Example 7b: Generalized Models for Ordinal Longitudinal Data using SAS GLIMMIX, STATA MEOLOGIT, and MPLUS (last proportional odds model only)

This example comes from real data collected over 12 days in 91 nursing home patients who were hospitalized. Day 0 is the first day of hospitalization. Each day an assessment as to the patient's level of delirium was conducted by hospital staff, in which 0 = no delirium, 1 = some delirium, and 2 = full delirium. We will first examine the pattern of change across days, and then see if cognitive status (as measured by MMSE centered at 16, SD = 7) predicts intercept and time slope differences in levels of delirium.

del012	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0.None	190	37.25	190	37.25
1.Some	239	46.86	429	84.12
2.Full	81	15.88	510	100.00

Model 1: Empty Means, Single-Level Ordinal Model for None (=0), Some (=1), or Full (=2) Delirium

$$\text{Logit}\left(y_{ti} > 0\right) = \gamma_{001}$$
$$\text{Logit}\left(y_{ti} > 1\right) = \gamma_{002}$$

DESCENDING means that we will predict the logit of the higher category in each submodel (which is not the default, but I think it makes more sense).

```
TITLE1 "SAS Model 1: Empty Means, Single-Level Ordinal Model";
TITLE2 "GLIMMIX Cumulative Logit Link, Multinomial Conditional Distribution";
PROC GLIMMIX DATA=work.alldata NOCLPRINT NAMELEN=100 GRADIENT METHOD=QUAD;
     CLASS patient_ID;
     MODEL del012 (DESCENDING) = / SOLUTION DDFM=BW LINK=CLOGIT DIST=MULT;
     ESTIMATE "Intercept for y > 0 ( gamma001)" intercept  0  1 / ILINK;
     ESTIMATE "Intercept for y < 1 (-gamma001)" intercept  0 -1 / ILINK;
     ESTIMATE "Intercept for y > 1 ( gamma002)" intercept  1  0 / ILINK;
     ESTIMATE "Intercept for y < 2 (-gamma002)" intercept -1  0 / ILINK; RUN;

* STATA Model 1: Empty Means, Single-Level Ordinal Model
* MEOLOGIT Cumulative Logit Link, Multinomial Conditional Distribution
* No option to reverse the model intercepts, so they are backwards
meologit del012 ,
    estat ic, n(91),
    nlcom 1/(1+exp( 1*(_b[cut1:_cons]))) // intercept for y>0 in probability
    nlcom 1/(1+exp(-1*(_b[cut1:_cons]))) // intercept for y<1 in probability
    nlcom 1/(1+exp( 1*(_b[cut2:_cons]))) // intercept for y>1 in probability
    nlcom 1/(1+exp(-1*(_b[cut2:_cons]))) // intercept for y<2 in probability
```

SAS Output:

Response Profile

Ordered Value	del012	Total Frequency
1	2.Full	81
2	1.Some	239
3	0.Non	190

SAS Note: The GLIMMIX procedure is modeling the probabilities of levels of del012 having lower Ordered Values in the Response Profile table.

Confusing, but this means we are predicting up, not down….

Convergence criterion (ABSGCONV=0.00001) satisfied.

Fit Statistics

-2 Log Likelihood	1035.58
AIC (smaller is better)	1039.58
AICC (smaller is better)	1039.60
BIC (smaller is better)	1048.05
CAIC (smaller is better)	1050.05
HQIC (smaller is better)	1042.90

What table is missing that would normally be here?

Parameter Estimates

Effect	del012	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient
Intercept	2.Full	-1.6670	0.1211	508	-13.76	<.0001	1.7E-14
Intercept	1.Some	0.5213	0.09159	508	5.69	<.0001	-759E-16

Estimates

In Probability

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Mean	Standard Error Mean
Intercept for y > 0 (gamma001)	0.5213	0.09159	508	5.69	<.0001	**0.6275**	0.02141
Intercept for y < 1 (-gamma001)	-0.5213	0.09159	508	-5.69	<.0001	**0.3725**	0.02141
Intercept for y > 1 (gamma002)	-1.6670	0.1211	508	-13.76	<.0001	**0.1588**	0.01619
Intercept for y < 2 (-gamma002)	1.6670	0.1211	508	13.76	<.0001	**0.8412**	0.01619

Intercept1 = logit of probability of y > 0 = exp(0.521) / (1+exp(0.521)) = .6274 (=.4686+.1588)
Intercept2 = logit of probability of y > 1 = exp(−1.667) / (1+exp(−1.667)) = .1588

Probability(y=0) = 1 – intercept1 = 1 − .6274 = .3725
Probability(y=1) = intercept1 – intercept2 = .6274 − .1588 = .4686
Probability(y=2) = intercept2 − 0 = .1588 − 0 = .1588

These will exactly match the observed frequencies prior to adding any random effects.

STATA Output—model intercepts are backwards (as was the default in SAS GLIMMIX):

```
Ordered logistic regression                          Number of obs     =        510
                                                      chi2()           =          .
Log likelihood = -517.78973                          Prob > chi2       =          .
------------------------------------------------------------------------------
      del012 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       /cut1 |  -.5212969   .0915869    -5.69   0.000     -.7008039   -.3417899
       /cut2 |   1.667008   .1211474    13.76   0.000      1.429563    1.904452
------------------------------------------------------------------------------

.         estat ic, n(91),
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |         91         .  -517.7897       2    1039.579   1044.601
-----------------------------------------------------------------------------
               Note: N=91 used in calculating BIC.

.    nlcom 1/(1+exp( 1*(_b[cut1:_cons]))) // intercept for y>0 in probability
------------------------------------------------------------------------------
      del012 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _nl_1 |    .627451    .021409    29.31   0.000     .5854901    .6694119
------------------------------------------------------------------------------

.     nlcom 1/(1+exp(-1*(_b[cut1:_cons]))) // intercept for y<1 in probability
------------------------------------------------------------------------------
      del012 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _nl_1 |    .372549    .021409    17.40   0.000     .3305881    .4145099
------------------------------------------------------------------------------

.     nlcom 1/(1+exp( 1*(_b[cut2:_cons]))) // intercept for y>1 in probability
------------------------------------------------------------------------------
```

```
      del012 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _nl_1 |   .1588235   .0161851     9.81   0.000     .1271013    .1905458
------------------------------------------------------------------------------

.     nlcom 1/(1+exp(-1*(_b[cut2:_cons]))) // intercept for y<2 in probability
------------------------------------------------------------------------------
      del012 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _nl_1 |   .8411765   .0161851    51.97   0.000     .8094542    .8728987
------------------------------------------------------------------------------
```

