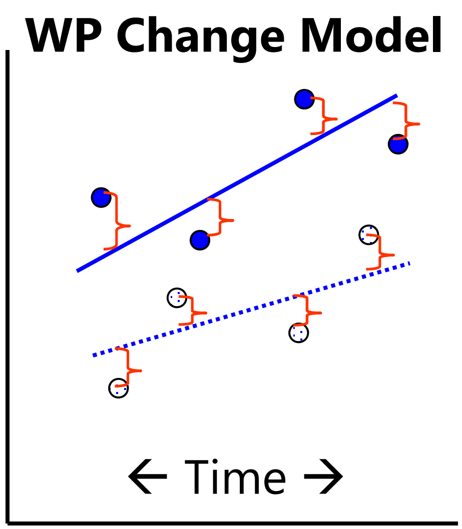
# Time-Varying Predictors of Within-Person Fluctuation

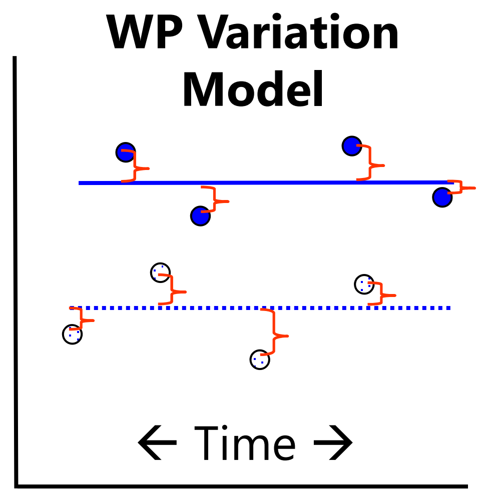
- Today's topics:
  - Time-varying predictors that fluctuate over time
  - Fixed level-1 effects using person-Mean-Centering (PMC)
    - Or "Variable-Based-Centering" more broadly
  - Fixed level-1 effects using grand-Mean-Centering (GMC)
    - Or "Constant-Based-Centering" more broadly
  - Interactions and random effects when using Person-MC vs. Grand-MC

# The Joy of Time-Varying Predictors

- TV predictors predict leftover **WP (residual) variation**:




If model for time works, then residuals should look like this →




- Modeling time-varying predictors is complicated because they represent an **aggregated effect**:
  - Effect of the *between-person* variation in the predictor $x_{ti}$ on Y
  - Effect of the *within-person* variation in the predictor $x_{ti}$ on Y
  - For now we are assuming the predictor $x_{ti}$ only **fluctuates** over time…
    - *We will need a **different model** when $x_{ti}$ changes individually over time…*

# The Joy of Time-Varying Predictors

- Time-varying (TV) predictors usually carry 2 kinds of effects because they are really 2 predictor variables, not 1

- Example: Stress measured daily
  - Some days are worse than others:
    - **WP variation in stress** *(represented as deviation from own mean)*
  - Some people just have more stress than others all the time:
    - **BP variation in stress** *(represented as person mean predictor over time)*

- Can quantify each source of variation with an **ICC**
  - ICC = (BP variance) / (BP variance + WP variance)
  - **ICC > 0?** TV predictor has **BP** variation (so it *could* have a BP effect)
  - **ICC < 1?** TV predictor has **WP** variation (so it *could* have a WP effect)

# Between-Person vs. Within-Person Effects

- Between-person and within-person effects in SAME direction
  - Stress → Health?
    - **BP: People with more chronic stress than other people may have worse general health than people with less chronic stress**
    - **WP: People may feel worse than usual when they are currently under more stress than usual (regardless of what "usual" is)**

- Between-person and within-person effects in OPPOSITE directions
  - Exercise → Blood pressure?
    - **BP: People who exercise more often generally have lower blood pressure than people who are more sedentary**
    - **WP: During exercise, blood pressure is higher than during rest**

- Variables have different **meanings** at different levels!
- Variables have different **scales** at different levels

# 3 Kinds of Fixed Effects for TV Predictors

- **Is the Level-2 Between-Person (BP) effect significant?**
  - Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
- **Is the Level-1 Within-Person (WP) effect significant?**
  - If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance ($\sigma^2_e$)?
- **Are BP and WP effects different : Is there a level-2 contextual effect?**
  - After controlling for the absolute value of TV predictor at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond)?
  - If there is no contextual effect, then the BP and WP effects of the TV predictor show *convergence*, such that their effects are of equivalent magnitude

# Modeling TV Predictors (labeled as $x_{ti}$)

- **Level-2 effect of $x_{ti}$:**
  - The level-2 effect of $x_{ti}$ is usually represented by the person's mean of time-varying $x_{ti}$ across time (labeled as **PMx$_i$** or $\bar{\mathbf{X}}_{\mathbf{i}}$)
  - PMx$_i$ should be centered at a <u>CONSTANT</u> (grand mean or other) so that 0 is meaningful, just like any other time-invariant predictor

- **Level-1 effect of $x_{ti}$ can be included two different ways:**
  - "**Group-mean-centering**" → "**<u>person-mean-centering</u>**" in longitudinal, in which level-1 predictors are centered using a <u>level-2 VARIABLE</u>
  - "**Grand-mean-centering**" → level-1 predictors are centered using a <u>CONSTANT</u> (not necessarily the grand mean; it's just called that)
  - Note that these 2 choices do NOT apply to the level-2 effect of $x_{ti}$
    - But the interpretation of the level-2 effect of $x_{ti}$ WILL DIFFER based on which centering method you choose for the level-1 effect of $x_{ti}$!

# Person-Mean-Centering (P-MC)

- In P-MC, we partition the TV predictor $x_{ti}$ into 2 variables that directly represent its BP (level-2) and WP (level-1) sources of variation, and **include these variables as the predictors instead**:

- **Level-2, PM predictor = person mean of $\mathbf{x_{ti}}$**
  - $\mathbf{PMx_i = \bar{X}_i} - \boldsymbol{C}$
  - $\mathrm{PMx_i}$ is centered at constant $C$, chosen for meaningful 0 (e.g., sample mean)
  - $\mathrm{PMx_i}$ is positive? Above sample mean → "more than other people"
  - $\mathrm{PMx_i}$ is negative? Below sample mean → "less than other people"

- **Level-1, WP predictor = deviation from person mean of $\mathbf{x_{ti}}$**
  - $\mathbf{WPx_{ti} = x_{ti} - \bar{X}_i}$ *(note: uncentered person mean $\bar{\boldsymbol{X}}_{\boldsymbol{i}}$ is used to center $x_{ti}$)*
  - $\mathrm{WPx_{ti}}$ is NOT centered at a constant; is centered at a VARIABLE
  - $\mathrm{WPx_{ti}}$ is positive? Above your own mean → "more than usual"
  - $\mathrm{WPx_{ti}}$ is negative? Below your own mean → "less than usual"

# Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 $x_{ti}$**

→ WP and BP Effects directly through **separate** parameters

**$x_{ti}$ is person-mean-centered into $WPx_{ti}$, with $PMx_i$ at L2:**

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$

$WPx_{ti} = x_{ti} - \bar{X}_i$ → it has only Level-1 WP variation

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$PMx_i = \bar{X}_i - C$ → it has only Level-2 BP variation

