exemplars to watch out for are given on the next slides
- The problem in each is a lack of differentiation of sources (piles) of variance, and thus what their effects mean
  - If the path model variables have not been de-trended for person mean differences (and for any individual change over time), then **all estimated paths will be smushed BP/WP to some degree**

A Model that Needs to Die*

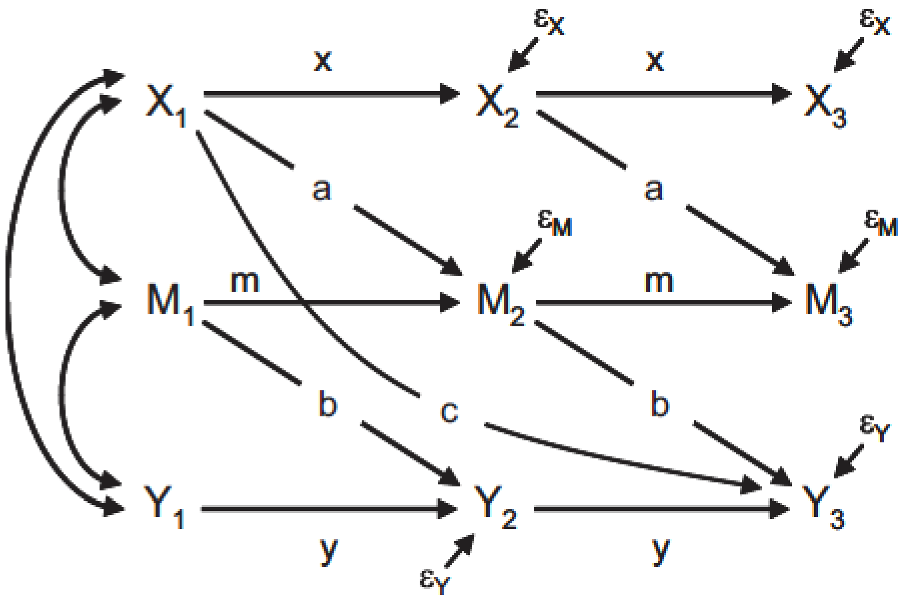


Autoregressive cross-legged panel model

* Emphasis mine, logic and picture provided by Berry & Willoughby (2017, *Child Development*)

- Logic: by including auto-regressive paths (B1 and B2) to "control" for previous occasions, the cross-lagged paths (B3 and B4) then represent effects of "change" on each variable in predicting the other (so they are "longitudinal" predictions)
- Reality: by allowing only one path, it smushes effects across sources of variance—BP intercept, BP slope(s), WP residual; autoregressive paths between occasions do NOT control for BP differences (assumes an AR(1) correlation model over time)

And take this one with it*…



Longitudinal mediation model

X= IV, M= mediator, Y= DV

* My point of view only, picture provided by Maxwell & Cole (2007, *Psychological Methods*)

- Logic: mediation should time to occur, so indirect effects should be specified across occasions (as before, of "change")
- Agreed, but if these variables haven't been de-trended for all sources of BP variance, then the *b* and *c* paths are smushed
- And what about BP mediation? Capturing BP variances in the same model would allow examination of that, too, right?

How to Fix It: Translating MLM's Variance Partitioning into Single-Level SEM

- "**Random effects**" = "pile of variance" = "variance components"
  - Random effects represent **person*something interaction terms** that create person-caused sources of covariance over time
  - Random intercept → person*intercept (person "main effect")
  - Random linear time slope → person*time interaction
- Random effects are the same thing as **latent variables**
  - Latent variable = unobservable ability or trait, created by sources of **common variance** across items (or time-specific outcomes here)
  - Latent variables for BP differences can be interpreted as "general tendency" (random intercept) and "propensity to change" (random time slope)
  - Model-based way of de-trending longitudinal outcomes to distinguish BP from WP sources of information (and examine all kinds of relations)
  - Uses "wide" data structure in which each occasion = separate variable

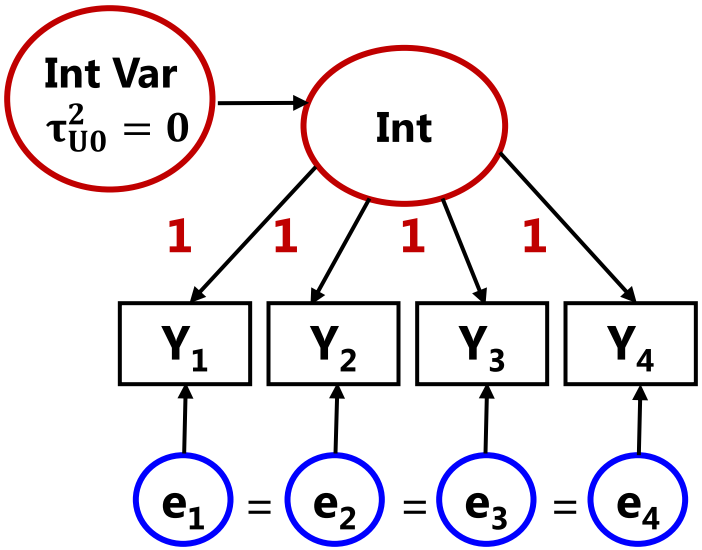
MLM as seen through Confirmatory Factor Analysis (CFA)

- **CFA model: $y_{is} = \mu_i + \lambda_i F_s + e_{is}$** (SEM is just relations among F's)
  - Observed response for item *i* (→ outcome at time *t*) and subject *s*
    = intercept of item *i* (μ)
    + subject *s*'s latent trait/factor (F), item-weighted by λ
    + error (e) of item *i* and subject *s*

- Four big differences when using CFA/SEM for longitudinal change:
  - Usually two factors for "level" and "change" (intercept and slope):
    $y_{ti} = (\gamma_{00} + U_{0i}) + (\gamma_{10} + U_{1i})time_{ti} + e_{ti}$ → so the U's are the F's
  - The separate **item (time-specific outcome) intercepts μ_i** cannot be identified from the "intercept" factor and therefore **must be fixed to 0**
  - The **factor loadings λ_i** for how each outcome is predicted by the latent factor are usually pre-determined by **how much time as passed**, and are fixed to the difference in time that corresponds to the **type of change** (e.g., linear, quadratic, piecewise)
  - Item (time-specific outcome) **residual variances should be constrained equal** (not default, but changes in variance over time should be captured by random slopes)