Model 2: Empty Means, Two-Level Ordinal Model for None (=0), Some (=1), or Full (=2) Delirium

$$\text{Logit}\left(y_{ti} > 0\right) = \gamma_{001} + U_{0i}$$
$$\text{Logit}\left(y_{ti} > 1\right) = \gamma_{002} + U_{0i}$$

COVTEST does a LRT for the contents of the G matrix as specified to be either 0 or . (where . means estimate it as specified). Below, the 0 value asks SAS to hold the random intercept variance to 0 and estimate that model in order to do a LRT test against our model.

```
TITLE1 "SAS Model 2: Empty Means, Two-Level Ordinal Model";
TITLE2 "GLIMMIX Cumulative Logit Link, Multinomial Conditional Distribution";
PROC GLIMMIX DATA=work.alldata NOCLPRINT NAMELEN=100 GRADIENT METHOD=QUAD(QPOINTS=15);
     CLASS patient_ID;
     MODEL del012 (DESCENDING) = / SOLUTION DDFM=BW LINK=CLOGIT DIST=MULT;
     RANDOM INTERCEPT / TYPE=UN SUBJECT=patient_ID;
     ESTIMATE "Intercept for y > 0 ( gamma001)" intercept  0  1 / ILINK;
     ESTIMATE "Intercept for y < 1 (-gamma001)" intercept  0 -1 / ILINK;
     ESTIMATE "Intercept for y > 1 ( gamma002)" intercept  1  0 / ILINK;
     ESTIMATE "Intercept for y < 2 (-gamma002)" intercept -1  0 / ILINK;
     COVTEST "Need Random Intercept?"  0; * Test if G matrix (1,1)=0;
RUN; TITLE1; TITLE2;

* STATA Model 2: Empty Means, Two-Level Ordinal Model
* MEOLOGIT Cumulative Logit Link, Multinomial Conditional Distribution
* No option to reverse the model intercepts, so they are backwards
meologit del012 , || PATIENT_ID: , covariance(unstructured) intpoints(15),
    nlcom 1/(1+exp( 1*(_b[cut1:_cons]))) // intercept for y>0 in probability
    nlcom 1/(1+exp(-1*(_b[cut1:_cons]))) // intercept for y<1 in probability
    nlcom 1/(1+exp( 1*(_b[cut2:_cons]))) // intercept for y>1 in probability
    nlcom 1/(1+exp(-1*(_b[cut2:_cons]))) // intercept for y<2 in probability
```

SAS Output:

Fit Statistics

-2 Log Likelihood	978.26
AIC (smaller is better)	984.26
AICC (smaller is better)	984.31
BIC (smaller is better)	991.79
CAIC (smaller is better)	994.79
HQIC (smaller is better)	987.30

Model-scale ICC for the correlation of occasions from the same person:

$$\text{ICC} = \frac{1.2547}{1.2547 + 3.29} = .2761$$

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Gradient
UN(1,1)	PATIENT_ID	1.2547	0.3480	-0.00002

Note that there is only ONE random intercept variance!

Solutions for Fixed Effects

Effect	del012	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient
Intercept	2.Full	-2.0423	0.1920	89	-10.63	<.0001	-0.00008
Intercept	1.Some	0.6855	0.1640	89	4.18	<.0001	0.000062

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Mean (In Probability)	Standard Error Mean
Intercept for y > 0 (gamma001)	0.6855	0.1640	89	4.18	<.0001	0.6650	0.03653
Intercept for y < 1 (-gamma001)	-0.6855	0.1640	89	-4.18	<.0001	0.3350	0.03653
Intercept for y > 1 (gamma002)	-2.0423	0.1920	89	-10.63	<.0001	0.1148	0.01952
Intercept for y < 2 (-gamma002)	2.0423	0.1920	89	10.63	<.0001	0.8852	0.01952

Tests of Covariance Parameters
Based on the Likelihood

Label	DF	-2 Log Like	ChiSq	Pr > ChiSq	Note
Need Random Intercept?	1	1035.58	57.32	<.0001	MI

MI: P-value based on a mixture of chi-squares.

COVTEST results: −2LL given is for a model without the random intercept variance

Model 3: Saturated Means, Two-Level Ordinal Model (for descriptive purposes)

LSMEANS is not available for ordinal outcomes, so the ESTIMATES below request the mean per day.

```
TITLE1 "SAS Model 3: Saturated Means, Two-Level Ordinal Model";
PROC GLIMMIX DATA=work.alldata NOCLPRINT NAMELEN=100 GRADIENT METHOD=QUAD(QPOINTS=15);
     CLASS patient_ID day;
      MODEL del012 (DESCENDING) = day / SOLUTION DDFM=BW LINK=CLOGIT DIST=MULT;
      RANDOM INTERCEPT / TYPE=UN SUBJECT=patient_ID;
      ESTIMATE "Intercept for y>0 Day 0"  intercept 0 1 day 1 0 0 0 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 1"  intercept 0 1 day 0 1 0 0 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 2"  intercept 0 1 day 0 0 1 0 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 3"  intercept 0 1 day 0 0 0 1 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 4"  intercept 0 1 day 0 0 0 0 1 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 5"  intercept 0 1 day 0 0 0 0 0 1 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 6"  intercept 0 1 day 0 0 0 0 0 0 1 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 7"  intercept 0 1 day 0 0 0 0 0 0 0 1 0 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 8"  intercept 0 1 day 0 0 0 0 0 0 0 0 1 0 0 0 0;
      ESTIMATE "Intercept for y>0 Day 9"  intercept 0 1 day 0 0 0 0 0 0 0 0 0 1 0 0 0;
      ESTIMATE "Intercept for y>0 Day 10" intercept 0 1 day 0 0 0 0 0 0 0 0 0 0 1 0 0;
      ESTIMATE "Intercept for y>0 Day 11" intercept 0 1 day 0 0 0 0 0 0 0 0 0 0 0 1 0;
      ESTIMATE "Intercept for y>0 Day 12" intercept 0 1 day 0 0 0 0 0 0 0 0 0 0 0 0 1;
      ESTIMATE "Intercept for y>1 Day 0"  intercept 1 0 day 1 0 0 0 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 1"  intercept 1 0 day 0 1 0 0 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 2"  intercept 1 0 day 0 0 1 0 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 3"  intercept 1 0 day 0 0 0 1 0 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 4"  intercept 1 0 day 0 0 0 0 1 0 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 5"  intercept 1 0 day 0 0 0 0 0 1 0 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 6"  intercept 1 0 day 0 0 0 0 0 0 1 0 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 7"  intercept 1 0 day 0 0 0 0 0 0 0 1 0 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 8"  intercept 1 0 day 0 0 0 0 0 0 0 0 1 0 0 0 0;
      ESTIMATE "Intercept for y>1 Day 9"  intercept 1 0 day 0 0 0 0 0 0 0 0 0 1 0 0 0;
      ESTIMATE "Intercept for y>1 Day 10" intercept 1 0 day 0 0 0 0 0 0 0 0 0 0 1 0 0;
      ESTIMATE "Intercept for y>1 Day 11" intercept 1 0 day 0 0 0 0 0 0 0 0 0 0 0 1 0;
      ESTIMATE "Intercept for y>1 Day 12" intercept 1 0 day 0 0 0 0 0 0 0 0 0 0 0 0 1; RUN;
```

```
* STATA Model 3: Saturated Means, Two-Level Ordinal Model
meologit del012 i.day, || PATIENT_ID: , covariance(unstructured) intpoints(15),
    margins day, pred(xb),            // day-specific differences
    marginsplot, noci xdimension(day)  // plot predicted, no CI
```