$\gamma_{10}$ = WP main effect of having more $x_{ti}$ than usual

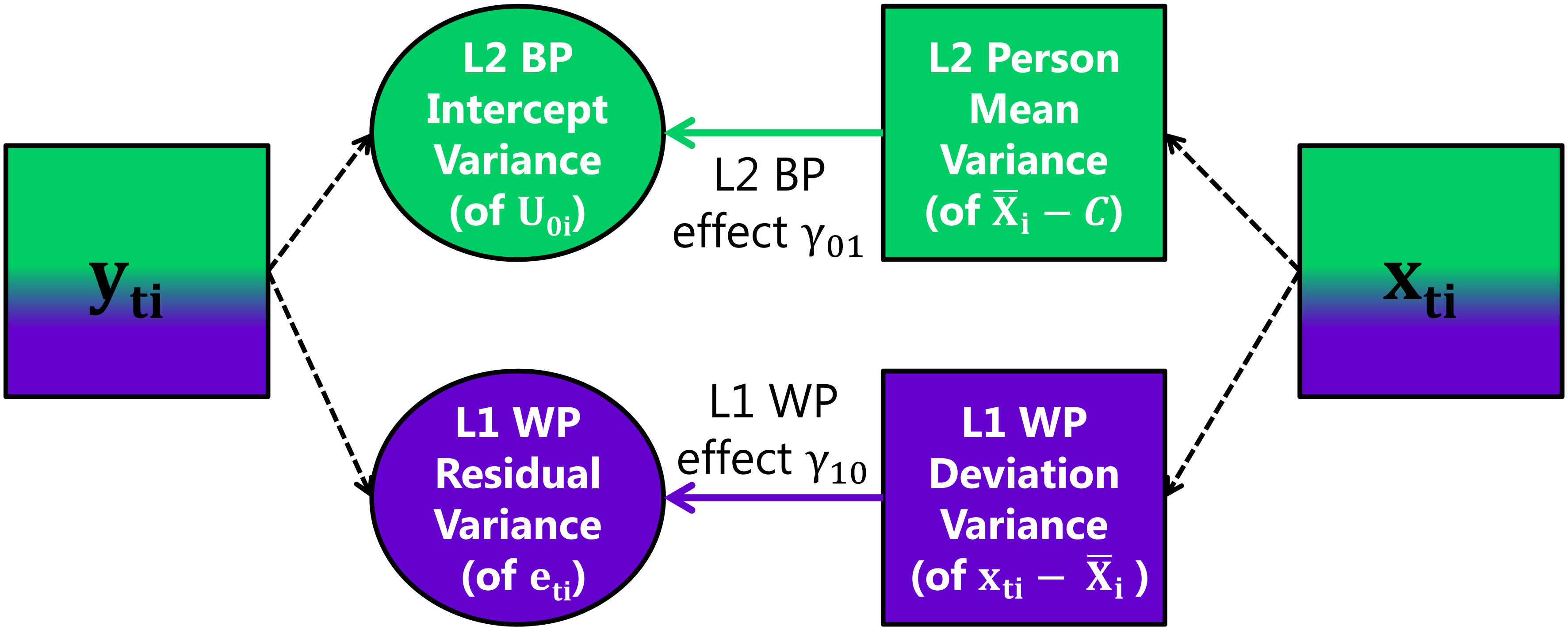
$\gamma_{01}$ = BP main effect of having more $\bar{X}_i$ than other people

Because $WPx_{ti}$ and $PMx_i$ are uncorrelated, each gets the total effect for its level (WP=L1, BP=L2)

# Person-Mean-Centering

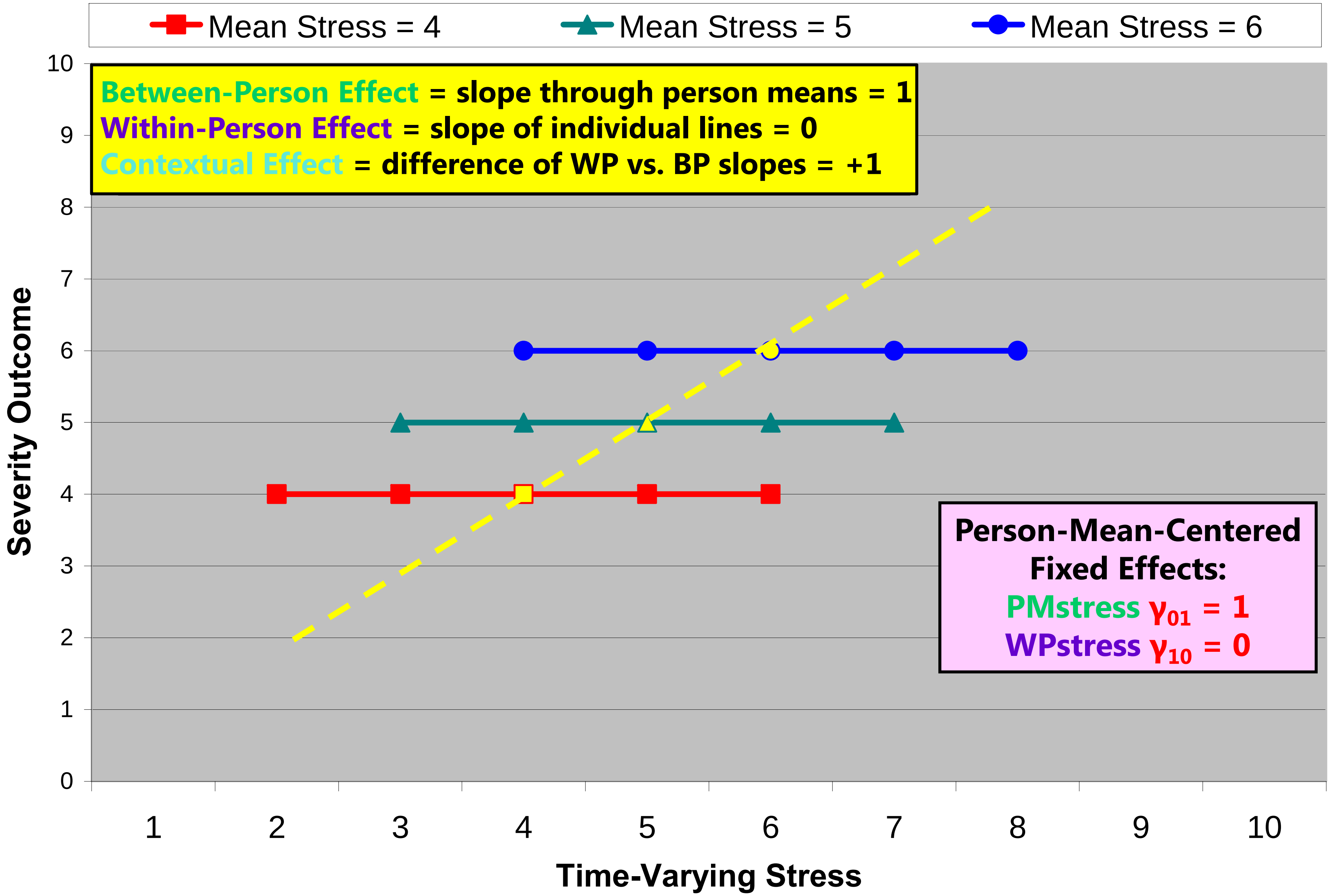
**Model-based** partitioning of $y_{ti}$ outcome variance into **variance components**:

**Brute-force** partitioning of $x_{ti}$ predictor variance into **observed variables**:

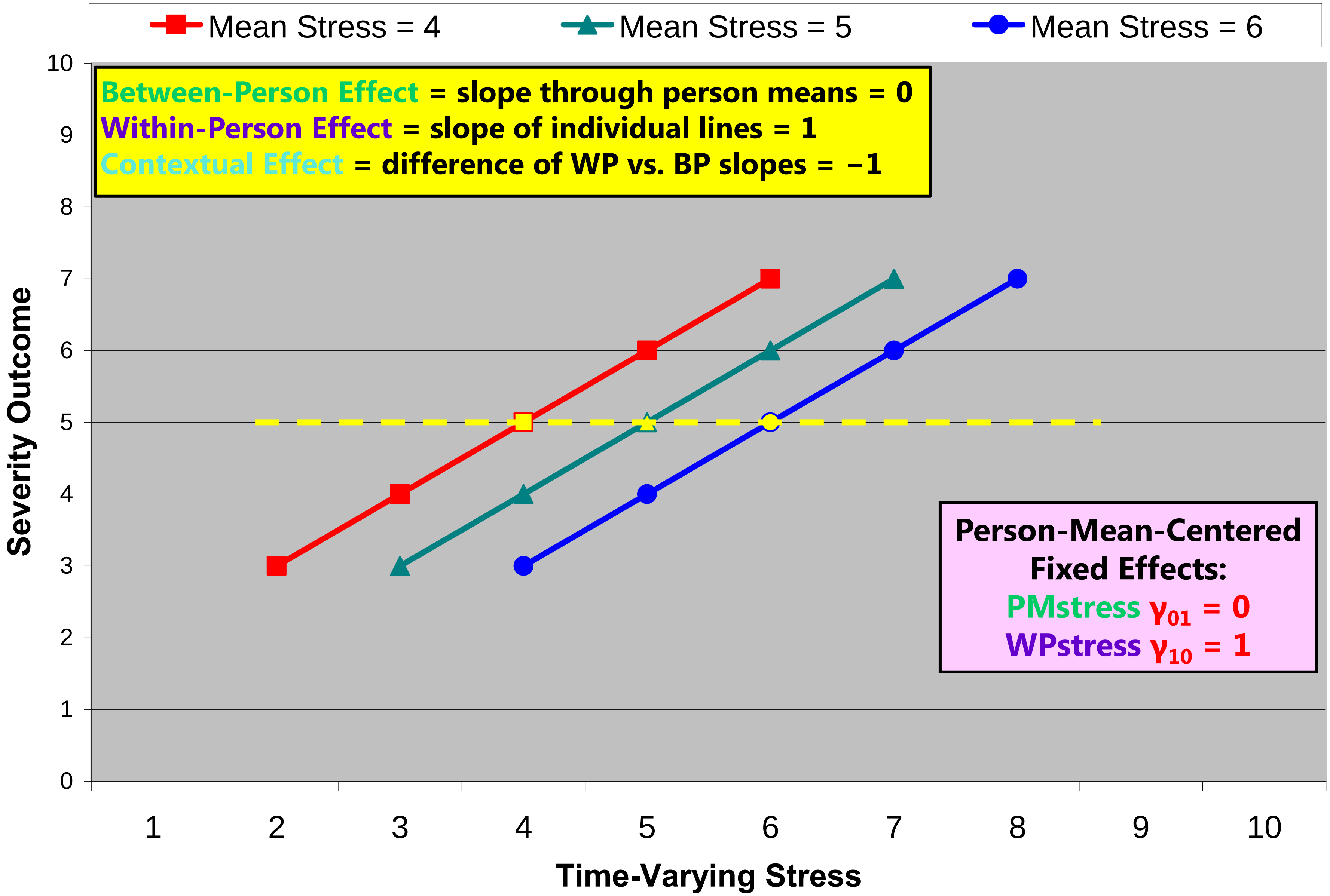



Why not let the model make variance components for $x_{ti}$, too?
This is the basis of multivariate MLM (or "multilevel SEM"): stay tuned…

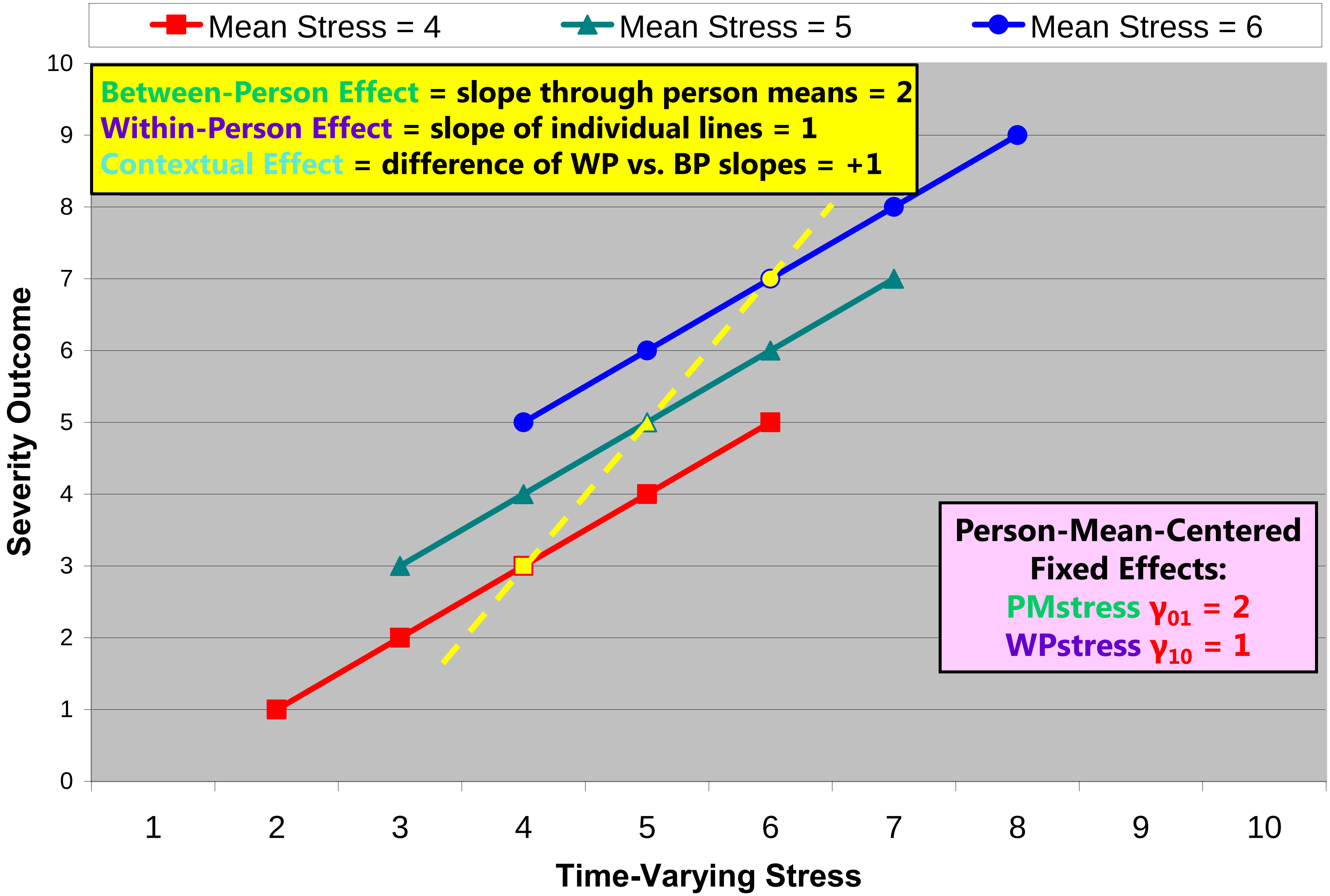
# ALL Between-Person Effect, NO Within-Person Effect