Random Effects as Latent Variables

- **BP model: e_{ti}-only model for the variance**
  - $y_{ti} = \gamma_{00} + e_{ti}$



Mean of the intercept factor = fixed intercept γ_{00}

Loadings of intercept factor = 1 (all occasions contribute equally)

Item intercepts = 0 (always)

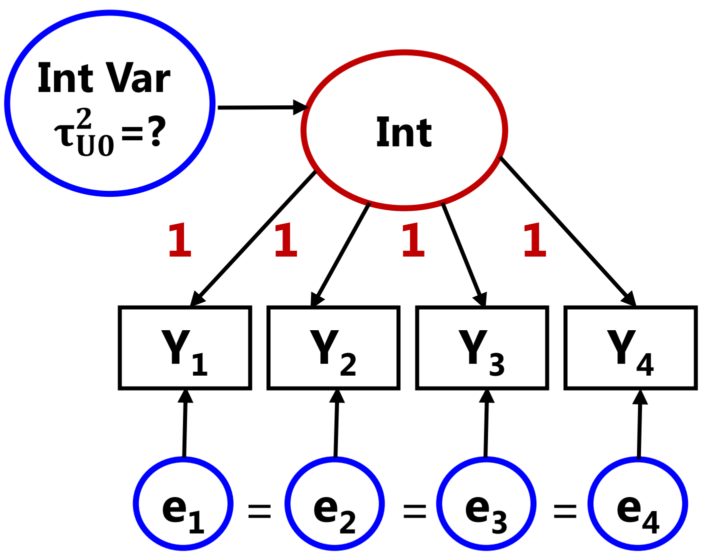
Variance of intercept factor = 0 so far

Residual variance (e) is assumed to be equal across occasions

  - After controlling for the *fixed* intercept, residuals are assumed uncorrelated: **this is a single-level model**

Random Effects as Latent Variables

- **+WP model: U_{0i} + e_{ti} model for the variance**
  - **$y_{ti} = \gamma_{00} + U_{0i} + e_{ti}$**



Mean of the intercept factor = fixed intercept γ_{00}

Loadings of intercept factor= 1 (all occasions contribute equally)

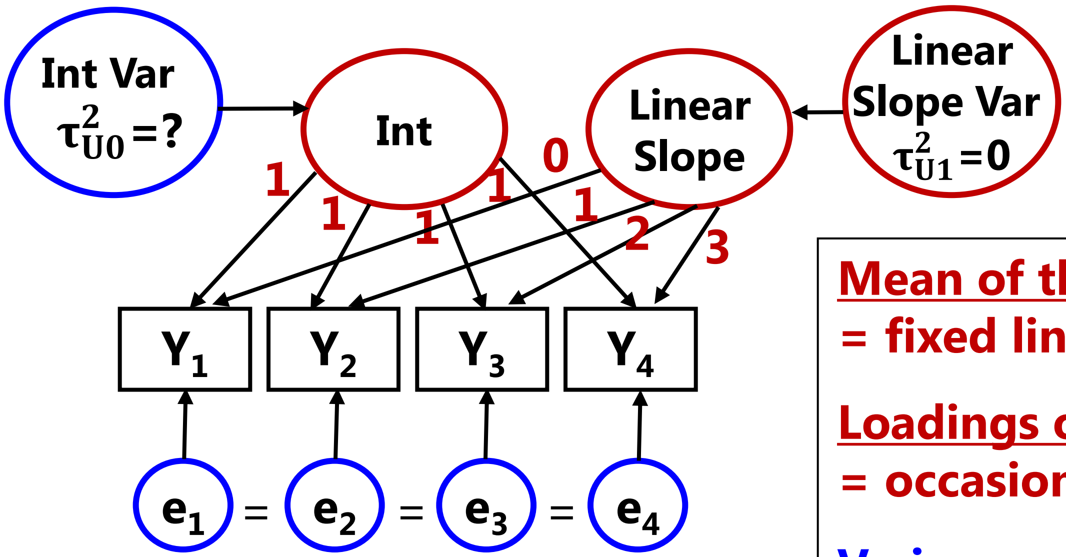
Variance of intercept factor = random intercept variance

Residual variance (e) is assumed to be equal across occasions

  - After controlling for the *random* intercept, residuals are assumed uncorrelated: **now two piles of variance** (what we would call an "**empty means, random intercept model**)

Random Effects as Latent Variables

- **Fixed linear time, random intercept model:**
  - $\mathbf{y_{ti} = \gamma_{00} + (\gamma_{10}Time_{ti}) + U_{0i} + e_{ti}}$



Mean of the linear slope factor
= fixed linear slope γ_{10}

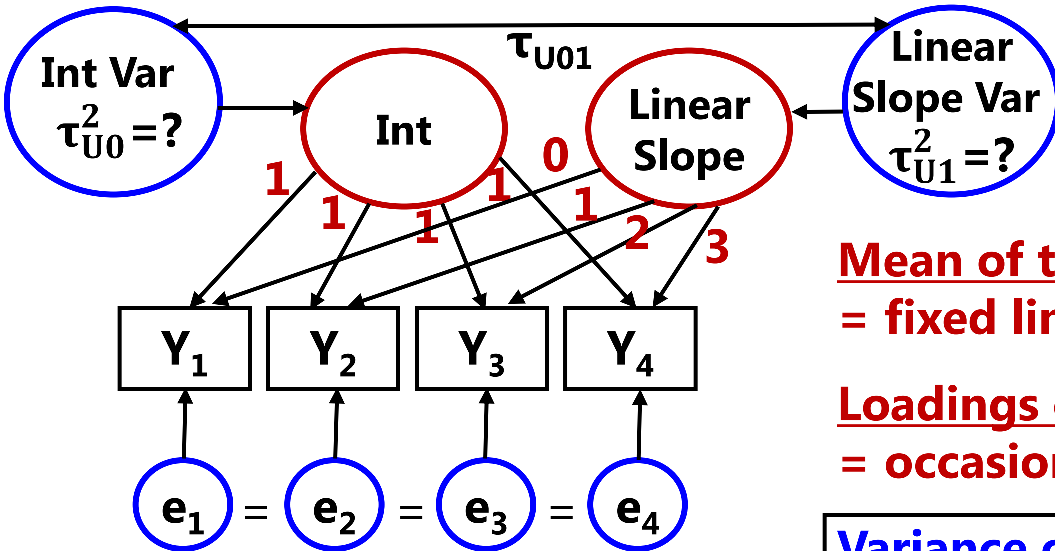
Loadings of linear slope factor
= occasions (keep real time)