Fit Statistics

-2 Log Likelihood	962.97
AIC (smaller is better)	992.97
AICC (smaller is better)	993.94
BIC (smaller is better)	1030.63
CAIC (smaller is better)	1045.63
HQIC (smaller is better)	1008.16

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Gradient
UN(1,1)	PATIENT_ID	1.3765	0.3733	0.00119

Solutions for Fixed Effects

Effect	del012	day: Days Since Hospitalization	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient
Intercept	2.Full		-3.0206	0.6836	89	-4.42	<.0001	-0.00018
Intercept	1.Some		-0.2132	0.6619	89	-0.32	0.7481	-0.00006
day		0	1.4344	0.7781	407	1.84	0.0660	-0.00064
day		1	0.9891	0.6947	407	1.42	0.1553	0.000097
day		2	1.2278	0.6907	407	1.78	0.0762	0.000141
day		3	0.8201	0.6929	407	1.18	0.2372	0.00013
day		4	1.1304	0.6985	407	1.62	0.1064	0.000157
day		5	0.3527	0.7175	407	0.49	0.6233	-0.00002
day		6	0.2854	0.7308	407	0.39	0.6963	0.000129
day		7	0.9837	0.7432	407	1.32	0.1864	0.001072
day		8	0.7707	0.7429	407	1.04	0.3001	0.000145
day		9	1.2647	0.7696	407	1.64	0.1011	-0.00168
day		10	0.3860	0.8141	407	0.47	0.6356	0.000177
day		11	-0.04530	0.9373	407	-0.05	0.9615	-8.53E-6
day		12	0	.	.	.	.	.

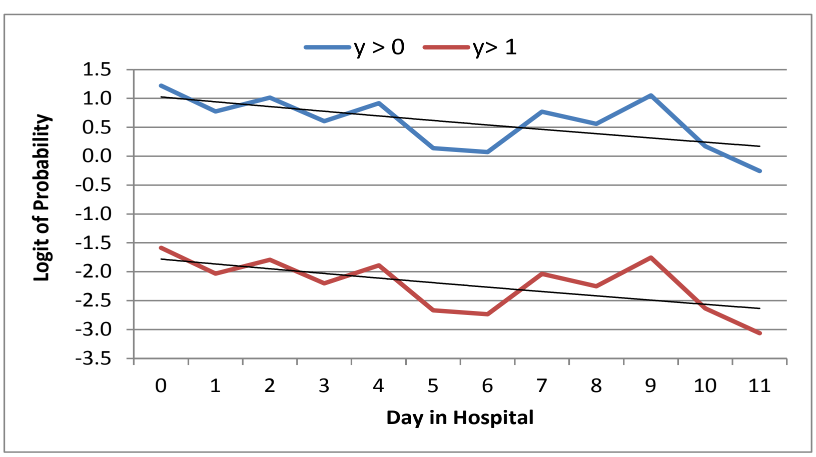
Type III Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
day	12	406	1.23	0.2597

Estimate for each day's mean = Intercept for Day 12 + day-specific difference from fixed effects solution

Estimates (truncated and re-arranged to save space)

Label	Estimate	Standard Error	Label	Estimate	Standard Error
Intercept for y>0 Day 0	1.2212	0.4358	Intercept for y>1 Day 0	-1.5863	0.4397
Intercept for y>0 Day 1	0.7759	0.2754	Intercept for y>1 Day 1	-2.0315	0.2946
Intercept for y>0 Day 2	1.0147	0.2699	Intercept for y>1 Day 2	-1.7928	0.2809
Intercept for y>0 Day 3	0.6069	0.2752	Intercept for y>1 Day 3	-2.2005	0.2954
Intercept for y>0 Day 4	0.9172	0.2982	Intercept for y>1 Day 4	-1.8903	0.3093
Intercept for y>0 Day 5	0.1395	0.3495	Intercept for y>1 Day 5	-2.6679	0.3763
Intercept for y>0 Day 6	0.07225	0.3805	Intercept for y>1 Day 6	-2.7352	0.4083
Intercept for y>0 Day 7	0.7705	0.4085	Intercept for y>1 Day 7	-2.0369	0.4211
Intercept for y>0 Day 8	0.5575	0.4177	Intercept for y>1 Day 8	-2.2499	0.4342
Intercept for y>0 Day 9	1.0515	0.4790	Intercept for y>1 Day 9	-1.7559	0.4854
Intercept for y>0 Day 10	0.1728	0.5540	Intercept for y>1 Day 10	-2.6346	0.5736
Intercept for y>0 Day 11	-0.2585	0.7402	Intercept for y>1 Day 11	-3.0659	0.7578
Intercept for y>0 Day 12	-0.2132	0.6619	Intercept for y>1 Day 12	-3.0206	0.6836



Although the df=12 multivariate Wald test for day was not significant, it looks like there could be a negative linear trend.

Note the predicted logit lines are completely parallel, as predicted by the model.

This parallelism is also known as "proportional odds" (but it holds for logits, too).