# NO Between-Person Effect, ALL Within-Person Effect



# Between-Person Effect > Within-Person Effect



# Within-Person Fluctuation Model with **Person-Mean-Centered Level-1 $x_{ti}$**

→ WP and BP Effects directly through **separate** parameters

**$x_{ti}$ is person-mean-centered into $WPx_{ti}$, with $PMx_i$ at L2:**

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(WPx_{ti}) + e_{ti}$

**$WPx_{ti} = x_{ti} - \bar{X}_i$ → it has only Level-1 WP variation**

**$PMx_i = \bar{X}_i - C$ → it has only Level-2 BP variation**

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i) + U_{1i}$

**$U_{1i}$ is a random slope for the WP effect of $x_{ti}$**

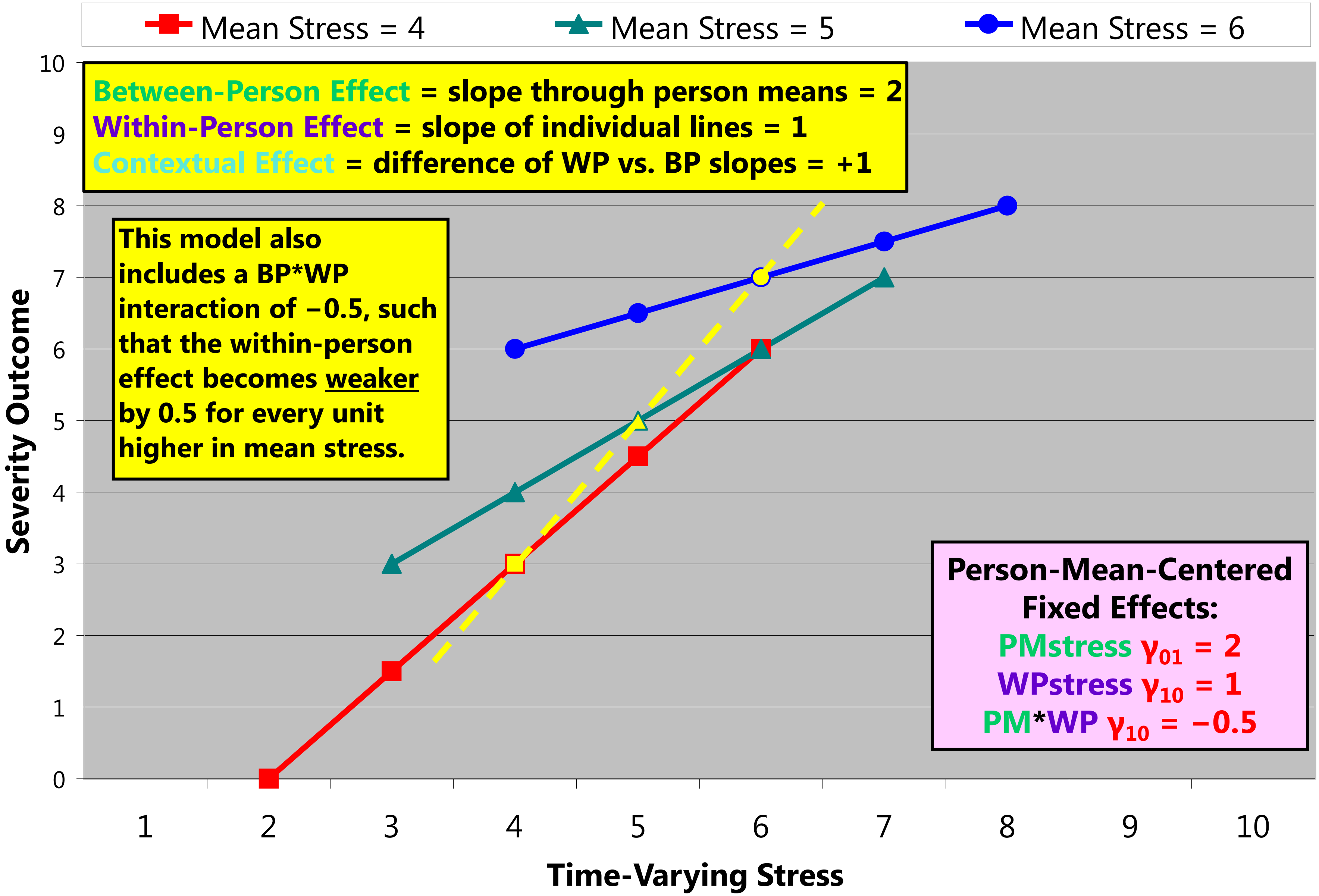
**$\gamma_{10}$ = WP simple main effect of having more $x_{ti}$ than usual *for* $PMx_i = 0$**

**$\gamma_{01}$ = BP simple main effect of having more $\bar{X}_i$ than other people for people at their own mean ($WPx_{ti} = x_{ti} - \bar{X}_i$ → 0)**

**$\gamma_{11}$ = BP*WP interaction: how the effect of having more $x_{ti}$ than usual differs by how much $\bar{X}_i$ you have**

Note: this model should also test $\gamma_{02}$ for $PMx_i * PMx_i$ (stay tuned)

# Between-Person x Within-Person Interaction



# 3 Kinds of Fixed Effects for TV Predictors

- **First 2 effects Person-Mean-Centering tells us <u>directly</u>:**

- **Is the Level-2 Between-Person (BP) effect significant?**
    - Are people with higher predictor values <u>than other people</u> (*on average over time*) also higher on Y <u>than other people</u> (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
    - This would be indicated by a significant fixed effect of $\mathbf{PMx_i}$
    - Note: this is NOT controlling for the absolute value of $x_{ti}$ at each occasion

- **Is the Level-1 Within-Person (WP) effect significant?**
    - If you have higher predictor values <u>than usual</u> (*at this occasion*), do you also have higher outcomes values <u>than usual</u> (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance ($\sigma^2_e$)?
    - This would be indicated by a significant fixed effect of $\mathbf{WPx_{ti}}$
    - Note: this is represented by the <u>relative</u> value of $x_{ti}$, NOT the <u>absolute</u> value of $x_{ti}$

# 3rd Kind of Effect for TV Predictors

- **What Person-Mean-Centering DOES NOT tell us directly:**

- **Are BP and WP effects different : Is there a level-2 contextual effect?**
  - After controlling for the absolute value of the TV predictor at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond just the time-specific value of the predictor)?
  - If there is no contextual effect, then the BP and WP effects of the TV predictor show ***convergence***, such that their effects are of equivalent magnitude

- **To answer this question about the level-2 contextual effect for the incremental contribution of the person mean, we have two options:**
  - Ask for the contextual effect via an ESTIMATE statement in SAS (or TEST in SPSS, or NEW in Mplus, or LINCOM in STATA): $\mathbf{WPx_{ti}}\ \ -1\ \ \mathbf{PMx_i}\ \ 1$
  - Use "**grand-mean-centering**" for time-varying $x_{ti}$ instead: $\mathbf{TVx_{ti}} = \mathbf{x_{ti}} - \boldsymbol{C}$
    → **centered at a CONSTANT, NOT A LEVEL-2 VARIABLE**
    - Which constant only matters for what the reference point is; it could be the grand mean or other

# Why the Difference in the Level-2 Effect? Remember Regular Old Regression…

- In this model: $Y_i = \beta_0 + \beta_1(X_{1i}) + \beta_2(X_{2i}) + e_i$
- If $X_{1i}$ and $X_{2i}$ **ARE NOT** correlated:
  - $\beta_1$ is **ALL the relationship** between $X_{1i}$ and $Y_i$
  - $\beta_2$ is **ALL the relationship** between $X_{2i}$ and $Y_i$

- If $X_{1i}$ and $X_{2i}$ **ARE** correlated:
  - $\beta_1$ is **different than** the full relationship between $X_{1i}$ and $Y_i$
    - "Unique" effect of $X_{1i}$ *controlling for* $X_{2i}$ or *holding* $X_{2i}$ *constant*
  - $\beta_2$ is **different than** the full relationship between $X_{2i}$ and $Y_i$
    - "Unique" effect of $X_{2i}$ *controlling for* $X_{1i}$ or *holding* $X_{1i}$ *constant*

- Hang onto that idea…

# Person-MC vs. Grand-MC for Time-Varying Predictors

| Level 2 | | Original | Person-MC Level 1 | Grand-MC Level 1 |
|---|---|---|---|---|
| $\bar{X}_i$ | $PMx_i = \bar{X}_i - 5$ | $x_{ti}$ | $WPx_{ti} = x_{ti} - \bar{X}_i$ | $TVx_{ti} = x_{ti} - 5$ |
| 3 | −2 | 2 | −1 | −3 |
| 3 | −2 | 4 | 1 | −1 |
| 7 | 2 | 6 | −1 | 1 |
| 7 | 2 | 8 | 1 | 3 |

Same $PMx_i$ goes into the model using either way of centering the level-1 variable $x_{ti}$

Using **Person-MC**, $WPx_{ti}$ has NO level-2 BP variation, so it is not correlated with $PMx_i$

Using **Grand-MC**, $TVx_{ti}$ STILL has level-2 BP variation, so it is STILL CORRELATED with $PMx_i$

**So the effects of $PMx_i$ and $TVx_{ti}$ when included together under Grand-MC will be different than their effects would be if they were by themselves…**