Variance of linear slope factor
= 0

  - After controlling for the *fixed linear slope and random intercept*, residuals are assumed uncorrelated

Random Effects as Latent Variables

- **Random linear model:**
  - $\mathbf{y_{ti} = \gamma_{00} + (\gamma_{10}Time_{ti}) + U_{0i} + (U_{1i}Time_{ti}) + e_{ti}}$



Mean of the linear slope factor
= fixed linear slope γ_{10}

Loadings of linear slope factor
= occasions (keep real time)

Variance of linear slope factor
= random slope variance (and covariance with random intercept

  - After controlling for the *random linear slope and random intercept*, residuals are assumed uncorrelated: **now three piles of variance** to be predicted (BP int, BP slope, WP res)

Adding Level-2 Predictors

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(Time_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(Group_i) + U_{0i}$

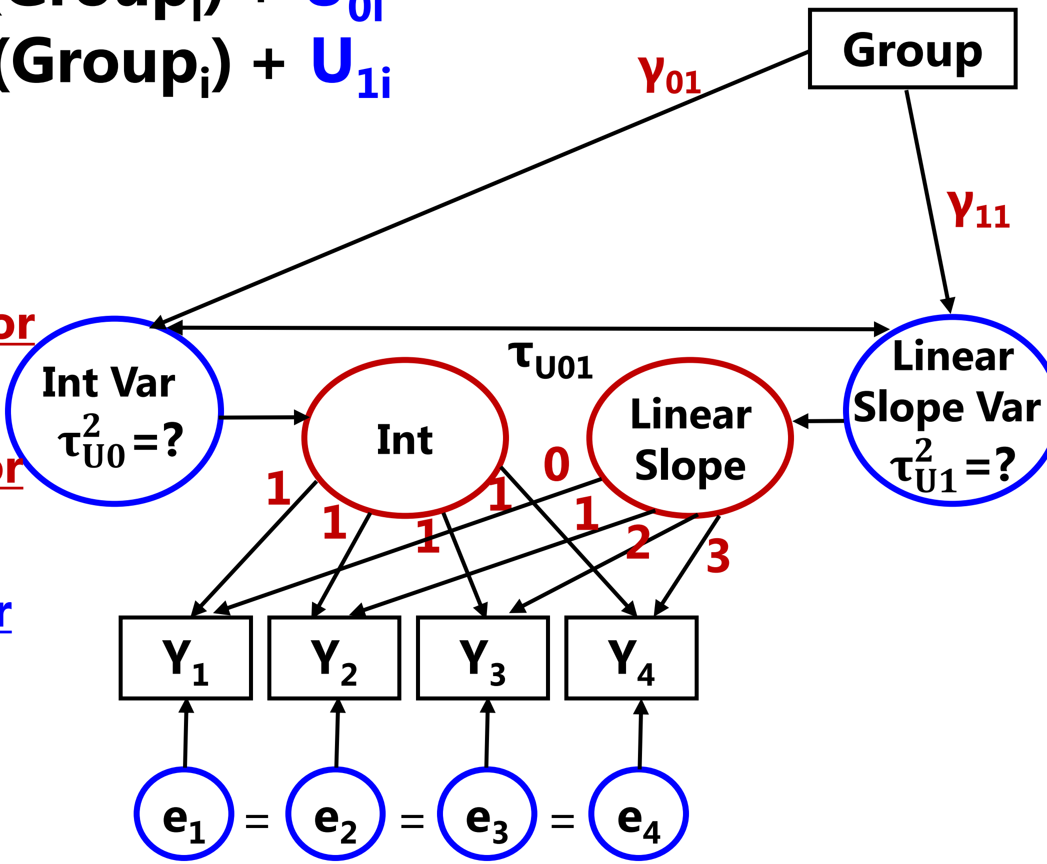
$\beta_{1i} = \gamma_{10} + \gamma_{11}(Group_i) + U_{1i}$