Model 4: Adding a Fixed Linear Slope for Days Since Hospital Admission

$$\mathrm{Logit}\left(y_{ti} > 0\right) = \gamma_{001} + \left(\gamma_{10}\right)\left(\mathrm{Day}_{ti}\right) + U_{0i}$$
$$\mathrm{Logit}\left(y_{ti} > 1\right) = \gamma_{002} + \left(\gamma_{10}\right)\left(\mathrm{Day}_{ti}\right) + U_{0i}$$

```
TITLE1 "SAS Model 4: Add Fixed Linear Time to Two-Level Ordinal Model";
PROC GLIMMIX DATA=work.alldata NOCLPRINT NAMELEN=100 GRADIENT METHOD=QUAD(QPOINTS=15);
       CLASS patient_ID;
       MODEL del012 (DESCENDING) = day / SOLUTION DDFM=BW LINK=CLOGIT DIST=MULT;
       RANDOM INTERCEPT / TYPE=UN SUBJECT=patient_ID;
       ESTIMATE "Predicted Intercept for y>0 at Day 0" intercept 0 1 day 0 / ILINK;
       ESTIMATE "Predicted Intercept for y>0 at Day 1" intercept 0 1 day 1 / ILINK;
       ESTIMATE "Predicted Intercept for y>0 at Day 2" intercept 0 1 day 2 / ILINK;
       ESTIMATE "Predicted Intercept for y>1 at Day 0" intercept 1 0 day 0 / ILINK;
       ESTIMATE "Predicted Intercept for y>1 at Day 1" intercept 1 0 day 1 / ILINK;
       ESTIMATE "Predicted Intercept for y>1 at Day 2" intercept 1 0 day 2 / ILINK;
       ESTIMATE "Nonsense ILINK (un-logit) for day slope"  day 1 / ILINK; RUN;

* STATA Model 4: Add Fixed Linear Time to Two-Level Ordinal Model
meologit del012 c.day, || PATIENT_ID: , covariance(unstructured) intpoints(15),
       estat ic, n(91),
       estimates store FixLin,  // save for LRT
        nlcom -1*_b[cut1:_cons] + 0*_b[day]   // logit y>0 at day 0
        nlcom -1*_b[cut1:_cons] + 1*_b[day]   // logit y>0 at day 1
        nlcom -1*_b[cut1:_cons] + 2*_b[day]   // logit y>0 at day 2
        nlcom -1*_b[cut2:_cons] + 0*_b[day]   // logit y>1 at day 0
        nlcom -1*_b[cut2:_cons] + 1*_b[day]   // logit y>1 at day 1
        nlcom -1*_b[cut2:_cons] + 2*_b[day]   // logit y>1 at day 2
        nlcom 1/(1+(exp(-1*(1*_b[cut1:_cons] + 0*_b[day])))) // predicted prob at day 1
        nlcom 1/(1+(exp(-1*(1*_b[cut1:_cons] + 1*_b[day])))) // predicted prob at day 1
        nlcom 1/(1+(exp(-1*(1*_b[cut1:_cons] + 2*_b[day])))) // predicted prob at day 2
```

Fit Statistics

-2 Log Likelihood	973.70
AIC (smaller is better)	981.70
AICC (smaller is better)	981.78
BIC (smaller is better)	991.75
CAIC (smaller is better)	995.75
HQIC (smaller is better)	985.75

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Gradient
UN(1,1)	PATIENT_ID	1.3074	0.3573	-6.99E-6

Note that the random intercept variance is greater than in the empty means model (=1.25) because fixed linear day cannot reduce residual variance…

Solutions for Fixed Effects

Effect	del012	Estimate	Standard Error	DF	t Value	Pr > |t|	Gradient
Intercept	2.Full	-1.7920	0.2249	89	-7.97	<.0001	2.794E-7
Intercept	1.Some	0.9610	0.2116	89	4.54	<.0001	-0.00002
day		-0.07314	0.03452	418	-2.12	0.0347	-0.00012

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Mean	Standard Error Mean
Predicted Intercept for y>0 at Day 0	0.9610	0.2116	418	4.54	<.0001	0.7233	0.04234
Predicted Intercept for y>0 at Day 1	0.8879	0.1920	418	4.62	<.0001	0.7085	0.03966
Predicted Intercept for y>0 at Day 2	0.8148	0.1771	418	4.60	<.0001	0.6931	0.03768
Predicted Intercept for y>1 at Day 0	-1.7920	0.2249	418	-7.97	<.0001	0.1428	0.02753
Predicted Intercept for y>1 at Day 1	-1.8651	0.2097	418	-8.89	<.0001	0.1341	0.02435
Predicted Intercept for y>1 at Day 2	-1.9382	0.1995	418	-9.72	<.0001	0.1258	0.02194
Nonsense ILINK (un-logit) for day slope	-0.07314	0.03452	418	-2.12	0.0347	0.4817	0.008618

NOTE: You can "un-logit" (through the inverse link) model-predicted outcomes to get predicted probabilities. But you CANNOT "un-logit" slopes that represent a one-unit difference in logits (which is NOT the same as a one-unit difference in probability). For example, here we see that the intercept difference between days (at −0.07314) is equal in logits, but this does not translate into equal distances in probability.

Model 5: Adding Random Linear Slope for Days Since Hospital Admission

$$\text{Logit}\left(y_{ti} > 0\right) = \gamma_{001} + \left(\gamma_{10} + U_{1i}\right)\left(\text{Day}_{ti}\right) + U_{0i}$$
$$\text{Logit}\left(y_{ti} > 1\right) = \gamma_{002} + \left(\gamma_{10} + U_{1i}\right)\left(\text{Day}_{ti}\right) + U_{0i}$$

```
TITLE1 "SAS Model 5: Add Random Linear Time to Two-Level Ordinal Model";
PROC GLIMMIX DATA=work.alldata NOCLPRINT NAMELEN=100 GRADIENT METHOD=QUAD(QPOINTS=15);
       CLASS patient_ID;
       MODEL del012 (DESCENDING) = day / SOLUTION DDFM=BW LINK=CLOGIT DIST=MULT;
       RANDOM INTERCEPT day / TYPE=UN SUBJECT=patient_ID;
       COVTEST "Need Random Slope?"  . 0 0; * Leave (1,1), test if (2,1) and (2,2) =0; RUN;

* STATA Model 5: Add Random Linear Time to Two-Level Ordinal Model
meologit del012 c.day, || PATIENT_ID: day, covariance(unstructured) intpoints(15),
       estat ic, n(91),
       estimates store RandLin,  // save for LRT
       lrtest RandLin FixLin     // LRT for random slope
```

```
           Fit Statistics
-2 Log Likelihood             966.27
AIC  (smaller is better)      978.27
AICC (smaller is better)      978.43
BIC  (smaller is better)      993.33
CAIC (smaller is better)      999.33
HQIC (smaller is better)      984.34

             Covariance Parameter Estimates
Cov                                     Standard
Parm       Subject        Estimate         Error     Gradient
UN(1,1)    PATIENT_ID       1.9373        0.7856     -0.00005
UN(2,1)    PATIENT_ID      -0.1871        0.1484      -0.0004
UN(2,2)    PATIENT_ID      0.06592       0.04296     -0.00092

                              Solutions for Fixed Effects
                                   Standard
Effect       del012     Estimate        Error        DF     t Value     Pr > |t|     Gradient
Intercept    2.Full      -1.8576       0.2582        89       -7.20       <.0001      -0.0002
Intercept    1.Some       1.0682       0.2437        89        4.38       <.0001     0.000104
day                     -0.09879      0.05501       418       -1.80       0.0732     -0.00088

                      Tests of Covariance Parameters
                         Based on the Likelihood
Label                   DF    -2 Log Like      ChiSq    Pr > ChiSq    Note
Need Random Slope?       2         973.70       7.44        0.0153    MI
MI: P-value based on a mixture of chi-squares.
```