# **WRONG WAY:** Within-Person Fluctuation Model with $\mathbf{x_{ti}}$ **represented at Level 1 Only:** → WP and BP Effects are **Smushed Together**

**$\mathbf{x_{ti}}$ is grand-mean-centered into $\mathbf{TVx_{ti}}$, WITHOUT $\mathbf{PMx_i}$ at L2:**

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(\mathbf{TVx_{ti}}) + \mathbf{e_{ti}}$

**$\mathbf{TVx_{ti} = x_{ti} - \mathit{C}}$ → it still has both Level-2 BP and Level-1 WP variation**

Level 2: $\beta_{0i} = \gamma_{00} + \mathbf{U_{0i}}$

$\beta_{1i} = \gamma_{10}$

**$\gamma_{10}$ = *smushed* WP and BP effects**

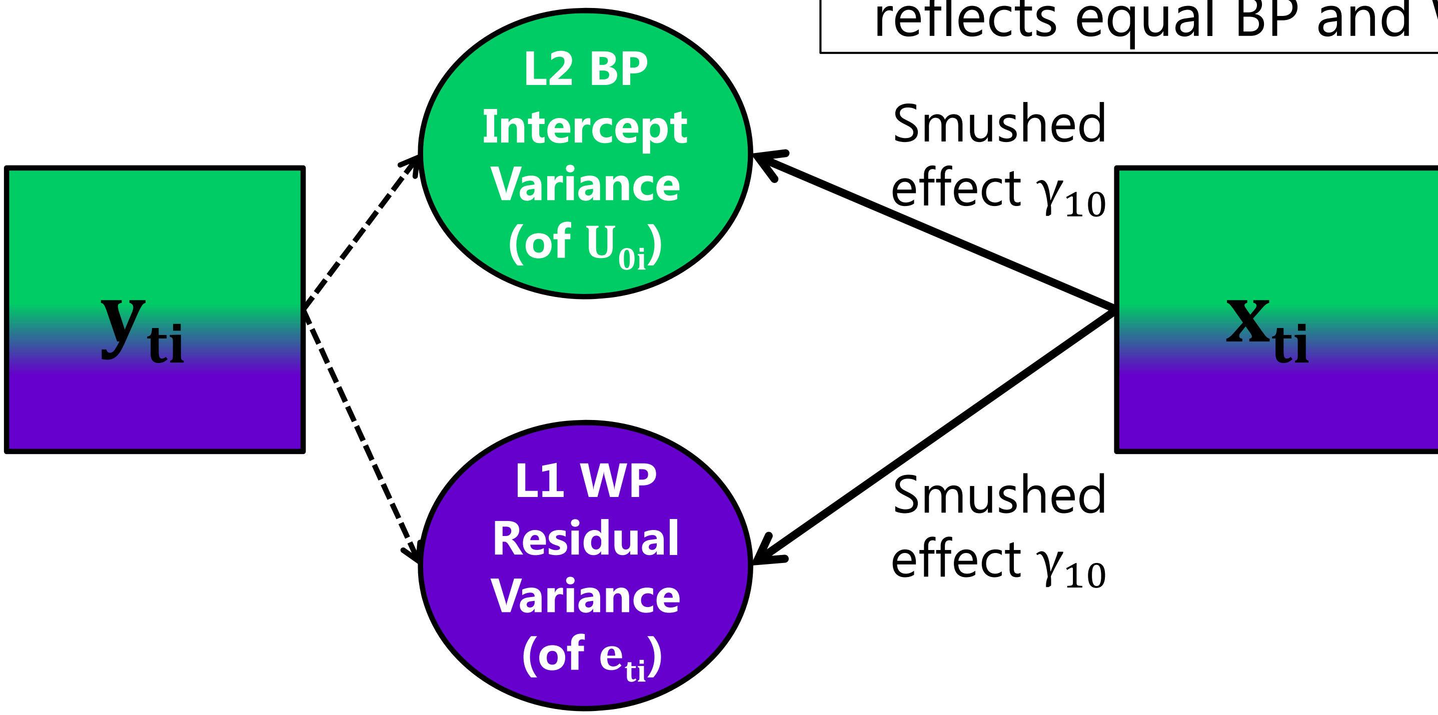
**Because $\mathbf{TVx_{ti}}$ still contains its original 2 different kinds of variation (BP and WP), its 1 fixed effect has to do the work of 2 predictors!**

**A *smushed* effect is also referred to as the *convergence*, *conflated*, or *composite* effect**

# Grand-Mean-Centering: Smushing

**Model-based** partitioning of $y_{ti}$ outcome variance into **variance components**:

**Original $x_{ti}$ has not been partitioned** AND it has only **one fixed effect** coefficient in the model. Thus, that smushed effect reflects equal BP and WP effects.



# Convergence (Smushed) Effect of a Time-Varying Predictor

$$\text{Convergence Effect: } \gamma_{\text{conv}} \approx \frac{\dfrac{\gamma_{BP}}{SE_{BP}^2} + \dfrac{\gamma_{WP}}{SE_{WP}^2}}{\dfrac{1}{SE_{BP}^2} + \dfrac{1}{SE_{WP}^2}}$$

Adapted from Raudenbush & Bryk (2002, p. 138)

- **The convergence effect will often be closer to the within-person effect** (due to larger level-1 sample size and thus smaller SE)

- **It is the rule, not the exception, that between and within effects differ** (Snijders & Bosker, 1999, p. 52-56, and personal experience!)

- However—when grand-mean-centering a time-varying predictor, **convergence is testable** by including a **contextual effect (carried by the person mean)** for how the **BP effect** differs from the **WP effect**…

# Within-Person Fluctuation Model with **Grand-Mean-Centered Level-1 $x_{ti}$**

→ Model tests difference of WP vs. BP effects (So it's been fixed!)

**$x_{ti}$ is grand-mean-centered into $TVx_{ti}$, <u>WITH</u> $PMx_i$ at L2:**

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}(TVx_{ti}) + e_{ti}$

**$TVx_{ti} = x_{ti} - C$ → it still has both Level-2 BP and Level-1 WP variation**

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

**$PMx_i = \bar{X}_i - C$ → it has only Level-2 BP variation**

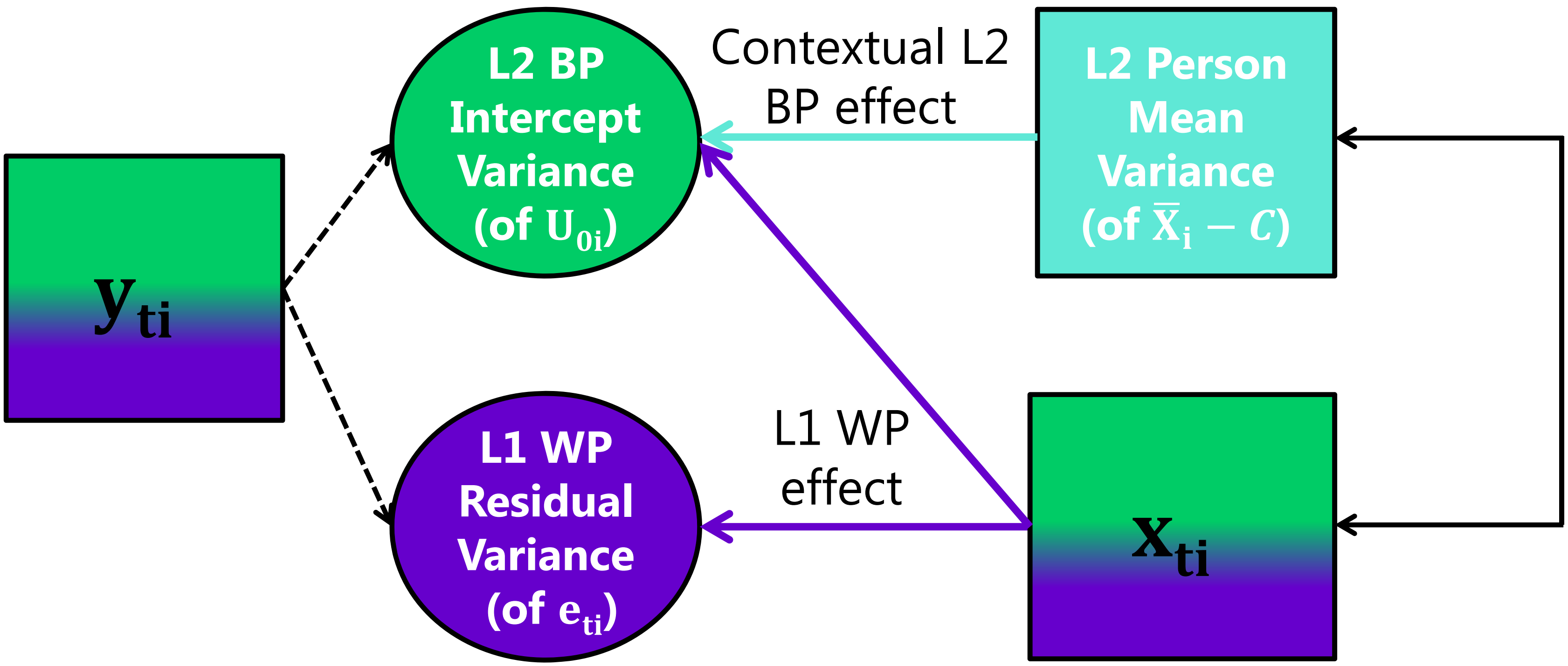
**$\gamma_{10}$ becomes the WP effect → *unique* level-1 effect after controlling for $PMx_i$**