Mean of the intercept factor
= fixed intercept γ_{00}

Mean of the linear slope factor
= fixed linear slope γ_{10}

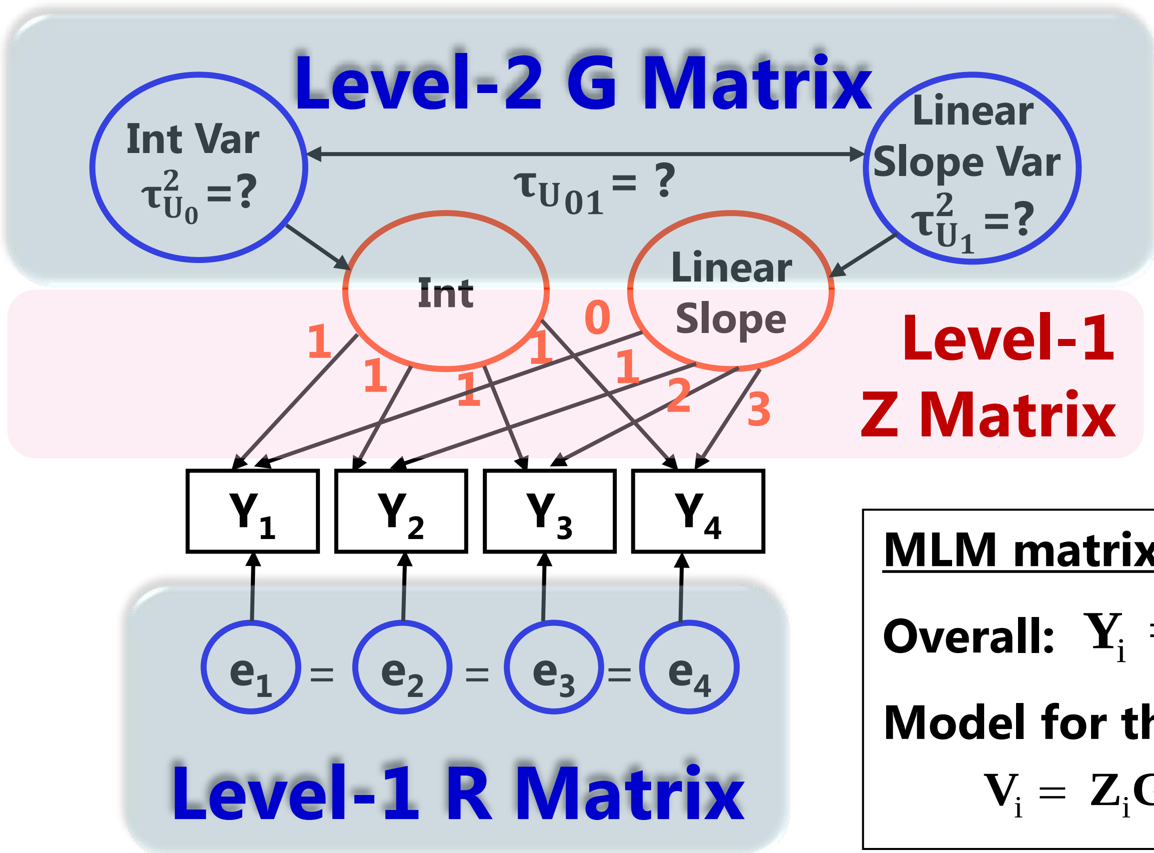
Loadings of linear slope factor
= occasions (keep real time)

Variance of linear slope factor
= random slope variance



Summary: Random Linear Time Model as Latent Variables in SEM

$$y_{ti} = \gamma_{00} + (\gamma_{10}Time_{ti}) + U_{0i} + (U_{1i}Time_{ti}) + e_{ti}$$



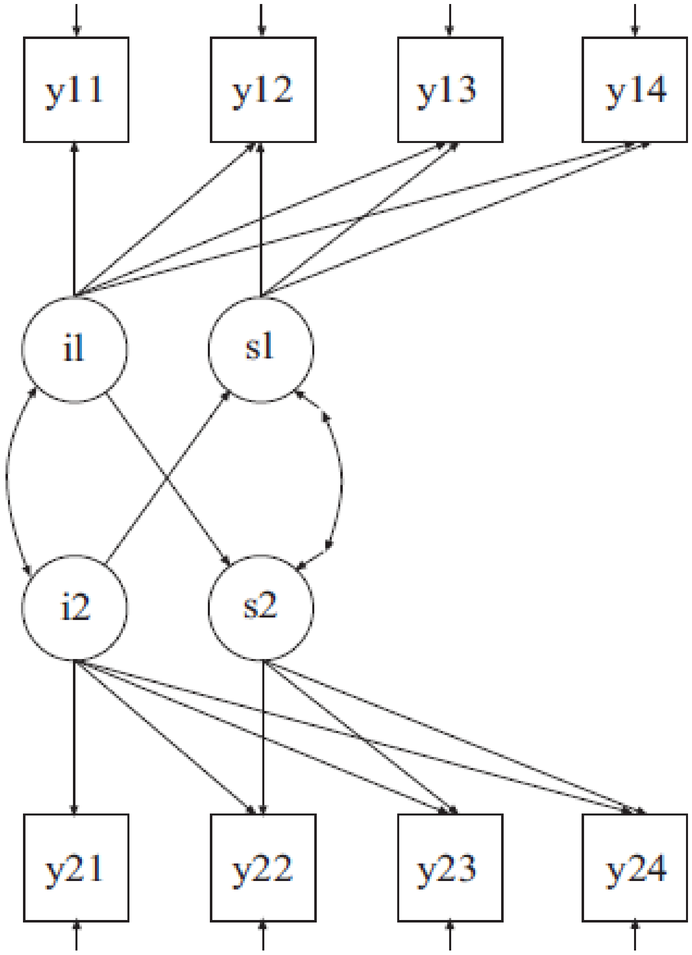
MLM matrix version of model

Overall: $\mathbf{Y}_i = \mathbf{X}_i\boldsymbol{\gamma} + \mathbf{Z}_i\mathbf{U}_i + \mathbf{E}_i$

Model for the Variance:

$$\mathbf{V}_i = \mathbf{Z}_i\mathbf{G}_i\mathbf{Z}_i^T + \mathbf{R}_i$$

Multivariate MLM as Single-Level SEM



This diagram is from the Mplus v. 8 Users Guide example 6.13.

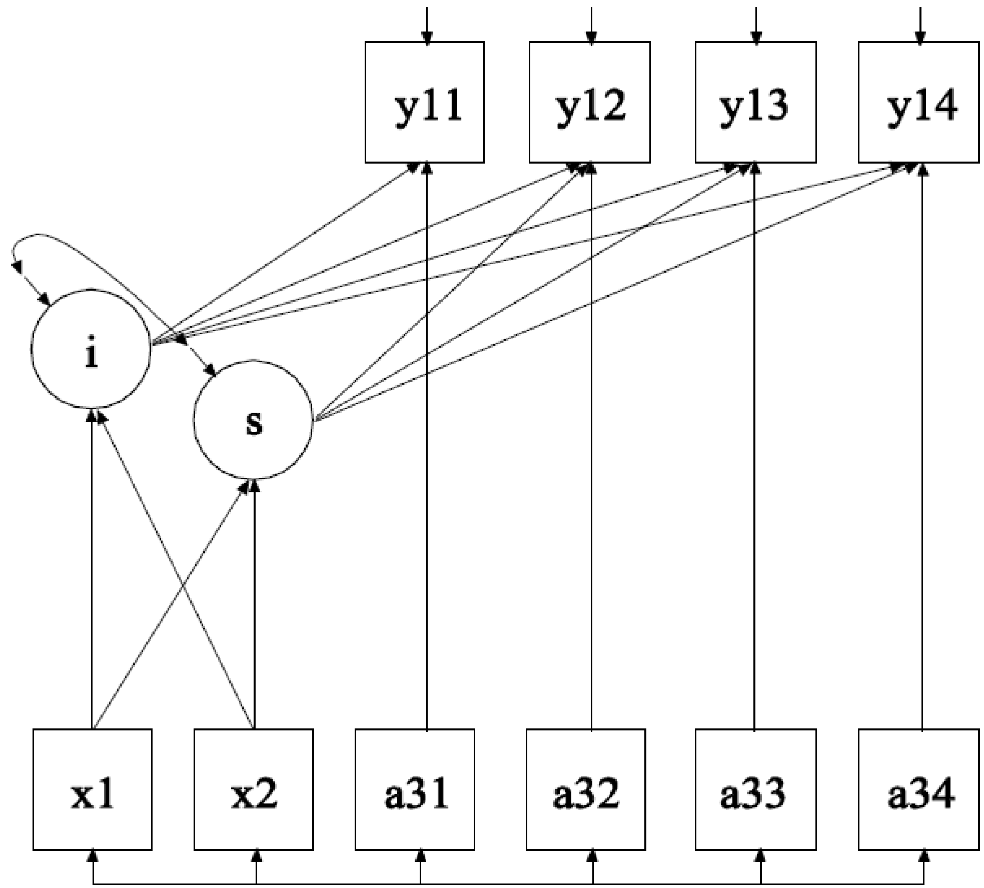
The **two-headed arrows** between the intercept factors (i1 and i2) and between the slope factors (s1 and s2) convey undirected **covariances**.