Model 6: Adding Effects of MMSE (Level-2 Predictor) on Intercept and Linear Time Slope

$$\text{Logit}\left(y_{ti} > 0\right) = \gamma_{001} + \left(\gamma_{10} + U_{1i}\right)\left(\text{Day}_{ti}\right) + \gamma_{01}\left(\text{MMSE}_i - 16\right) + \gamma_{11}\left(\text{MMSE}_i - 16\right)\left(\text{Day}_{ti}\right) + U_{0i}$$
$$\text{Logit}\left(y_{ti} > 1\right) = \gamma_{002} + \left(\gamma_{10} + U_{1i}\right)\left(\text{Day}_{ti}\right) + \gamma_{01}\left(\text{MMSE}_i - 16\right) + \gamma_{11}\left(\text{MMSE}_i - 16\right)\left(\text{Day}_{ti}\right) + U_{0i}$$

```
TITLE1 "SAS Model 6: Add Effects of MMSE on Intercept and Slope to Two-Level Ordinal Model";
PROC GLIMMIX DATA=work.alldata NOCLPRINT NAMELEN=100 GRADIENT METHOD=QUAD(QPOINTS=15);
      CLASS patient_ID;
      MODEL del012 (DESCENDING) = day mmse16 day*mmse16 / DDFM=BW SOLUTION LINK=CLOGIT DIST=MULT;
      RANDOM INTERCEPT day / TYPE=UN SUBJECT=patient_ID; RUN;

* STATA Model 6: Add Effects of MMSE on Intercept and Linear Time Slope
xtmelogit del012 c.day c.mmse16 c.mmse16#c.day, ///
      ||  PATIENT_ID: day , covariance(unstructured) intpoints(15),
      estat ic, n(91)
```

SAS Output:

```
           Fit Statistics
-2 Log Likelihood             941.85
AIC  (smaller is better)      957.85
AICC (smaller is better)      958.14
BIC  (smaller is better)      977.94
CAIC (smaller is better)      985.94
HQIC (smaller is better)      965.96
```

```
NOTE: GCONV convergence criterion satisfied.
FROM THE LOG:
NOTE: At least one element of the (projected) gradient
is greater than 1e-3.
```

```
             Covariance Parameter Estimates
Cov                                 Standard
Parm       Subject       Estimate      Error     Gradient

UN(1,1)    PATIENT_ID      1.5528     0.6970     -0.0012
UN(2,1)    PATIENT_ID     -0.1970     0.1418    -0.00588
UN(2,2)    PATIENT_ID     0.06289    0.04045    -0.00421
```

```
                              Solutions for Fixed Effects
                                    Standard
Effect        del012     Estimate      Error      DF    t Value    Pr > |t|    Gradient
Intercept     2.Full      -2.0024     0.2614      88      -7.66      <.0001    -0.00085
Intercept     1.Some       0.9388     0.2427      88       3.87      0.0002     0.00126
day                       -0.1255    0.05860     417      -2.14      0.0328    0.007006
mmse16                   -0.08364    0.03233      88      -2.59      0.0113    -0.01616
day*mmse16               -0.00822   0.008001     417      -1.03      0.3048    -0.05209
```

STATA Output:

```
Mixed-effects ologit regression                 Number of obs      =       510
Group variable:      PATIENT_ID                 Number of groups   =        91
Integration method: mvaghermite                 Integration pts.   =        15
                                                Wald chi2(3)       =     26.71
Log likelihood = -470.92603                     Prob > chi2        =    0.0000
------------------------------------------------------------------------------
      del012 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         day |  -.1254878   .0586055    -2.14   0.032    -.2403525   -.0106231
      mmse16 |  -.0836333   .0323286    -2.59   0.010    -.1469963   -.0202704
c.mmse16#c.day |  -.0082202   .0080008    -1.03   0.304    -.0239015    .0074611
-------------+----------------------------------------------------------------
       /cut1 |  -.9388694   .2427417    -3.87   0.000    -1.414634   -.4631045
       /cut2 |   2.002329    .261361     7.66   0.000     1.490071    2.514587
-------------+----------------------------------------------------------------
PATIENT_ID   |
    var(day)|   .0628849   .0404465                      .0178266    .2218322
  var(_cons)|   1.552793   .6969916                      .6442291    3.742714
-------------+----------------------------------------------------------------
PATIENT_ID   |
cov(_cons,day)|  -.1970328   .1418016    -1.39   0.165    -.4749588    .0808932
------------------------------------------------------------------------------
```

```
LR test vs. ologit model: chi2(3) = 45.03                 Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
.         estat ic, n(91)
Akaike's information criterion and Bayesian information criterion
-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |         91         .   -470.926       8    957.8521   977.9389
-----------------------------------------------------------------------------
```

This model makes what is referred as the "proportional odds" assumption. This means that although we have estimated separate intercepts for the two sub-models, we have constrained the fixed slopes for the predictor effects to be the same for each submodel (as well as the random effects). Testing this assumption requires programming a non-proportional odds model in SAS NLMIXED instead.