**$\gamma_{01}$ *becomes* the contextual effect that indicates <u>how the BP effect differs from the WP effect</u>**
**→ *unique* level-2 effect after controlling for $TVx_{ti}$**
**→ does usual level matter beyond current level?**

# Grand-Mean-Centering + PM

**Model-based** partitioning of $y_{ti}$ outcome variance into **variance components**:

Original $x_{ti}$ is not partitioned, but person mean $\mathbf{\bar{X}_i - \mathit{C}}$ is added to allow an extra (different) effect at L2.




Because original $x_{ti}$ still has BP variance, it still carries part of the BP effect…

# Person-MC and Grand-MC Models are Equivalent Given a Fixed Level-1 Main Effect Only

**Person-MC:** $WPx_{ti} = x_{ti} - PMx_i$

**Level-1:** $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

**Level-2:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$

$\rightarrow y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

**Composite Model:**
← In terms of P-MC
← In terms of G-MC

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**Grand-MC:** $TVx_{ti} = x_{ti}$

**Level-1:** $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

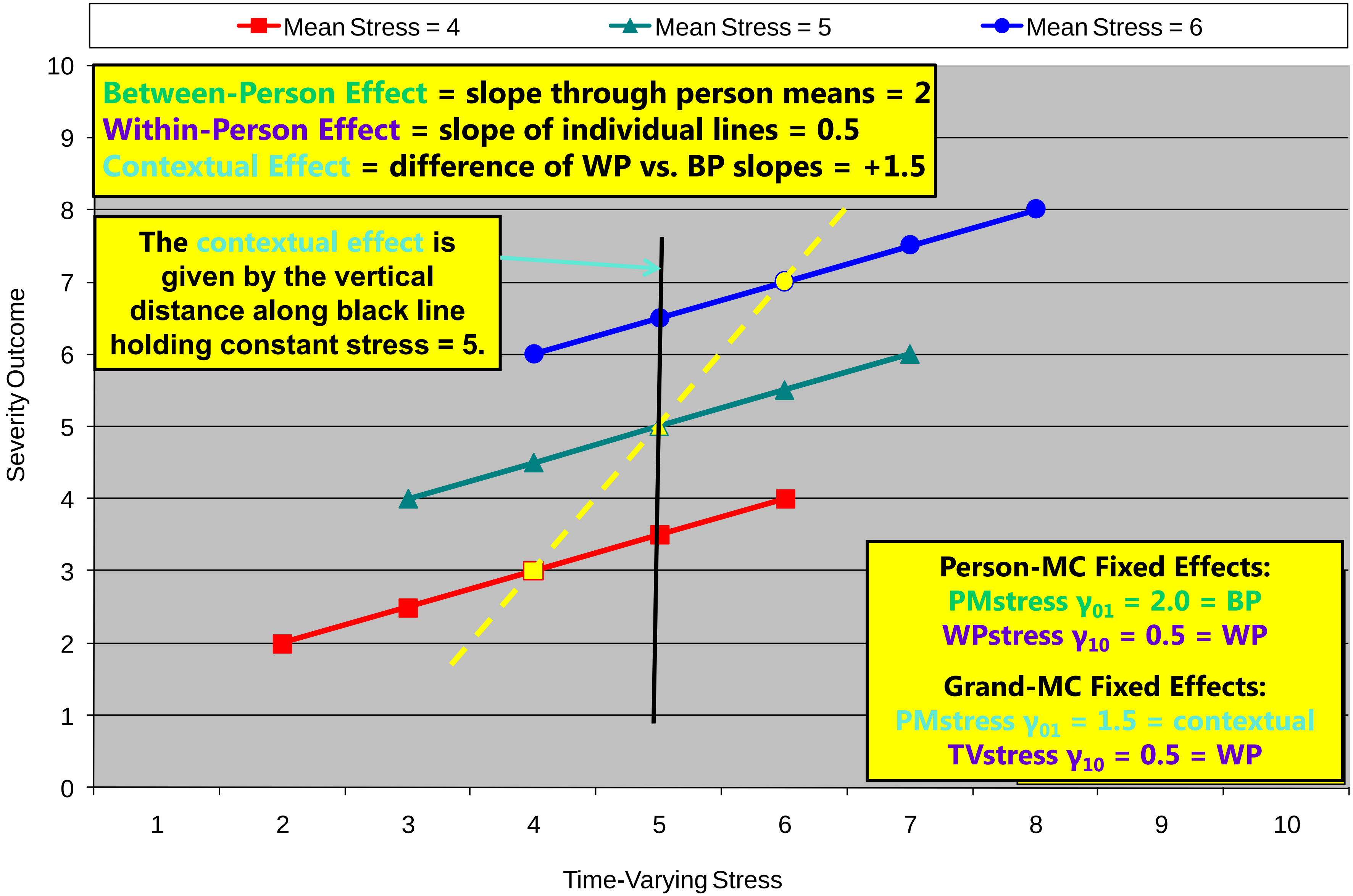
**Level-2:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10}$

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$

| Effect | P-MC | G-MC |
|---|---|---|
| Intercept | $\gamma_{00}$ | $\gamma_{00}$ |
| WP Effect | $\gamma_{10}$ | $\gamma_{10}$ |
| Contextual | $\gamma_{01} - \gamma_{10}$ | $\gamma_{01}$ |
| BP Effect | $\gamma_{01}$ | $\gamma_{01} + \gamma_{10}$ |

# P-MC vs. G-MC: Interpretation Example



# Summary: 3 Fixed Effects for TV Predictors

- **Is the Between-Person (BP; Level 2) effect significant?**
  - Are people with higher predictor values than other people (*on average over time*) also higher on Y than other people (*on average over time*), such that the person mean of the TV predictor accounts for level-2 random intercept variance ($\tau^2_{U_0}$)?
  - Given directly by level-2 effect of $\mathrm{PMx_i}$ if using Person-MC for the level-1 predictor (or can be requested via ESTIMATE if using Grand-MC for the level-1 predictor)

- **Is the Within-Person (WP; Level 1) effect significant?**
  - If you have higher predictor values than usual (*at this occasion*), do you also have higher outcomes values than usual (*at this occasion*), such that the within-person deviation of the TV predictor accounts for level-1 residual variance ($\sigma^2_e$)?
  - Given directly by the level-1 effect of $\mathrm{WPx_{ti}}$ if using Person-MC —*OR* — given directly by the level-1 effect of $\mathrm{TVx_{ti}}$ if using Grand-MC and including $\mathrm{PMx_i}$ at level 2 (without $\mathrm{PMx_i}$, the level-1 effect of $\mathrm{TVx_{ti}}$ if using Grand-MC is the smushed effect)