The **single-headed arrows** from i1 to s2 and from i2 to s1 are **directed regressions** which convey directionality (although this model **fits equivalently** whether one uses directed regressions or covariances among the latent factors).

Summary: Random Effects Phrased as Latent Variables in Single-Level SEM

- Random effects are person-specific sources of covariation among outcomes over time—these are the same as latent variables
  - Time-specific outcomes become "items" in factor analysis
  - Factor loadings convey time span and pattern of change
    - You can use individually varying time loadings for unbalanced data—via TSCORES in Mplus—which means absolute fit assessment is not provided
  - Fixed effect = latent variable mean ("mean" → "intercept" if predicted)
  - Random effect variance = latent variable variance ("variance" → "residual" variance if predicted)
  - Covariances among random effects in multivariate MLM can also be phrased as directed regressions (in "truly" multivariate MLM, M-SEM, or SEM)
- For univariate or multivariate longitudinal models with only level-2 predictors, MLM → single-level SEM with no real problem
  - **This is NOT true for time-varying predictors, the specification of which are *still* frequently misunderstood in single-level SEMs**

Time-Varying Predictors in Single-Level SEM: What *Not* to Do

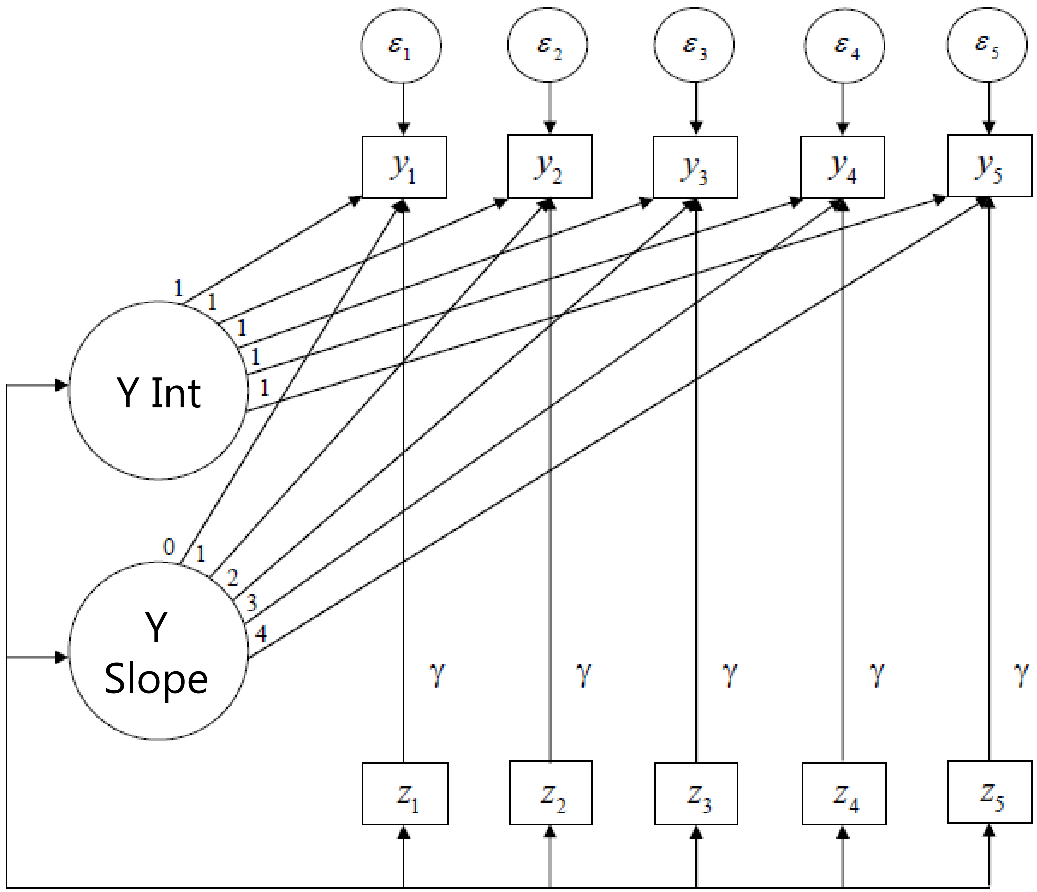


This diagram is from the Mplus v. 8 Users Guide example 6.10.

Although the y11-y14 outcomes are predicted by latent intercept and slope factors (separating two kinds of BP variance from WP variance), this is not the case for the a31-a34 outcomes.

Consequently, in the model shown here, the a→y paths will be smushed effects.