Model 7: Two-Level Ordinal Model with NON-PROPORTIONAL Fixed Effects

$$\text{Logit}\left(y_{ti}>0\right)=\gamma_{001}+\left(\gamma_{101}+U_{1i}\right)\left(\text{Day}_{ti}\right)+\gamma_{011}\left(\text{MMSE}_i-16\right)+\gamma_{111}\left(\text{MMSE}_i-16\right)\left(\text{Day}_{ti}\right)+U_{0i}$$
$$\text{Logit}\left(y_{ti}>1\right)=\gamma_{002}+\left(\gamma_{102}+U_{1i}\right)\left(\text{Day}_{ti}\right)+\gamma_{012}\left(\text{MMSE}_i-16\right)+\gamma_{112}\left(\text{MMSE}_i-16\right)\left(\text{Day}_{ti}\right)+U_{0i}$$

```
TITLE1 "Ordinal Mixed Non-Proportional Model for MMSE";
PROC NLMIXED DATA=work.alldata METHOD=GAUSS TECH=QUANEW QPOINTS=15 GCONV=1e-12;
       * Must list all parms to be estimated here with start values;
       * B01 and B02 = intercepts for each equation;
       * Bs = fixed effects, now separate per equation;
       * Vs = variance components in order of G matrix;
       PARMS B01=.6 B02=-1.6
             B11day=0 B21mmse=0 B31mmseday=0
             B12day=0 B22mmse=0 B32mmseday=0
             V11=1 V21=-.2 V22=.05;
       * Linear predictor written as single-level equation for y>0 and y>1;
       Y1 = B01 + B11day*day + U1*day + B21mmse*mmse16 + B31mmseday*mmse16*day + U0;
       Y2 = B02 + B12day*day + U1*day + B22mmse*mmse16 + B32mmseday*mmse16*day + U0;
       * Model for probability of response - writing it the shorter way;
            IF (del=0) THEN P =  1 - (1/(1 + EXP(-Y1)));
       ELSE IF (del=1) THEN P =      (1/(1 + EXP(-Y1))) - (1/(1 + EXP(-Y2)));
       ELSE IF (del=2) THEN P =                            (1/(1 + EXP(-Y2)));
       LL = LOG(P);
       MODEL del ~ GENERAL(LL);
       * Random intercept and linear slope;
       RANDOM U0 U1 ~ NORMAL([0,0],[V11,V21,V22]) SUBJECT=patient_ID;
       * Testing proportional odds;
        ESTIMATE "Linear Time Slope Difference"         B11day     - B12day;
       ESTIMATE "MMSE on Intercept Difference"         B21mmse    - B22mmse;
       ESTIMATE "MMSE on Linear Time Slope Difference" B31mmseday - B32mmseday;
CONTRAST "Overall Proportional Odds Test" B11day-B12day, B21mmse-B22mmse, B31mmseday-B32mmseday;
RUN;
```

NOTE: GCONV convergence criterion satisfied (and no error messages in the log!)

Fit Statistics

-2 Log Likelihood	932.7
AIC (smaller is better)	954.7
AICC (smaller is better)	955.2
BIC (smaller is better)	982.3

Previous fit statistics from proportional odds version of the model:

Fit Statistics

-2 Log Likelihood	941.85	−2ΔLL(3) = 9.15, p <.03
AIC (smaller is better)	957.85	so proportional odds
AICC (smaller is better)	958.14	is broken somewhere
BIC (smaller is better)	977.94	

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
B01	0.8480	0.2430	89	3.49	0.0008	0.05	0.3651	1.3308	-1.81E-7
B02	-1.7275	0.3090	89	-5.59	<.0001	0.05	-2.3416	-1.1135	1.067E-7

B11day	-0.1065	0.06068	89	-1.75	0.0828	0.05	-0.2270	0.01412	-3.64E-7
B21mmse	-0.08158	0.03649	89	-2.24	0.0279	0.05	-0.1541	-0.00908	2.13E-7
B31mmseday	-0.01468	0.009376	89	-1.57	0.1211	0.05	-0.03331	0.003954	2.69E-6
B12day	-0.1525	0.07644	89	-2.00	0.0491	0.05	-0.3044	-0.00065	6.875E-7
B22mmse	-0.06753	0.03809	89	-1.77	0.0796	0.05	-0.1432	0.008148	1.048E-6
B32mmseday	-0.00260	0.009486	89	-0.27	0.7849	0.05	-0.02145	0.01625	-1.67E-6
V11	1.2819	0.6499	89	1.97	0.0517	0.05	-0.00944	2.5733	1.263E-7
V21	-0.1637	0.1313	89	-1.25	0.2158	0.05	-0.4246	0.09721	7.953E-7
V22	0.06395	0.03903	89	1.64	0.1049	0.05	-0.01361	0.1415	1.978E-6

Additional Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Lower	Upper
Linear Time Slope Difference	0.04608	0.06441	89	0.72	0.4762	-0.08190	0.1741
MMSE on Intercept Difference	-0.01405	0.03828	89	-0.37	0.7145	-0.09011	0.06202
MMSE on Linear Time Slope Difference	-0.01208	0.008537	89	-1.41	0.1606	-0.02904	0.004884

Contrasts

Label	Num DF	Den DF	F Value	Pr > F
Proportional Odds Test	3	89	2.67	0.0521

Neither MMSE*day interaction is significant, so we can remove them and test the remaining fixed slopes for proportional odds.

Model 8: Two-Level Ordinal Model with NON-PROPORTIONAL Fixed Effects Removing Nonsignificant MMSE*Linear Day Interactions

$$\text{Logit}\left(y_{ti}>0\right)=\gamma_{001}+\left(\gamma_{101}+U_{1i}\right)\left(\text{Day}_{ti}\right)+\gamma_{011}\left(\text{MMSE}_i-16\right)+U_{0i}$$
$$\text{Logit}\left(y_{ti}>1\right)=\gamma_{002}+\left(\gamma_{102}+U_{1i}\right)\left(\text{Day}_{ti}\right)+\gamma_{012}\left(\text{MMSE}_i-16\right)+U_{0i}$$

```
TITLE1 "SAS Model 8: Two-Level Ordinal Model with NON-PROPORTIONAL Fixed Effects, No MMSE*Day";
PROC NLMIXED DATA=work.alldata METHOD=GAUSS TECH=QUANEW QPOINTS=15 GCONV=1e-12;
       * Must list all parms to be estimated here with start values;
       * B01 and B02 = intercepts for each equation;
       * Bs = fixed effects;
       * Vs = variance components in order of G matrix;
       PARMS B01=.6 B02=-1.6 B11day=0 B21mmse=0 B12day=0 B22mmse=0
             V11=1 V21=-.2 V22=.05;
       * Linear predictor - written as single-level equation;
       Y1 = B01 + U0 + B11day*day + B21mmse*mmse16 + U1*day;
       Y2 = B02 + U0 + B12day*day + B22mmse*mmse16 + U1*day;
       * Model for probability of response - writing it the shorter way;
            IF (del=0) THEN P =  1 - (1/(1 + EXP(-Y1)));
       ELSE IF (del=1) THEN P =      (1/(1 + EXP(-Y1))) - (1/(1 + EXP(-Y2)));
       ELSE IF (del=2) THEN P =                           (1/(1 + EXP(-Y2)));
       LL = LOG(P);
       MODEL del ~ GENERAL(LL);
       * Random intercept and linear slope;
       RANDOM U0 U1 ~ NORMAL([0,0],[V11,V21,V22]) SUBJECT=patient_ID;
       * Testing proportional odds;
        ESTIMATE "Linear Time Slope Difference"   B11day  - B12day;
       ESTIMATE "MMSE on Intercept Difference"   B21mmse - B22mmse;
       CONTRAST "Overall Proportional Odds Test" B11day-B12day, B21mmse-B22mmse; RUN;
```

NOTE: GCONV convergence criterion satisfied (and no error messages in the log!)