- **Are the BP and WP Effects different: Is there a level-2 contextual effect?**
  - After controlling for the absolute value of TV predictor value at each occasion, is there still an incremental contribution from having a higher person mean of the TV predictor (i.e., does one's general tendency predict $\tau^2_{U_0}$ above and beyond)?
  - Given directly by level-2 effect of $\mathrm{PMx_i}$ if using Grand-MC for the level-1 predictor (or can be requested via ESTIMATE if using Person-MC for the level-1 predictor)

# Variance Accounted For By Level-2 Predictors

- **Fixed effects of level 2 predictors *by themselves*:**
  - Level-2 (BP) main effects reduce level-2 (BP) random intercept variance
  - Level-2 (BP) interactions also reduce level-2 (BP) random intercept variance

- **Fixed effects of *cross-level interactions* (level 1* level 2):**
  - If the interacting level-1 predictor is <u>random</u>, any cross-level interaction with it will reduce its corresponding level-2 BP random slope variance (that line's U)
  - If the interacting level-1 predictor <u>not random</u>, any cross-level interaction with it will reduce the level-1 WP residual variance instead
    - This is because the level-2 BP random slope variance would have been created by decomposing the level-1 residual variance in the first place
    - The level-1 effect would then be called "**systematically varying**" to reflect a compromise between "fixed" (all the same) and "random" (all different)—it's not that each person needs his or her own slope, but that the slope varies systematically across people as a function of a known person predictor (and not otherwise)

# Variance Accounted For By Level-1 Predictors

- **Fixed effects of level 1 predictors *by themselves*:**
  - Level-1 (WP) main effects reduce Level-1 (WP) residual variance
  - Level-1 (WP) interactions also reduce Level-1 (WP) residual variance

- **What happens at level 2 depends on what kind of variance the level-1 predictor has:**
  - If the level-1 predictor ALSO has level-2 variance (e.g., Grand-MC predictors), then its level-2 variance will also likely reduce level-2 random intercept variance
  - If the level-1 predictor DOES NOT have level-2 variance (e.g., Person-MC predictors), then its reduction in the level-1 residual variance will cause an INCREASE in level-2 random intercept variance
    - Same thing happens with Grand-MC level-1 predictors, but you don't generally see it
  - It's just an artifact that the estimate of true random intercept variance is:

    True $\tau^2_{U_0}$ = observed $\tau^2_{U_0} - \frac{\sigma^2_e}{n}$ $\rightarrow$ so if only $\sigma^2_e$ decreases, $\tau^2_{U_0}$ increases

# Variance Accounted for… For Real

- **Pseudo-$R^2$** is named that way for a reason… piles of variance can shift around, such that it can actually be negative
  - Sometimes a sign of model mis-specification
  - Hard to explain to readers when it happens!

- **One last simple alternative: Total $R^2$**
  - Generate model-predicted y's from fixed effects only (NOT including random effects) and correlate with observed y's
  - Then square correlation → total $R^2$
  - Total $R^2$ = total reduction in overall variance of y across levels
  - Can be "unfair" in models with large unexplained sources of variance

- MORAL OF THE STORY: Specify EXACTLY which kind of pseudo-$R^2$ you used—give the formula and the reference!!

# The Joy of Interactions Involving Time-Varying Predictors

- <u>**Must consider interactions with both its BP and WP parts:**</u>
- Example: Does time-varying stress ($x_{ti}$) interact with sex ($Sex_i$)?

- <u>Person-Mean-Centering:</u>
  - $\mathbf{WPx_{ti}} * \mathbf{Sex_i}$ → Does the WP stress effect differ between men and women?
  - $\mathbf{PMx_i} * \mathbf{Sex_i}$ → Does the BP stress effect differ between men and women?
    - Not controlling for current levels of stress
    - If forgotten, then $\mathbf{Sex_i}$ moderates the stress effect only at level 1 (WP, not BP)

- <u>Grand-Mean-Centering:</u>
  - $\mathbf{TVx_{ti}} * \mathbf{Sex_i}$ → Does the WP stress effect differ between men and women?
  - $\mathbf{PMx_i} * \mathbf{Sex_i}$ → Does the *contextual* stress effect differ b/t men and women?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of $\mathbf{PMx_i}$, the interaction of $\mathbf{TVx_{ti}} * \mathbf{Sex_i}$ would still be smushed

# Interactions with Time-Varying Predictors: Example: TV Stress ($x_{ti}$) by Gender ($Sex_i$)

**Person-MC:** $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti} - PMx_i)$

---

**Grand-MC:** $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(Sex_i)$

Composite: $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$

# Interactions Involving Time-Varying Predictors Belong at Both Levels of the Model

**On the left below → Person-MC:** $WPx_{ti} = x_{ti} - PMx_i$

$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti} - PMx_i)$

← **Composite model written as Person-MC**

$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + (\gamma_{03} - \gamma_{11})(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$

← **Composite model written as Grand-MC**

**On the right below → Grand-MC:** $TVx_{ti} = x_{ti}$

$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(Sex_i) + \gamma_{03}(Sex_i)(PMx_i) + \gamma_{11}(Sex_i)(x_{ti})$

After adding an interaction for $Sex_i$ with stress at both levels, then the Person-MC and Grand-MC models are equivalent

**Intercept:** $\gamma_{00} = \gamma_{00}$ **BP Effect:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Contextual:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

**WP Effect:** $\gamma_{10} = \gamma_{10}$ **BP*Sex Effect:** $\gamma_{03} = \gamma_{03} + \gamma_{11}$ **Contextual*Sex:** $\gamma_{03} = \gamma_{03} - \gamma_{11}$

**Sex Effect:** $\gamma_{20} = \gamma_{20}$ **BP*WP or Contextual*WP is the same:** $\gamma_{11} = \gamma_{11}$

# Intra-variable Interactions

- **Still must consider interactions with both its BP and WP parts!**
- Example: Interaction of TV stress ($x_{ti}$) with person mean stress ($PMx_i$)

- Person-Mean-Centering:
  - $\mathbf{WPx_{ti}} * \mathbf{PMx_i}$ → Does the WP stress effect differ by overall stress level?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the BP stress effect differ by overall stress level?
    - Not controlling for current levels of stress
    - If forgotten, then $\mathbf{PMx_i}$ moderates the stress effect only at level 1 (WP, not BP)

- Grand-Mean-Centering:
  - $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ → Does the WP stress effect differ by overall stress level?
  - $\mathbf{PMx_i} * \mathbf{PMx_i}$ → Does the *contextual* stress effect differ by overall stress?
    - Incremental BP stress effect *after controlling for current levels of stress*
    - If forgotten, then although the level-1 main effect of stress has been un-smushed via the main effect of $\mathbf{PMx_i}$, the interaction of $\mathbf{TVx_{ti}} * \mathbf{PMx_i}$ would still be smushed

# Intra-variable Interactions:
## Example: TV Stress ($x_{ti}$) by Person Mean Stress ($PMx_i$)

**Person-MC:** $WPx_{ti} = x_{ti} - PMx_i$

**Level-1:** $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

**Level-2:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

**Composite:** $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$

---

**Grand-MC:** $TVx_{ti} = x_{ti}$

**Level-1:** $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

**Level-2:** $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{02}(PMx_i)(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}(PMx_i)$

**Composite:** $y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

# Intra-variable Interactions:
## Example: TV Stress ($x_{ti}$) by Person Mean Stress ($PMx_i$)

**On the left below → Person-MC:** $WPx_{ti} = x_{ti} - PMx_i$

$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti} - PMx_i)$

← **Written as Person-MC**

$y_{ti} = \gamma_{00} + (\gamma_{01} - \gamma_{10})(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ (\gamma_{02} - \gamma_{11})(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