Time-Varying Predictors in Single-Level SEM: What *Not* to Do (continued)

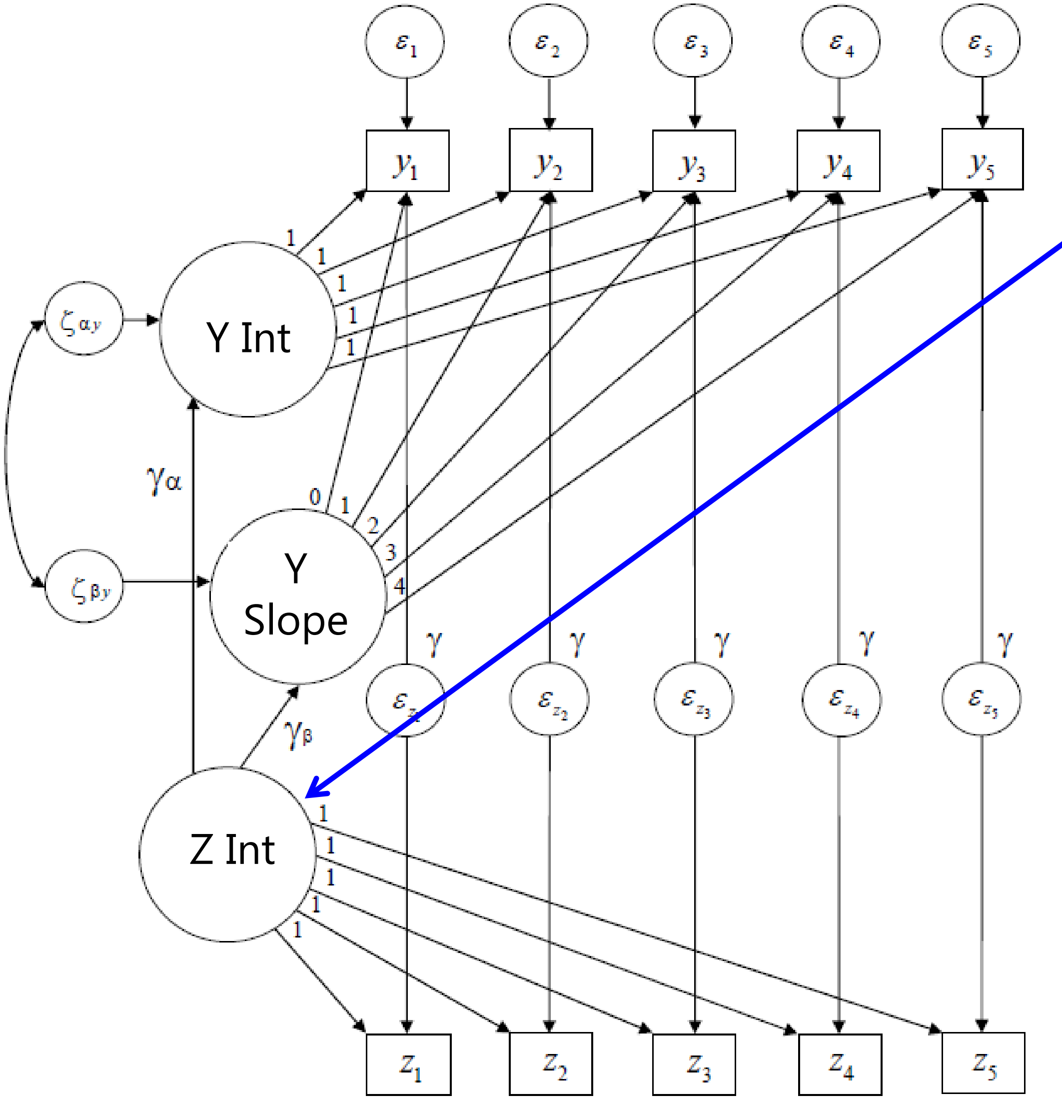


This diagram is from Curran et al. (2012). The time-varying predictors z1-z5 boxes have directed effects onto the y1-y5 outcomes at the same time.

If you constrain these paths to be equal (as γ), you get **a smushed effect** (they call it an "aggregate" effect).

If you add covariances of the z's with the intercept, γ then becomes **the WP effect**. But the BP effect is not in here! And you cannot add PMz to get it like in MLM because it will be redundant (→ ipsative).

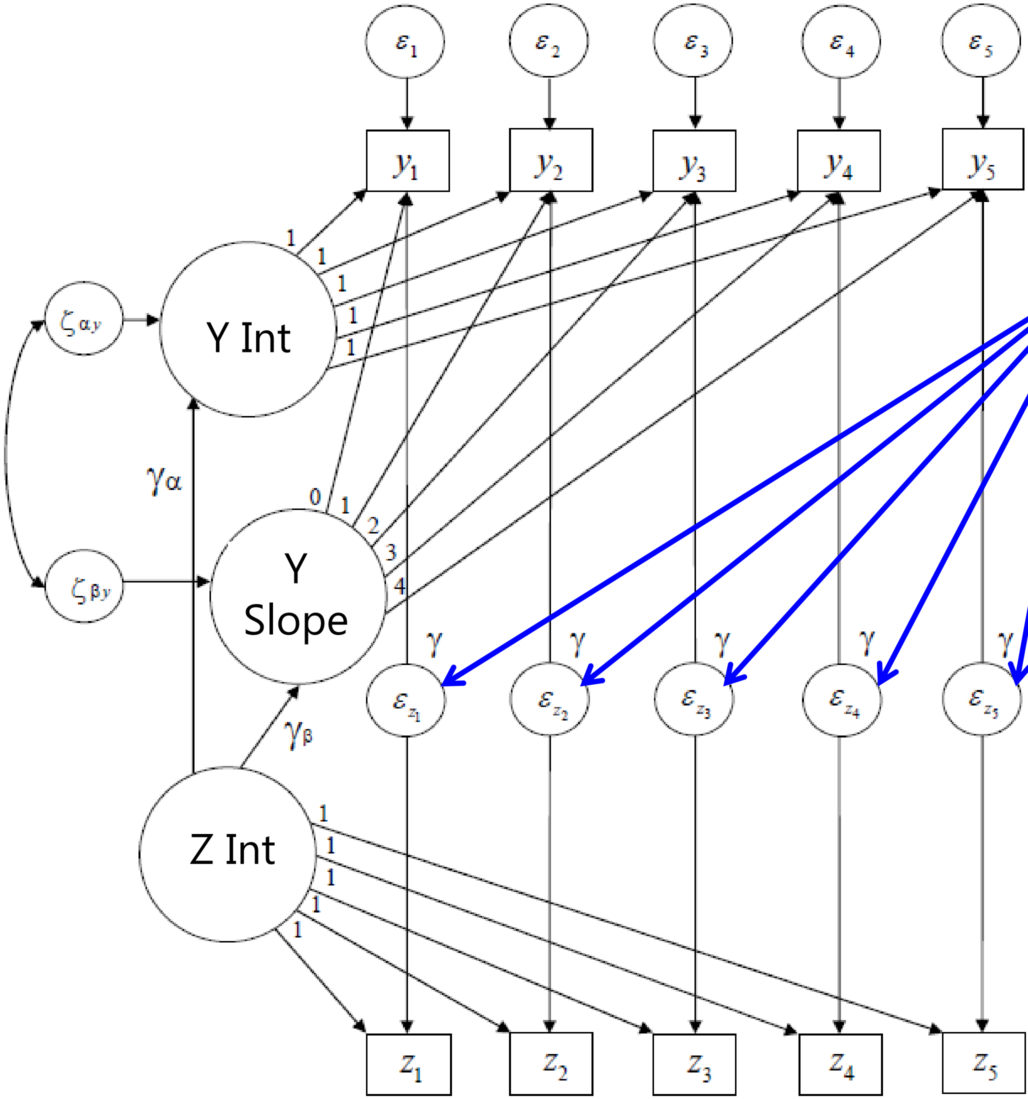
How to Fix It (by Curran et al., 2012)