Fit Statistics

-2 Log Likelihood	936.1
AIC (smaller is better)	954.1
AICC (smaller is better)	954.4
BIC (smaller is better)	976.7

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
B01	0.8102	0.2414	89	3.36	0.0012	0.05	0.3305	1.2898	-2.98E-7
B02	-1.7729	0.2935	89	-6.04	<.0001	0.05	-2.3560	-1.1897	3.463E-7

B11day	-0.08620	0.05541	89	-1.56	0.1233	0.05	-0.1963	0.02389	-2.74E-6
B21mmse	-0.1294	0.02461	89	-5.26	<.0001	0.05	-0.1783	-0.08051	2.046E-6
B12day	-0.1321	0.06746	89	-1.96	0.0534	0.05	-0.2661	0.001970	2.472E-6
B22mmse	-0.07120	0.02584	89	-2.76	0.0071	0.05	-0.1225	-0.01985	-4.38E-6
V11	1.3939	0.6673	89	2.09	0.0396	0.05	0.06800	2.7199	1.427E-8
V21	-0.1681	0.1308	89	-1.29	0.2021	0.05	-0.4280	0.09179	6.383E-7
V22	0.05676	0.03734	89	1.52	0.1320	0.05	-0.01742	0.1309	1.598E-6

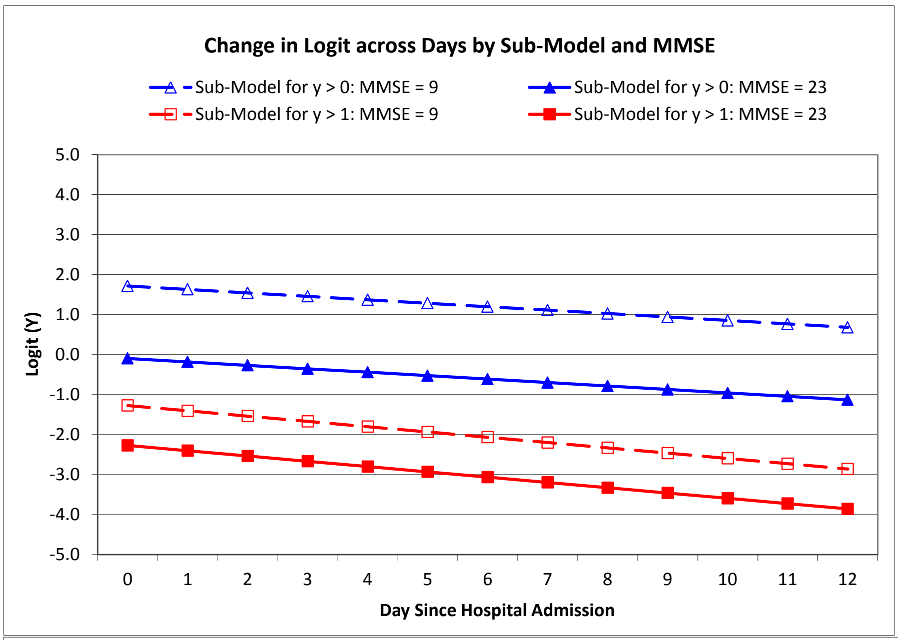
Additional Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Lower	Upper
Linear Time Slope Difference	0.04586	0.06274	89	0.73	0.4668	-0.07881	0.1705
MMSE on Intercept Difference	-0.05820	0.02398	89	-2.43	**0.0172**	-0.1058	-0.01056

Contrasts

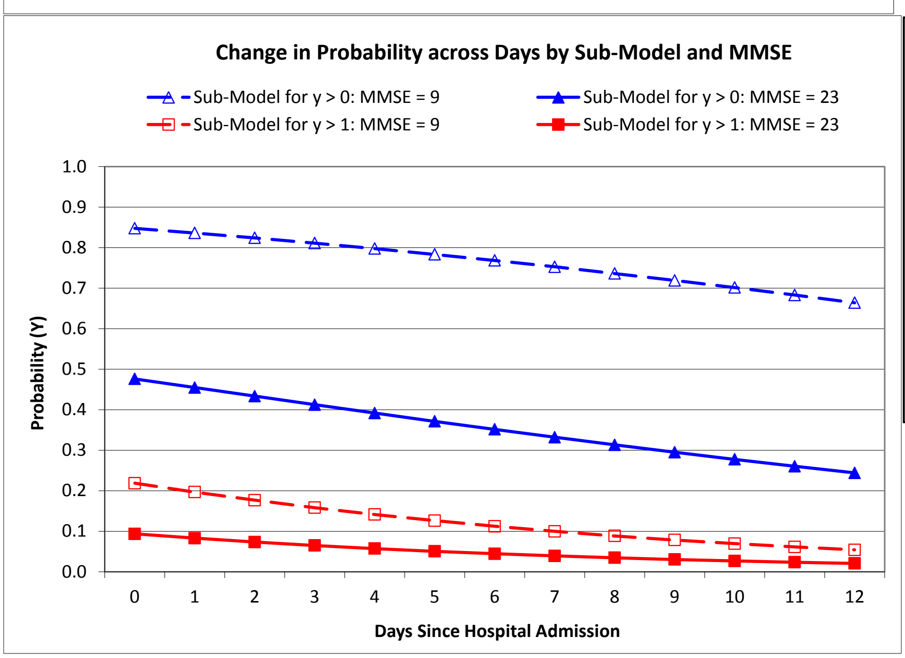
Label	Num DF	Den DF	F Value	Pr > F
Overall Proportional Odds Test	2	89	3.52	0.0338

It looks like MMSE has a more negative effect on y>0 than y>1.



There is a *linear* relationship between day and the predicted *logits*.

Also, the distance between the lines is constant over time (consistent with only a main effect of MMSE).



There is a *NON-linear* relationship between day and the predicted *probabilities*, even though day only has a linear effect.

Likewise, the distance between the lines is *not* constant, even though there is no interaction of MMSE*day in the model.