← **Written as Grand-MC**

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**On the right below → Grand-MC:** $TVx_{ti} = x_{ti}$

$y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + e_{ti}$
$+ \gamma_{02}(PMx_i)(PMx_i) + \gamma_{11}(PMx_i)(x_{ti})$

After adding an interaction for $PMx_i$ with stress at both levels, then the Person-MC and Grand-MC models are equivalent

---

**Intercept:** $\gamma_{00} = \gamma_{00}$ **BP Effect:** $\gamma_{01} = \gamma_{01} + \gamma_{10}$ **Contextual:** $\gamma_{01} = \gamma_{01} - \gamma_{10}$

**WP Effect:** $\gamma_{10} = \gamma_{10}$ **BP² Effect:** $\gamma_{02} = \gamma_{02} + \gamma_{11}$ **Contextual²:** $\gamma_{02} = \gamma_{02} - \gamma_{11}$

**BP*WP or Contextual*WP is the same:** $\gamma_{11} = \gamma_{11}$

# When Person-MC ≠ Grand-MC: Random Effects of TV Predictors

**Person-MC:** $WPx_{ti} = x_{ti} - PMx_i$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti} - PMx_i) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to $PMx_i$ is removed from the random slope in Person-MC.

$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti} - PMx_i) + U_{0i} + U_{1i}(x_{ti} - PMx_i) + e_{ti}$

---

**Grand-MC:** $TVx_{ti} = x_{ti}$

Level-1: $y_{ti} = \beta_{0i} + \beta_{1i}(x_{ti}) + e_{ti}$

Level-2: $\beta_{0i} = \gamma_{00} + \gamma_{01}(PMx_i) + U_{0i}$

$\beta_{1i} = \gamma_{10} + U_{1i}$

Variance due to $PMx_i$ is still part of the random slope in Grand-MC. So these models cannot be made equivalent.

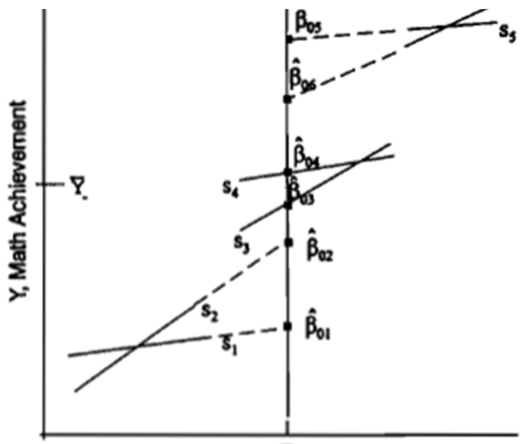
$\rightarrow y_{ti} = \gamma_{00} + \gamma_{01}(PMx_i) + \gamma_{10}(x_{ti}) + U_{0i} + U_{1i}(x_{ti}) + e_{ti}$

# Random Effects of TV Predictors

- **Random intercepts** mean different things under each model:
  - **Person-MC** → Individual differences at $\mathbf{WPx_{ti}}$ **=0** (that everyone has)
  - **Grand-MC** → Individual differences at $\mathbf{TVx_{ti}}$**=0** (that not everyone has)
- **Differential shrinkage of the random intercepts** results from differential reliability of the intercept data across models:
  - Person-MC → Won't affect shrinkage of slopes unless highly correlated
  - Grand-MC → Will affect shrinkage of slopes due to forced extrapolation
- As a result, the **random slope variance may be too small** when using Grand-MC rather than Person-MC
  - Problem worsens with greater ICC of TV Predictor (more extrapolation)
  - Anecdotal example using clustered data was presented in Raudenbush & Bryk (2002; chapter 6)

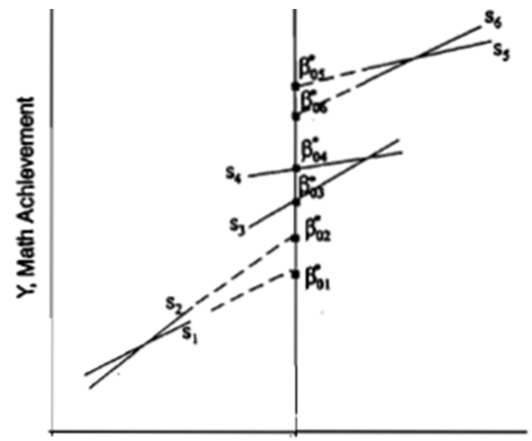
# Bias in Random Slope Variance

**OLS Per-Person Estimates**




**Level-1 X**

**EB Shrunken Estimates**




**Level-1 X**

Top right: Intercepts and slopes are homogenized in Grand-MC because of intercept extrapolation

Bottom: Downwardly-biased random slope variance in Grand-MC relative to Person-MC

| *Unconditional Results* | *Conditional Results* |
|---|---|
| **Person-MC** | |
| $\hat{\mathbf{T}} = \begin{bmatrix} 8.68 & 0.05 \\ 0.05 & 0.68 \end{bmatrix}$ $\hat{\sigma}^2 = 36.70$ | $\hat{\mathbf{T}} = \begin{bmatrix} 2.38 & 0.19 \\ 0.19 & \boxed{0.15} \end{bmatrix}$ $\hat{\sigma}^2 = 36.70$ |
| **Grand-MC** | |
| $\hat{\mathbf{T}} = \begin{bmatrix} 4.83 & -0.15 \\ -0.15 & 0.42 \end{bmatrix}$ $\hat{\sigma}^2 = 36.83$ | $\hat{\mathbf{T}} = \begin{bmatrix} 2.41 & 0.19 \\ 0.19 & \boxed{0.06} \end{bmatrix}$ $\hat{\sigma}^2 = 36.74$ |

# Modeling Time-Varying <u>Categorical</u> Predictors

- Person-MC and Grand-MC really only apply to *continuous* TV predictors, but the need to consider BP and WP effects applies to *categorical* TV predictors too
- Binary level-1 predictors do not lend themselves to Person-MC
  - e.g., $x_{ti}$ = 0 or 1 per occasion, person mean = .50 across occasions → impossible values
  - If $x_{ti} = 0$, then $WPx_{ti} = 0 - .50 = -0.50$; If $x_{ti} = 1$, then $WPx_{ti} = 1 - .50 = 0.50$
  - Better: Leave $x_{ti}$ uncentered and include person mean as level-2 predictor (results ~ Grand-MC)
- For >2 categories, person means of multiple dummy codes starts to break down, but we can think about types of people, and code BP effects accordingly
- <u>Example: Dementia present/not at each time point?</u>
  - **BP effects →** Ever diagnosed with dementia (no, yes)?
    - People who will eventually be diagnosed may differ prior to diagnosis (a BP effect)
  - **TV effect →** Diagnosed with dementia at each time point (no, yes)?
    - Acute differences of before/after diagnosis logically can only exist in the "ever" people
- Other examples: Mentor status, father absence, type of shift work (AM/PM)

# Wrapping Up: Person-MC vs. Grand-MC

- Time-varying predictors carry at least two potential effects:
  - Some people are higher/lower than other people → BP, level-2 effect
  - Some occasions are higher/lower than usual → WP, level-1 effect

- BP and WP effects almost always need to be represented by two or more model parameters, using either:
  - *Person-mean-centering* ($\mathrm{WPx}_{ti}$ and $\mathrm{PMx}_{i}$): WP ≠ 0?, BP ≠ 0?
  - *Grand-mean-centering* ($\mathrm{TVx}_{ti}$ and $\mathrm{PMx}_{i}$): WP ≠ 0?, BP ≠ WP?
  - Both yield equivalent models if the level-1 WP effect is fixed, but not if the level-1 WP effect is random
    - Grand MC → *absolute* effect of $x_{ti}$ varies randomly over people
    - Person MC → *relative* effect of $x_{ti}$ varies randomly over people
    - Use prior theory and empirical data (ML AIC, BIC) to decide