The z1-z5 time-varying predictors now have their own intercept factor, which directly represents their BP intercept variance.

So the **BP intercept effect** is given by $\boldsymbol{\gamma_\alpha}$, and **the WP effect** is now given by $\boldsymbol{\gamma}$ from the residuals of z1-z5 →y1-y5.

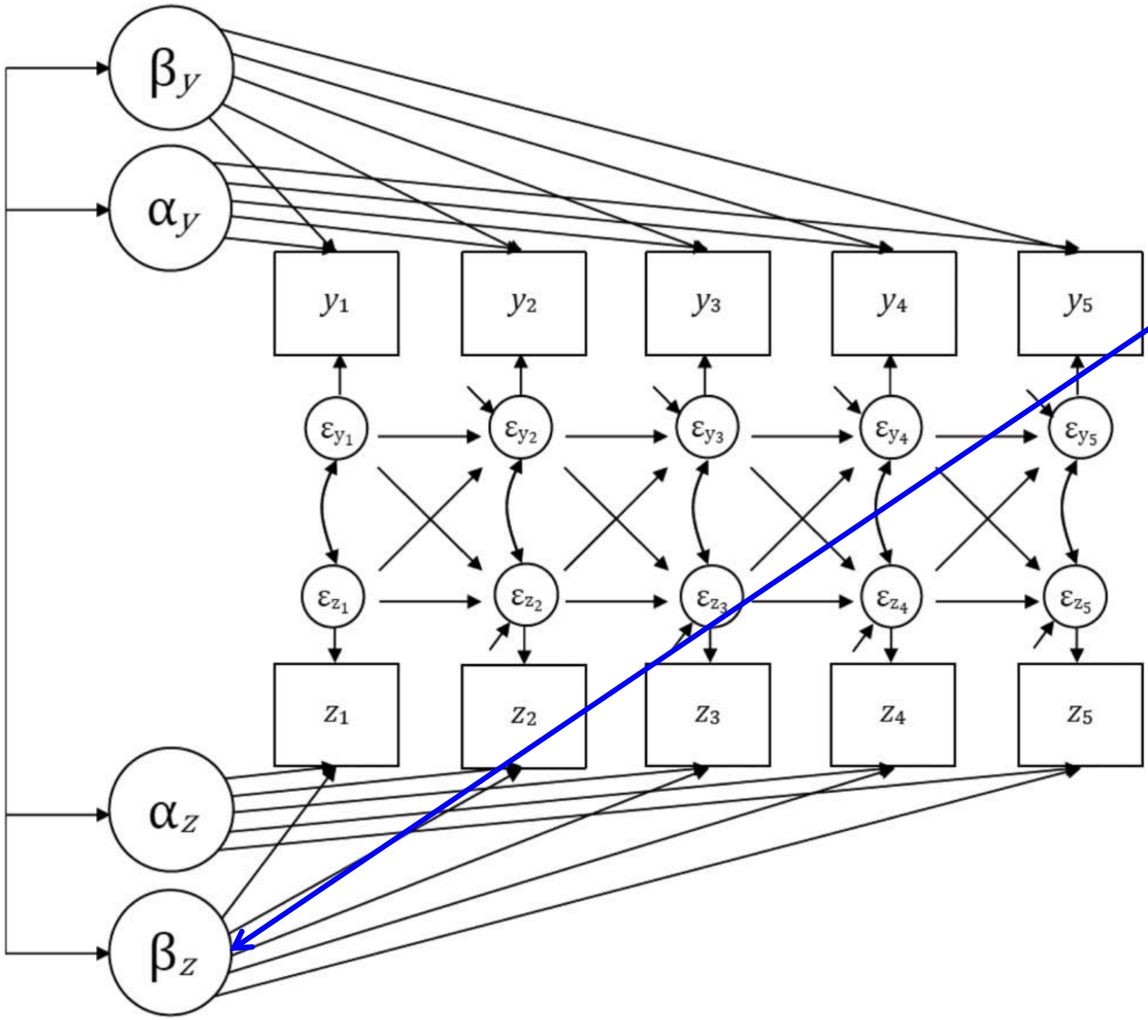
How to Fix It (by Curran et al., 2012)



In addition to adding an intercept factor for z1-z5, they added "**structured residuals**," such that the residual variance for each z variable was transferred to a new "latent" variable (LZ).

The LZ variables then predict the y1-y5 residuals directly to create the **level-1 WP effect**. Through this modification, the Z →Y intercept path is the **level-2 *total* between-person intercept effect**.