Mplus Syntax and Output for <u>the last proportional odds model 6</u> (using observed variables as predictors rather than latent)—results are very similar to SAS. I do not think non-proportional odds, cumulative logit models can be fit by Mplus (or at least I don't know how).

```
TITLE:  2-Level Model for Occasions within Persons Predicting Ordinal Delirium;
DATA:    FILE = delirium.csv;     ! Can just list file if in same directory;
         FORMAT = free;           ! FREE or FIXED format;
         TYPE = individual;       ! Individual or matrix data as input;

VARIABLE:
! List of ALL variables in stacked data file, in order;
! Mplus does NOT know what they used to be called, though;
    NAMES ARE Patient day mmse16 del;
! List of ALL variables used in model (DEFINED variables at end);
    USEVARIABLES ARE day mmse16 del;
! Outcomes that are binary/ordinal;
    CATEGORICAL ARE del;
! Missing data codes (here, -999);
    MISSING ARE ALL (-999);
! Identify upper-level nesting;
    CLUSTER = Patient;
! Predictor variables with variation ONLY within at level 1;
    WITHIN = day;
! Predictor variables with variation ONLY between at level 2;
    BETWEEN = mmse16;

ANALYSIS:    TYPE IS TWOLEVEL RANDOM;       ! 2-level model with random slopes;
             ESTIMATOR IS ML;             ! Can also use MLR for non-normality;

MODEL:
!!! MODEL 6
! Level-1, time-level model;
%WITHIN%
! NO residual variance is estimated for del at level 1;
    LinDay | del ON day;      ! B1s effect of linear time;
! Level-2, patient-level model;
%BETWEEN%
    del;                     ! Random intercept variance (is default);
    [del$1 del$2];           ! Fixed "thresholds" (is intercept*-1);
    [LinDay];                ! Fixed linear time slope;
    LinDay;                  ! Random linear time slope;
    del WITH LinDay;         ! Covariance of intercept and linear time slope;
    del ON mmse16;           ! MMSE --> intercept;
    LinDay ON mmse16;        ! MMSE --> linear time slope;
```

```
MODEL FIT INFORMATION

Number of Free Parameters                        8

Loglikelihood
          H0 Value                        -470.926

Information Criteria
          Akaike (AIC)                     957.853
          Bayesian (BIC)                   991.728
          Sample-Size Adjusted BIC         966.335
            (n* = (n + 2) / 24)

MODEL RESULTS
                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Within Level

Between Level

 LINDAY     ON
    MMSE16            -0.008      0.008     -1.046      0.296

 DEL        ON
    MMSE16            -0.083      0.032     -2.570      0.010

 DEL      WITH
    LINDAY            -0.197      0.142     -1.387      0.165

 Intercepts
    LINDAY            -0.126      0.059     -2.141      0.032

 Thresholds
    DEL$1             -0.939      0.243     -3.868      0.000
    DEL$2              2.003      0.261      7.661      0.000

 Residual Variances
    DEL                1.552      0.696      2.229      0.026
    LINDAY             0.063      0.041      1.553      0.120


BRANT WALD TEST FOR PROPORTIONAL ODDS
                                   Degrees of
                      Chi-Square     Freedom    P-Value
  DEL
    Overall test           6.096           2      0.047
    DAY                    0.095           1      0.758
    MMSE16                 5.805           1      0.016
```

Sample Results Section

The extent to which cognition could predict change during a hospital stay (of 0–12 days) in delirium status was examined was examined in a series of multilevel models. The sample included 510 daily observations, which were modeled as nested at level 1 within their 91 patients at level 2, such that patient differences were captured via random effects. In the full sample, 37.25% of the daily observations had no delirum, 46.86% had some delirum, and 15.88% had full deilium. The ordinal delirium status outcome (0 = none, 1= some, 2 = full) was predicted using a cumulative logit link function and multinomial conditional outcome distribution. The overall model was parameterized to create two simultaneously estimated submodels: one predicting delirum > none, and another predicting delirium > some. Notably, in this approach, separate fixed intercepts are specified for each submodel, but all other parameters are assumed equal across submodels (i.e., the proportional odds assumption).

Model parameters were estimated via full-information marginal maximum likelihood (MML) using adaptive Gaussian quadrature with 15 points of integration per random effect dimension in SAS GLIMMIX. Accordingly, all fixed effects should be interpreted as unit-specific (i.e., as the fixed effect specifically for patients in which the corresponding random effect = 0). The significance of fixed effects was evaluated with Wald tests (i.e., the *t*-test of the ratio of each estimate to its standard error using between–within denominator degrees of freedom), whereas the significance of random effects was evaluated via likelihood ratio tests (i.e., $-2\Delta LL$ with degrees of freedom equal to the number of new random effects variances and covariances).

As derived from an empty means, random intercept model, time-varying delirium had an intraclass correlation of ICC = .276, indicating that 27.6% of the variance in delirium status was between patients, which was significant, $-2\Delta LL(1) = 57.32$, $p < .0001$. The first submodel's intercept for the expected logit (log-odds) of delirium greater than none was 0.6855 (or probability = .665), and the second submodel's intercept for the expected logit of delirium greater than some was −2.042 (or probability = .115). To ascertain the expected pattern of change across the 13 days, a model with a random intercept for patient and saturated means (i.e., with a separate offset relative to the reference day 12 for each previous day) was then estimated. Results indicated a noisy, but mostly linear decrease in the log-odds across days. Accordingly, starting from the empty means, random intercept model, we first added a fixed linear slope for change per day, which indicated a significant decrease in the log-odds of delirium for each submodel of 0.073 per day ($p < .04$). We then added a random slope for linear change per day, which was significant, $-2\Delta LL(2) = 7.44$, $p < .0001$, indicating patients differed in their linear rates of change. Although the fixed linear effect of day became nonsignificant at this step ($p < .08$), it was retained given the significant random linear effect of day. Fixed and random quadratic effects of change per day were also examined and found to be nonsignificant (not shown); thus, the random linear model of change was retained.

We then sought to predict patient differences in their intercepts and linear rates of change by pateint cognition at hospital entry (as measured by the Mini-Mental Status Exam centered at 16). Its simple main effect (as evaluated at day 0) indicated that the log-odds of delirium decreased significantly by 0.084 per unit increase in cognition. The interaction of cognition by day indicated that the linear rate of decrease in the log-odds per day became nonsignificantly more negative (stronger) by 0.008 per unit increase in cognition. However, it is important to recognize that this model assumes that three of its fixed effects–for linear day, cognition, and cognition by linear day—have equivalent effects in each submodel (i.e., they are the same when predicting delirium greater than none as when predicting delirium greater than some). We tested this assumption by allowing separate effects per submodel within SAS NLMIXED (with all other estimation options as described above). Model fit improved significantly, $-2\Delta LL(3) = 9.15$, $p < .03$, indicating some difference in the fixed effects across submodels, although the specific contrasts between each fixed effect across submodels were nonsignificant. Given that the cognition by day interaction was nonsignificant in either submodel, we removed these two fixed effects and re-estimated the model. Results are shown in Table X and depicted in Figure X. The fixed linear effect of day was nonsignificantly negative, equivalently so, in both submodels. The effect of cognition on the intercept was significantly negative in both submodels, but significantly more negative (stronger) in predicting delirium > none.