Another Example of Structured Residuals

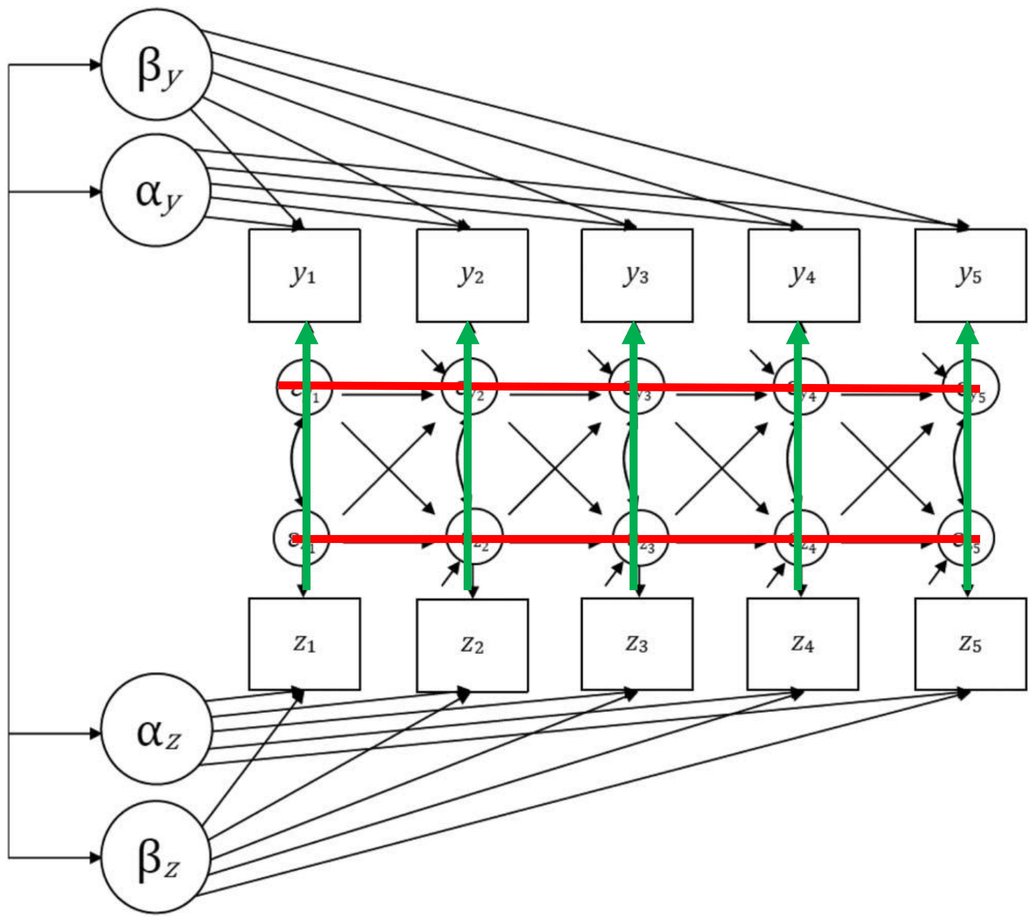


From Curran et al. (2014; *Journal of Consulting and Clinical Psychology*)

If z1-z5 has individual differences in change over time instead of just fluctuation, **just add a slope factor for z1-z5**—then you'd be back to multivariate multilevel model we began with.

When **using level-1 structured residuals,** all paths among the intercept and slope factors will represent their **total level-2 BP effects**.

How To Fix It Without Structured Residuals



IF you predict the y1-y5 residuals directly from z1-z5 (without structured residuals), **that effect is still the level-1 WP effect.**

The problem (in Mplus) is that some of the paths among the intercept and slope factors become **BP contextual effects** instead. These include paths for intercept → intercept and slope → slope, but not for intercept → slope (or slope → intercept).

In either version, you can still get the missing L2 effect (BP total or BP contextual) by requesting a NEW effect in MODEL CONSTRAINT.

When to Use Each: Multivariate MLM vs Single-Level SEM

- Models and software are logically separate, but (current) software restrictions may make it so one version is easier than the other for specifying certain types of models

- “Truly” Multivariate MLM (e.g., MLM side of Mplus):
  - Uses stacked data, so *contemporaneous* level-1 is explicit, which easily allows for random effects of level-1 predictors, mediation, and/or measurement models at each level of analysis
  - However: be careful of otherwise equivalent Mplus models whose L2 parameters change interpretation with different version of the syntax!

- Single-Level SEM (e.g., SEM side of Mplus):
  - Uses wide data structure, so level-1 parameters must be specified through constraints across multiple observed variables, which assumes balanced time (Mplus Tscores that allows individually varying times for growth models is not relevant for WP fluctuation models)

When to Use Each: Multivariate MLM vs Single-Level SEM

- Models requiring access to level-1 observations *at different occasions* across DVs can be easier to do in single-level SEM

- Single-Level SEM (e.g., SEM side of Mplus):
  - All occasions are accessible at once, which means that patterns of residual covariance over time can be easily included (via constraints)
  - Lagged residual relationships across DVs can be easily included (e.g., time 1 X → time 2 Y, time 1 Y → time 2 X), just make sure to not smush!

- Multivariate MLM (e.g., MLM side of Mplus):
  - Uses stacked data, so it doesn't have access to previous occasions' information stored on different rows (which needs to be unsmushed)
    - Mplus 8 allows auto-regressive relations, but only as specified as directed paths (not residual covariances) and only by using Bayes MCMC estimation

Summary: Multivariate Longitudinal Modeling

- Models and software are logically separate
  - No single approach/program can do everything you want; software options will always vary in what is possible and in how conveniently each model can be specified
- Univariate MLM:
  - Easy to specify in many widely available software packages; has REML estimation
  - Limited to multivariate multilevel models whose outcome relations can be phrased as covariances (in L2 G or L1 R), not directed paths (as needed in mediation)
- "Truly" Multivariate MLM (or M-SEM):
  - Trickier to specify correctly; available in many fewer (expensive) packages
  - More flexible for adding levels of analysis or specifying level-specific associations
- Single-Level SEM:
  - Harder to phrase level-specific associations, largely built for balanced data
  - Easier to specify relations across different occasions as variables than as rows of data